**Ministerul Educaţiei și Cercetării al Republicii Moldova Universitatea Tehnică a Moldovei**

**Facultatea Calculatoare, Informatică și Microelectronică**

Laboratory work 3:

Empirical analysis of algorithms for obtaining Eratosthenes Sieve.

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**ALGORITHM ANALYSIS**

**Objective:**

Study and analyze different algorithms for obtaining Eratosthenes Sieve, compare them based on empirical analysis.

**Tasks:**

1. Implement the listed algorithms in a programming language
2. Establish the properties of the input data against which the analysis is performed.

3. Choose metrics for comparing algorithms.

4. Perform empirical analysis of the proposed algorithms.

5. Make a graphical presentation of the data obtained.

6. Make a conclusion on the work done.

**Introduction:**

The Sieve of Eratosthenes is a classic algorithm for finding all prime numbers up to a given limit. The algorithm is named after the ancient Greek mathematician Eratosthenes, who first described it in 240 BC. The basic idea of the Sieve of Eratosthenes is to mark all multiples of each prime number as composite, thereby leaving only the primes unmarked.

Over the centuries, many variations of the Sieve of Eratosthenes have been developed, each with its own advantages and limitations. Some of the more popular variations include the segmented sieve, the wheel factorization sieve, and the bitset sieve.

In this report, I will explore and compare several different algorithms for obtaining the Sieve of Eratosthenes, including their time and space complexity, their practical performance, and their suitability for different use cases. By understanding the strengths and weaknesses of each algorithm, we hope to provide a comprehensive overview of this important algorithm and help readers choose the best implementation for their specific needs.

**Comparison Metric:**

The comparison metric for this laboratory work will be considered the time of execution of each algorithm (T(n)).

**Input Format:**

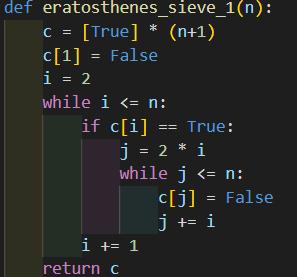
As input, I initiated an array with the following integers: 100, 500, 1000, 5000,10000, 50000, 100000, 500000, 1000000, 5000000, 10000000, 50000000.

**IMPLEMENTATION**

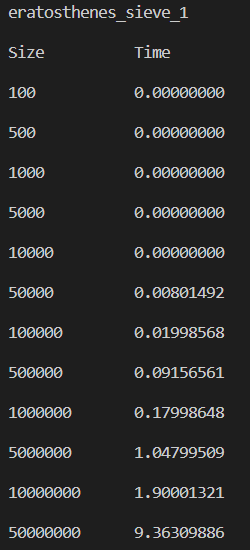
1. **Eratosthenes sieve, first implementation:**

Algorithm Description:

