Problem 1. Let $\Sigma = \{0, 1\}$. Give a PDA for each of the following languages (give a high-level description, and the formal PDA if you want):

1. The language of strings whose seventh-last symbol is a 1.
2. The language consisting of strings with the same number of 0s as 1s.
3. The language consisting of strings that are palindromes.
4. The language of *valid counter string*. Let $\Sigma = \{p, m\}$. We say that a string $w \in \Sigma^*$ is a *valid counter string* if, when read from left to right, and interpreting p as adding 1 and m as subtracting 1, and we start from 0, the running sum is non-negative. For instance, ppm and pm and $ppmmp$ are valid counter strings, but $pmmp$ and mpp are not valid counter string.

Note that if you use acceptance by empty-stack, then you must include ϵ in the languages.

Problem 2. Show that recognition by empty-stack and recognition by accepting states are equally powerful (modulo the caveat that empty-stack PDAs must accept ϵ).

That is, if M accepts by final state, then we can build M' that accepts by empty-stack such that $L(M) = L(M') \cup \{\epsilon\}$. And conversely, if M accepts by empty-stack we can build M' that accepts by final state such that $L(M) = L(M')$.

Problem 3. Find a CFG that generates the language of valid counter-strings.

Problem 4. Consider the following decision problem: given a PDA M, decide whether or not $L(M) \neq \emptyset$. Show that this problem can be solved in polynomial time (in the size of M).

Problem 5. Consider the following problem: given nondeterministic finite automata M_1, M_2 over the alphabet Σ decide if there exists a string $w \in \Sigma^*$ and a positive integer $n \geq 1$ such that $w \in L(M_1)^n \cap L(M_2)^n$. Show that this problem can be solved in polynomial time (in the sizes of the NFAs).

Problem 6. Call a language ϵ-free if it does not include ϵ. Define a variant of PDAs that captures the ϵ-free CFLs and has the following attributes:

1. it does not have epsilon transitions (i.e., a PDA must scan a symbol from Σ at every step),
2. it cannot accept ϵ.
3. it has empty-stack acceptance.

Problem 7. Show that the intersection of a CFL and a regular language is a CFL.