

Data Warehouse Systems

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1 Definition

GI-Group Definition: “A Data Warehouse is a database which (from a technical point of view) integrates data from different (heterogeneous) data sources and (from an economic point of view) provides the user with this data for business analysis purposes. Frequently, but not necessarily, a historization of data takes place.”

Inman's Definition: “A data warehouse is a subject-oriented, integrated, time-variant and non-volatile collection of data in support of management's decision making process.” **Integrated** means the data is collected from multiple (separate) sources, and compiled into a single source of truth. **Time-variant** means that the data is accurate at the time it was compiled (ie. the transactions took place). This is very useful for observing changing trends. **Non-volatile** means the compiled data in the DWS is not changed (often), but only queried.

A DWS is a static copy of accumulated transaction data used for analysis.

Data Warehouses are not single products, but a system comprised of multiple interacting components, most of which are usually bought from third-parties.

1.1 Operational Data Stores vs. Data Warehouses

Operational Data Stores (ODSs) are used in the day to day transactions of a business. They are very fast, use a single data source, are used by multiple users concurrently and mostly perform small transactions (e.g. sales). Think of the database interacting with Point of Sale terminals in a supermarket.

Data Warehouse (DWs) are very large databases used to store accumulated data. They are usually only accessed by single users and even then mostly only for reads. Data is fed into DWs periodically (e.g. daily or weekly), and then usually not changed afterwards. This means that locks can be optimized differently for DWs than for ODSs. The access patterns are also very different from ODSs, with range queries being the norm.

A full comparison is given in Table 1.

ODSs are regular databases. DWs are larger, and optimized for range-queries and rare modifications.

ODSs are used for Online Transaction Processing (OLTP), and DWs for Online Analytics Processing (OLAP).

	ODS	DW
Data Sources	mostly only one	many
Data Volume	MB-GB	GB-TB-PB
Access	Single Tuple accesses	Range queries
Up-to-dateness	Up to date	(Possibly) outdated
Use	Input output by employees	Evaluation by analysts/managers
Number of users	Many	few
Response time	ms-s	s-min-h

Table 1: Comparison between ODSs and DWs

2 Reference Architecture

The goal of a reference architecture is to provide a fundamental, abstract, implementation independent visualization of the DW. It is useful for comparing DW models, systems and components and can be used for planning specifications and implementations of a DW system. A reference architecture should provide an overview of the operators (functions) and operands (databases, data), as well as the **data-flow** required for the functions and the **control-flow** required for the underlying processes.

A reference architecture is a model of a DW system.

2.1 Requirements of a DW architecture:

- **Isolation:** the DW should be independent of its data sources after the data has been imported.
- **Persistency:** after importing the data the DW should suffice as a permanent storage
- **Flexibility of use:** arbitrary evaluations should be possible
- **Scalability:** it should be easy to integrate new data sources over time
- **Efficiency:** repeating tasks should be easy to automate
- **Uniqueness** of data structures, access rights and processes
- **Orientation** of the system towards the analysis of data (ie. optimizations for range queries)

2.2 Static View of a DW Architecture

This is a view of the static components themselves, ie. the databases or the component extracting data from data sources. The system components include:

- **Interface components:** DW manager, Metadata manager
- **Databases:** DW, Base DB, Metadata DB
- **Functional components:** Monitor, Extraction, Transformation, Loading and Analysis components

The static view is (somewhat) equivalent to the white blobs in Figure 1.

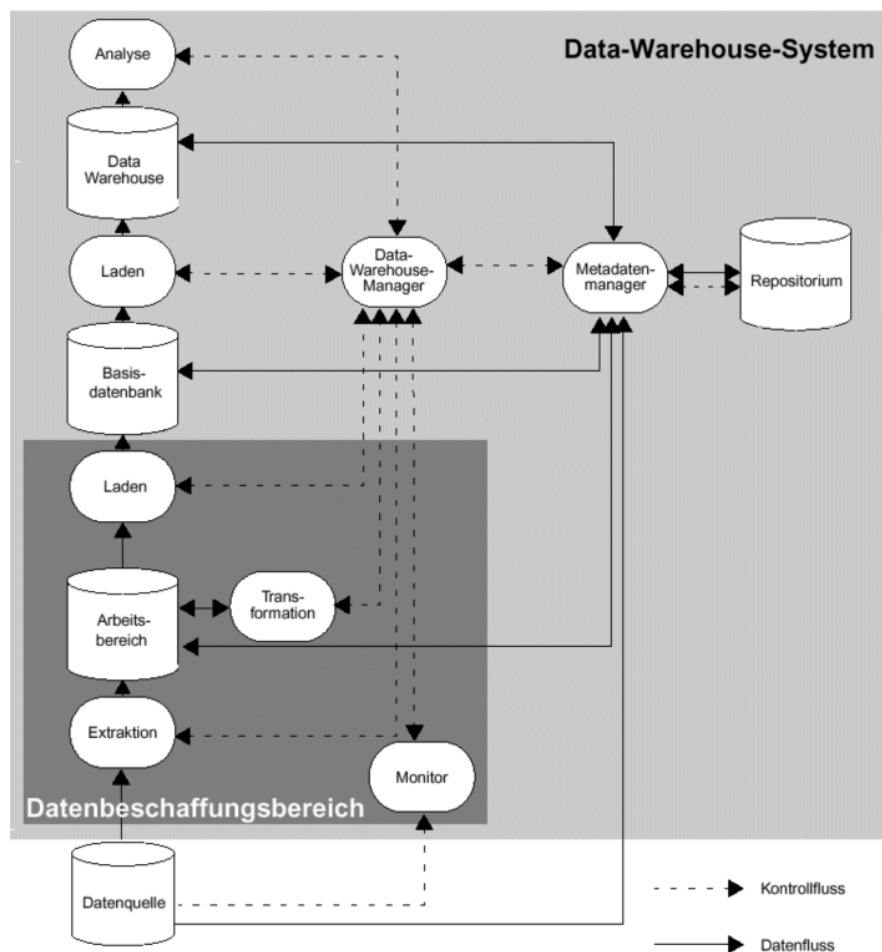


Figure 1: The data and control flow from a data source to the DW

2.3 Dynamic View of a DW Architecture

The dynamic view contains the data flow (comprised of main- and metadata flow) as well as the control flow. It is depicted by the arrows in Figure 1, with the solid arrows being data-flow and the dashed arrows being control-flow.

2.4 Components of a DW System

All the DW components do exactly what their name suggests. Figure 1 is in principle enough to fully understand Section 2.

The **monitor** handles the extraction from the data source(s), sometimes filtering out irrelevant data. Relevant data is extracted into the (temporary) **working area**, where it is *transformed* (cleaned, integrated, etc.). To reduce potential consistency issues caused by changing dimension data (e.g. product names changing), the data can then be loaded into a **base database**. This database contains

the raw transformed data, which needs to be combined with metadata to be meaningful. The base database is application independent and modelled based on the relations in the data. Finally, the data can be combined with metadata and loaded into the **data warehouse** which is modelled specifically for the application (analysis). The metadata is kept in a separate metadata **repository**.

