Brown Mathematical Contest for Modeling 2022

Combating Urban Heat Islands with Strategic Vegetation

A Case Study on Durham, NC

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1 Introduction

1.1 Problem Explanation

The issue of global warming has been recognized for decades by scientists, and since the turn of the century, increasingly so by the general public as well. However, the increased awareness has not translated into meaningful and effective solutions.

Both the lack of decisive action at a government level and a sense of "one person isn't going to make a difference" at an individual level has kept us on a trajectory of ever-increasing temperature records. This has only been exacerbated by coverage of such "records" by various outlets as simple milestones to be reached, numbing the public to the real and personal effects of global warming.

Then, given our historical inability as a people to take impactful action at a larger scale, it is critical that communities and governments at a more localized level work to at least mitigate the impacts of global warming. A particular phenomenon that we can seek to address is the Urban Heat Island effect.

1.2 Existing Literature

The Urban Heat Island effect represents the phenomenon of modern city construction trapping excess heat throughout the day, then releasing the heat at night. Specifically, the effect is brought about by common construction materials, such as concrete and cement on roads and sidewalks, as well as clay and metal used in housing and roofing [Kim07]. These materials, although great for building with, have the unfortunate downside of being excellent trappers of heat.

The prevalence of these materials in urban and suburban areas keeps temperatures elevated even during the cooler night hours, especially in the summer months. The result is that the eventual consequences of global warming, such as heat-related illnesses and associated hospitalizations, are exacerbated.

One potential solution to UHI is the use of trees and vegetation to reduce temperatures. Research indicates that higher levels of vegetation, especially in the form of trees, are effective at reducing temperatures [Kur+94].

As an extension, additional research indicates that planting trees may be a cost-effective solution to UHI consequences. By reducing hospitalization rates for heat-related illnesses, the planting of trees can actually help cities save money in the long run [McP+05].

We now note that vegetation appears to be a potentially effective solution in combating UHI. The remainder of this paper will then assume planting vegetation as a viable solution method. With this is mind, we will instead turn our attention to the question of how a city may be able to most effectively utilize green spaces as a means to alleviate the temperature elevations cause by UHI.

1.3 Problem Restatement

The goal of this paper is then to address the following:

1. Explore trends in Durham's demographic, geographic, and meteorological data to determine the interactions between the features

2. Apply these interactions to create a model that optimally allocates a limited project budget to census tracts for greenery development

3. Identify a metric for utility per dollar efficiency with the goal of advising decision-makers with a recommendation of the optimal project budget

2 Initial Exploration

The data set provided described 193 measurements of nighttime temperature reduction throughout Durham, North Carolina. In conjunction with these mentions, the data set indicated the corresponding percent below two times the poverty level, percent green coverage, and latitude-longitude coordinates. In the following sections, we discuss the results of the initial exploratory analysis we conducted which revealed several trends critical to our core modeling work.

2.1 Spatial Relations

Given a data set of measurements associated with coordinates, we investigated geographic patterns within the data ranging from the distribution of city tracts themselves to geographic relations with temperature and greenery coverage. Preliminary analysis of spatial trends and underlying patterns gives way to important modeling decisions later on.

2.1.1 Location Frequency

First, we generated a kernel density estimation from the given coordinates along with the marginal distributions of longitudes and latitudes.

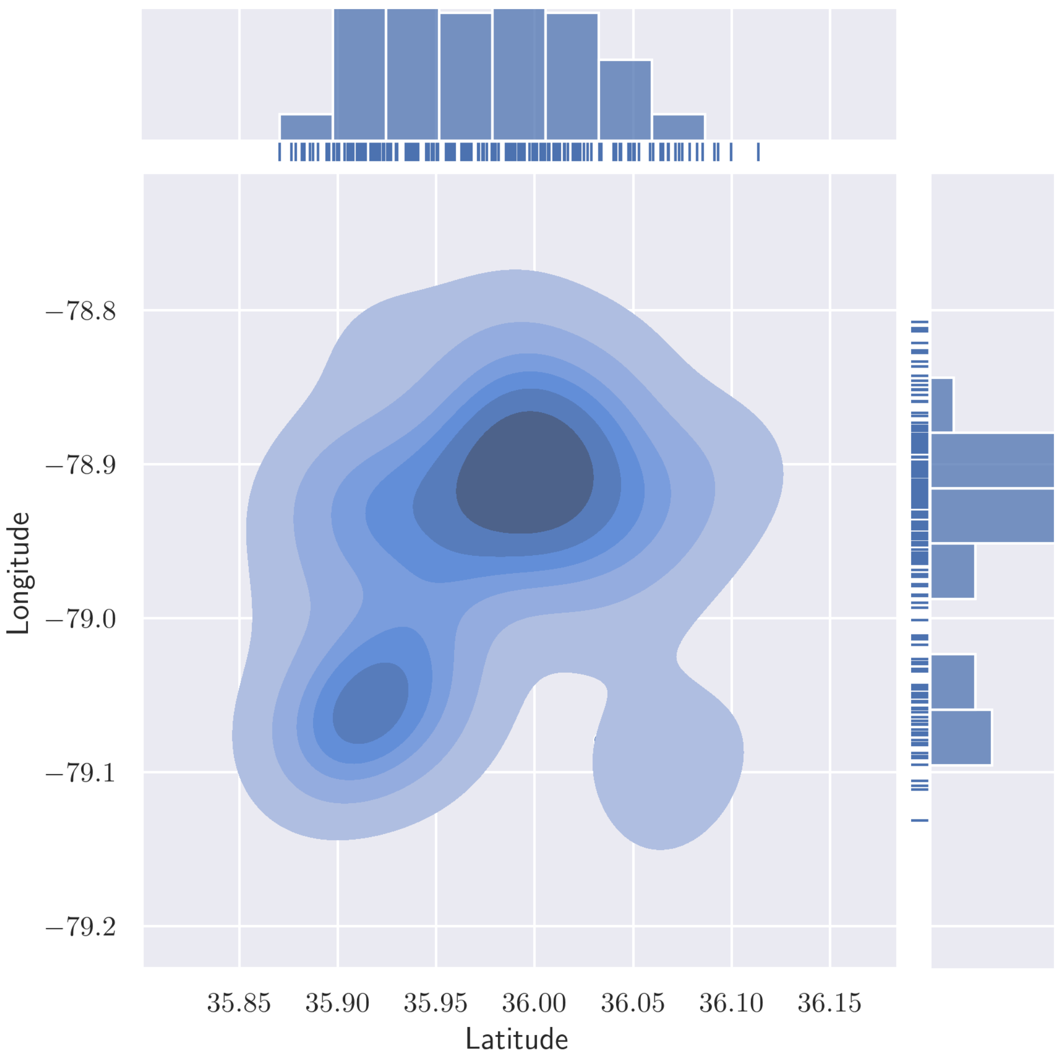


Figure 1: Tract density across the city

Figure 1 reveals that the coordinates mainly concentrate around two regions and are otherwise evenly spread around with smaller localized areas of concentration. These two concentrated regions are more specifically the area around $(35.92°, -79.05°)$ and $(35.98°, -78.90°)$. Contextualized with a map of the local area, we can note that the upper-right cluster coincides with Duke's campus and downtown Durham, whereas the lower-left cluster coincides with UNC's Chapel Hill campus.

2.1.2 Green Space

Next, we explore the distribution of greenery coverage over the city—the primary factor in temperature reduction and hence, UHI effects.

