Auto-regressive AR(2) Random Process

M.Tognolini HEIG-VD 2022 MA-StatDig

Autoregressive random process (CompEx 3.4)

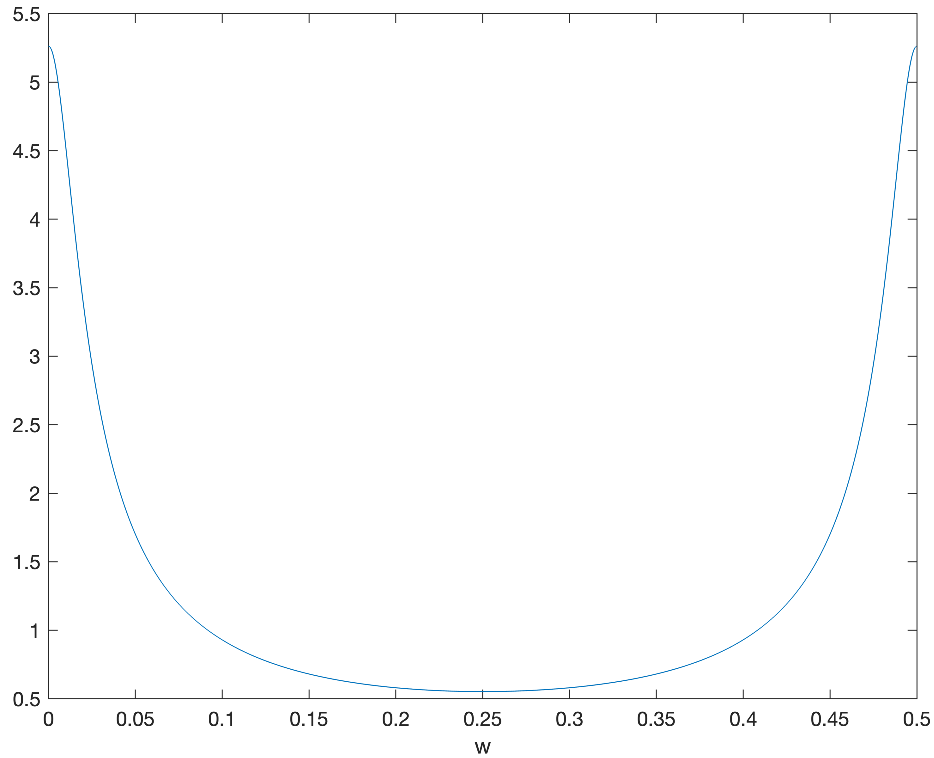
We consider the autoregressive AR(2) random process given by the difference equation as follow:

$$x(n) = a(1)x(n-1) + a(2)x(n-2) + b(0)v(n) \qquad (1)$$

Here v(n) is a unit variance white noise.

