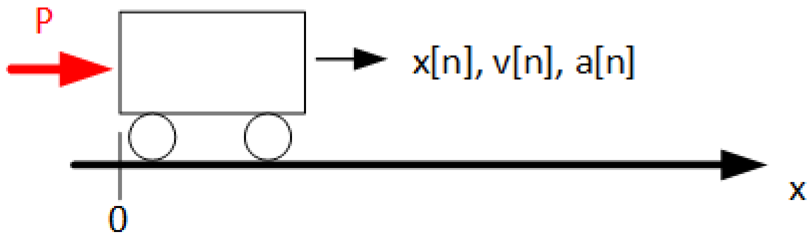
Chapitre 7 : Optimum filters

Ex7.6 : Kalman filter - Truck on rail

Consider a truck on frictionless, straight rails. Initially, the truck is stationary at position 0, but it is buffeted this way and that by a random uncontrolled forces with non zero mean (trend). We measure the position of the truck every Δt seconds, but these measurements are imprecise



We want to maintain a model of where the truck is (position $x[n]$) and what is its velocity ($v[n]$) of a truck every Δt [s].

$$X[n] = \begin{bmatrix} x[n] \\ v[n] \end{bmatrix}$$

The truck is pushed by a uniformely random force which creates a $\mu_a = 2 \left[\frac{m}{s^2}\right]$ mean accelaration with a standard deviation of $\sigma_a = 2 \left[\frac{m}{s^2}\right]$. The sampling period is 100 [ms].

The position of the truck is measured with a high noise with a standard deviation of $\sigma_v = 100\ [m]$

Generate trajectory

Parameters

Based on the description, the following parameters are defined:

```
param = 1×5 table
```

	dt	AccelerationMean	SigmaA	SigmaV	N
1	0.1000	0.5000	2	100	1000

Generation

We can write the motion equations:

$$\begin{cases} x[n] & = x[n-1] + v[n-1] \cdot \Delta t + \frac{1}{2}(\Delta t)^2 a[n-1] \\ v[n] & = v[n-1] + \Delta t \cdot a[n-1] \end{cases}$$

They can be written in matrix shape:

$$\begin{bmatrix} x[n] \\ v[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x[n-1] \\ v[n-1] \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t \\ \Delta t \end{bmatrix} a[n-1]$$

or in compact shape:

$$\boldsymbol{X}[n] = \boldsymbol{F} \cdot \boldsymbol{X}[n-1] + \boldsymbol{B} \cdot a[n-1]$$

where a[n] is a uniformy distributed noise with non nul mean μ_a and variance of σ_a.

And the only the position of the truck is measured.

$$z[n] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x[n] \\ v[n] \end{bmatrix} + \mathrm{vn}[n]$$

Or in compact shape:

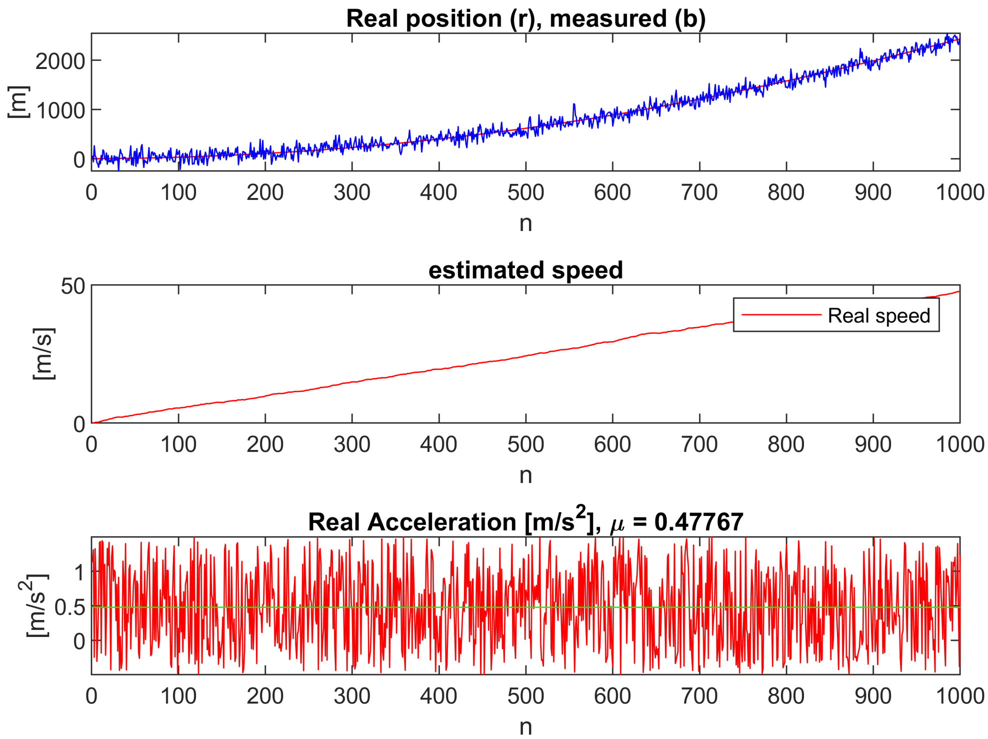
$$z[n] = \boldsymbol{H} \cdot \boldsymbol{X}[n] + \mathrm{vn}[n]$$

Where vn[n] is the measurement noise which is normally distributed with zero mean and a variance of σ_v^2.

```
F=
    1.0000    0.1000
         0    1.0000

B=
    0.0050
    0.1000

H=
     1     0
```

Generation

Based on the previous equations, the truck position is generated.



Kalman Filter

Kalman matrices

The previous already defined the matrices **F**, **B**, and **H**.

Only the covariance matrix for the process noise and the measurement are missing. The covariance matrix of the process is :

$$Q_w = G \cdot G^T \sigma_a^2$$

Where G is the input-model for the unknown input. For this example, the matrix **G** is equal to **B**.

Finally, the covariance matrix of the measurement noise is:

$$Q_v = \sigma_v^2$$

```
Qw=
    0.0001    0.0020
    0.0020    0.0400

Qv=
       10000
```

Inital condition

And the initial conditions are defined as null as the truck is at a known position.

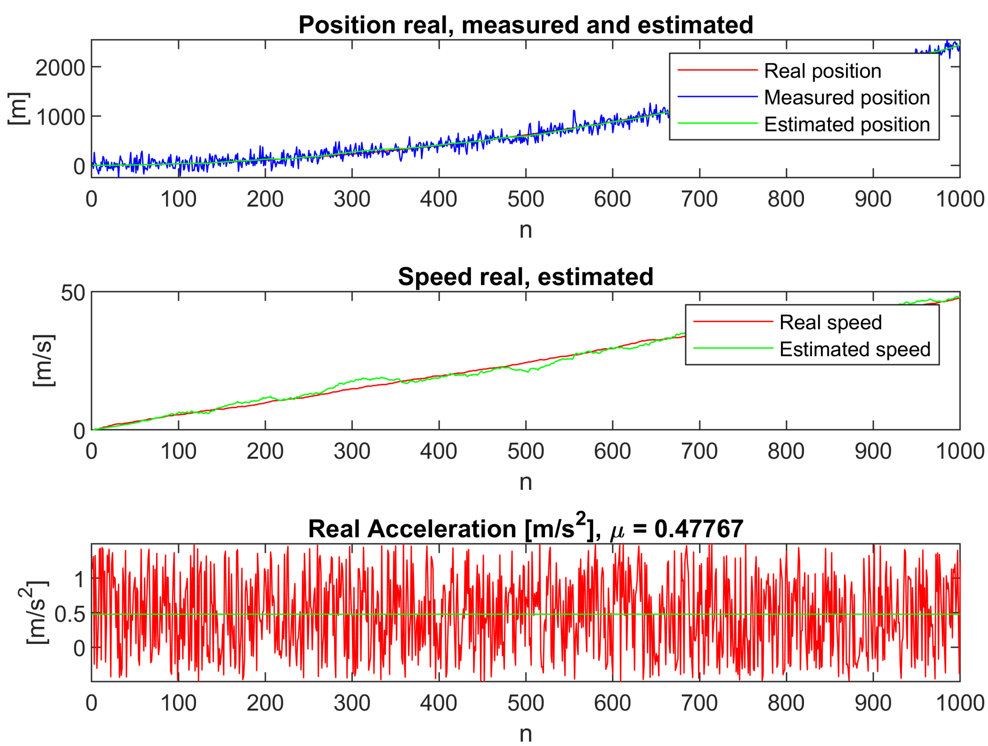
$$\hat{X}[0|0] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$P[0|0] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

