UNIVERSITAS INDONESIA

Veritas, Probitas, Iustitia

FAKULTAS ILMU KOMPUTER

# PENGANTAR STATISTIKA

## 03. Probability and Sample - Distribution of Sample Mean


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# LEARNING OBJECTIVES

- Define the distribution of sampling means
- Describe distribution by shape, expected value, and standard error
- Describe the location of sample mean $M$ by $z$-score
- Determine probabilities corresponding to the sample mean using $z$-scores, unit normal table

# What you need from prev. chapters

- Random sampling (Chapter 6)
- Probability and the normal distribution (Chapter 6)
- z-Scores (Chapter 5)

# Samples and Population

- The location of a score in a sample or in a population can be represented with a *z*-score
- Researchers typically want to study entire samples rather than single scores
- Sample provides estimate of the population
- Tests involve transforming sample mean to a *z*-score

# Sampling Error

- Error does not indicate a mistake was made
- Sampling error is the natural discrepancy, or the amount of error, between a sample statistic and its corresponding population
  parameter
- Samples are variable; two samples are *very, very rarely identical*

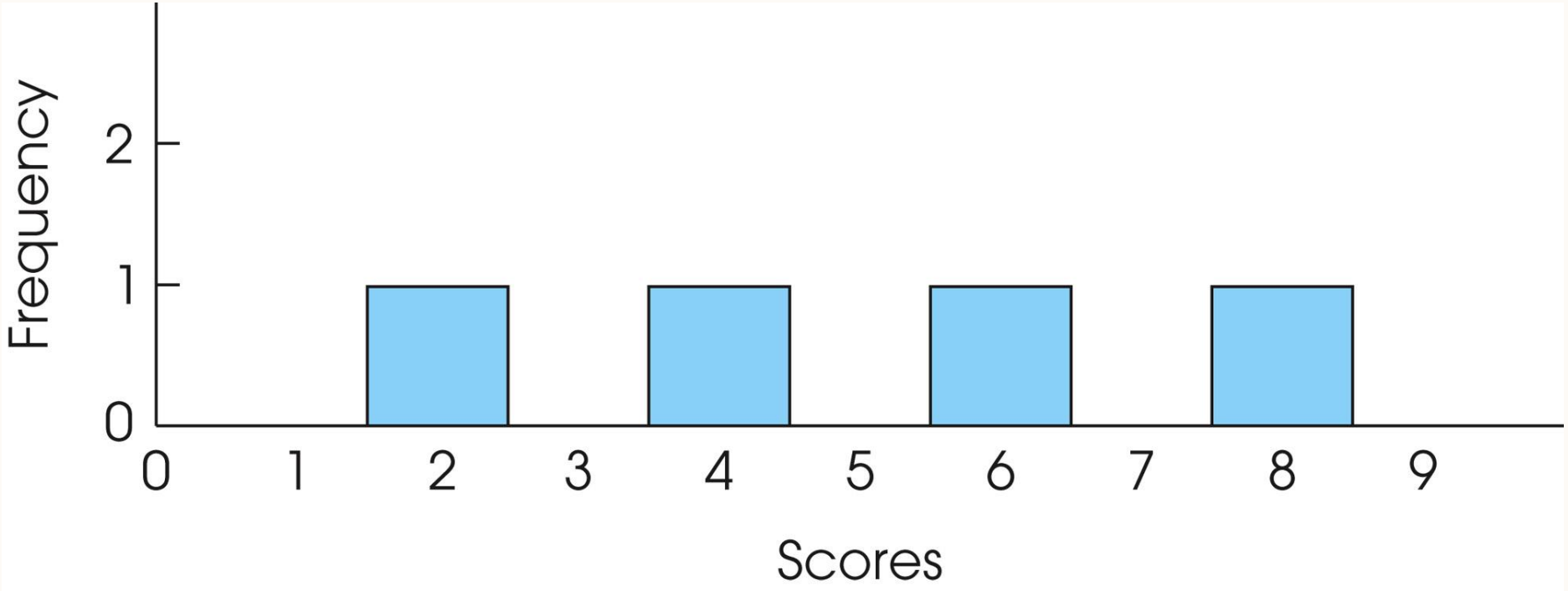
# Distribution of Sample Means

- Samples differ from each other
  - Given a random sample it is unlikely that sample means would always be the same
  - Sample means differ from each other
- The distribution of sample means is the collection of sample means for all the possible random samples of a particular size ($n$) that can be obtained from a population

