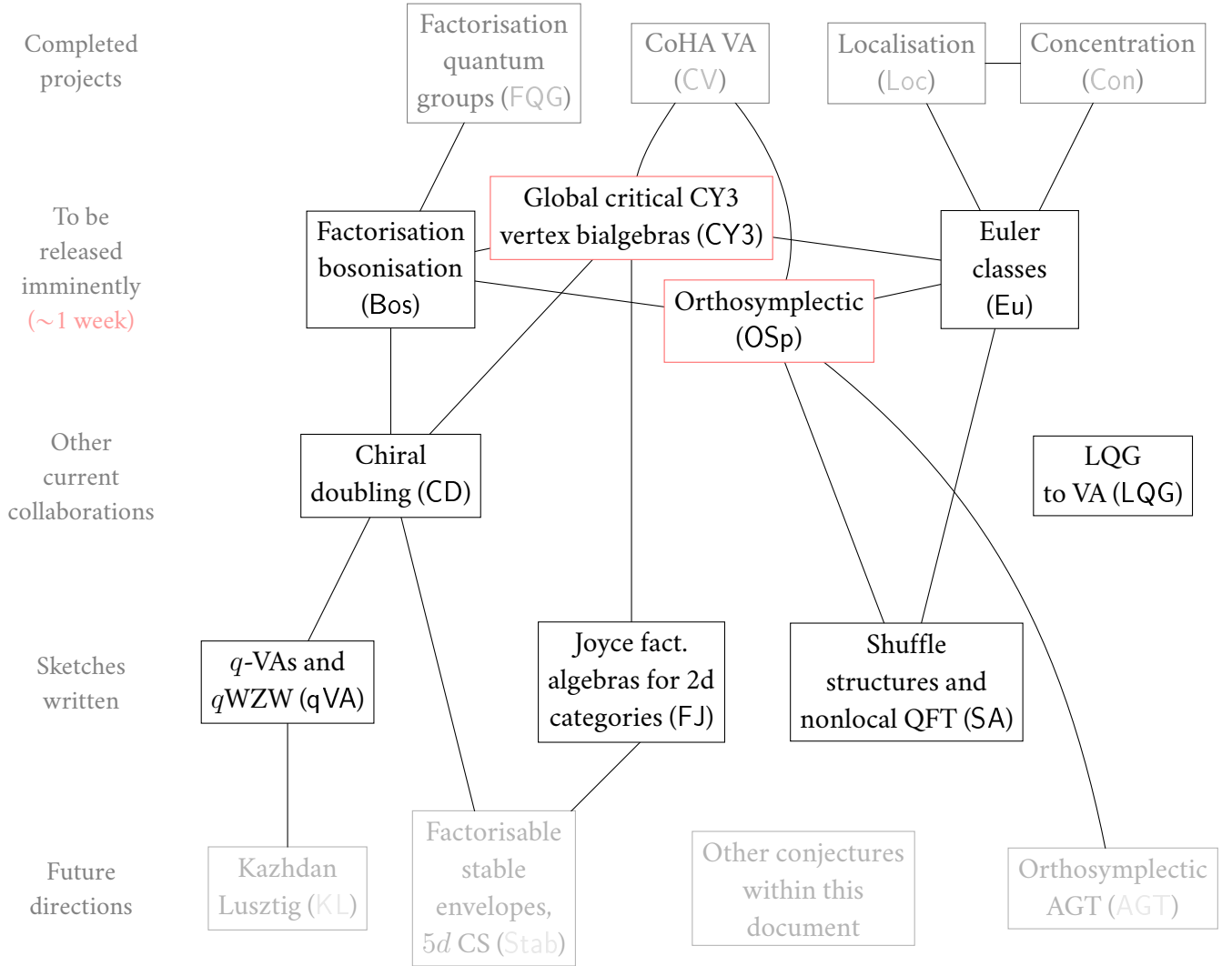


RESEARCH PLANS (7 PAGES)

ALEXEI LATYNTSEV

See section 2 for a the list of papers and papers in preparation corresponding to the following projects.



