definition convergence example x^n, 1/2 lines, $sqrt(x^2 + 1/n)$, rationals definition uniform convergence example x/n with restriction

Theorem 0.1. *f_n are uniformly convergent and continuous then f is continuous*

Theorem 0.2. *$f_n : [a, b] \to \mathbb{R}$ uniformly convergent if f_n is Riemann integrable then f is Riemann integrable*

$$\int_a^b \underbrace{\lim f_n}_{f} = \lim \int_a^b f_n$$

Lemma 1. *metric spaces and $f_n : X \to X'$ are uniformly convergent if f_n is bounded then f is bounded.*

Definition: $\mathcal{B}(X, \mathbb{R})$: all bounded function from X to $\mathbb{R}$. $d(f, g) = \sup \{|f(x) - g(x)| : x \in X\}$

Definition: $\|f\| = \sup\{|f(x)| : x \in X\}$ therefore $d(f, g) = \|f - g\|$

Proposition 0.3. *$(\mathcal{B}, d)$ is a complete metric space.*

Definition: $\mathcal{C}_b(X, \mathbb{R})$ and $\mathcal{C}_b(X, \mathbb{R})$ closed subset of $\mathcal{B}(X, \mathbb{R})$

exampele: any compact metric space

Theorem 0.4. *f_n are differentiable functions*

1. *f_n' are uniformly convergent to g*
2. *f_n are point convergent*

Proposition 0.5. *$f_n : [a, b] \to \mathbb{R}$ consider $\sum f_n$*

1. *f_n are riemann integrable and the series uniformly convergent then the $\sum f_n$ is riemann integrable and*

$$\int_a^b \sum f_n = \sum \int_a^b f_n$$

2. *Similarly for derivative*

Definition (Wierstrass M test): This is super cool

power series and convergence radius of convergence of integral/derivative of power series is equal to the radius of convergence of the originalo series.

Theorem 0.6. *in the convergence cirle the power series is inifinitely integrable and differentiable, and coefficients are taylor coefficients*

analytical definition proof of analytical means analytical in interval using a pair sequence