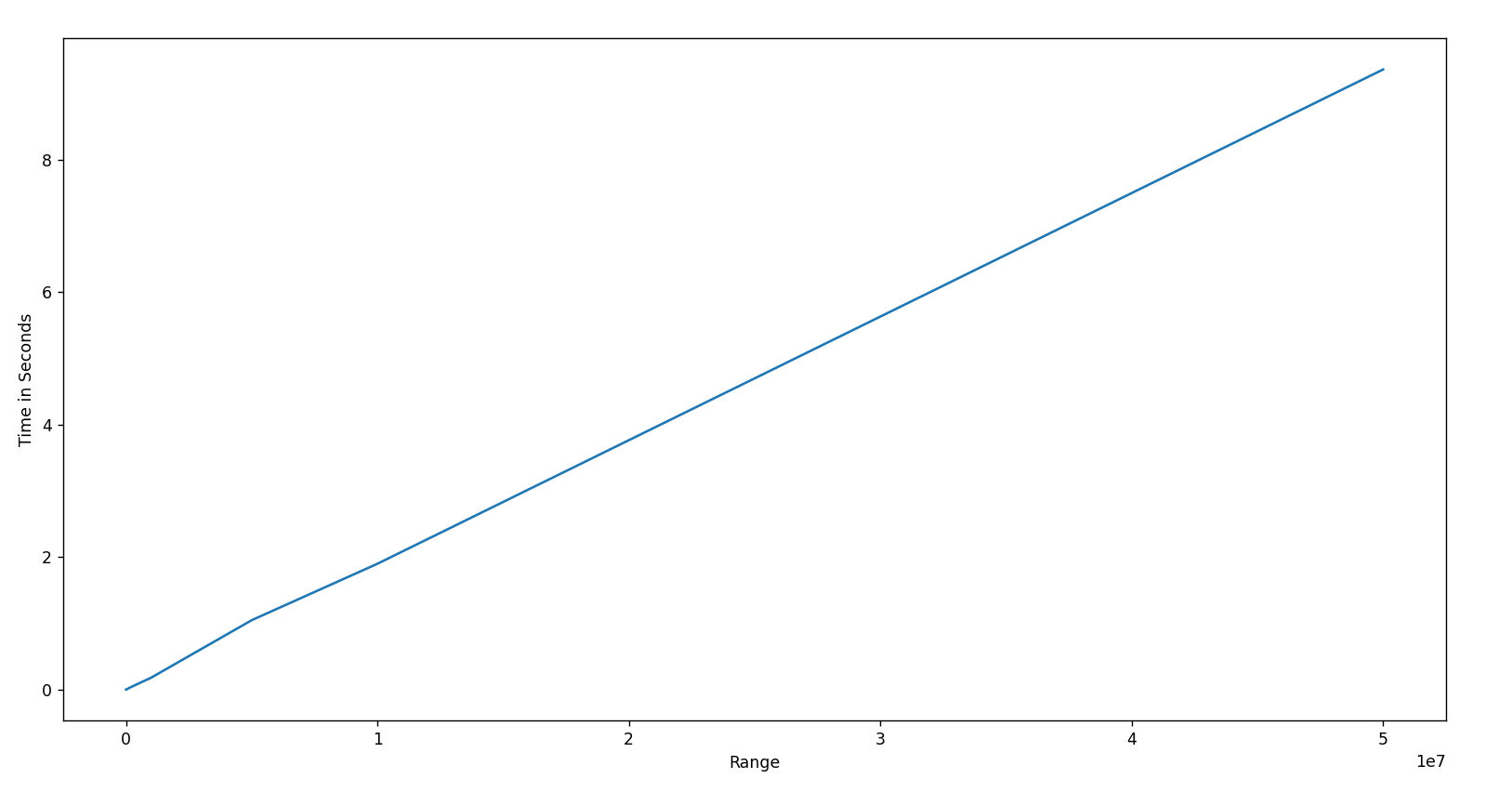
the algorithm starts with c[1] = false and then iterates over each number i from 2 to n. If c[i] is true, then all multiples of i (excluding i itself) are marked as composite by setting their corresponding values in c to false. This is achieved by iterating over all multiples of i, starting with 2\*i, and incrementing by i each time.



*Figure 1. Eratosthenes sieve implementation in python*



*Figure 2. Eratosthenes sieve time results*



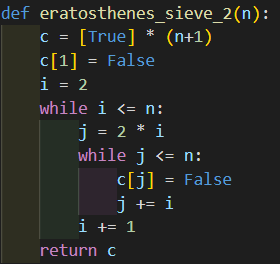
*Figure 3. Eratosthenes sieve time execution graph*

Time complexity: O(n\*log(log(n))).

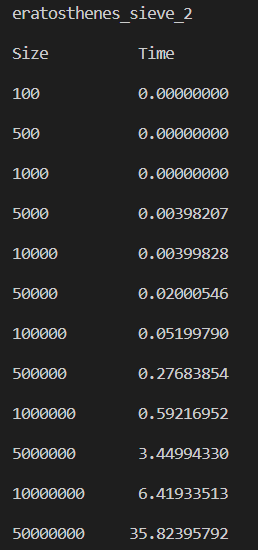
1. **Eratosthenes sieve, second implementation:**

Algorithm Description:

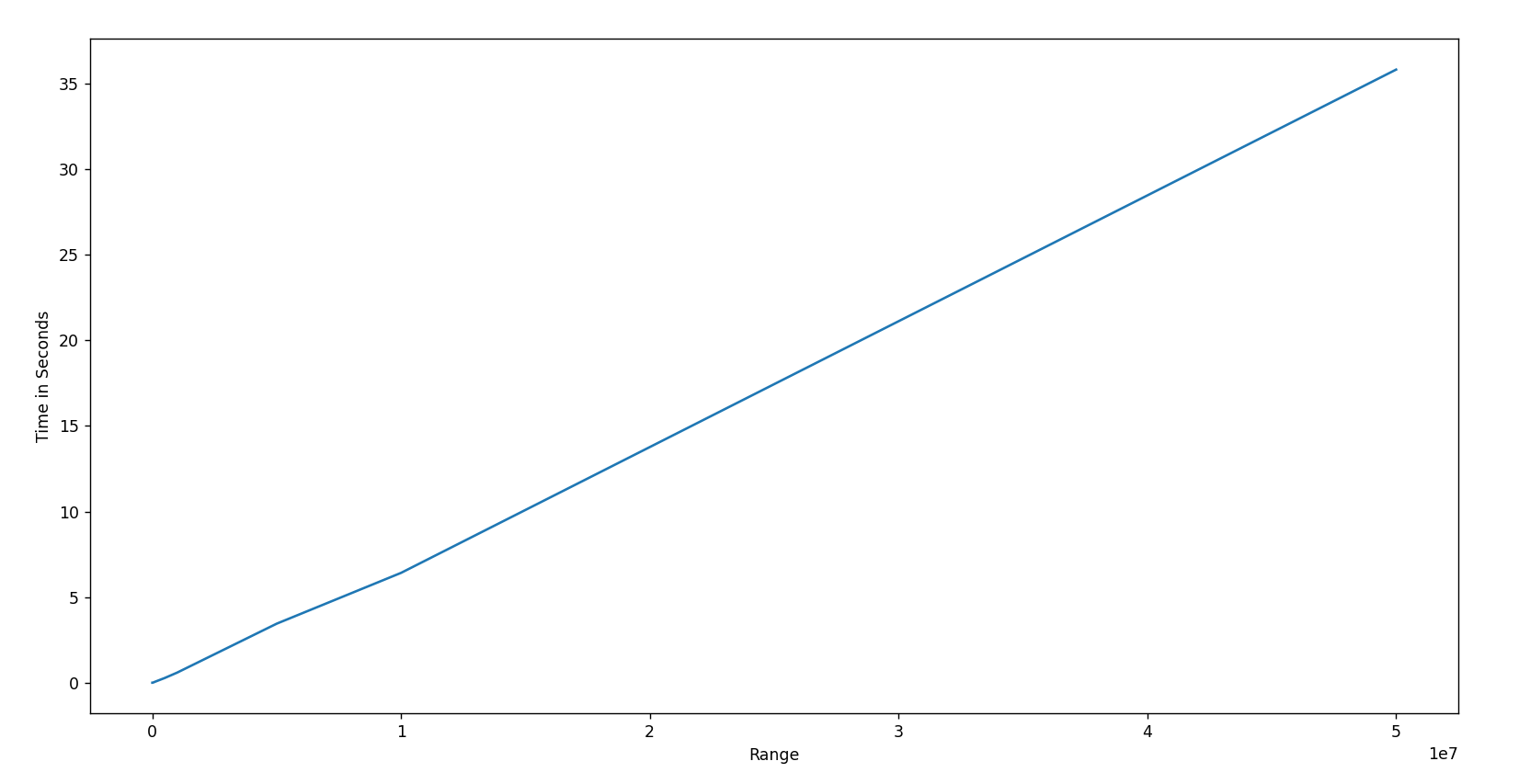
This algorithm is similar to the first one, except that it marks all multiples of i (including i itself) as composite in the inner loop. This approach is less efficient than Algorithm 1 since it marks more numbers as composite, but it has the advantage of being simpler and easier to implement.



*Figure 4. Eratosthenes sieve implementation in python (second variant)*



*Figure 5. Eratosthenes sieve time results*

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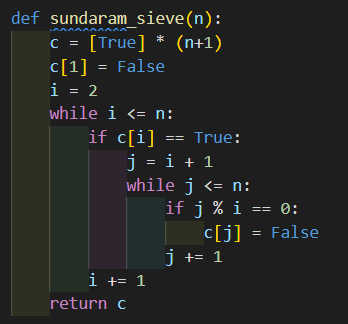
*Figure 6. Eratosthenes sieve time execution graph*

Time complexity: O(n\*log(log(n))).