2.5 Data Sources

The selection of data sources for the data warehouse is a defining factor for the quality of the finished data warehouse. Sources must be selected based on their **relevance** and the **quality of their data**. Factors for relevance are:

- Purpose of the data warehouse
- Availability of the source data (technical, organisational, legal)
- Cost of acquisition

Factors for quality are:

- Correctness
- Consistency
- Completeness
- Comprehensibility (Metadata, documentation)

Data sources can also be classified based on a number of characteristics; examples include origin, time, usage (metadata, base-data), ...

2.6 Control Components

Important control components are:

The **data warehouse manager** handles initialization, control and management of all functions and other components. It is the main control component and manages the entire work- and data-flow.

The **metadata manager** does exactly what the name says. It manages metadata, including start and end dates of validity (important for renames and the like) and links between the data warehouse manager and the metadata repository.

Monitors are used (one per data source) to monitor for updates and extract new data. They can either directly feed relevant data to the work area, or simply notify the manager of updates and wait for scheduled extraction. Techniques for extracting are *trigger-based* (periodically, ...), *replication-based* (store changed tuples in special relation), *timestamp-based*, *log-based*, *snapshot-based* (operate on deltas, high implementation effort but sometimes only possibility for legacy systems).

2.7 Work Area

It is a temporary storage area required for the process of data transformation, and the central storage for **ETL-components**.

2.8 ETL-Components

ETL stands for extraction, transformation and loading, and describes the components needed in order to fill the data warehouse with data originally from the data sources. As the data sources are heterogeneous (e.g. different currencies, inconsistent schemas, ...) the data needs to be cleaned and annotated with metadata in order to be useful. This cleanup is done in the ETL process.

Extraction Component

Controls the periodical transmission of source data to the working area. The extraction time determines the analysis accuracy and depends on the analysis goals. Examples for times the extraction can take place are:

- periodical
- on demand
- event-driven
- immediately on updates

The extraction strategy dictates the extraction techniques.

Transformation Component

Transformation is the process of reshaping the schema (column names, ...) as well as cleaning the data and unifying data types, currencies, unit formats (times, ...) etc.

During this process the data is also checked for integrity violation, illegal values (consistency and plausibility checks), redundancy, incomprehensible and inconsistent values, as well as missing and NULL values. All these checks are part of the code cleanup.

The schema transformation that is done by this component is necessary due to a heterogeneity of the data sources. Because different data sources have different views, legal requirements or simply different models their schemas can vary wildly. Schema transformation requires expert knowledge and is hard to automate, as a transformation script has to be created for every single data source. It also requires the setup of the metadata database.

Figure 2 shows the individual steps of the transformation process.

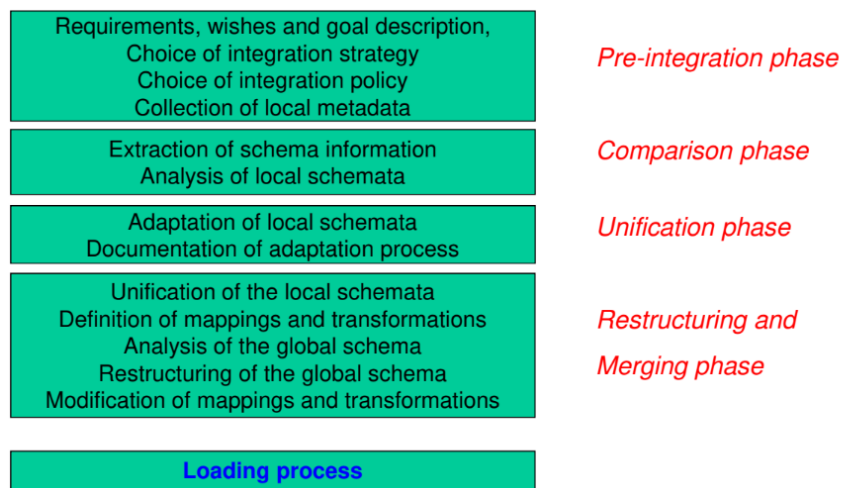


Figure 2: The steps of the transformation process

Loading Component

This component takes care of loading the transformed data into the database(s). If there is no base database the analytics specific data is transmitted to the data warehouse from the working area. If such a database does exist, the cleaned data is loaded into it, and a version of the data made specifically for analysis is sent to the data warehouse. The analysis version might include pre-aggregated results.

2.9 Base Database

The base database acts as a **central data storage**. It stores the cleaned data at the highest resolution (lowest granularity) while staying completely application neutral. It functions as a buffer between the data sources and data warehouses.

In a situation where there are multiple DWs the base database functions as the **single source of truth**, thus saving the DWs from gathering their data from each data source themselves. This reduces complexity from $\mathcal{O}(m \cdot n)$ to $\mathcal{O}(m + n)$.

The documentation of the base database must ensure traceability for the entire data flow from the sources to the base database. This includes the ETL process and points of possible human intervention in case the automated system fails. The documentation of the ETL processes must be **exact** so they can be reproduced in the next iteration.

The metadata repository as well as the base database must be **available** for the DWs to function properly if they are the single source of truth.

2.10 Data Warehouse

The DW is specifically organized for analysis. This includes ordering along multiple dimensions. In order to speed up this sorting, DWs have moved to multi-dimensional representations of their data.

Special requirements for a database management system (DBMS) of a DW include:

- bulk loading
- access interface for analysis tools
- Optimization and tuning for frequent queries (indices, materialized views, ...)

2.11 Data Marts

Data marts provide a partial view of the DW. They can either access the DW for data (requiring a permanent connection) or save their relevant part themselves. Both variants come with a tradeoff of availability versus integrity.

Splitting the DW into multiple data marts allows departments to operate independent of each other, as well as distributing the workload and required storage.

Data marts can be categorized by geography, organisation or function. While they can be kept independent from the main DW by gathering the data directly after the ETL process, this causes a lot of integrity problems and **should be avoided**.

2.12 Metadata Repository

The metadata repository contains a description of the **entire** DW system. This includes information about schemata, data-types, formats, but also the setup, maintenance and administration of the DWS itself. It is vital for understanding the data, and without it, the base database is essentially worthless.

The repository is managed by the metadata manager component.

2.13 Analysis tools

Analysis tools are also called Business Intelligence (BI) tools, and operate on the DW. They can be classified by their intended use into reporting tools, OLAP tools (used for interactive data analysis), and data mining tools (used for finding patterns in the data).

