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Math 440

1, 2, 3, 4, 5

1.

a.

$$y(0) = 0.50$$

$$y(0.2) = y_0 + h * f(t_0, y_0) = 0.5 + (0.2)(0.5 - 0^2 + 1) = 0.8$$

$$y(0.4) = y_1 + h * f(t_1, y_1) = 0.8 + (0.2)(0.8 - (0.2)^2 + 1) = 1.152$$

b.

$$y(0) = 0.50$$

$$y(0.1) = y_0 + h * f(t_0, y_0) = 0.5 + (0.1)(0.5 - 0^2 + 1) = 0.65$$

$$y(0.2) = y_1 + h * f(t_1, y_1) = 0.65 + (0.1)(0.65 - (0.1)^2 + 1) = 0.814$$

$$y(0.3) = y_2 + h * f(t_2, y_2) = 0.814 + (0.1)(0.814 - (0.2)^2 + 1) = 0.9914$$

$$y(0.4) = y_3 + h * f(t_3, y_3) = 0.9914 + (0.1)(0.9914 - (0.3)^2 + 1) = 1.18154$$

c.

$$|y_1 - t^2 + 1 - (y_2 - t^2 + 1)| \leq L|y_1 - y_2|$$

$$|y_1 - y_2| \leq L|y_1 - y_2|$$

Therefore: $L = 1$

$$|y_1 - w_1| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_1-0)(1)} - 1) = 0.2(e^{0.2} - 1) = 0.044281$$

$$|y_2 - w_2| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_2-0)(1)} - 1) = 0.2(e^{0.4} - 1) = 0.098365$$

$$|y_3 - w_3| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_3-0)(1)} - 1) = 0.2(e^{0.6} - 1) = 0.16442$$

$$|y_4 - w_4| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_4-0)(1)} - 1) = 0.2(e^{0.8} - 1) = 0.24511$$

$$|y_5 - w_5| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_5-0)(1)} - 1) = 0.2(e^1 - 1) = 0.34366$$

$$|y_6 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_6-0)(1)} - 1) = 0.2(e^{1.2} - 1) = 0.46402$$

$$|y_7 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_7-0)(1)} - 1) = 0.2(e^{1.4} - 1) = 0.61104$$

$$|y_8 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_8-0)(1)} - 1) = 0.2(e^{1.6} - 1) = 0.79061$$

$$|y_9 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_9-0)(1)} - 1) = 0.2(e^{1.8} - 1) = 1.0099$$

$$|y_{10} - w_{10}| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_{10}-0)(1)} - 1) = 0.2(e^2 - 1) = 1.2778$$

2.

$$i(0) = 0$$

$$i(0.1) = i_0 + h * f(t_0) = 0 + (0.1)(f(0)) = -0.12024$$

$$i(0.2) = i_1 + h * f(t_1) = -0.12024 + (0.1)(f(0.1)) = -0.22153$$

$$i(0.3) = i_2 + h * f(t_2) = -0.22153 + (0.1)(f(0.2)) = -0.30059$$

$$i(0.4) = i_3 + h * f(t_3) = -0.30059 + (0.1)(f(0.3)) = -0.35511$$

$$\begin{aligned}
i(0.5) &= i_4 + h * f(t_4) = -0.35511 + (0.1)(f(0.4)) = -0.38385 \\
i(0.6) &= i_5 + h * f(t_5) = -0.38385 + (0.1)(f(0.5)) = -0.38663 \\
i(0.7) &= i_6 + h * f(t_6) = -0.38663 + (0.1)(f(0.6)) = -0.36430 \\
i(0.8) &= i_7 + h * f(t_7) = -0.36430 + (0.1)(f(0.7)) = -0.31867 \\
i(0.9) &= i_8 + h * f(t_8) = -0.31867 + (0.1)(f(0.8)) = -0.25240 \\
i(1) &= i_9 + h * f(t_9) = -0.25240 + (0.1)(f(0.9)) = -0.27709
\end{aligned}$$