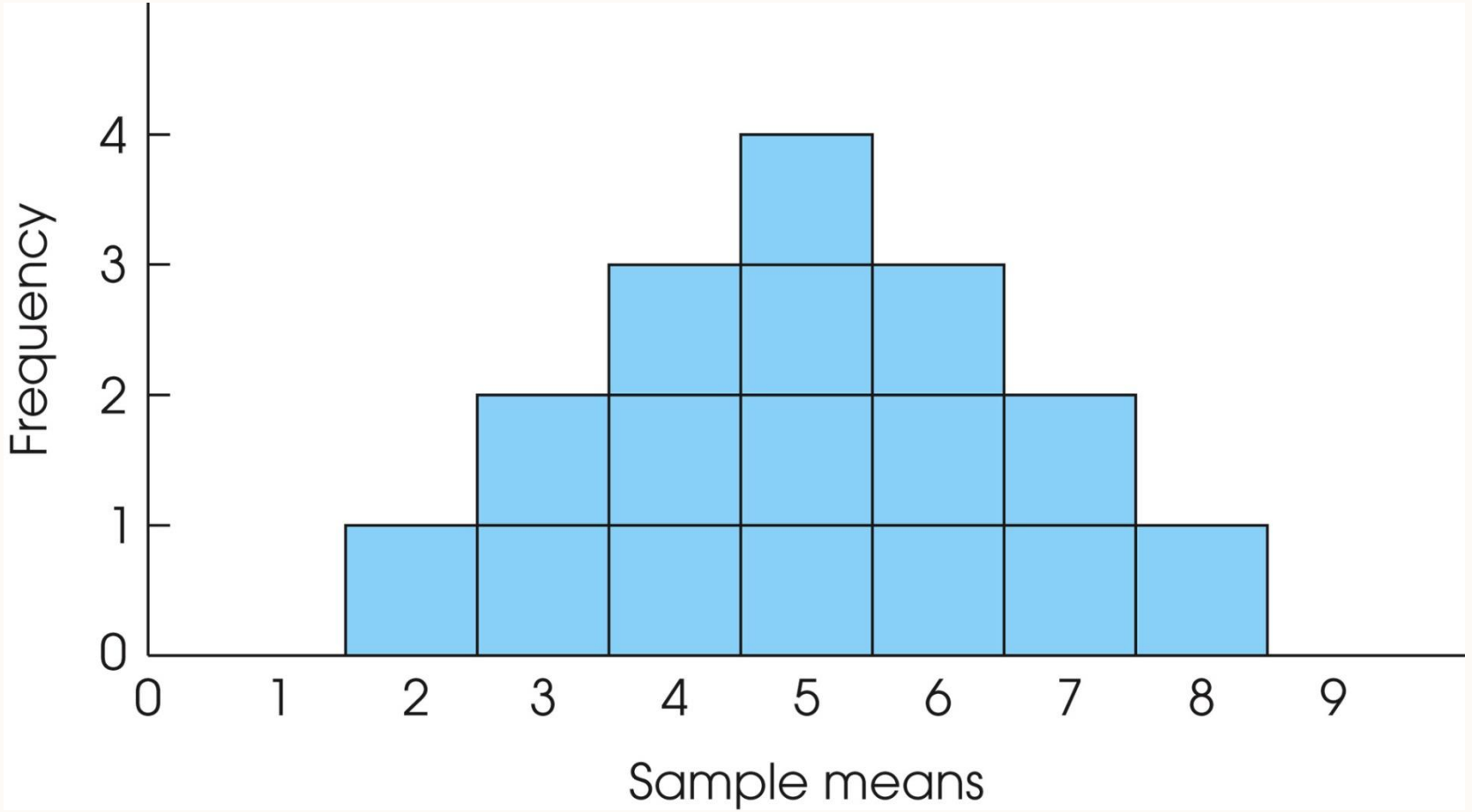
# Sampling distribution

- Distributions in earlier chapters were distributions of scores from samples
- "Distribution of Sample Means" is called a sampling distribution
  - The "Distribution of Sample Means" is a special kind of population
  - It is a distribution of sample means obtained by selecting all the possible samples of a specific size ($n$) from a population

# Figure 7.1 Population Frequency Distribution Histogram



# Figure 7.2 Distribution of Sample Means ($n$=2)



# Important Characteristics of Distributions of Sample Means

- The sample means pile up around the population mean
- The distribution of sample means is approximately normal in shape

# Central Limit Theorem

- Applies to any population with mean $\mu$ and standard deviation $\sigma$
- Distribution of sample means approaches a normal distribution as $n$ approaches infinity
- Distribution of sample means for samples of size $n$ will have a mean of $\mu_M$
- Distribution of sample means for samples of size $n$ will have a standard deviation = $\sigma / \sqrt{n}$

# Shape of the Distribution of Sample Means

- The distribution of sample means is almost perfectly normal in either of two conditions
  - The population from which the samples are selected is a normal distribution
    or
  - The number of scores ($n$) in each sample is relatively large—at least 30

# Expected Value of M

- Mean of the distribution of sample means is $\mu_M$ and has a value equal to the mean of the population of scores, $\mu$
- Mean of the distribution of sample means is called the ***expected value*** of *M*
- *M* is an *unbiased statistic* because $\mu_M$ , the expected value of the distribution of sample means is the value of the population mean, $\mu$

# Standard Error of M

- Variability of a distribution of *scores* is measured by the standard deviation
- Variability of a distribution of sample *means* is measured by the standard deviation of the sample means, and is called the *standard error of M* and written as $\sigma_M$
- In journal articles or other textbooks, the standard error of *M* might be identified as "standard error," "*SE*," or "*SEM*"

# Standard Error of M

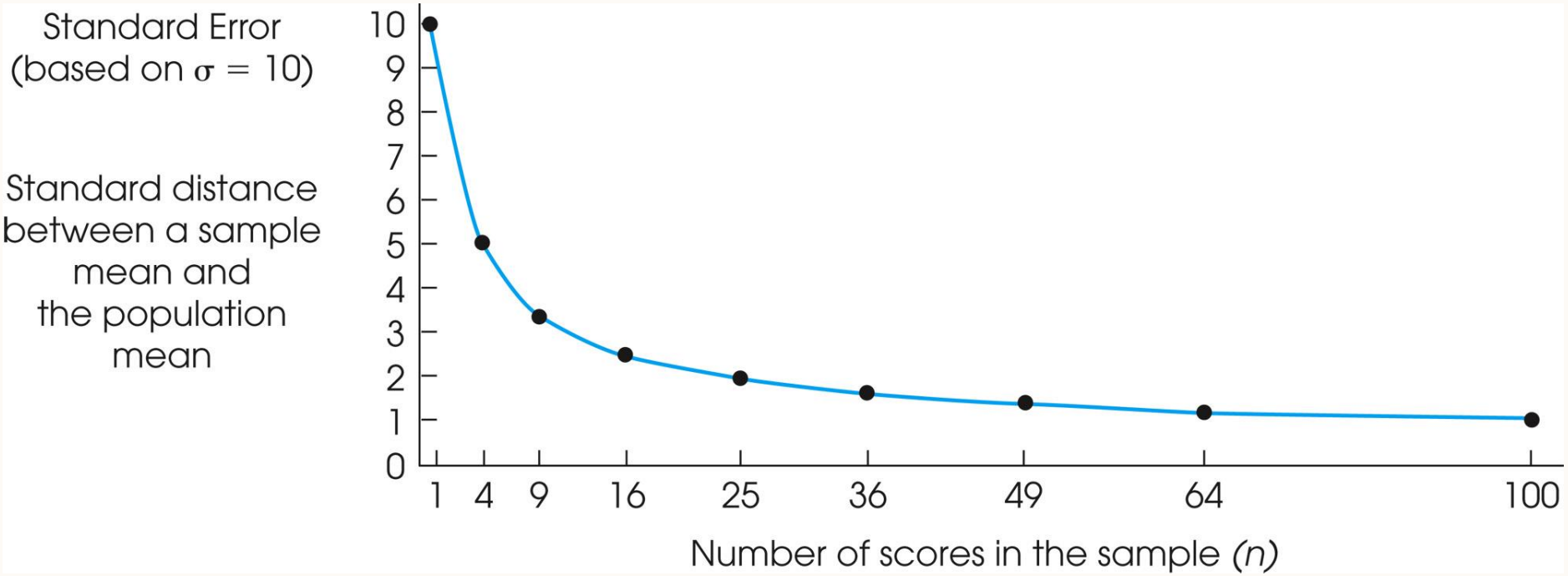
- The *standard error of M* is the standard deviation of the distribution of sample means
- The *standard error of M* provides a measure of how much distance is expected on average between M and $\mu$