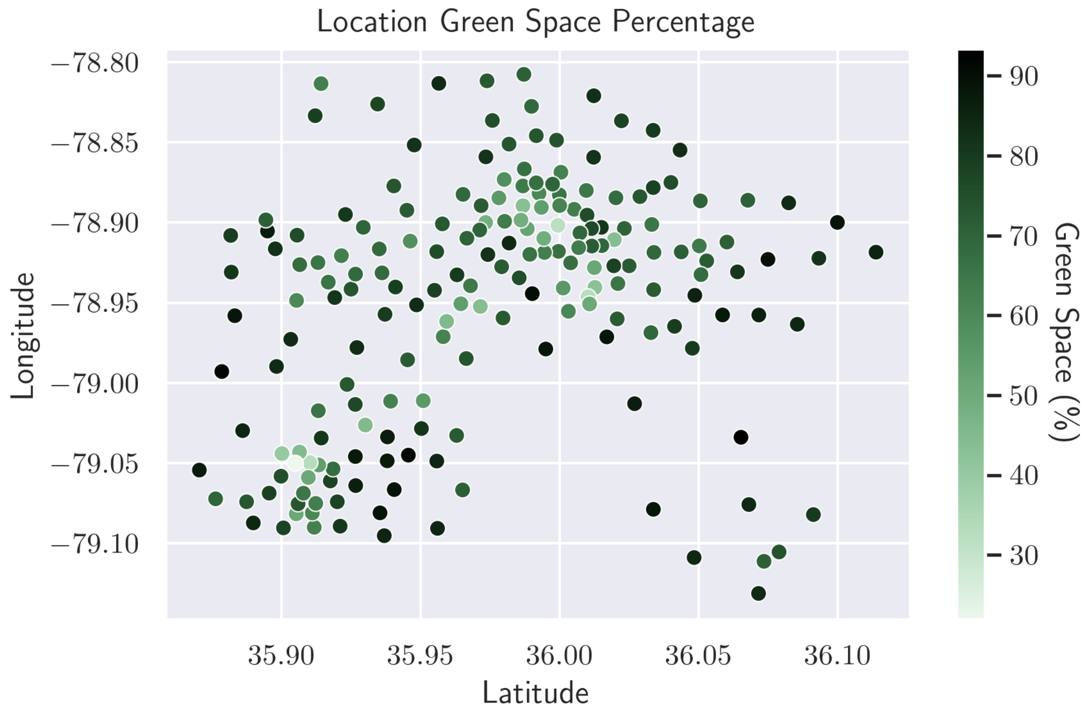


Figure 2: Green space (%) across the city

From Figure 2, we observe that along the outskirts of the city, the percentage of green space is higher and around the two hypothesized population centers, the green space coverage is lower. This makes sense as cities have less space for trees and have more manmade structures that decrease this percentage.

2.1.3 Average Temperature Reduction

The following visualization 3 displays the distribution of the average nighttime temperature reduction across the city tracts.

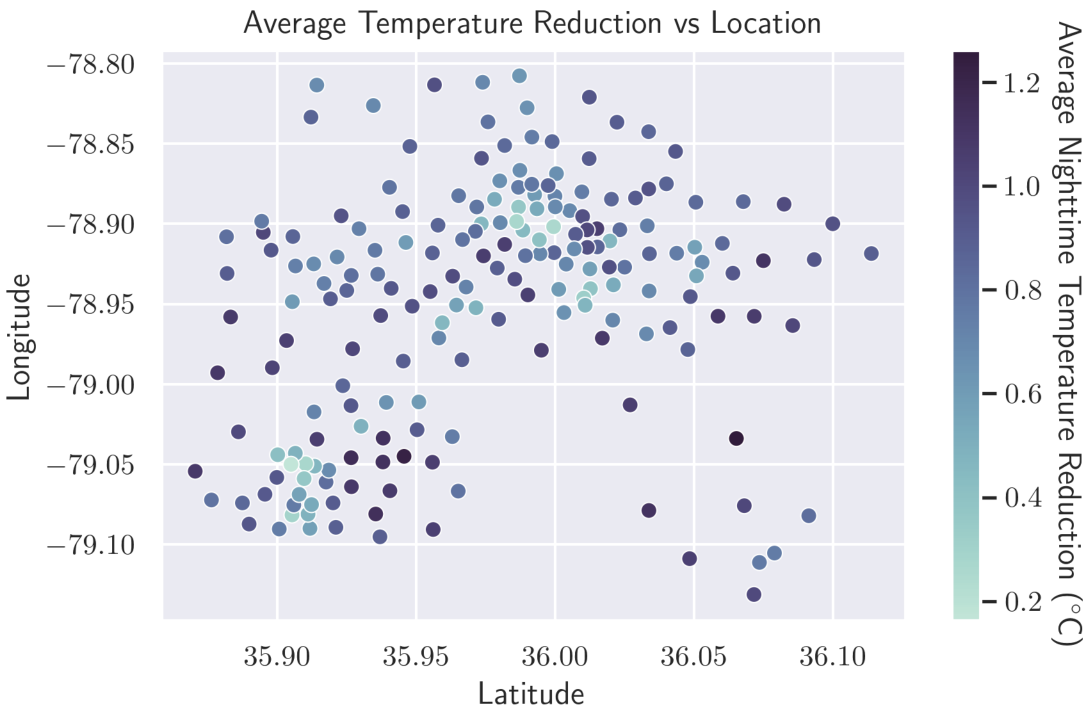


Figure 3: Green space (%) across the city

Yet again, we notice the concentration of extrema in the average temperature reduction in population centers; clearly the surrounding areas are generally cooler from a larger magnitude in temperature reduction.

2.2 Green Space and Average Temperature Drop

From the exploration earlier, we observe that there is a relationship between green space and average temperature change; that they both take on peak values within population centers. Further analysis with an ordinary linear regression reveals a strong linear relation, with green space percentages correlating strongly with average nighttime temperature changes. Another observation is that most of the sample data points are in regions above 60% greenery and approximately have greater than 0.6°C of average temperature drop.

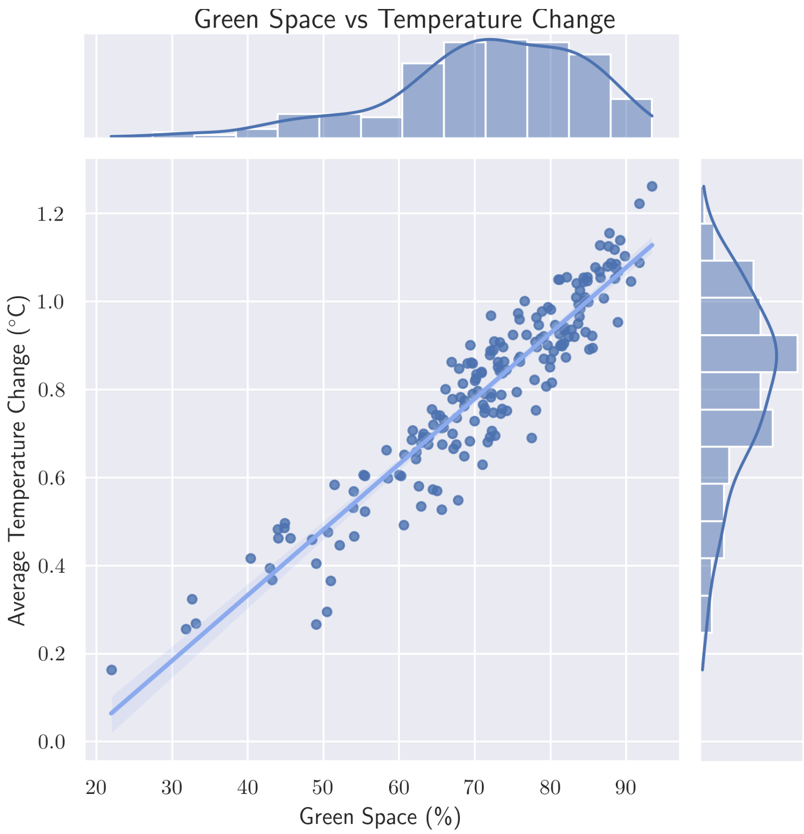


Figure 4: Regressing Temperature Change on Greenery

The given regression line is as follows:

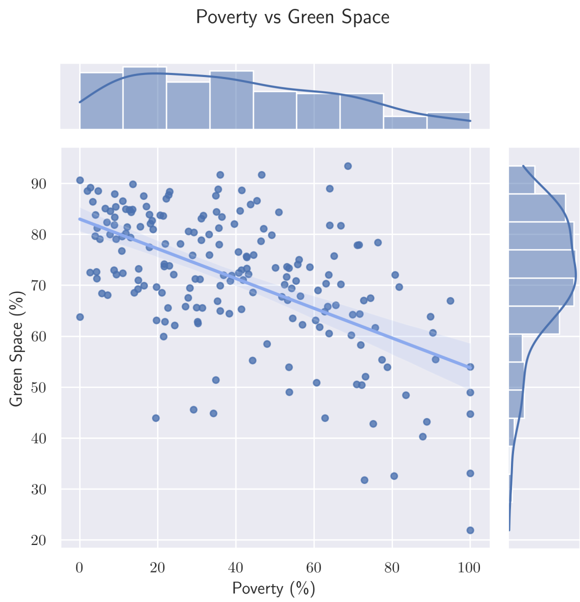
$$\hat{t} = 0.0148g - 0.26 \tag{1}$$

where $\hat{t}$ is the predicted average nighttime temperature decrease in Celsius and g is the percentage of greenery coverage. That is, for every increase in the percentage of green space, the magnitude of the average temperature drop at night increases by $0.0148°C$.

