UNIVERSITAS INDONESIA

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FAKULTAS ILMU KOMPUTER

PENGANTAR STATISTIKA

Introduction to the *t* Statistic

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Chapter 9 Learning Outcomes

1

- Know when to use *t* statistic instead of *z*-score hypothesis test

2

- Perform hypothesis test with *t*-statistics

3

- Evaluate effect size by computing Cohen's *d*, percentage of variance accounted for (r^2), and/or a confidence interval

Tools You Will Need

- Sample standard deviation (Chapter 4)
- Standard error (Chapter 7)
- Hypothesis testing (Chapter 8)

9.1 Review Hypothesis Testing with *z*-Scores

- Sample mean (*M*) estimates (& approximates) population mean (μ)
- Standard error describes how much difference is reasonable to expect between *M* and μ.
- either or
- $$\sigma_M = \frac{\sigma}{\sqrt{n}} \qquad \sigma_M = \sqrt{\frac{\sigma^2}{n}}$$

z-Score Statistic

- Use *z*-score statistic to quantify inferences about the population.

$$z = \frac{M - \mu}{\sigma_M} = \frac{\text{obtained difference between data and hypothesis}}{\text{standard distance between M and } \mu}$$

- Use unit normal table to find the critical region if *z*-scores form a normal distribution
  - When $n \geq 30$ or
  - When the original distribution is approximately normally distributed

Problem with z-Scores

- The z-score requires more information than researchers typically have available
- Requires knowledge of the population standard deviation σ
- Researchers usually have only the sample data available

Introducing the *t* Statistic

- t statistic is an alternative to z
- t might be considered an "approximate" z
- Estimated standard error (s_M) is used as in place of the real standard error when the value of σ_M is unknown

Estimated standard error

- Use s^2 to estimate σ^2
- Estimated standard error:

$$\textit{estimated}\ \text{standard error} = s_M = \frac{s}{\sqrt{n}} \text{ or } \sqrt{\frac{s^2}{n}}$$

- Estimated standard error is used as estimate of the real standard error when the value of σ_M is unknown.

The *t*-Statistic

- The *t*-statistic uses the estimated standard error in place of σ_M

$$t = \frac{M - \mu}{s_M}$$

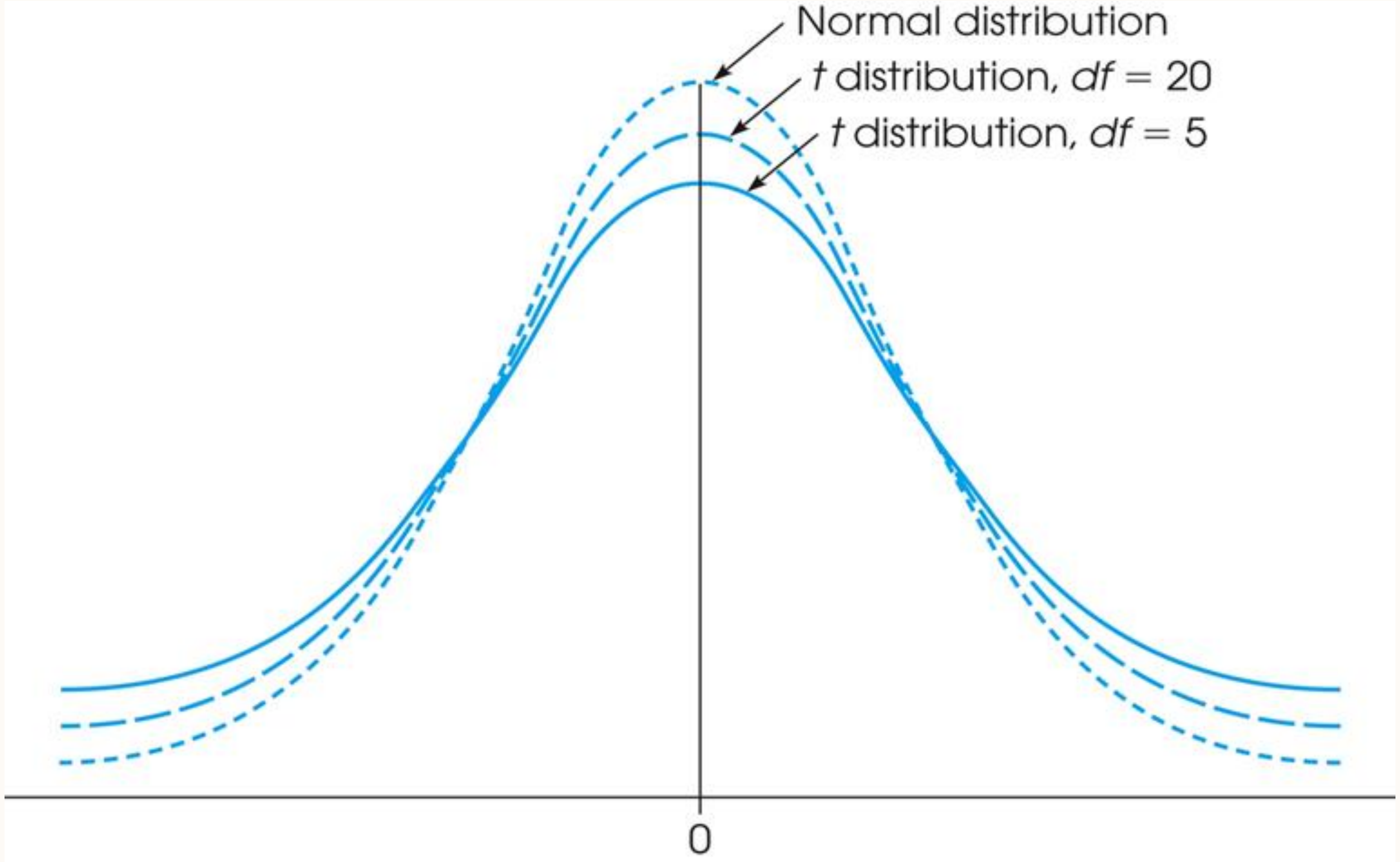
- The t statistic is used to test hypotheses about an unknown population mean μ when the value of σ is also unknown