# Standard Error Magnitude

- Law of large numbers: the larger the *sample size*, the more probable it is that the sample mean will be close to the population mean

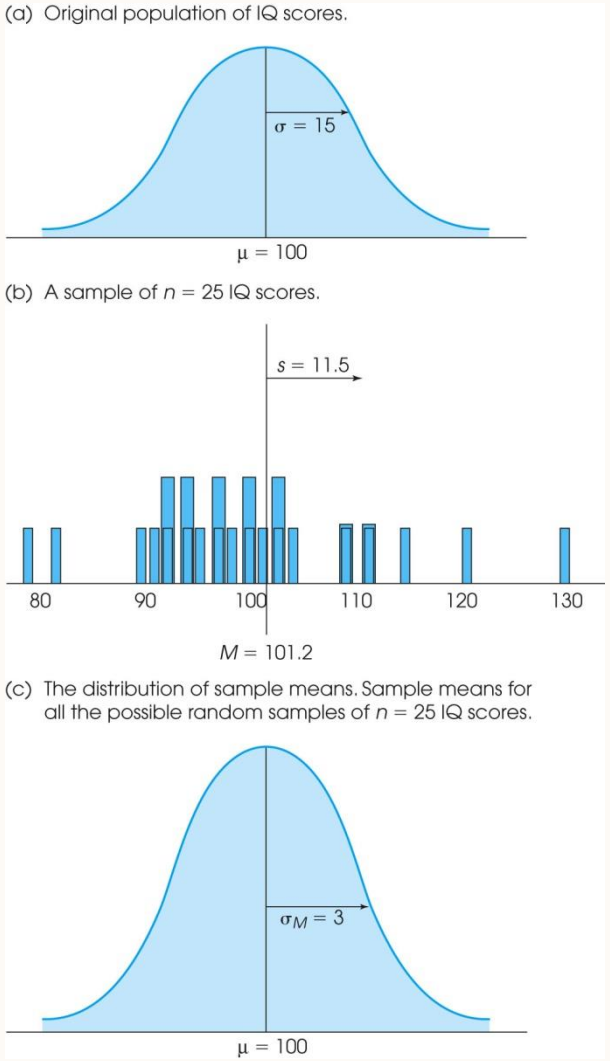
- Population variance: The smaller the *variance* in the population, the more probable it is that the sample mean will be close to the population mean

