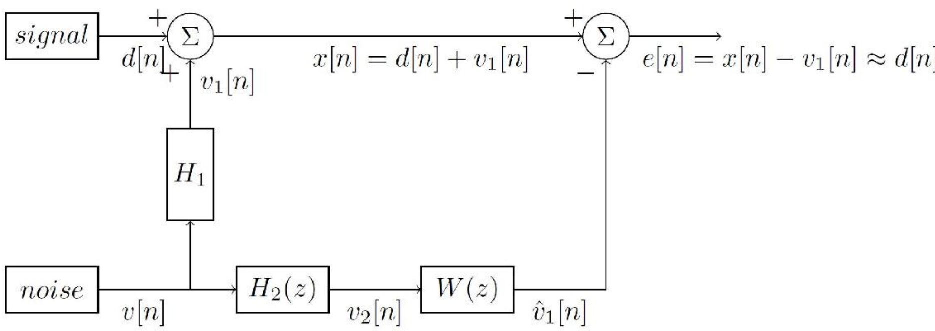
Chapitre 7 : Optimum filters

FIR Wiener : Noise Cancellation (7.2.6)

The noise cancellation problem is described by the following figure:



Suppose the desired signal d[n] is given by :

$$d[n] = \sin(n\omega_0 + \phi\ )$$

with $\omega_0 = 0.05\pi$ and the related noise sequence $v_1[n]$ and $v_2[n]$ are AR(1) processes driven by the same noise signal $v[n]$.

$$v_1[n] = 0.8v_1[n-1] + v[n]$$

$$H_1(z) = \frac{1}{1 - 0.8z^{-1}}$$

$$v_2[n] = -0.6v_2[n-1] + v[n]$$

$$H_2(z) = \frac{1}{1 + 0.6z^{-1}}$$

Where v[n] is zero-mean, unit variance white noise that is clearly uncorrelated with d[n].

Goal is to design a FIR Wiener filter of orders 6 and 12 to remove the noise.

Signals creation

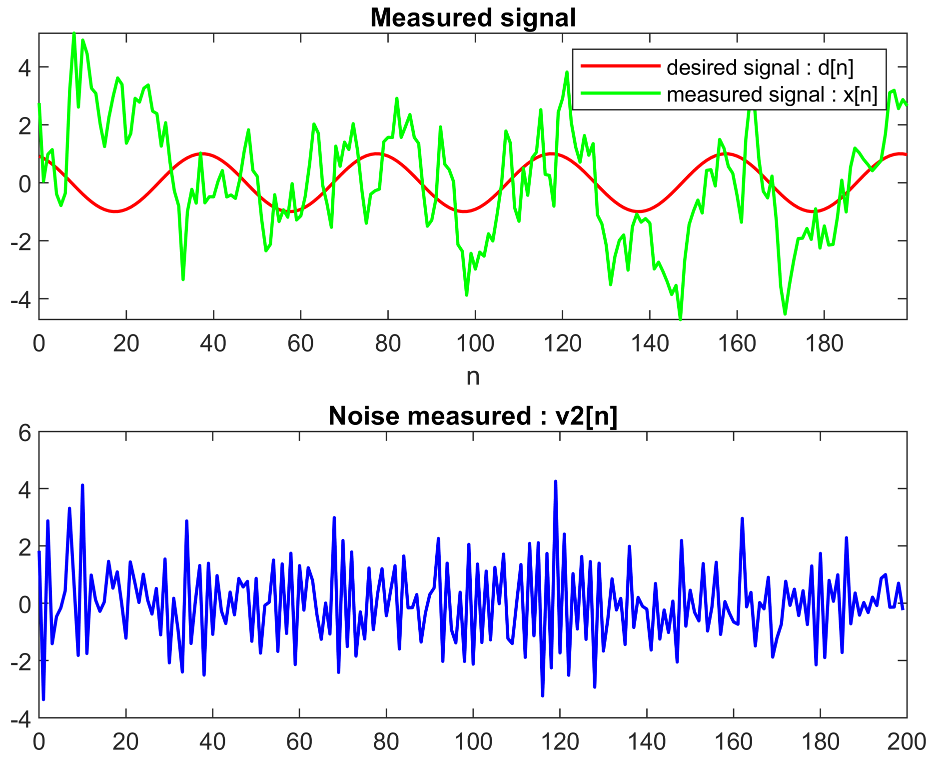
Parameters

```
H1_a = -0.8000
H2_a = 0.6000
```

