Supervised Learning - Foundations Project: ReCell

Problem Statement

Business Context

Buying and selling used phones and tablets used to be something that happened on a handful of online marketplace sites. But the used and refurbished device market has grown considerably over the past decade, and a new IDC (International Data Corporation) forecast predicts that the used phone market would be worth $52.7bn by 2023 with a compound annual growth rate (CAGR) of 13.6% from 2018 to 2023. This growth can be attributed to an uptick in demand for used phones and tablets that offer considerable savings compared with new models.

Refurbished and used devices continue to provide cost-effective alternatives to both consumers and businesses that are looking to save money when purchasing one. There are plenty of other benefits associated with the used device market. Used and refurbished devices can be sold with warranties and can also be insured with proof of purchase. Third-party vendors/platforms, such as Verizon, Amazon, etc., provide attractive offers to customers for refurbished devices. Maximizing the longevity of devices through second-hand trade also reduces their environmental impact and helps in recycling and reducing waste. The impact of the COVID-19 outbreak may further boost this segment as consumers cut back on discretionary spending and buy phones and tablets only for immediate needs.

Objective

The rising potential of this comparatively under-the-radar market fuels the need for an ML-based solution to develop a dynamic pricing strategy for used and refurbished devices. ReCell, a startup aiming to tap the potential in this market, has hired you as a data scientist. They want you to analyze the data provided and build a linear regression model to predict the price of a used phone/tablet and identify factors that significantly influence it.

Data Description

The data contains the different attributes of used/refurbished phones and tablets. The data was collected in the year 2021. The detailed data dictionary is given below.

- brand_name: Name of manufacturing brand
- os: OS on which the device runs
- screen_size: Size of the screen in cm
- 4g: Whether 4G is available or not
- 5g: Whether 5G is available or not

- main_camera_mp: Resolution of the rear camera in megapixels
- selfie_camera_mp: Resolution of the front camera in megapixels
- int_memory: Amount of internal memory (ROM) in GB
- ram: Amount of RAM in GB
- battery: Energy capacity of the device battery in mAh
- weight: Weight of the device in grams
- release_year: Year when the device model was released
- days_used: Number of days the used/refurbished device has been used
- normalized_new_price: Normalized price of a new device of the same model in euros
- normalized_used_price: Normalized price of the used/refurbished device in euros

Importing necessary libraries

In [89]:
```
# Libraries to help with reading and manipulating data
import numpy as np
import pandas as pd
import datetime as dt

# Libraries to help with data visualization
import matplotlib.pyplot as plt
import seaborn as sns

# split the data into train and test
from sklearn.model_selection import train_test_split

# to build linear regression_model
import statsmodels.api as sm

# to check model performance
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score

# to compute VIF
from statsmodels.stats.outliers_influence import variance_inflation_factor

# to check normalcy
import pylab
import scipy.stats as stats

# to check homoscedasticity
import statsmodels.stats.api as sms
from statsmodels.compat import lzip
```

In [2]:
```
# Set standard styling; writer has slight vision deficiency
# Paper context selected for readability in turned in .html format, but was originally
custom_palette = sns.color_palette('colorblind')
sns.set(style='whitegrid', context='paper', palette= custom_palette)
sns.set(rc={'grid.color': 'gray', 'grid.alpha': 0.5})
sns.palplot(custom_palette)
```

Loading the dataset

In [3]:
```
from google.colab import files
import io

try:
    uploaded
except NameError:
    uploaded = files.upload()

df= pd.read_csv(io.BytesIO(uploaded['used_device_data.csv']))
```

Choose Files No file chosen Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable.

```
Saving used_device_data.csv to used_device_data.csv
```

In [6]:
```
##convert to html snippet
#%%shell
#jupyter nbconvert --to html ///content/SupervisedLearning_ReCell.ipynb
```

Data Overview

Shape

In [7]:
```
# check the shape of the data
df.shape
```

Out[7]: (3454, 15)

In [8]:
```
df.duplicated().sum()
```

Out[8]: 0

There are 3454 rows of data, with 15 starting columns. There are no duplicate rows, so for now that will reamin our shape!

First few rows and data types

In [9]:
```
# check the first 5 rows of the data
df.head()
```

Out[9]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory	ram
0	Honor	Android	14.50	yes	no	13.0	5.0	64.0	3.0
1	Honor	Android	17.30	yes	yes	13.0	16.0	128.0	8.0
2	Honor	Android	16.69	yes	yes	13.0	8.0	128.0	8.0
3	Honor	Android	25.50	yes	yes	13.0	8.0	64.0	6.0
4	Honor	Android	15.32	yes	no	13.0	8.0	64.0	3.0

In [10]:

```
# check column types and number of values
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 3454 entries, 0 to 3453
Data columns (total 15 columns):
 #   Column                 Non-Null Count  Dtype  
---  ------                 --------------  -----  
 0   brand_name             3454 non-null   object 
 1   os                     3454 non-null   object 
 2   screen_size            3454 non-null   float64
 3   4g                     3454 non-null   object 
 4   5g                     3454 non-null   object 
 5   main_camera_mp         3275 non-null   float64
 6   selfie_camera_mp       3452 non-null   float64
 7   int_memory             3450 non-null   float64
 8   ram                    3450 non-null   float64
 9   battery                3448 non-null   float64
 10  weight                 3447 non-null   float64
 11  release_year           3454 non-null   int64  
 12  days_used              3454 non-null   int64  
 13  normalized_used_price  3454 non-null   float64
 14  normalized_new_price   3454 non-null   float64
dtypes: float64(9), int64(2), object(4)
memory usage: 404.9+ KB
```

- All data types are as expected
- There are some missing values that will need treatment
- The 4g and 5g columns appear to be yes/no objects, but can check the unique values during EDA
- At this time, all columns appear relavent and worth including in the initial model.
- Release year is a numeric value - but the feature that is more intuitive is how many years ago it was released. Lets go ahead and update that feature.

Early feature engineering - years_since_release

In [11]:

```
# Early Feature Engineering - release_year to years_since_release

# create a copy of data so that any edits from here will not change the original uploa
df1 = df.copy()

# To make more robust, defining a current year variable instead of hard coding "2023"
```

```
current_year = dt.datetime.now().year

# create new feature
df1['years_since_release'] = current_year - df1['release_year']

# remove old feture
df1.drop(columns=['release_year'], inplace = True)

# rearrange column order. This is purely convenience -
# it puts the used price on the bottom of some later key visualizations for less scrol
column_order_temp = list(df1.columns)
column_order_temp[12], column_order_temp[14] = column_order_temp[14], column_order_tem
df1 = df1[column_order_temp]

# check new dataframe info
df1.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 3454 entries, 0 to 3453
Data columns (total 15 columns):
 #   Column                 Non-Null Count  Dtype  
---  ------                 --------------  -----  
 0   brand_name             3454 non-null   object 
 1   os                     3454 non-null   object 
 2   screen_size            3454 non-null   float64
 3   4g                     3454 non-null   object 
 4   5g                     3454 non-null   object 
 5   main_camera_mp         3275 non-null   float64
 6   selfie_camera_mp       3452 non-null   float64
 7   int_memory             3450 non-null   float64
 8   ram                    3450 non-null   float64
 9   battery                3448 non-null   float64
 10  weight                 3447 non-null   float64
 11  days_used              3454 non-null   int64  
 12  years_since_release    3454 non-null   int64  
 13  normalized_new_price   3454 non-null   float64
 14  normalized_used_price  3454 non-null   float64
dtypes: float64(9), int64(2), object(4)
memory usage: 404.9+ KB
```

In [12]:
```
# check the first 5 rows of the new dataframe
df1.head()
```

Out[12]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory	ram
0	Honor	Android	14.50	yes	no	13.0	5.0	64.0	3.0
1	Honor	Android	17.30	yes	yes	13.0	16.0	128.0	8.0
2	Honor	Android	16.69	yes	yes	13.0	8.0	128.0	8.0
3	Honor	Android	25.50	yes	yes	13.0	8.0	64.0	6.0
4	Honor	Android	15.32	yes	no	13.0	8.0	64.0	3.0

Basic Statistical Summary

In [13]:
```
df1.describe(include="all").T
```

Out[13]:

	count	unique	top	freq	mean	std	min	25%
brand_name	3454	34	Others	502	NaN	NaN	NaN	NaN
os	3454	4	Android	3214	NaN	NaN	NaN	NaN
screen_size	3454.0	NaN	NaN	NaN	13.713115	3.80528	5.08	12.7
4g	3454	2	yes	2335	NaN	NaN	NaN	NaN
5g	3454	2	no	3302	NaN	NaN	NaN	NaN
main_camera_mp	3275.0	NaN	NaN	NaN	9.460208	4.815461	0.08	5.0
selfie_camera_mp	3452.0	NaN	NaN	NaN	6.554229	6.970372	0.0	2.0
int_memory	3450.0	NaN	NaN	NaN	54.573099	84.972371	0.01	16.0
ram	3450.0	NaN	NaN	NaN	4.036122	1.365105	0.02	4.0
battery	3448.0	NaN	NaN	NaN	3133.402697	1299.682844	500.0	2100.0
weight	3447.0	NaN	NaN	NaN	182.751871	88.413228	69.0	142.0
days_used	3454.0	NaN	NaN	NaN	674.869716	248.580166	91.0	533.5
years_since_release	3454.0	NaN	NaN	NaN	8.034742	2.298455	4.0	6.0
normalized_new_price	3454.0	NaN	NaN	NaN	5.233107	0.683637	2.901422	4.790342
normalized_used_price	3454.0	NaN	NaN	NaN	4.364712	0.588914	1.536867	4.033931

- No extraoridnay values in this chart
- The normalized columns do appear to be normally distributed, while most other have a shift.
- The phones that have near 0GB of RAM, near 0BG of ROM, and near 0MP on the two main cameras may be outliers. However, the phones do go back to 2013.

First look at missing values

In [14]:
```
df1.isnull().sum()
```

Out[14]:
```
brand_name                 0
os                         0
screen_size                0
4g                         0
5g                         0
main_camera_mp           179
selfie_camera_mp           2
int_memory                 4
ram                        4
battery                    6
weight                     7
days_used                  0
years_since_release        0
normalized_new_price       0
normalized_used_price      0
dtype: int64
```

Observation:

This initial look shows that:

- One variable, main_camera_mp is missing in 179 of the 3454 data points.
- There are several variables with only a handful of missing values: selfie_camera_mp, int_memory, ram, battery, weight.
- Will come back to these after EDA - we will impute, remove, or otherwise treat based on EDA discoveries.

Exploratory Data Analysis (EDA)

Custom Functions for EDA / Visualization

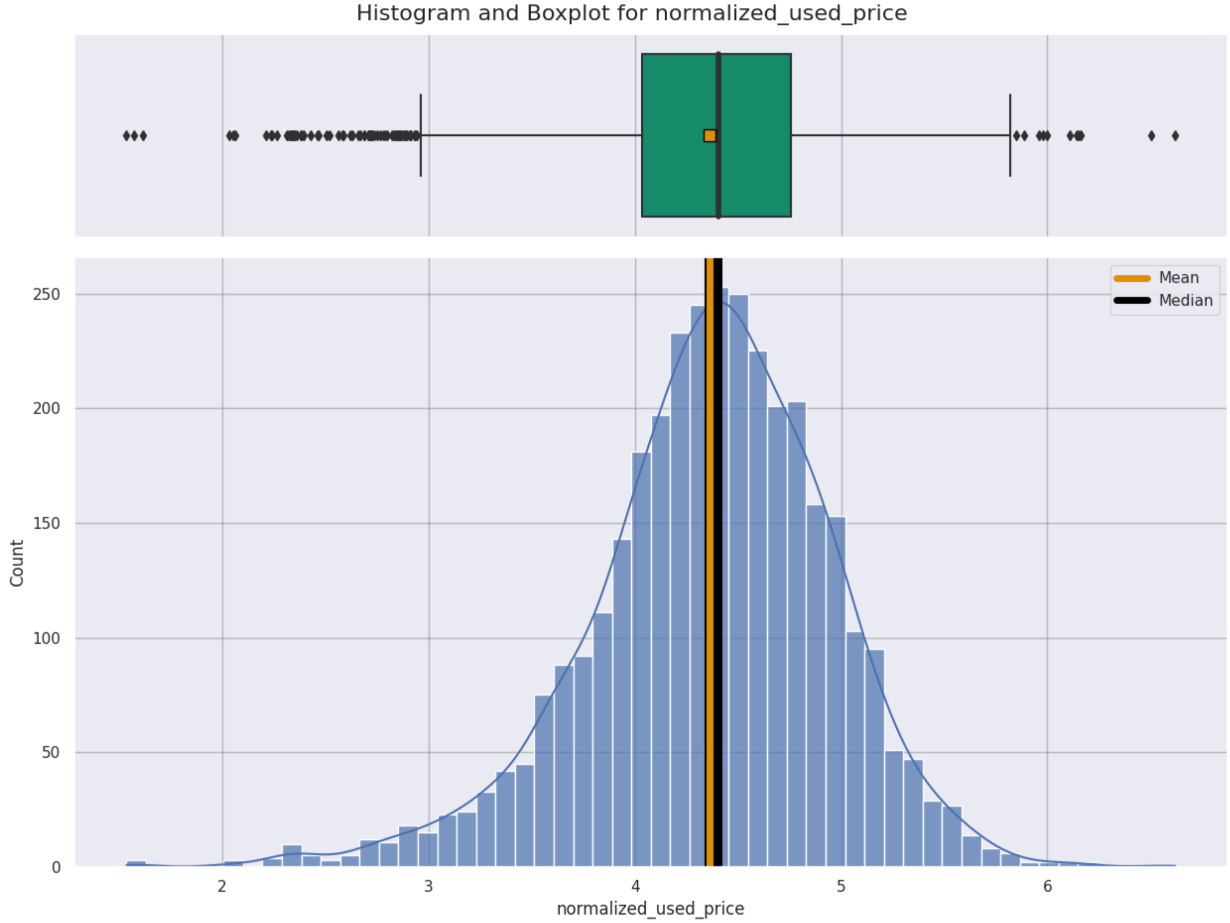
In [15]:
```
def histogram_boxplot(data, feature, figsize=(15, 10), kde=False, bins=None):
    """
    Boxplot and histogram combined

    data: dataframe
    feature: dataframe column
    figsize: size of figure (default (15,10))
    kde: whether to show the density curve (default False)
    bins: number of bins for histogram (default None)
    """

    # creating the 2 subplots
    f2, (ax_box2, ax_hist2) = plt.subplots(
        nrows=2,  # Number of rows of the subplot grid= 2
        sharex=True,  # x-axis will be shared among all subplots
        gridspec_kw={"height_ratios": (0.25, 0.75), "hspace": 0.05, "top": 0.95},
        figsize=figsize,
    )

    #create a title
    f2.suptitle("Histogram and Boxplot for " + feature, fontsize=16)

    # boxplot will be created and a square will indicate the mean value of the column
    sns.boxplot(
```

```
        data=data, x=feature, ax=ax_box2, showmeans=True,
        meanprops={"marker":"s", "markersize" : 8, "markerfacecolor": custom_palette[1
        medianprops = {'linewidth':4},
        color = custom_palette[2]
    )

    #remove duplicate x label
    ax_box2.set_xlabel("")

    #create histogram, with consideration of bins
    sns.histplot(
        data=data, x=feature, kde=kde, ax=ax_hist2, bins=bins, alpha = 0.7
    ) if bins is not None else sns.histplot(
        data=data, x=feature, kde=kde, ax=ax_hist2, alpha = 0.7
    )

    # add mean and median to histogram
    ax_hist2.axvline(
        data[feature].mean(), color = 'black', linestyle= '-', linewidth = 8
    ) # a pseudo border for the mean line

    ax_hist2.axvline(
        data[feature].mean(), color = custom_palette[1], linestyle= '-', linewidth = 5
    )

    ax_hist2.axvline(
        data[feature].median(), color = 'black', linestyle= '-', linewidth = 5, label
    )

    # Add a legend
    ax_hist2.legend(loc='upper right')
```

Targeted EDA Questions

1. What does the distribution of normalized used device prices look like?

In [16]:
```
### histogram and boxplot showing target variable - used prices
histogram_boxplot(df1, "normalized_used_price", kde = True)
```



The used prices are normalized, as the name of the variable implies. The Mean, Median, and Mode are all around the same value of ~4.4. There are many outliers less than the Q1 value, with values between 1.53 and 4.03. The middle 50% of the data is in a very narrow space relative to the actual range of the data.

2. What percentage of the used device market is dominated by Android devices?

In [17]:
```
df1['os'].value_counts(1)
```

Out[17]:
```
Android    0.930515
Others     0.039664
Windows    0.019398
iOS        0.010423
Name: os, dtype: float64
```

Android is 93% of the Market. Which is suprising to me as an Android user myself - the emote reactions between iOS and Android at time of writing still dont work perfectly and generate duplicate messages. This tells me not to trust my own intuition about this market data - I am not a SME and my experience is not standard.

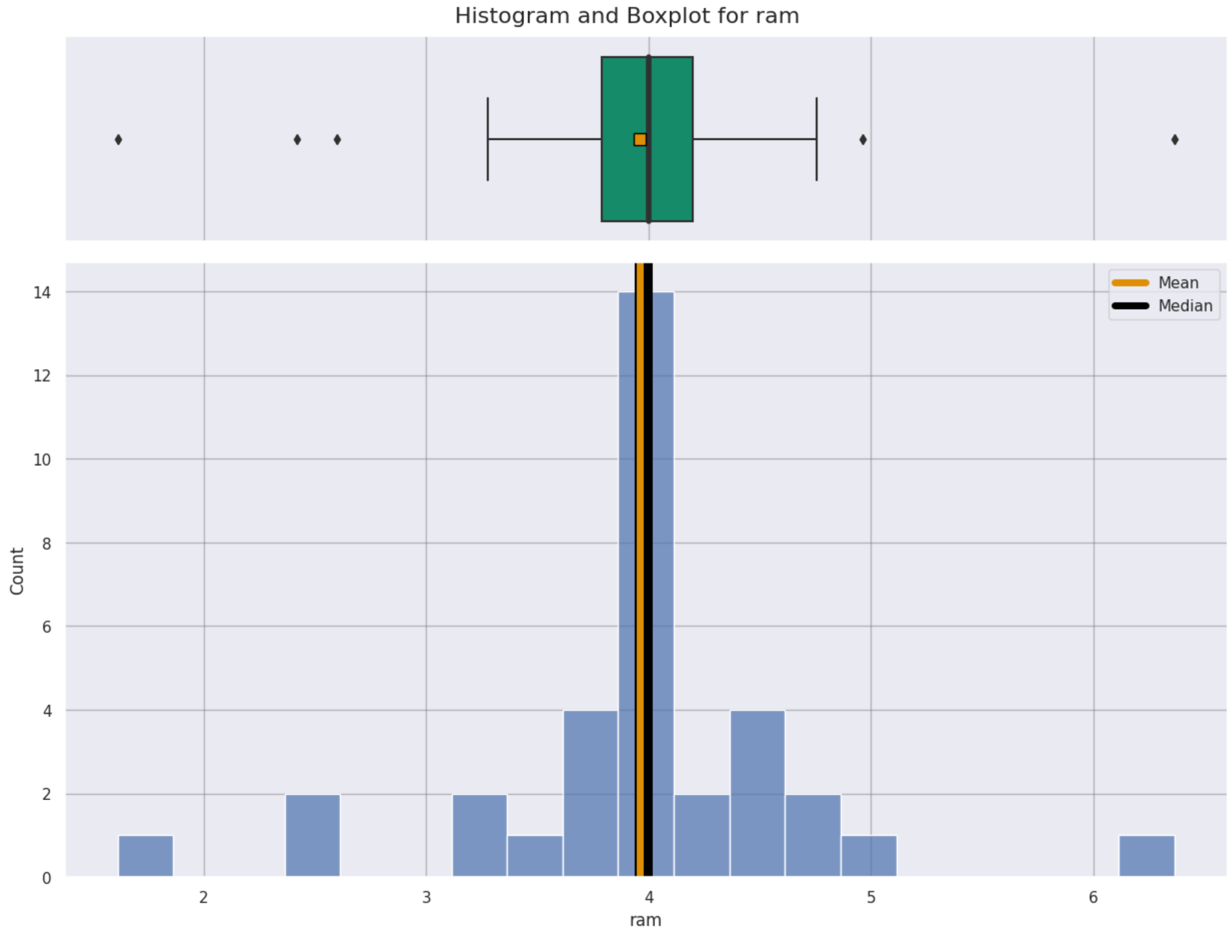
3. The amount of RAM is important for the smooth functioning of a device. How does the amount of RAM vary with the brand?