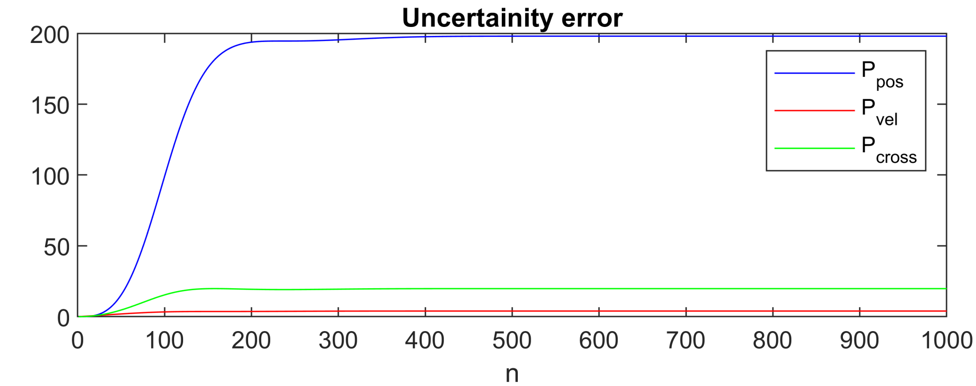
```
Xhat0=
     0
     0

PInit=
     0     0
     0     0
```

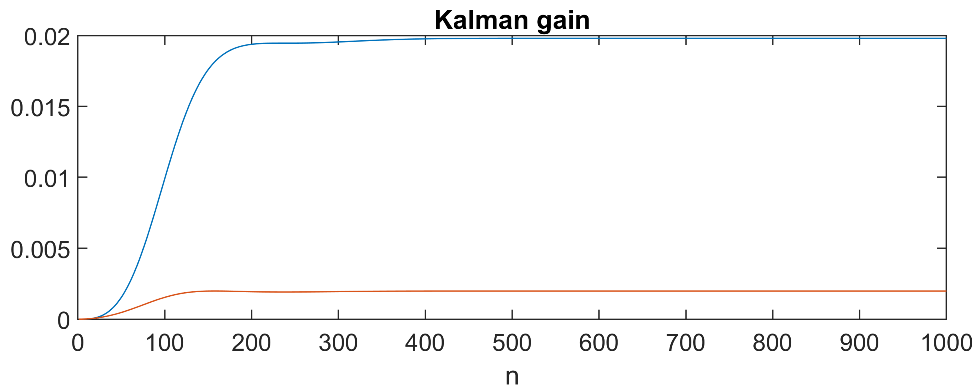
Apply Kalman filter

Finally, the kalman filter can be applied. It gives the following estimation. The u input vector is simply a constant with the average mean value of the input acceleration.



The Kalman gain and the estimated errors accross time can be also shown.


Uncertainity error
200
150
100
50
0
0
100
200
300
400
500
600
700
800
900
1000
n
P_{pos}
P_{vel}
P_{cross}


Kalman gain
0.02
0.015
0.01
0.005
0
0
100
200
300
400
500
600
700
800
900
1000
n