# Conditional independance on extremal linear latent model

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In this document, we want to define a new notion on conditional independence for a particular model and see if it corresponds to the "classic" extremal conditional independence.

# 1 The latent linear model

## 1.1 Presentation

Let's consider a random vector $X$ of $\mathbb{R}^d$ such that we have the following representation :

$$X = MZ + \varepsilon$$

where $Z$ represents a unobservable random vector of $\mathbb{R}^K$ (identified as the latent variable), $M$ is a $d \times K$ matrix and $\varepsilon \in \mathbb{R}^d$ a random noise.

For the next assumptions, we will use the same context as (Boulin 2024), and so :

- $K$ is not known and the dimensional parameters $d$ and $K$ can increase and be larger than $n$, the number of observation.
- all the components of the random vector $Z$ are asymptotically independent with a tail indexx equal to one. It means that we can express the exponential measure as below :

$$\Lambda_Z = \sum_{k=1}^{K} \delta_0 \otimes \cdots \otimes \Lambda_{Z_k} \otimes \cdots \otimes \delta_0, \qquad \Lambda_{Z_k}(dy) = y^{-2} dy$$

- the random noise $\varepsilon$ possess a distribution with a tail that is lighter than the factors (*what does it mean ?*).

### 1.2 Induced properties

Theses assumptions give to the random vector $X$ the regular variation property and bring also a spectral measure $\Phi$ which have a discrete representation :

$$\Phi(.) = \sum_{k=1}^{K} ||A_{.k}||\delta_{\frac{A_{.k}}{||A_{.k}||}}(.),$$

with $\delta_x(.)$ the Dirac measure on $x$.

*I don't really see why all of this is true...*

Thus, we can compute the limits for the maxima $n$ replications of $X$

## 2 Conditionnal independance over $Z$

### 2.1 Heuristic and definition

### 2.2 Properties

### 2.3 Extremal independance

### 2.4 Extremal conditionnal independance

#### References

Boulin, Alexis. 2024. "Variable Clustering of Multivariate Time Series According to the Dependence of Their Extremes." PhD thesis, Université Côte d'Azur.