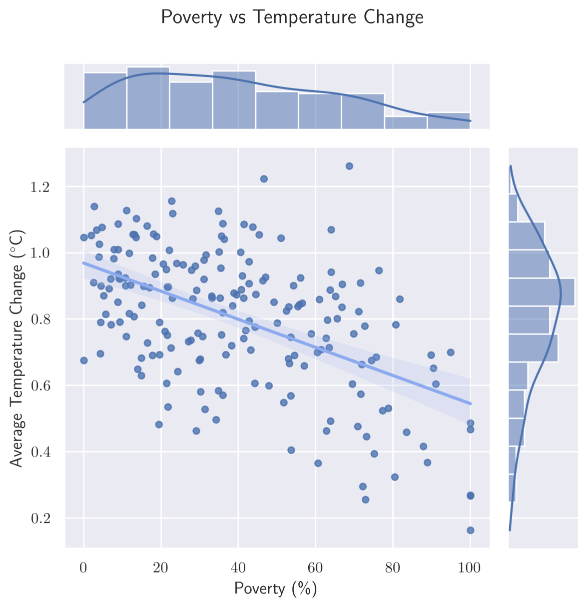
This relationship can be explained by the physical role vegetation plays in temperature regulation. During the day, different surfaces absorb and hold onto heat differently. For example, materials such as concrete and roofs absorb and hold a lot more energy compared to vegetative surfaces such as grass and trees. This can be due to a variety of factors such as evapotranspiration, shading, color, etc. Critical to our modelling effort is how these surfaces release heat and alter the micro climate around city islands; that is, areas with more trees hold on to less heat and therefore releases more heat than their counterparts with less vegetation.

2.3 Poverty

Now we explore relations between low-income neighborhoods and UHI effects. This is because lower-income populations have less access to tools such as reliable air conditioning and healthcare. Because of this, we believe the additional utility that can be derived from planting trees in lower-income neighborhoods may be an effective strategy in maximizing the value of planting trees to reduce UHI (maximize utility). We explore relationships between poverty and measurements of green space and temperature reduction. In the following plots, we graph the percent green space and average temp change against the percent poverty of each location.



(a) Poverty vs Green Space



(b) Poverty vs Average Temperature Drop

Figure 5: Poverty relations

2.4 Takeaways

Below are some of the biggest conclusions we draw from our initial exploratory analysis from which we can introduce factors in modeling.

- The spatial distribution of data points in the data set concentrates around two major groups that appear to be population centers
- Average nighttime temperature reduction has a strong positive linear relationship with green space coverage percentage.
- Lower income areas have less green space coverage and average temperature drop at night time, causing UHI effects to disproportionately affect the neighborhoods in question.

3 Model

In this section, we take the findings from our initial exploratory analysis and reconcile them with our task at hand to motivate the development of a model for the mitigation of UHI effects on Durham.

3.1 Notation

Table 1: Notation

Symbol	Definition
$\mu(\cdot)$	utility function
$\mathcal{X}$	set of individual tracts in Durham
$\mathcal{D}$	set of greenery planting distribution functions
Δt_{red}	change in average night time temperature reduction
$\Delta x^{(s)}$	change in greenery under $s \sim \mathcal{D}$
c_x	cost per square mile
f	increment of funds
g	percent green coverage
A_x	area of a tract
B	total budget
α	base cost of planting
λ	additional cost of planting in low greenery area
i	time index
p	percent poverty
$\hat{t}$	predicted average temperature drop from regression

3.2 Motivation and Breakdown

Our task is to quantify the effect of planting greenery on the reduction of UHI effects over time; in order to do so, we construct a model under which we can optimize the city's utility as a function of planting greenery. As we seek to analyze and mitigate the effects of UHI which can take the form of air conditioning concerns, hospitalizations, and health concerns, we chose to establish a notion of city utility as a direct measurement of UHI effects. Further, as low-income areas are disproportionately affected by UHI, we seek a metric that prioritizes balanced utility across the entire city's residents rather than one which is able to maximize utility by polarizing resource allocation. Lastly, our recommendation should be actionable and feasible—that is, we are constrained by factors such as monetary cost to the city government.

Taking these considerations into account, we present the following utility model in order to best benefit the city in relation to the maximization of uniform city resident utility:

$$U_{\mathcal{X}} = \left(\prod_{x \in \mathcal{X}} \mu(x) \right)^{\frac{1}{|\mathcal{X}|}} \tag{2}$$

where $x \in \mathcal{X}$ are the geographical regions (tracts) of Durham and $\mu(\cdot)$ is the governing utility function. It is apparent that we employ the geometric mean of individual tract utilities in order to discourage skewed utility maximization by asymmetric resource allocation as the geometric mean exhibits central tendency.

Then to account for the budgeting constraints that the Durham government will face in executing the greenery planting scheme, we introduce the following constraint function:

$$\sum_{x \in \mathcal{X}} \Delta x^{(s)} \cdot (\alpha + \lambda \mathbb{1}_{\{x_g \leq 0.6\}}) \leq B \tag{3}$$

where $\Delta x^{(s)}$ is the change in greenery percentage under some greenery planting distribution s for a tract x, α is the base cost incurred by planting in a region, λ the additional cost incurred for planting in a low greenery region, and x_g is the initial greenery percentage of the region x. Essentially, the constraint function is the sum of the costs incurred to produce greenery percentage increases in every tract of Durham; note that there is an additional cost for planting in regions that are not sufficiently "green".

Putting these two components together results in our final optimization problem, specified as:

$$\begin{aligned} \max_{s \sim \mathcal{D}} \quad & \left(\prod_{x \in \mathcal{X}} \mu(x^{(s)}) \right)^{\frac{1}{|\mathcal{X}|}} \\ \text{subject to} \quad & \sum_{x \in \mathcal{X}} \Delta x^{(s)} \cdot (\alpha + \lambda \mathbb{1}_{\{x_g \leq 0.6\}}) \leq B \end{aligned} \tag{4}$$

where $s \sim \mathcal{D}$ is the particular greenery planting distribution function employed.

3.3 Assumptions

1. **Different Vegetation, Same Effect**: The model does not make the distinction that different vegetation may have different effects on the UHI effect. In reality, vegetation such as grass and shrubs do not provide the extensive shade that trees provide, their contribution to evaporation also mitigates the UHI effect. Therefore, we simplify and assume that all vegetation mitigates the UHI effect.

2. **Financial and Infrastructural Barriers to Planting**: The model makes the assumption that the only limiting factors to increasing greenery are financial costs and the existing infrastructure in the target area. Financial constraints limit us to selecting only a small subset of regions to increase vegetation because city budgets can be limited. Infrastructure constraints make it more difficult to greenify areas with a smaller percentage of existing greenery. This is because there are additional costs such as initial soil and groundwork to develop a growing environment.

3. **Localized Effects**: We assume that the greenery only has a localized effect on UHI. The green percent of one coordinate does not affect the average temperature drop of nearby coordinates.

   Note: While the effects of temperature reduction are localized the utility gain may not be localized. Because the model employs a geometric mean to measure the city's overall utility, the utilities of each individual city have an effect on the utility.

4. **Cooler Temperature, Happier Inhabitants**: The model focuses on cities with high heat so we assume that additional drops in temperature always may the inhabitants happy. We also consider any increases in average temperature drop to be negative utility.

5. **Sustained Costs**. We assume that the entirety of the cost for planting greenery is upfront and there are no sustained maintenance costs after the planting is done.

6. **Constant Fertility**. We assume that nothing distinguishes the fertility of a location besides the initial greenery. The model does not account for biological and geographic factors that may otherwise influence planting.

3.4 Additional Data

In order to better contextualize the provided data, while also supplementing it with other key factors, we utilized US census data from [Cen] and [Mtg]. We utilized the longitude and latitude coordinate pairings provided in conjunction with the census geocoder platform to associate the given data entries to census tract numbers. This then enabled us to enrich our data with population and area measurements for each tract obtained from the census interactive map. We believed that these two features were important considerations, the former for determining planting prioritization and the latter fro determining planting costs. Combined together, this data enabled our model to more accurately consider project efficiency, which we deemed to be critical in the context of a real-world problem.

3.5 Change in Average Temperature Reduction at Night Time

During the day, different surfaces absorb and hold onto heat differently. For example, materials such as concrete and roofs absorb and hold a lot more energy compared to vegetative surfaces such as grass and trees. This can be due to a variety of factors such as evapotranspiration, shading, color, etc. What is important is how these surfaces release heat at night and alter the microclimate around city islands.