Degrees of freedom

- Computation of sample variance requires computation of the sample mean first.
  - Only n-1 scores in a sample are independent
  - Researchers call n-1 the degrees of freedom

- Degrees of freedom
  - Noted as df
  - $df = n-1$

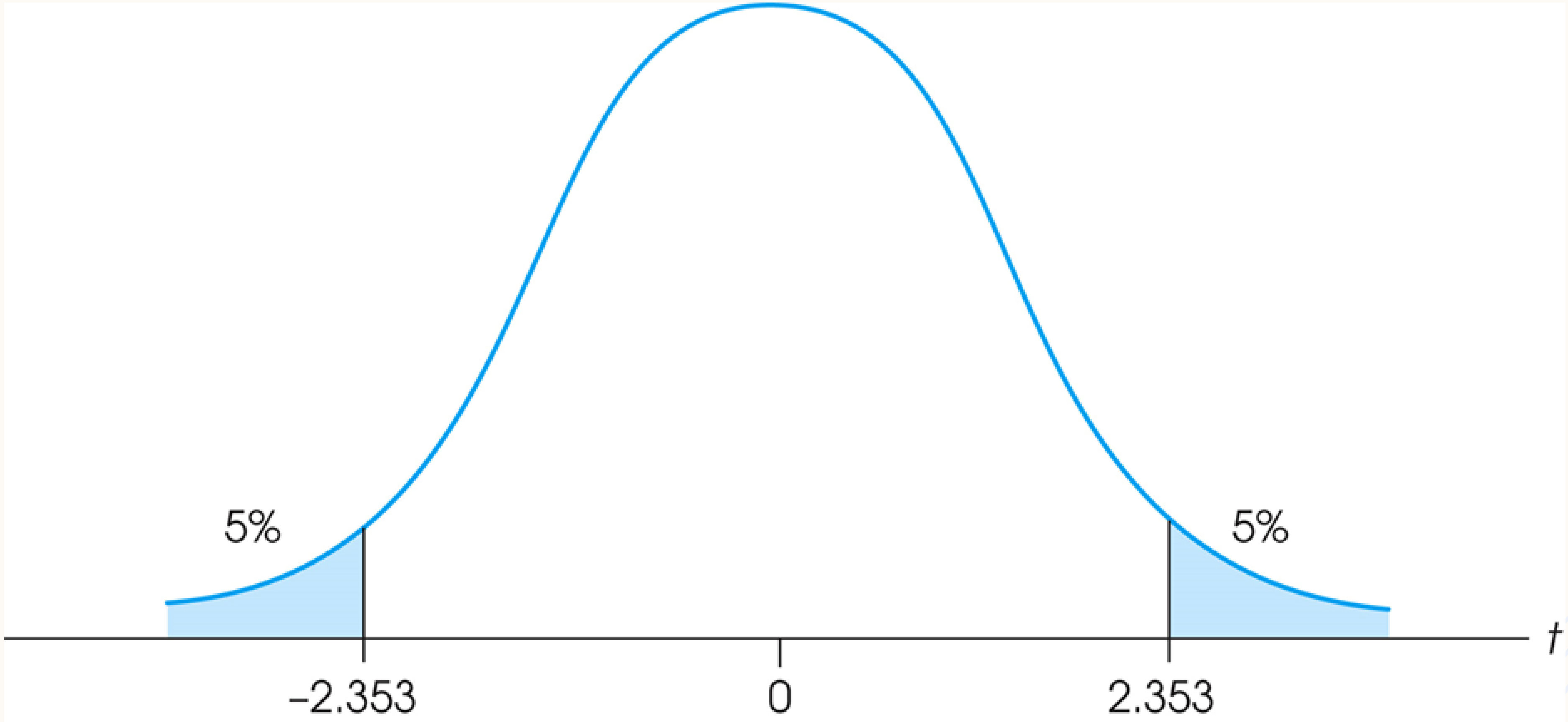
Distributions of the *t*



The *t* Distribution

- Family of distributions, one for each value of degrees of freedom
- Approximates the shape of the normal distribution
  - Flatter than the normal distribution
  - More spread out than the normal distribution
  - More variability ("fatter tails") in *t* distribution
- Use Table of Values of *t* in place of the Unit Normal Table for hypothesis tests

