Contents

Chapter 1

Logic

1.1 Proposition Logic

A logical **proposition** is a statement that is either true or false.

1.2 Operators

Definition (Negation): Let p be a proposition, then its negation $\neg p$, read as "not p", has the opposite truth value of p.

p	$\neg p$
T	F
F	T

Definition (Conjunction): Let p and q be two propositions, then their conjunction $p \wedge q$, read as "p and q", is true only when both p and q are true, it is false otherwise.

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Conjunction and not operators are **universal**, also called **functionally complete**, in the sense that every boolean function $f : B^n \to B$ can be generated with these two operators. Note that, every boolean function $f : B^n \to B$ for $n \geq 2$ can be generated with binary boolean functions thus, it only suffice to show that every binary and unary boolean function can be generated with $\{\wedge, \neg\}$.

Definition (Disjunction): Let p and q be two propositions, then their disjunction $p \vee q$, read as "p or q", is false only when both p and q are false, and true otherwise.

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Similary, $\{\vee, \neg\}$ is functionally complete.

Definition (Exclusive or): Let p and q be two propositions, then their exclusive or $p \oplus q$, read as "p xor q", is true only when exactly one the p or q is true, it is false otherwise.

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

Curiously, $\{\oplus, \neg\}$ is not universal.

1.3 Predicate Logic

Predicate logic is an extention of propositional logic. A predicate is a statement that may be true or false depending on the value of its variable. The collection of value that a variable x can take is called x's **universe of discourse**. **Quantifiers** allow us to quantify how many objects in the universe of discourse satisfy a given predicate.

Definition: Universal quantifier $\forall$ asserts that for all x in the universe of discourse the predicate is true.

Definition: Existential quantifire $\exists$ asserts that there exists a x in the universe of discourse that the predicate is true for that x.