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Math 440
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1.
a. Not a cubic spline. The derivative of $x^3 + x - 1$ at $x = 1$ is $3(1)^2 + 1 = 4$ while the derivative of $-(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1$ at $x = 1$ is $-3((1)-1)^2 + 6((1)-1) + 3 = 3$.
b. Is a cubic spline. At $x = 1$: $2x^3+x^2 4x+5 = 12$, $6x^2+2x+4 = 12$, and $12x+2 = 14$ matches $(x-1)^3+7(x-1)^2+12(x-1)+12 = 12$, $3(x-1)^2+14(x-1)+12 = 12$, and $6(x-1)+14 = 14$

2.
a. Is a cubic spline. At $x = 1$: $1 + 2(1) + 3(1)^2 + 4(1)^3 = 10$, $2 + 6(1) + 12(1)^2 = 20$, and $6 + 24(1) = 30$ matches $10 + 20((1) - 1) + 15((1) - 1)^2 + 4((1) - 1)^3 = 10$, $20 + 30((1) - 1) + 12((1) - 1)^2 = 20$, $30 + 24((1) - 1) = 30$.
b. Not natural because $S_1''(1) = 30 \neq 0$. Not parabolically terminated because $d_1 \neq 0$. Not not-a-knot because $S_1'''(1) = 6 \neq S_2'''(2)$.

3.
a. $S_1''(x_1) = S_2''(x_1)$, so $\frac{9}{2}x = 2c - \frac{9}{2}(x-1)$ at $x = 1$. Therefore $\frac{9}{2} = 2c$ and $c = 9$. Not natural because $S_1''(1) \neq 0$. Not parabolically terminated because $d_1 \neq 0$. Not not-a-knot because $S_1'''(1) \neq S_2'''(2)$.
b. $S_1''(x_1) = S_2''(x_1)$, so $8 = 2c$. Therefore $c = 4$. Parabolically terminated since $d_1 = 0 = d_2$.
c. $S_2(x_2) = S_3(x_2)$, so $-1+c(x-1)+\frac{1}{2}(x-1)^2-(x-1)^3 = 1+\frac{1}{2}(x-2)-\frac{5}{2}(x-2)^2-(x-2)^3$ at $x = 2$. Therefore $-1 + c + \frac{1}{2} - 1 = 1$ and $c = \frac{5}{2}$. Not natural because $S_1''(1) \neq 0$. Not parabolically terminated because $d_1 \neq 0$. Not not-a-knot because $S_1'''(1) \neq S_2'''(2)$.

4.
For k_1: $S_1(x_1) = S_2(x_1)$, so $4 + k_1 x + 2x^2 - \frac{1}{6}x^3 = 1 - \frac{4}{3}(x-1) + k_2(x-1)^2 - \frac{1}{6}(x-1)^3$ at $x = 1$. Therefore $4 + k_1 + 2 - \frac{1}{6} = 1$ and $k_1 = \frac{-29}{6}$.
For k_2: $S_2(x_2) = S_3(x_2)$, so $1-\frac{4}{3}(x-1)+k_2(x-1)^2-\frac{1}{6}(x-1)^3 = 1+k_3(x-2)+(x-2)^2-\frac{1}{6}(x-2)^3$ at $x = 2$. Therefore $1 - \frac{4}{3} + k_2 - \frac{1}{6} = 1$ and $k_2 = \frac{3}{2}$.
For k_3: $S_3'(x_2) = S_2'(x_2)$, so $k_3 + 2(x-2) - \frac{1}{2}(x-2)^2 = \frac{-4}{3} + 3(x-1) - \frac{1}{2}(x-1)^2$ at $x = 2$. Therefore $k_3 = \frac{-4}{3} + 3 - \frac{1}{2} = \frac{7}{6}$.
Not natural since $S_1''(1) \neq 0$. Not parabolically terminated since $d_1 \neq 0$. Not not-a-knot because $S_1'''(1) = 4 \neq S_2'''(2)$.

7.
a.
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

Therefore $c_1 = 0, c_2 = \frac{3}{2}, c_3 = 0$.

$$S(n) = \begin{cases} \frac{1}{2}x + \frac{1}{2}x^3, & \text{on } [0,1] \\ 1 + 2(x-1) + \frac{3}{2}(x-1)^2 - \frac{1}{2}(x-1)^3, & \text{on } [1,2] \end{cases}$$

b.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3}{2} \\ 0 \end{bmatrix}$$

Therefore $c_1 = 0, c_2 = \frac{3}{2}, c_3 = 0$.

$$S(n) = \begin{cases} 1 - x + \frac{1}{4}x^3, & \text{on } [-1,1] \\ 1 + 2(x-1) + \frac{3}{2}(x-1)^2 - \frac{1}{2}(x-1)^3, & \text{on } [1,2] \end{cases}$$

9.

