A manual for MA2007B

Applications of Geometry and Topology for Data Science

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Introduction to Topology 1.

Topology was first studied in...

1.1. Basic concepts and Examples

Topology is the study of properties that are preserved under continuous functions in a generic way, for this reason, we need to work with objects that are proper to the space and these objects are subsets. The set theory is needed to properly understand topology because it is the main language of the theory, if you need a refresh of set theory you can see the reference.

Definition 1.1.1 (Topological Space) *Let X be a set and τ a collection of subsets of X. We say that τ is a* topology *for X is it satisfies:*

1. *If the total and the empty set belong to τ.*

$$\emptyset, X \in \tau.$$

2. *For every subcollection of elements of τ, let say $\{U_\alpha\}_{\alpha \in A}$, we have that its union is an element of τ.*

$$\bigcup_{\alpha \in U} U_\alpha \in \tau.$$

3. *For every finite subcollection of elements of τ, let say $\{U_j\}_{j=1}^{n}$, we have that its intersection is an element of τ.*

$$\bigcap_{j=1}^{n} U_j \in \tau.$$

The elements of τ are called open sets *of X and the pair (X, τ) is called a* topological space*.*

This is not the first time that we see the concept of openness, for example in calculus we already worked with open intervals, open boxes (the real plane or space), and open disks (complex numbers). Topology theory intends to generalize this concept to general sets.

Example 1.1.1 (Trivial topology) Let X be any set and consider the collection $\tau = \{\emptyset, X\}$. τ is a topology for X and it is called the trivial topology.

This is not the best topology to understand properties of X but it is a good counterexample when you want to generalize topological constructions.

Example 1.1.2 (Discrete topology) Let X be any set and consider the collection $2^X = \{\text{all possible subsets of } X\}$. τ is a topology for X and it is called the discrete topology.

Note that each element of X is an open set on this topology.

Example 1.1.3 (Co-finite topology) Let X be any set and consider the collection $\tau_{cf} = \{U \subset X : X \setminus U \text{ is finite}\}$, i.e., the complement of U in X is a finite set of points. τ is a topology for X and it is called the co-finite topology.

Example 1.1.4 (Spaces with different topologies) Let $X = \{1, 3, 5, 7\}$ and consider the collections

$$
\begin{aligned}
\tau_1 &= \{\varnothing, \{1\}, \{5\}, \{7\}, \{1,5\}, \{1,7\}, \{5,7\}, \{1,5,7\}, X\}. \\
\tau_2 &= \{\varnothing, \{1\}, \{3\}, \{7\}, \{1,3\}, \{1,7\}, \{3,7\}, \{1,3,7\}, X\}.
\end{aligned}
$$

Both (X, τ_1) and (X, τ_2) are topological spaces but they are not the same space. For example, the open set $\{1, 5\}$ is not part of (X, τ_2) and $\{1, 3\}$ is not part of (X, τ_1).

Therefore in order to obtain the same topological space we need to find exactly the same open sets in both topologies.

Second Chapter 2.

Appendix

Some more blindtext A.