In [18]:
```
# Look at the stats summary of the RAM grouped by Brand Name first, then we can visual
mean_ram_by_brand = df1.groupby(by = ['brand_name'])['ram'].mean().sort_values()
print(mean_ram_by_brand.describe(), end='\n\n')
print(mean_ram_by_brand)
```

```
count    34.000000
mean      3.962049
std       0.787467
min       1.613636
25%       3.790803
50%       4.000000
75%       4.198341
max       6.363636
Name: ram, dtype: float64

brand_name
Celkon        1.613636
Nokia         2.420294
Infinix       2.600000
Lava          3.277778
Karbonn       3.353448
Alcatel       3.407025
Micromax      3.679487
Spice         3.750000
Others        3.777888
BlackBerry    3.829545
Lenovo        3.885965
Acer          3.901961
Gionee        3.933036
LG            3.936567
Motorola      3.943396
Coolpad       3.954545
HTC           4.000000
Panasonic     4.000000
Apple         4.000000
XOLO          4.000000
Microsoft     4.000000
ZTE           4.023214
Asus          4.049180
Sony          4.069767
Realme        4.195122
Samsung       4.199413
Meizu         4.451613
Google        4.533333
Xiaomi        4.583333
Honor         4.603448
Huawei        4.655378
Vivo          4.756410
Oppo          4.961240
OnePlus       6.363636
Name: ram, dtype: float64
```

- There are 34 brands in this data set, with a mean RAM of 3.96 and a range of 4.75.
- This means the devices with the highest RAM have almost 4 times as much RAM as the devices with the least RAM.
- The standard deviation of 0.78 and the closeness of the mean and Q2 value propose the distribution of average RAMs is somewhat normally distributed, lets take a look.

In [19]:
```
### histogram and boxplot to show distribution of average RAMs across all brands
mean_ram_df = mean_ram_by_brand.reset_index()
histogram_boxplot(mean_ram_df, 'ram')
```



I would certainly want to ask the SME at this point for guidance on how to treat these outliers. The OnePlus devices with a mean of 6.36 are so far ahead of the group - possibly a niche company? Same with the bottom three brands; Celkon, Nokia, and Infinix. I said earlier that I didnt want to trust my experience with devices, but it is an Internet joke that Nokia devices live forever. I wonder if these three brands also have an older mean age.

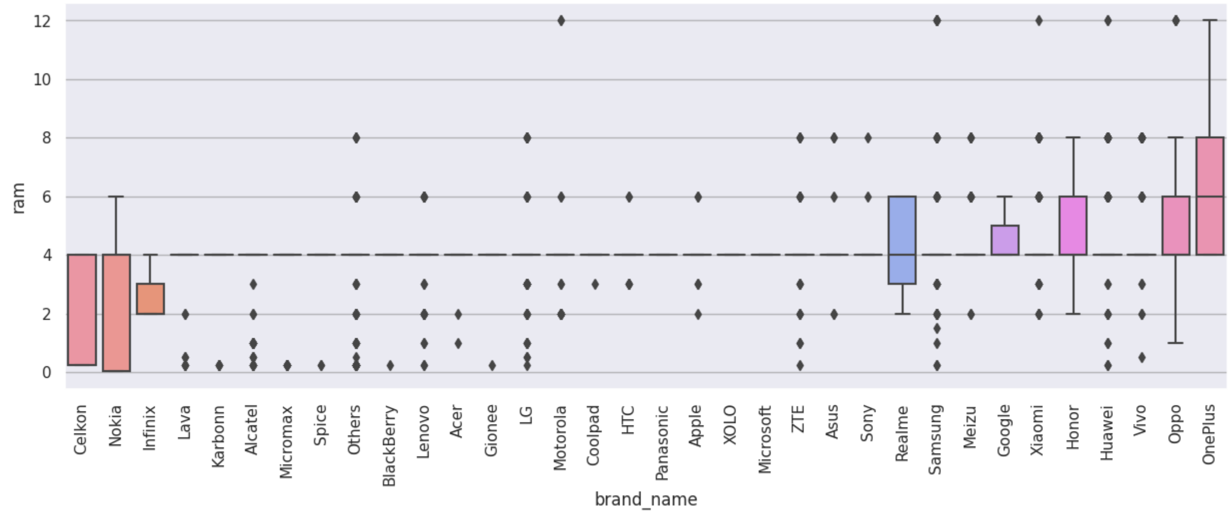
In [20]:
```
# Quick look at years since release by brand
mean_year_by_brand = df1.groupby(by = ['brand_name'])['years_since_release'].mean().sc
print(mean_year_by_brand)
```

```
brand_name
Spice         11.0
Celkon        11.0
Karbonn       11.0
XOLO          10.0
Micromax      10.0
Acer          10.0
Lava           9.0
Microsoft      9.0
Gionee         9.0
Alcatel        9.0
Others         9.0
HTC            9.0
BlackBerry     9.0
Sony           9.0
Asus           9.0
Panasonic      8.0
Lenovo         8.0
Samsung        8.0
ZTE            8.0
LG             8.0
Coolpad        8.0
Nokia          8.0
Apple          7.0
Huawei         7.0
Meizu          7.0
Motorola       7.0
Oppo           7.0
Xiaomi         6.0
Vivo           6.0
Google         6.0
OnePlus        6.0
Honor          6.0
Realme         5.0
Infinix        4.0
Name: years_since_release, dtype: float64
```

- Reaffirming that my intution/lived experience with cell phones is not helpful here.
- The Infinix brand has the most average releases, Nokia is in the middle and Celkon are the oldest.
- Thus the three Brands with average RAMs below the average are not because of the age of those devices.
- Lets looks a a box plot with the brand names next

In [21]:

```
# Box plots show the outliers by brand the best
# Brands are sorted from lowest mean RAM to highest mean RAM
plt.figure(figsize=(15, 5))
sns.boxplot(df1, x="brand_name", y="ram", order=mean_ram_by_brand.index)
plt.xticks(rotation=90)
plt.show()
```



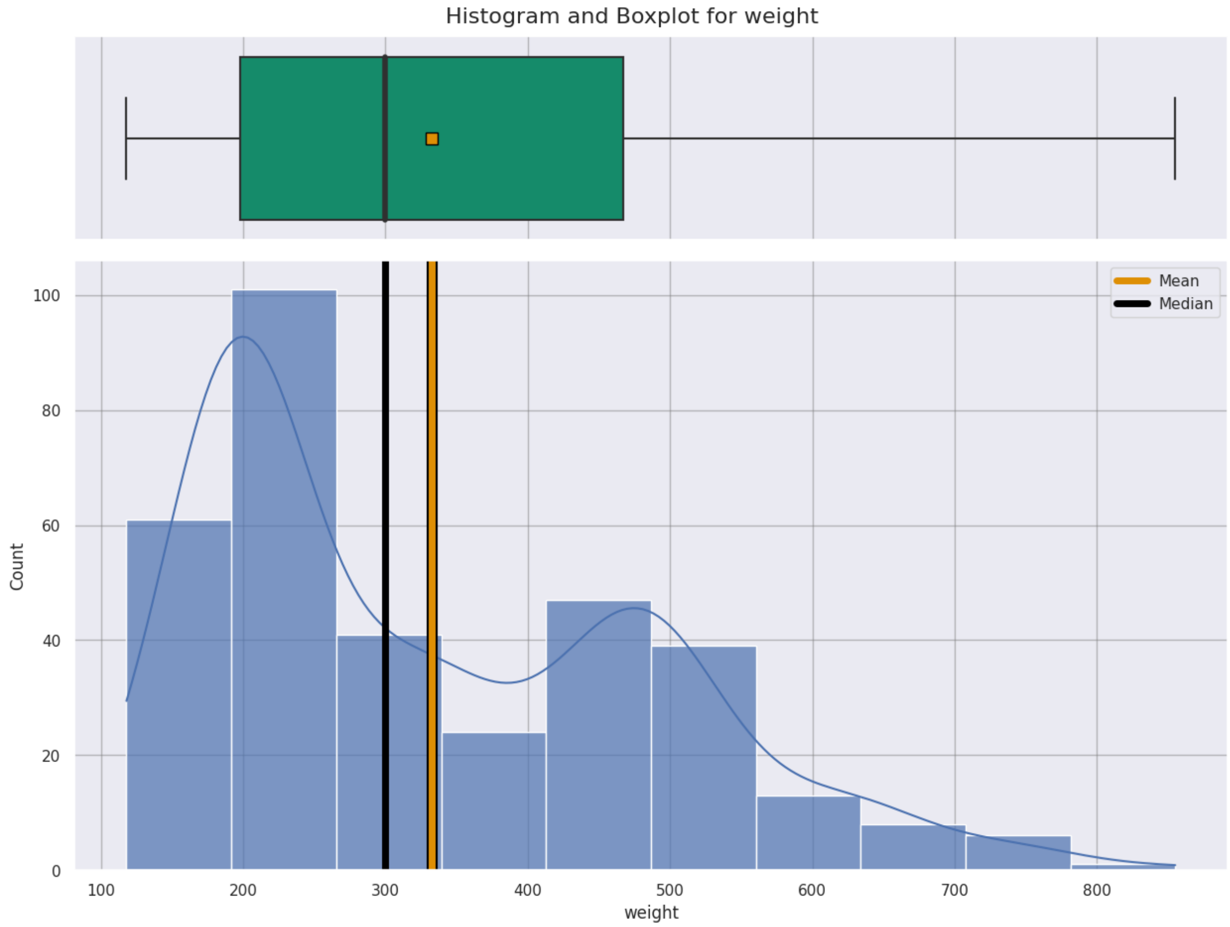
- The brand-mean and brand-median are both ~ 4 GB of RAM.
- This visualization suggests that brand behavior of device production with respect to RAM is highly variable. The number of outliers and collapsed boxes suggests that most brands intentionally produce a high variety of devices.
- The exceptions to this are the companies who tend to produce devices with similar RAM profiles. These are easily spotted by the size of their tails or presence of no outliers.
  - Celkon, Nokia, and Infinix focus on devices with less than brand-average RAM
  - Realme, Google, Honor all have a window of RAM near the brand-mean and no outliers.
  - OnePlus makes stunnigly above-average RAM devices.

4. A large battery often increases a device's weight, making it feel uncomfortable in the hands. How does the weight vary for phones and tablets offering large batteries (more than 4500 mAh)?

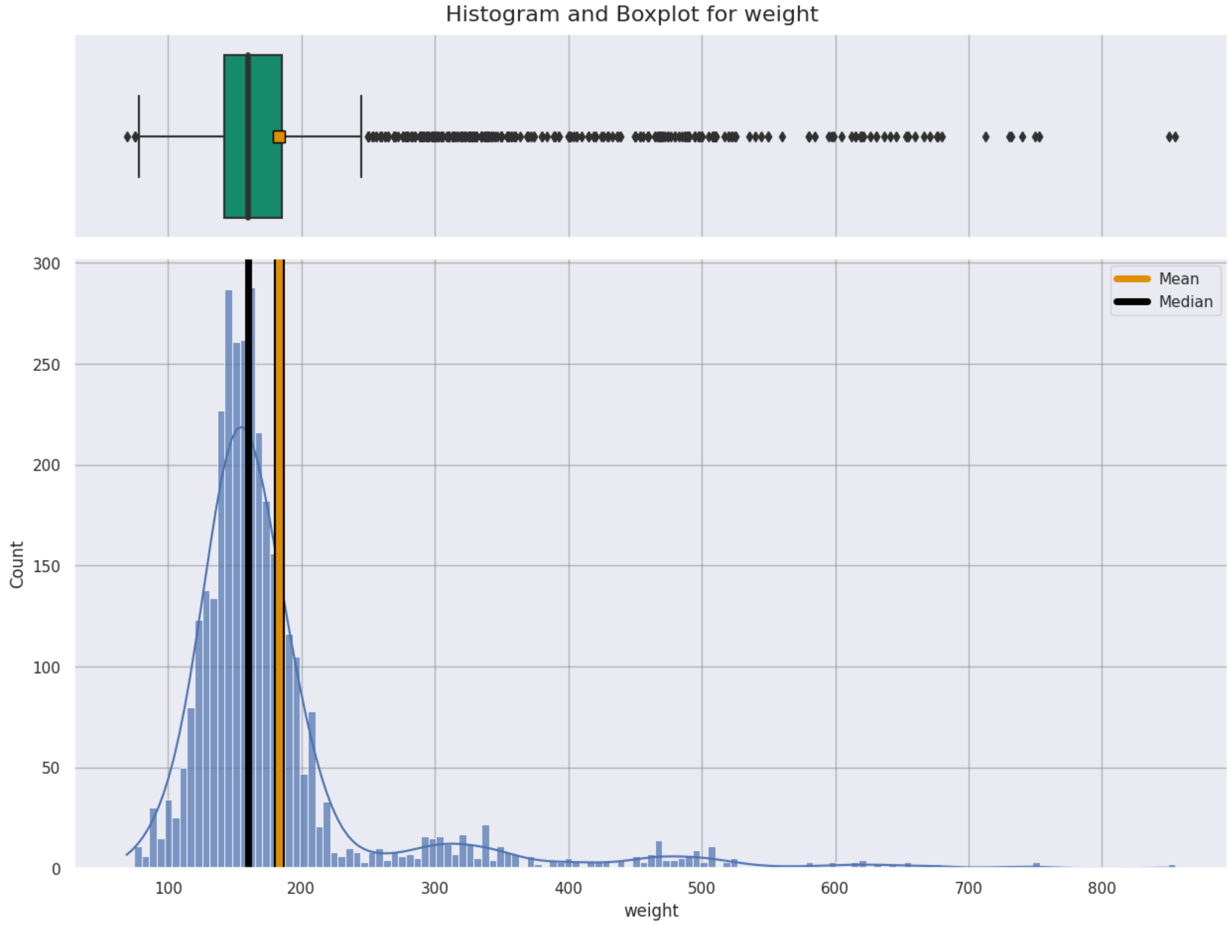
In [22]:
```
#new dataframe
df_big_batteries = df1[df1['battery'] > 4500]
df_big_batteries['weight'].describe()
```

Out[22]:
```
count    341.000000
mean     332.275660
std      155.501832
min      118.000000
25%      198.000000
50%      300.000000
75%      467.000000
max      855.000000
Name: weight, dtype: float64
```

In [23]:
```
histogram_boxplot(df_big_batteries, 'weight', kde = True)
```



In [24]:
```
histogram_boxplot(df1, 'weight', kde = True)
```



These two graphs compare the distribution of ALL device weights, and the distribution of devices weights whose batteries are greater than 4500.

What we see here is that the mode and general shapes of the distributions stay the same - but shited to the right. These large battery devices make up the just right of the curve data of the original distribution. The general shape of this distribution of the big_battery phones is the same; long right tail with a somehewat significant multi-modal distribution, and a mean greater than the median. The big_battery devices are more cetnralized, thus have less identfied outliers. Though they are the outliers of the orginial data set.

5. Bigger screens are desirable for entertainment purposes as they offer a better viewing experience. How many phones and tablets are available across different brands with a screen size larger than 6 inches?

In [25]:
```
# new datafrome;
df_screen_inches = df1.copy()

# Convert screen size from cm to inches (1 cm = 0.393701 inches)
df_screen_inches['screen_size'] = df1['screen_size'] * 0.393701

#count
df_screen_inches[df_screen_inches['screen_size'] > 6]['brand_name'].value_counts()
```

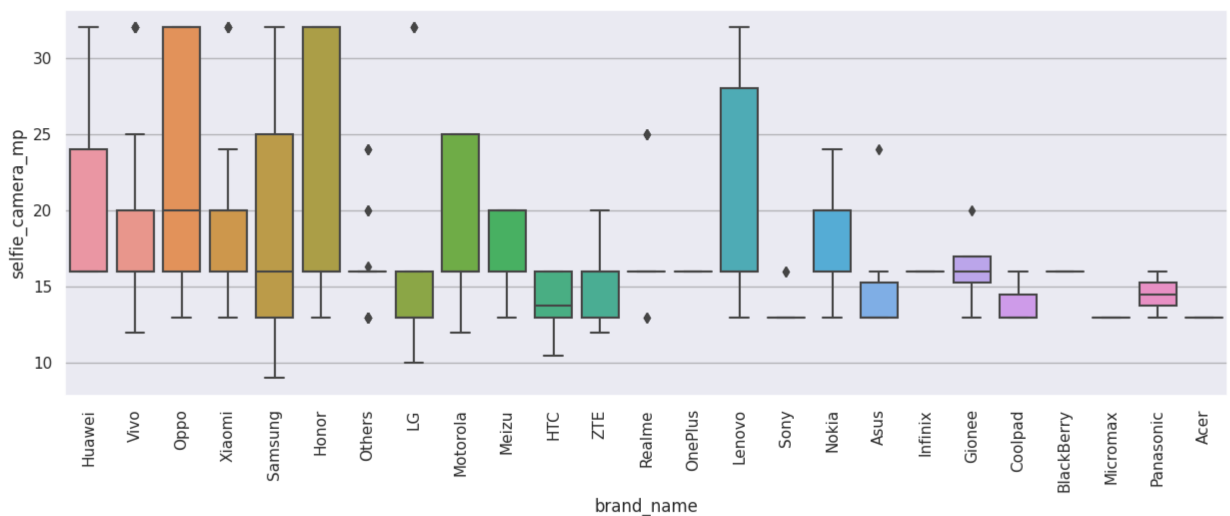
Out[25]:
```
Huawei       157
Samsung      131
Others       118
Vivo          82
Oppo          78
Lenovo        76
Honor         72
Xiaomi        69
LG            63
Asus          51
Motorola      43
Realme        41
Alcatel       34
ZTE           25
Apple         24
Sony          23
Meizu         21
Acer          20
Nokia         19
OnePlus       16
HTC           14
Micromax      11
Infinix       10
Gionee         8
Google         7
XOLO           4
Panasonic      3
Coolpad        3
Karbonn        2
Spice          2
Microsoft      1
Name: brand_name, dtype: int64
```

6. A lot of devices nowadays offer great selfie cameras, allowing us to capture our favorite moments with loved ones. What is the distribution of devices offering greater than 8MP selfie cameras across brands?

