Assignment 1, Report

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1 Implementation

Implementing the different solvers was fairly straight-forward. The different update procedures between the solvers are as follows:

1.1 Forward Euler

1. Calculate forces for t
2. Calculate acceleration for t
3. Calculate position for $t+h$
4. Calculate velocity for $t+h$

1.2 Symplectic Euler

The only difference between the forward and symplectic Euler is the order of the calculations. 1. Calculate position for $t+h$ 2. Calculate forces for t (using $x(t+h)$) 3. Calculate acceleration for t 4. Calculate position for $t+h$

Note: we used the variant of the symplectic Euler which uses the new position, not the new velocity.

1.3 Leapfrog

The leapfrog solver works in two steps: 1. Update the velocity at $t-h/2$ 2. Update the position at t

This provides better accuracy ($\boldsymbol{O}(h3)$). For calculating the damping force, the velocity for t is approximated. It is unclear how this affects the accuracy.

Note: because we cannot perform calculations in half-steps in our framework, we halved the step size and updated the positions and velocities every other update respectively. This means that instead of using dt we use $2\ dt$ for full update steps, and dt for the initialization step.

1.4 Midpoint

The midpoint solver works as follows: 1. Calculate positions and velocities for $t+h/2$ 2. Calculate forces for $t+h/2$ using the new positions and veloicities 3. Calculate the positions for $t+h$ based off of the position for t and the velocity for $t+h/2$ 4. Calculate the velocities for $t+h$ similarly

1.5 Changes to the code

Most of the changes we implemented were in the **Exercise.cpp**. However, based on OOP considerations, we decided to implement the application of force in the **Spring.cpp**, to save us the check if a point is contained in a spring. This does not affect solver behaviour, as the calculation of spring forces is fixed for all methods.

In addition, we also added internal damping for the springs, based on the internal friction that occurs when deforming the spring. Spring damping is calculated based on a spring damping coefficient and the relative velocity of the connected mass-points. Only relative velocity parallel to the direction of the spring is taken into account for this damping. The default spring damping coefficient is 0.

2 Stability

This section provides an analysis of the stability for the different solvers and different test cases. We describe a system as stable if it comes to rest at some point and the total energy shrinks. An unstable system results in an "explosion", where all non-fixed mass points shoot out of view. We could not observe any difference regarding the testcase. A(n) (un)stable system was (un)stable for all 3 testcases(spring, hanging, falling). In general the stepsize and spring stiffness had the biggest impact on stability. Since there are infinitely many possible scenarios we decided to start with the default values and from there alter stepsize and spring stiffness seperately up to a change in stability. Default stepsize: 0.003, stiffness: 60

1. **Forward Euler**: Unstable for the default parameters. Stable for stepsize <= 0.001 or stiffness <= 20

2. **Symplectic Euler**: Stable for the default parameters. Unstable for stepsize >= 0.1 or stiffness >= 69000

3. **Leapfrog**: Stable for the default parameters. Unstable for stepsize >= 0.1 or stiffness >= 69000

4. **Midpoint**: Stable for the default parameters. Unstable for stepsize >= 0.04 or stiffness >=2500

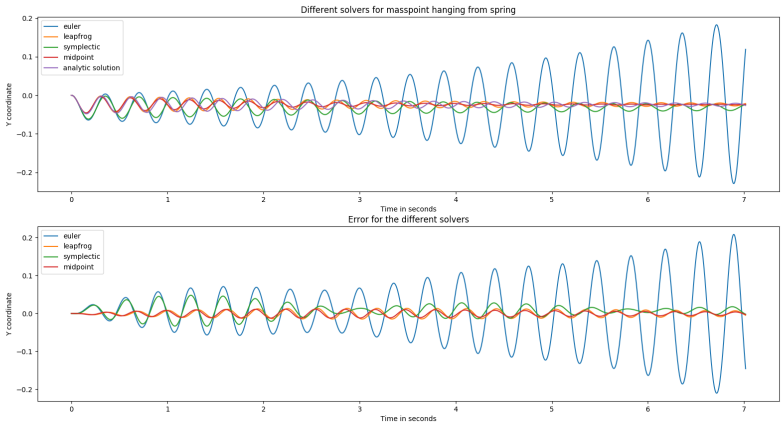
Result: The forward Euler solver is by far the least stable. The Symplectic Euler and Leapfrog solvers are very robust and very similar in their stability behaviour. The Midpoint solver, while being the most accurate is not as stable as the Symplectic Euler or the Leapfrog. It is very susceptible instabilities at larger step sizes.

3 Comparison to exact solution

The first plot in the following figure shows the behaviour of the different solvers, as well as the analytical solution. The analytical solution was obtained using the start values $x(0) = 0$ and $x'(0) = 0$.

As visible, the most divergence from the analytical solution is in frequency. The solvers that comes closest, however, are the leapfrog and midpoint solvers. These are also the solvers with the highest order of accuracy. Interesting to note is the fact that the approximation of velocity for the calculation of acceleration in the leapfrog solver does not cause greater error, as this is the solver that most accurately approximates the decay of amplitude in the oscillation.

The second plot shows the absolute error of the different solver methods compared to the analytical solution. Improtant to note here is the fact that the error stems from a difference in frequency, not amplitude. Thus the error is better observed in the plot of amplitudes.


Different solvers for masspoint hanging from spring
euler
leapfrog
symplectic
midpoint
analytic solution
Y coordinate
Time in seconds
Error for the different solvers
euler
leapfrog
symplectic
midpoint
Y coordinate
Time in seconds