The *t* distribution for



9.2 Hypothesis tests with the *t* statistic

- The one-sample *t* test statistic (assuming the Null Hypothesis is true)

$$t = \frac{\text{sample mean - population mean}}{\text{estimated standard error}} = \frac{M - \mu}{s_M} = 0$$

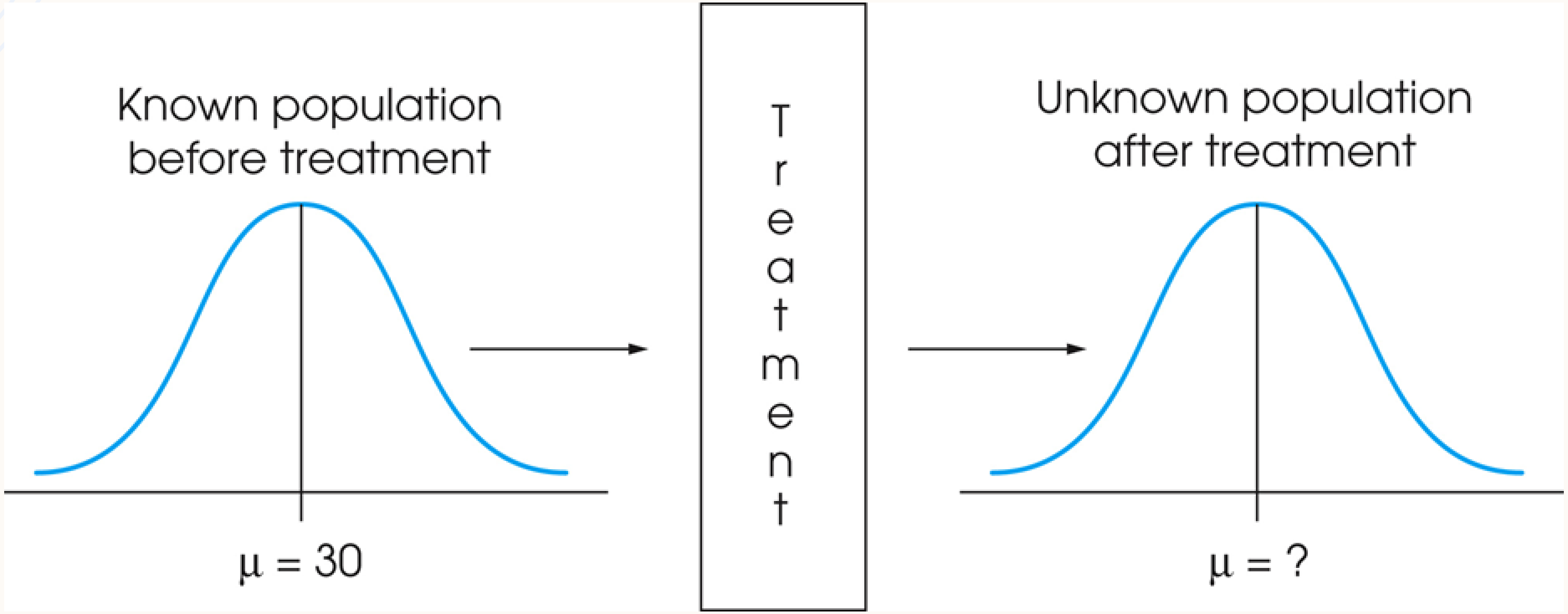


Figure 9.3 Basic experimental situation for t statistic

Hypothesis Testing: Four Steps

- State the null and alternative hypotheses and select an alpha level
- Locate the critical region using the *t* distribution table and *df*
- Calculate the *t* test statistic
- Make a decision regarding H_0 (null hypothesis)