In [26]:
```
#start with the counts
selfie_devices_counts = df1[df1['selfie_camera_mp'] >8]['brand_name'].value_counts()
selfie_devices_counts
```

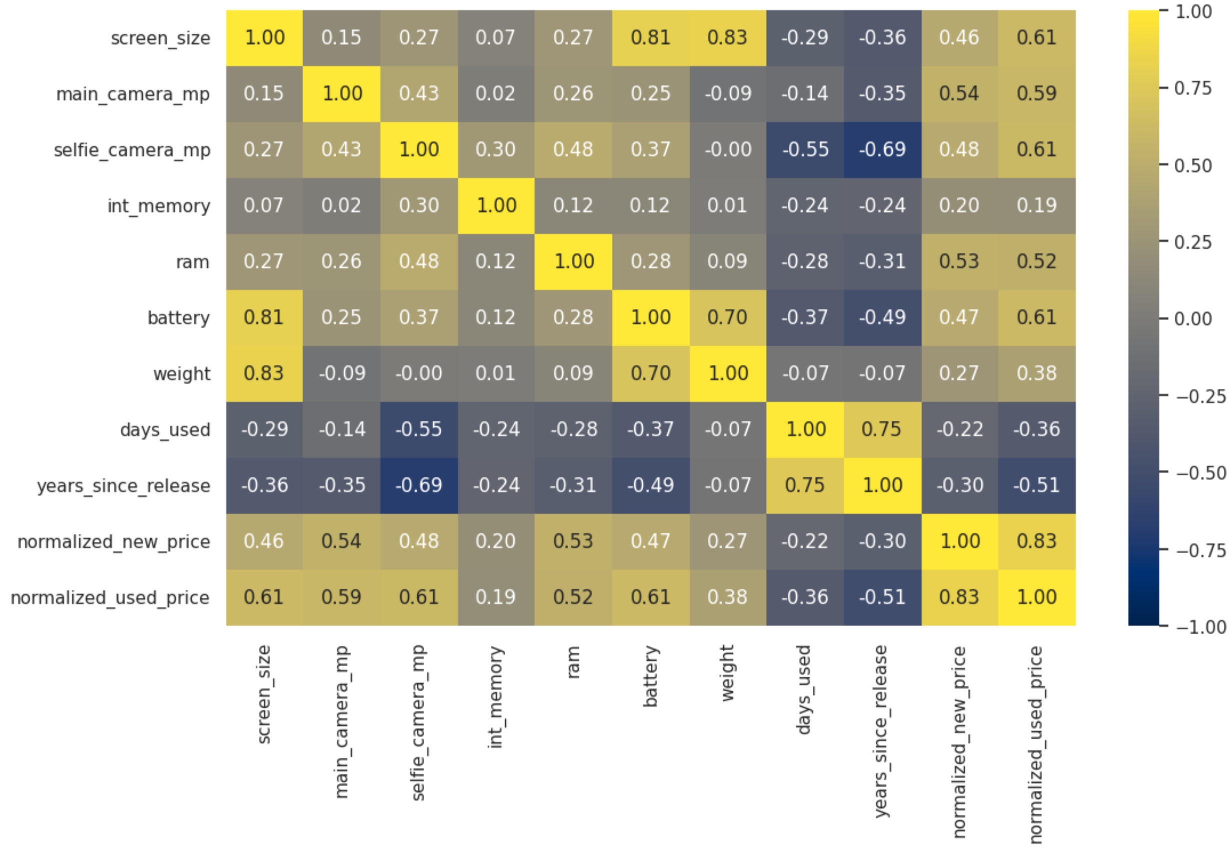
Out[26]:
```
Huawei        87
Vivo          78
Oppo          75
Xiaomi        63
Samsung       57
Honor         41
Others        34
LG            32
Motorola      26
Meizu         24
HTC           20
ZTE           20
Realme        18
OnePlus       18
Lenovo        14
Sony          14
Nokia         10
Asus           6
Infinix        4
Gionee         4
Coolpad        3
BlackBerry     2
Micromax       2
Panasonic      2
Acer           1
Name: brand_name, dtype: int64
```

In [27]:
```
# Box plot here, sorted by count of devices with 8mp+ selfie cameras
plt.figure(figsize=(15, 5))
sns.boxplot(df1[df1['selfie_camera_mp'] >8], x="brand_name", y="selfie_camera_mp", orc
plt.xticks(rotation=90)
plt.show()
```



7. Which attributes are highly correlated with the normalized price of a used device?

In [28]:
```
#list of numerical colums
num_cols = df1.select_dtypes(include = np.number).columns.tolist()

#heatmap to show correlations
plt.figure(figsize = (12,7))
sns.heatmap(df1[num_cols].corr(), annot = True, vmin = -1, vmax = 1, fmt=".2f", cmap='
plt.show()
```



- `used_price` has positive correlations with the following features, in increasing level of correlation:
  - RAM (0.52)
  - Main camera mp (0.59)
  - Screen size, Selfie camera mp, and Battery (0.61)
  - New price (0.83)
- And it has negative correlations with the following featrues, in increasing level of correlation:
  - days_used (-0.36)
  - years_since_release (-0.51)

Non-Question Based EDA

Below is a small collection of EDA that is either specifically referenced later or was instrumental in my understanding for model building. Additional EDA can be found here for reference as needed: appendix section.

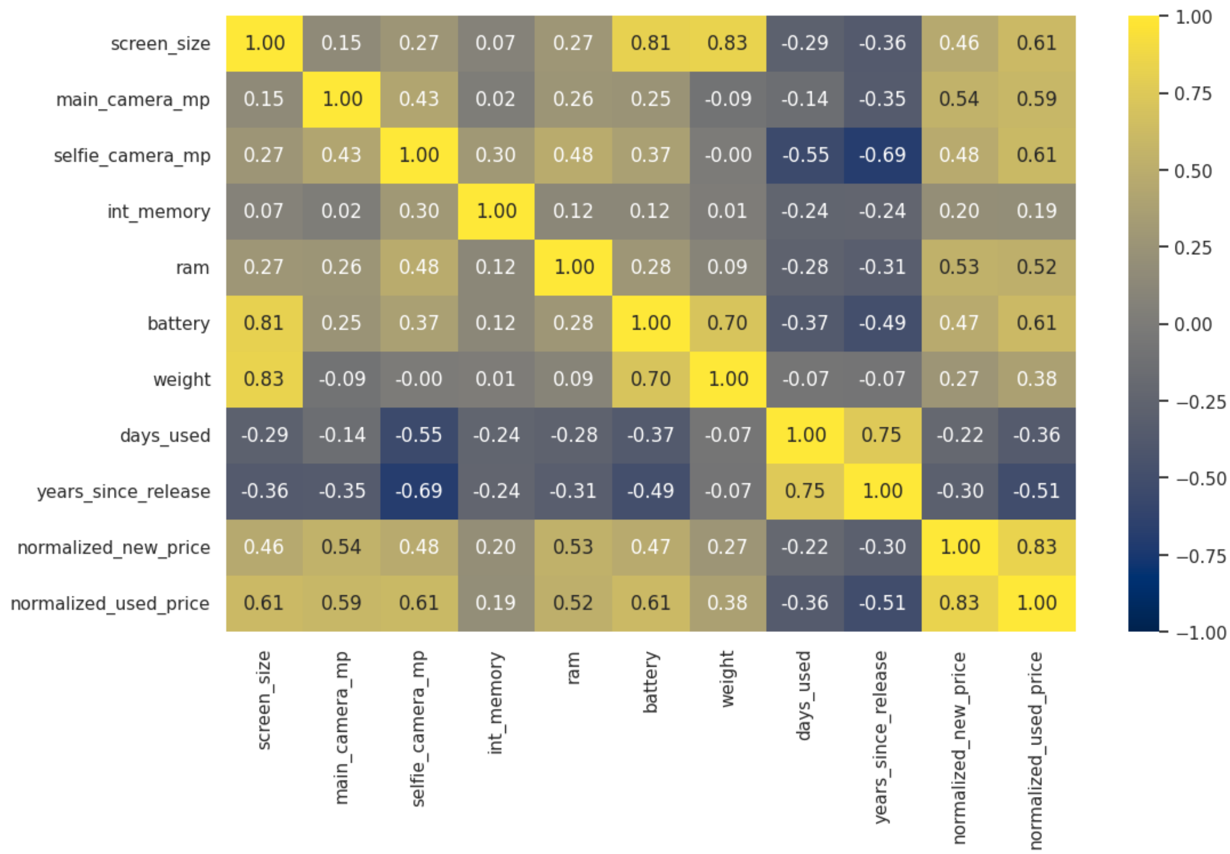
In [29]:
```
#Are there any observations in the categorical data that are functionally representing
#Create dataframe of just objects columns
df1_objs = df1.select_dtypes(exclude=['int', 'float'])
#Print each unique observation in every category, with the percent of the category it
for col in df1_objs.columns:
  cat_percents = df1_objs[col].value_counts(normalize = True) *100
  print(cat_percents, "\n")
```

```
Others        14.533874
Samsung        9.872611
Huawei         7.266937
LG             5.819340
Lenovo         4.950782
ZTE            4.053272
Xiaomi         3.821656
Oppo           3.734800
Asus           3.532137
Alcatel        3.503185
Micromax       3.387377
Vivo           3.387377
Honor          3.358425
HTC            3.184713
Nokia          3.068906
Motorola       3.068906
Sony           2.489867
Meizu          1.795020
Gionee         1.621309
Acer           1.476549
XOLO           1.418645
Panasonic      1.360741
Realme         1.187030
Apple          1.129126
Lava           1.042270
Celkon         0.955414
Spice          0.868558
Karbonn        0.839606
Coolpad        0.636943
BlackBerry     0.636943
Microsoft      0.636943
OnePlus        0.636943
Google         0.434279
Infinix        0.289519
Name: brand_name, dtype: float64

Android    93.051534
Others      3.966416
Windows     1.939780
iOS         1.042270
Name: os, dtype: float64

yes    67.602779
no     32.397221
Name: 4g, dtype: float64

no     95.599305
yes     4.400695
Name: 5g, dtype: float64
```

- There are a lot `brands` in this data set, which will make a lot of dummy variables. The question set above showed that `brand` behaviors do result in different performances in the numerical variables, so the brand names will stay for now.
  - `Others` is the largest brand category, reprsenting 14.5% of the total data
  - Followed by Samsung, Huawei, and LG each individually above 5% of the total data

- There are 9 brands that each are less than 1% of the total data. However, what this data set doesnt really include is the volume or inventory by brand. Just because, for example, Google only has their Pixel line of 15-18 phones, doesnt necessarily mean they represent less than 1% of the used device sales.
- The `os`, `4g`, and `5g` categorical variables all have reasonable values.
  - The top operating system is Android by far - with 93% of the dataset
  - ~67% of the phones have 4g connectivity
  - While only 4.4% of phones have 5g connectivity. The last phones in the data set are 3 years old as of writing, so this particular categorical variable will likely catch up with 4g when the data catches up with 2023 and beyond.

In [30]:
```
#list of numerical colums
num_cols = df1.select_dtypes(include = np.number).columns.tolist()

#heatmap to show correlations
plt.figure(figsize = (12,7))
sns.heatmap(df1[num_cols].corr(), annot = True, vmin = -1, vmax = 1, fmt=".2f", cmap='
plt.show()
```



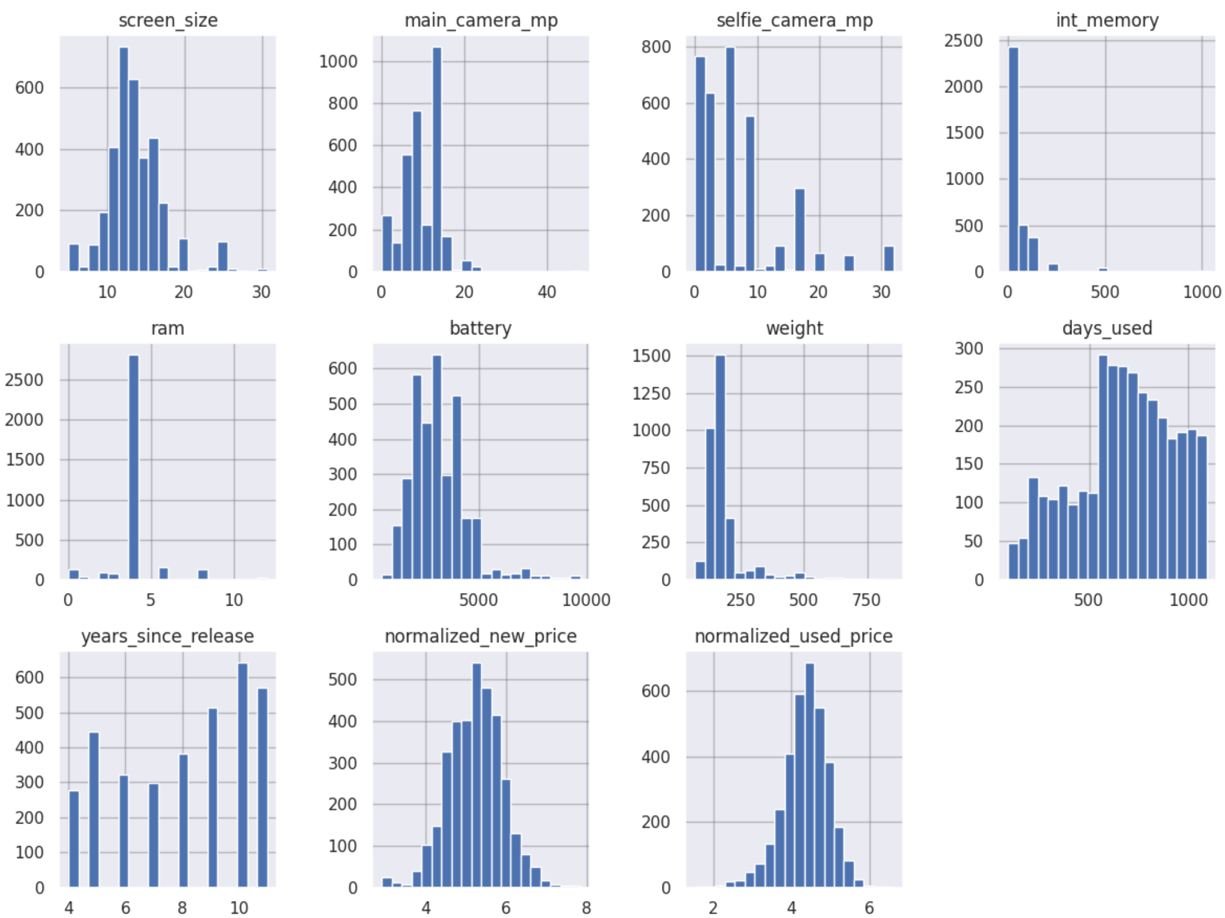
- `used_price` has positive correlations with the following features, in increasing level of correlation:
  - RAM (0.52)
  - Main camera mp (0.59)
  - Screen size, Selfie camera mp, and Battery (0.61)
  - New price (0.83)

- `screen_size`, `battery`, and `weight` form a trio, likely indicating that larger screen sizes tend to require more battery power and increased battery capacity tends to increase device weight.

- The positive correlation between `days_used` and `years_since_release` is to be expected. As the number of years since a device's release increases, the number of days it has been used also tends to increase.

- `selfie_camera_mp` and `years_since_release` exhibit a high negative correlation. This suggests that selfie cameras have improved the most rapidly of all the features in the last 10 years.

In [31]:
```
#Histograms of all numeric data to get feel for distributions and spot any values that
df1[num_cols].hist(figsize = (12,9), layout = (3,4), bins = 20)
plt.tight_layout()
plt.show()
```



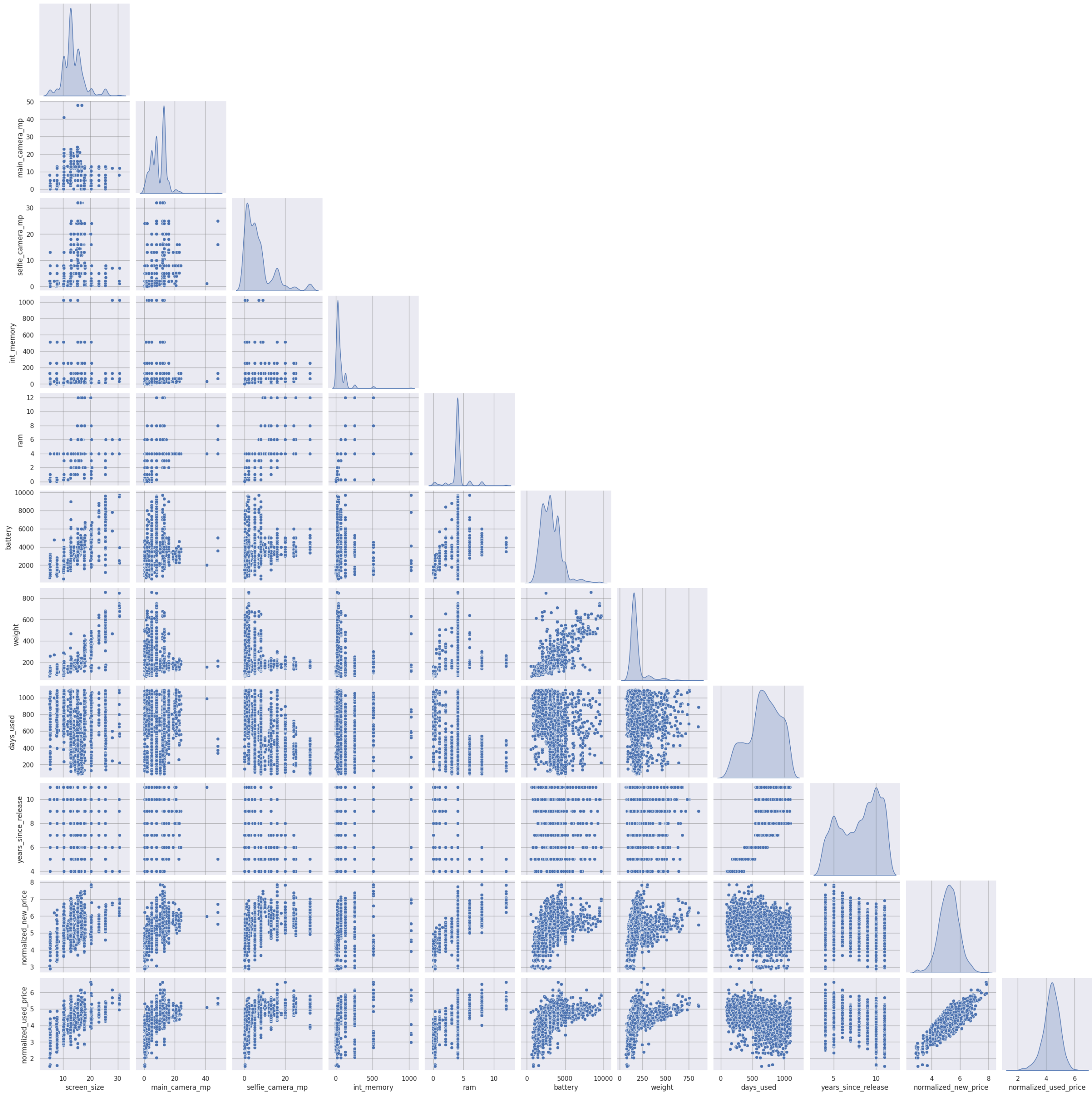
- First want to draw attention to the y-axis scales.
  - `ram` and `internal memory` both have a high frequency value, representing the majority of the data. This implies that they values to not change much when other features change.
  - `weight` and `main camera mp` also have a clear higher frequency group, representing about half the data set.