SigmaN	N	p	H1_a	H2_a
1	200	6 12	-0.8	0.6

Signal generation

Based on the previous parameters, the signals are generated.



Wiener solving

Filter design

The Wiener-Hopf equations for the noise cancellation described by figure are given by :

$$\boldsymbol{R}_{v_2,v_2}\boldsymbol{w} = \boldsymbol{r}_{v_1,v_2}$$

As we assume that the noise $v_2[n]$ is uncorrelated with $d[n]$:

$$r_{v_1,v_2}[k] = E\left\{v_1[n]v_2{}^*[n-k]\right\} = E\left\{[x[n]-d[n]]v_2{}^*[n-k]\right\} = E\{x[n]v_2[n-k]\}$$

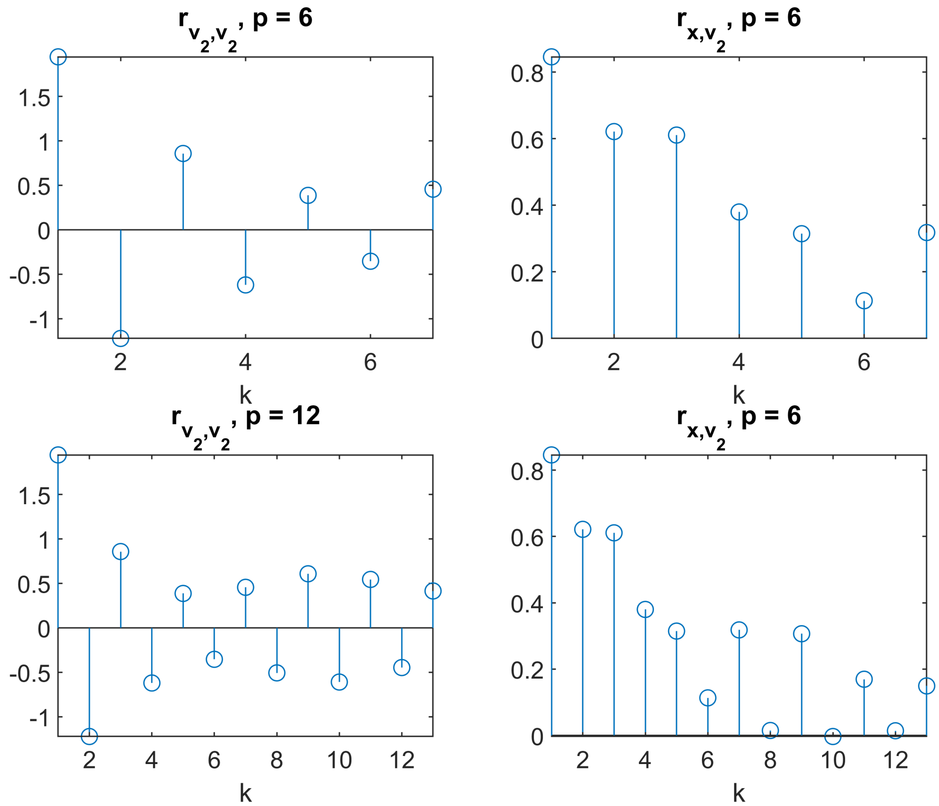
Then, the Wiener-Hopf equations become:

$$\boldsymbol{R}_{v_2,v_2}\boldsymbol{w} = \boldsymbol{r}_{x,v_2}$$

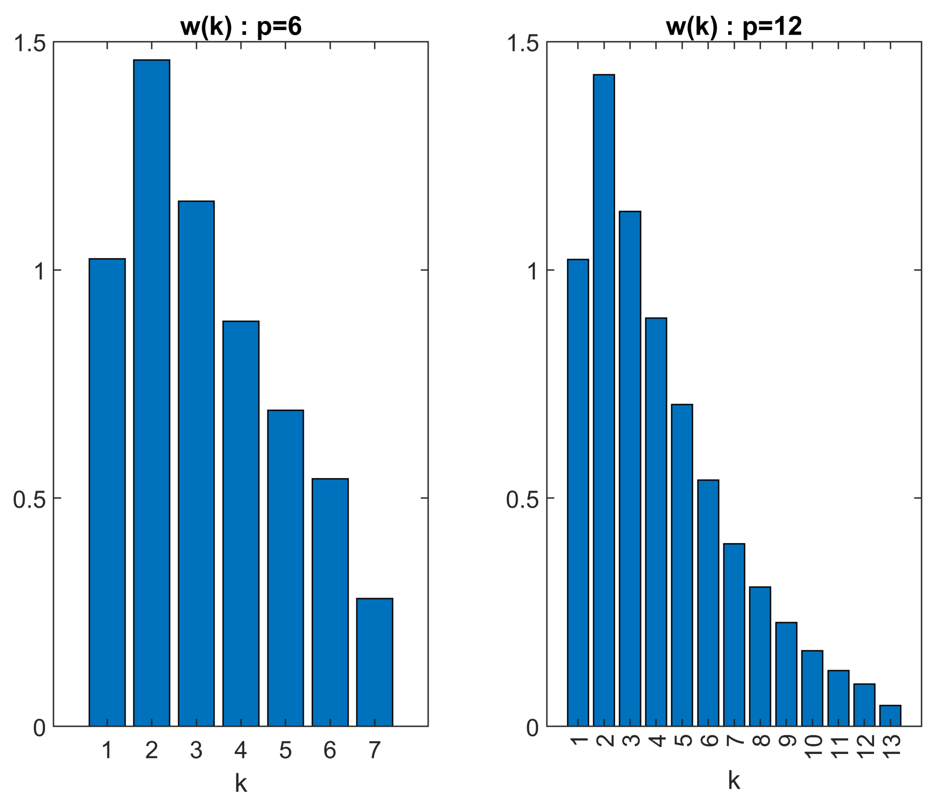
And the solution is:

$$\boldsymbol{w} = \boldsymbol{R}_{v_2,v_2}^{-1}\boldsymbol{r}_{x,v_2}$$

The different correlation and cross-correlation are (for two different orders):