Let $\hat{t}$ be the predicted average temperature reduction given a green percentage. Then the following formula is the predicted average temperature drop given that the green percentage changes from time step $i-1$ to i.

$$\Delta t_{red} = \hat{t}_i - \hat{t}_{i-1} \tag{5}$$

3.6 Planting costs

Planting costs are not the same across areas with different amounts of initial greenery. Planting trees in the middle of urban cities will be more expensive because there is a lack of initial infrastructure. Planting in areas with a low percentage of green coverage likely means there are additional costs, ranging across soil work, irrigation, maintenance, etc...

To account for this, we use the following formula to describe the cost of planting trees per square mile.

$$c_x = \alpha + \lambda \mathbb{1}_{\{x_g \leq 0.6\}} \tag{6}$$

As the percentage of green coverage increases past 60%, the cost to plant becomes cheaper. Similarly, areas of low green coverage will cost more than the base price α because of additional infrastructural costs λ. We set our value for α at 500 and λ at 128. This value is represented with the unit of thousands of dollars, indicating that we approximated our base cost of vegetation per square mile at \$500000 and additional cost at \$128000. We derive this number from statistics provided by [Sat13] and [Icp], which gave estimates for tree density and planting costs, respectively. The scaled product of the estimates gave us \$250000. However, because we were not able to ascertain details regarding the planting conditions under these estimates were made, we made the assumption to let \$250000 be the cost of planting in an area with average green space ($g = 0.5$), giving an α of 500. Using a similar derive λ using statistics and conversions sourced from [Kim07].

3.7 Effect of funds and percent green coverage

To find the new green coverage after additional funds we use the following formula, Where f is an increment of injected funds, c is the cost per square mile, g_{i-1} is the fraction of greenery previously, and A is the area of the region

$$g_i = \left(\frac{\frac{f}{c_x} + g_{i-1} \cdot A_x}{A_x} \right) \tag{7}$$

3.8 Utility Function

3.8.1 Assumptions

1. **Impoverished Neighborhoods Benefit more from UHI Reduction**. Lower-income neighborhoods benefit more from UHI reduction than higher-income counterparts because of the law of diminishing marginal utility. Lower-income areas have a larger number of heat-related hospitalization as well as worse access to healthcare. Mitigating UHI effects for lower-income areas should theoretically increase utility more since it deals with issues of needs instead of wants. Additionally., high-income areas have access to reliable air conditioning, so their comfort is less sensitive to external factors such as temperature.

2. **Equal Utility Gain for Individuals**. We assume that increases in greenery increase the utility of every individual the same in a region. As a result, we assume that utility also linearly scales with population.

The utility function captures the utility gained from planting greenery in a tract given its population, temperature reduction, percent poverty, and existing greenery.

3.8.2 Temperature Reduction

We assume that UHI effects are purely negative utility. Increasing the magnitude of the average temperature drop means decreasing UHI effects. Therefore actions that maximize average nighttime temperature drop Δt_{red} increase utility μ_x. Additionally, we assume that average temperature drop experiences diminishing marginal utility. Therefore we choose to represent this relationship using a natural logarithm.

3.8.3 Percent Poverty

The higher the percent poverty of an area, the more utility that area stands to gain from additional funds to plant trees. We assume that the relationship between poverty percentage as a decimal and utility is an exponential relationship.

3.8.4 Population

A larger population means there are more people to benefit from. Therefore the model attempts to capture a positive relationship between population and utility.

3.8.5 Final Equation

Having considered the assumptions about low-income neighborhoods, temperature reduction effects, and population, we construct the following to serve as the utility function for a single tract.

$$\mu_x = S_x \times \ln(1 + \Delta t_{red}) \times e^{p_x} \tag{8}$$

3.9 Optimization Algorithm

In order to solve the optimization problem we present, we employ a pseudo-gradient descent algorithm in which funding is iteratively distributed so as to maximize the utility and obey the constraints at every iteration.

Algorithm 1 Optimization algorithm

funds $\leftarrow B$
while funds > 0 **do**
    `curr_util` $\leftarrow \left(\prod_{x\in\mathcal{X}} \mu(x)\right)^{\frac{1}{|\mathcal{X}|}}$
    $x_i \leftarrow \arg\max_{x\in\mathcal{X}} \Delta(U_{\mathcal{X}}|_{\text{Fund } x})$
    Allocate fund injection to x_i ▷ Inject tract with best geometric mean benefit
    `curr_util` $\leftarrow U'_{\mathcal{X}}$ ▷ $U'_{\mathcal{X}}$ is the geometric mean after injection
    funds $\leftarrow$ funds $- \Delta x_i^{(s)} \cdot (\alpha + \lambda \mathbb{1}_{\{x_{i,g} \leq 0.6\}})$
end while

We note that our method is not a full gradient descent in the sense that we do not compute the instantaneous change in utility for each unit of funding. Rather, we break the total available funding down into increments, which we sequentially distribute to various tracts. This distribution is determined by measuring the increase in the city's utility as a whole for each offering of funds, then committing the offering to the tract which maximizes this utility increase.

We set this increment to \$10000 as a balancing point, with the consideration that perfecting the distribution down to a much greater precision is not meaningful. By this, we mean that being off by a higher precision increment, say \$100, from the supposed optimum, does not hurt the conclusions that we can derive from our model. Rather, we suggest that our model's projections are estimates, given that the actual planting of greenery is possible only in certain increments (such as the fact that planting half of a park does not make much sense).

In return for this negligible loss of accuracy, we gain a greater level of computational efficiency, which makes our model scale much better with more data. Whether that data comes in the form of optimizing funding at a federal level or at a local level with more localized temperature readings, our model will not have difficulty in giving accurate results in a timely sensible manner.

4 Results

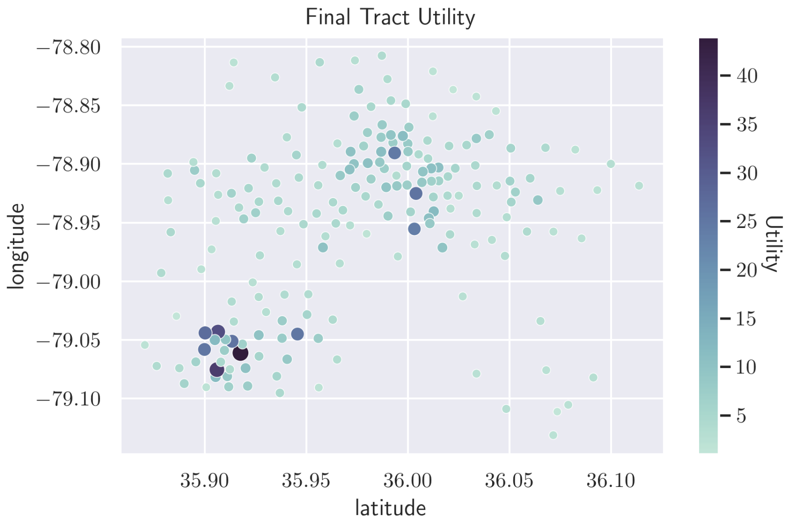
4.1 Analysis

The results of the utility optimization algorithm are shown graphically in figure 6a. Most notably, we can see that the majority of tracts exhibit uniform utility with some outliers which correspond to tracts with high population density which can easily be correlated in figure 6a. This phenomenon is explained by the fact that the utility function is maximized by an almost-uniform allocation of greenery; however, the most densely populated areas benefit exponentially more from the greenery addition due to their marginal return rates being at their peak.

In figure 8, we observe that most locations benefit somewhere between 0 to 10 utility points, with a few locations that benefit much more(20 to 40 utility points). Additionally, we see that the few points that have higher utility correspond to higher poverty areas.

Spatially the areas of interest for this project should be located around the coordinates $(35.92°, -79.05°)$ and $(35.98°, -78.90°)$.

(a) Tract Density



(b) Final Tract Utility

Figure 6: Population Density and Utility by Tract

Figure 7 displays the behavior of marginal utility over the optimization process. Clearly, there is a demonstration of diminishing marginal utility; the total utility across Durham increases over the period of 150 injections of funds for planting greenery, then the money no longer acts as efficiently—the marginal utility gain for the same amount of money begins to decrease. Assuming an initial budget of 2 million dollars and allocating funds in $\$10,000$ increments, the city of Durham should stop allocating funds at around 1.4 million after beginning to realize diminishing marginal returns in order to optimize both monetary and utility aspects simultaneously.

