Random Process Experiments

M.Tognolini HEIG-VD 2022 MA-StatDig

Random Phase sinusoid (ex 3.3.1)

As random process consider a random phase sinusoid:

$x(n) = Asin(n\omega_0 + \phi)$ (eq1)

Here n is the sample number ω_0 is a constant value in rad/sample and ϕ is the random variable in this process, and it is uniform distributed from $-\pi \cdots + \pi$ the probability density function is given by :

$$f_\phi(\alpha) = \begin{cases} (2\pi)^{-1} & \alpha \in [-\pi \cdots \pi] \\ 0 & \text{otherwise} \end{cases}$$ (eq2)

First define NR values of ϕ vit the function rand() (uniform distribution) then define the N samples of each realization of the eq1.

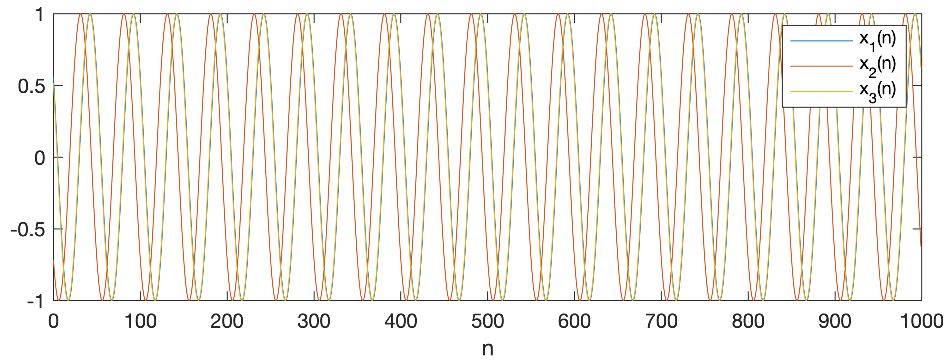
```
N = 1000
NR = 100
A = 1
F0 = 0.0200
testrand = 0.8147
```

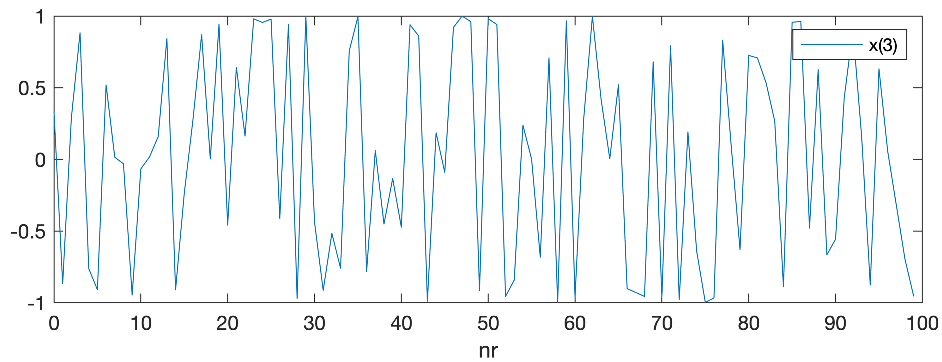
The size of the x(n) is :

```
ans = 1×2
       100        1000
```

Each line is a realization and each column is a sample

Plot the time sequence for realizations from nr = 1:3 and plot the sample x(3) fo all the R realizations:





We could estimate the mean of the process $m_x(n) = E\{x(n)\}$

For each sample value of n we could compute the arithmetic mean over all the realizations, in théory with a infinite number of réalizations $m_x(n) = 0$

Estimate the mean of the process x(n) :

```
mxn = 1×1000
    0.0389    0.0310    0.0225    0.0137    0.0047   -0.0044   -0.0134   -0.0222 ···
```

In this case we notice the mean of the process is very close to 0 so the process is called zero mean process.

And th variance of the process:

```
varxn = 1×1000
    0.5197    0.5221    0.5233    0.5231    0.5216    0.5190    0.5153    0.5107 ···
```

the theory is confirmed.