OLAP systems

These systems are used for online analytics. Cobb (inventor of database management), defines twelve rules for a good OLAP system, which are extended in the lecture slides to 18:

1. Conceptual multi-dimensional view
2. Transparency for access from multiple data sources (Transparency in this case means that the extra steps taken for the access to a different data source are invisible (transparent) to the user, not that the access is easy to understand)
3. Flexible access possibilities
4. Same response time in report creation (in regard to queries on different dimensions)
5. Client-Server architecture
6. Equality of dimensions
7. Adaptive administration of sparse data cubes
8. Multi-user operation
9. Unlimited, cross-dimensional operations
10. Intuitive data handling
11. Flexible reporting
12. Unlimited number of dimensions/aggregation levels
13. Easy data integration
14. Support of different analysis models
15. Separation of analysis and operative data
16. Separation of storage areas
17. Differentiation between NULL and non-existent values
18. Handling of missing values

relational	multidimensional
Conceptual schema	Conceptual schema
ER	mE/R, mUML
Logical schema	Logical schema
Relations	Dimensions, Cube
Physical schema	Physical schema
Index structures, B-trees	Relations(Fact table, ...), MD-Structures

Figure 3: A comparison of relational and multi-dimensional modeling methods

3 Multi-dimensional Data Modeling

As we usually want aggregations along multiple dimensions in DWSs, we want our (conceptual) model to represent this. Therefore we need to adapt existing modeling methods for multiple dimensions. While our representation of the data is multi-dimensional, internally all data is still stored in relational databases, so our adapted modeling methods are very related to traditional relational modeling methods. A comparison can be found in Figure 3.

Multi-dimensional modeling works just like regular modeling. If you understand ER diagrams, creating mER diagrams is easy, just add multiple hierarchies. An example can be found in Figure 4.

3.1 Schemas and Instances

A classification schema (of a dimension) is a set D of classification levels: $(\{D_0, \dots, D_{top}\}, \rightarrow)$. Together with their dependency operator \rightarrow they become a partially ordered set (the partial order allows for multiple parallel hierarchies). Dimension elements are occurrences of the lowest classification level D_0 and occurrences of higher classification levels are hierarchy nodes. Three examples for a classification schema can be found in Figure 4. Every path from D_0 to D_{top} defines a classification hierarchy, and an instance of a dimension is the set of all classification hierarchies.

3.2 Data cubes

As the highest number of dimensions we can easily visualize as a geometrical shape is three, data cubes are a common representation of higher-dimensional data. The cube consists of data cells which contain 1 to n measures, and the location in the cube along its three axes represents the values along three dimensions (e.g. geographical, time and product).

Cube Schemas and Instances

A cube schema $W[G, M]$ consists of the granularity G and the set of measures M . For example, if we were to store the sales of an article per store and per day, G would be the set of article, store and day. Our measures M would be the sales and maybe additionally the turnover.

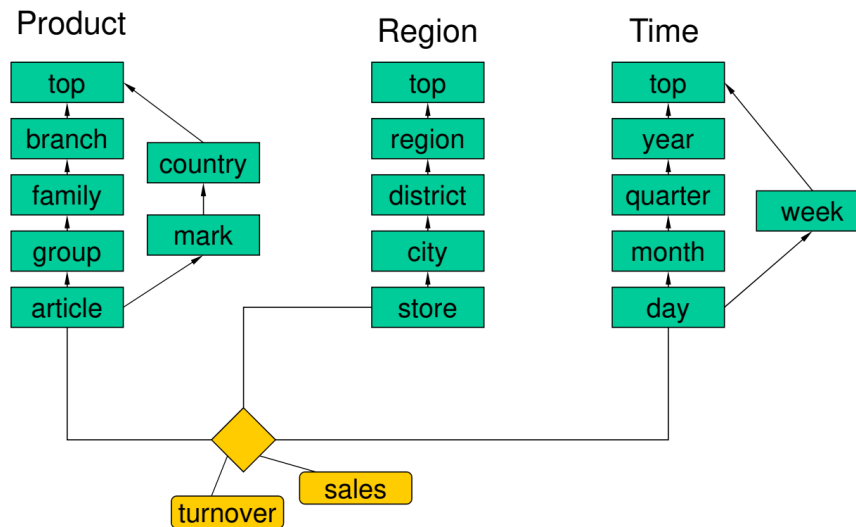


Figure 4: An example for a mER diagram with multiple classification schemes

An instance of a cube W contains all cells from the definition domain of the cube $W = \text{dom}(G) \times \text{dom}(M)$. In general, not all cells have a value. Figure 5 shows an instance of a cube including classification hierarchies.

Features of Measures

Measures have a name, domain and aggregation type. The domain is roughly equivalent to their data type, but restrictions are possible (e.g. negative cost).

Aggregation types are:

- **FLOW**: arbitrarily aggregationable (Turnover, Sales)
- **STOCK**: not temporally summable (Stock)
- **Value per unit (VPU)**: not summable (Price, taxes)

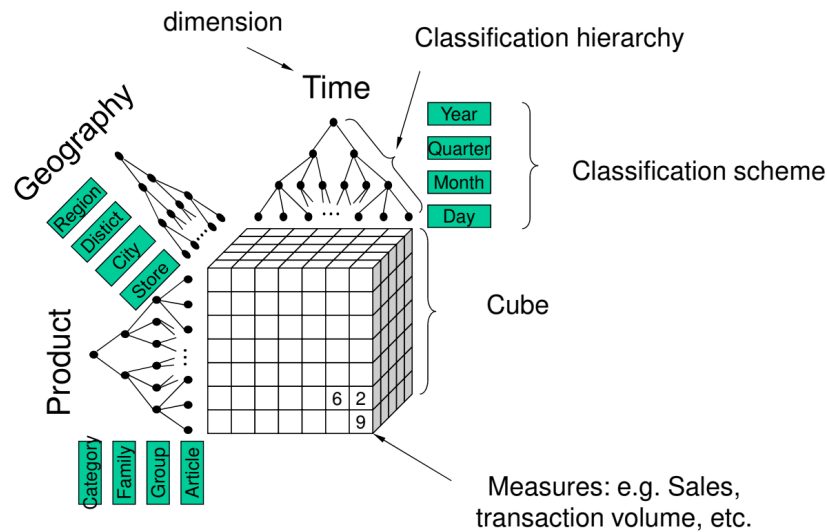
Multi-dimensional Operations

By representing our data as a cube aggregations become easy to understand operations:

- **Pivoting**: rotate the cube along one or multiple axes
- **Roll up, Drill down, Drill across**: A roll up is the aggregation of data along one or multiple axes, a drill down is the opposite. A drill across performs analysis over a spectrum of dimensional values.
- **Slice and Dice**: this is just a visual representation of filtering

Aggregation

Aggregation is a change of granularity through some function. Aggregation functions map a set of values to a single one, and can be done via cumulation (sums, averages) or ranking.



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Figure 5: An instance of a data cube

Summability of measures plays a huge role in aggregation, as it dictates which types of analysis are and aren't possible. Disjunctivity, Completeness and Type Compatibility are necessary for aggregation. Disjunctivity and Completeness are themselves necessary attributes of classification hierarchies. Completeness means that all elements must be contained in the hierarchy, and Disjunctivity means no element can be contained in two classifications in the same hierarchy. The type compatibility is dependent on the aggregation type of the variable in question.