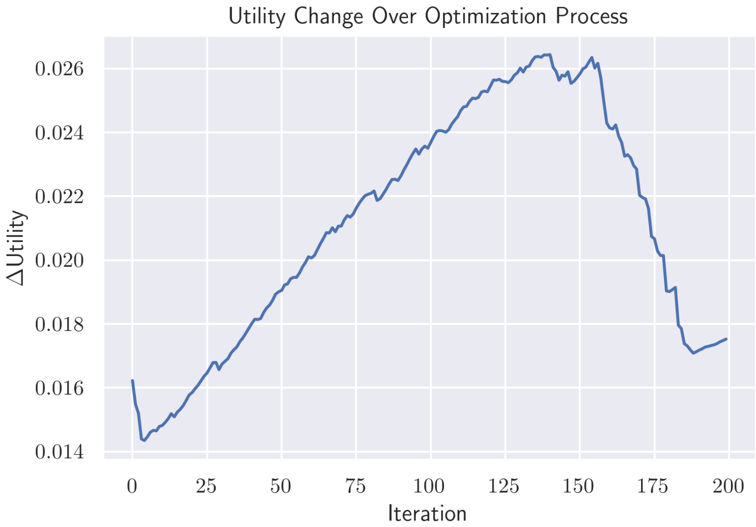


Figure 7: Marginal Utility across Optimization Process

Further, from figure 8 we make a rather interesting observation: most of the utility across tracts are in the $[0, 10]$ range; however, there are some outlier neighborhoods which have high final utilities. Diving deeper into this, we found that these utilities all fell outside of 2 standard deviations from the mean in final utility and corresponded to the tracts at city centers. This emphasizes the fact that the most utility is gained from allocating greenery planting efforts to the tracts which have high population density which are generally correspondent with low-income zones.

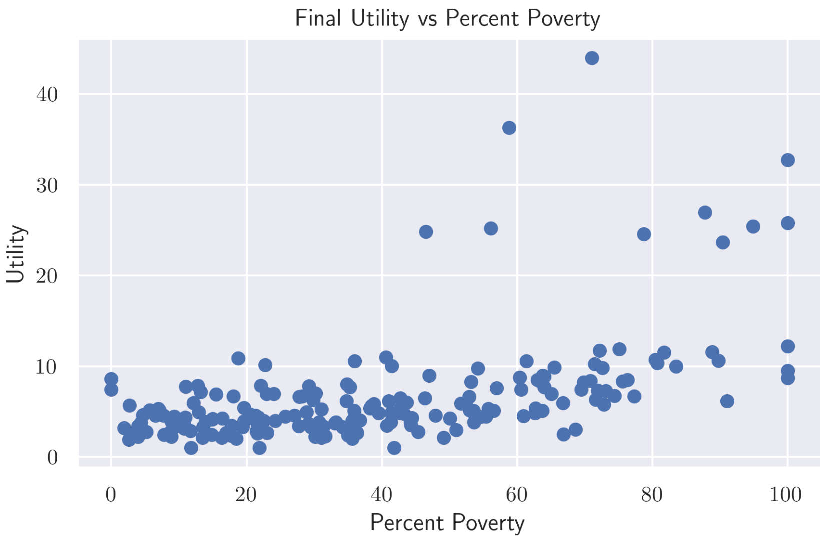


Figure 8: Final Utility with respect to Poverty

5 Strengths and Weaknesses

5.1 Strengths

1. **Generalization**: As we have not made any constricting assumptions which only pertain to the city of Durham, our model can be generalized to any other city. Therefore, using this model, other city governments can create policies to target areas that maximize utility

and social equity across the city for a variety of purposes given their individual utility and cost functions.

2. **Flexibility of Utility**: While we measure the overall utility of the city using a geometric mean, we have the flexibility to change the utility function $\mu(\cdot)$. In particular, the utility function can be changed in order to incorporate additional inputs or model different relationships between inputs. Because of this, we can create different utility functions that have different financial, environmental, or social intents.

3. **Social Equity**: Considering that one of the focuses that the city of Durham would like to address in a solution is the disproportionate impact of UHI effects on low-income neighborhoods, one of our model's strengths is that it works directly towards this goal. More specifically, the utility function which we seek to maximize is the geometric mean of all city tract utilities—this results in our model favoring providing greenery allocation to lower-income neighborhoods as their marginal utility gain under this regime is considerably greater than that of an affluent neighborhood if it were given the same allocation. As a result, the optima of this utility function imply the uniform distribution of final utility values across all neighborhoods, meaning that we prioritize increasing the utility of low-income populations prior to providing additional utility to populations that are already faring well.

5.2 Weaknesses

1. **Interpretability**: Because the primary goal of the model is to produce explainable results that can inform policy action, interpretability is important. While having flexibility in the utility function may be a good thing to test out different utility valuations, when the number of inputs is large, the projected utilities may potentially be difficult to interpret because of the large number of variable dependencies.

   However, the model remains fully effective when tasked with developing the optimal strategy for planting distribution. This is because comparisons of utility can be made without the context of a meaningful unit of measurement. This weakness applies when cities are using the model to determine whether a proposal is worth the money. However, given the difficulty of quantizing some sense of individual happiness or satisfaction, we leave this aspect of decision-making to other models developed on data better suited for such measurements.

2. **Simplifying the Impact of Trees and UHI reduction**. The addition of trees and reduction of UHI effects have extended environmental and health impacts that our model does not explicitly highlight. The addition of trees can provide improvements to air quality, stress reduction, etc. which can all increase the utility of planting trees.

6 Potential Improvements and Future Work

Clearly, under a recommendation regime in which we use data to gather insights and make predictions about trends, our model can be further improved and generalized through the inclusion and use of larger datasets as well as additional relevant data. Some examples of these data include healthcare and climate trends within the tracts or similar cities such that we can better inform our modeling procedure. However, there are a couple of other improvements and directions of viable future work, which we explore in the following sections.

6.1 Potentially Correlated Resource Basket Allocation Schemes

Another potential area of continuation would be to research the development of utility and optimization schemes in the case where the utility is not only effected by a singular component, i.e. greenery percentage, but rather a basket of components which may or may not be correlated. This adds a new dimension of nuance in the freedom to weight priority not only across tracts, but also between products in the basket as well. Further, correlation could open doors for more efficient paths to optima that may not be achievable by the modeling of a single product; for example, the effect of choosing to employ two products, say greenery and solar panels, has a multiplicative effect on the reduction of UHI effects with only a slight increase in cost.

This expansion of research would lead to the more difficult task of optimization in high dimensions but at the benefit of much greater flexibility and applicability—rarely are governments only focusing on the effect on a single independent factor on the region; rather, there are often realistically multiple different elements which must be finely balanced.

6.2 Robust Resource Allocation Algorithms

A possible and applicable improvement is the development of robust resource allocation algorithms. During the collection of data for analysis, one observation we made was that some of the available government data were either outdated or poorly maintained; for example, data entries could be missing for multiple tracts. The loss or generally, corruption, of the data upon which the allocation scheme is decided would be detrimental to any practical application, particularly for cities that may not frequently maintain their data. Hence, we are interested in developing corruption-robust allocation algorithms which are guaranteed to discover global optima regardless of data poisoning. Formally, we seek to construct provably robust algorithms for optimization on ϵ-corrupted data similar to the meta-algorithm for robust stochastic optimization explored in [Da19].

Clearly, a direct benefit of this development would be that our work could be abstracted to be useful to any city which maintains some semblance of relevant data records. Further, generally robust algorithms would go beyond just that of UHI effect mitigation; we would be able to apply it under a variety of different planning schemes ranging from park placement to road development and everything in between.

6.3 Contextualizing the Utility Gained from Vegetation

Our current model compares the various possible distributions of funding and greenery with the goal finding the optimum, where the utility of the city as a whole is maximized. Because we are making comparisons within the framework of the model, the fact that our model's representation of utility cannot be quantized is not problematic. However, if we are to try using the resulting utility figures to conclusively say that a project will be worth implementing at a certain cost, a roadblock is met.

Because of this, we are not able to put a hard dollar count on the value being added by the model's proposed optimal project, despite knowing that it is the best possible distribution.

Therefore, had we had more time, our team would have looked into possibly utilizing another data set to quantify utility gained in terms of a more interpretable unit. Even without it, we are able to answer important questions, but we believe that adding this element would have enabled us to give a more nuanced recommendation of a project.