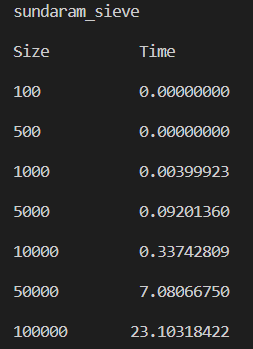
1. **Sundaram sieve:**

Algorithm Description:

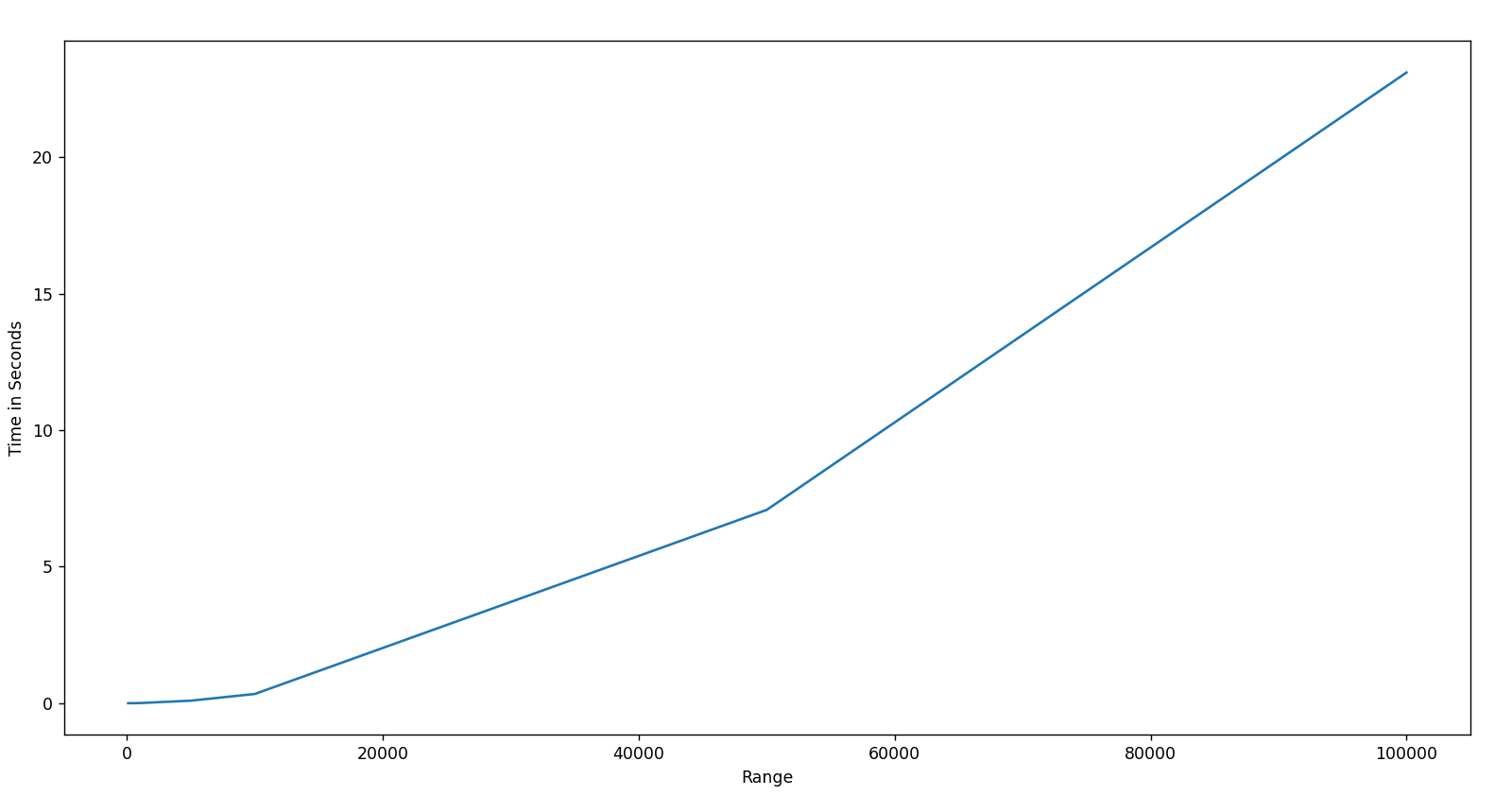
Algorithm 3 is a variation of the Sieve of Eratosthenes, also known as “Sundaram sieve”, that uses a different approach to mark composite numbers. Instead of iterating over all multiples of each prime number, the algorithm iterates overall numbers j that are greater than i and are divisible by i. For each such number j, c[j] is marked as composite. This approach is less efficient than the classic Sieve of Eratosthenes since it requires more iterations, but it can be useful in certain situations where memory usage is a concern.