3.3 Schema Creation and Updates

According to Kimball this should be done in four steps:

1. Selection of business process
2. Selection of the granularity
3. Selection of the dimensions (useful for functional dependencies)
4. Selection of the measures

In my opinion one should select the dimensions before the granularity, but maybe that's just me

Schema updates are very tedious, as the metadata has to be kept consistent even with the schema changes. The approach by Chameni and Stock suggests versioning the classification hierarchies and storing the timestamps in a validity matrix. The schema can also evolve, with most of it staying the same as the old version. **This is dangerous territory.**

4 Implementation of the MD Model

We need to find a way of storing our MD model. We could use a classical relational database, which would be very scalable, but we would have to transform all queries into relational form.

Alternatively we could create a multi-dimensional database, which would have easier querying, but would scale very badly due to empty cells.

We could also combine both approaches, giving us a tradeoff between the two.

4.1 Relational Storage

This requires three layers: an RDBMS (relational DBMS), storing the data and performing the queries, an OLAP server transforming the OLAP operations into SQL, and a presentation layer, which the user interacts with.

The problem is we have to find a suitable storage method while keeping the cardinality and consistent performance over all dimensions. As a start, we define every cube cell via a single tuple. This tuple contains the measure data and references to the dimension data. There are multiple ways of storing the dimension data.

Snowflake Schema

The snowflake schema stores dimensional data in a tree of references. An example is shown in Figure 6. This schema is fully normalized, and has thus no memory overhead. However, queries require joins over a lot of tables, making them slow (some DBMS optimizations can help here).

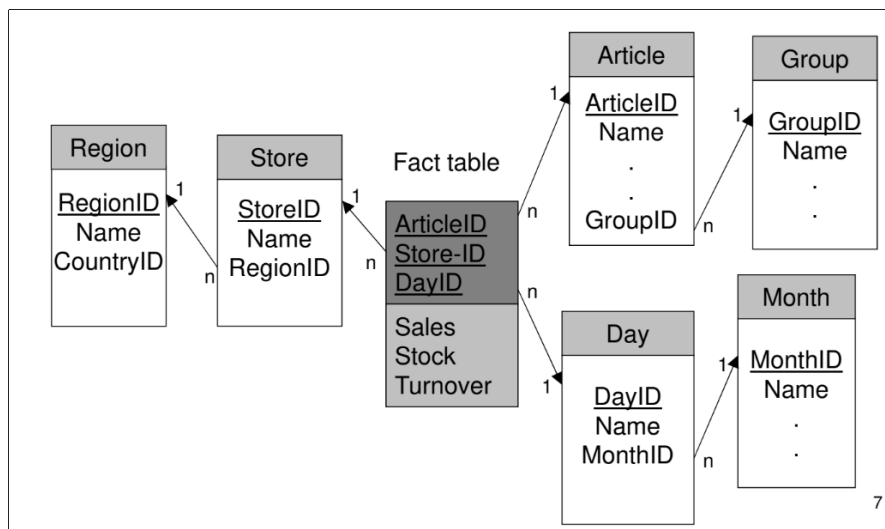


Figure 6: An example of a snowflake schema (the 7 is a page number)

Star Schema

The star schema is basically a snowflake schema that is not normalized. The data in the dimension tables is potentially very redundant, which can lead to increased memory consumption. The joins in the query are faster (because there are fewer) with the star schema. An example is shown in Figure 7.

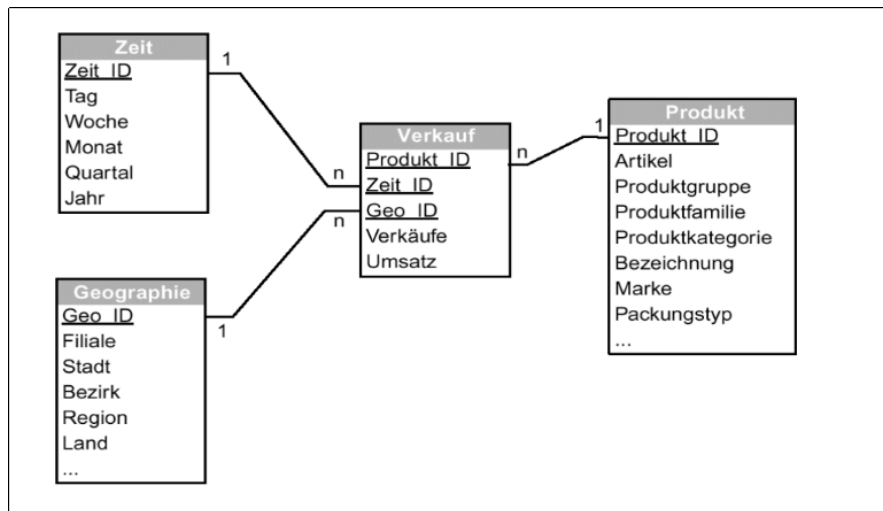


Figure 7: An example of the star schema

If multiple cubes are necessary, just reuse and combine parts you already have. This leads to a galaxy representation

4.2 MD Queries

A common pattern is the star join pattern. You simply join the fact table with the dimension table through the indices stored in the fact table. Because grouping along multiple dimensions is a frequent use-case in DWs, modern DBMSs have special methods for that. I will explain them in the order that I found easiest to understand. All of this can be done with simple group by clauses and unions, but that is an absolute nightmare for larger numbers of tables, and should be avoided where possible.

Grouping Sets

A grouping set is used in the **GROUP BY** clause of a query. It is a set of sets defining the different groupings. Figure 8 shows a very good example of how they work (for anyone interested it was taken from the postgresql manual on grouping sets).

A grouping set of $((a), (b, c), ())$ will aggregate over rows with the same value in column a , then over all rows with identical b and c columns, and then over all. It will display all results in the same table.

Rollup

A rollup of (a, b, c) is equivalent to a grouping set of $((a, b, c), (a, b), (a), ())$. The rollup operator always adds the aggregate over all rows. If this is not desired, the row should be filtered.

```
=> SELECT * FROM items_sold;
brand | size | sales
-----+-----+-----
Foo   | L    | 10
Foo   | M    | 20
Bar   | M    | 15
Bar   | L    | 5
(4 rows)
```

```
=> SELECT brand, size, sum(sales) FROM items_sold GROUP BY GROUPING SETS ((brand), (size), ());
brand | size | sum
-----+-----+-----
Foo   |      | 30
Bar   |      | 20
      | L    | 15
      | M    | 35
      |      | 50
(5 rows)
```

Figure 8: An example of grouping sets

Cube

The cube operator calculates the power set of the supplied column set, and uses that as a grouping set. For example, a cube of (a, b, c) becomes a grouping set of $(a, b, c), (a, b), (a, c), (b, c), (a), (b), (c), ()$.