7 Conclusion

As cities around the world continue to rise and develop in an era of global warming, UHI effects can be dangerous to human health during the summer months. Additionally, UHI effects are a particular concern to low-income populations without reliable cooling and restricted access to healthcare. As a result, city governments such as Durham, NC actively seek to develop policies for planting greenery to combat UHI effects and generally, increase population health and happiness.

With the use of data sourced from the city about a variety of components such as neighborhood poverty compositions, current greenery coverage, and population, we construct a model which measures city utility as a proxy for the effects of UHI. Further, we take into consideration the importance of financial and social constraints that the city may realistically face. Coupled with this, we devise an optimization scheme in order to fully realize the effects of greenery planting on overall Durham utility and discover the best allocation for greenery which maximizes the temperature drop in the city.

Using the results from our model, we are able to geographically map out the optimal tracts to plant greenery in order to maximize the overall utility of the city. Specifically, the city of Durham should allocate approximately 1.3 to 1.5 million dollars to greenery planting efforts in the regions around coordinates $(35.92°, -79.05°)$ and $(35.98°, -78.90°)$.

8 Proposal to the Durham Government

Dear Mayor Elaine O'Neal,

Our research team writes to you with the hopes of aiding the city of Durham in addressing the growing concerns over the Urban Heat Island effect. Although Durham is not the most heavily populated of cities, it nevertheless suffers from the fallout effects of global warming. We concur with the city's recognition in that, although it may not be able to single-handedly resolve the issue of rising global temperatures, efforts can still be taken to protect the local population from its effects.

To this end, our team proposes a city-wide project to plant trees and vegetation as an endeavor to combat UHI effects. Past research by the EPA as well as other institutions indicates that increases in vegetations levels can reduce the health dangers of summer heat waves by both providing shade for heat-trapping materials such as concrete and inducing evapotranspiration processes that expedite the release of heat throughout the day. We find this to be a cost-effective plan in comparison to major green infrastructure projects. We find this to be important since our plan allows for immediate action to be taken throughout the entire city, as opposed to the decades-long rollout of expensive small-scale construction projects.

We recognize that although the city wishes to do as much as is in its power to help the residents, the resources available to be committed to this project may be limited. Therefore, we surmised, a plan of action to mitigate the effects of UHI must consider not only the effectiveness of a project, but also its efficiency. With this goal in mind, our team developed a model that works to optimize the distribution of available green funds to the various census tracts belonging to the city.

To specify, the distribution of funds created by our model emphasizes two key points.

The first is that regions with higher rates of poverty are prioritized for funding, all the while maintaining high efficiency in terms of budget-to-temperature reduction. The model does so with the recognition that the heat effects of UHI have the most severe impacts on lower-income neighborhoods with less access to insulation, air-conditioning, and regular medical care.

The second is that our model is able to account for population and area data, which allows for us to quantify more accurately the reduction in temperature achieved through a given level of funding. A distribution without these data points may struggle to consider the relative impacts of helping a large number of people at a high cost versus helping a few people at a low cost, but due to the design of our model, we are able to capture these vital comparisons and make optimal decisions.

With these considerations, our model was able to come up with a cost-efficient project designed to combat the effects of UHI in Durham. By following the model's suggestions, we are confident that with a projected budget proposal of \$1.5 million, it will be possible to add over 1800 acres of impactful greenery throughout the city. Thanks to the model's optimization of funding distribution, this translates to an effective reduction of nighttime temperatures by slightly more than 0.8 degrees Celsius. By current standards of global warming, such a reduction would mitigate the effect of four decades of warming, making this a powerful investment which will pay dividends for many years.

Considering past expenditures by the city on green infrastructure projects, the cost of this project appears to be very much in line with what the city can afford. Furthermore, by following the recommended distributions of our model, the city will be able to strategically distribute the planting across the census tracts as to maximize the utility gain experienced by the residents of Durham.

Our team looks forward to working with the city of Durham and its residents to create a greener and healthier community.

Sincerely, Team 01

References

[Cen] *Census Geocoder*. URL: https://geocoding.geo.census.gov/geocoder/geographies/coordinates?form.

[Da19] Ilias Diakonikolas and et al. *Sever: A robust meta-algorithm for stochastic optimization*. May 2019. URL: https://arxiv.org/pdf/1803.02815.pdf.

[Icp] *Seedling Tree planting*. July 2021. URL: https://www.potomacriver.org/resources/get-involved/water/seedling-tree-planting/.

[Kim07] H H Kim. *Urban heat island*. 2007. URL: https://www.tandfonline.com/doi/abs/10.1080/01431169208904271.

[Kur+94] D M Kurn et al. "The potential for reducing urban air temperatures and energy consumption through vegetative cooling". In: (May 1994). DOI: 10.2172/10180633. URL: https://www.osti.gov/biblio/10180633.

[McP+05] Greg McPherson et al. *Municipal Forest Benefits and Costs in Five US Cities*. Dec. 2005. URL: https://www.fs.usda.gov/psw/publications/mcpherson/psw_2005_mcpherson003.pdf.

[Mtg] *Story map series*. URL: https://mtgis-portal.geo.census.gov/arcgis/apps/MapSeries/index.html?appid=2566121a73de463995ed2b2fd7ff6eb7.

[Sat13] Michelle Satterfield. *Low Pole Stew Reoff*. Feb. 2013. URL: https://www.fs.usda.gov/Internet/FSE_DOCUMENTS/stelprd3794596.pdf.