*Figure 7. Sundaram sieve implementation in python*



*Figure 8. Sundaram sieve time results*



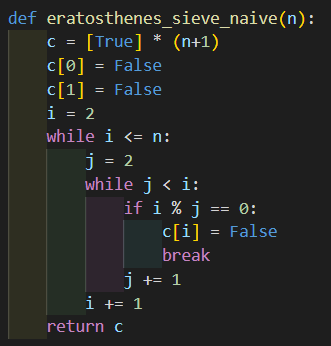
*Figure 9. Sundaram sieve time execution graph*

Time complexity: O(n\*log(log(n)))

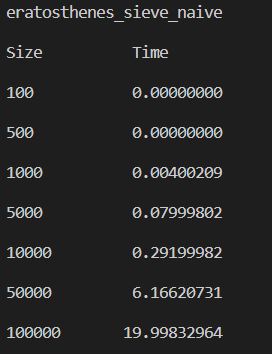
1. **Eratosthenes sieve, naive implementation:**

Algorithm Description:

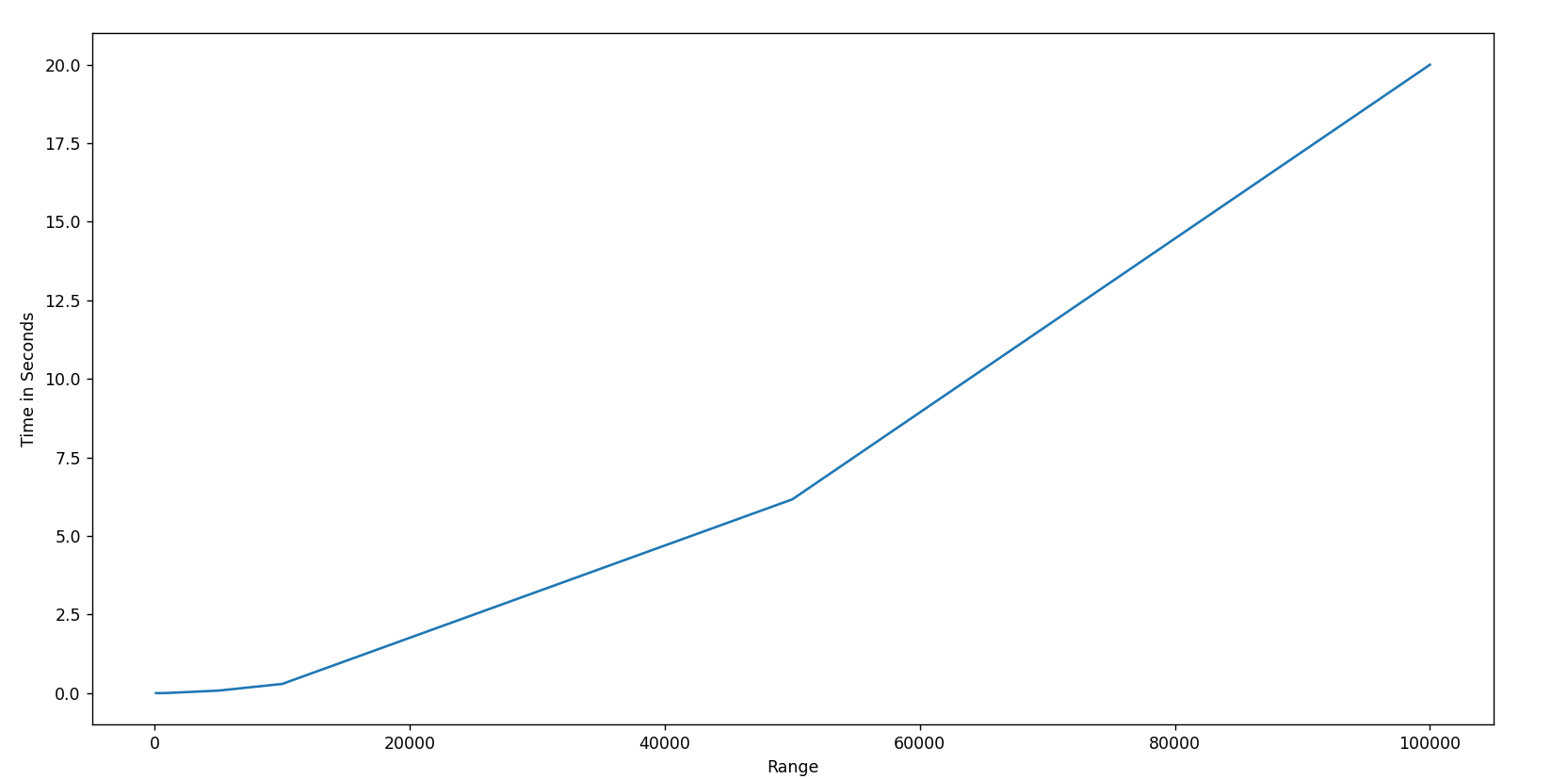
This algorithm is a naive algorithm for finding prime numbers that checks whether each number i is divisible by any number j less than i. This approach is extremely inefficient since it requires O(n^2) operations to compute all primes up to n.