4.
a.

```

midPoint.m
1 function midPoint(f, a, b, n, alpha)
2 h = (b - a) ./ n;
3 t = a:h:b;
4
5 w(1) = alpha;
6 printf("y(%f) = %f\n", t(1), w(1));
7 for i = 1:n
8     w(i+1) = w(i) + h .* f(t(i) + (h ./ 2), w(i) + (h ./ 2) .* w(i));
9     printf("y(%f) = %f\n", t(i+1), w(i+1));
10 end
11
runge_kutta.m
1 function runge_kutta(f, a, b, n, alpha)
2 h = (b - a) ./ n;
3 t = a:h:b;
4
5 w(1) = alpha;
6 printf("y(%f) = %f\n", t(1), w(1));
7 for i = 1:n
8     k1 = h .* f(t(i), w(i));
9     k2 = h .* f(t(i) + (h ./ 2), w(i) + 0.5 .* k1);
10    k3 = h .* f(t(i) + (h ./ 2), w(i) + 0.5 .* k2);
11    k4 = h .* f(t(i+1), w(i) + k3);
12
13    w(i+1) = w(i) + (1 ./ 6) .* (k1 + 2 .* k2 + 2 .* k3 + k4);
14    printf("y(%f) = %f\n", t(i+1), w(i+1));
15 end
16
Command Window
>> di
di =
@t) 0.3 .* ddt (t) + (1 / 1.4) .* dt (t) + (1 / 1.7) .* y (t)
>> midPoint(di, 0, 1, 10, 0)
y(0.000000) = 0.000000
y(0.100000) = -0.111238
y(0.200000) = -0.201767
y(0.300000) = -0.268778
y(0.400000) = -0.310498
y(0.500000) = -0.326215
y(0.600000) = -0.316270
y(0.700000) = -0.282006
y(0.800000) = -0.225675
y(0.900000) = -0.150317
y(1.000000) = -0.059610
>>

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5.
a.

```

midPoint.m
1 function runge_kutta(f, a, b, n, alpha)
2 h = (b - a) ./ n;
3 t = a:h:b;
4
5 w(1) = alpha;
6 printf("y(%f) = %f\n", t(1), w(1));
7 for i = 1:n
8     k1 = h .* f(t(i), w(i));
9     k2 = h .* f(t(i) + (h ./ 2), w(i) + 0.5 .* k1);
10    k3 = h .* f(t(i) + (h ./ 2), w(i) + 0.5 .* k2);
11    k4 = h .* f(t(i+1), w(i) + k3);
12
13    w(i+1) = w(i) + (1 ./ 6) .* (k1 + 2 .* k2 + 2 .* k3 + k4);
14    printf("y(%f) = %f\n", t(i+1), w(i+1));
15 end
16
runge_kutta.m
1 function runge_kutta(f, a, b, n, alpha)
2 h = (b - a) ./ n;
3 t = a:h:b;
4
5 w(1) = alpha;
6 printf("y(%f) = %f\n", t(1), w(1));
7 for i = 1:n
8     k1 = h .* f(t(i), w(i));
9     k2 = h .* f(t(i) + (h ./ 2), w(i) + 0.5 .* k1);
10    k3 = h .* f(t(i) + (h ./ 2), w(i) + 0.5 .* k2);
11    k4 = h .* f(t(i+1), w(i) + k3);
12
13    w(i+1) = w(i) + (1 ./ 6) .* (k1 + 2 .* k2 + 2 .* k3 + k4);
14    printf("y(%f) = %f\n", t(i+1), w(i+1));
15 end
16
Command Window
>> f = @(t, y) y - t.^ 2 + 1
f =
@(t, y) y - t.^ 2 + 1
>> runge_kutta(f, 0, 0.4, 4, 0.5)
y(0.000000) = 0.500000
y(0.100000) = 0.657414
y(0.200000) = 0.829298
y(0.300000) = 1.015070
y(0.400000) = 1.214087
>> |

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b.

$$y(0.1) - w_1 = (0.1 + 1)^2 - \frac{1}{2}e^{0.1} - 0.657414 = 5.4096e^{-7}$$

$$y(0.2) - w_2 = (0.2 + 1)^2 - \frac{1}{2}e^{0.2} - 0.829298 = 6.2092e^{-7}$$

$$y(0.3) - w_3 = (0.3 + 1)^2 - \frac{1}{2}e^{0.3} - 1.015070 = 5.9621e^{-7}$$

$$y(0.4) - w_4 = (0.4 + 1)^2 - \frac{1}{2}e^{0.4} - 1.214087 = 6.5118e^{-7}$$