Regularization

Week 04 - Day 04

Yesterday - Overfitting

This morning - Overfitting

Today - ???

Yesterday - Overfitting

This morning - Overfitting

Today - Overfitting

Linear models

Linear models: overfit? underfit?

Linear models = simple models = low variance

Sometimes Linear Models overfits!

Too many irrelevant features

Correlated features

Too many irrelevant features

Correlated features

What’s the name?

Too many irrelevant features

Correlated features

Multicollinearity!

Solution = Regularization!

Smallest Error

Name of the process to find the smallest error?

Optimization

What do we want to optimize?

The error!

Error definition?

Error = SUM((real - predicted)**2)

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Linear model

What’s the formula for the prediction?

(e.g. 2 parameters x1 and x2)

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n} \left(y_i - \left(\beta_0 + \sum_{j=1}^{p} \beta_j x_j \right) \right)^2$$

Regularization

Optimize error

Optimize error+penalty

(penalty for complexity)

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n} \left(y_i - \left(\beta_0 + \sum_{j=1}^{p} \beta_j x_j \right) \right)^2$$

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n}\left(y_i - \left(\beta_0 + \sum_{j=1}^{p}\beta_j x_j\right)\right)^2$$

$$\sum_{i=1}^{n}\left(y_i - \left(\beta_0 + \sum_{j=1}^{p}\beta_j x_j\right)\right)^2 + \lambda_2 \sum_{j=1}^{p}\beta_j^2$$

Penalty

What’s the penalty?

Sum of coefficients**2!

(beta1**2 + beta2**2 + beta3**2)

We optimize error+penalty

Result

Smaller coefficients (smaller variance)

Or

Coefficients = 0 (less variables)

Different types of penalty

$$\text{beta1}^{**}2 + \text{beta2}^{**}2 + \text{beta3}^{**}2$$

Vs.

$$|\text{beta1}| + |\text{beta2}| + |\text{beta3}|$$

Ridge

beta1**2 + beta2**2 + beta3**2

Vs.

|beta1| + |beta2| + |beta3|

Lasso

Ridge = smaller coefficients

Lasso = zeroed coefficients

Parameters Tuning

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n}\left(y_i - \left(\beta_0 + \sum_{j=1}^{p}\beta_j x_j\right)\right)^2$$

$$\sum_{i=1}^{n}\left(y_i - \left(\beta_0 + \sum_{j=1}^{p}\beta_j x_j\right)\right)^2 + \lambda_2 \sum_{j=1}^{p}\beta_j^2$$

Penalty

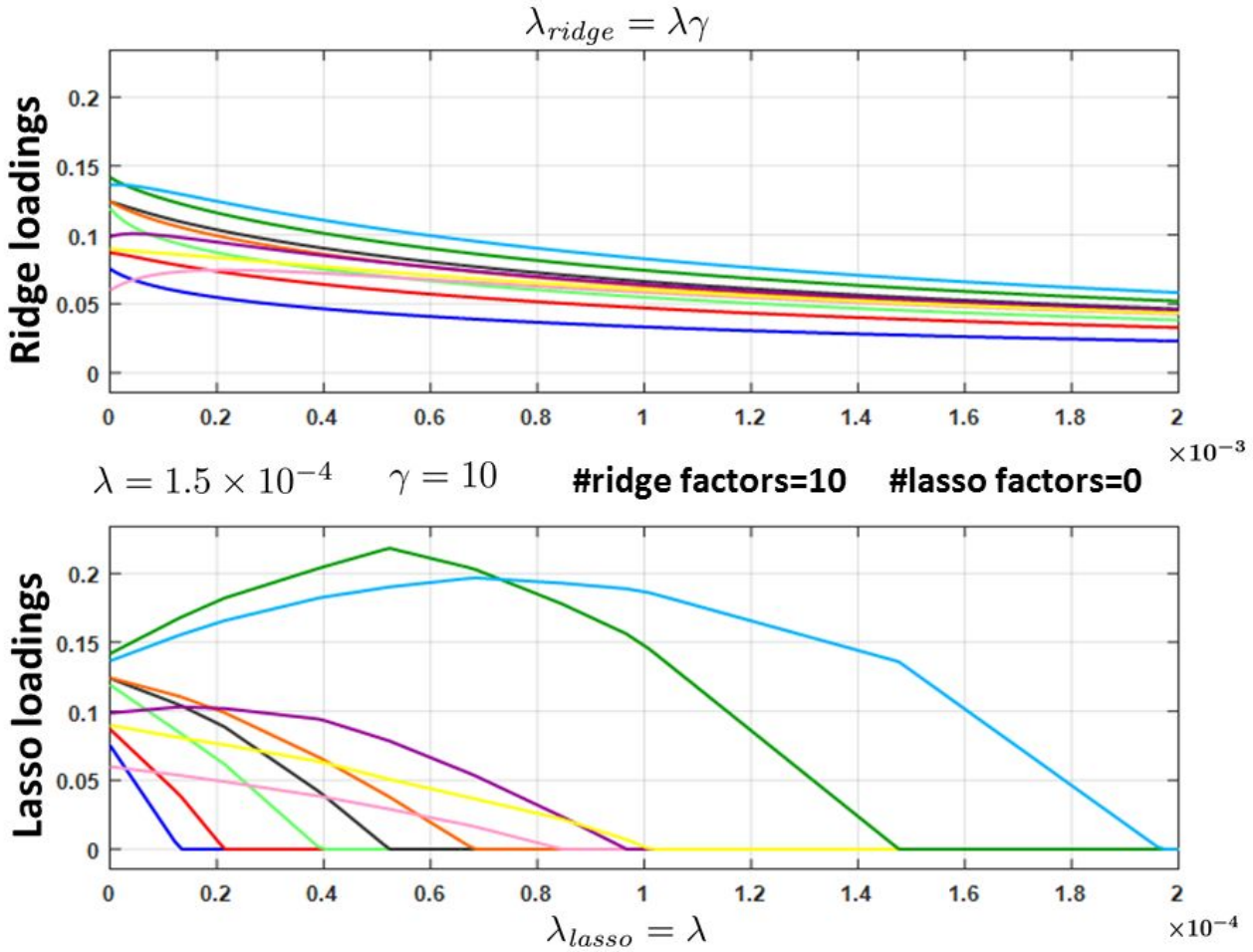
$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n}\left(y_i - \left(\beta_0 + \sum_{j=1}^{p}\beta_j x_j\right)\right)^2$$

$$\sum_{i=1}^{n}\left(y_i - \left(\beta_0 + \sum_{j=1}^{p}\beta_j x_j\right)\right)^2 + \lambda_2 \sum_{j=1}^{p}\beta_j^2$$

Parameter to tune

Penalty


$\lambda_{ridge} = \lambda\gamma$
Ridge loadings
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$\times 10^{-3}$
$\lambda = 1.5 \times 10^{-4}$
$\gamma = 10$
#ridge factors=10
#lasso factors=0
Lasso loadings
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$\times 10^{-4}$
$\lambda_{lasso} = \lambda$

Coefficients: Normalized or not?

Yes!

We don’t want to penalise the “scale”

Elastic Net

Elastic net = lasso + ridge

Practical advices

Try all models!

Play with parameters tuning!

Sklearn implements these models

Summary

- 3 new models (lasso, ridge, elastic net)
- Fight overfitting + multicollinearity
- Optimize error + penalty
- We need to tune the (hyper)parameters