A **ROLLUP** performs a rollup along a single dimension, a **CUBE** does so along all combinations of dimensions.

Other useful functions

The `GROUPING()` function tells us whether a given column was aggregated, and if multiple columns are supplied, it tells us the aggregations in a bitmask fashion.

There also usually exist nicer functions for handling date data-types.

The OVER Clause

This clause allows us to include aggregations with the unaggregated data. Figure 9 shows an example. `OVER` can also be used for window functions. This is actually very well explained in the `POSTGRES` documentation, and a very nice example is shown in Figure 10. Window functions can also be used for ranking with the `RANK()` and `DENSE_RANK()` functions.

5 Versioning

Versioning can be very important in DWS systems. For organizational or legal reasons, deleting entries can sometimes not be an option. In those cases we need to add some measure of declaring a tuple invalid.

There are in general four methods of versioning:

- Overwriting old values
- Adding a version number
- Tuple timestamps

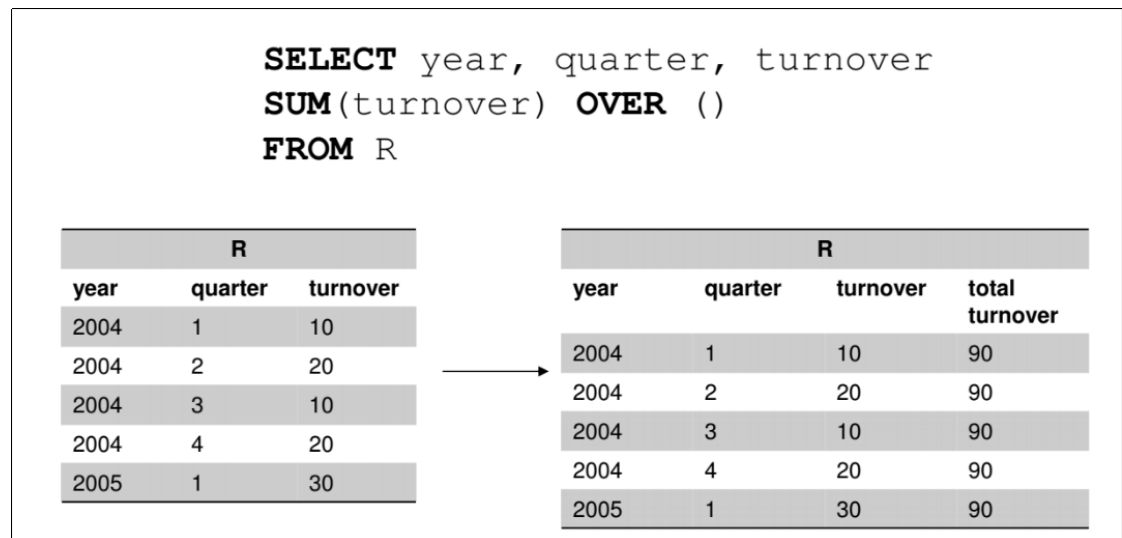


Figure 9: An example of the OVER clause

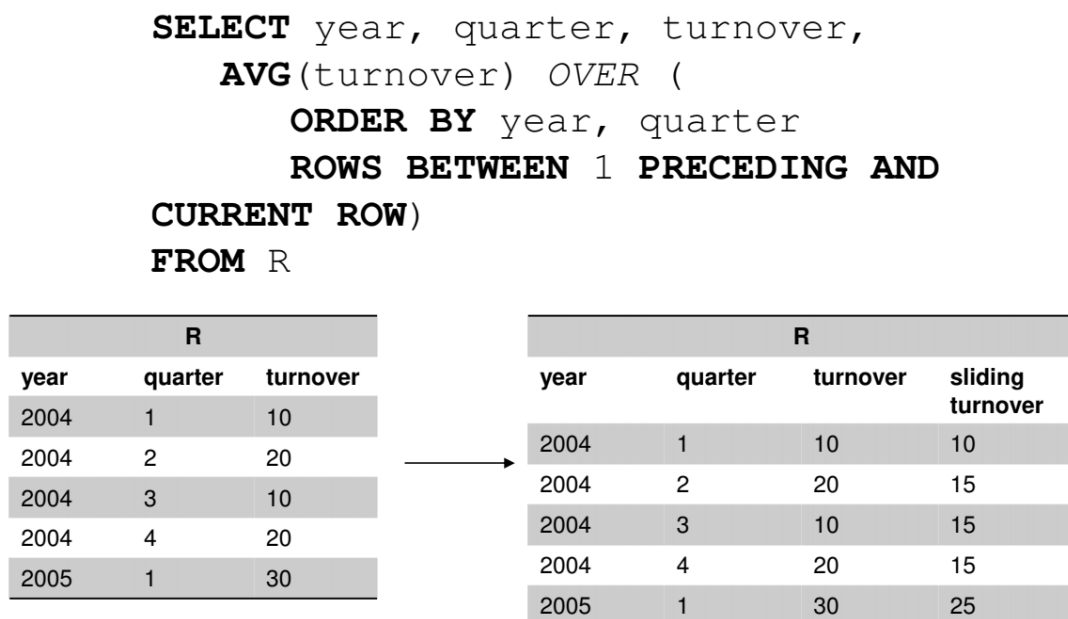


Figure 10: Using OVER for sliding window computation

- Attribute timestamps

Where overwriting old values leads to data loss and is thus sometimes unacceptable.

Attribute timestamping adds a start and end validity date to every attribute (leading to non-first-normal-form tables), while timestamping saves a start and end date for every tuple. Because we need the beginning and end time of both validity and the transaction, we need a total of four additional columns per timestamped attribute or tuple. Adding timestamps to tuples can lead to significant memory overhead due to the redundant saving of unchanging tuple parts.

5.1 Schema Updates

Sometimes changes to the schema become necessary. These are **a lot harder** to handle than instance updates, because they could force us to remodel the entire DW. The two main methods of handling them are schema versioning and schema evolution. For schema evolution old queries are still possible, but for a new schema this might not be the case. The steps necessary for adapting a schema to certain changes are shown in Figure 11.

Operator	Schema evolution (no new relations)	Schema versioning (new relations or adaptation)
<i>insert classification-level</i>	Add new tuple on meta-level; Adaptation of the instances of the object level	Add new tuple on meta-level, Adaptation of existing tuples on metalevel
<i>delete classification-level</i>	Delete existing tuple on meta-level; Adaptation of the instances of the object level	Change existing tuples on meta-level
<i>insert measure</i>	Add new tuple on meta-level; Modify instances	Add new tuple on meta-level
<i>delete measure</i>	Delete existing tuple on meta-level; Instances are deleted automatically	Change existing tuple on metalevel
<i>insert dimension</i>	Add new tuple on meta-level; Instances change	Add new tuple on meta-level
<i>delete dimension</i>	Delete existing tuple on meta-level; aggregate instances	Change existing tuple; adjust existing tuple on meta-level
<i>modify granularity</i>	Change existing tuple on meta-level; aggregate/deaggregate instances	Add new tuple on meta-level; change existing tuple

Figure 11: Necessary steps for schema updates

6 Multi-dimensional Storage

The best abstract model for storing things in MD fashion is a cube. However, we do not only consider a three-dimensional cube, but an n dimensional hypercube. For visualization purposes we can reduce it to three dimensions, and our operations will work on other dimensions just the same. We now need two (different) data structures for the dimensions (for addressing) and for the cube itself.