A Data

Tract	Avg Temp Reduction ($^{\circ}C$)	Poverty (%)	Greenery (%)	Longitude	Latitude	Population	Area (mi^2)
101	0.847667	56.50623886	67.91999817	-78.88460488	36.02022844	3282	1.3
101	0.907226	42.75788348	73.30000305	-78.8838644	36.02885952	3282	1.3
102	0.825103	52.4137931	70.12000275	-78.90368904	36.02326272	4430	1.5
102	0.741116	41.38785626	65.33000183	-78.90122109	36.03312664	4430	1.5
200	0.742552	62.67837541	64.83999634	-78.88002715	36.00959211	2893	1.2
200	0.690315	54.52436195	63.52999878	-78.89177338	36.00522303	2893	1.2
200	0.960217	28.82797732	75.87000275	-78.89539737	36.00986406	2893	1.2
301	0.46284	62.7430911	43.97000122	-78.91069399	36.01965863	2644	0.6
301	0.861752	28.18897638	69.48000336	-78.9144127	36.01516696	2644	0.6
301	0.967994	24.01574803	72.12999725	-78.91467542	36.01156011	2644	0.6
302	1.049724	18.77934272	81.01999664	-78.90302361	36.01501127	3672	0.7
302	1.001626	40.60606061	76.55999756	-78.90378381	36.01146324	3672	0.7
302	0.901283	54.21760391	69.40000153	-78.9065127	36.00731602	3672	0.7
401	0.791461	9.309791332	72.15000153	-78.9270117	36.02498007	2564	1.3
401	0.964205	25.72178478	78.13999939	-78.92698965	36.01946431	2564	1.3
401	0.527745	31.35435993	65.61000061	-78.9380664	36.02093514	2564	1.3
402	0.583461	34.79749002	51.41999817	-78.92812022	36.01248291	2966	0.5
500	0.699625	94.90131579	67	-78.9251309	36.00395296	4192	0.8
500	0.862927	35.24451939	66.87999725	-78.91797012	35.99957401	4192	0.8
500	0.700161	60.39325843	63.16999817	-78.91869384	35.99454215	4192	0.8
500	0.801081	65.52984166	66.11000061	-78.91985634	35.98927044	4192	0.8
600	0.887973	63.72575799	72.05999756	-78.92759374	35.97914725	5507	2.7
600	0.973152	42.69911504	75.70999908	-78.93451135	35.98545882	5507	2.7
600	1.046455	0	90.62000275	-78.94435096	35.9901614	5507	2.7
700	0.405412	53.62731152	49.04000092	-78.90991749	35.99429973	3701	1.3
700	1.05485	13.1713555	84.33999634	-78.91283432	35.98182027	3701	1.3
700	1.056046	18.04961506	82.15000153	-78.91998861	35.97419721	3701	1.3
900	0.714413	63.44086022	65.87000275	-78.88273371	35.99989799	1783	0.4
900	0.685738	75.61576355	61.66999817	-78.88940023	36.00001732	1783	0.4
1001	0.604393	69.48640483	60.22000122	-78.88180753	35.99260041	3879	0.9
1001	0.755617	69.8212408	64.29000092	-78.87720685	35.98677939	3879	0.9
1001	0.662885	71.90553746	58.29999924	-78.87317152	35.98005686	3879	0.9
1002	0.658896	56.9870484	62.25999832	-78.8686807	36.00045423	6140	1.3
1002	0.666354	53.19767442	67.13999939	-78.8665987	35.9872941	6140	1.3
1002	0.778827	72.65469062	67.01999664	-78.87526444	35.9915803	6140	1.3
1002	0.860054	81.72661871	69.70999908	-78.87609385	35.99746096	6140	1.3
1100	0.53156	78.75706215	53.91999817	-78.89059429	35.99344063	4151	0.5
1100	0.36741	88.85350318	43.20999908	-78.88954956	35.98681091	4151	0.5
1301	0.691264	89.78032474	63.84000015	-78.89925132	35.98021488	1432	0.3
1303	0.458997	83.50951374	48.45999908	-78.89999771	35.97339507	4052	0.6
1303	0.819855	35.96214511	69.97000122	-78.90469176	35.9712845	4052	0.6
1304	0.797337	63.00990099	70.37999725	-78.90981025	35.96665755	2781	0.7
1400	0.782782	80.72601556	72.08999634	-78.88946963	35.97177444	2746	0.7
1400	0.523759	77.32831609	55.43000031	-78.88466168	35.97816905	2746	0.7
1501	0.604256	91.07142857	55.47000122	-78.9408338	36.0012675	3430	1.1
1505	0.394118	75.10548523	42.86000061	-78.9404539	36.01267863	3951	0.4
1505	0.323652	80.45977011	32.61000061	-78.94629352	36.01023767	3951	0.4
1504	0.476014	70.84314649	50.56000137	-78.9507799	36.01076508	3033	0.5
1504	0.651928	90.42553191	60.66999817	-78.95535577	36.00315489	3033	0.5
1503	0.67518	0	63.77999878	-78.91566928	36.00684428	1861	0.2
1601	0.984319	17.7916234	83.90000153	-78.88776574	36.08235058	6117	10.2
1601	0.953767	35.4534005	88.88999939	-78.89994653	36.09986231	6117	10.2
1603	1.103366	13.56321839	89.83999634	-78.92298932	36.07489442	6846	9.9
1603	0.935656	8.861283644	81.83000183	-78.92229571	36.09319282	6846	9.9
1603	0.894654	17.00819672	85.51999664	-78.91851126	36.1137441	6846	9.9
1604	1.068514	3.292410714	86.44000244	-78.95766979	36.071607	6655	9.5
1604	1.010037	7.8125	84.54000092	-78.96344471	36.08545042	6655	9.5
1705	0.747754	30.74433657	71.22000122	-78.9183273	36.04371464	5794	2.9
1705	0.692939	36.76177837	71.98999786	-78.94187263	36.03379182	5794	2.9
1705	0.859366	60.9375	69.01999664	-78.91864768	36.03390484	5794	2.9
1706	1.085551	41.44775248	88.66000366	-78.9713135	36.01694508	5252	2.3
1706	0.706876	43.20407364	72.19999695	-78.96008238	36.02069777	5252	2.3
1713	0.869305	33.04410701	80.01999664	-78.97840775	36.0477092	3407	2.6
1713	0.682891	14.85549133	69.29000092	-78.96864847	36.03282282	3407	2.6
1713	0.901471	10.78838174	79.58999634	-78.96473368	36.04130667	3407	2.6
1712	1.087439	8.905852417	87.94000244	-78.95762662	36.05857513	4150	3.6
1712	0.922806	9.154383243	85.44999695	-78.9453842	36.04860978	4150	3.6
1708	0.838048	55.37563048	70.88999939	-78.88613455	36.06766424	4895	2.5
1709	0.868299	65.06666667	75.73999786	-78.8750199	36.04000198	8149	3.2
1709	0.94696	76.30878438	78.38999939	-78.87823386	36.03365542	8149	3.2
1709	0.835505	66.7867036	70.19999695	-78.88652254	36.05059593	8149	3.2
1710	0.840033	38.48372476	70.88999939	-78.91219129	36.06013808	4646	1.5
1710	0.57332	71.62162162	64.43000031	-78.9145901	36.05021232	4646	1.5
1711	0.926821	47.02495202	81.12000275	-78.93076679	36.06392348	4748	1.5
1711	0.680667	52.93141593	71.69999695	-78.92392559	36.05288004	4748	1.5
1711	0.548649	51.68674699	67.77999878	-78.93246551	36.05082383	4748	1.5
1806	0.897439	21.89086294	78.18000031	-78.83660112	36.02221759	6760	13.7
1801	0.90524	66.84350133	81.75	-78.85931742	36.01220683	8293	17.7
1801	0.920544	31.09565217	83.09999847	-78.85483203	36.04329163	8293	17.7
1801	0.851617	49.15773354	79.90000153	-78.84251652	36.03361343	8293	17.7
1802	0.844253	55.75666164	74.13999939	-78.84866603	35.99889007	7548	3.5
1802	0.745898	53.54239257	73.41999817	-78.84586139	35.99160239	7548	3.5
1802	0.837337	52.98165138	73.63999939	-78.85113013	35.98180695	7548	3.5
1802	0.942132	64.01718582	81.80000305	-78.85908235	35.97348191	7548	3.5
1806	0.935838	18.46075173	82.73000336	-78.82097701	36.01228499	6760	13.7
1810	0.794621	42.71356784	75.51999664	-78.83638804	35.97581384	3550	1.2
1811	0.629451	14.92905614	71.01000214	-78.8076303	35.98714263	6378	2.4
1811	0.648923	13.91927575	68.58999634	-78.82759443	35.98989004	6378	2.4
1808	0.695296	4.250681199	72.66000366	-78.81167798	35.97389765	11498	7
1808	0.966031	20.55114433	83.77999878	-78.81326896	35.95656667	11498	7
1809	0.873087	39.55685891	82.04000092	-78.85169051	35.94771253	7316	4.7
1809	0.690678	17.78151261	77.47000122	-78.82617191	35.93456517	7316	4.7
2007	0.98227	7.744636316	80.01000214	-78.932641	35.96310444	5080	2.3

