GRAVITY RELATED DERIVATIONS

Stem Astronomy Club
Academic committee
Academic 23
LaTeX

Contents

1 Symbols meaning

F is force
m is mass
a is acceleration
v is velocity
t is one cycle time
r is the distance
G is gravitational constant

2 Kepler's Law

1- Newton's Second Law

$$F = ma \tag{1}$$

Because the gravitational force is a centripetal one, then it will have a centripetal acceleration:

$$F = m\frac{v^2}{r} \tag{2}$$

since the motion of the body is circular, the covered distance in one complete cycle will be a circumference of a circle

$$F = m\frac{\left(\frac{2\pi r}{t}\right)^2}{r} \tag{3}$$

From Newton's law of gravity

$$F = G\frac{Mm}{r^2} \tag{4}$$

from equation 3 and 4 we get:

$$\begin{aligned} \frac{m4\pi^2 r}{t^2} &= G\frac{Mm}{r^2} \\ \frac{4\pi^2 r}{t^2} &= G\frac{M}{r^2} \\ \frac{4\pi^2 r}{t^2} &= G\frac{M}{r^2} \\ 4\pi^2 r^3 &= GMt^2 \\ \frac{4\pi^2 r^3}{GM} &= t^2 \end{aligned} \tag{5}$$

by substituting r with the semi-major a, and t with the orbital period P

$$P^2 = \frac{4\pi^2}{GM}a^3 \tag{6}$$

(Kepler's law)

3 Orbital Velocity

from eq. 2 and eq. 4

$$\begin{aligned} m\frac{v^2}{r} &= G\frac{Mm}{r^2} \\ v^2 &= \frac{GM}{r} \\ v &= \sqrt{\frac{GM}{r}} \end{aligned} \tag{7}$$

4 Escape Velocity

the potential energy due to gravity equals:

$$E = G\frac{Mm}{r} \tag{8}$$

the Kinetic energy of the body equals:

$$E = \frac{1}{2}mv^2 \tag{9}$$

for the body to have an escape velocity (least velocity required to leave its orbit),the potential energy should equal the kinetic energy. from equations 8 and 9 we get:

$$\begin{aligned} G\frac{Mm}{r} &= \frac{1}{2}mv^2 \\ \frac{2GM}{r} &= v^2 \\ v_{esc} &= \sqrt{\frac{2GM}{r}} \end{aligned} \tag{10}$$

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