6.1 Dimensions

Abstractly viewed, a dimension is an ordered list of values (along that dimension). These values have a fixed domain. By ordering the dimension values we can create a mapping from dimension value to index and vice versa. If we know all dimension values of a cell, we can then calculate its position in memory. The order of the dimensions can be chosen arbitrarily and is fixed after setup of the data cube. It has far reaching consequences concerning queries on the cube.

There are multiple ways for storing complex records in cubes:

- **Complex cells:** These store the entire tuple of measures in the cell designated by the dimension values
- **Multiple cubes with flat cells:** Create one cube per measure ("Multi-Cube" approach)
- **Measure dimension:** Push (some) measures into the dimensions, requires all measures to be of the same data type

While single cube approaches are easier to understand, they can create large memory overhead because of large areas (volumes) in the cube with NULL values. Multi-cube approaches limit this memory consumption, but require joins for data consolidation.

6.2 Classification Hierachies and Aggregations

An optimization approach is to store both the dimension values and all nodes of higher classification levels in the cube. This means that all cells in a certain category are aggregated (for example during updates) and this aggregation is also stored in the cube, reducing the query time for future aggregations. Such aggregations can also be done only for parts of the cube.

Unfortunately, this increases the size of the cube, which may or may not be an issue. For small cubes it is better to just calculate these aggrrgations at query time, while for larger cubes this can become vital. Storing aggregations only for some hierarchy levels (e.g. every second) can create an optimal middle ground between both approaches.

6.3 Partial and Virtual Cubes

Query results for all queries on a cube are also cubes, thus allowing the same optimizations and techniques. Also, views created on the cube are themselves (virtual) cubes and allow for the same processing as real cubes.

6.4 Storing MD-data in MD-arrays

A more direct approach to saving MD-data is saving them in arrays. For any amount of dimensions we need to linearize the data for storage. This makes storage very compact, compared to tables. A large part of the performance stems from the order the dimensions are linearized in. Figure 12 shows an example of such a linearization.

Calculating the index based on the dimension values is then very simple and follows the formula:

$$I = \sum_{i=1}^n v_i \prod_{j=2}^i |D_{j-1}| \quad (1)$$

So for three dimensions we get $I = v_1 + v_2 \cdot |D_1| + v_3 \cdot |D_1| \cdot |D_2|$. Index calculation can be sped up using the Horner Schema.

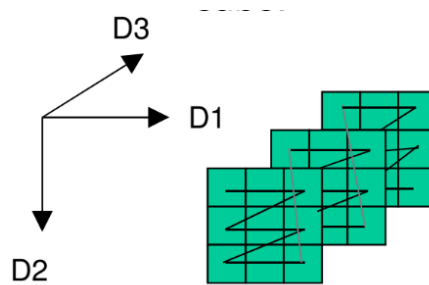


Figure 12: An example of MD-array linearization

Sparse vs Dense Population

The filling level is given by the number of filled cells, divided by the number of total cells. Dense population is given when the filling level is high, sparse population is given when it is low. For dense populations, array storage is more efficient than relational storage. For sparse populations, we can make some adjustments to the array saving method to greatly increase storage efficiency.

One method is to not store empty areas in sparsely populated arrays. If the empty cells would occupy an entire memory block, you can just skip it. This requires additional calculations for indices.

There is also the possibility of storing sparsely populated dimensions on another level than densely populated ones.

Limitations of MD-Storage

MD-storage methods do have some limitations. It can be expensive to maintain the full MD-array (due to null-values), and not saving NULL areas makes it more like a relational approach. Inserting new values can cause **a lot** of memory shifting in order to keep the order on indices consistent. There is also no current standard for MD-storage, as opposed to SQL, and all query languages are proprietary.

6.5 Target Figures

Target figures can be difficult to store in MD-arrays, as they are only known in spares quality (e.g. only per store per month), and need to be filled in later. We would however still be able to calculate with them in aggregations (e.g. prognosing yearly totals for the running year). This requires the DW to maintain the target figures itself, often with (goal) tracking. The target figures can also be stored along with actual measures in the cube cells.

6.6 Access Control

Access control is a tricky issue for OLAP systems, as we want to generate accurate reports without the user obtaining specific information he or she should not usually access. A user can also obtain specific cell values by using tracker queries and set algebra. An example of a tracker query is shown in Figure 13. Most systems only have a simple rights system which does not protect against tracker queries. A possible countermeasure would be to recognize tracker queries and forbid the user from accessing the results. This could however disrupt the workflow and make generating all desired reports impossible, due to queries resembling tracker queries by chance.

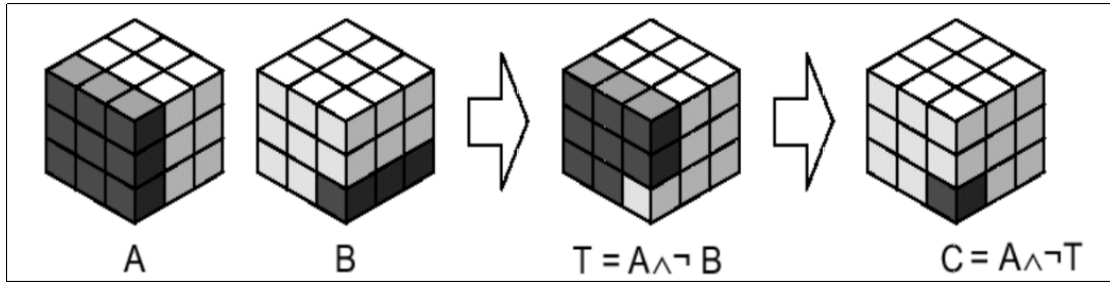


Figure 13: An example of tracker queries

7 Index Structures for the MD data model

As covered briefly in Section 6.4, we want our data structure to support certain operations efficiently. Because we cannot move the **entire** DW on inserts (and for other performance reasons), we cannot use native MD-arrays as storage method. As relational storage methods have become the norm, and with that highly optimized, they are also used for DWs. In order to use them efficiently for DW queries (range queries, aggregations, ...), we need to do some optimization on the index structure.

We want to store our data in a way, such that data which is in close proximity to each other in **any** dimension is in close proximity in memory. This greatly reduces the number of disk accesses we need for range queries, as it increases the likelihood of tuples being on the same memory block. As memory controllers access one memory page at a time anyway, having more tuples on the same page decreases query time.