# Figure 7.3 Standard Error and Sample Size Relationship



# Figure 7.4 Distributions of Scores vs. Sample Means

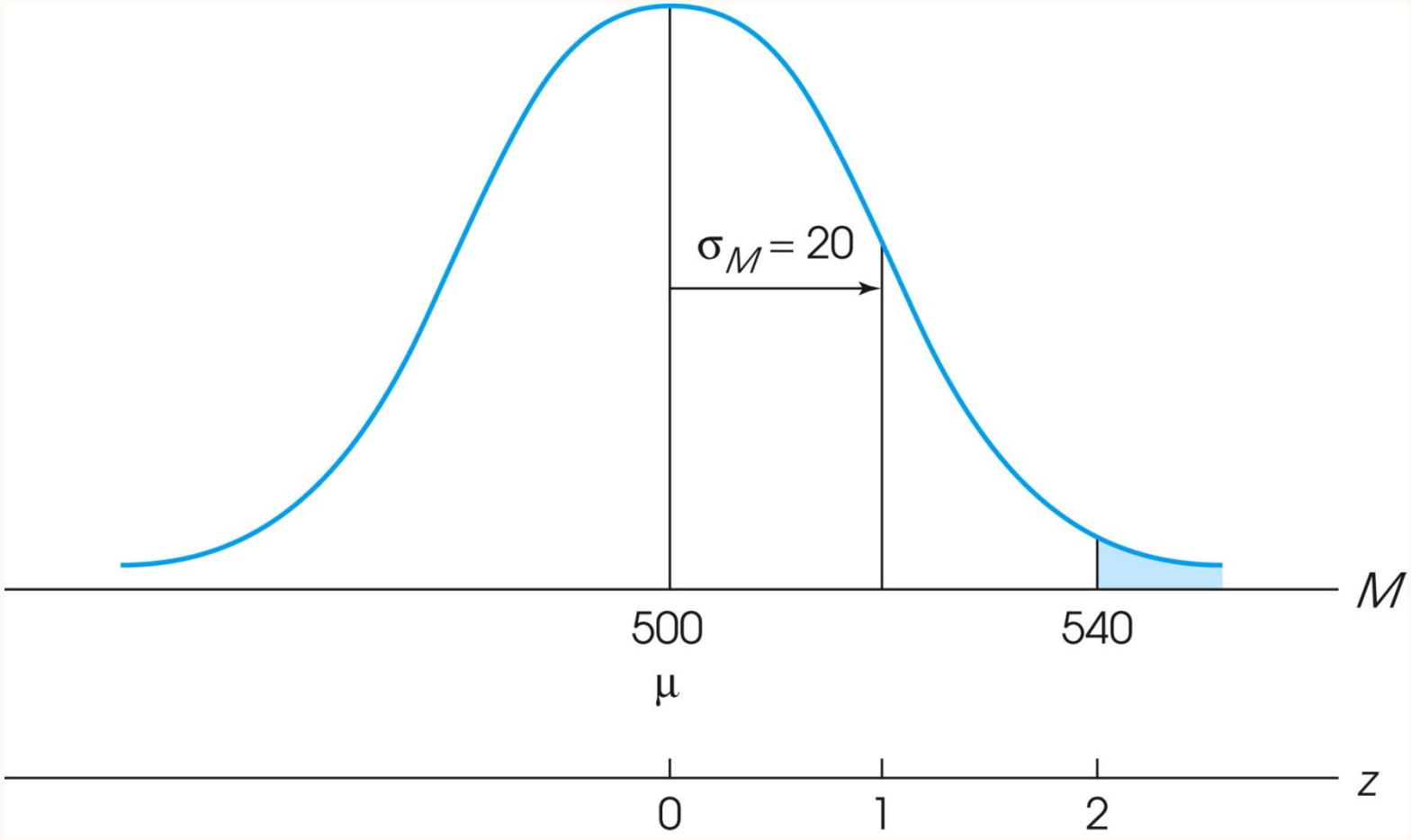




# Probability and the Distribution of Sample Means

- Primary use of the distribution of sample means is to find the probability associated with any particular sample (sample mean)
- Proportions of the normal curve are used to represent probabilities
- A $z$-score for the sample mean is computed

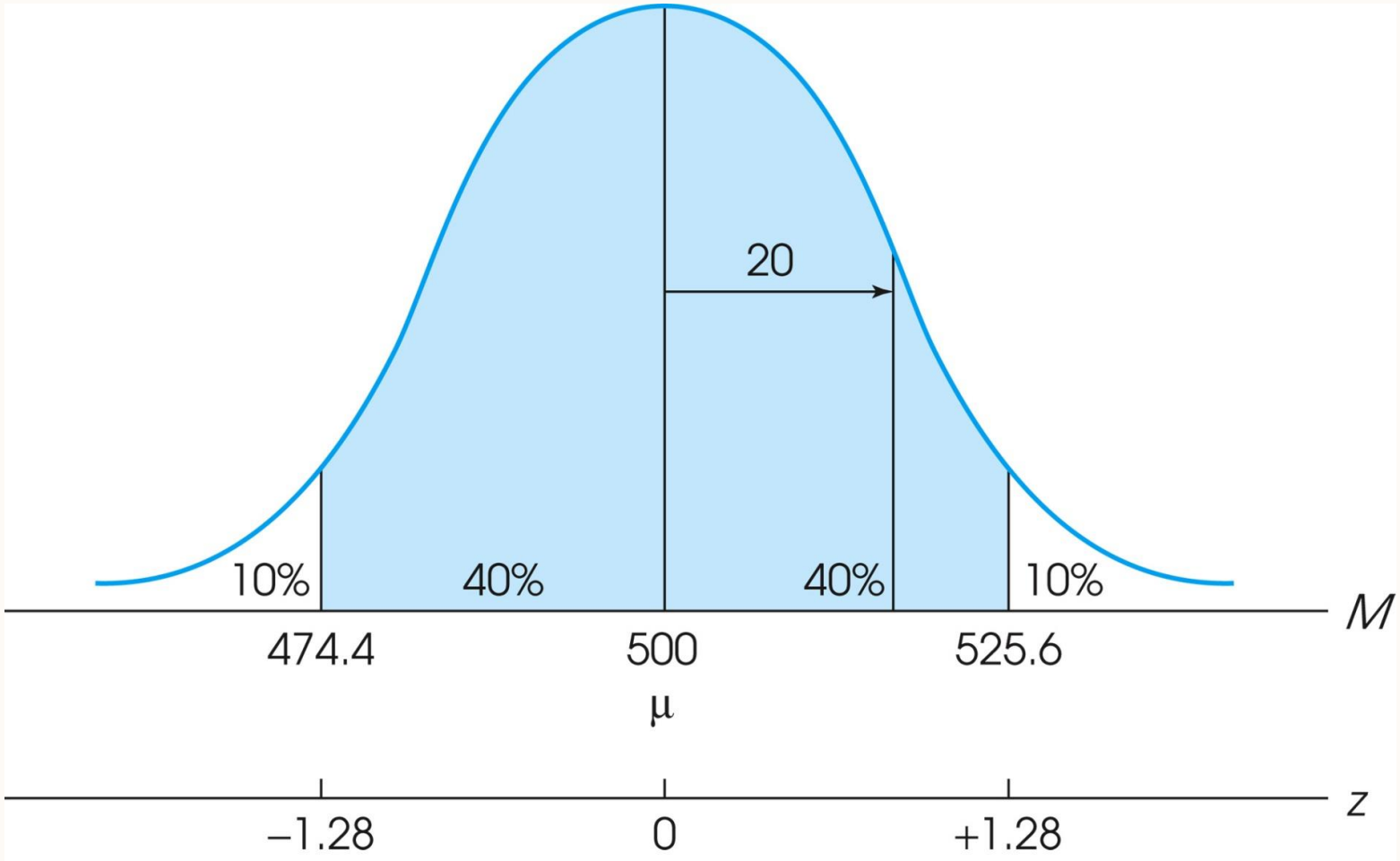
# Figure 7.5 Distribution of Sample Means for $n = 25$



