Anders Lykkehoy
April 25, 2017
Math 440
1, 2, 3, 4, 5

1.
a.
$y(0) = 0.50$
$y(0.2) = y_0 + h * f(t_0, y_0) = 0.5 + (0.2)(0.5 - 0^2 + 1) = 0.8$
$y(0.4) = y_1 + h * f(t_1, y_1) = 0.8 + (0.2)(0.8 - (0.2)^2 + 1) = 1.152$

b.
$y(0) = 0.50$
$y(0.1) = y_0 + h * f(t_0, y_0) = 0.5 + (0.1)(0.5 - 0^2 + 1) = 0.65$
$y(0.2) = y_1 + h * f(t_1, y_1) = 0.65 + (0.1)(0.65 - (0.1)^2 + 1) = 0.814$
$y(0.3) = y_2 + h * f(t_2, y_2) = 0.814 + (0.1)(0.814 - (0.2)^2 + 1) = 0.9914$
$y(0.4) = y_3 + h * f(t_3, y_3) = 0.9914 + (0.1)(0.9914 - (0.3)^2 + 1) = 1.18154$

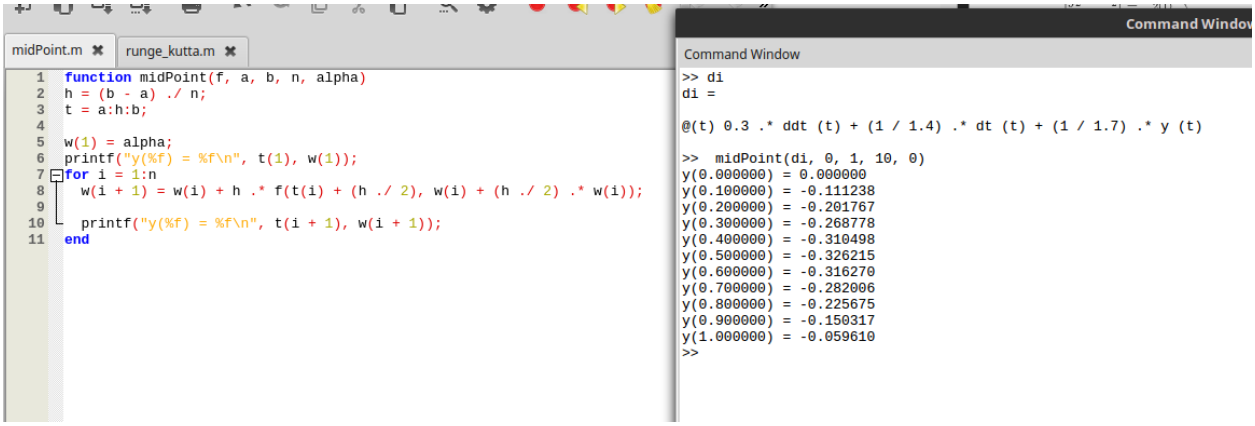
c.
$|y_1 - t^2 + 1 - (y_2 - t^2 + 1)| \leq L|y_1 - y_2|$
$|y_1 - y_2| \leq L|y_1 - y_2|$
Therefore: $L = 1$

$|y_1 - w_1| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_1-0)(1)} - 1) = 0.2(e^{0.2} - 1) = 0.044281$
$|y_2 - w_2| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_2-0)(1)} - 1) = 0.2(e^{0.4} - 1) = 0.098365$
$|y_3 - w_3| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_3-0)(1)} - 1) = 0.2(e^{0.6} - 1) = 0.16442$
$|y_4 - w_4| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_4-0)(1)} - 1) = 0.2(e^{0.8} - 1) = 0.24511$
$|y_5 - w_5| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_5-0)(1)} - 1) = 0.2(e^{1} - 1) = 0.34366$
$|y_6 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_6-0)(1)} - 1) = 0.2(e^{1.2} - 1) = 0.46402$
$|y_7 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_7-0)(1)} - 1) = 0.2(e^{1.4} - 1) = 0.61104$
$|y_8 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_8-0)(1)} - 1) = 0.2(e^{1.6} - 1) = 0.79061$
$|y_9 - w_6| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_9-0)(1)} - 1) = 0.2(e^{1.8} - 1) = 1.0099$
$|y_10 - w_10| \leq \frac{(0.2)(2)}{2(1)}(e^{(t_10-0)(1)} - 1) = 0.2(e^{2} - 1) = 1.2778$

