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Math 440
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1.
a.

0	0	$\frac{2}{\pi}$	$\frac{-4}{\pi^2}$
$\frac{\pi}{2}$	1	$\frac{-2}{\pi}$	
π	0		

$P_2(x) = 0 + \frac{2}{\pi}(x-0) + \frac{-4}{\pi^2}(x-0)(x-\frac{\pi}{2}) = \frac{2}{\pi}x + \frac{-4}{\pi^2}x(x-\frac{\pi}{2})$
b.$P_2(\frac{\pi}{4}) = \frac{2}{\pi}(\frac{\pi}{4}) + \frac{-4}{\pi^2}(\frac{\pi}{4})(\frac{\pi}{4}-\frac{\pi}{2}) = \frac{1}{2} + \frac{1}{4} = 0.75$
c.$\frac{(\frac{\pi}{4})-0)((\frac{\pi}{4})-\frac{\pi}{2})((\frac{\pi}{4})-\pi)}{3!}|1| = 0.2422365366$
d. $sin(\frac{\pi}{4}) - P_2(\frac{\pi}{4}) = -0.042893$. This is within our error bound.

2.
a.

1	0	$\ln 2$	$\frac{-\ln 2}{6}$
2	$\ln 2$	$\frac{\ln 2}{2}$	
4	$\ln 4$		

$P_2(x) = \ln(2)(x-1) + \frac{-\ln 2}{6}(x-1)(x-2)$
7 b. $P_2(3) = \ln(2)((3)-1) + \frac{-\ln 2}{6}((3)-1)((3)-2) = 1.1552$
c. $\frac{(3-1)(3-2)(3-4)}{3!}|1| = \frac{1}{3}$
d. $\ln 3 - P_2(3) = -0.056633$. This is within our error bound.

4.
a.$\frac{(1-0)(1-2)(1-4)(1-6)(1-8)(1-10)}{6!}|\frac{720}{5^7}| = 0.012096$
b. $\frac{(5-0)(5-2)(5-4)(5-6)(5-8)(5-10)}{6!}|\frac{720}{5^7}| = 0.0028800$

1.
a.
$x_1 = \cos(\frac{2(1)-1}{6}\pi) = \cos(\frac{1}{6}\pi)$
$x_2 = \cos(\frac{2(2)-1}{6}\pi) = \cos(\frac{1}{2}\pi)$
$x_3 = \cos(\frac{2(3)-1}{6}\pi) = \cos(\frac{5}{6}\pi)$
$x_4 = \cos(\frac{2(4)-1}{6}\pi) = \cos(\frac{7}{6}\pi)$
$x_5 = \cos(\frac{2(5)-1}{6}\pi) = \cos(\frac{3}{2}\pi)$
$x_6 = \cos(\frac{2(6)-1}{6}\pi) = \cos(\frac{11}{6}\pi)$
b.
$x_1 = 2\cos(\frac{2(1)-1}{4}\pi) = 2\cos(\frac{1}{4}\pi)$

