Anders Lykkehoy
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Math 440
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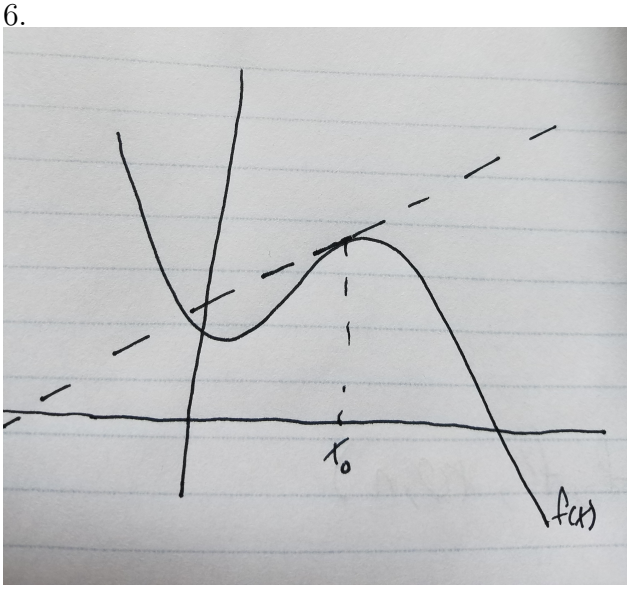
6.



8.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$
$$x_{i+1} = x_i - \frac{ax_i - b}{a}$$
$$x_{i+1} = x_i - x_i - \frac{b}{a}$$
$$x_{i+1} = \frac{-b}{a}$$

9.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$
$$x_{i+1} = x_i - \frac{x_i^2 - A}{2x_i}$$
$$x_{i+1} = \frac{2x_i^2}{2x_i} - \frac{x_i^2 - A}{2x_i}$$
$$x_{i+1} = \frac{2x_i^2 - x_i^2 + A}{2x_i}$$
$$x_{i+1} = \frac{x_i^2 + A}{2x_i}$$
$$x_{i+1} = \frac{\frac{x_i^2}{x_i} + \frac{A}{x_i}}{2}$$
$$x_{i+1} = \frac{x_i + \frac{A}{x_i}}{2}$$

11.

Using the function $f(x) = x^n - A$ to estimate the positive nth root of A, we get that $f`(x) = nx^{n-1}$ and $f``(x) = n(n-1)x^{n-2}$. Since $f`(\sqrt[n]{A}) = n(\sqrt[n]{A})^{n-1} \neq 0$, and $\frac{f`(\sqrt[n]{A})}{2f``(\sqrt[n]{A})} = \frac{n(\sqrt[n]{A})^{n-1}}{n(n-1)(\sqrt[n]{A})^{n-2}} =$ the newton method quadratically converges for the base function.

12.

$x_{50} = 562949953421312.00000000$

```
46 | 35184372088832.00000000 | 70368744177664.00000000 | 0.00000000 | 35184372088832.00000000
47 | 70368744177664.00000000 | 140737488355328.00000000 | 0.00000000 | 70368744177664.00000000
48 | 140737488355328.00000000 | 281474976710656.00000000 | 0.00000000 | 140737488355328.00000000
49 | 281474976710656.00000000 | 562949953421312.00000000 | 0.00000000 | 281474976710656.00000000
50 | 562949953421312.00000000 | 1125899906842624.00000000 | 0.00000000 | 562949953421312.00000000
>>
```

1.

