Geometry of a Metric Space

Conference session 03 - Lecture 2

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Objectives of the Session

- Enter the favorite sets into a metric space.
- Define the notion of neighborhood in a metric space.
- Set the relationship between neighbourhoods and open assemblies in a metric space.
- Characterize the equivalence of metrics visually.

Motivation.

> Suppose you have a set of store locations data and want to group them into regions to optimize logistics. What is an important piece in this problem?

Motivation

Algorithms like the *K-means* or *DBSCAN* require a notion of closeness between the points in order to be able to group them together. And this notion of closeness can be defined through a metric depending on the nature of our data.

How are the paths displayed on the plane depending on metric?

Predilected sets

Neighborhoods or Open Balls

> **Definition.** Be sure to go (X, d) a metric space and be it $p * X$ One point. The set
>
> $$B_r(p) == *\text{-}\ x * X : d(\text{-}\ x, p) * r*$$
>
> it's called a **Open neighbourhood** or **Open ball** of p with radio r.
>
> How neighborhoods look depending on the metric you choose, for example in $\mathbb{R}^2$?

Sphere

Definition. Be sure to go (X, d) a metric space and be it p * X One point. The set

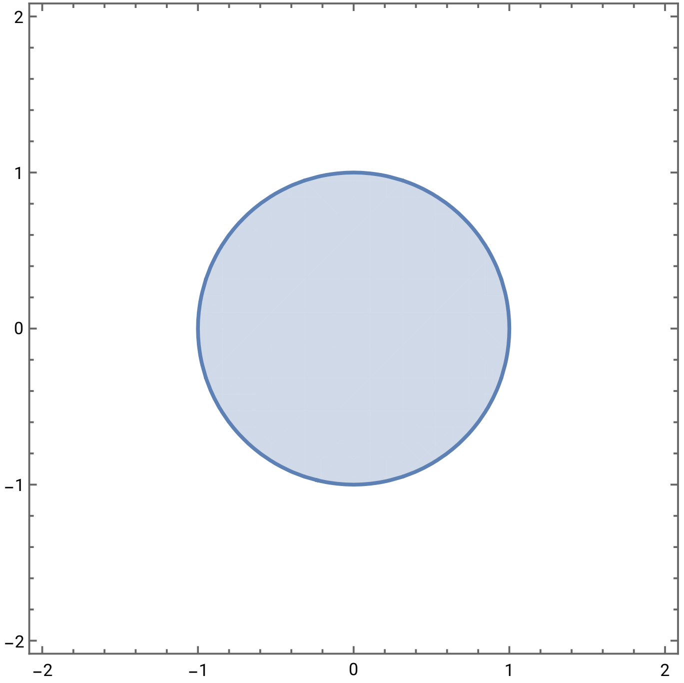
$$- Yes._r(p) == *- x * X : d(- x, p) == r*$$

it's called a of p with radio r.

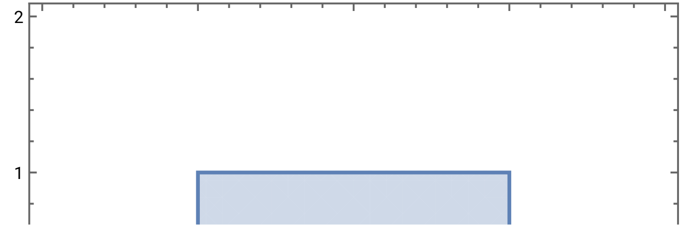
How these definitions are visualized in $\mathbb{R}^2$?

```
In[•]:= RegionPlot[(x)^2 + (y)^2 <= 1, {x, -2, 2}, {y, -2, 2}]
```

Out[•]=




```
In[•]:= RegionPlot[Max[Abs[x], Abs[y]] <= 1, {x, -2, 2}, {y, -2, 2}]
```



Visually how the metrics are compared in $\mathbb{R}^2$?

Analytically as the metrics are compared in $\mathbb{R}^2$