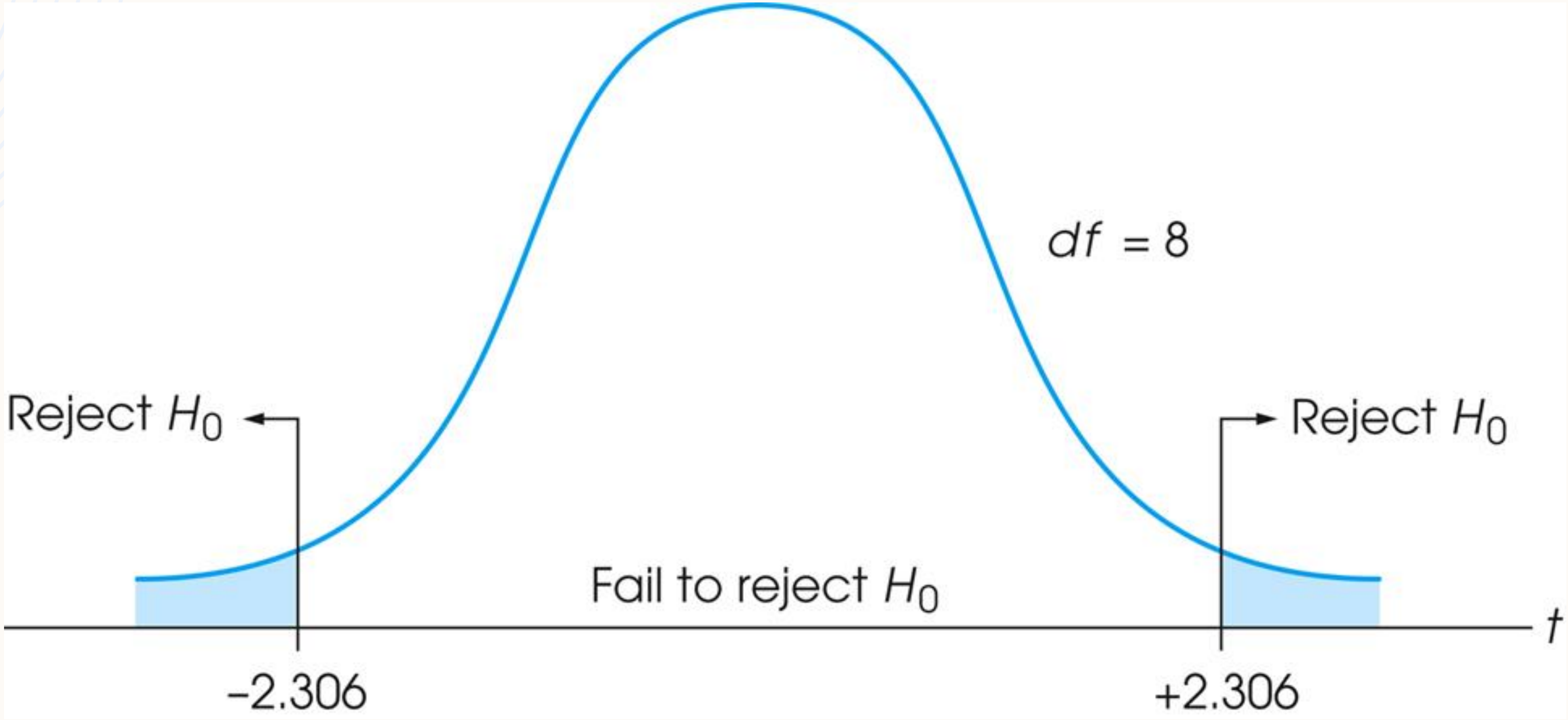


Figure 9.4 Critical region in the *t* distribution for α = .05 and *df* = 8

Assumptions of the *t* test

- Values in the sample are independent observations. The population sampled must be normal.
    - With large samples, this assumption can be violated without affecting the validity of the hypothesis test.

Learning Check

- When n is small (less than 30), the t distribution

____

A
- is almost identical in shape to the normal z distribution

B
- is flatter and more spread out than the normal z distribution

C
- is taller and narrower than the normal z distribution

D
- cannot be specified, making hypothesis tests impossible

Learning Check - Answer

- When n is small (less than 30), the t distribution _______

A
- is almost identical in shape to the normal z distribution

B
- is flatter and more spread out than the normal z distribution

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- is taller and narrower than the normal z distribution

D
- cannot be specified, making hypothesis tests impossible

Learning Check

- Decide if each of the following statements is True or False

T/F

- By chance, two samples selected from the same population have the same size ($n = 36$) and the same mean ($M = 83$). That means they will also have the same t statistic.