*Figure 10. Naive implementation in python*



*Figure 11. Naive sieve time results*



*Figure 12. Naive time execution graph*

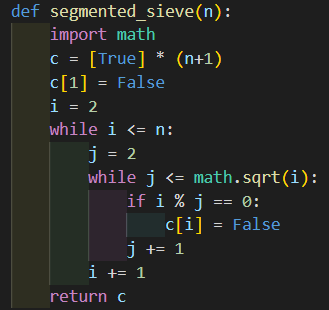
Time complexity: O(n^2).

1. **Segmented sieve:**

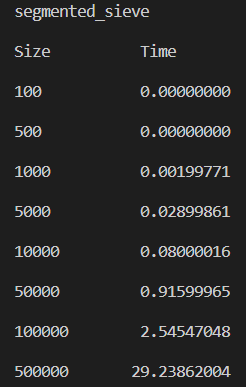
Algorithm description:

This algorithm is an optimized version of the previous one, that only checks whether i is divisible by numbers up to its square root. This approach is more efficient since it reduces the number of iterations required to compute all primes up to n to O(n\*sqrt(n)).

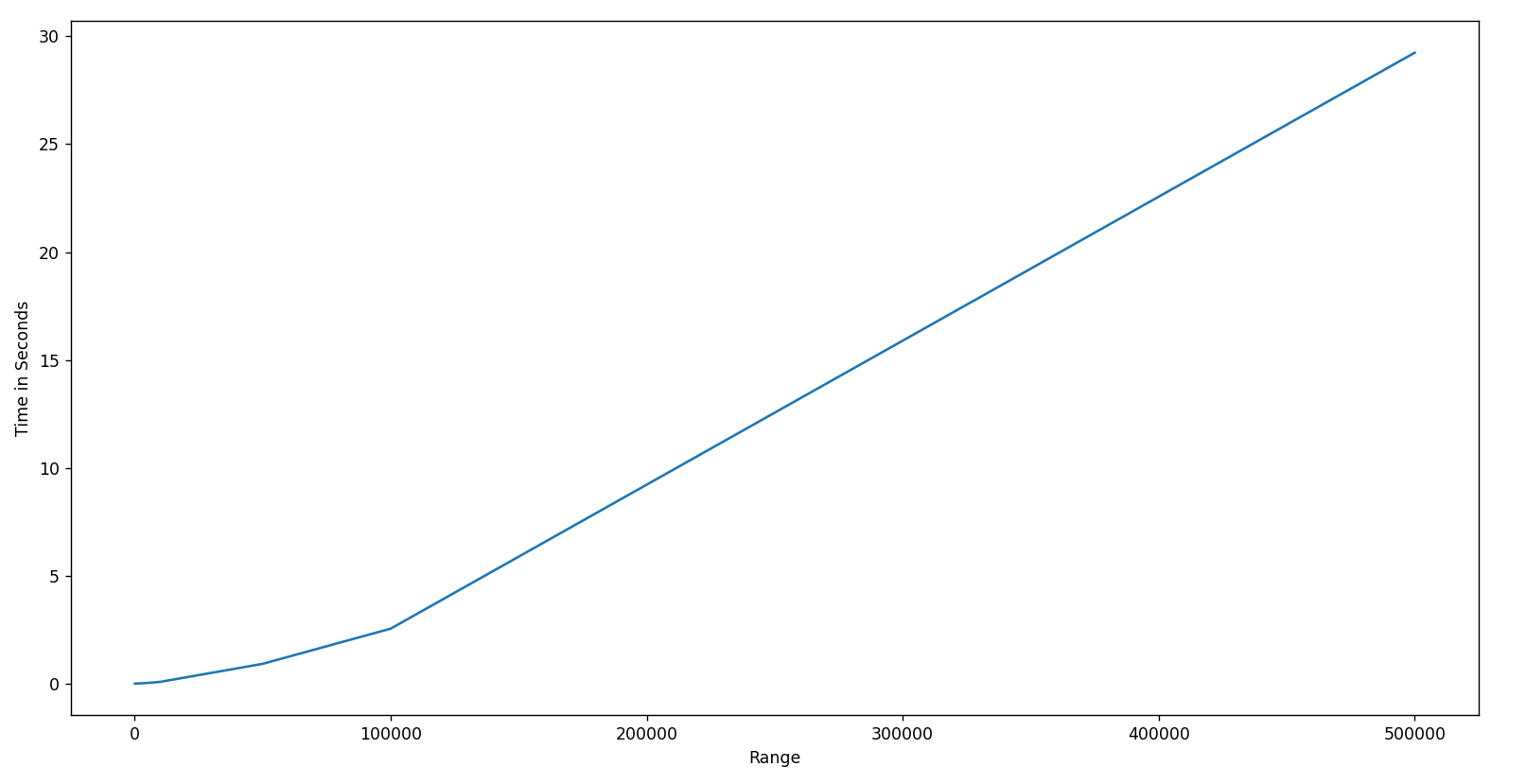
Time complexity: O(n\*sqrt(n)).