- The only features showing a normal distribution are the `new price` and the `used price`, with `screen size` a possible thrid feature but not nearly as clean as the price features.
- The `days used` feature has two buckets of values; phones used less than 550 days, and phones used more than 150 days. Something interesting happens to the cell phone usage from around the age of ~18 months
- If I had access to a SME I would check in on some of these features with 0 values. Are there phones from 2013+ without these components?
  - `main_camera_mp` & `selfie_camera_mp` - these correlate with eachother, and price. Outliers would have a large effect, so likely replace with medians?
  - `int_memory` - lets pull the statistics for this. If they are mostly 0s this feature then replacing with median will likely replace with a 0.
  - `ram` - could replace with mean grouped by brand

  > For now, choosing to leave these as they are since I can't be sure the 0 values are not valid data points for the model to consider - but may return to this after checking the model's performance.

In [32]:
```
#pairplot the numerical columns to visualize
sns.pairplot(df1[num_cols], diag_kind='kde', corner = True)
#this was really why I rearranged the columns earlier, makes this easier to read for m
```

Out[32]:
```
<seaborn.axisgrid.PairGrid at 0x7ee63bf08c40>
```



- There does not seem to be any linear relationships between used_price and the other than new_price.
- screen_size has some slight linearity with weight and battery
- weight and battery also have some linearity. My guess is that these three will show up again later in the multicollinearity work.
- There generally does not seem to be much linearity in this dataset outside of that, and several assumptions of classical linear regression seem to be violated. We will test that soon.

Data Preprocessing

Missing Value Treatment

We saw earlier that the features with missing values were:

- selfie_camera_mp
- int_memory
- ram
- battery
- weight
- and largest group of missing: main_camera_mp.

Additionally, we saw 0-values in positions that probably are not likely. Barring a SME to consult, we chose to leave those values as they were until we could test the model performance.

Lets work through these one by one. First:

> selfie_camera_mp

In [33]:
```
# Selfie Camera missing values
df1[df1['selfie_camera_mp'].isna()]
```

Out[33]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory	ra
1080	Google	Android	15.32	yes	no	12.2	NaN	64.0	
1081	Google	Android	12.83	yes	no	12.2	NaN	64.0	

A quick search shows that these are probably the Pixel 3 and 3 XL, since Google only has one main line of cell phones, and those were the two released in 2018. They both had 8 mp selfie cameras. Source

In [34]:
```
#replace the two selfie camera NaN with the value 8
df1['selfie_camera_mp'].fillna(8, inplace= True)
```

> int_memory

We know from EDA that int_memory is a interger variable that is highly skewed and with a stong mode. It correlates best with `main camera mp`. All of the belwo are Nokia phones. Thus, we will impute with the median value to limit the effecot of outliers on this variable.

In [35]:
```
df1[df1['int_memory'].isna()]
```

Out[35]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory	rar
117	Nokia	Others	5.18	yes	no	0.3	0.0	NaN	0.0
2035	Nokia	Others	5.18	no	no	5.0	0.0	NaN	0.0
2064	Nokia	Others	5.18	no	no	0.3	0.0	NaN	0.0
2092	Nokia	Others	7.62	no	no	5.0	0.0	NaN	0.0

In [36]:
```
#replace four internal memory NaN with median of the internal memory of Nokia phones b
df1["int_memory"] = df1["int_memory"].fillna(
    value=df1[df1['brand_name'] == 'Nokia'].groupby(["brand_name", "main_camera_mp"])[
)
```

> RAM

We know from EDA that `RAM` is functionally a integer variable with a stong mode. `RAM` correlates strongest with the two `price` features. All of the below data points are again Nokia phones. We will chose the median for Nokia Phones for these missing values.

In [37]:
```
df1[df1['ram'].isna()]
```

Out[37]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory	rar
114	Nokia	Others	5.18	no	no	0.3	0.0	0.06	Na
335	Nokia	Others	5.18	no	no	0.3	0.0	0.10	Na
2059	Nokia	Others	5.18	no	no	0.3	0.0	0.06	Na
2090	Nokia	Others	7.62	no	no	5.0	0.0	0.06	Na

In [38]:
```
#replace four ram NaN with median of the ram of Nokia phones
df1["ram"] = df1["ram"].fillna(
    value=df1[df1['brand_name'] == 'Nokia'].groupby(["brand_name"])['ram'].transform('
)
```

> battery

We know from EDA that `battery` is a lightly skewed continuous variable with strong positive correlations with `screen size` and `weight`. We will chose the median `battery` capacity.

In [39]:
```
df1[df1['battery'].isna()]
```

Out[39]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory
1829	Meizu	Android	12.83	yes	no	13.0	5.0	16.00
1831	Meizu	Android	12.83	yes	no	20.7	5.0	16.00
1832	Meizu	Android	13.61	yes	no	20.7	2.0	16.00
1962	Microsoft	Windows	25.55	no	no	5.0	3.5	32.00
2058	Nokia	Others	5.18	no	no	0.3	0.0	0.06
2059	Nokia	Others	5.18	no	no	0.3	0.0	0.06

In [40]:
```
#replace battery NaN with median of the batteries
df1['battery'] = df1['battery'].fillna(
```

```
    value = df1['battery'].median()
)
```

> weight

We know from EDA that `weight` is a highly skewed continuous variable with a strong mode. It positively correlates strongly with `battery` capacity and `screen size`. We will again chose the median `weight`.

In [41]:
```
df1[df1['weight'].isna()]
```

Out[41]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory
3002	XOLO	Android	12.70	yes	no	13.0	5.0	32.0
3003	XOLO	Android	12.83	yes	no	8.0	5.0	16.0
3004	XOLO	Android	12.70	no	no	8.0	2.0	32.0
3005	XOLO	Android	10.29	no	no	5.0	0.3	32.0
3006	XOLO	Android	12.70	no	no	5.0	0.3	16.0
3007	XOLO	Windows	12.70	no	no	8.0	2.0	32.0
3008	XOLO	Android	12.70	no	no	8.0	5.0	32.0

In [42]:
```
#replace battery NaN with median of the batteries
df1['weight'] = df1['weight'].fillna(
    value=df1[df1['brand_name'] == 'XOLO'].groupby(["brand_name"])['weight'].transform
)
```

> main_camera_mp

Finally the largest missing group. We know from EDA that `main camera mp` is decently scewed and strongly positively correlates with the `price` variables. The price variables are continuous such that many class values only have single data point membership, so grouping on them as is will not help. Instead we will group_by the `selfie camera mp` as those two features share a correlation of 0.43

In [43]:
```
df1[df1['main_camera_mp'].isna()]
```

Out[43]:

	brand_name	os	screen_size	4g	5g	main_camera_mp	selfie_camera_mp	int_memory	ra
59	Infinix	Android	17.32	yes	no	NaN	8.0	32.0	
60	Infinix	Android	15.39	yes	no	NaN	8.0	64.0	
61	Infinix	Android	15.39	yes	no	NaN	8.0	32.0	
62	Infinix	Android	15.39	yes	no	NaN	16.0	32.0	
63	Infinix	Android	15.29	yes	no	NaN	16.0	32.0	
...	...	...	...	...	...	...	...	...	
3411	Realme	Android	15.34	yes	no	NaN	16.0	64.0	
3412	Realme	Android	15.32	yes	no	NaN	16.0	64.0	
3413	Realme	Android	15.32	yes	no	NaN	25.0	64.0	
3448	Asus	Android	16.74	yes	no	NaN	24.0	128.0	
3449	Asus	Android	15.34	yes	no	NaN	8.0	64.0	

179 rows × 15 columns

In [44]:

```
# replace main_camra_mp NaN with median of the main camera mps
df1['main_camera_mp'] = df1['main_camera_mp'].fillna(
    value=df1.groupby('selfie_camera_mp')['main_camera_mp'].transform('median')
)

# Last Check
df1.isnull().sum()
```

Out[44]:

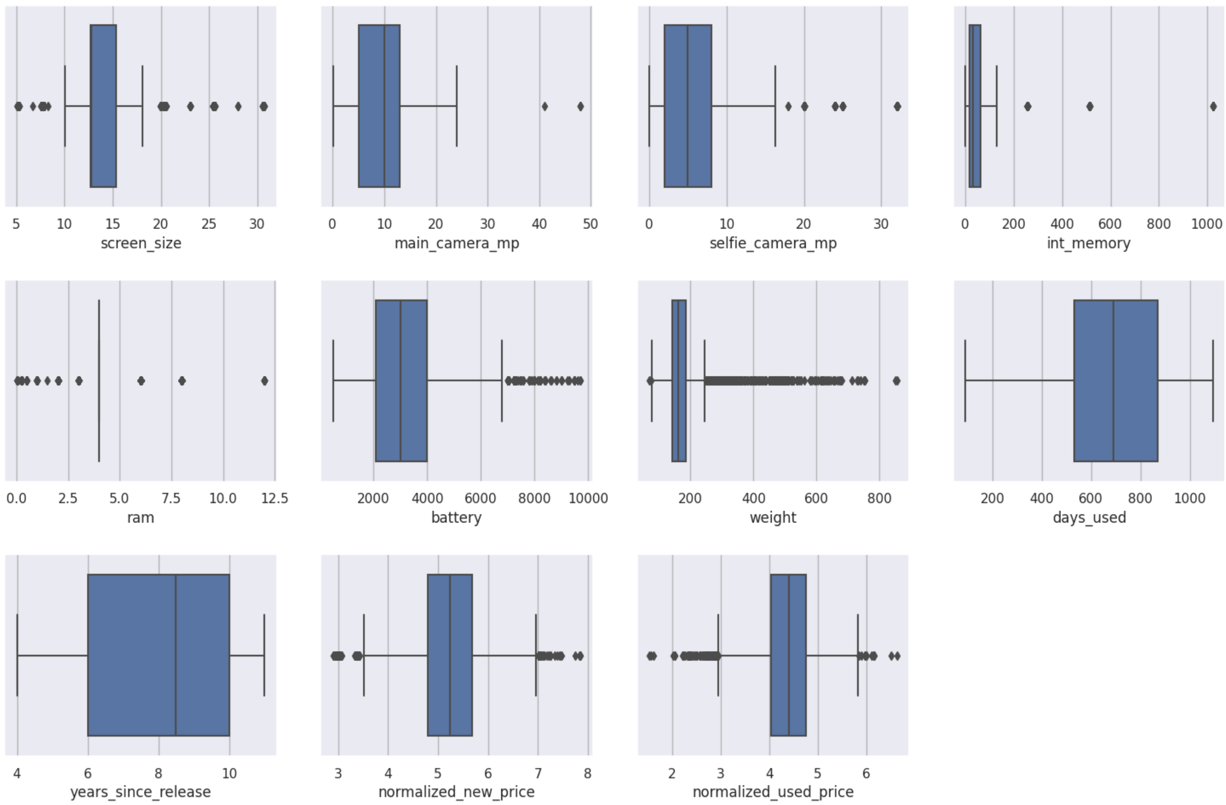
```
brand_name              0
os                      0
screen_size             0
4g                      0
5g                      0
main_camera_mp          0
selfie_camera_mp        0
int_memory              0
ram                     0
battery                 0
weight                  0
days_used               0
years_since_release     0
normalized_new_price    0
normalized_used_price   0
dtype: int64
```

No additional NaN values appear at this time.

Outlier Detection and Treatment

We already know there are going to be a fair number of outliers from our EDA work. The question is if any of the outliers need treatment.

In [45]:
```
# outlier detection using boxplot
num_cols = df1.select_dtypes(include=np.number).columns.tolist()

plt.figure(figsize=(15, 10))

for i, variable in enumerate(num_cols):
    plt.subplot(3, 4, i + 1)
    sns.boxplot(data=df1, x=variable)
    plt.tight_layout(pad=2)

plt.show()
```



- I have no reason to reject the outliers at this time. Technical specs have a wide variety, particulary over 10 years of development. However if the performance data for the model is unsatisfactory, this is one place I will return to see what can be optimized.

Prepare data for modeling

Order we will take here:

1. Create Independent and Dependent Variables
2. Add Constant to Dependent Variable
3. Create Dummy Variables
4. Split Data

In [46]:
```
# create new dataframes for variables
X = df1.drop(['normalized_used_price'], axis=1)
y = df1['normalized_used_price']
```

In [47]:
```
# add intercept to data
X = sm.add_constant(X)
```

In [48]:
```
# Create Dummy Variables for Categorical features

# drop_first=True will drop the first variable of each type. This will become the base
X = pd.get_dummies(
    X,
    columns=X.select_dtypes(include=["object", "category"]).columns.tolist(),
    drop_first=True,
)
X.head()
```

Out[48]:

	const	screen_size	main_camera_mp	selfie_camera_mp	int_memory	ram	battery	weight	days_u
0	1.0	14.50	13.0	5.0	64.0	3.0	3020.0	146.0	
1	1.0	17.30	13.0	16.0	128.0	8.0	4300.0	213.0	
2	1.0	16.69	13.0	8.0	128.0	8.0	4200.0	213.0	
3	1.0	25.50	13.0	8.0	64.0	6.0	7250.0	480.0	
4	1.0	15.32	13.0	8.0	64.0	3.0	5000.0	185.0	

5 rows × 49 columns

In [49]:
```
# Splitting data in train and test sets
# because our target variable is continuous, there are many values in the Y data set t
# thus we cant stratify this data set.
# choosing random_state=1 so this is replicable
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.30, random_state=1
)
```

In [50]:
```
# check to see if split worked correctly
print("Number of rows in train data =", X_train.shape[0])
print("Number of rows in test data =", X_test.shape[0])
```

```
Number of rows in train data = 2417
Number of rows in test data = 1037
```

Model Building - Linear Regression

In [51]:
```
olsmodel = sm.OLS(y_train, X_train).fit()
print(olsmodel.summary())
```

```
                             OLS Regression Results
================================================================================
Dep. Variable:     normalized_used_price   R-squared:                       0.845
Model:                               OLS   Adj. R-squared:                  0.841
Method:                    Least Squares   F-statistic:                     268.2
Date:                   Fri, 12 Jan 2024   Prob (F-statistic):               0.00
Time:                           20:00:13   Log-Likelihood:                 121.93
No. Observations:                   2417   AIC:                            -145.9
Df Residuals:                       2368   BIC:                             137.9
Df Model:                             48
Covariance Type:               nonrobust
====================================================================================
====
                              coef    std err          t      P>|t|      [0.025      0.
975]
------------------------------------------------------------------------------------
----
const                       1.3814      0.075     18.497      0.000       1.235
1.528
screen_size                 0.0245      0.003      7.195      0.000       0.018
0.031
main_camera_mp              0.0206      0.002     13.675      0.000       0.018
0.024
selfie_camera_mp            0.0135      0.001     12.042      0.000       0.011
0.016
int_memory                  0.0001   6.98e-05      1.678      0.094   -1.98e-05
0.000
ram                         0.0237      0.005      4.604      0.000       0.014
0.034
battery                 -1.691e-05   7.28e-06     -2.321      0.020   -3.12e-05   -2.62
e-06
weight                      0.0010      0.000      7.445      0.000       0.001
0.001
days_used                4.221e-05   3.09e-05      1.367      0.172   -1.84e-05
0.000
years_since_release        -0.0232      0.005     -5.090      0.000      -0.032      -
0.014
normalized_new_price        0.4306      0.012     35.006      0.000       0.407
0.455
brand_name_Alcatel          0.0154      0.048      0.324      0.746      -0.078
0.109
brand_name_Apple           -0.0051      0.147     -0.035      0.972      -0.294
0.284
brand_name_Asus             0.0157      0.048      0.327      0.744      -0.078
0.110
brand_name_BlackBerry      -0.0540      0.070     -0.768      0.442      -0.192
0.084
brand_name_Celkon          -0.0468      0.066     -0.706      0.480      -0.177
0.083
brand_name_Coolpad          0.0216      0.073      0.296      0.767      -0.121
0.165
brand_name_Gionee           0.0453      0.058      0.785      0.433      -0.068
0.159
brand_name_Google          -0.0313      0.085     -0.369      0.712      -0.197
0.135
brand_name_HTC             -0.0125      0.048     -0.259      0.796      -0.107
0.082
brand_name_Honor            0.0323      0.049      0.657      0.511      -0.064
0.129
brand_name_Huawei          -0.0019      0.045     -0.042      0.967      -0.089
```

```
0.085
brand_name_Infinix        0.0618      0.093      0.662      0.508      -0.121
0.245
brand_name_Karbonn        0.0945      0.067      1.407      0.160      -0.037
0.226
brand_name_LG            -0.0128      0.045     -0.283      0.777      -0.102
0.076
brand_name_Lava           0.0251      0.062      0.402      0.688      -0.097
0.147
brand_name_Lenovo         0.0458      0.045      1.013      0.311      -0.043
0.135
brand_name_Meizu         -0.0123      0.056     -0.219      0.827      -0.122
0.098
brand_name_Micromax      -0.0339      0.048     -0.707      0.479      -0.128
0.060
brand_name_Microsoft      0.0957      0.088      1.082      0.279      -0.078
0.269
brand_name_Motorola      -0.0098      0.050     -0.198      0.843      -0.107
0.088
brand_name_Nokia          0.0705      0.052      1.361      0.174      -0.031
0.172
brand_name_OnePlus        0.0700      0.077      0.904      0.366      -0.082
0.222
brand_name_Oppo           0.0150      0.048      0.314      0.753      -0.079
0.109
brand_name_Others        -0.0076      0.042     -0.181      0.856      -0.090
0.075
brand_name_Panasonic      0.0540      0.056      0.967      0.334      -0.056
0.164
brand_name_Realme         0.0334      0.062      0.542      0.588      -0.087
0.154
brand_name_Samsung       -0.0312      0.043     -0.722      0.471      -0.116
0.054
brand_name_Sony          -0.0605      0.051     -1.198      0.231      -0.160
0.039
brand_name_Spice         -0.0150      0.063     -0.237      0.813      -0.139
0.109
brand_name_Vivo          -0.0140      0.048     -0.290      0.772      -0.109
0.081
brand_name_XOLO           0.0153      0.055      0.279      0.780      -0.092
0.123
brand_name_Xiaomi         0.0875      0.048      1.816      0.069      -0.007
0.182
brand_name_ZTE           -0.0050      0.047     -0.105      0.916      -0.098
0.088
os_Others                -0.0483      0.033     -1.471      0.141      -0.113
0.016
os_Windows               -0.0201      0.045     -0.445      0.657      -0.109
0.069
os_iOS                   -0.0640      0.147     -0.437      0.662      -0.351
0.223
4g_yes                    0.0540      0.016      3.400      0.001       0.023
0.085
5g_yes                   -0.0707      0.032     -2.243      0.025      -0.132      -
0.009
==============================================================================
Omnibus:                      223.629   Durbin-Watson:                   1.909
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              420.797
Skew:                          -0.621   Prob(JB):                     4.22e-92
Kurtosis:                       4.624   Cond. No.                     1.78e+05
```

```
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
ified.
[2] The condition number is large, 1.78e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Initial Regression Result Interpretation

1. **Adjusted. R-squared**: It reflects the fit of the model.
   - Adjusted R-squared values generally range from 0 to 1, where a higher value generally indicates a better fit, assuming certain conditions are met.
   - In our case, the value for adj. R-squared is **0.842**, which is a great starting point.

1. ***const* coefficient**: It is the Y-intercept.
   - It means that if all the predictor variable coefficients are zero, then the expected output (i.e., Y) would be equal to the *const* coefficient.
   - In our case, the value for `const` coefficient is **1.3582**

1. **Coefficient of a predictor variable**: It represents the change in the output Y due to a change in the predictor variable (everything else held constant).
   - In our case, the coefficient of `normalized new price` is **0.4307** which is quite large, but if you are the person selling or trading in your used device you might not think so.

Model Performance Check

Running a performance check no, prior to multicollinearity, gives us a comparison starting point gor our main metrics. The metrics we will be using to judge the model today are;

- RMSE
- MAE
- R^2
- Adjusted R^2
- MAPE - this will be interesting as several of our features are equal to 0, there are many extreme values and thus outliers in the data set. We will need to be careful with this metric, particularly early.

In [52]:
```python
# We need a few custom functions to find the state metrics
# Then we will summarize those metrics in a chart