The second order moments of the process

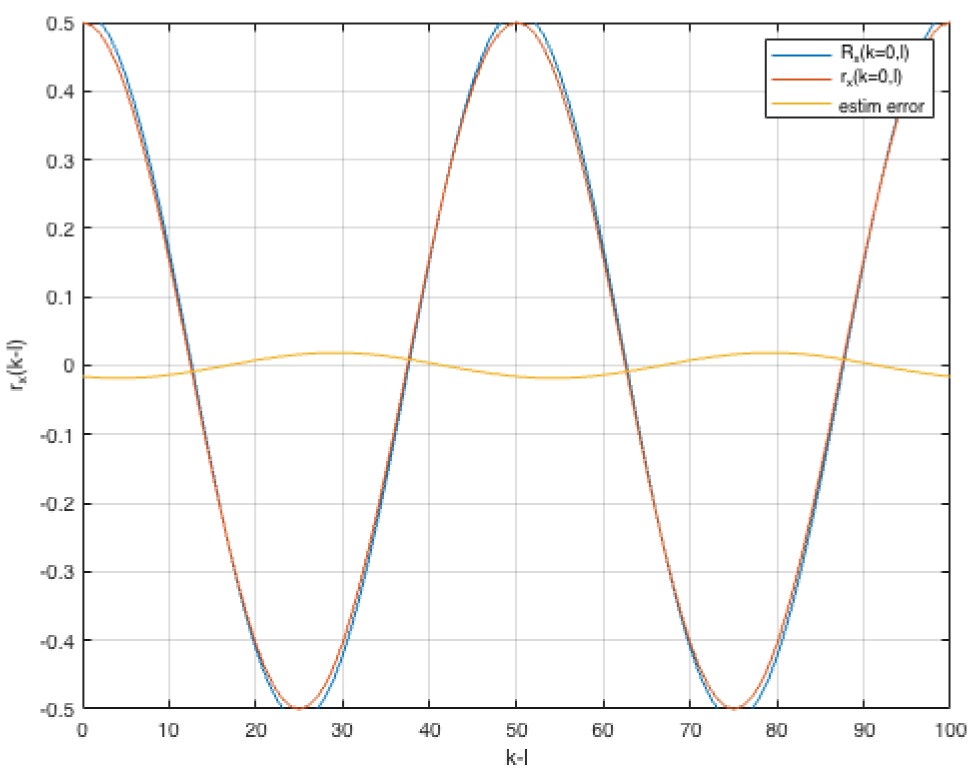
We could compute the autocorrelation of the process x(n) theoretically $r_x(k,l) = E\{x(k)x(l)\}$ between two samples k, l over all the realizations and this is in theory given by : $r_x(k,l) = E\{x(k)x(l)\} = \frac{1}{2}A^2 cos[(k-l)\omega_0]$

In order to verify it estimate numerically with the matrix of data xn the autocorrelation matrix Rxx , that is the product between samples number k for each realization by sample l for each realization the scalar product $\hat{r}_x(k,l) = \frac{1}{NR}x(k)^T x(l)$ multiplying latrix xn themselves we obtain the matrix Rxx who is symmetric.

The autocorrelation matrix is equal to the covariance matrix because the process is zero mean.

```
Rx = 1000×1000
    0.5160    0.5131    0.5021    0.4832    0.4567    0.4229    0.3825    0.3361 ···
    0.5131    0.5179    0.5144    0.5028    0.4833    0.4562    0.4219    0.3809
    0.5021    0.5144    0.5185    0.5145    0.5023    0.4823    0.4546    0.4197
    0.4832    0.5028    0.5145    0.5181    0.5134    0.5007    0.4801    0.4519
    0.4567    0.4833    0.5023    0.5134    0.5164    0.5113    0.4981    0.4770
    0.4229    0.4562    0.4823    0.5007    0.5113    0.5138    0.5082    0.4946
    0.3825    0.4219    0.4546    0.4801    0.4981    0.5082    0.5103    0.5043
    0.3361    0.3809    0.4197    0.4519    0.4770    0.4946    0.5043    0.5061
    0.2844    0.3340    0.3783    0.4166    0.4484    0.4732    0.4904    0.4999
    0.2281    0.2817    0.3308    0.3748    0.4128    0.4443    0.4688    0.4859
      ⋮
```

