Conditional independance on extremal linear latent model

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Contents

In this document, we want to define a new notion on conditional independence for a particular model and see if it corresponds to the "classic" extremal conditional independence.

1 The latent linear model

1.1 Presentation

Let's consider a random vector X of $\mathbb{R}^d$ such that we have the following representation :

$$X = MZ + \varepsilon$$

where Z represents a unobservable random vector of $\mathbb{R}^K$ (identified as the latent variable), M is a $d \times K$ matrix and $\varepsilon \in \mathbb{R}^d$ a random noise.

For the next assumptions, we will use the same context as (Boulin 2024), and so : - K is not known and the dimensional parameters d and K can increase and be larger than n, the number of observation.

- all the components of the random vector Z are asymptocally independent with a tail indexx equal to one. It means that we can express the exponential measure as below :

$$\Lambda_Z = \sum_{k=1}^{K} \delta_0 \otimes \cdots \otimes \Lambda_{Z_k} \otimes \cdots \otimes \delta_0, \qquad \Lambda_{Z_k}(dy) = y^{-2}dy$$

- the random noise ε possess a distribution with a tail that is lighter than the factors (*what does it mean ?*).

1.2 Induced properties

Theses assumptions give to the random vector X the regular variation property and bring also a spectral measure Φ which have a discrete representation

$$\Phi(.) = \sum_{k=1}^{K} ||A_{.k}||\delta_{\frac{A_{.k}}{||A_{.k}||}}(.),$$

with $\delta_x(.)$ the Dirac measure on x.

I don't really see why all of this is true...

2 Conditionnal independance over Z

2.1 Heuristic and definition

2.2 Properties

2.3 Extremal independance

2.4 Extremal conditionnal independance

References

Boulin, Alexis. 2024. "Variable Clustering of Multivariate Time Series According to the Dependence of Their Extremes." PhD thesis, Université Côte d'Azur.