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Math 440
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1.
a. Given the continuous function $f(x) = x^3 - 9$, we know there is a root in the interval $[2, 3]$ We know this because $f'(x)$ is positive in the given interval, and $f(2)$ is negative while $f(3)$ is positive.
b. Given the continuous function $f(x) = 3x^3 + x^2 - x - 5$, we know there is a root in the interval $[1, 2]$. This is because $f'(x)$ is positive in the given interval, and $f(1)$ is negative while $f(2)$ is positive.
c. Given the continuous function $f(x) = \cos^2(x) - x + 6$, we know there is a root in the interval $[6, 7]$. This is because $f'(x)$ is negative in the given interval, and $f(6)$ is positive while $f(7)$ is negative.

2.
a. Given the continuous function $f(x) = x^5 + x - 1$ we know there is a root in the interval $[0, 1]$. This is because $f'(x)$ is positive in the given interval, and $f(0)$ is negative while $f(1)$ is positive.
b. Given the continuous function $f(x) = \sin(x) - 6x - 5$ we know there is a root in the interval $[-1, 0]$. This is because $f'(x)$ is negative over the given interval, and $f(-1)$ is positive while $f(0)$ is negative.
c. Given the continuous function $f(x) = \ln(x)x^2 - 3$ we know there is a root in the interval $[1, 2]$. This is because $f'(x)$ is positive in the given interval, and $f(1)$ is negative while $f(2)$ is positive.

5.
a. There exists a root in the interval $[2, 3]$
b.$\frac{3-2}{2^{n+1}} \leqslant \frac{1}{2}10^{-10}$
$\frac{1}{2^n} \leqslant 10^{-10}$
$2^n \leqslant 10^6$
$n \geqslant \frac{\ln(10^{10})}{\ln(2)}$
$n \geqslant 33.219$
$n = 34$

1.
a. Using octave the estimate of the root is at $x = 2.080083$
b. Using octave the estimate of the root is at $x = 1.169725$
c. Using octave the estimate of the root is at $x = 6.776092$

2.
a. Using octave the estimate of the root is at $x = 0.75487766$
b. Using octave the estimate of the root is at $x = -0.97089893$
c. Using octave the estimate of the root is at $x = 1.59214293$

3.
a. On the interval $[-2, -1]$, there exists a root $x = -1.641784$. On the interval $[-1, 0]$, there exists a root $x = -0.168255$. On the interval $[1, 2]$, there exits a root $x = 1.810038$
b.On the interval $[-2, -1]$, there exists a root $x = -1.023482$. On the interval $[-0.5, 0.5]$, there exists a root $x = 0.163822$. On the interval $[0.5, 1.5]$, there exits a root $x = 0.788941$
c.On the interval $[-1.5, -0.5]$, there exists a root $x = -0.818094$. On the interval $[-0.5, 0.5]$, there exists a root $x = 0$. On the interval $[0.5, 1.5]$, there exits a root $x = 0.506308$

4.
a. Using the function $f(x) = x^2 - 2$ an estimate of the root is $x = 1.414214$ starting with an interval of $[1, 2]$ over 27 steps.
b. Using the function $f(x) = x^2 - 3$ an estimate of the root is $x = 1.732051$ starting with an interval of $[1, 2]$ over 27 steps.
c. Using the function $f(x) = x^2 - 5$ an estimate of the root is $x = 2.236068$ starting with an interval of $[2, 3]$ over 27 steps.

5.
a. Using the function $f(x) = x^3 - 2$ an estimate of the root is $x = 1.259921$ starting with an interval of $[1, 2]$ over 27 steps.
b. Using the function $f(x) = x^3 - 3$ an estimate of the root is $x = 1.442250$ starting with an interval of $[1, 2]$ over 27 steps.
c. Using the function $f(x) = x^3 - 5$ an estimate of the root is $x = 1.709976$ starting with an interval of $[1, 2]$ over 27 steps.

6. Using the bisection method on the function $f(x) = \sin(x) - \cos(x)$ on the interval of $[0, 1]$, we find there is a root at approximately $x = 0.785398$.

7. To find where the $\det(A) = 1000$, we can use the function:

$$f(x) = \det(A) - 1000 = x^4 - 202x^2 - 1404x - 3475$$

Using the bisection theorem on the interval $[-18, -17]$ we can find a root of $x = -17.188498$. Using the theorem on the interval $[9, 10]$ gives us the root $x = 9.708299$. Substituting the root $x = -16.556659$ into the matrix we find that the $\det(A) = 1000.00413888$, and substituting the root $x = 9.708299$ we find that the $\det(A) = 999.99949746$.