Index structures can be classified as follows:

- **Clustering:** Clustering index structures keep neighboring tuples in close proximity in memory. We can further distinguish between tuple clustering (tuples are clustered on a memory page) and page clustering (pages are stored in tuple order). Page clustering is very expensive, as it requires page reordering; however, it allows for prefetching in queries. In non-clustering index structures the tuples are stored in random order.
- **Dimensionality:** States the number of attributes of the underlying relation used for calculating the index key.
- **Symmetry:** In symmetrical index structures the order of index attributes is not relevant for performance of the index structure.
- **Dynamic Behaviour:** The cost of updating the index structure on dynamic changes.

The following subsections will cover the index structures. I will not go into too much detail, but will rather provide a refresher. Most of the index structures are actually very intuitive, and most of them are very well explained by wikipedia.

7.1 B-trees

B-trees are used in regular OLTP databases. They provide a method for generating a **guaranteed balanced tree** with low depth. Insertion in B-trees is fairly simple. You add the new element where it would belong (by order of its primary key), and if the node is full, you split it, moving the middle element to the parent node. If there is no parent node, you create one. An example is shown in Figure 14.

Deleting from a B-tree is done by removing the element from its containing node. If the node is now too empty, we combine it with one of the neighboring siblings. This might cause a split and a reorganization of the index structure, which makes deleting from B-trees tedious. Luckily, this is a non-issue in DWs, as we never really delete anything.

B*-trees

In order to achieve greater in order traversal speed, we push the elements all into the leaves, with the internal nodes only acting as organizers. If we now store a reference to the next (and previous) leaf node in every leaf, we can get very high in order traversal speed.

Problems with B-trees

B-trees can only work with a single index. Even when combining multiple dimension values into a single index like in MD-arrays, we can only efficiently traverse them in a single dimension. B-trees are **asymmetrical**.

7.2 Grid-Files

Grid files are essentially a grid of references to buckets. The dimensions of the (n-dimensional) grid represent the domain of the dimension values in our DW. After calculating the position of the element in the grid via the dimension values, the element is stored in the bucket referenced at the target location. Because the buckets are limited in size, splitting might become necessary. This is done by splitting the grid row (or column or n-dimensional equivalent) into two halves, and creating a new bucket. All non-full buckets in the same row can stay as they were, as we can simply replicate the reference to them. This means that once we split a bucket in a row, all other buckets in the same row now have two references to them. Should one of them also become too full, we can create a second bucket without splitting the row again. If a bucket becomes empty we can simply combine it with the "neighboring" bucket, and store the same reference in two grid cells.

Row splitting in grid files is an expensive operation, as all other references have to be moved in memory for the grid file to stay performant. Apart from this performance issue, they offer great clustering and symmetry properties.

7.3 R-trees

R-trees are optimized for elements that cannot be accurately represented as a single point. It is mainly used for geographical objects (in 2-D, 3-D or n-D), but can be used for arbitrary multidimension data. They work as follows: Each leaf node approximates its represented element via its bounding rectangle. Each intermediate node represents its children via their bounding rectangle. These bounding rectangles are generally not disjoint, and so traversing multiple paths through the tree might become necessary for queries. Each node contains between m (lower bound) and M (upper bound) elements. Figure 15 shows an example of an R-tree and its 2-D representation. Because the branching in every level is at least m , the height of the tree is bound by the logarithm to base m .

Algorithms for R-trees

Searching is done by traversing the tree and selecting path(s) whose bounding rectangles contain the search point. For range queries the same thing is done but with intersections with the search rectangle.