# A *z*-Score for Sample Means

- Sign tells whether the location is above (+) or below (-) the mean
- Number tells the distance between the location and the mean in standard deviation (standard error) units
- *z*-formula: $$z = \frac{M - \mu}{\sigma_M}$$

# Figure 7.6 Middle 80% of the Distribution of Sample Means



# Learning Check

- A population has $\mu = 60$ with $\sigma = 5$; the distribution of sample means for samples of size $n = 4$ selected from this population would have an expected value of _____

A • 5

B • 60

C • 30

D • 15

# Learning Check - Answer

- A population has $\mu = 60$ with $\sigma = 5$; the distribution of sample means for samples of size $n = 4$ selected from this population would have an expected value of _____

| | |
|---|---|
| A | • 5 |
| B | • **60** |
| C | • 30 |
| D | • 15 |

# Learning Check

- Decide if each of the following statements is True or False

T/F

- The shape of a distribution of sample means is always normal

T/F

- As sample size increases, the value of the standard error decreases

# Learning Check - Answer

**False**

- The shape is normal *only* if the population is normal or $n \geq 30$

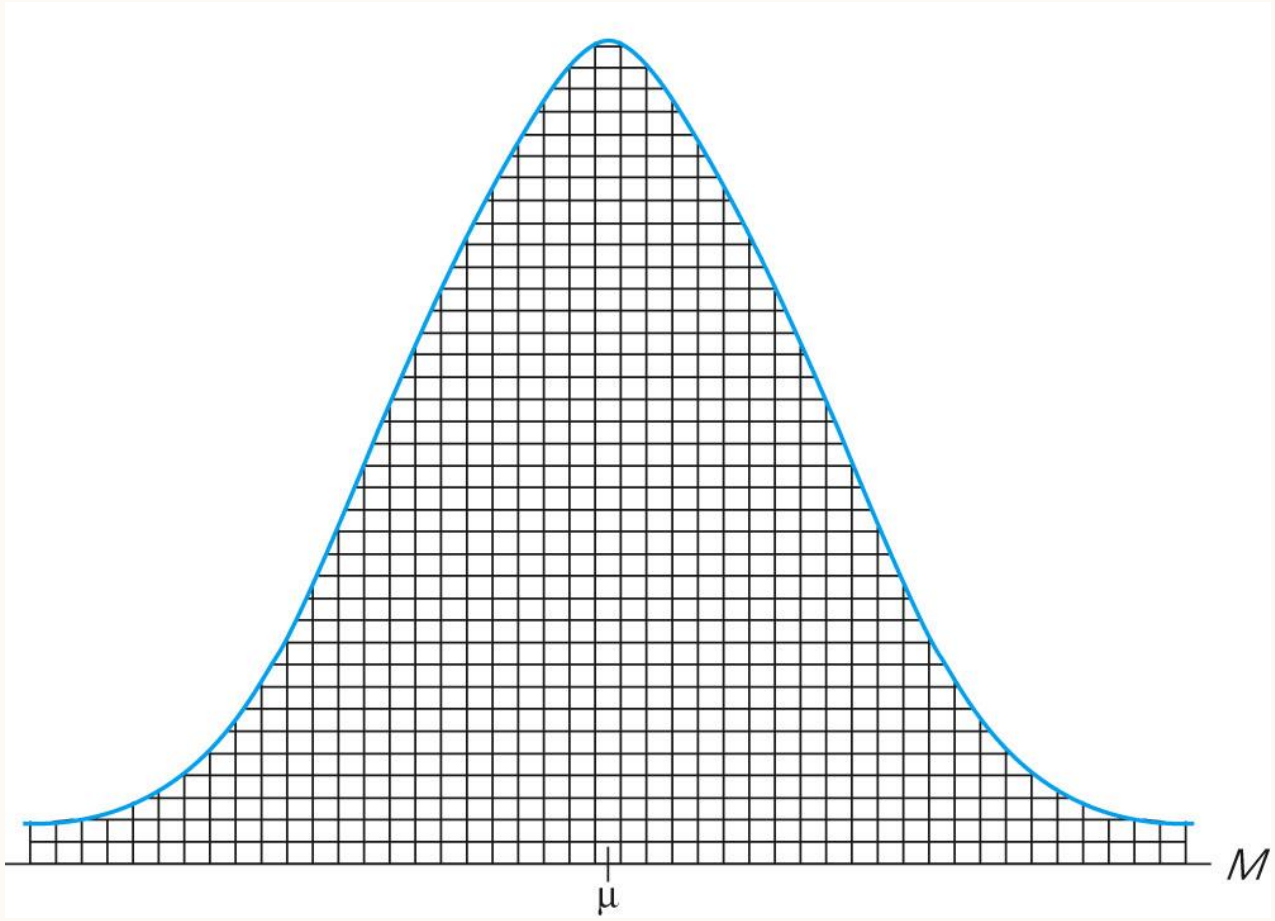
**True**

- Sample size is in the denominator of the equation so as *n* grows larger, standard error decreases