# function to compute adjusted R-squared
def adj_r2_score(predictors, targets, predictions):
    r2 = r2_score(targets, predictions)
    n = predictors.shape[0]
    k = predictors.shape[1]
    return 1 - ((1 - r2) * (n - 1) / (n - k - 1))
```

```python
# function to compute MAPE
def mape_score(targets, predictions):
    return np.mean(np.abs(targets - predictions) / targets) * 100


# function to compute different metrics to check performance of a regression model
def model_performance_regression(model, predictors, target):
    """
    Function to compute different metrics to check regression model performance

    model: regressor
    predictors: independent variables
    target: dependent variable
    """

    # predicting using the independent variables
    pred = model.predict(predictors)

    r2 = r2_score(target, pred)  # to compute R-squared
    adjr2 = adj_r2_score(predictors, target, pred)  # to compute adjusted R-squared
    rmse = np.sqrt(mean_squared_error(target, pred))  # to compute RMSE
    mae = mean_absolute_error(target, pred)  # to compute MAE
    mape = mape_score(target, pred)  # to compute MAPE

    # creating a dataframe of metrics
    df_perf = pd.DataFrame(
        {
            "RMSE": rmse,
            "MAE": mae,
            "R-squared": r2,
            "Adj. R-squared": adjr2,
            "MAPE": mape,
        },
        index=[0],
    )

    return df_perf
```

We can compare these metrics for our Training and Test performance data. The inside the custom function **model_performance_regression**, it calculates the predictions using the model and the independent variable. These predictions are then compared to the actual target data to test performance.

In [53]:

```python
# checking model performance on train set (seen 70% data)
print("Training Performance\n")
olsmodel_train_perf = model_performance_regression(olsmodel, X_train, y_train)
print(olsmodel_train_perf, "\n\n")

# checking model performance on test set (seen 30% data)
print("Test Performance\n")
olsmodel_test_perf = model_performance_regression(olsmodel, X_test, y_test)
print(olsmodel_test_perf)
```

```
Training Performance

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.230067  0.180513   0.844638        0.841422  4.331739


Test Performance

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.238749  0.185047   0.841962        0.834117  4.509903
```

- The train and test values for all metrics are comprable - this is evidence that our model is not overfitting to the train data set.
- The training R^2 is 0.841, so the model is also not underfitting.
- MAE suggests that the model can currently predict the normalized used cost of a device with a mean error of 0.18 on the test data
- The MAPE of 4.5 on the rest data means that we are able to predict the cost of a used device within 4.5%

Multicollinearity

This fucntion will produce a chart of VIFs for each feature.

In [54]:
```
def checking_vif(predictors):
    vif = pd.DataFrame()
    vif["feature"] = predictors.columns

    # calculating VIF for each feature
    vif["VIF"] = [
        variance_inflation_factor(predictors.values, i)
        for i in range(len(predictors.columns))
    ]
    return vif
```

In [55]:
```
vif_df = checking_vif(X_train).sort_values(by = "VIF")
vif_df
```

Out[55]:

	feature	VIF
22	brand_name_Infinix	1.287613
18	brand_name_Google	1.321784
4	int_memory	1.364595
32	brand_name_OnePlus	1.437127
16	brand_name_Coolpad	1.467947
23	brand_name_Karbonn	1.573730
45	os_Windows	1.595359
14	brand_name_BlackBerry	1.633748
39	brand_name_Spice	1.688904
25	brand_name_Lava	1.712289
15	brand_name_Celkon	1.775033
48	5g_yes	1.812955
44	os_Others	1.855966
29	brand_name_Microsoft	1.869450
36	brand_name_Realme	1.946486
17	brand_name_Gionee	1.951267
35	brand_name_Panasonic	2.106401
41	brand_name_XOLO	2.138134
27	brand_name_Meizu	2.179506
5	ram	2.256876
2	main_camera_mp	2.301882
47	4g_yes	2.466277
8	days_used	2.660219
3	selfie_camera_mp	2.809386
38	brand_name_Sony	2.942954
10	normalized_new_price	3.133184
30	brand_name_Motorola	3.273871
13	brand_name_Asus	3.331892
20	brand_name_Honor	3.340503
28	brand_name_Micromax	3.363600
11	brand_name_Alcatel	3.405706
19	brand_name_HTC	3.410113
31	brand_name_Nokia	3.472893
40	brand_name_Vivo	3.650779

	feature	VIF
42	brand_name_Xiaomi	3.719750
43	brand_name_ZTE	3.797320
33	brand_name_Oppo	3.971141
6	battery	4.082852
26	brand_name_Lenovo	4.558770
24	brand_name_LG	4.849897
9	years_since_release	4.905339
21	brand_name_Huawei	5.983851
7	weight	6.400694
37	brand_name_Samsung	7.539937
1	screen_size	7.673096
34	brand_name_Others	9.710730
46	os_iOS	11.782043
12	brand_name_Apple	13.055131
0	const	249.500215

To remove multicollinearity

1. Drop every column one by one that has a VIF score greater than 5. However, the dummy variables can be ignores.
2. Look at the adjusted R-squared and RMSE of all these models.
3. Drop the variable that makes the least change in adjusted R-squared.
4. Check the VIF scores again.
5. Continue till you get all VIF scores under 5.

Let's define a function that will help us do this.

In [56]:
```
def treating_multicollinearity(predictors, target, high_vif_columns):
    """
    Checking the effect of dropping the columns showing high multicollinearity
    on model performance (adj. R-squared and RMSE)

    predictors: independent variables
    target: dependent variable
    high_vif_columns: columns having high VIF
    """
    # empty lists to store adj. R-squared and RMSE values
    adj_r2 = []
    rmse = []

    # build ols models by dropping one of the high VIF columns at a time
    # store the adjusted R-squared and RMSE in the lists defined previously
    for cols in high_vif_columns:
```

```
        # defining the new train set
        train = predictors.loc[:, ~predictors.columns.str.startswith(cols)]

        # create the model
        olsmodel = sm.OLS(target, train).fit()

        # adding adj. R-squared and RMSE to the lists
        adj_r2.append(olsmodel.rsquared_adj)
        rmse.append(np.sqrt(olsmodel.mse_resid))

    # creating a dataframe for the results
    temp = pd.DataFrame(
        {
            "col": high_vif_columns,
            "Adj. R-squared after_dropping col": adj_r2,
            "RMSE after dropping col": rmse,
        }
    ).sort_values(by="Adj. R-squared after_dropping col", ascending=False)
    temp.reset_index(drop=True, inplace=True)

    return temp
```

In [57]:

```
# Since the very beginning, the high correlation between battery, screen size, and wei
#lets start with those as they are the highest non - dummy VIF values
col_list = ['screen_size', 'weight', 'battery']
res = treating_multicollinearity(X_train, y_train, col_list)
res
```

Out[57]:

	col	Adj. R-squared after_dropping col	RMSE after dropping col
0	battery	0.841196	0.232650
1	screen_size	0.838092	0.234913
2	weight	0.837847	0.235090

The original adjusted R^2 wass 0.841. The smallest drop in R^2 was for the battery feature. This one will be removed first. Then we will check VIF again:

In [58]:

```
col_to_drop = "battery"
X_train2 = X_train.loc[:, ~X_train.columns.str.startswith(col_to_drop)]
X_test2 = X_test.loc[:, ~X_test.columns.str.startswith(col_to_drop)]

# Check VIF now
vif = checking_vif(X_train2)
print("VIF after dropping ", col_to_drop)
vif.sort_values(by = "VIF")
```

```
VIF after dropping  battery
```

Out[58]:

	feature	VIF
21	brand_name_Infinix	1.279237
17	brand_name_Google	1.321437
4	int_memory	1.364594
31	brand_name_OnePlus	1.437122
15	brand_name_Coolpad	1.467752
22	brand_name_Karbonn	1.572098
44	os_Windows	1.593486
13	brand_name_BlackBerry	1.633610
38	brand_name_Spice	1.688748
24	brand_name_Lava	1.712280
14	brand_name_Celkon	1.773537
47	5g_yes	1.812602
43	os_Others	1.848024
28	brand_name_Microsoft	1.869345
35	brand_name_Realme	1.939327
16	brand_name_Gionee	1.939603
34	brand_name_Panasonic	2.106374
40	brand_name_XOLO	2.137841
26	brand_name_Meizu	2.179345
5	ram	2.256738
2	main_camera_mp	2.279383
46	4g_yes	2.434749
7	days_used	2.659405
3	selfie_camera_mp	2.807978
37	brand_name_Sony	2.941446
9	normalized_new_price	3.118989
29	brand_name_Motorola	3.273572
12	brand_name_Asus	3.330590
19	brand_name_Honor	3.340290
27	brand_name_Micromax	3.363010
10	brand_name_Alcatel	3.401027
18	brand_name_HTC	3.409155
30	brand_name_Nokia	3.470357
39	brand_name_Vivo	3.649537

	feature	VIF
41	brand_name_Xiaomi	3.706349
42	brand_name_ZTE	3.797315
32	brand_name_Oppo	3.970779
25	brand_name_Lenovo	4.552290
8	years_since_release	4.759563
23	brand_name_LG	4.847958
6	weight	5.767609
20	brand_name_Huawei	5.982128
1	screen_size	7.225586
36	brand_name_Samsung	7.537389
33	brand_name_Others	9.710713
45	os_iOS	11.771080
11	brand_name_Apple	13.048774
0	const	249.176046

Still have `screen_size` and `weight` VIF values greater than 5, so we will keep going:

In [59]:
```
col_list = ['screen_size', 'weight']
res = treating_multicollinearity(X_train, y_train, col_list)
res
```

Out[59]:

	col	Adj. R-squared after_dropping col	RMSE after dropping col
0	screen_size	0.838092	0.234913
1	weight	0.837847	0.235090

The original adjusted R^2 wass 0.841. The smallest drop in R^2 was for the screen_size feature. This one will be our 2nd removed variable. Then we will check VIF again:

In [60]:
```
col_to_drop = "screen_size"
X_train3 = X_train2.loc[:, ~X_train2.columns.str.startswith(col_to_drop)]
X_test3 = X_test2.loc[:, ~X_test2.columns.str.startswith(col_to_drop)]

# Check VIF now
vif = checking_vif(X_train3)
print("VIF after dropping ", col_to_drop)
vif
```

```
VIF after dropping  screen_size
```

Out[60]:

	feature	VIF
0	const	215.009771
1	main_camera_mp	2.270808
2	selfie_camera_mp	2.803120
3	int_memory	1.362429
4	ram	2.256720
5	weight	1.366582
6	days_used	2.645792
7	years_since_release	4.491322
8	normalized_new_price	3.061021
9	brand_name_Alcatel	3.401010
10	brand_name_Apple	12.977666
11	brand_name_Asus	3.326216
12	brand_name_BlackBerry	1.632320
13	brand_name_Celkon	1.772977
14	brand_name_Coolpad	1.467318
15	brand_name_Gionee	1.933948
16	brand_name_Google	1.319297
17	brand_name_HTC	3.396478
18	brand_name_Honor	3.339774
19	brand_name_Huawei	5.980170
20	brand_name_Infinix	1.279234
21	brand_name_Karbonn	1.572074
22	brand_name_LG	4.832443
23	brand_name_Lava	1.711954
24	brand_name_Lenovo	4.549408
25	brand_name_Meizu	2.176385
26	brand_name_Micromax	3.358649
27	brand_name_Microsoft	1.867442
28	brand_name_Motorola	3.261685
29	brand_name_Nokia	3.450731
30	brand_name_OnePlus	1.437095
31	brand_name_Oppo	3.965431
32	brand_name_Others	9.648248
33	brand_name_Panasonic	2.105542

	feature	VIF
34	brand_name_Realme	1.938129
35	brand_name_Samsung	7.523065
36	brand_name_Sony	2.933767
37	brand_name_Spice	1.683307
38	brand_name_Vivo	3.649127
39	brand_name_XOLO	2.137716
40	brand_name_Xiaomi	3.703978
41	brand_name_ZTE	3.788114
42	os_Others	1.620630
43	os_Windows	1.593485
44	os_iOS	11.640619
45	4g_yes	2.434455
46	5g_yes	1.807984

The VIF for weight dropped drammatically to 1.366. This means that our model only needs to look at weight, the screen_size and battery capacity were too multicollinear.

The only remaining VIF points greater than 5 are dummy variables. The years_since_release VIF is equal to 4.49 which is close to the traditional cut of of a VIF value of 5. We can come back to this if necessary.

In [61]:
```
olsmodel3 = sm.OLS(y_train, X_train3).fit()
print(olsmodel3.summary())
```

```
                              OLS Regression Results
==================================================================================
Dep. Variable:     normalized_used_price   R-squared:                       0.841
Model:                               OLS   Adj. R-squared:                  0.838
Method:                    Least Squares   F-statistic:                     273.0
Date:                   Fri, 12 Jan 2024   Prob (F-statistic):               0.00
Time:                           20:00:16   Log-Likelihood:                 95.609
No. Observations:                   2417   AIC:                            -97.22
Df Residuals:                       2370   BIC:                             174.9
Df Model:                             46
Covariance Type:               nonrobust
======================================================================================
====
                            coef    std err          t      P>|t|      [0.025      0.
975]
--------------------------------------------------------------------------------------
----
const                     1.5641      0.070     22.326      0.000       1.427
1.701
main_camera_mp            0.0209      0.002     13.812      0.000       0.018
0.024
selfie_camera_mp          0.0138      0.001     12.160      0.000       0.012
0.016
int_memory             9.798e-05   7.05e-05      1.390      0.165   -4.03e-05
0.000
ram                       0.0237      0.005      4.558      0.000       0.013
0.034
weight                    0.0017   6.24e-05     26.523      0.000       0.002
0.002
days_used              2.836e-05   3.11e-05      0.911      0.362   -3.27e-05    8.94
e-05
years_since_release      -0.0287      0.004     -6.504      0.000      -0.037      -
0.020
normalized_new_price      0.4402      0.012     35.822      0.000       0.416
0.464
brand_name_Alcatel        0.0188      0.048      0.391      0.696      -0.076
0.113
brand_name_Apple          0.0616      0.148      0.415      0.678      -0.229
0.352
brand_name_Asus           0.0016      0.048      0.033      0.974      -0.093
0.097
brand_name_BlackBerry    -0.0690      0.071     -0.972      0.331      -0.208
0.070
brand_name_Celkon        -0.0504      0.067     -0.752      0.452      -0.182
0.081
brand_name_Coolpad        0.0149      0.074      0.203      0.839      -0.130
0.159
brand_name_Gionee         0.0137      0.058      0.236      0.814      -0.100
0.128
brand_name_Google        -0.0577      0.085     -0.675      0.499      -0.225
0.110
brand_name_HTC           -0.0307      0.049     -0.631      0.528      -0.126
0.065
brand_name_Honor          0.0356      0.050      0.716      0.474      -0.062
0.133
brand_name_Huawei        -0.0091      0.045     -0.203      0.839      -0.097
0.079
brand_name_Infinix        0.0433      0.094      0.461      0.645      -0.141
0.228
brand_name_Karbonn        0.1013      0.068      1.494      0.135      -0.032
```

