Основной блок задач

№1

1) $$A_1A_2 = \begin{pmatrix} 0+i & -i\cdot 0+0 \\ 0+0 & 0 + -i \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$A_2A_3 = \begin{pmatrix} 0+0 & 0+i \\ i+0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$A_1\cdot A_3 = \begin{pmatrix} 0+0 & 0-1 \\ 1+0 & 0+0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$A_1A_2A_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} i+0 & 0+0 \\ 0+0 & 0+i \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$A_1A_1 = \begin{pmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{pmatrix} = \cancel{\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A_2A_2 = \begin{pmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{pmatrix} = \cancel{\begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A_3A_3 = \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

№2

$$A_3\cdot u_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$A_1 + iA_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$(A_1+iA_2)u_1 = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+0 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A_1 - iA_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$

$$(A_1-iA_2)u_1 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+0 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$A_3 \cdot u_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+0 \\ 0-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$(A_1 + i A_2) u_2 = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+2 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$(A_1 - i A_2) u_2 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+0 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

3)

$$B_1 = \begin{pmatrix} \sqrt{\frac{i}{2}} & \sqrt{\frac{1}{2i}} \\ \sqrt{\frac{1}{2i}} & \sqrt{\frac{i}{2}} \end{pmatrix}$$

№2

$$\frac{5i}{8-2i} = \frac{5i(8+2i)}{64+4} = \frac{40i-10}{68}$$

$$\overline{\frac{5i}{8-2i}} = \frac{-20i \mp 5}{34}$$

$$\left|\frac{-5-20i}{34}\right| = \sqrt{\frac{25+400}{34^2}} = \sqrt{\frac{12,5}{34}} = \sqrt{\frac{25}{68}} = \frac{5}{2\sqrt{17}}$$

$$\varphi = -\pi + \operatorname{arctg}\frac{-20}{-5} = -\pi + \operatorname{arctg} 4$$

Применим формулу Эйлера:

$$\frac{e^{i\omega t}}{e^{-\lambda t}} = \frac{\cos\omega t + i\sin\omega t}{e^{-\lambda t}}$$

$$\overline{\frac{e^{i\omega t}}{e^{-\lambda t}}} = \frac{\cos\omega t - \sin\omega t}{e^{-\lambda t}}$$

$$\left|\frac{\cos\omega t - \sin\omega t}{e^{-\lambda t}}\right| = \sqrt{\frac{\cos^2\omega t + \sin^2\omega t}{e^{-\lambda t}}} = \sqrt{e^{\lambda t}}$$

$$\varphi = \omega t$$

$$z^6 + 1 = 0 \iff z^6 - i^2 = 0 \Rightarrow (z^3 - i)(z^3 + i) = 0$$

$$(z^3 + i^3)(z^3 - i^3) = 0 \iff (z+i)(z^2 - zi - 1)(z - i)(z^2 + zi - 1)$$

$z^2 - zi - 1$

$D = i^2 + 4 = 3$

$z_{1,2} = \frac{i \pm \sqrt{3}}{2}$

$z^2 + zi - 1$

$D = i^2 + 4 = 3$

$z_{1,2} = \frac{-i \pm \sqrt{3}}{2}$

$$\left[\begin{array}{l} z = i \\ z = -i \\ z = \frac{\sqrt{3}}{2} - \frac{i}{2} \\ z = -\frac{\sqrt{3}}{2} - \frac{i}{2} \\ z = \frac{\sqrt{3}}{2} + \frac{i}{2} \\ z = -\frac{\sqrt{3}}{2} + \frac{i}{2} \end{array}\right.$$

Мнимые части:

1

-1 самое маленькое

$-\frac{1}{2}$

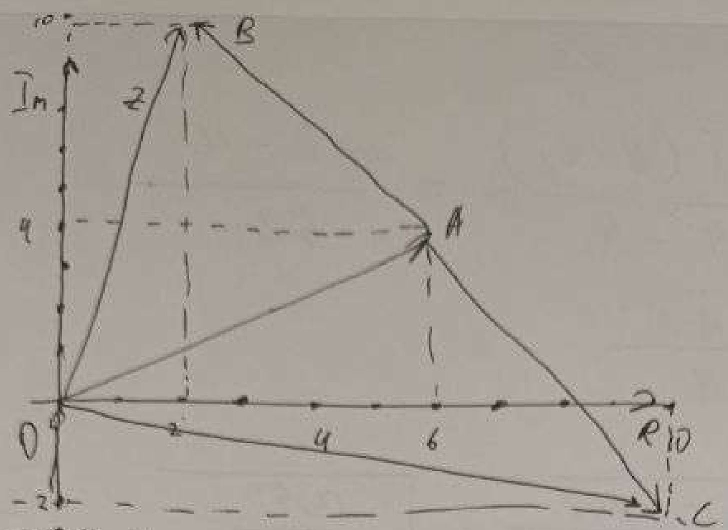
$-\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

Ответ: $z = -i$

б)



Докажем, что $\angle AOB = 45^\circ$ и $\angle AOC$ тоже

$BA = OA = AC = \sqrt{36+16}$

$BA \perp OA$ $\left(\frac{4}{6} \cdot \left(-\frac{6}{4}\right) = -1 \right.$ − признак $\perp$ прямых $\left.\right) \Rightarrow$

$\Rightarrow \Delta BAO$ и ΔOAC − п/у и р/б $\Rightarrow \angle BOA = \angle AOC = 45^\circ$

$OB = OC = \sqrt{100+4} = \sqrt{104}$ $\quad AC = \sqrt{52}$

Переведём OA в OB: $(6+4i)(a+bi) = (2+10i)$

$$\begin{cases} 6a - 4b = 2 \\ 4a + 6b = 10 \end{cases} \Rightarrow \begin{matrix} a = 1 \\ b = 1 \end{matrix}$$

Переведём OA в OC:

$(6+4i)(a+bi) = -2i + 10$

$$\begin{cases} 6a - 4b = 10 \\ 4a + 6b = -2 \end{cases} \Rightarrow \begin{matrix} a = 1 \\ b = -1 \end{matrix}$$

Значит есть 2 варианта: $\dfrac{(1+i)\sqrt{52}}{\sqrt{104}} = \dfrac{1+i}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}$

или $\dfrac{(1-i)\sqrt{52}}{\sqrt{104}} = \dfrac{1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}$

$N3$

1. $$\left.\begin{array}{l} a = \frac{d^2x}{dt^2} \\ F_{сопр} = -b\frac{dx}{dt} \\ F_{упр} = -kx \end{array}\right\} \Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

2. $\omega_0 = \sqrt{\frac{k}{m}}$ $\qquad \gamma = \frac{b}{2m}$

$$x(t) = A e^{-\gamma t}\cos\left(\sqrt{\omega_0^2 - \gamma^2}\, t + \varphi\right)$$

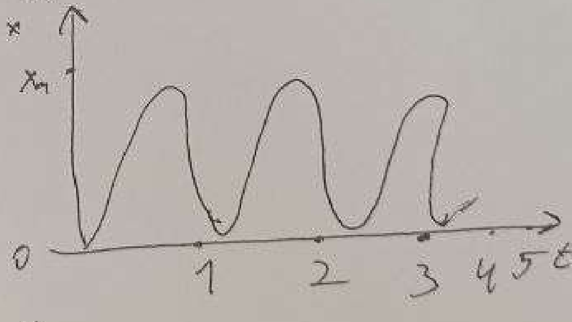
3. $x(0) = A\cos(\varphi) = x_0$

$v(0) = -A\gamma\cos\varphi + A\sqrt{\omega_0^2-\gamma^2}\cdot\sin(\varphi) = v_0$