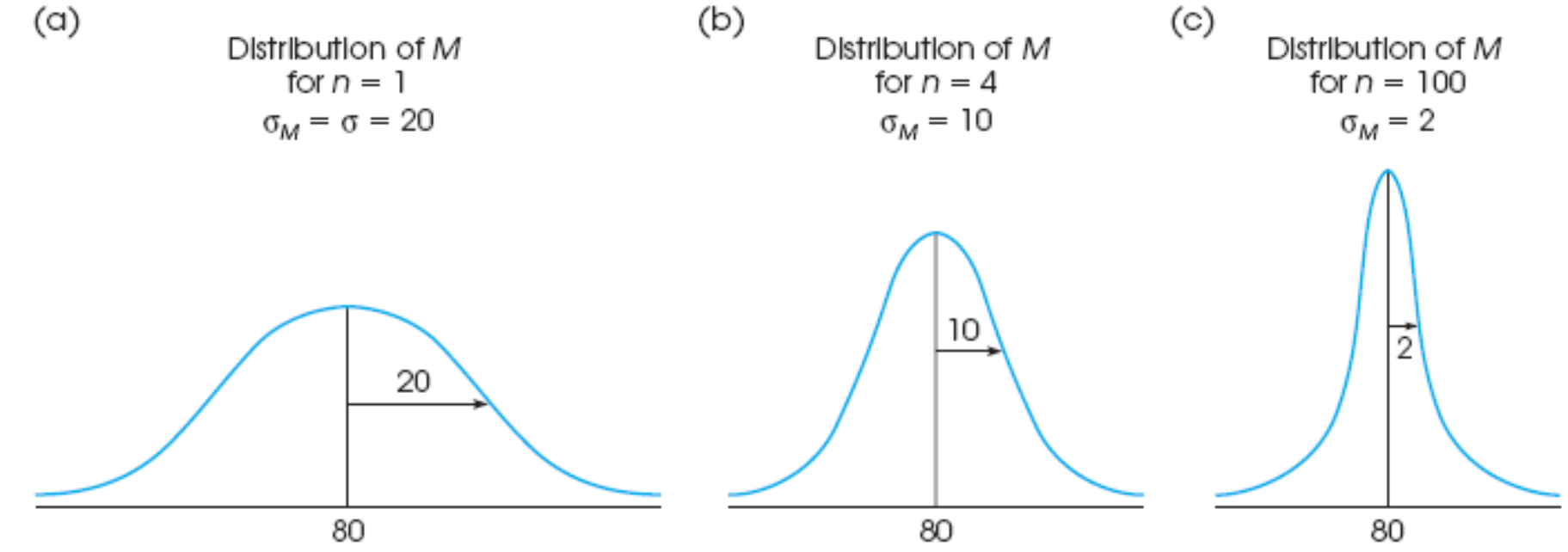
# More about Standard Error

- There will usually be discrepancy between a sample mean and the true population mean
- This discrepancy is called *sampling error*
- The amount of sampling error varies across samples
- The variability of sampling error is measured by the standard error of the mean

# Figure 7.7 Example of typical distribution of sample means



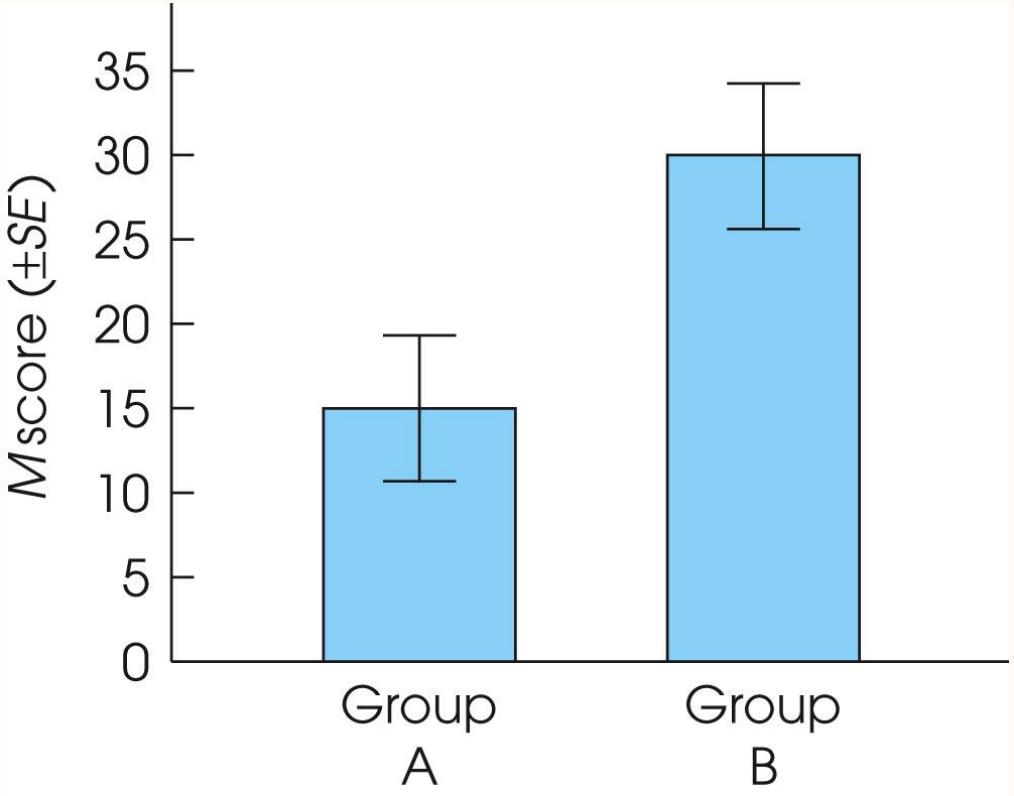
# Figure 7.8 Distribution of Sample Means when $n = 1$, 4, and 100