Each element of Rx is an esrimate of $\hat{r}_x(k,l)$ and is close to $r_x(k,l)$ for large NR realizations. If we plot the first line of Rxx $R_x(1,l) = \hat{r}_x(k=0,l)$ and we compare to the plot of the theoric fuction $r_x(k,l)$ for k=0 we have:



We could observe that if we increase the number of sample per realization we don't reduce the estimation error of the correlation rx , but if we increase the number of realizations NR the error is reduced.

Autocorrelation of a sum of process (Exemple 3.3.3)

Consider the exemple in slide 45. y(n) is a sum of 2 random processes: A sum of M sinosoids with a random phase like the process x(n) above and the white gaussian noise v(n).

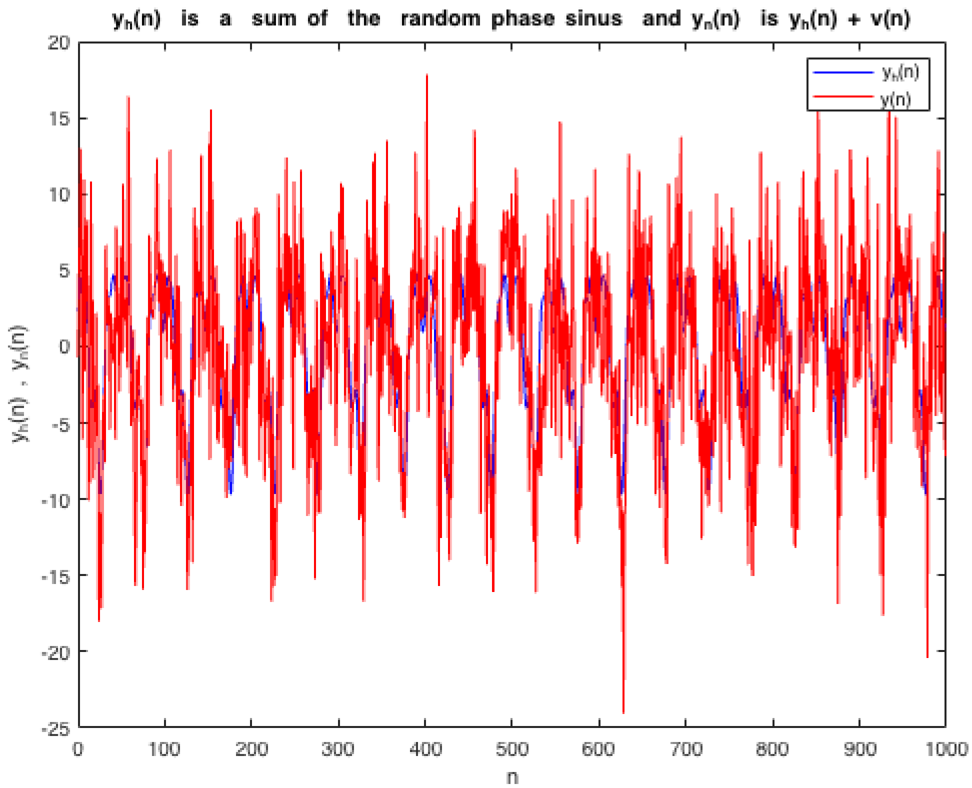
$$y(n) = y_h(n) + v(n) = \left[\sum_{m=1}^{M} A_m sin(n\omega_m + \phi_m)\right] + v(n)$$

In the exemple $A_m = A$ and $\omega_m = \omega_0$ are constants with m , and ϕ_m a random variable uniformly distributed like in the above exemple.