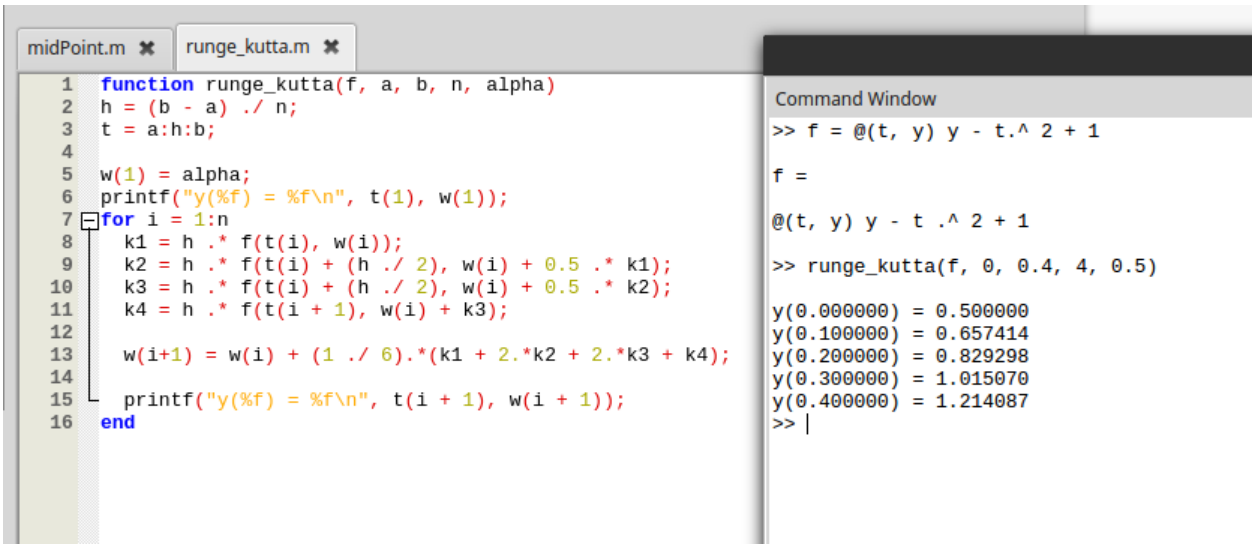
2.
$i(0) = 0$
$i(0.1) = i_0 + h * f(t_0) = 0 + (0.1)(f(0)) = -0.12024$
$i(0.2) = i_1 + h * f(t_1) = -0.12024 + (0.1)(f(0.1)) = -0.22153$
$i(0.3) = i_2 + h * f(t_2) = -0.22153 + (0.1)(f(0.2)) = -0.30059$
$i(0.4) = i_3 + h * f(t_3) = -0.30059 + (0.1)(f(0.3)) = -0.35511$

$i(0.5) = i_4 + h * f(t_4) = -0.35511 + (0.1)(f(0.4)) = -0.38385$
$i(0.6) = i_5 + h * f(t_5) = -0.38385 + (0.1)(f(0.5)) = -0.38663$
$i(0.7) = i_6 + h * f(t_6) = -0.38663 + (0.1)(f(0.6)) = -0.36430$
$i(0.8) = i_7 + h * f(t_7) = -0.36430 + (0.1)(f(0.7)) = -0.31867$
$i(0.9) = i_8 + h * f(t_8) = -0.31867 + (0.1)(f(0.8)) = -0.25240$
$i(1) = i_9 + h * f(t_9) = -0.25240 + (0.1)(f(0.9)) = -0.27709$

4.
a.



5.
a.



b.
$y(0.1) - w_1 = (0.1 + 1)^2 - \frac{1}{2}e^{0.1} - 0.657414 = 5.4096e^{-7}$

$$y(0.2) - w_2 = (0.2+1)^2 - \tfrac{1}{2}e^{0.2} - 0.829298 = 6.2092e^{-7}$$
$$y(0.3) - w_3 = (0.3+1)^2 - \tfrac{1}{2}e^{0.3} - 1.015070 = 5.9621e^{-7}$$
$$y(0.4) - w_4 = (0.4+1)^2 - \tfrac{1}{2}e^{0.4} - 1.214087 = 6.5118e^{-7}$$