

Умножение  

$$\begin{array}{r} 25 \\ \times 31 \\ \hline 25 \\ 75 \\ \hline 775 \end{array}$$
 .

$$(2 \cdot 10 + 5)(3 \cdot 10 + 1)$$

$$= 2 \cdot 10 \cdot 3 \cdot 10 + 5 \cdot 3 \cdot 10 + 5 \cdot 1$$

$$= \underline{2 \cdot 3 \cdot 10^2} + \underline{5 \cdot 3 \cdot 10} + \underline{5 \cdot 1} + \underline{2 \cdot 1 \cdot 10}$$

При умножении  
число попарно умножается

Однозначные цифры:  
 $x, y : \text{Наго}$     $x \cdot y$

$x = a \cdot 10^r + b$ ;  $y = c \cdot 10^s + d$

$x \cdot y = (a \cdot 10^r + b) \cdot (c \cdot 10^s + d) = ac \cdot 10^{r+s} + ad \cdot 10^r + bc \cdot 10^s + bd$

$= \underbrace{ac \cdot 10^{r+s}}_{t_1} + \underbrace{10^r(ad + bc)}_{t_2} + \underbrace{bd}_{t_3} \rightarrow \text{суммируется } n^2$

$\underbrace{(a+b)(d+c)}_{(a+b)(d+c)} = ad + ac + bd + bc$

умножение  
однозначных чисел

Регуларизация:  $x = ac + bd$   
 $y = (a+b)(c+d) - ac - bd$

$$x \cdot y = t_1 \cdot 10^{2r} + 10^r(t_3 - t_1 - t_2) + t_2$$

Как мы змогли навчитись переписать?

$$x = 1736$$

$$y = 2054$$

$$r_x = 4$$

$$r_y = 4$$

$$r^* = \frac{\max(r_x, r_y)}{2} = 2$$

$$\downarrow$$

$$x = 17 \cdot 10^2 + 36$$

$$y = 20 \cdot 10^2 + 54$$

$$x = 17 \quad y = 20$$

$$x = 36 \quad y = 54$$

$$(1) \quad t_1 = a \cdot c = 17 \cdot 20 \rightarrow \text{переписать}$$

$$t_2 = b \cdot d = 36 \cdot 54 \rightarrow \text{переписать}$$

$$t_3 = (a+b)(c+d) = 53 \cdot 74 \rightarrow \text{переписать} \quad x = 53 \quad y = 74$$

return  $t_1 \cdot 10^2 + 10 \cdot (t_3 - t_2 - t_1) + t_2 \rightarrow$  базовый  
вариант.

На каком змані зможна переписати  $t_1, t_2, t_3$

①

$$689 \cdot 3564$$

$$r_{\max} = 4$$

$$r/2 = 2$$

$$a = 6 \quad c = 35$$

$$b = 89 \quad d = 64$$

$$\bullet t_1 = ac; t_2 = bd$$

$$\bullet t_3 = (a+b)(c+d)$$

$$6 \cdot 10^2 + t_2 + 10 \cdot (t_3 - t_2 - t_1) \quad \text{return } (2455596)$$

②

$$t_1 = 6 \cdot 35$$

$$r_{\max} = 2$$

$$r/2 = 1$$

$$a = 0 \quad c = 3$$

$$b = 6 \quad d = 5$$

- $t_1 = ac$ ,  $t_2 = bd$
- $t_3 = (a+b)(c+d)$

$$t_1 \cdot 10^2 + t_2 + 10 \cdot (t_3 - t_2 - t_1) \\ = 210 \quad \text{return}(210)$$

$$t_1 = 0 \cdot 3$$

$$r_{\max} = 1$$

return(0)

$$t_2 = 6 \cdot 5$$

$$r_{\max} = 1$$

return(30)

$$t_3 = 6 \cdot 8$$

$$r_{\max} = 1$$

return(48)

$$t_2 = 89 \cdot 64$$

$$r_{\max} = 2$$

$$a = 8 \quad c = 6$$

$$b = 9 \quad d = 4$$

$$\bullet t_1 = ac, t_2 = bd$$

$$\bullet t_3 = (a+b)(c+d)$$

$$t_1 \cdot 10^2 + t_2 + 10 \cdot (t_3 - t_2 - t_1) \\ = 89 \cdot 64 \quad \text{return } (8964)$$

$$t_1 = 8 \cdot 6 \quad t_2 = 9 \cdot 4 \quad t_3 = 17 \cdot 10$$

$$\text{return}(48)$$

$$\text{return}(36)$$

$$r_{\max} = 2 \quad r/2 = 1$$

$$a = 1 \quad c = 1$$

$$b = 7 \quad d = 0$$

$$\bullet t_1 = ac, t_2 = bd,$$

$$\bullet t_3 = (a+b)(c+d)$$

$$t_1 \cdot 10^2 + t_2 + 10 \cdot (t_3 - t_2 - t_1) \\ = 170 \quad \text{return } (170)$$