$x_2 = 2\cos(\frac{2(2)-1}{4}\pi) = 2\cos(\frac{3}{4}\pi)$
$x_3 = 2\cos(\frac{2(3)-1}{4}\pi) = 2\cos(\frac{5}{4}\pi)$
$x_4 = 2\cos(\frac{2(4)-1}{4}\pi) = 2\cos(\frac{7}{4}\pi)$
c.
$x_1 = \frac{1}{2}(4+12) + \frac{1}{2}(12-4)\cos(\frac{2(1)-1}{6}\pi) = 8 + 4\cos(\frac{1}{6}\pi)$
$x_2 = 8 + 4\cos(\frac{2(2)-1}{6}\pi) = 8 + 4\cos(\frac{1}{2}\pi)$
$x_3 = 8 + 4\cos(\frac{2(3)-1}{6}\pi) = 8 + 4\cos(\frac{5}{6}\pi)$
$x_4 = 8 + 4\cos(\frac{2(4)-1}{6}\pi) = 8 + 4\cos(\frac{7}{6}\pi)$
$x_5 = 8 + 4\cos(\frac{2(5)-1}{6}\pi) = 8 + 4\cos(\frac{3}{2}\pi)$
$x_6 = 8 + 4\cos(\frac{2(6)-1}{6}\pi) = 8 + 4\cos(\frac{11}{6}\pi)$
d.
$x_1 = \frac{1}{2}(-0.3+0.7) + \frac{1}{2}(0.7-(-0.3))\cos(\frac{2(1)-1}{5}\pi) = \frac{1}{5} + \frac{1}{2}\cos(\frac{1}{5}\pi)$
$x_2 = \frac{1}{5} + \frac{1}{2}\cos(\frac{2(2)-1}{5}\pi) = \frac{1}{5} + \frac{1}{2}\cos(\frac{3}{5}\pi)$
$x_3 = \frac{1}{5} + \frac{1}{2}\cos(\frac{2(3)-1}{5}\pi) = \frac{1}{5} + \frac{1}{2}\cos(\pi)$
$x_4 = \frac{1}{5} + \frac{1}{2}\cos(\frac{2(4)-1}{5}\pi) = \frac{1}{5} + \frac{1}{2}\cos(\frac{7}{5}\pi)$
$x_5 = \frac{1}{5} + \frac{1}{2}\cos(\frac{2(5)-1}{5}\pi) = \frac{1}{5} + \frac{1}{2}\cos(\frac{9}{5}\pi)$

2.
a. Upper bound $\leq \frac{(\frac{1-(-1)}{2})^6}{2^{6-1}} = \frac{1^6}{2^5} = \frac{1}{2^5}$
b. Upper bound $\leq \frac{(\frac{2-(-2)}{2})^4}{2^{4-1}} = \frac{2^4}{2^3} = 2$
c. Upper bound $\leq \frac{(\frac{12-(4)}{2})^6}{2^{6-1}} = \frac{4^6}{2^5} = 128$
d. Upper bound $\leq \frac{(\frac{0.7-(-0.3)}{2})^5}{2^{5-1}} = \frac{(\frac{1}{2})^6}{2^4} = \frac{1}{2^10}$

3.
$|f(x) - Q_5(x)| = \frac{|(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)|}{6!}|e^1| = \frac{(\frac{1-(-1)}{2})^6}{6!2^5}e = \frac{1}{6!2^10}e = 0.0000036869$
Up to 5 decimal places would be correct for $Q_5(x)$ on this interval.

5.
$|f(x) - Q_3(x)| = \frac{|(x-x_1)(x-x_2)(x-x_3)(x-x_4)|}{4!}|\sin(\frac{\pi}{2})| = \frac{(\frac{2-0}{2})^4}{4!2^3}1 = \frac{1}{4!2^3} = 0.0052083$

8.
$T_n(x) = \cos(n\arccos x)$ Therefore $T_n(0) = \cos(n\arccos 0) = \cos(n\frac{\pi}{2})$

9.
a. $T_999(-1) = \cos(999\arccos(-1)) = \cos(999\pi) = -1$
b. $T_1000(-1) = \cos(1000\arccos(-1)) = \cos(1000\pi) = 1$
c. $T_999(0) = \cos(999\arccos 0) = \cos(999\frac{\pi}{2}) = 0$
d. $T_1000(0) = \cos(1000\arccos 0) = \cos(1000\frac{\pi}{2}) = 1$
e. $T_999(\frac{-1}{2}) = \cos(999\arccos\frac{-1}{2}) = \cos(999\frac{2\pi}{3}) = 1$

f. $T_1000(\frac{-1}{2}) = \cos(1000 \arccos \frac{-1}{2}) = \cos(1000\frac{2\pi}{3}) = \frac{-1}{2}$