T/F

- Compared to a z-score, a hypothesis test with a t statistic requires less information about the population

Learning Check - Answers

False

- The two t values are unlikely to be the same; variance estimates (s^2) differ between samples

True

- The t statistic does not require the population standard deviation; the z-test does

9.3 Measuring Effect Size

- Hypothesis test determines whether the treatment effect is greater than chance
  - No measure of the size of the effect is included
  - A very small treatment effect can be statistically significant
- Therefore, results from a hypothesis test should be accompanied by a measure of effect size

Cohen's *d*

- Original equation included population parameters
- Estimated Cohen's *d* is computed using the sample standard deviation

$$\text{estimated } d = \frac{\text{mean difference}}{\text{sample standard deviation}} = \frac{M - \mu}{s}$$

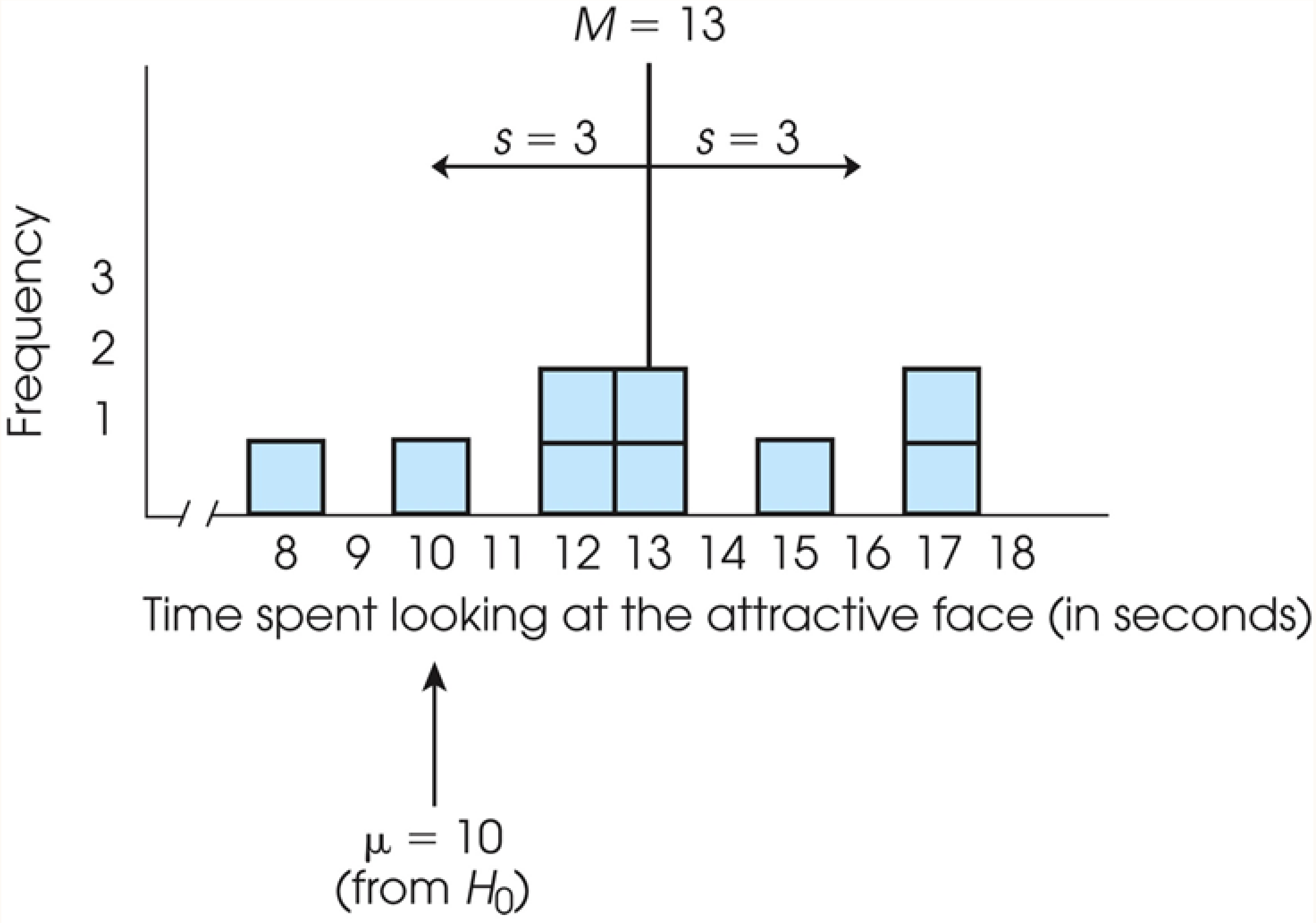

Figure 9.5
Distribution for Examples 9.1 & 9.2

Percentage of variance explained

- Determining the amount of variability in scores explained by the treatment effect is an alternative method for measuring effect size.

$$r^2 = \frac{\text{variability accounted for}}{\text{total variability}} = \frac{t^2}{t^2 + df}$$

- $r^2 = 0.01$ small effect
- $r^2 = 0.09$ medium effect
- $r^2 = 0.25$ large effect

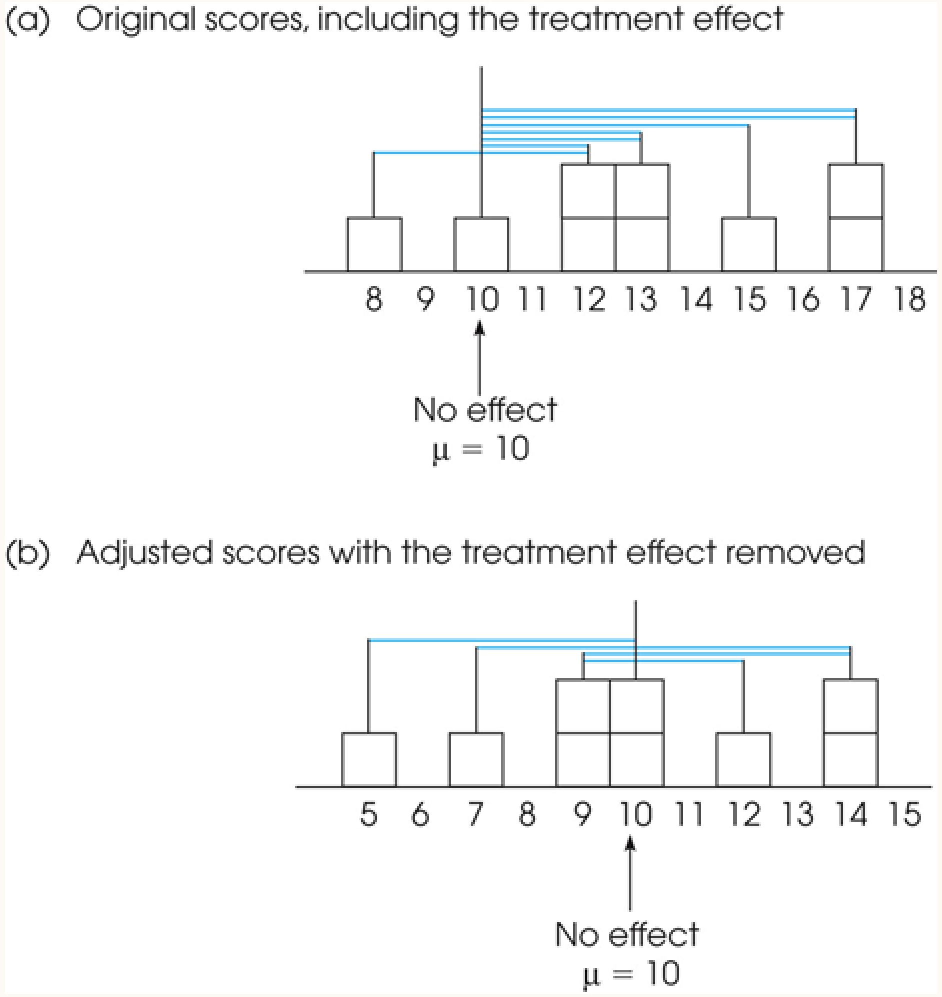

Figure 9.6 Deviations with and without the treatment effect

Confidence Intervals for Estimating μ

- Alternative technique for describing effect size
- Estimates μ from the sample mean (M)
- Based on the reasonable assumption that M should be "near" μ
- The interval constructed defines "near" based on the estimated standard error of the mean (s_M)
- Can confidently estimate that μ should be located in the interval

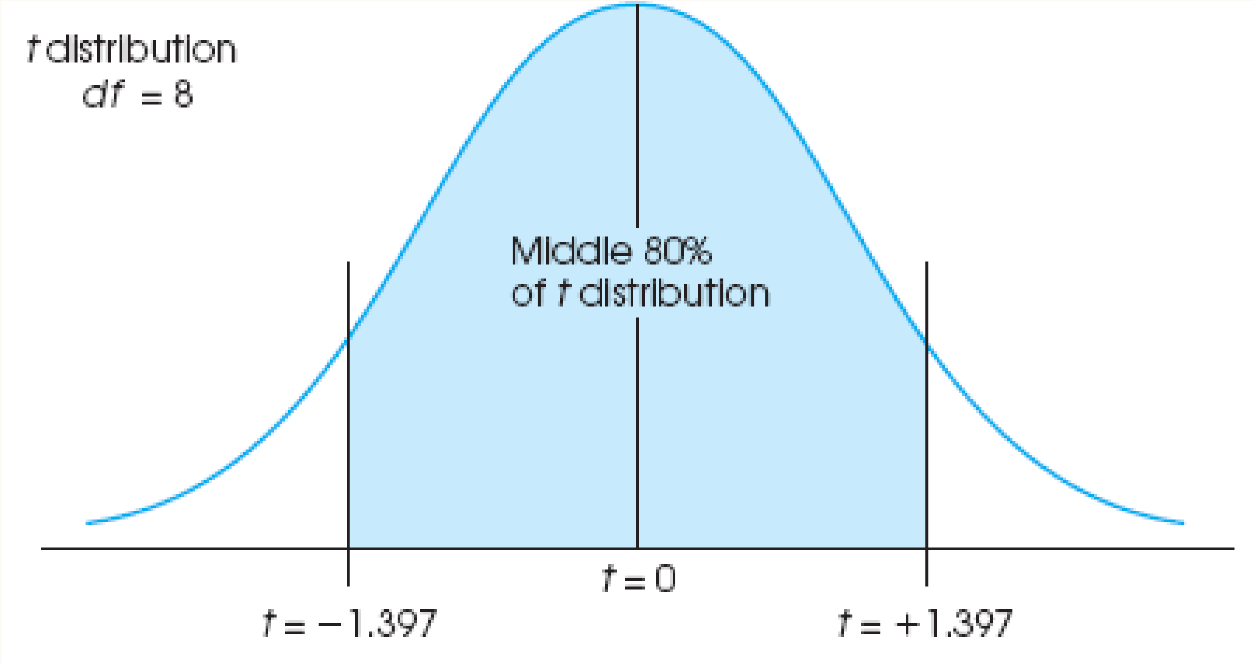

Figure 9.7
t Distribution with *df* = 8

Confidence Intervals for Estimating μ (Continued)

- Every sample mean has a corresponding *t:*

$$t = \frac{M - \mu}{s_M}$$

- Rearrange the equations solving for μ:

$$\mu = M \pm t s_M$$

Confidence Intervals for Estimating μ (continued)

- In any t distribution, values pile up around $t = 0$
- For any α we know that $(1 - \alpha)$ proportion of t values fall between $\pm\, t$ for the appropriate df
- E.g., with $df = 9$, 90% of t values fall between ± 1.833 (from the t distribution table, $\alpha = .10$)
- Therefore we can be 90% confident that a sample mean corresponds to a t in this interval

Confidence Intervals for Estimating μ (continued)

- For any sample mean M with s_M
- Pick the appropriate degree of confidence (80%? 90%? 95%? 99%?) 90%
- Use the t distribution table to find the value of t (For $df = 9$ and $\alpha = .10$, $t = 1.833$)
- Solve the rearranged equation
- $\mu = M \pm 1.833(s_M)$
- Resulting interval is centered around M
- Are 90% confident that μ falls within this interval

Factors Affecting Width of Confidence Interval

- Confidence level desired
  - More confidence desired increases interval width
  - Less confidence acceptable decreases interval width
- Sample size
  - Larger sample□ smaller SE□ smaller interval
  - Smaller sample□ larger SE□ larger interval

In the Literature

- Report whether (or not) the test was "significant"
  - "Significant" □ H_0 rejected
  - "Not significant" □ failed to reject H_0
- Report the *t* statistic value including *df*, e.g., $t(12) = 3.65$
- Report significance level, either
  - $p < \text{alpha}$, e.g., $p < .05$ or
  - Exact probability, e.g., $p = .023$

9.4 Directional Hypotheses and One-tailed Tests

- *Non-directional* (two-tailed) test is most commonly used
- However, directional test may be used for particular research situations
- Four steps of hypothesis test are carried out
  - The critical region is defined in just one tail of the t distribution.

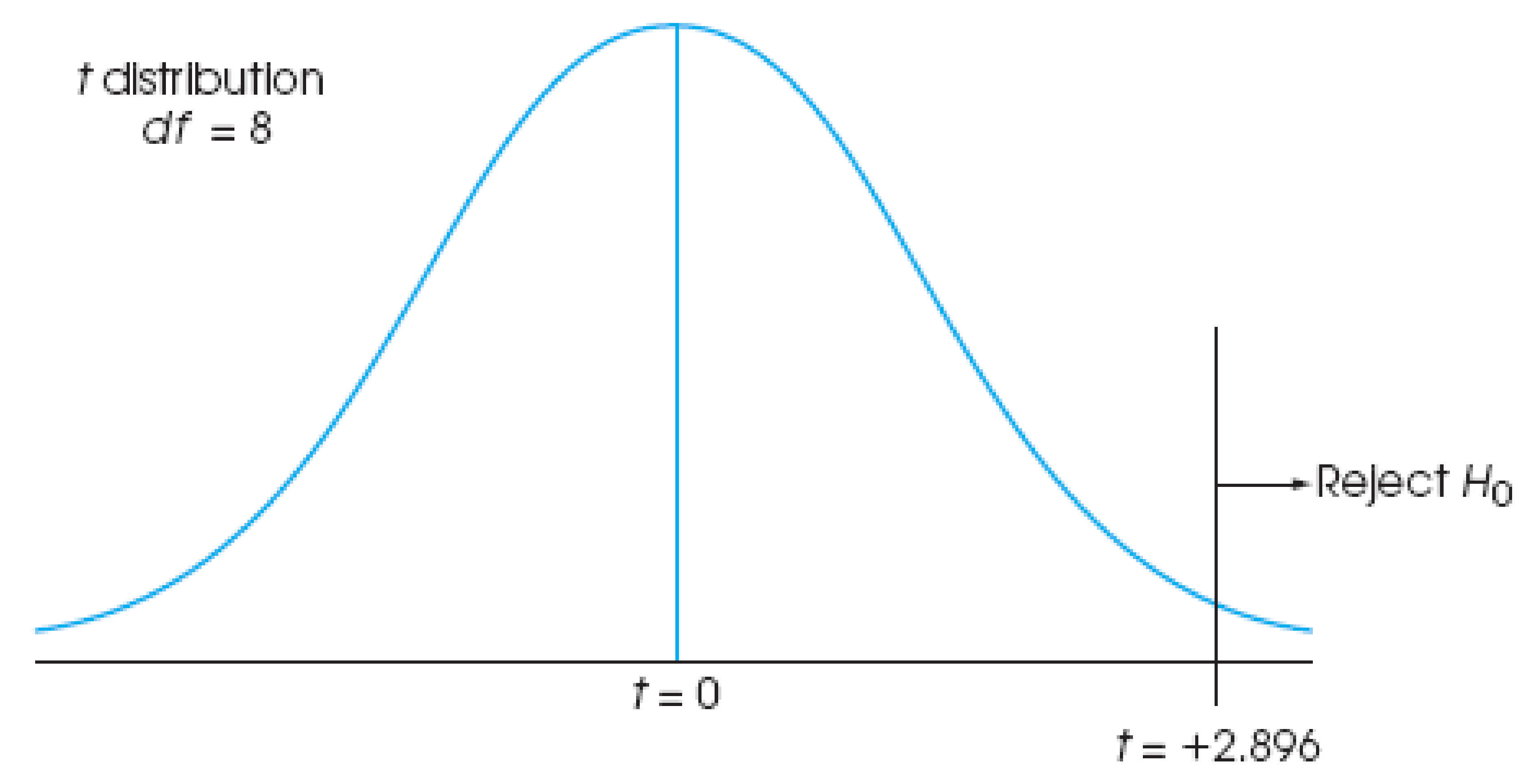

Figure 9.8
One-tailed Critical Region

Learning Check

- The results of a hypothesis test are reported as follows: $t(21) = 2.38$, $p < .05$. What was the statistical decision and how big was the sample?

A
- The null hypothesis was rejected using a sample of $n = 21$

B
- The null hypothesis was rejected using a sample of $n = 22$

C
- The null hypothesis was not rejected using a sample of $n = 21$

D
- The null hypothesis was not rejected using a sample of $n = 22$

Learning Check - Answer

- The results of a hypothesis test are reported as follows: $t(21) = 2.38$, $p < .05$. What was the statistical decision and how big was the sample?

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- The null hypothesis was not rejected using a sample of $n = 21$

D
- The null hypothesis was not rejected using a sample of $n = 22$

Learning Check

- Decide if each of the following statements is True or False

T/F	• Sample size has a great influence on measures of effect size
T/F	• When the value of the *t* statistic is near 0, the null hypothesis should be rejected

Learning Check - Answers

False

- Measures of effect size are not influenced to any great extent by sample size

False

- When the value of t is near 0, the difference between M and μ is also near 0

THANK YOU