```
0.234
brand_name_LG           -0.0325      0.046     -0.710      0.478      -0.122
0.057
brand_name_Lava          0.0195      0.063      0.309      0.757      -0.104
0.143
brand_name_Lenovo        0.0341      0.046      0.747      0.455      -0.055
0.124
brand_name_Meizu        -0.0275      0.057     -0.486      0.627      -0.139
0.084
brand_name_Micromax     -0.0471      0.048     -0.975      0.330      -0.142
0.048
brand_name_Microsoft     0.0779      0.089      0.873      0.383      -0.097
0.253
brand_name_Motorola     -0.0314      0.050     -0.627      0.531      -0.130
0.067
brand_name_Nokia         0.0472      0.052      0.903      0.367      -0.055
0.150
brand_name_OnePlus       0.0674      0.078      0.861      0.389      -0.086
0.221
brand_name_Oppo          0.0020      0.048      0.041      0.968      -0.093
0.097
brand_name_Others       -0.0306      0.042     -0.721      0.471      -0.114
0.053
brand_name_Panasonic     0.0460      0.056      0.814      0.416      -0.065
0.157
brand_name_Realme        0.0143      0.062      0.230      0.818      -0.108
0.136
brand_name_Samsung      -0.0459      0.044     -1.052      0.293      -0.132
0.040
brand_name_Sony         -0.0755      0.051     -1.481      0.139      -0.175
0.024
brand_name_Spice        -0.0410      0.064     -0.642      0.521      -0.166
0.084
brand_name_Vivo         -0.0196      0.049     -0.401      0.689      -0.116
0.076
brand_name_XOLO          0.0110      0.055      0.198      0.843      -0.098
0.120
brand_name_Xiaomi        0.0724      0.049      1.492      0.136      -0.023
0.168
brand_name_ZTE          -0.0208      0.048     -0.435      0.664      -0.115
0.073
os_Others               -0.1318      0.031     -4.253      0.000      -0.193      -
0.071
os_Windows              -0.0166      0.046     -0.364      0.716      -0.106
0.073
os_iOS                  -0.1591      0.147     -1.081      0.280      -0.448
0.130
4g_yes                   0.0510      0.016      3.200      0.001       0.020
0.082
5g_yes                  -0.0805      0.032     -2.533      0.011      -0.143      -
0.018
==============================================================================
Omnibus:                      235.226   Durbin-Watson:                   1.906
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              441.109
Skew:                          -0.649   Prob(JB):                     1.64e-96
Kurtosis:                       4.642   Cond. No.                     3.82e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
```

```
ified.
[2] The condition number is large, 3.82e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

In [62]:

```
# checking model performance on train set (seen 70% data)
print("Training Performance - Model 1\n")
olsmodel_train_perf = model_performance_regression(olsmodel, X_train, y_train)
print(olsmodel_train_perf, "\n\n")

# checking model performance on test set (seen 30% data)
print("Test Performance- Model 1\n")
olsmodel_test_perf = model_performance_regression(olsmodel, X_test, y_test)
print(olsmodel_test_perf, "\n\n")

# checking model performance on train set (seen 70% data)
print("Training Performance- Model 3\n")
olsmodel_train_perf = model_performance_regression(olsmodel3, X_train3, y_train)
print(olsmodel_train_perf, "\n\n")

# checking model performance on test set (seen 30% data)
print("Test Performance - Model 3\n")
olsmodel_test_perf = model_performance_regression(olsmodel3, X_test3, y_test)
print(olsmodel_test_perf)
```

```
Training Performance - Model 1

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.230067  0.180513   0.844638        0.841422  4.331739


Test Performance- Model 1

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.238749  0.185047   0.841962        0.834117  4.509903


Training Performance- Model 3

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.232586  0.181761   0.841218        0.838068  4.370381


Test Performance - Model 3

      RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.24301  0.188755    0.83627         0.82849  4.605767
```

Based on this data - Model 1 (all data) is actually stronger in predictive accuracy than Model 3 (remvoe two multicollinear points). However, we are not done quite yet, we will look at p-values next. The benefit of Model 3 is that it is less complex and thus easier to interpret. It is possible that Model 1 was slightly overfit, and now Model 3 is slightly underfit. Further, the drop in these performance measurements, while noticible, are actually still quite small.

We shall carry on in the process with knowledge of the trade off between performance and complexity.

Analysis of high p-value variables - Independence of Features

Next, a similar iterative process to drop variables with high p-values. A "high" p-value is defined as greater that 0.05, a chosen significant level. The null hypothese is that each independent feature is not significant. Thus, a p-value greater that 0.05 confirms thathypothese - the feature is not significant and can be removed.

This iterates similarly in concept to how we approached the VIF values for multicolinearity - we will remove one variable at a time and recalculate the model to see the effect on other p-values. This repeates until all features have p-values less than 0.05, until all independent features are significant.

However, two big changes: we can now remove dummy variables without concern and we can write a while loop to step through the interation for us. The below function will produce a list of features that are selected as significant. We then fit a new model to just those features.

In [63]:
```
# initial list of columns
predictors = X_train3.copy()
cols = predictors.columns.tolist()

# setting an initial max p-value
max_p_value = 1

while len(cols) > 0:
    # defining the train set
    X_train_aux = predictors[cols]

    # fitting the model
    model = sm.OLS(y_train, X_train_aux).fit()

    # getting the p-values and the maximum p-value
    p_values = model.pvalues
    max_p_value = max(p_values)

    # name of the variable with maximum p-value
    feature_with_p_max = p_values.idxmax()

    if max_p_value > 0.05:
        cols.remove(feature_with_p_max)
    else:
        break

selected_features = cols
print(selected_features)
```

```
['const', 'main_camera_mp', 'selfie_camera_mp', 'ram', 'weight', 'years_since_releas
e', 'normalized_new_price', 'brand_name_Karbonn', 'brand_name_Samsung', 'brand_name_S
ony', 'brand_name_Xiaomi', 'os_Others', 'os_iOS', '4g_yes', '5g_yes']
```

We not take that to make Model #4:

In [64]:
```
# Select only the features selected as significant from data set 3:
X_train4 = X_train3[selected_features]
X_test4 = X_test3[selected_features]
```

```
# and create Model #4
olsmodel4 = sm.OLS(y_train, X_train4).fit()
print(olsmodel4.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     normalized_used_price   R-squared:                       0.839
Model:                               OLS   Adj. R-squared:                  0.838
Method:                    Least Squares   F-statistic:                     894.9
Date:                   Fri, 12 Jan 2024   Prob (F-statistic):               0.00
Time:                           20:00:17   Log-Likelihood:                 79.741
No. Observations:                   2417   AIC:                            -129.5
Df Residuals:                       2402   BIC:                            -42.63
Df Model:                             14
Covariance Type:               nonrobust
==================================================================================
===
                            coef    std err          t      P>|t|      [0.025      0.9
75]
----------------------------------------------------------------------------------
---
const                     1.5776      0.052     30.217      0.000       1.475       1.
680
main_camera_mp            0.0209      0.001     14.620      0.000       0.018       0.
024
selfie_camera_mp          0.0139      0.001     12.921      0.000       0.012       0.
016
ram                       0.0218      0.005      4.382      0.000       0.012       0.
032
weight                    0.0017   6.01e-05     27.655      0.000       0.002       0.
002
years_since_release      -0.0285      0.003     -8.393      0.000      -0.035      -0.
022
normalized_new_price      0.4414      0.011     39.298      0.000       0.419       0.
463
brand_name_Karbonn        0.1157      0.055      2.113      0.035       0.008       0.
223
brand_name_Samsung       -0.0372      0.016     -2.255      0.024      -0.070      -0.
005
brand_name_Sony          -0.0663      0.030     -2.173      0.030      -0.126      -0.
006
brand_name_Xiaomi         0.0808      0.026      3.143      0.002       0.030       0.
131
os_Others                -0.1265      0.027     -4.636      0.000      -0.180      -0.
073
os_iOS                   -0.0888      0.045     -1.967      0.049      -0.177      -0.
000
4g_yes                    0.0507      0.015      3.359      0.001       0.021       0.
080
5g_yes                   -0.0663      0.031     -2.160      0.031      -0.126      -0.
006
==============================================================================
Omnibus:                      247.525   Durbin-Watson:                   1.900
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              489.073
Skew:                          -0.659   Prob(JB):                    6.30e-107
Kurtosis:                       4.765   Cond. No.                     2.42e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
ified.
[2] The condition number is large, 2.42e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Check the performance;

In [65]:
```
# checking model performance on train set (seen 70% data)
print("Training Performance - Model 1\n")
olsmodel_train_perf = model_performance_regression(olsmodel, X_train, y_train)
print(olsmodel_train_perf, "\n\n")

# checking model performance on test set (seen 30% data)
print("Test Performance- Model 1\n")
olsmodel_test_perf = model_performance_regression(olsmodel, X_test, y_test)
print(olsmodel_test_perf, "\n\n")

# checking model performance on train set (seen 70% data)
print("Training Performance- Model 4\n")
olsmodel_train_perf = model_performance_regression(olsmodel4, X_train4, y_train)
print(olsmodel_train_perf, "\n\n")

# checking model performance on test set (seen 30% data)
print("Test Performance - Model 4\n")
olsmodel_test_perf = model_performance_regression(olsmodel4, X_test4, y_test)
print(olsmodel_test_perf)
```

```
Training Performance - Model 1

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.230067  0.180513   0.844638        0.841422  4.331739


Test Performance- Model 1

       RMSE       MAE  R-squared  Adj. R-squared      MAPE
0  0.238749  0.185047   0.841962        0.834117  4.509903


Training Performance- Model 4

       RMSE       MAE  R-squared  Adj. R-squared     MAPE
0  0.234118  0.182771   0.839119        0.838114  4.39565


Test Performance - Model 4

       RMSE       MAE  R-squared  Adj. R-squared     MAPE
0  0.241513  0.186707   0.838281        0.835905  4.55824
```

Analysis of Performance Metrics

Once again we see that Model 1 slighlty outperforms Model 4. However, Model 4 is significantly less complex and the drop in performance is not significant. Model 4 is still performing very well. Here is a more detailed look at what each of these Performance Metrics actually mean;

- **RMSE** is a measure of minimization of prediction errors. Said another way, a lower RMSE means that a model is minimizing errors. Our target variable is normalized (which we will prove shortly) with a minimum of 1.5 and a maximum of 6.2. The RMSE of 0.24 on that scale is about a 6% error swing. Thus the margins of the buisness model must keep that in mind for risk calculations.

- **MAE** is similar, it is the absoulte error. A smaler MAE means that the predictions are closer to the "true" values. MAE is 0.19, which again on our target variable range is about a 4% error.

- **MAPE** takes the MAE and instead considers the percent error, as already discussed!

- **Adjusted R^2 and R^2** work together to comment on complexity. Adjusted R^2 penalizes models with to omuch complexity because of the inclusion of the number of predictors (features) included in the model. When it decreases, it means that variables removed did indeed have some explanatory power. In this case the decrease is very small. It is also worth noting that the two variables removed by multicollinearity; `screen_size` and `battery` had more variability than `weight`, a feature whose values had a strong modal tendency. This means that removing `weight` would have a greater effect on the model, but we did lose some nuance with the removal of the more variable features of the trio.

Checking Linear Regression Assumptions

We will be checking the following Linear Regression assumptions:

1. **Linearity of variables**

2. **Independence of error terms**

3. **Normality of error terms**

4. **No Heteroscedasticity**

Linearity and Independence

We will plot the fitted values vs the residuals for model 4. We are hoping to see no pattern in the data - showing that the residuals are independent and the model is linear.

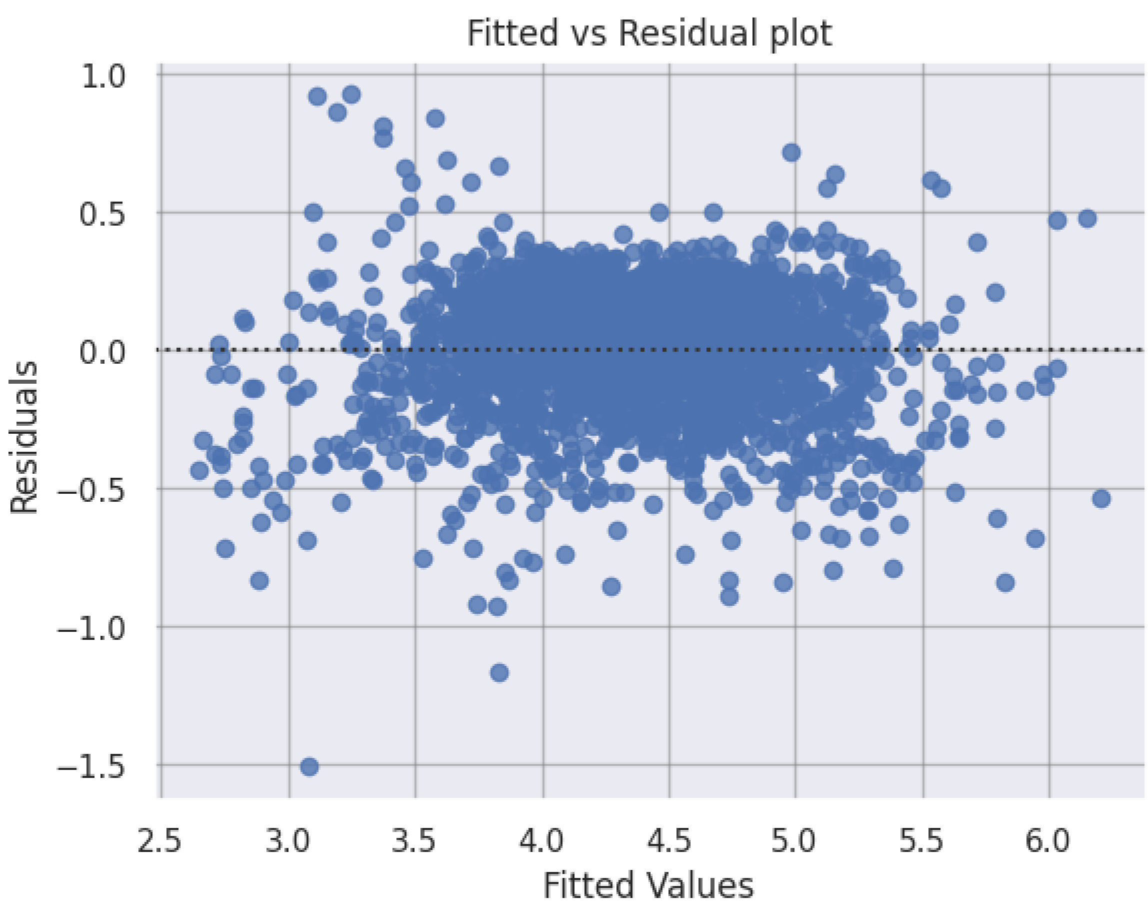
In [66]:
```
# let us create a dataframe with actual, fitted and residual values
df_pred = pd.DataFrame()

df_pred["Actual Values"] = y_train  # actual values
df_pred["Fitted Values"] = olsmodel4.fittedvalues  # predicted values
df_pred["Residuals"] = olsmodel4.resid  # residuals

df_pred.head()
```

Out[66]:

	Actual Values	Fitted Values	Residuals
3026	4.087488	3.867212	0.220276
1525	4.448399	4.600978	-0.152578
1128	4.315353	4.286047	0.029306
3003	4.282068	4.195229	0.086839
2907	4.456438	4.489618	-0.033180

In [67]:

```
# let's plot the fitted values vs residuals

sns.residplot(
    data=df_pred, x="Fitted Values", y="Residuals"
)
plt.xlabel("Fitted Values")
plt.ylabel("Residuals")
plt.title("Fitted vs Residual plot")
plt.show()
```



The scatter plot shows the distribution of residuals (errors) vs fitted values (predicted values).

If there exist any pattern in this plot, we consider it as signs of non-linearity in the data and a pattern means that the model doesn't capture non-linear effects.

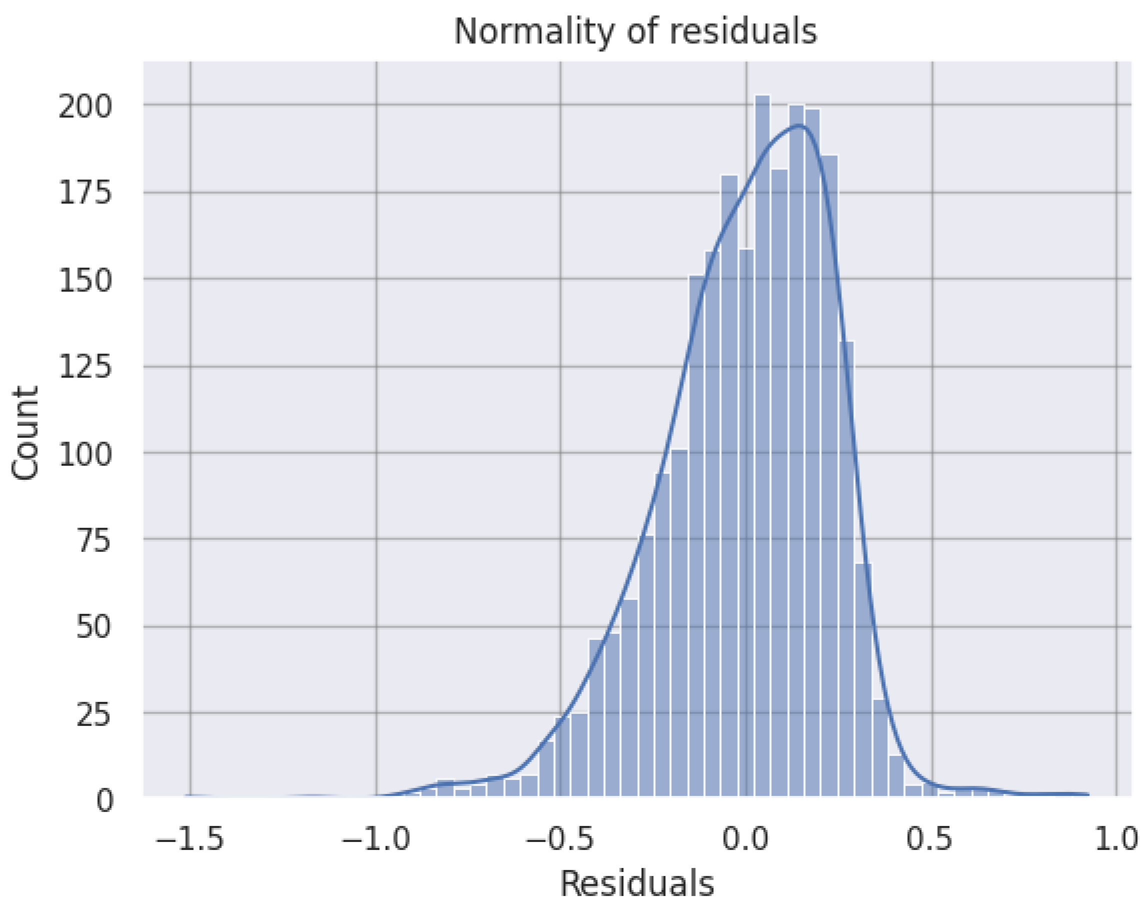
We see no real pattern in the plot above it - it is generally ovoid but there isnt a clear curve or funnel. But we will also test the homoscedasticity of the model in a moment to help build the case for the strengths or limitations of the model. For now, the assumptions of linearity and independence are mediocre-ly satisfied. This is likely due to the large number of zeros in our data set - which could be fixed with some time with a SME or it could be a reality of this market.

Test for Normality

An assumption of this model is that the residuals are normally distributed - this allows appropriate sized and stability in confidence intervals. In order to rest the assumption, we will look at three indicators of normality; a Histogram, a Q-Q plot, and a Shapiro-Wilk test.

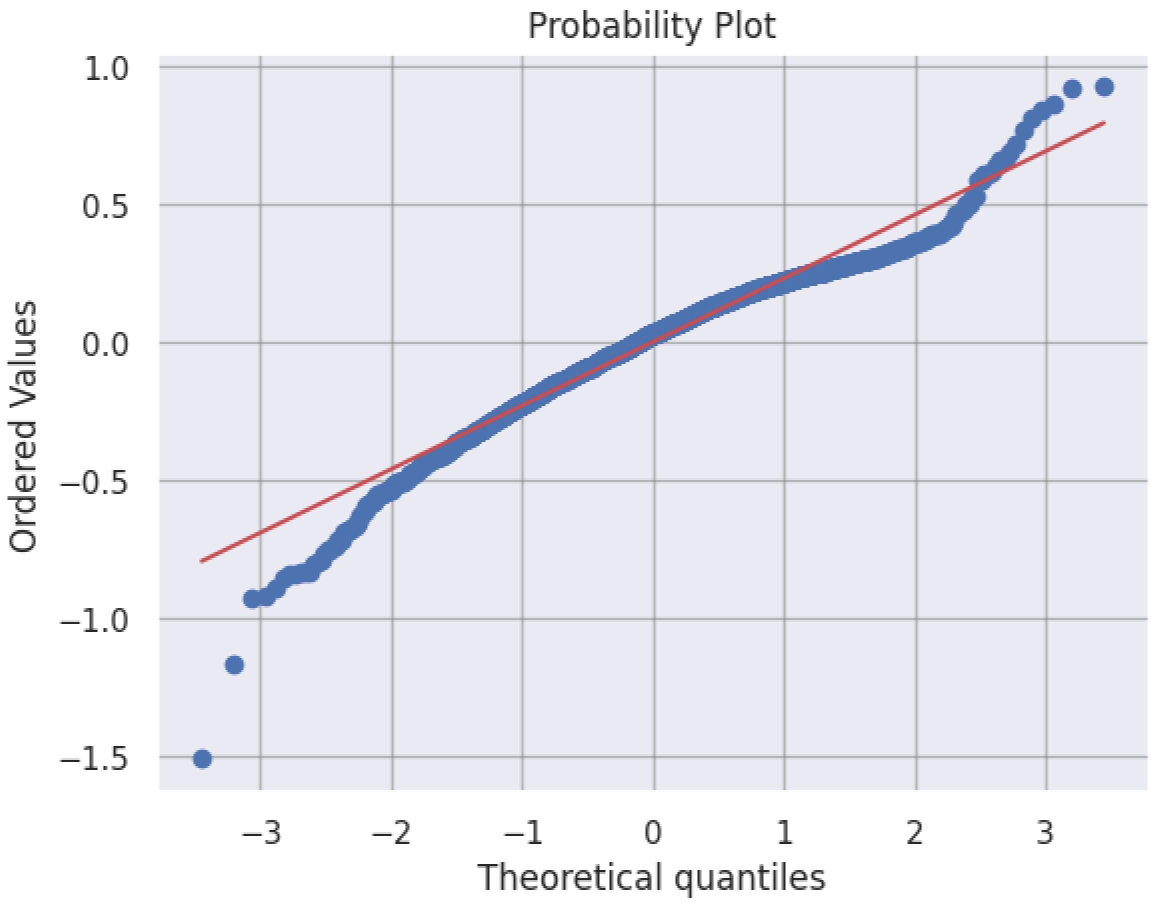
We already know there are unusual data points in the data set, so this discussion will be how crucial further discussions of those unusual data points will be.

In [68]:
```
sns.histplot(data=df_pred, x="Residuals", kde=True)
plt.title("Normality of residuals")
plt.show()
```



Its more bell shaped than I expected! There are moredata points on the left of the data, but the data is nearly symmetric about residuals = 0.0.

In [69]:
```
# Q-Q plot
stats.probplot(df_pred["Residuals"], dist="norm", plot=pylab)
plt.show()
```



The tails on this data is pretty rough - evidence that a quality SME is worth the investement.

In [70]:
```
# Shapiro-Wilk test
stats.shapiro(df_pred["Residuals"])
```

Out[70]:
```
ShapiroResult(statistic=0.9674226641654968, pvalue=5.645825118302675e-23)
```

The p-value is significantly less than 0.05, thus the residuals are NOT normal as per the Shapiro - Wilk test. Generally speaking we can say that the assumption is once again only mediocre-ly satisfied, and further data prep work will increase the normalcy of the residuals.

Test for Homoscedasticity

Whereas the previous section worked with the assumption that the magnitude of the residuals were normally distrubuted, homoscedasticity looks at the distribution of the residuals across the regression line itself. The presence of outliers, a known issue in this data, reduces homoscedasticity.

The goldfeldquant test has a null hypothese that the residuals are homoscedastic, so we are looking for p-value greater than 0.05.

In [71]:
```
name = ["F statistic", "p-value"]
test = sms.het_goldfeldquandt(df_pred["Residuals"], X_train4)
lzip(name, test)
```

Out[71]:
```
[('F statistic', 1.0088616644571273), ('p-value', 0.43944597704581817)]
```

This assumption is satisfied! This is confidence building that the outliers in the data, especially as the represent specific phone models, are not harming the model. That isnt to say that the model wouldnt be improved by their treatment, however.

Summary

The assumptions of linear regression have been, at least at a very low level, satisfied. Improvement of the data preperation, particulary in working with outliers and the presence of 0-values, would improve these metrics.

Final Model

Lets print the final model one more time and look at some sample predictions.

In [72]:
```
# create new "final" independent variables
X_train_final = X_train4.copy()
X_test_final = X_test4.copy()

# this is equvalent to Model #4
olsmodel_final = sm.OLS(y_train, X_train_final).fit()
print(olsmodel_final.summary())
```

```
                            OLS Regression Results                            
==================================================================================
Dep. Variable:     normalized_used_price   R-squared:                       0.839
Model:                               OLS   Adj. R-squared:                  0.838
Method:                    Least Squares   F-statistic:                     894.9
Date:                   Fri, 12 Jan 2024   Prob (F-statistic):               0.00
Time:                           20:00:18   Log-Likelihood:                 79.741
No. Observations:                   2417   AIC:                            -129.5
Df Residuals:                       2402   BIC:                            -42.63
Df Model:                             14                                         
Covariance Type:               nonrobust                                         
======================================================================================
===
                          coef    std err          t      P>|t|      [0.025      0.9
75]
--------------------------------------------------------------------------------------
---
const                   1.5776      0.052     30.217      0.000       1.475       1.
680
main_camera_mp          0.0209      0.001     14.620      0.000       0.018       0.
024
selfie_camera_mp        0.0139      0.001     12.921      0.000       0.012       0.
016
ram                     0.0218      0.005      4.382      0.000       0.012       0.
032
weight                  0.0017   6.01e-05     27.655      0.000       0.002       0.
002
years_since_release    -0.0285      0.003     -8.393      0.000      -0.035      -0.
022
normalized_new_price    0.4414      0.011     39.298      0.000       0.419       0.
463
brand_name_Karbonn      0.1157      0.055      2.113      0.035       0.008       0.
223
brand_name_Samsung     -0.0372      0.016     -2.255      0.024      -0.070      -0.
005
brand_name_Sony        -0.0663      0.030     -2.173      0.030      -0.126      -0.
006
brand_name_Xiaomi       0.0808      0.026      3.143      0.002       0.030       0.
131
os_Others              -0.1265      0.027     -4.636      0.000      -0.180      -0.
073
os_iOS                 -0.0888      0.045     -1.967      0.049      -0.177      -0.
000
4g_yes                  0.0507      0.015      3.359      0.001       0.021       0.
080
5g_yes                 -0.0663      0.031     -2.160      0.031      -0.126      -0.
006
==============================================================================
Omnibus:                      247.525   Durbin-Watson:                   1.900
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              489.073
Skew:                          -0.659   Prob(JB):                    6.30e-107
Kurtosis:                       4.765   Cond. No.                     2.42e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
ified.
[2] The condition number is large, 2.42e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

In [73]:
```
# predictions on the test set
pred = olsmodel_final.predict(X_test_final)

df_pred_test = pd.DataFrame({"Actual": y_test, "Predicted": pred})
df_pred_test.sample(10, random_state=1)
```

Out[73]:

	Actual	Predicted
1995	4.566741	4.384396
2341	3.696103	4.003929
1913	3.592093	3.644629
688	4.306495	4.103986
650	4.522115	5.112641
2291	4.259294	4.398014
40	4.997685	5.459388
1884	3.875359	4.052498
2538	4.206631	4.036213
45	5.380450	5.228828

I would say this falls pretty firmly in the "good, not great" category!

In [74]:
```
print("Final Model Perofmance Metrics: Train vs Test")
# checking model performance on train set (seen 70% data)
print("Training Performance\n")
olsmodel_final_train_perf = model_performance_regression(
    olsmodel_final, X_train_final, y_train
)
print(olsmodel_final_train_perf, "\n\n")

# checking model performance on test set (seen 30% data)
print("Test Performance\n")
olsmodel_final_test_perf = model_performance_regression(
    olsmodel_final, X_test_final, y_test
)
print(olsmodel_final_test_perf)
```

```
Final Model Perofmance Metrics: Train vs Test
Training Performance

       RMSE       MAE  R-squared  Adj. R-squared     MAPE
0  0.234118  0.182771   0.839119        0.838114  4.39565



Test Performance

       RMSE       MAE  R-squared  Adj. R-squared     MAPE
0  0.241513  0.186707   0.838281        0.835905  4.55824
```

These values did not shift much though the process.

- The model is able to explain ~84% of the variation in the data.

- The train and test RMSE and MAE are low (relative to the scale of the target variable) and comporable. The model is not overfitting.
- The MAPE on the test set suggests we can predict within 4.6% of the normalized used prices.
- Hence, we propose this model as a great starting tool for the buisness discussions to follow.

Actionable Insights and Recommendations

> **Next steps in Data Analysis and Modeling Work**

1. Connecting with a subject matter expert to weigh in on the outliers, missing value treatment would be very helpful. For example, the outlier of the 1TB internal memory phone, while not common, is certainly an available devide in America. Should those outliers continue to stay?

2. Also, based ont he Business Recommendations made below, other data cleaning work could be done. For example, if the company decides to only focus on used phones from the last 4 years, we can create a new model that focuses on those parameters.

3. Update the dataset through at least 2022 and add a few more features to the data set:

    - Based on the days used data, many people sell their devices at the 18 month mark. Thus the data set should be updated to include phones released 18 months ago.
    - Inventory of available used devices of that model - instead of each row of data being a unique build; each row of the data set is an actual phone that was available and its selling price. This would be a huge data set, and the model straignthens with more data.
    - Market specific categoricals; for example India is accelearting its 5g access presently, so the 5g phones will likely go up in demand now that the technology is support. More 4g phones will be available for resale as users upgrade.

> **Buisness Recommendation Based on Current model**

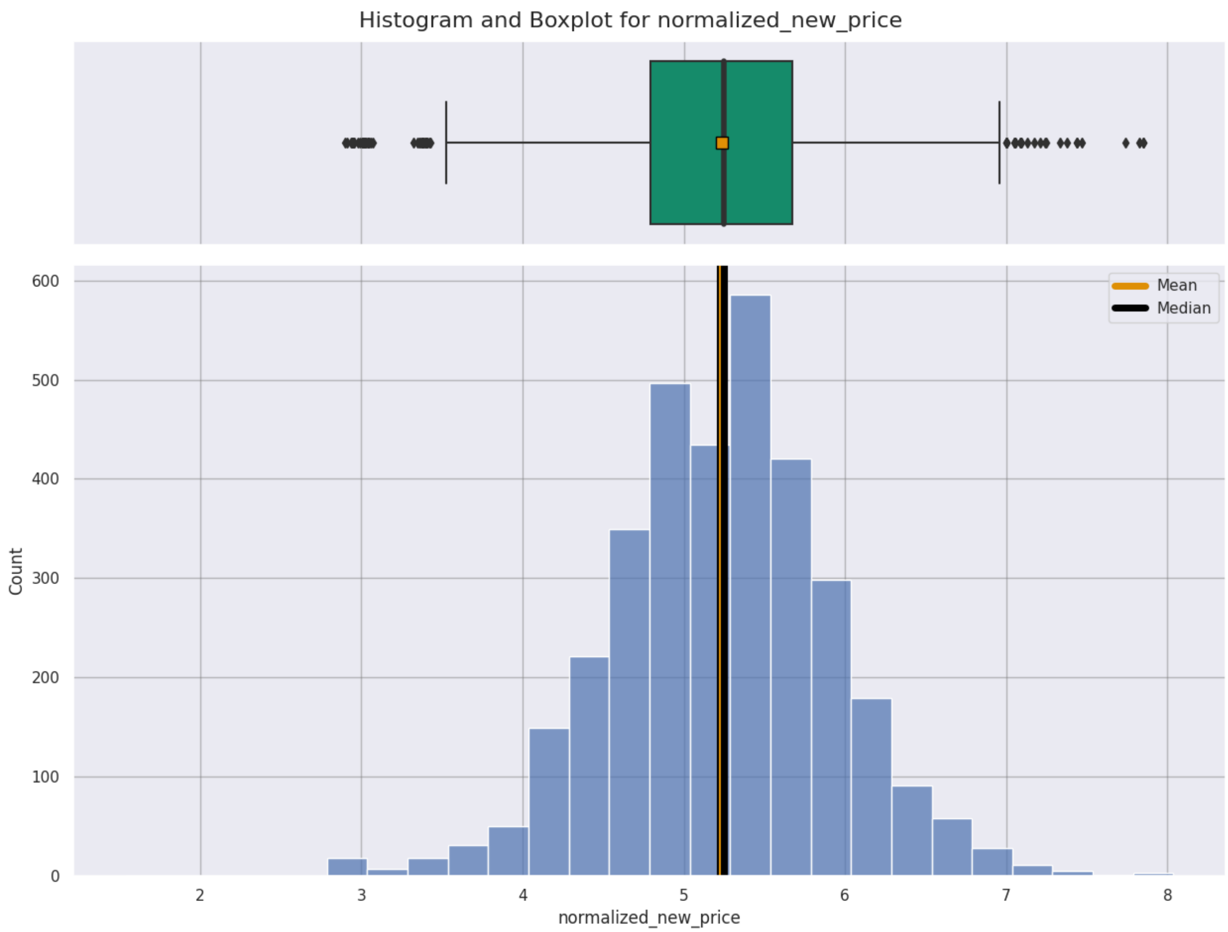
1. I suggest as a buisness model to try to focus on keeping inventory of used devices that have non-zero values for the following features:
    - main camera mega pixles
    - selfie camera mega pixels
    - internal memory
    - RAM
2. Consider focusing the buisness model, taking into consideration sourcing, technical support, repairs, resale strategy, regional partners, partnerships with connection providers, e-commerce vs brick and morter etc. Here are some focuses of devices that, when the categories are more limited may straighthen the model:

- Consider focuses only on a few key brands. Our model suggests that Karbonn and Xiaomi might be good choices. However other buisness factors like repair costs, inventory, regional availabilty etc become relevant.
- selfie camera mega pixels - consider only devices above the Q1 value of this data set
- internal memory - consider only devices above the Q1 value of this data set
- RAM - consider only devices in the 4 GB+ range
- Market specific choices: not all phones brands or individual technologies are available worldwide.

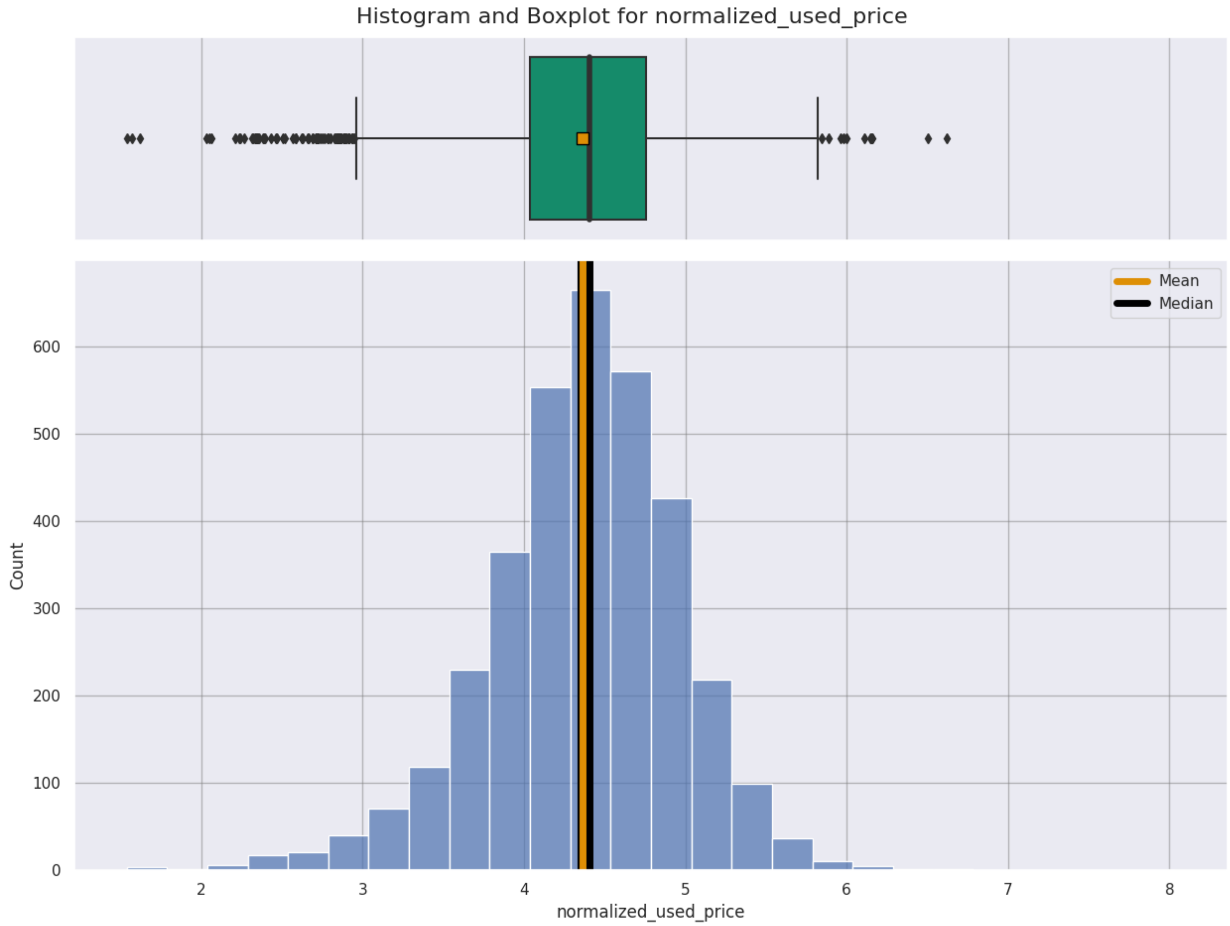
Appendix: Detailed Exploratory Data Analysis (EDA)