$S'(0) = S_1'(0) = b_1$. Since $S_1(x_1) = S_2(x_1)$, $3 + b_1 + 1 = 1$ and $b_1 = -3$.

$S'(3) = S_2'(3) = b_2 + 6(2) - 6(2) = b_2$. Since $S_1'(x_1) = S_2'(x_1)$, $-3 + 3 = b_2$ so $b_2 = 0$.

13.

a. For b_1: Since $S_1(x_1) = S_2(x_1)$, $-1 + b_1 - \frac{5}{9} + \frac{5}{9} = 0$. Therefore $b_1 = 1$.

For c_3: Since $S_2''(x_2) = S_3''(x_2)$, so $\frac{20}{9} - 4(x-1) = 2c_3 - \frac{2}{3}(x-2)$ at $x = 2$. Therefore $\frac{-16}{9} = 2c_3$ and $c_3 = \frac{-8}{9}$.

b. Not natural because $S_1''(0) \neq 0$

c.

14.

a. $S_1''(x_1) = S_2''(x_1)$, so $3x = 2c + 2d(x-2)$ at $x = 2$. Therefore $6 = 2c$ and $c = 3$.

b. Yes. $S_2''(x_3) = 0$, so $6 + 2d(x-2) = 0$ at $x = 3$. Therefore $6 + 2d = 0$ and $d = -3$.

1.

a.

$$S(n) = \begin{cases} 3 + \frac{8}{3}x - \frac{2}{3}x^3, & \text{on } [0,1] \\ 5 + \frac{2}{3}(x-1) - 2(x-1)^2 + \frac{1}{3}(x-1)^3, & \text{on } [1,2] \\ 4 - \frac{8}{3}(x-2) - (x-2)^2 + \frac{1}{3}(x-2)^3, & \text{on } [2,3] \end{cases}$$

b.

$$S(n) = \begin{cases} 3 + 2.56289(x+1) - 0.56289(x+1)^3, & \text{on } [-1,0] \\ 5 + 0.87421x - 1.68868x^2 + 0.31761x^3, & \text{on } [0,3] \\ 1 - 0.68239(x-3) + 1.16981(x-3)^2 - 0.48742(x-3)^3, & \text{on } [3,4] \\ 1 + 0.194969(x-4) - 0.292453(x-4)^2 + 0.097484(x-4)^3, & \text{on } [4,5] \end{cases}$$

2.

a.

$$S(n) = \begin{cases} 3 + \frac{23}{6}x - 2x^2 + \frac{1}{6}x^3, & \text{on } [0,1] \\ 5 + \frac{1}{3}(x-1) - \frac{3}{2}(x-1)^2 + \frac{1}{6}(x-1)^3, & \text{on } [1,2] \\ 4 - \frac{13}{6}(x-2) - (x-2)^2 + \frac{1}{6}(x-2)^3, & \text{on } [2,3] \end{cases}$$

b.

$$S(n) = \begin{cases} 3 + 3.88667(x+1) - 2.15(x+1)^2 + 0.26333(x+1)^3, & \text{on } [-1,0] \\ 5 + 0.37667x - 1.36x^2 + 0.26333x^3, & \text{on } [0,3] \\ 1 - 0.67333(x-3) + 1.01(x-3)^2 - 0.33667(x-3)^3, & \text{on } [3,4] \\ 1 + 0.33667(x-4) - 0.33667(x-4)^3, & \text{on } [4,5] \end{cases}$$

5.

$$S(n) = \begin{cases} 1 + 4.6786x^2 - 2.6786x^3, & \text{on } [0,1] \\ 3 + 1.3214(x-1) - 3.3571(x-1)^2 + 2.0357(x-2)^3, & \text{on } [1,2] \\ 3 + 0.71429(x-2) + 2.75(x-2)^2 - 2.46429(x-2)^3, & \text{on } [2,3] \\ 41.1786(x-3) - 4.6429(x-3)^2 3.8214(x-3)^3, & \text{on } [3,4] \end{cases}$$

6.

$$S(n) = \begin{cases} 1 - 2x + 8.1429x^2 - 4.1429x^3, & \text{on } [0,1] \\ 3 + 1.8571(x-1) - 4.2857(x-1)^2 + 2.4286(x-1)^3, & \text{on } [1,2] \\ 3 + 0.57143(x-2) + 3(x-2)^2 - 2.57143(x-2)^3, & \text{on } [2,3] \\ 4 - 1.1429(x-3) - 4.7143(x-3)^2 + 3.8571(x-3)^3, & \text{on } [3,4] \end{cases}$$