First create y(n) process using the matrix previously defined xn and use the function randn to generate the white gaussian noise.

with M = 5 and wm = w0/m and Am = A0/m with phim a uniform distrubutes phase like in ex 3.1.1 , compute the sum of the harmonic process yh(n) and the gaussian noise v(n) produced with the function randn() with a std of A0.

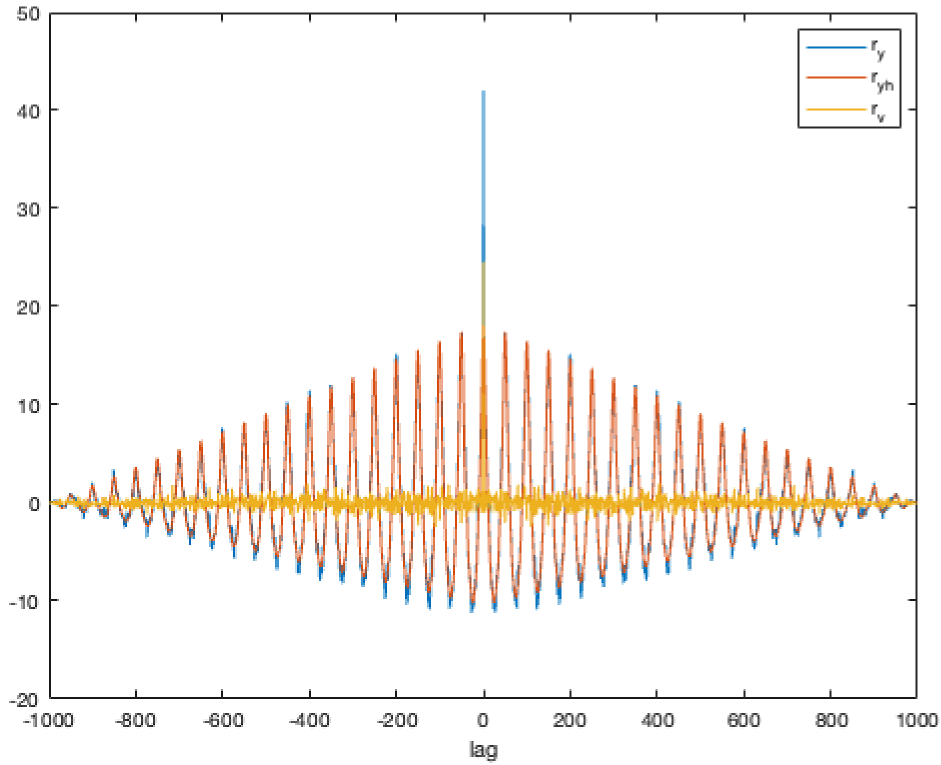
```
M = 5

wm = 1×5
    0.1257    0.2513    0.3770    0.5027    0.6283

A0 = 5

Am = 1×5
    5.0000    2.5000    1.6667    1.2500    1.0000

phim = 5×1
    1.8490
   -1.1862
    0.1793
   -2.1008
    0.6408
```



If we compute the autocorrelation of the harmonic process $y_h(n)$ called $r_{yh}(k)$ and the autocorrealation of the noise $v(n)$ called $r_v(k)$ we try to show that the autocorrelation of $y(n)$ called $r_y(k)$ is the summ of the autocorrelation of y_h and v(n) :

$$r_y(k) = r_{yh}(k) + r_v(k)$$

that is the case if the 2 process are uncorrelated.




```
rv0 = 24.5480
ry0 = 42.0194
ryh0 = 18.2951
ans = 42.8431
```

The power spectrum

The power spectrum is given by the discrete-time Fourier transform of the autocorrelation sequence r_x in ex 3.3.1:

$$P_x(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} r_x(k)e^{-j\omega}$$

Since rx(k) is always symetric, the spectrum is real and non-negative $P_x(e^{j\omega}) \geq 0$.