4. $\omega_0 = \sqrt{\frac{4}{0,1}} = 2\sqrt{10}$ рад/с

$\gamma = \frac{0,2}{0,2} = 1\,с^{-1}$

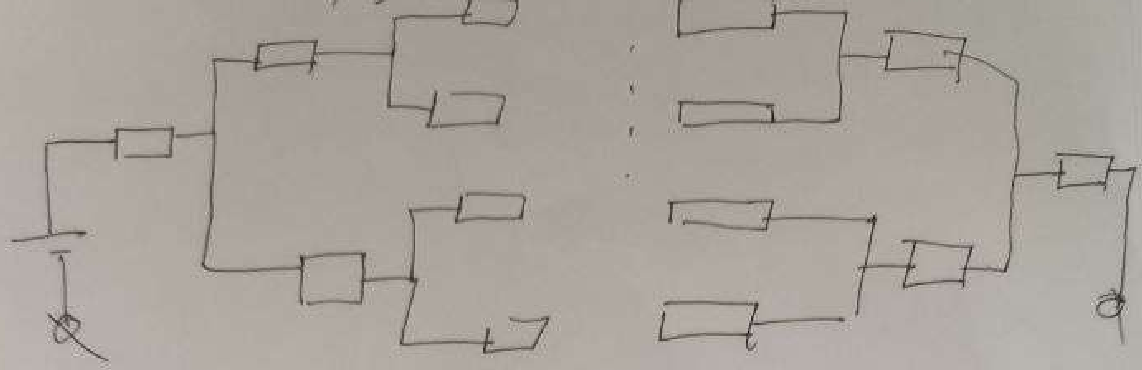
$\sqrt{\omega_0^2-\gamma^2} = \sqrt{39} = 6,24$ рад/с



$T = \frac{2\pi}{\sqrt{\omega_0^2-\gamma^2}} = \frac{2\pi}{6,24} \approx 1\,с$

$$\Delta E = \int_0^2 \frac{b}{2}\left(\frac{dx}{dt}\right)^2 dt$$

№ 5



$$R_N = \frac{R_{N-1}}{2} + 2$$

$$2R_N = R_{N-1} + 4$$

$$2R_N = \frac{R_{N-2}}{2} + 2 + 4$$

$$4R_N = R_{N-2} + 4 + 8$$

$$8R_N = R_{N-3} + 4 + 8 + 16$$

$$\vdots$$

$$2^i R_N = R_{N-i} + 2^2 + 2^3 + 2^4 + \ldots 2^{i+1} = R_{N-i} + 2^{i+2} - 1 - 1 - 2$$

$$2^i R_N = R_{N-i} + 2^{i+2} - 4$$ Подставим $i = N-1$

$$2^{N-1} R_N = R_1 + 2^{N+1} - 4$$

$$R_1 = 2R$$

$$R_N = \frac{2R}{2^{N-1}} + \frac{2^{N+1}}{2^{N-1}} - \frac{4}{2^{N-1}} = \frac{R}{2^{N-2}} + 4 - \frac{1}{2^{N-3}}$$

Ответ: $\frac{R}{2^{N-2}} + 4 - \frac{1}{2^{N-3}} = R_{\Sigma}$

5. $\cancel{t_{yy}} = \frac{d}{v_0} = \frac{0,02}{5} = 0,004\,c$

N 7

1) $V = I_1(R_1 + j\omega L_1) + j\omega M I_2$

$j\omega M I_1 = -I_2(R_2 + j\omega L_2)$

$I_2 = \frac{j\omega M V}{-\omega^2 M^2 - (R_1 + j\omega L_1)}$

$I_1 = +\frac{R_2 + j\omega L_2 (R_2 + j\omega L_2)}{\omega^2 M^2 + (R_1 + j\omega L_1) \cdot (R_2 + j\omega L_2)}$

2) $P_L = I_2^2 R_L$

$P_{ист} = I_1^2 R_1 + \frac{V^2}{R_{обм}}$

$\eta = \frac{P_L}{P_{ист}}$

3) $f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{(2\pi f_0)^2 L}$

$C_1 = \frac{1}{(2\pi \cdot 200 \cdot 10^3)^2 \cdot 100 \cdot 10^{-6}} = \frac{1}{16\pi^2 \cdot 1000000}$

$C_2 = \frac{1}{(2\pi \cdot 200 \cdot 10^3)^2 \cdot 40 \cdot 10^{-6}} = \frac{1}{64\pi^2 \cdot 100000}$

4) $\cancel{P_L = I_2^2, P_0 P_{ист}} =$

$P_L = I_2^2 \cdot 10$

$P_{ист} = I_1^2 R_1 + \frac{V^2}{R}$