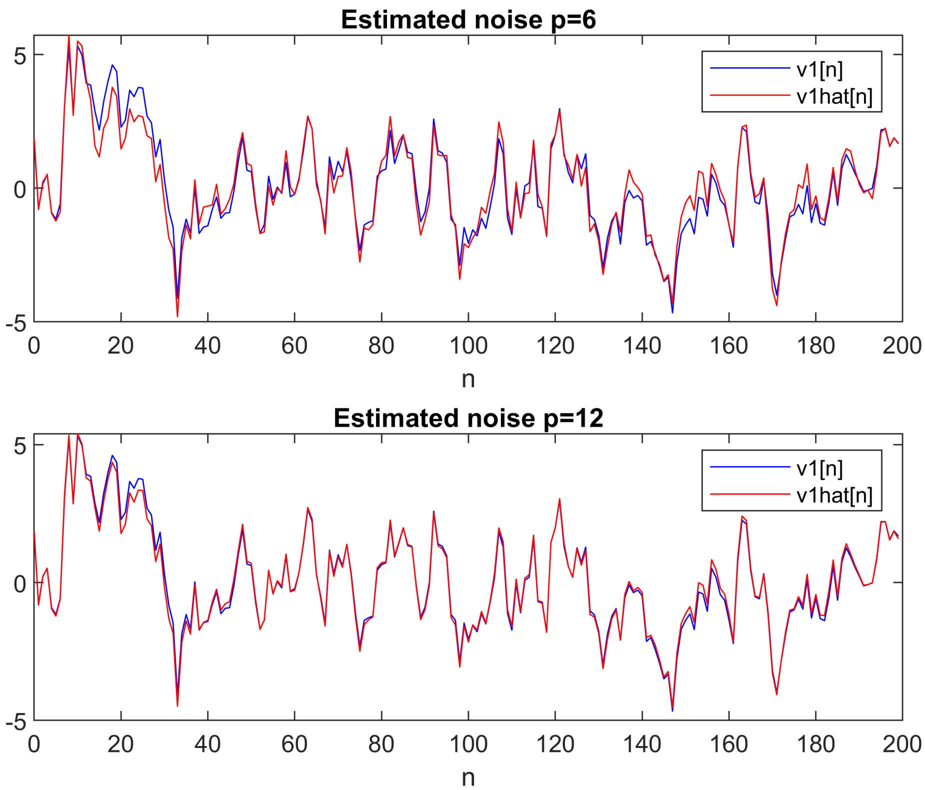


The Wiener coefficients obtained are:

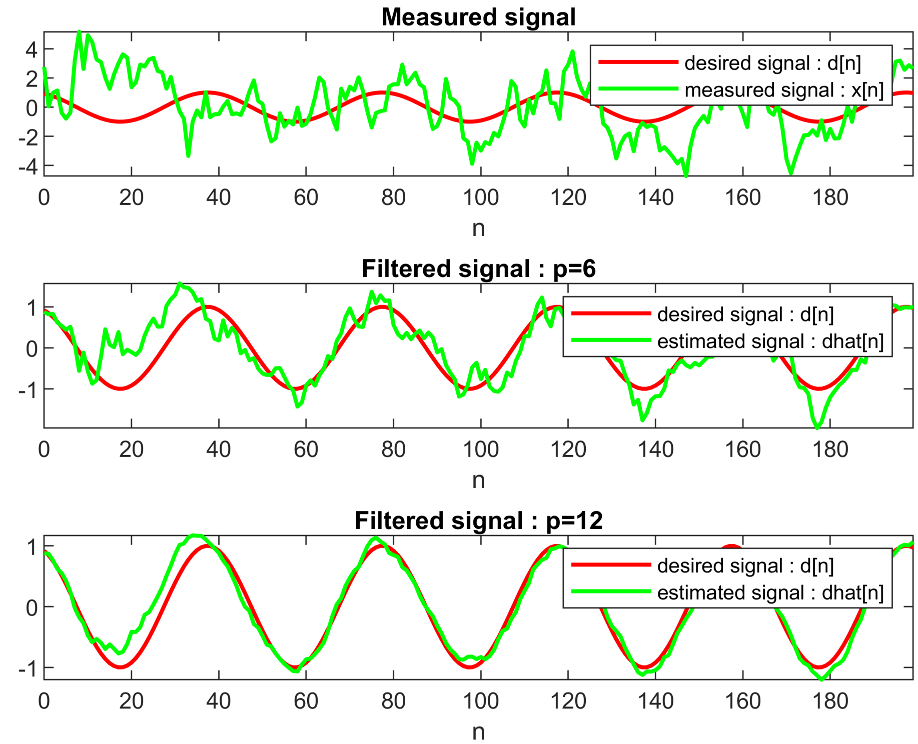


filtering

Estimate the noise v1 based on v2 and the Wiener filter designed



Finally, the estimated noise can be removed from the measured signal.



And the error on the signal reconstruction can be computed: