t Statistics: Example Case

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Hypothesis Testing

EXAMPLE 9.2

Infants, even newborns, prefer to look at attractive faces compared to less attractive faces (Slater et al., 1998). In the study, infants from 1–6 days old were shown two photographs of women's faces. Previously, a group of adults had rated one of the faces as significantly more attractive than the other. The babies were positioned in front of a screen on which the photographs were presented. The pair of faces remained on the screen until the baby accumulated a total of 20 seconds of looking at one or the other. The number of seconds looking at the attractive face was recorded for each infant. Suppose that the study used a sample of $n = 9$ infants and the data produced an average of $M = 13$ seconds for the attractive face with $SS = 72$. Note that all the available information comes from the sample. Specifically, we do not know the population mean or the population standard deviation.

STEP 1 **State the hypotheses and select an alpha level.** Although we have no information about the population of scores, it is possible to form a logical hypothesis about the value of μ. In this case, the null hypothesis states that the infants have no preference for either face. That is, they should average half of the 20 seconds looking at each of the two faces. In symbols, the null hypothesis states

$$H_0\colon \mu_{\text{attractive}} = 10 \text{ seconds}$$

The alternative hypothesis states that there is a preference and one of the faces is preferred over the other. A directional, one-tailed test would specify which of the two faces is preferred, but the nondirectional alternative hypothesis is expressed as follows:

$$H_1\colon \mu_{\text{attractive}} \neq 10 \text{ seconds}$$

We will set the level of significance at $\alpha = .05$ for two tails.

STEP 2 **Locate the critical region.** The test statistic is a t statistic because the population variance is not known. Therefore, the value for degrees of freedom must be determined before the critical region can be located. For this sample

$$df = n - 1 = 9 - 1 = 8$$

For a two-tailed test at the .05 level of significance and with 8 degrees of freedom, the critical region consists of t values greater than $+2.306$ or less than -2.306. Figure 9.4 depicts the critical region in this t distribution.

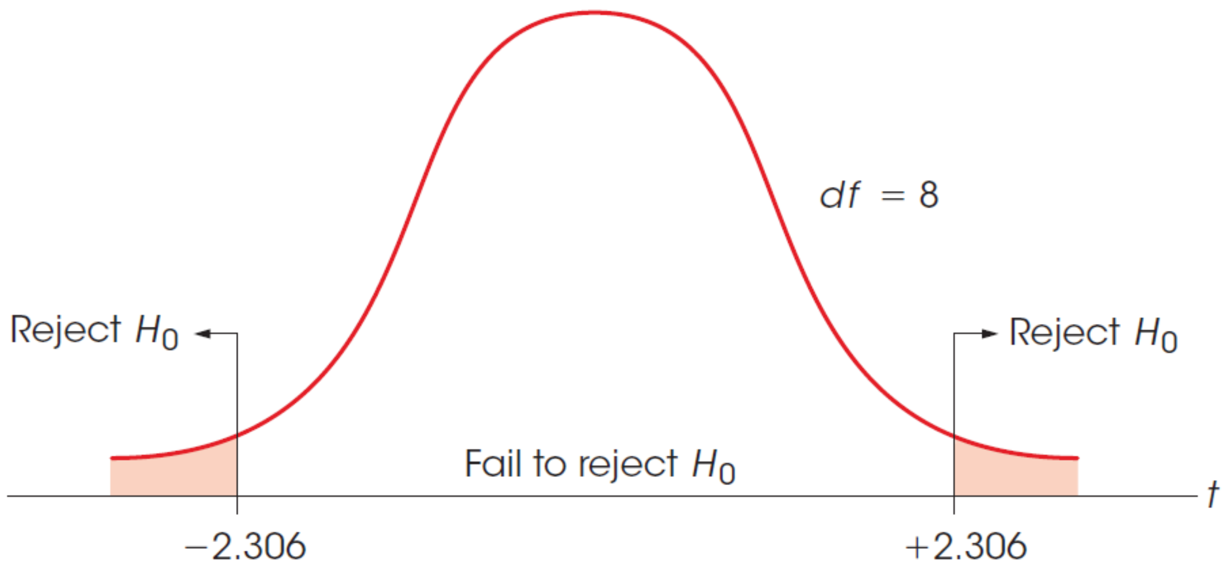


FIGURE 9.4
The critical region in the t distribution for $\alpha = .05$ and $df = 8$.

STEP 3 **Calculate the test statistic.** The t statistic typically requires more computation than is necessary for a z-score. Therefore, we recommend that you divide the calculations into a three-stage process as follows.

a. First, calculate the sample variance. Remember that the population variance is unknown, and you must use the sample value in its place. (This is why we are using a t statistic instead of a z-score.)

$$s^2 = \frac{SS}{n-1} = \frac{SS}{df} = \frac{72}{8} = 9$$

b. Next, use the sample variance (s^2) and the sample size (n) to compute the estimated standard error. This value is the denominator of the t statistic and measures how much difference is reasonable to expect by chance between a sample mean and the corresponding population mean.

$$s_M = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{9}{9}} = \sqrt{1} = 1$$

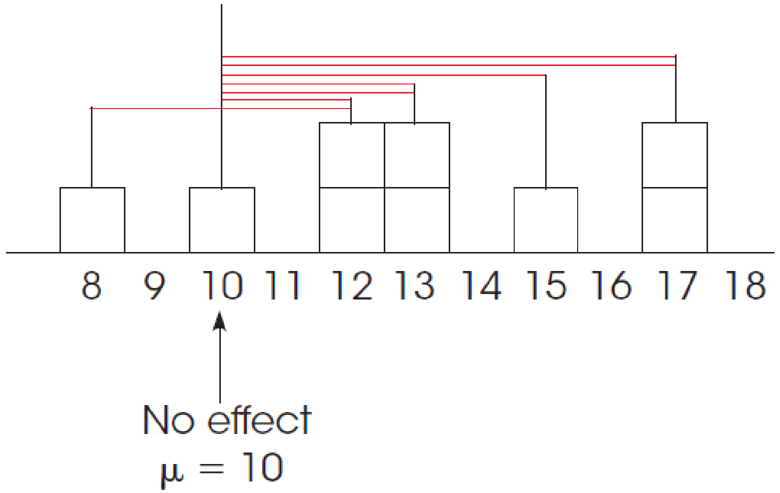
c. Finally, compute the t statistic for the sample data.

$$t = \frac{M - \mu}{s_M} = \frac{13 - 10}{1} = 3.00$$

STEP 4 **Make a decision regarding H_0.** The obtained t statistic of 3.00 falls into the critical region on the right-hand side of the t distribution (see Figure 9.4). Our statistical decision is to reject H_0 and conclude that babies do show a preference when given a choice between an attractive and an unattractive face. Specifically, the average amount of time that the babies spent looking at the attractive face was significantly different from the 10 seconds that would be expected if there were no preference. As indicated by the sample mean, there is a tendency for the babies to spend more time looking at the attractive face. ■

Percentage of Variance Accounted For by the Treatment (***r^2***)

(a) Original scores, including the treatment effect



(b) Adjusted scores with the treatment effect removed

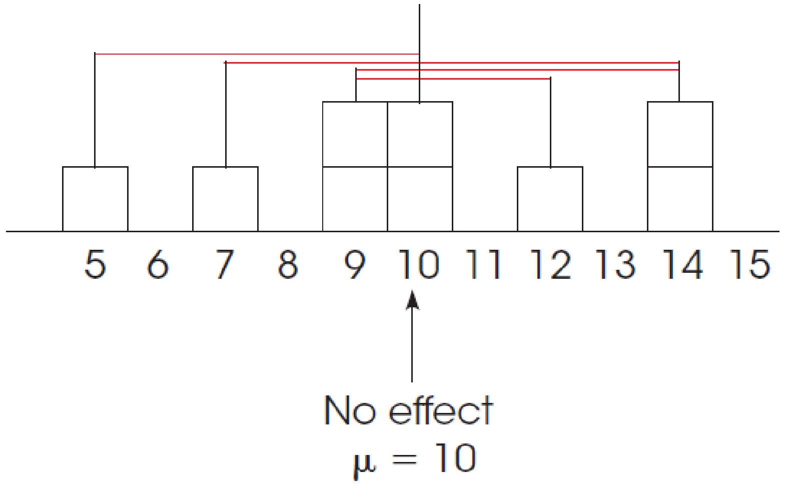


TABLE 9.2
Calculation of *SS*, the sum of squared deviations, for the data in Figure 9.6. The first three columns show the calculations for the original scores, including the treatment effect. The last three columns show the calculations for the adjusted scores after the treatment effect has been removed.

Calculation of *SS* including the treatment effect			Calculation of *SS* after the treatment effect is removed		
Score	Deviation from $\mu = 10$	Squared Deviation	Adjusted Score	Deviation from $\mu = 10$	Squared Deviation
8	−2	4	$8 - 3 = 5$	−5	25
10	0	0	$10 - 3 = 7$	−3	9
12	2	4	$12 - 3 = 9$	−1	1
12	2	4	$12 - 3 = 9$	−1	1
13	3	9	$13 - 3 = 10$	0	0
13	3	9	$13 - 3 = 10$	0	0
15	5	25	$15 - 3 = 12$	2	4
17	7	49	$17 - 3 = 14$	4	16
17	7	49	$17 - 3 = 14$	4	16
		$SS = 153$			$SS = 72$

$$\frac{\text{variability accounted for}}{\text{total variability}} = \frac{81}{153} = 0.5294 \quad (52.94\%)$$

Rather than computing r^2 directly by comparing two different calculations for SS, the value can be found from a single equation based on the outcome of the t test.

$$r^2 = \frac{t^2}{t^2 + df} \tag{9.5}$$

The letter r is the traditional symbol used for a correlation, and the concept of r^2 is discussed again when we consider correlations in Chapter 15. Also, in the context of t statistics, the percentage of variance that we are calling r^2 is often identified by the Greek letter omega squared (ω^2).

For the hypothesis test in Example 9.2, we obtained $t = 3.00$ with $df = 8$. These values produce

$$r^2 = \frac{3^2}{3^2 + 8} = \frac{9}{17} = 0.5294 \quad (52.94\%)$$