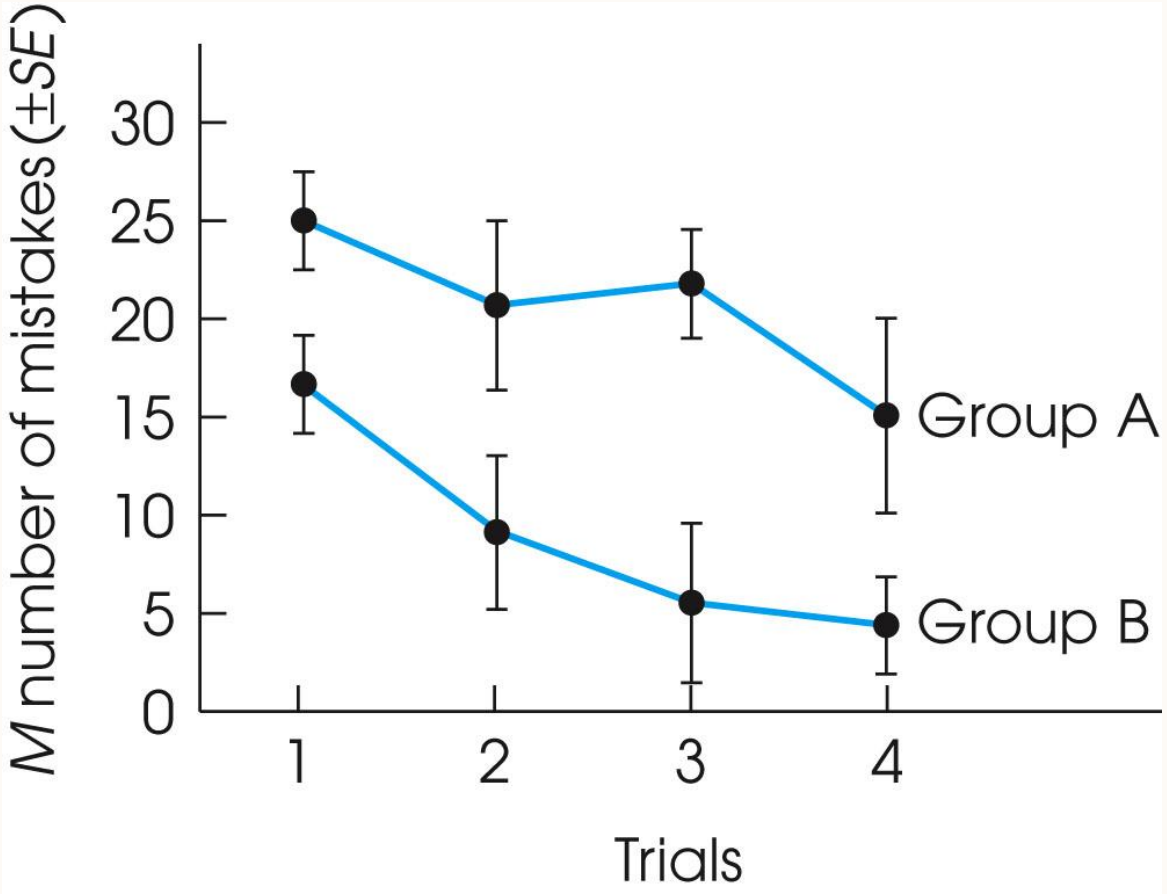
# In the Literature

- Journals vary in how they refer to the standard error but frequently use:
  - *SE*
  - *SEM*
- Often reported in a table along with *n* and *M* for the different groups in the experiment
- May also be added to a graph

# Figure 7.9
# Mean (±1 *SE*) in Bar Chart



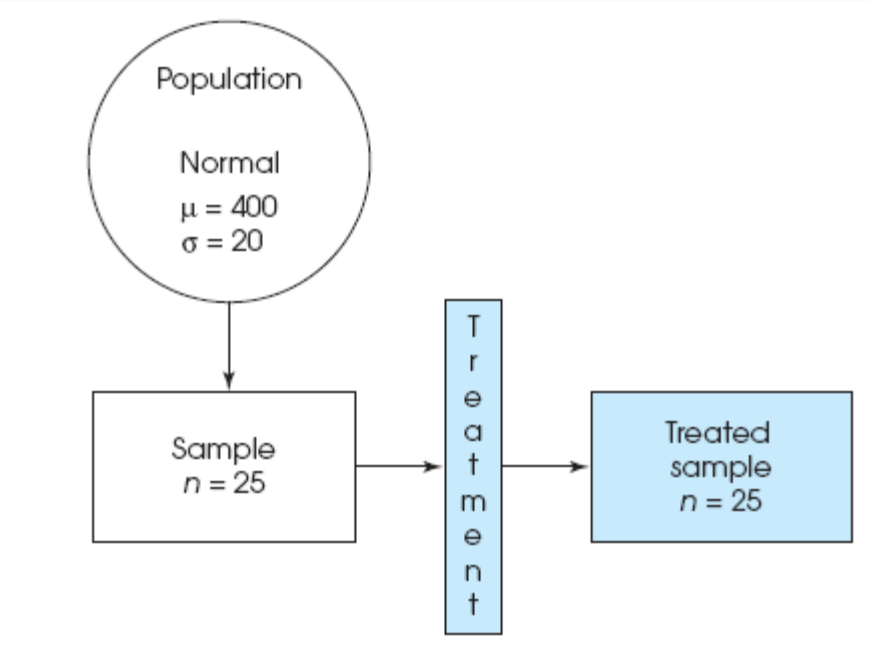
# Figure 7.10
# Mean (±1 *SE*) in Line Graph