In [75]:
```
# create an array for bins values to better compare the two visualizations for prices
# start at the minumum for the used price and maximum of the new price, with 0.25 step
bins_prices = np.arange(df1['normalized_used_price'].min(),df1['normalized_new_price']
```

In [76]:
```
histogram_boxplot(df1, "normalized_new_price", bins = bins_prices)
```

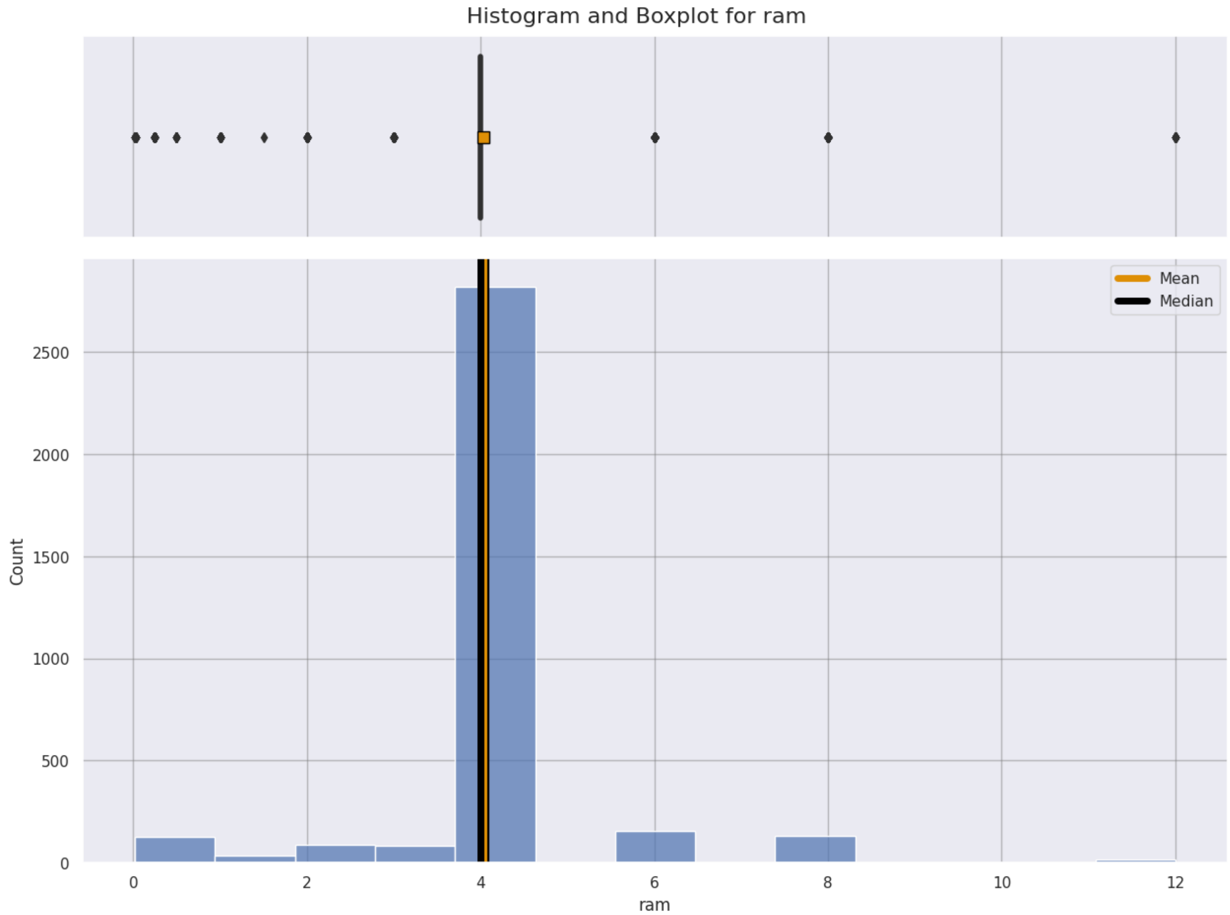


In [77]:
```
histogram_boxplot(df1, "normalized_used_price", bins = bins_prices)
```

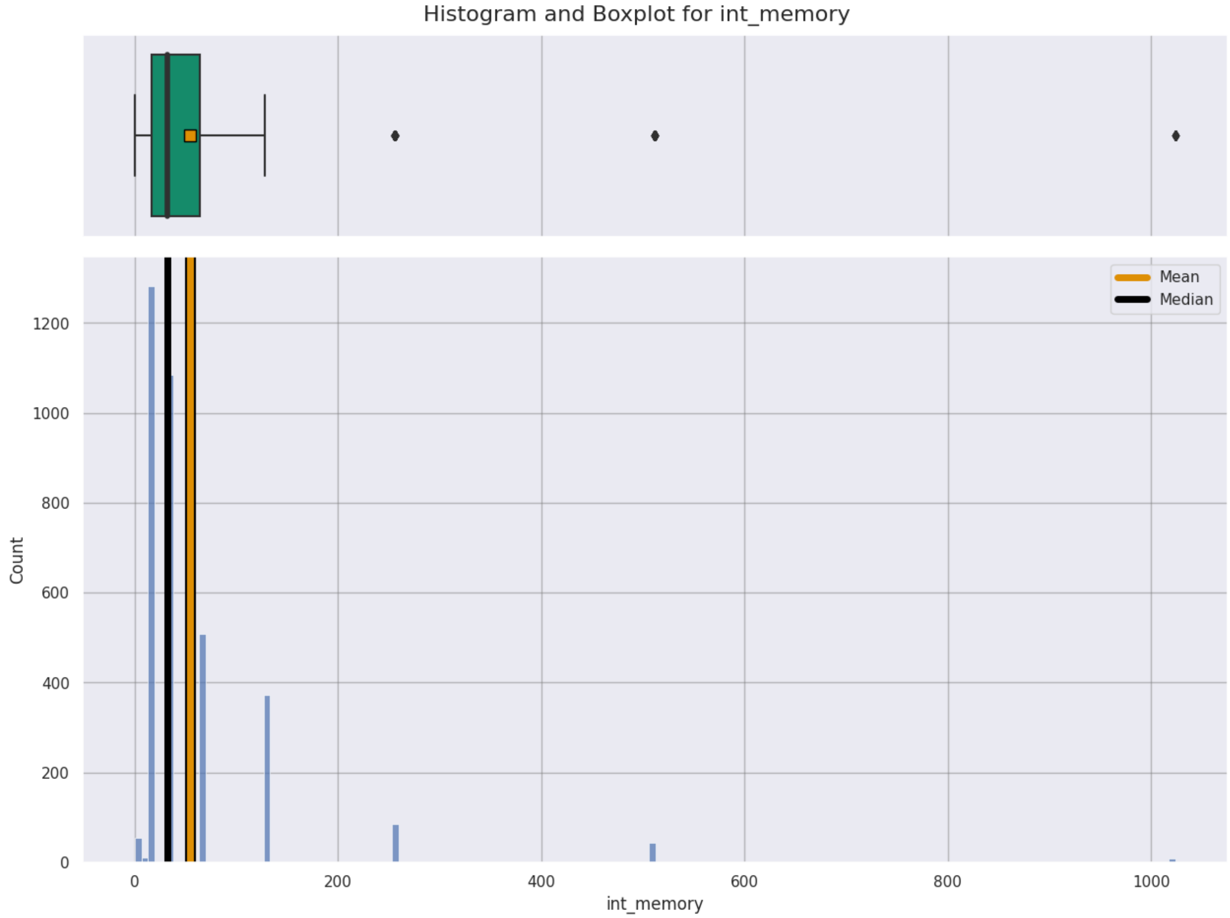


Observations: The used prices are visually slightly more normalized, and has more outliers less than the Q1 value. The new prices are such that the median and mean are less than the mode; the histogram is normalized but not purely symetrical.

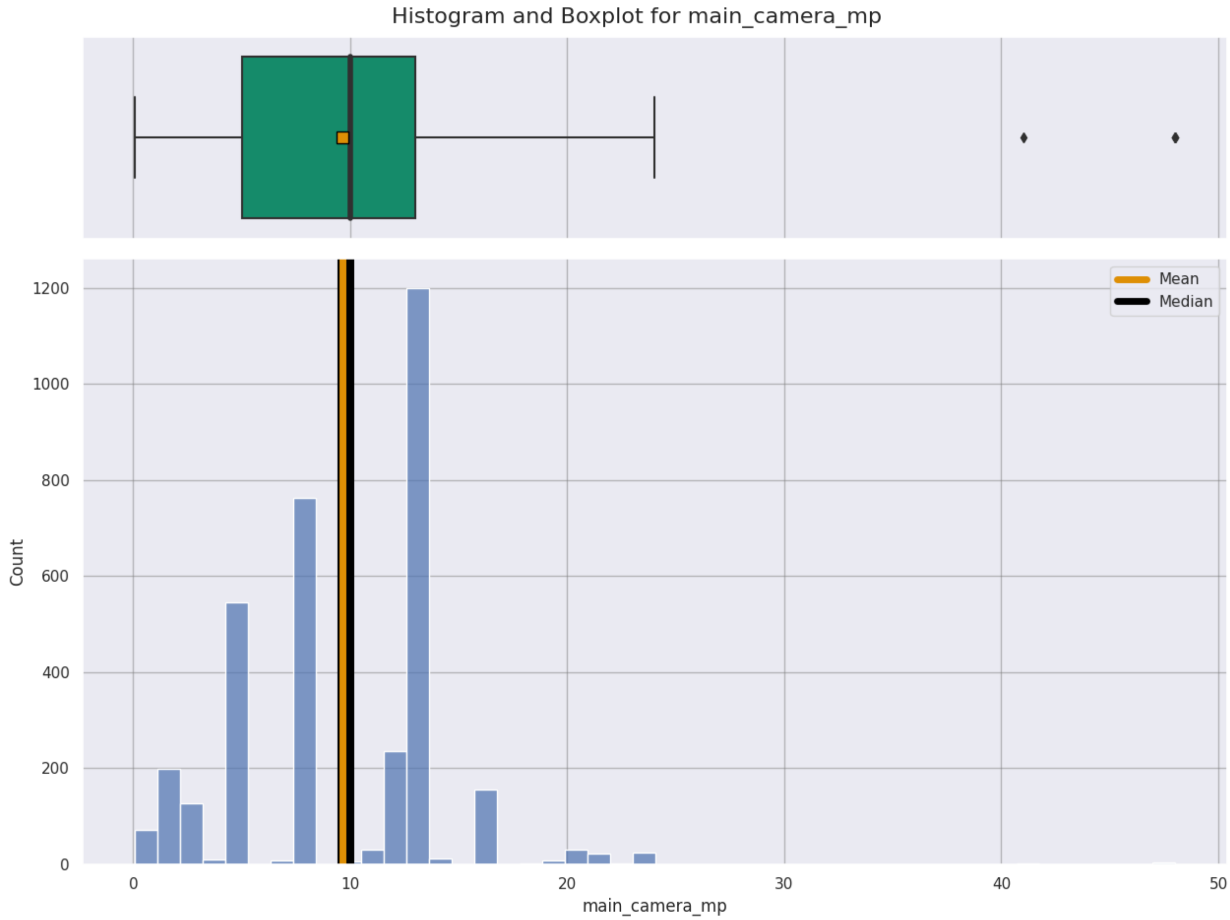
In [78]:
```
histogram_boxplot(df1, 'ram')
```



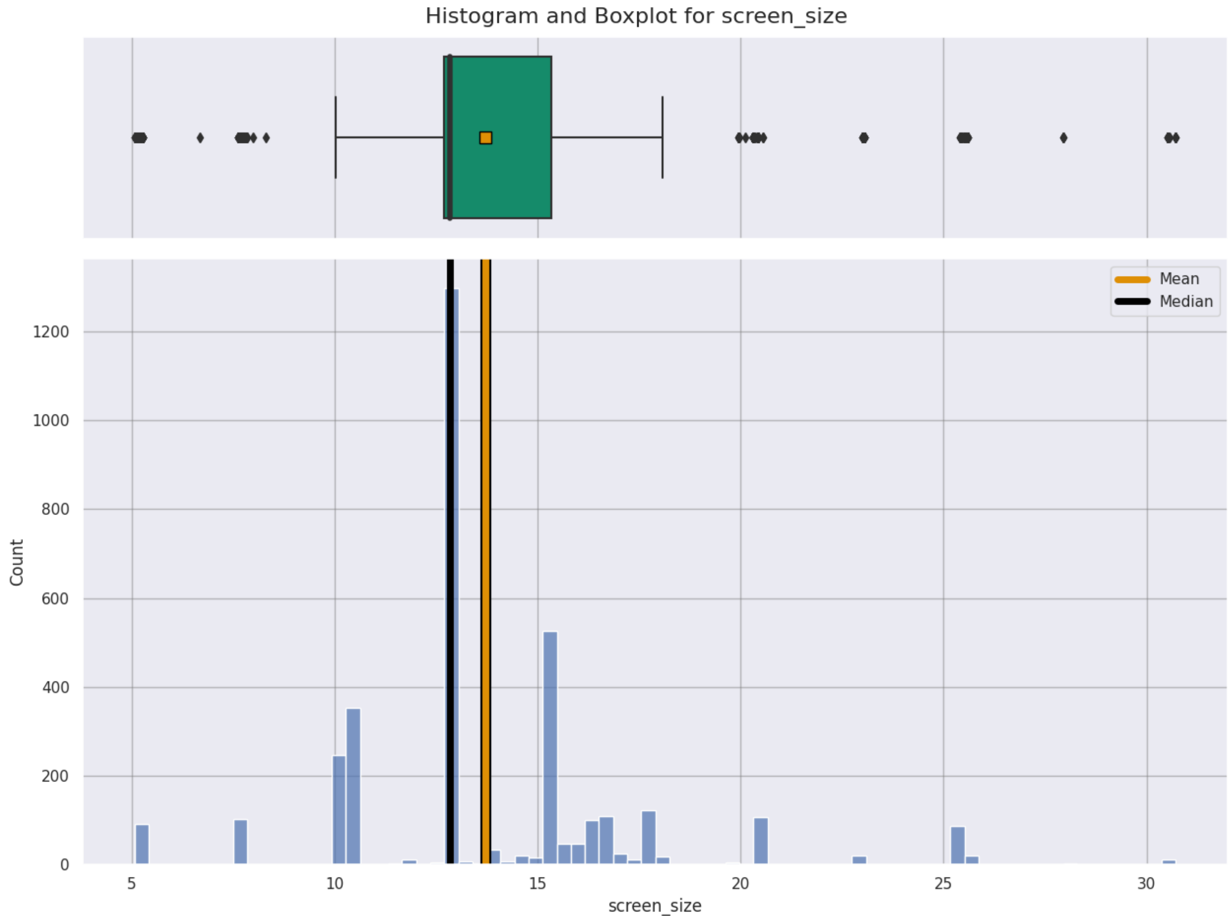
In [79]:
```
histogram_boxplot(df1, 'int_memory')
```



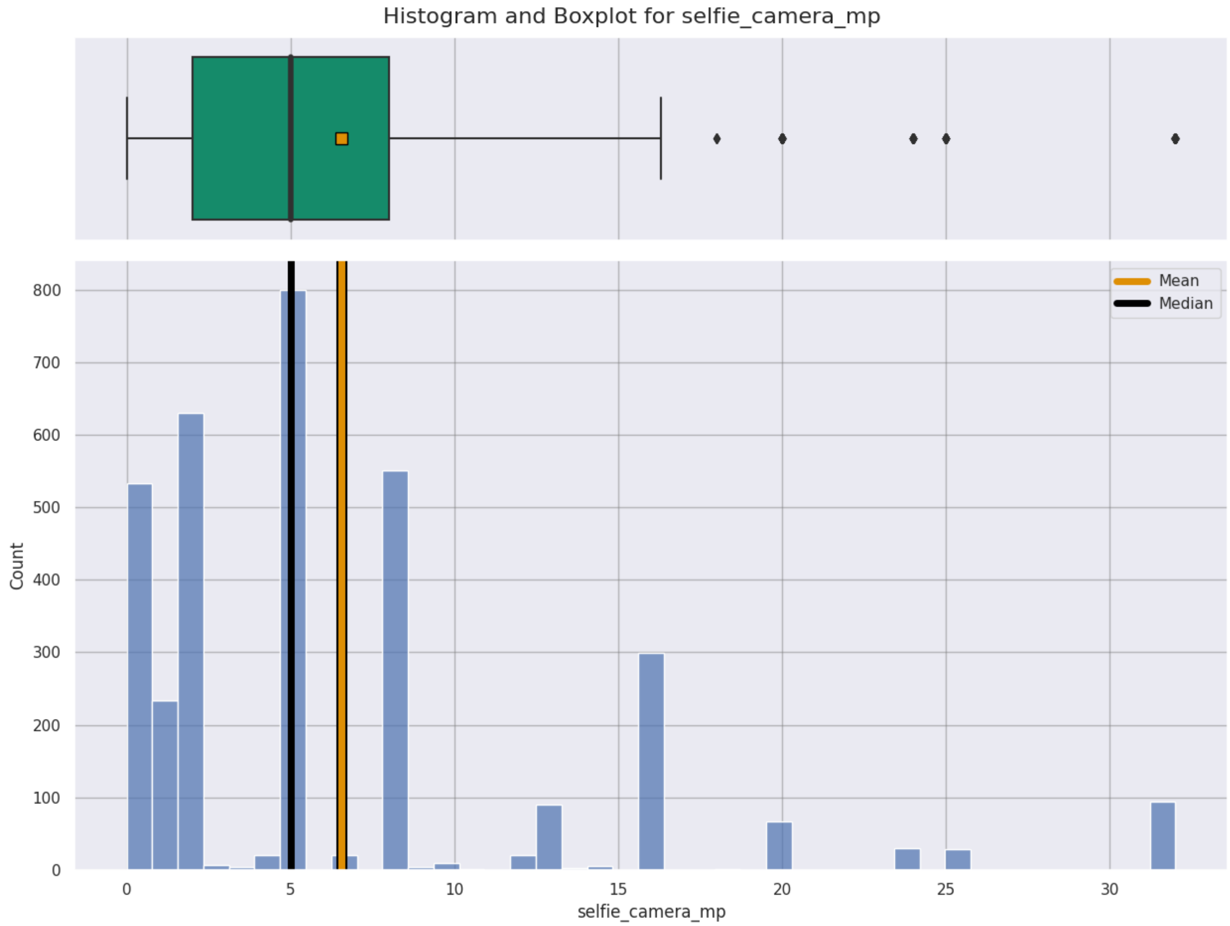
In [80]:
```
histogram_boxplot(df1, 'main_camera_mp')
```



In [81]:
```
histogram_boxplot(df1, 'screen_size')
```



In [82]:
```
histogram_boxplot(df1, 'selfie_camera_mp')
```


Histogram and Boxplot for selfie_camera_mp
Mean
Median
Count
800
700
600
500
400
300
200
100
0
0
5
10
15
20
25
30
selfie_camera_mp