If we take as exemple the previous harmonic process x(n) we could compute the power spectrum $P_x(e^{j\omega})$, compare the theoric rx and the numerical estimation of rx based on the marix Rx:

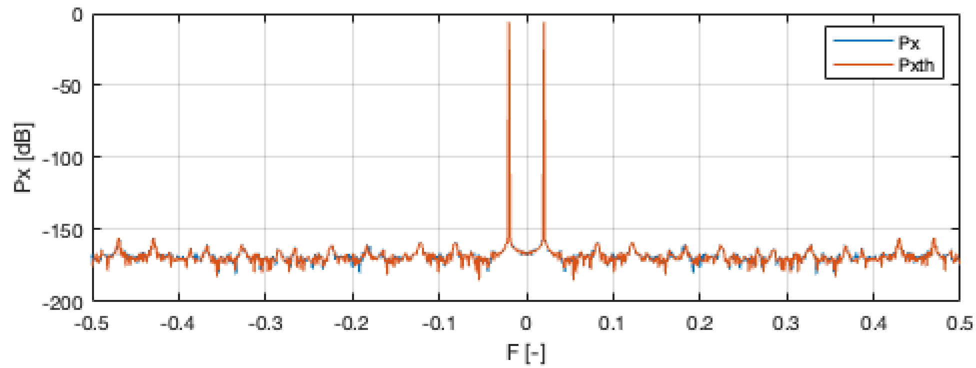
An interesting property is that the sum of the power spectrum is the power of the process , ths variance, verify with your data and the theoric ppectrum

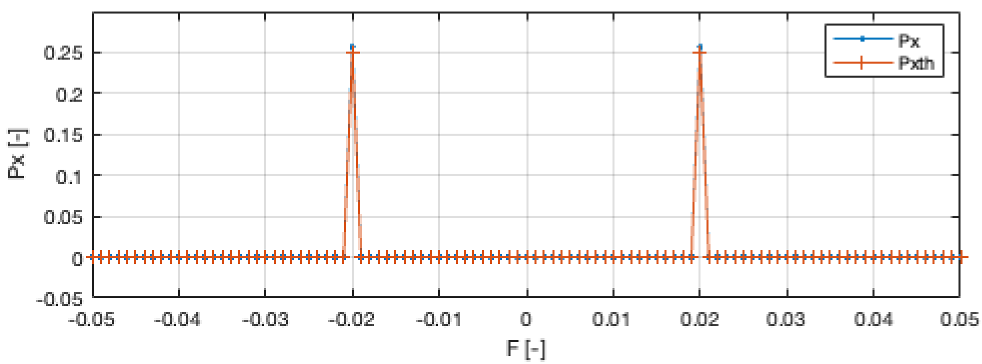
$$E\{|x(n)|^2\} = \int_{F=-1/2}^{+1/2} P_x(e^{j2\pi F})dF = \sum_{k=-N/2}^{+N/2} P_x(k)$$

```
Var_xth = 0.5000 + 0.0000i
```

```
Var_x = 0.5131 - 0.0000i
Varxn = 0.5197
```

Plot the spectrum amplitude in dB (10* log10 (abs(Px)))....