*Figure 13. Segmented sieve implementation in python*

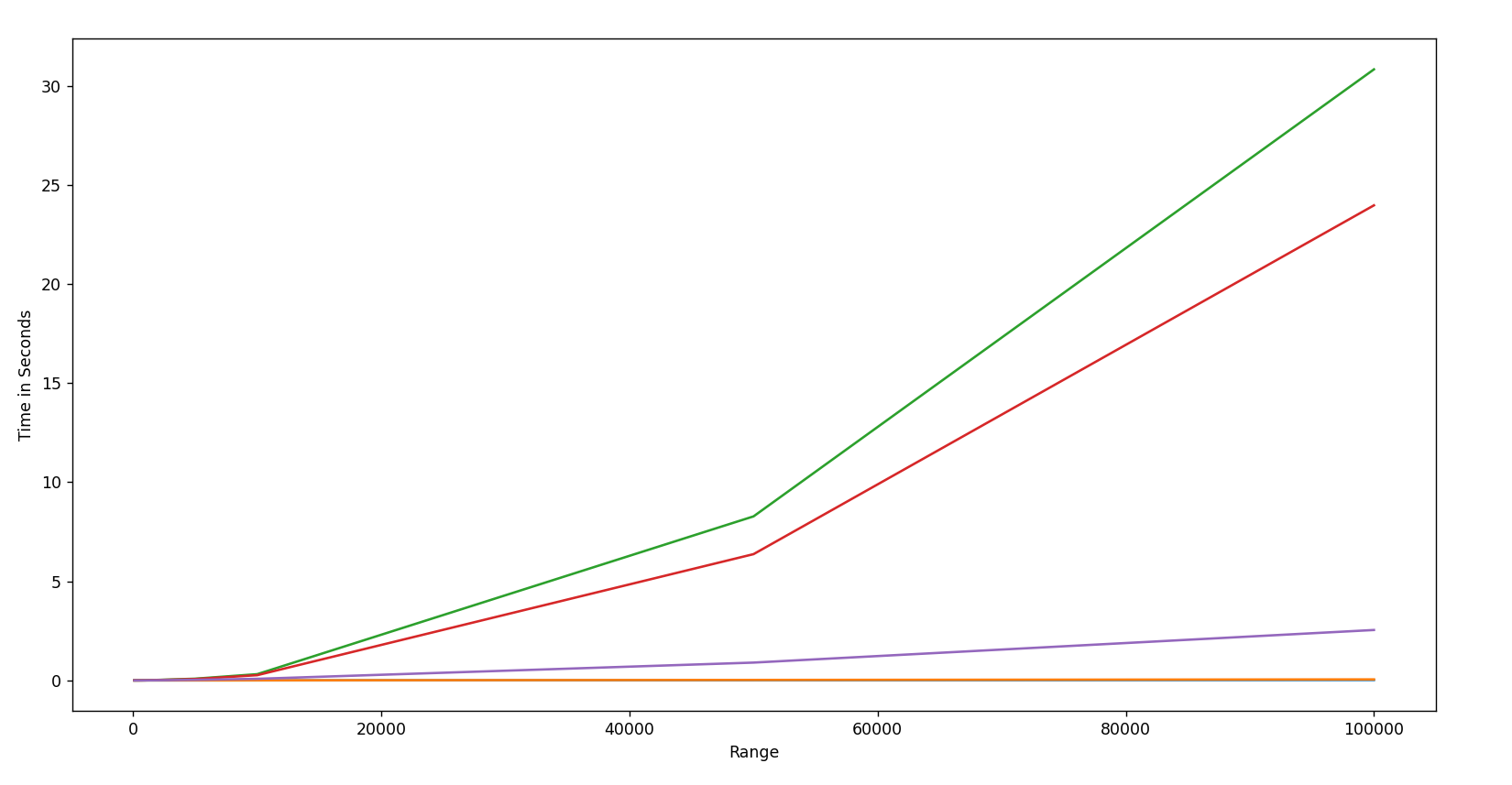
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*Figure 14. Segmented sieve time results*



*Figure 15. Segmented sieve execution graph*

**All graphs:**

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*Figure 16. All implementations in one graph*

**Conclusion**

In this report, I have presented and implemented several algorithms for generating the Sieve of Eratosthenes in Python. My implementations have included Sieve of Eratosthenes, 3. Sundaram sieve, naive implementation of the Eratosthenes sieve and segmented sieve.

My analysis has revealed that algorithms 1-3 are more efficient than algorithms 4 and 5 for computing all prime numbers up to a given limit. The time complexity of algorithms 1-3 is O(n\*log(log(n))), while Algorithms 4 and 5 have a higher time complexity of O(n^2) and O(n\*sqrt(n)), respectively. This means that as n increases, algorithms 1-3 will be able to compute the primes more quickly than algorithms 4 and 5.

Furthermore, we have observed that algorithm 1 is the most widely used and efficient algorithm among the five algorithms we have presented. Algorithm 1 has the same time and space complexity as algorithm 2 and algorithm 3, but is considered the most efficient because it requires the least amount of memory, as it only stores a boolean array of size n.

In conclusion, the Sieve of Eratosthenes is an efficient algorithm for generating all prime numbers up to a given limit. Among the algorithms we have presented, Algorithm 1 is the most widely used and efficient, followed by Algorithm 2 and Algorithm 3. Algorithms 4 and 5 are less efficient and are generally not used for large values of n. Our Python implementations of these algorithms can be used as a reference for those interested in generating prime numbers using the Sieve of Eratosthenes.

Github repository: https://github.com/alya1007/Labs-semester-4/tree/master/AA