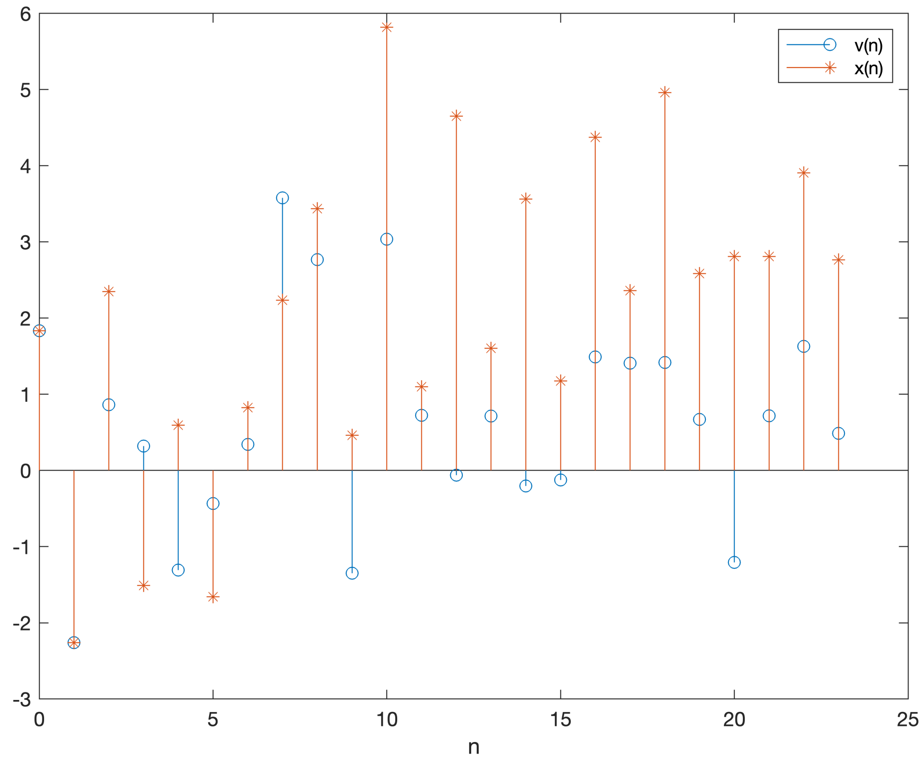
1. with a(1) = 0, a(2) = -0.81 and b(0) = 1 , generate   a24 sample of the random process x(n). Generate v(n) with the finction randn() , as   the gaussian white noise.

```
N = 24
testrand = 0.5377
vn = 1×24
    1.8339   -2.2588    0.8622    0.3188   -1.3077   -0.4336    0.3426    3.5784 ···
b = 1
a = 1×3
    1.0000         0   -0.8100
```



Compute x(n) with the filter function:

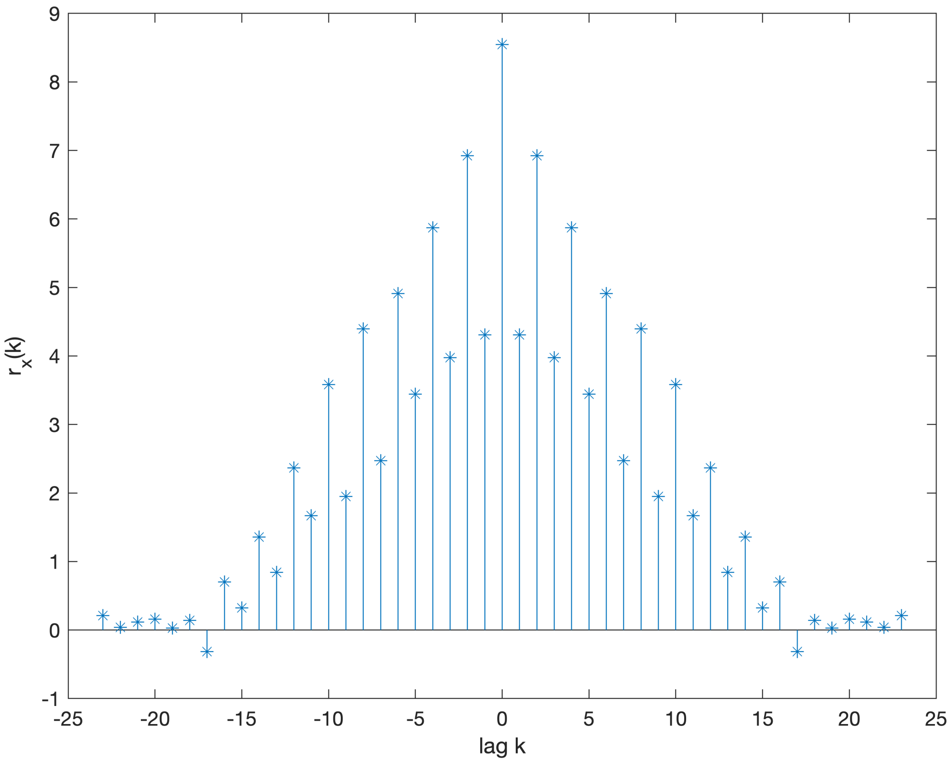
```
xn = 1×24
    1.8339   -2.2588    2.3476   -1.5109    0.5939   -1.6574    0.8237    2.2359 ···
```



