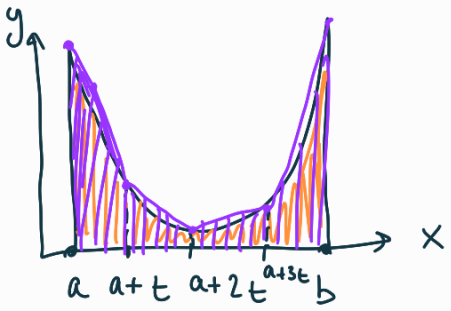
Вычисление площади под кривой методом трапеций.

$f(x) = x^2$

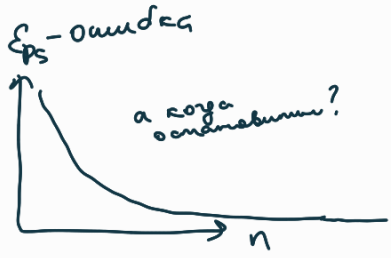


(n) - кол-во разбиений = 4

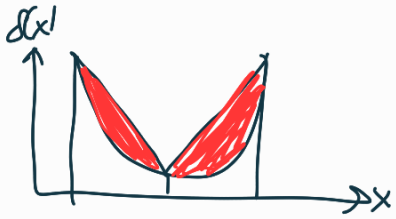
$$t = \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx \approx ?$$

- реальная площадь
- вычисленная методом трап.
- ошибка



Качество разбиения



(I) $\varepsilon = 0.01$. - заданная точность

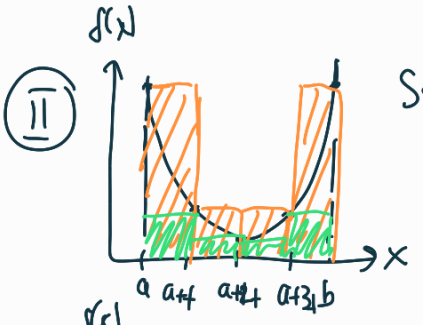
$f(x) = x^2$

$$\int_a^b x^2\,dx = \frac{x^3}{3}\Big|_a^b = \frac{b^3-a^3}{3} = I$$

$n_{start} = 10$: $eps = |S_{trap} - I| < \varepsilon \Rightarrow$ ОК.

не ок $\Rightarrow n = 2 \cdot n$

(II)

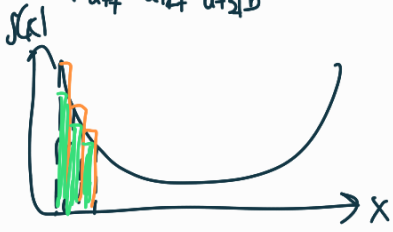


$$S_{sup} = \sum_{i=1}^{n} \sup_{x \in \Delta x_i} f(x) \cdot t$$

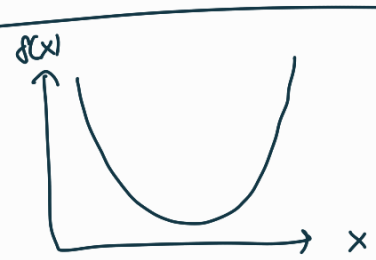
$$S_{inf} = \sum_{i=1}^{n} \inf_{x \in \Delta x_i} f(x) \cdot t$$

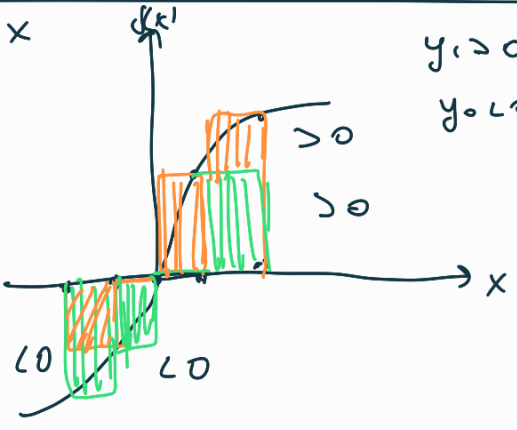
$|\Delta x_i| = t$

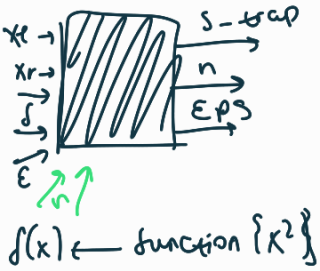
$$S_{trap} = \sum_{i=1}^{n} \frac{f(a+(i-1)\cdot t) + f(a+i\cdot t)}{2} \cdot t$$



$eps = |S_{sup} - S_{inf}| < \varepsilon \Rightarrow$ ОК



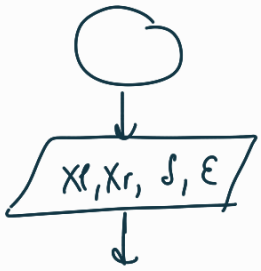




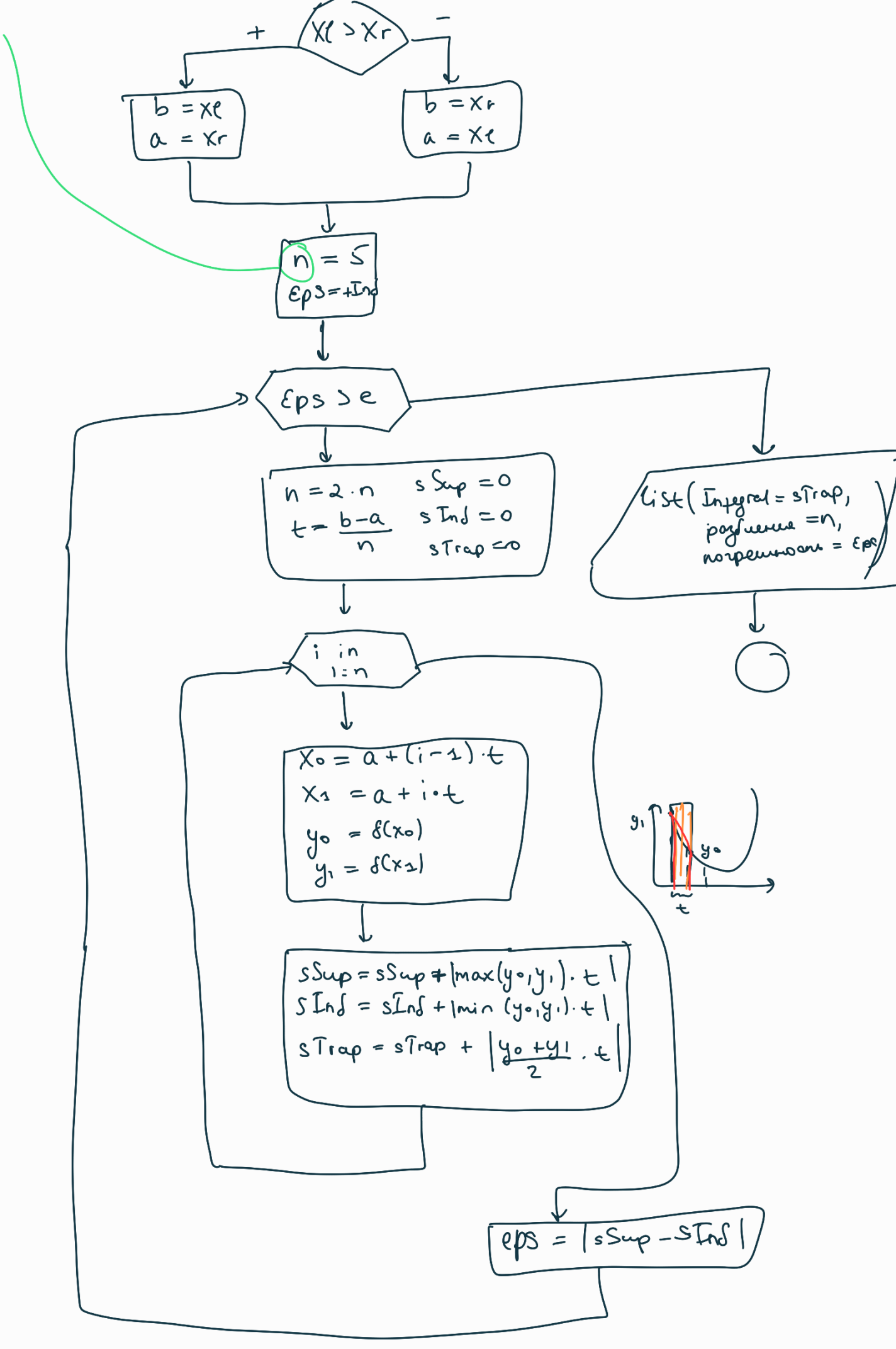
f(x) ← function {x^2}

ε - точность задания

Eps - в ходе вычислений



$$\begin{pmatrix} Xl = -2 & Xl = 2 \\ Xr = 2 & Xr = -2 \end{pmatrix}!$$


Xl > Xr
+
−
b = Xl
a = Xr
b = Xr
a = Xl
n = 5
eps = +Inf
eps > e
n = 2 · n
t = (b − a) / n
sSup = 0
sInf = 0
sTrap = 0
list(Integral = sTrap,
разбиение = n,
погрешность = eps
i in
1:n
x0 = a + (i − 1) · t
x1 = a + i · t
y0 = f(x0)
y1 = f(x1)
sSup = sSup + |max(y0, y1) · t|
sInf = sInf + |min(y0, y1) · t|
sTrap = sTrap + |(y0 + y1)/2 · t|
eps = |sSup − sInf|
y1
y0
t