a. $x_8 = 1.76929235$

```
1 | 1.00000000 | 4.00000000 | -3.00000000 | 3.00000000
2 | 4.00000000 | 2.82608696 | 54.00000000 | -1.17391304
3 | 2.82608696 | 2.14671901 | 14.91912550 | -0.67936794
4 | 2.14671901 | 1.84232628 | 3.59950729 | -0.30439274
5 | 1.84232628 | 1.77284764 | 0.56850886 | -0.06947864
6 | 1.77284764 | 1.76930140 | 0.02634489 | -0.00354624
7 | 1.76930140 | 1.76929235 | 0.00006684 | -0.00000904
8 | 1.76929235 | 1.76929235 | 0.00000000 | -0.00000000
>>
```

b. $x_6 = 1.67282170$

```
1 | 1.00000000 | 1.88258995 | -3.28171817 | 0.88258995
2 | 1.88258995 | 1.69064886 | 1.45309006 | -0.19194109
3 | 1.69064886 | 1.67295507 | 0.11364717 | -0.01769379
4 | 1.67295507 | 1.67282171 | 0.00084390 | -0.00013336
5 | 1.67282171 | 1.67282170 | 0.00000005 | -0.00000001
6 | 1.67282170 | 1.67282170 | -0.00000000 | 0.00000000
>>
```

c. $x_4 = 1.12998050$

```
1 | 1.00000000 | 1.13510383 | -0.44024719 | 0.13510383
2 | 1.13510383 | 1.12998867 | 0.01807458 | -0.00511516
3 | 1.12998867 | 1.12998050 | 0.00002879 | -0.00000817
4 | 1.12998050 | 1.12998050 | 0.00000000 | -0.00000000
>>
```

2.

a. $x_7 = 0.75487767$

```
1 | 0.00000000 | 1.00000000 | -1.00000000 | 1.00000000
2 | 1.00000000 | 0.83333333 | 1.00000000 | -0.16666667
3 | 0.83333333 | 0.76438212 | 0.23521091 | -0.06895122
4 | 0.76438212 | 0.75502487 | 0.02532928 | -0.00935725
5 | 0.75502487 | 0.75487770 | 0.00038629 | -0.00014717
6 | 0.75487770 | 0.75487767 | 0.00000009 | -0.00000004
7 | 0.75487767 | 0.75487767 | 0.00000000 | -0.00000000
8 | 0.75487767 | 0.75487767 | 0.00000000 | -0.00000000
>>
```

b. $x_4 = -0.97089892$

```
1 | 0.00000000 | -1.00000000 | -5.00000000 | -1.00000000
2 | -1.00000000 | -0.97096377 | 0.15852902 | 0.02903623
3 | -0.97096377 | -0.97089892 | 0.00035249 | 0.00006485
4 | -0.97089892 | -0.97089892 | 0.00000000 | 0.00000000
5 | -0.97089892 | -0.97089892 | 0.00000000 | 0.00000000
>>
```

c. $x_5 = 1.59214294$

```
1 | 1.00000000 | 1.66666667 | -2.00000000 | 0.66666667
2 | 1.66666667 | 1.59329292 | 0.28860340 | -0.07337375
3 | 1.59329292 | 1.59214322 | 0.00438522 | -0.00114971
4 | 1.59214322 | 1.59214294 | 0.00000106 | -0.00000028
5 | 1.59214294 | 1.59214294 | 0.00000000 | -0.00000000
>>
```

5.

Finding the root of the function $f(x) = \pi 10x^2 + \frac{2}{3}\pi x^3 - 400$, should give an approximation of the radius r for the silo. After 7 steps a root $x = 3.23618742 was found.$

```
1 | 1.00000000 | 6.30260399 | -366.48967836 | 5.30260399
2 | 6.30260399 | 4.17698907 | 1372.27596584 | -2.12561492
3 | 4.17698907 | 3.36866689 | 300.75420531 | -0.80832218
4 | 3.36866689 | 3.23943163 | 36.56849184 | -0.12923526
5 | 3.23943163 | 3.23618944 | 0.87368892 | -0.00324219
6 | 3.23618944 | 3.23618742 | 0.00054412 | -0.00000202
7 | 3.23618742 | 3.23618742 | 0.00000000 | -0.00000000
8 | 3.23618742 | 3.23618742 | 0.00000000 | 0.00000000
9 | 3.23618742 | 3.23618742 | 0.00000000 | 0.00000000
10 | 3.23618742 | 3.23618742 | 0.00000000 | 0.00000000
>>
```