2007	0.921427	9.288354898	79.08000183	-78.94024278	35.94098942	5080	2.3
2007	0.978127	31.08108108	78.86000061	-78.942257	35.95501794	5080	2.3
2008	0.950185	21.34502924	83.62999725	-78.95137509	35.94855411	3178	2
2008	0.987491	3.947368421	79.66000366	-78.95722904	35.93721218	3178	2
2009	0.776034	44.32907348	68.61000061	-78.88244962	35.9652651	4800	2.8
2009	0.863162	41.33333333	73.01000214	-78.9008426	35.95783596	4800	2.8
2009	0.874495	44.4588464	75.94000244	-78.91815724	35.95581689	4800	2.8
2013	0.920839	6.940700809	82.36000061	-78.91640517	35.89778038	4805	2
2013	1.007632	22.09543568	87.04000092	-78.90531536	35.89496788	4805	2
2013	0.807398	12.95210166	79.41999817	-78.90805909	35.88174562	4805	2
2015	0.720227	38.24657534	64.48000336	-78.93941779	35.9679464	5328	1.5
2015	0.569181	53.52422907	53.93999863	-78.95066325	35.96448014	5328	1.5
2015	0.482426	19.40532081	43.95000076	-78.95235328	35.97148471	5328	1.5
2032	0.707448	61.39981977	61.79000092	-78.97119197	35.95814188	3320	0.5
2031	0.46301	29.11330049	45.61000061	-78.96166856	35.95937928	2201	0.6
2030	1.052558	1.900584795	88.52999878	-78.97886642	35.99501904	4354	5.2
2030	0.910234	2.575660013	72.51000214	-78.95952188	35.97964637	4354	5.2
2029	0.896782	21.58167036	73.73000336	-78.98481277	35.96643061	3774	2.3
2033	0.850721	8.763277693	73.01000214	-79.00096892	35.92357953	4293	2.8
2034	0.889008	40.71573261	72.44000244	-78.98560992	35.94533335	6052	4
2034	1.009655	8.885163453	83.40000153	-78.97795279	35.92711239	6052	4
2019	1.04142	36.34377276	83.41000366	-78.97281259	35.9033351	5134	7.9
2019	1.08842	35.95068138	91.73000336	-78.99290699	35.87854232	5134	7.9
2019	1.025938	3.976143141	83.83999634	-78.98976682	35.89821934	5134	7.9
2020	1.076318	4.67404674	88.55000305	-78.95810003	35.88321462	8403	6.3
2020	0.900007	4.471903501	81.26000214	-78.93092616	35.88194935	8403	6.3
2021	0.606078	21.44886364	60	-78.94858209	35.90541503	4604	2.3
2021	0.736309	27.72795217	67.55999756	-78.92628536	35.90655887	4604	2.3
2022	0.675429	29.88764045	65.68000031	-78.92498671	35.91311583	4524	1
2022	0.692957	19.58708311	63.13000107	-78.92066611	35.92153273	4524	1
2023	0.899395	15.47518923	81.52999878	-78.94679155	35.91914586	2837	0.9
2023	0.732083	27.06367925	65.84999847	-78.93719692	35.91682012	2837	0.9
2024	0.782725	7.073954984	68.08000183	-78.93140217	35.93614511	7058	1.7
2024	0.863743	22.92490119	73.84999847	-78.94154598	35.92500813	7058	1.7
2024	0.814158	5.628847845	68.41000366	-78.93217396	35.92661731	7058	1.7
2025	0.762898	20.98512057	68.55000305	-78.91648173	35.93514892	6410	2.3
2025	0.728674	16.43035863	69.94000244	-78.90302379	35.92940893	6410	2.3
2025	0.947298	27.81954887	80.56999969	-78.89499009	35.92300065	6410	2.3
2026	0.598766	47.90136411	58.49000168	-78.91161586	35.94627425	6831	2.2
2026	0.878351	38.84958474	71.95999908	-78.89235133	35.94508285	6831	2.2
2035	0.788143	50.07974482	73.47000122	-78.87721375	35.94039015	6682	3.8
2036	0.757434	29.52054795	71.27999878	-78.8983602	35.89438982	4307	2.6
2036	0.863888	33.16725979	75.94999695	-78.9079154	35.90560737	4307	2.6
2038	0.870442	5.128205128	79.06999969	-78.83341244	35.9121195	5724	4.9
2038	0.679444	30.11928429	62.88999939	-78.81342249	35.91422154	5724	4.9
2200	0.255522	72.85499247	31.77000046	-78.90179059	35.99941187	3177	0.6
2300	0.446123	73.11827957	52.09999847	-78.90391404	35.98836063	2434	0.5
2300	0.26666	100	49.00999832	-78.89850212	35.98610664	2434	0.5
10707	0.580532	30.24193548	62.56999969	-79.0900113	35.91176594	3363	0.6
10707	0.492382	63.78539493	60.56999969	-79.08111188	35.91107436	3363	0.6
10708	0.295515	72.19935985	50.43999863	-79.08161311	35.90530426	2603	0.3
10708	0.755946	58.85111372	73.58000183	-79.07538607	35.90596098	2603	0.3
10710	0.752905	35.63714903	78.05000305	-79.09037983	35.90070863	3335	6.4
10709	0.930816	41.02449889	84.62999725	-79.08729273	35.8898653	1955	1.1
10705	0.886934	29.17948718	80.41000366	-79.0893357	35.92108659	5036	1.5
10705	0.909011	71.50621118	77.97000122	-79.07410599	35.92006031	5036	1.5
10705	0.570457	35.91885442	65.01000214	-79.06878779	35.90787356	5036	1.5
10705	0.53505	21.76870748	62.90000153	-79.07513353	35.91233007	5036	1.5
10706	0.891951	6.512605042	85.08999634	-79.09523927	35.93695029	3668	2.2
10706	1.139672	2.706843718	89.18000031	-79.08102088	35.93542729	3668	2.2
10904	1.261831	68.65671642	93.43000031	-79.03389837	36.06519345	3673	16.4
10902	1.054908	45.35232384	86.62000275	-79.01295215	36.02695298	6584	12.9
10902	1.118158	23.00985595	88.43000031	-79.07882715	36.03370428	6584	12.9
11001	0.816382	11.7396358	80.16999817	-79.08210287	36.09123666	4733	4.2
11002	1.002335	34.99308437	84.41000366	-79.07583765	36.06802991	4291	8.2
11001	0.751791	21.60278746	74.19999695	-79.10539222	36.07888479	4733	4.2
11002	0.7655	41.82879377	71.06999969	-79.11123321	36.07347412	4291	8.2
11002	0.99394	31.63716814	83.68000031	-79.13125081	36.07151326	4291	8.2
11106	1.044536	50.9762533	84.37000275	-79.10887611	36.04834707	2471	3.9
11208	1.046813	13.46085724	84.87999725	-79.09068678	35.95609831	5245	10.5
11207	0.790019	4.362801378	71.33000183	-79.06672769	35.96510476	4599	1.8
11206	0.747673	10.95768374	72.38999939	-79.03275312	35.96297023	4184	1.5
11206	0.60679	44.22187982	55.29000092	-79.01109837	35.95095432	4184	1.5
11300	0.822262	71.08585859	77.86000061	-79.06112661	35.91755449	3132	0.4
11400	1.156149	22.75334608	87.76999664	-79.04590046	35.92656293	3893	1.3
11400	0.675853	74.40510737	67.5	-79.05360914	35.91853932	3893	1.3
11500	1.050435	35.59742191	81.23000336	-79.03439056	35.91428411	1849	1.4
11601	0.467075	100	54.02999878	-79.05119511	35.91340217	2114	0.3
11601	0.269164	100	33.09000015	-79.04965692	35.91021633	2114	0.3
11602	0.486198	100	44.77999878	-79.04309904	35.90649278	6498	0.6
11602	0.162987	100	21.93000031	-79.05008345	35.90494336	6498	0.6
11602	0.416423	87.79495525	40.33000183	-79.04404864	35.90016694	6498	0.6
11700	0.365415	60.57951482	50.93999863	-79.05896635	35.90965866	5791	1
11700	0.924401	56.09756098	75.02999878	-79.05817906	35.89983487	5791	1
11800	1.069609	63.97790055	88.97000122	-79.06651689	35.9405322	3076	2
11800	1.077901	43.66341714	85.88999939	-79.06400732	35.92667699	3076	2
11903	1.055529	12.75862069	84.83000183	-79.04872702	35.95583562	2162	0.7
11904	1.222715	46.5392562	91.73000336	-79.04503341	35.94561257	3096	1.5
11904	1.125369	34.82972136	87.62000275	-79.04858851	35.93800534	3096	1.5
11902	0.903531	12.13235294	81.43000031	-79.02845695	35.9503089	4367	1.6
11902	1.127758	11.03038309	86.54000092	-79.03369366	35.9380982	4367	1.6
12101	0.642349	24.26653406	62.16999817	-79.01141515	35.9392307	3913	1.3
12103	0.712859	22.52934495	65.62000275	-79.01733808	35.91323711	2764	1
12101	0.496851	34.20647149	44.86000061	-79.02627327	35.93015239	3913	1.3
12102	0.924341	10.71044133	76.77999878	-79.01341952	35.92652677	1543	0.8
12201	0.998756	11.77474403	84.98999786	-79.0297625	35.88603298	3084	7.7
12202	0.916013	46.33792603	78.65000153	-79.06873474	35.89557033	5449	2.2
12202	0.841606	15.0055991	73.27999878	-79.0741095	35.88745941	5449	2.2
12202	0.790708	19.57943925	69.69000244	-79.07226721	35.87631929	5449	2.2

B Code

```python
# Majority of setup code omitted for brevity

def opt(
    x: np.ndarray,
    green_pct: np.ndarray,
    ar: np.ndarray,
    pop: np.ndarray,
    pvty_pct: np.ndarray,
    funds: int,
    injection: int,
    reg: LinearRegression,
) -> Tuple[np.ndarray, float, List]:
    """
    Optimize the utility function for geographic tracts
    Returns geometric mean
    """
    util_delta = []
    util_array = x
    while funds > 0:
        total_util = geo_mean(util_array)
        max_util_delta, max_util_idx = 0, -1
        for idx, u in enumerate(util_array):
            temp = util_array.copy()
            new_green = green_change(injection, green_pct[idx], ar[idx])
            if new_green > 95:
                continue
            else:
                temp[idx] += util(pop[idx], green_pct[idx], new_green, pvty_pct[idx], reg)
                curr_util = geo_mean(temp)
                delta = curr_util - total_util
                if delta >= max_util_delta:
                    max_util_delta = delta
                    max_util_idx = idx
        util_delta.append(total_util)
        ng = green_change(injection, green_pct[max_util_idx], ar[max_util_idx])
        util_array[max_util_idx] += util(pop[max_util_idx], green_pct[max_util_idx], ng, pvty_pct[max_util_idx], reg)
        green_pct[max_util_idx] = ng
        funds -= injection
    return util_array, geo_mean(util_array), util_delta
```