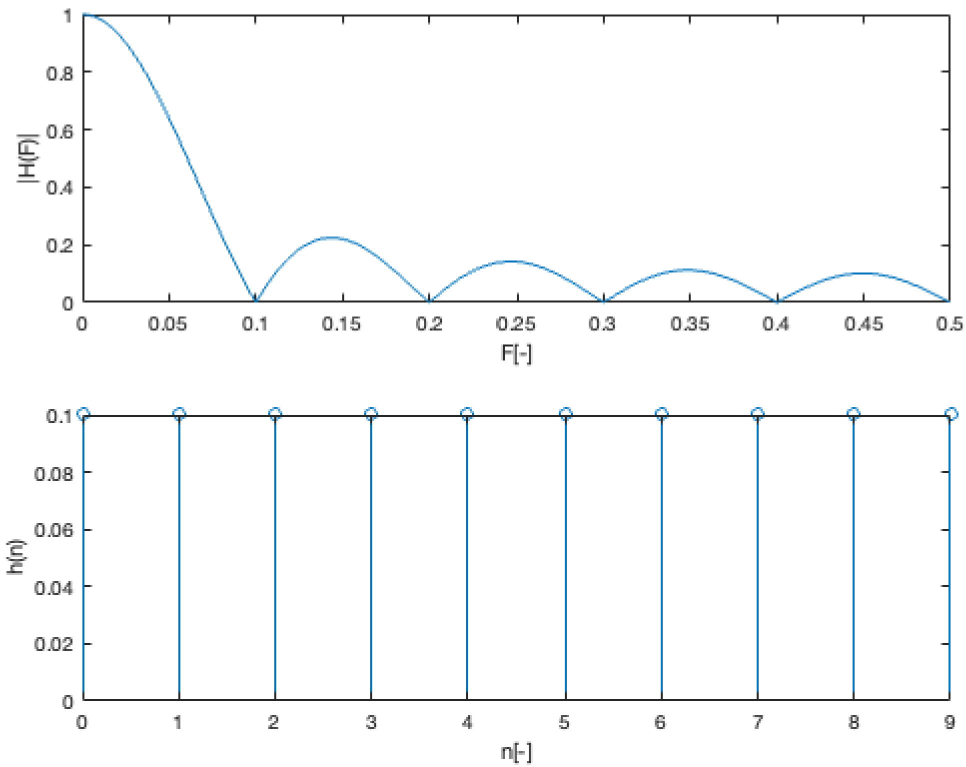

```
Lambdamax = 259.2701
Lambdamin = 7.8255e-18 - 4.8146e-17i
```

Filtering Random Process

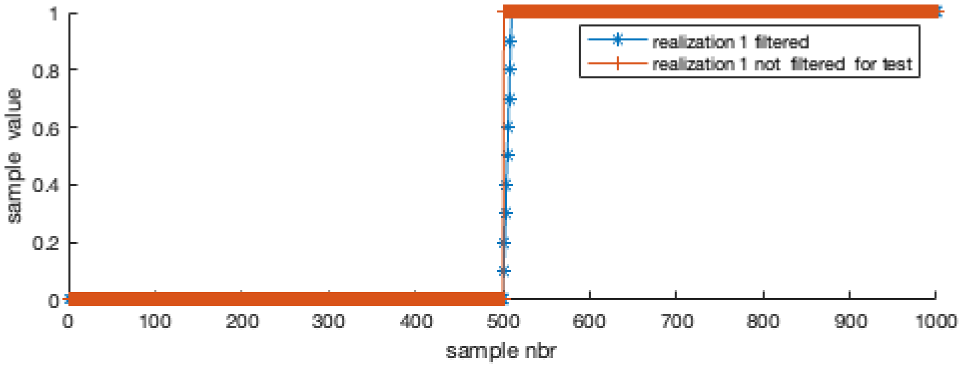
We consider a Gaussian noise as random process wn with variance $\sigma_w^2 = 1$ and we apply a discrete filter to this process, the filter is defined by his transfer function $H(z)$, and in our exemple is FIR 10 tap MA filter :