2. Estimate the autocorrelation sequence of x(n) using the sample autocorrelation :

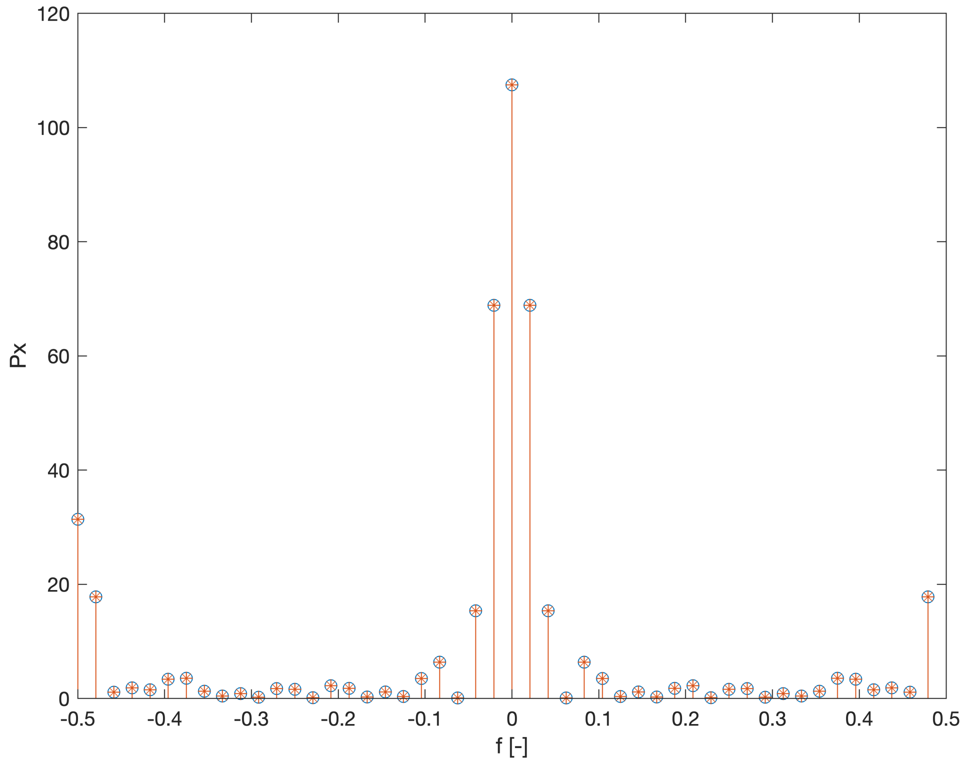
[rxhat,k] = SampleAutocorr(xn)

```
rxhat = 1×47
    5.0714    0.9180    2.8214    3.7992    0.7281    3.3481   -7.5824   16.8412 ···
k = 1×47
   -23   -22   -21   -20   -19   -18   -17   -16   -15   -14   -13   -12   -11 ···

rxhat = 1×47
    0.2113    0.0383    0.1176    0.1583    0.0303    0.1395   -0.3159    0.7017 ···
```



3. Estimate the power spectrum from the sample autocorrelation:

```
w = 1×48
   -3.1416   -3.0107   -2.8798   -2.7489   -2.6180   -2.4871   -2.3562   -2.2253 ···

Px2 = 1×48
   31.3891   17.8194    1.1140    1.8736    1.5195    3.3949    3.5446    1.2839 ···
```



4. From the esimation of the autocorrelation rxhat(k) use the Youle Walker equations to find the values of the coefficients a(1) and a(2) and b(0) and comment the accuracy of the estimation:

$$\begin{bmatrix} r_x(0) & r_x(-1) & r_x(-2) \\ r_x(1) & r_x(0) & r_x(-1) \\ r_x(2) & r_x(1) & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a(1) \\ a(2) \end{bmatrix} = \sigma_v \, |b(0)|^2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad (2)$$

In this case the process is real value and stationary so $r_x(-1) = r_x(1)$.

Take the values computed for rxhat for k = 0,1,2 and $\sigma_v^2 = 1$

```
Rx = 3×3
    8.5457    4.3075    6.9229
    4.3075    8.5457    4.3075
    6.9229    4.3075    8.5457

a2 = -0.7454

a1 = -0.1283

b0 = 2.8324
```