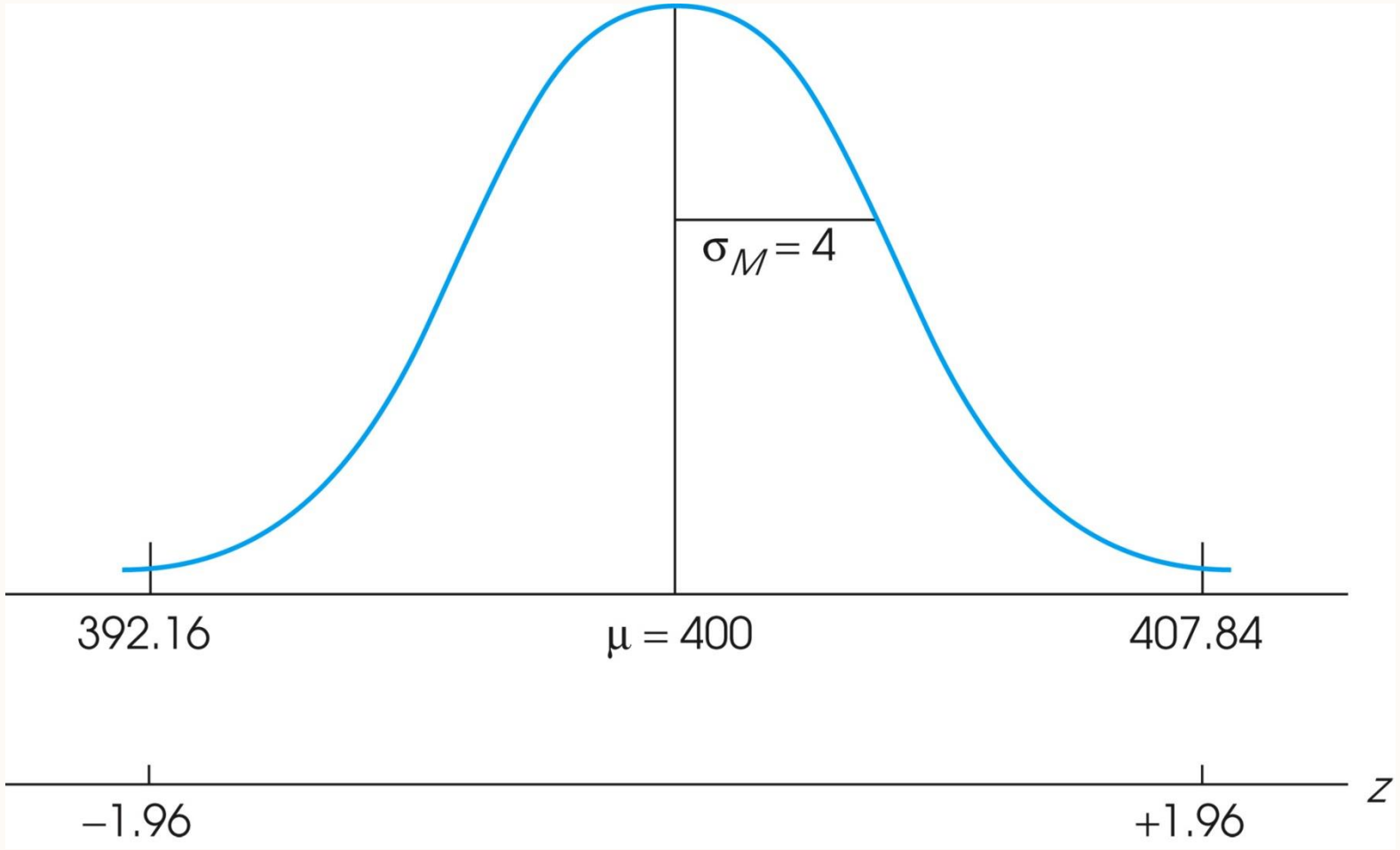
# Looking Ahead to Inferential Statistics

- Inferential statistics use sample data to draw general conclusions about populations
  - Sample information is not a perfectly accurate reflection of its population (sampling error)
  - Differences between sample and population introduce uncertainty into inferential processes
- Statistical techniques use probabilities to draw inferences from sample data

# Figure 7.11 Conceptualization of research study in Example 7.5



# Figure 7.12 Untreated Sample Means from Example 7.5



# Learning Check

- A random sample of $n = 16$ scores is obtained from a population with $\mu = 50$ and $\sigma = 16$. If the sample mean is $M = 58$, the $z$-score corresponding to the sample mean is ____?

| | |
|---|---|
| A | • $z = 1.00$ |
| B | • $z = 2.00$ |
| C | • $z = 4.00$ |
| D | • Cannot determine |

# Learning Check - Answer

- A random sample of $n = 16$ scores is obtained from a population with $\mu = 50$ and $\sigma = 16$. If the sample mean is $M = 58$, the $z$-score corresponding to the sample mean is ____?

| | |
|---|---|
| A | • $z = 1.00$ |
| B | • $z = 2.00$ |
| C | • $z = 4.00$ |
| D | • Cannot determine |

# Learning Check

- Decide if each of the following statements is True or False

T/F

- A sample mean with $z = 3.00$ is a fairly typical, representative sample

T/F

- The mean of the sample is always equal to the population mean

# Learning Check - Answers

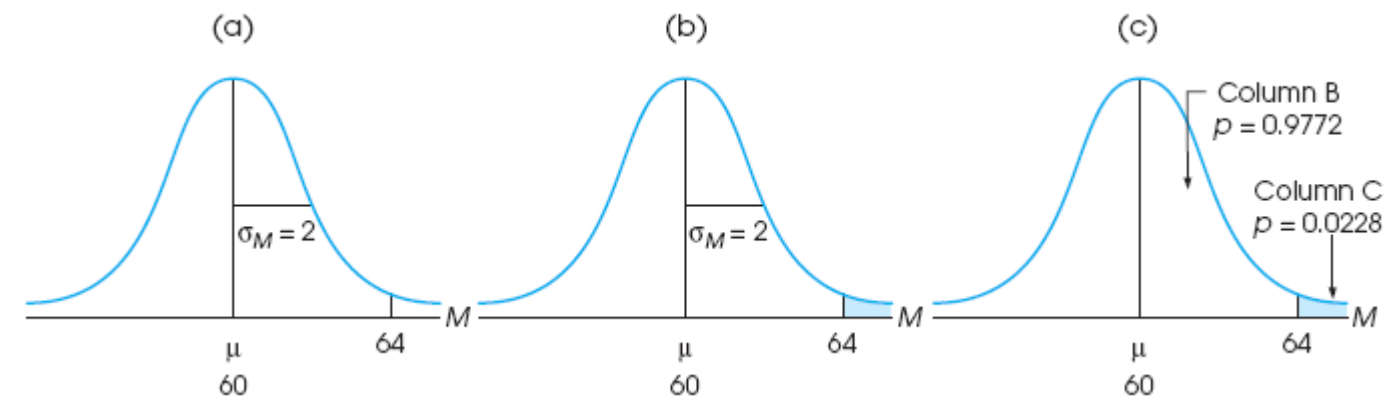
**False**

- A *z*-score of 3.00 is an extreme, or unlikely, *z*-score

**False**

- Individual samples will vary from the population mean

# Figure 7.13 Sketches of Distribution in Demonstration 7.1



# THANK YOU