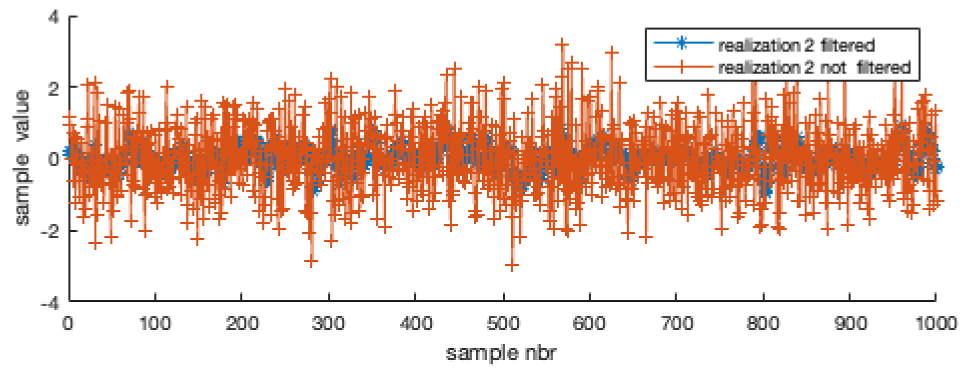
```
Num = 1x10
    0.1000    0.1000    0.1000    0.1000    0.1000    0.1000    0.1000    0.1000 ···

Den = 1x10
     1     0     0     0     0     0     0     0     0     0

num/den =

   0.1 z^9 + 0.1 z^8 + 0.1 z^7 + 0.1 z^6 + 0.1 z^5 + 0.1 z^4 + 0.1 z^3
   + 0.1 z^2 + 0.1 z + 0.1
   -------------------------------------------------------------------
                                   z^9
```

Compute the impulse response over the N samples and the frequency response .



Now we filter the random process wn with this filter to obtain the process zn :

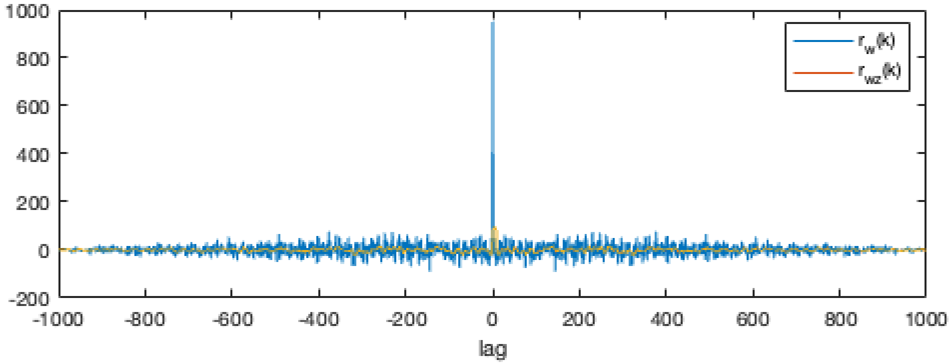




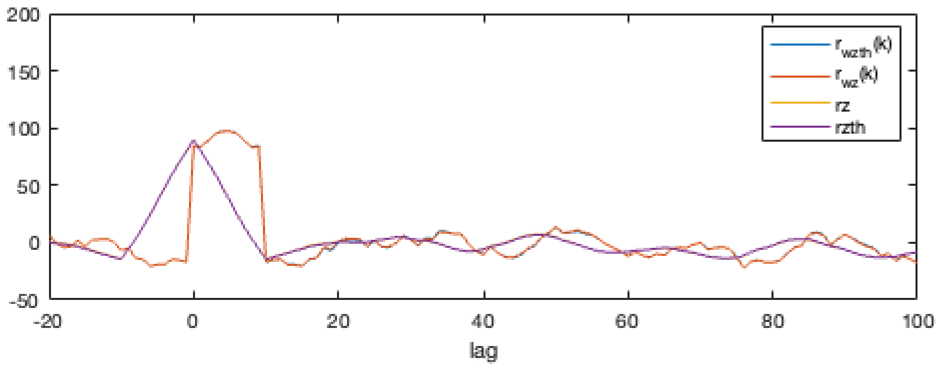
Compute the autocorrelation of the process wn and the process zn and the intercorrelation between wn and zn an test if :

$r_{wz}(k) = r_w(k) * h(k)$ and

$r_z(k) = r_w(k) * h(k) * h(-k)$



$r_w(k)$
$r_{wz}(k)$
lag



$r_{wzth}(k)$
$r_{wz}(k)$
rz
rzth
lag