$$t_3 = 95 \cdot 99$$

$$r_{\max} = 2 \quad r/2 = 1$$

но не comes

$$\text{return } (9405)$$

③

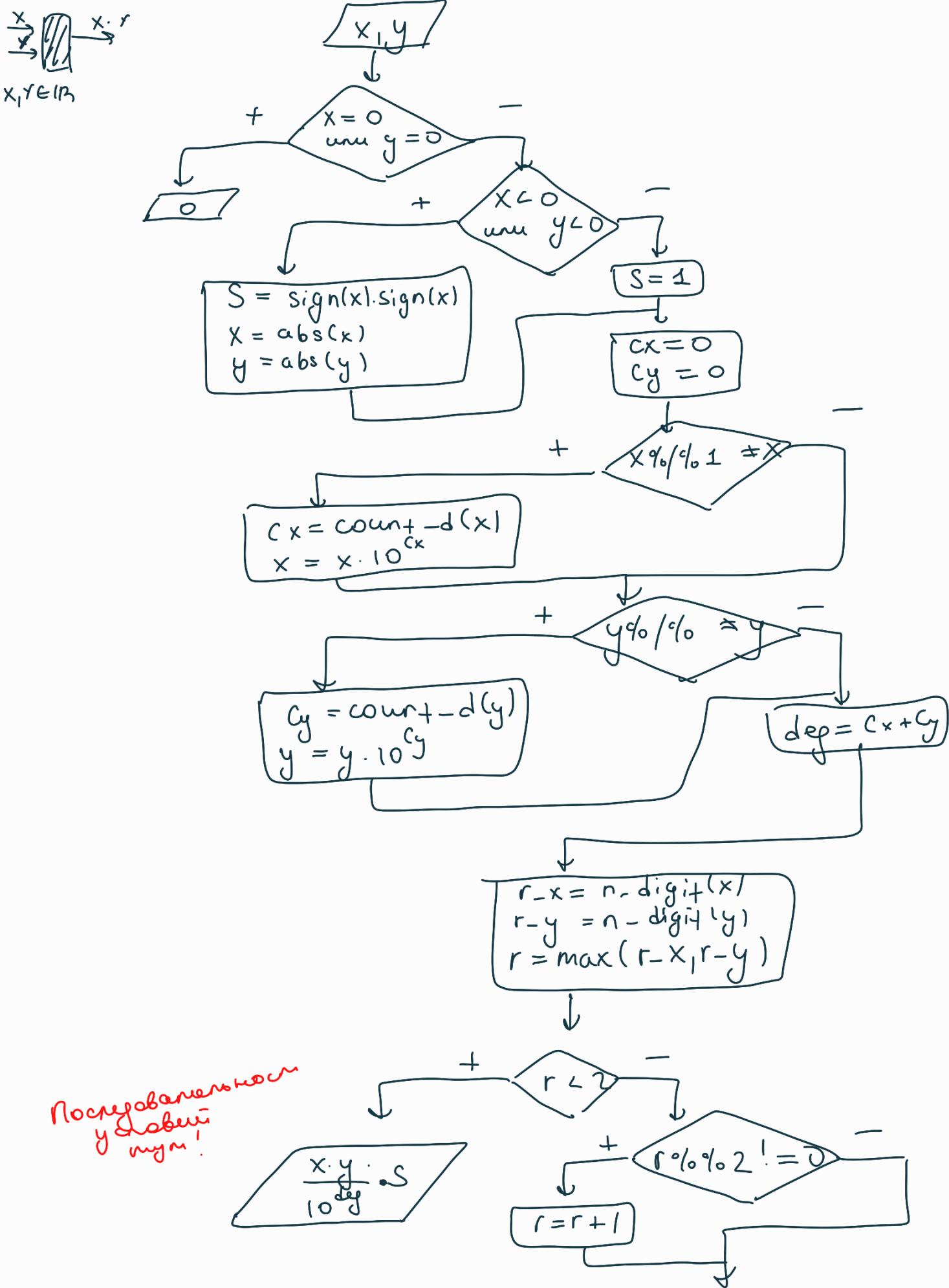


$$\deg = Cx + Cy = 3$$

Brouge agreement  
na 10<sup>deg</sup>

$$6.54 \cdot 70.5 = 461,070$$

## func-on multiplication ( $x, y$ )



$$\begin{aligned} a &= X \% / \% 10^{r/2} \\ b &= X \% \% 10^{r/2} \\ c &= Y \% / \% 10^{r/2} \\ d &= Y \% \% 10^{r/2} \end{aligned}$$

$$\begin{aligned} t_1 &= \text{multiplication}(a, c) \\ t_2 &= \text{multiplication}(b, d) \\ t_3 &= \text{multiplication}(a+b, c+d) \end{aligned}$$

we  
use  
more  
memory  
than  
usual  
way

$$\frac{t_1 \cdot 10^r + t_2 + (t_3 - t_2 - t_1) \cdot 10^{r/2} \cdot S}{10^{\deg}}$$

Оғарылған үшінде:  $n^2$   
Капақтау:  $O(n^{\log_2 3})$