Bayes' Theorem

Bayes' Theorem, named after 18th-century British mathematician **Thomas Bayes**, is a mathematical formula for determining **conditional probability**.

Conditional probability is the likelihood of an outcome occurring, based on a **previous outcome** having occurred in similar circumstances. Bayes' theorem provides a way to revise existing predictions or theories (update probabilities) given new or additional evidence.

There are four parts:

- **Posterior probability** (updated probability after the evidence is considered)
- **Prior probability** (the probability before the evidence is considered)
- **Likelihood** (probability of the evidence, given the belief is true)
- **Marginal probability** (probability of the evidence, under any circumstance)

Bayes Theorem Formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\underbrace{\boxed{P(A|B)}}_{\text{posterior}} = \underbrace{\boxed{P(A)}}_{\text{prior}} \times \frac{\overbrace{\boxed{P(B|A)}}^{\text{likelihood}}}{\underbrace{\boxed{P(B)}}_{\text{marginal}}}$$

where:

$P(A) =$ The probability of A occurring

$P(B) =$ The probability of B occurring

$P(A|B) =$ The probability of A given B

$P(B|A) =$ The probability of B given A

$P\left(A \cap B\right)) =$ The probability of both A and B occurri

Example: dangerous Fire

Let us say P(Fire) means how often there is fire, and P(Smoke) means how often we see smoke,

P(Fire|Smoke) means how often there is fire when we can see smoke
P(Smoke|Fire) means how often we can see smoke when there is fire

- **dangerous fires are rare (1%)**
- **but smoke is fairly common (10%) due to barbecues,**
- **and 90% of dangerous fires make smoke**

We can then discover the probability of **dangerous Fire when there is Smoke**:

P(Fire|Smoke) = (P(Fire) * P(Smoke|Fire))/ P(Smoke)

= (1% x 90%)/10%

= 9%

Example: Picnic Day

- **Oh no! 50% of all rainy days start off cloudy!**
- **But cloudy mornings are common (about 40% of days start cloudy)**
- **And this is usually a dry month (only 3 of 30 days tend to be rainy, or 10%)**

  Answer:
- **P(Rain) is Probability of Rain = 10%**
- **P(Cloud|Rain) is Probability of Cloud, given that Rain happens = 50%**
- **P(Cloud) is Probability of Cloud = 40%**

The chance of Rain given Cloud is written **P(Rain|Cloud)**

P(Rain|Cloud) = P(Rain) P(Cloud|Rain) / P(Cloud)

P(Rain|Cloud) = (*0.1 x 0.5) /* 0.4 = .125

Example : Just 4 Numbers

Imagine 100 people at a party, and you tally how many wear pink or not, and if a man or not, and get these numbers:

	Pink	notPink
Man	5	35
notMan	20	40

	Pink	notPink	
Man	5	35	40
notMan	20	40	60
	25	75	100

And calculate some probabilities:

- the probability of being a man is P(Man) = $\frac{40}{100}$ = 0.4
- the probability of wearing pink is P(Pink) = $\frac{25}{100}$ = 0.25
- the probability that a man wears pink is P(Pink|Man) = $\frac{5}{40}$ = 0.125

Imagine a pink-wearing guest leaves money behind ... was it a man? We can answer this question using Bayes' Theorem:

$$P(Man|Pink) = \frac{P(Man)\ P(Pink|Man)}{P(Pink)}$$

$$P(Man|Pink) = \frac{0.4 \times 0.125}{0.25} = 0.2$$

General

	B	notB	
A	s	t	s+t
notA	u	v	u+v
	s+u	t+v	s+t+u+v

- the overall probability of "A" is $P(A) = \frac{s+t}{s+t+u+v}$
- the probability of "B given A" is $P(B|A) = \frac{s}{s+t}$

$$P(A) \times P(B|A) = P(A)\,P(B|A)$$

$$\frac{s+t}{s+t+u+v} \times \frac{s}{s+t} = \frac{s}{s+t+u+v}$$

$$P(B) \times P(A|B) = P(B)\,P(A|B)$$

$$\frac{s+u}{s+t+u+v} \times \frac{s}{s+u} = \frac{s}{s+t+u+v}$$

$$P(B)\,P(A|B) = P(A)\,P(B|A)$$

$$P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}$$