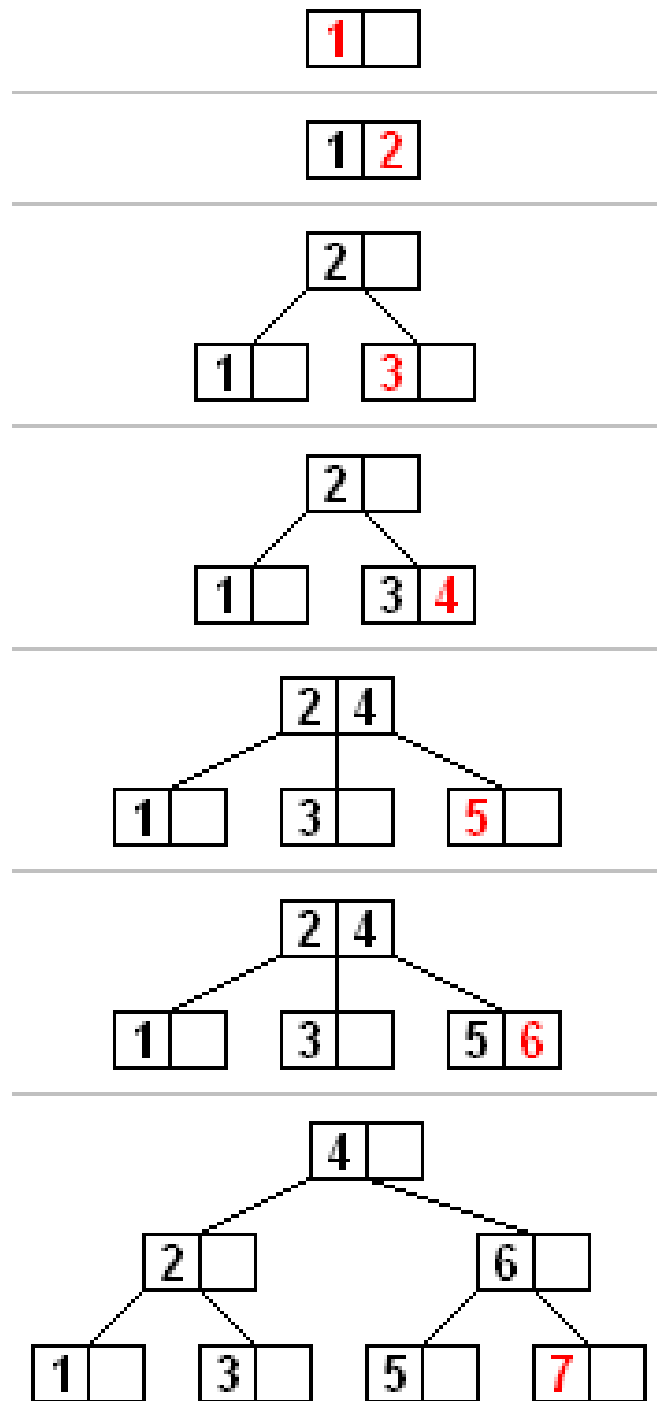


Figure 14: An example of insertion into a B-tree

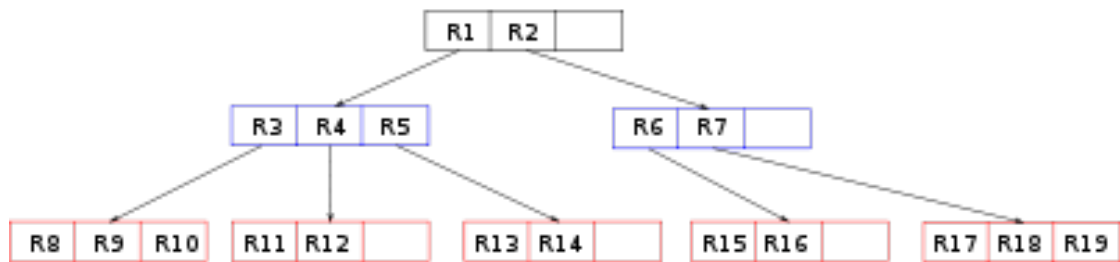
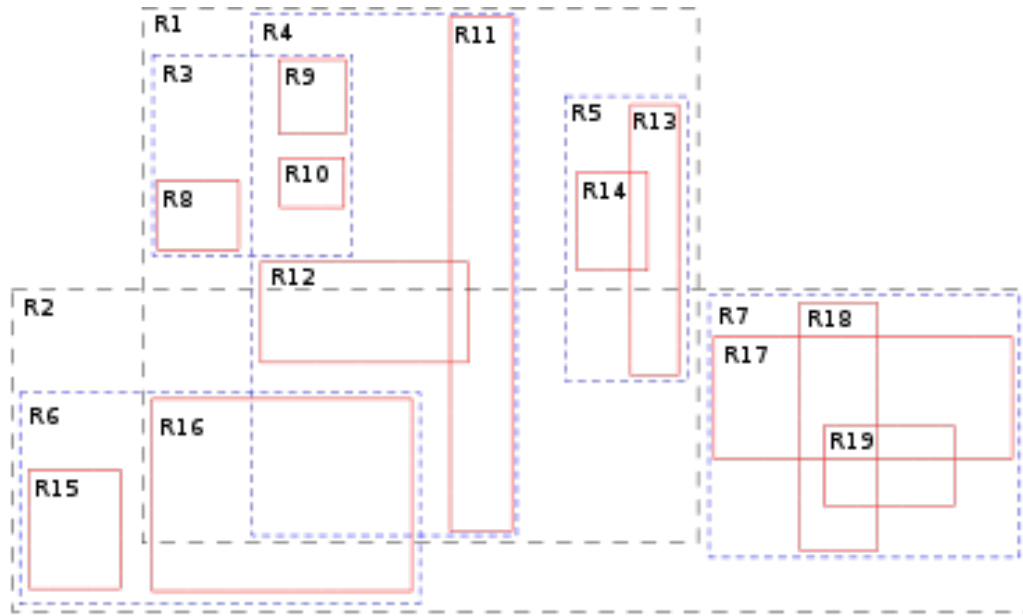


Figure 15: An example R-tree with $m = 2$ and $M = 3$

Insertion works almost the same way as in a B-tree. However, due to the potentially overlapping bounding rectangles, multiple candidates must be considered. The candidate which requires less increase in their bounding rectangle is chosen. Just like in B-trees, if a node is overfull, this causes a split. Splits are a bit more complex than in B-trees, and will be covered separately.

Deletion is done by simply removing the element from its containing node, and adjusting the bounding rectangle. If changes occur, they are propagated recursively upwards. Should an underflow happen we can combine sibling nodes. As opposed to B-trees, any siblings can be combined in R-trees. The combination with the smallest increase in bounding rectangle is chosen.

Splitting is a non-trivial problem, as finding the best way to split is expensive. There exists a quadratic algorithm which solves this. In the first step, all pairs of children are tested for their wastefulness, and the most wasteful pair is chosen as seeds for the new split nodes. (Wastefulness is calculated by the difference of the bounding rectangle area and the individual rectangle areas). This guarantees us a maximum distance between the seeds. Now we simply take a random other child and

add it to the node where it causes the lesser bounding rectangle increase.

There is also a heuristic linear cost algorithm, which works almost the same way but chooses the seeds differently. For each dimension, it records the normalized separation between the lowest high side and the highest low side. It then chooses the rectangles which have the highest separation along any dimension. The comparison is possible because the distances have been normalized (divided by the total length of the set along their respective dimension).

A very good graphic example can be found on the slidedeck for chapter 5, on pages 49 to 70.

Evaluation of the R-tree R-trees are a dynamic, height-balanced structure which is suitable for storing large amounts of data. If the data is distributed "well", they can provide rapid access. Well in this case means distinct bounding rectangles.

The rectangle approximation however can be rather imprecise, and the quality and speed of the search space is limited by the overlap of the rectangles. In the worst case the entire index space has to be searched. Update algorithms are also very costly, which makes R-trees not ideal for data which is highly dynamic.

7.4 R⁺-trees

These trees solve the search space overlap problem by "clipping" the bounding rectangles. If an element is contained in both rectangles, it (or a reference to it) is stored redundantly. A graphical example is shown in Figure 16. The elements are logically "cut" on insertion. Due to the redundant storing of elements, additional memory overhead can be incurred. This is somewhat balanced by the reduced query time, but the query time can also be reduced in different ways.

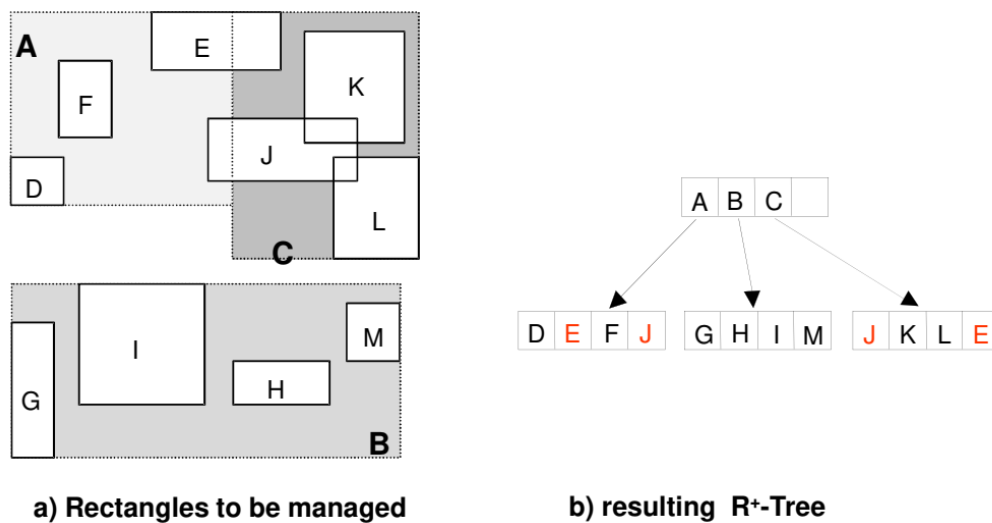


Figure 16: A graphical example for an R⁺-tree

7.5 R*-trees