This is a technical summary.

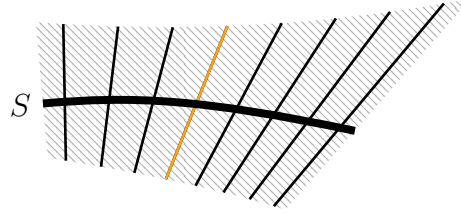
For an **abridged** (4 pp.) version, see <https://alyoshalatyntsev.github.io/planabridged/planabridged.pdf>.

For a **non-technical summary** (2 pp.), see <https://alyoshalatyntsev.github.io/plansummary/plansummary.pdf>.

1. Details of projects

1.1. Algebraic structures attached to Calabi-Yau-threefolds (CV, CY3, FJ, Stab)

This first series of projects asks: the assignment $Y \mapsto H^\bullet(\mathcal{M}_Y, \mathcal{P}_Y)$ of a CoHA [KS; KPS; YZ] to a three-Calabi-Yau manifold or category Y , what are all the algebraic structures it has? What are the relations between them? Can we recover known structures on Yangians, \mathcal{W} -algebras, etc.?



The general expectation is that it should *factorise* over Y . We focus on cases where $Y = K_S$ is the canonical bundle of an algebraic surface, and the *vertex* structure on the fibres.

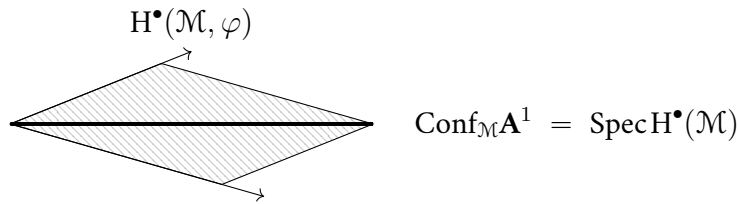
Global critical CY3 vertex quantum groups. Let \mathcal{M}_Y be a *global critical locus* inside a smooth ambient stack \mathcal{M} . For instance Y is K_{T^*C} , a local curve [KK], representations of quiver with potential; more generally or any *deformed CY3 completion*.

Theorem A. [CY3] *There is a vertex coproduct on the critical CoHA*

$$H^\bullet(\mathcal{M}, \varphi) \rightarrow H^\bullet(\mathcal{M}, \varphi) \hat{\otimes} H^\bullet(\mathcal{M}, \varphi)((z^{-1}))$$

which forms a quasitriangular vertex **bialgebra** in meromorphic braided category $H^\bullet(\mathcal{M})\text{-Mod}$.¹

Theorem B. [CY3] *View $H^\bullet(\mathcal{M}, \varphi)$ as a quasicoherent sheaf on the coloured configuration space:*



*There is a localised coproduct giving making this a quasitriangular **factorisation bialgebra** over $\text{Conf}_{\mathcal{M}} \mathbf{A}^1$.*

We obtain Theorem A from Theorem B by defining a **localised-to-vertex** functor between factorisation bialgebras on $\text{Conf}_{\mathcal{M}} \mathbf{A}^1$ and $\text{Ran} \mathbf{A}^1$. In the quiver case a factorisation bialgebra is the same a Davison localised bialgebra [Da], and

Theorem C. [CY3, CV] *In the quiver with potential case, the localised coproduct on the Yangian $H^\bullet(\mathcal{M}_Q, \varphi_W) \simeq Y_h(\mathfrak{n}_Q)$ agrees with the Davisons'/Yang-Zhao's. In the ADE case, the vertex product is **Drinfeld's** meromorphic coproduct.*

¹The formalism of braided factorisation categories is developed in FQG.

Relevance. The above is a first step to giving a *factorisable, basis-free* definition of $Y_h(\mathfrak{g})\text{-Mod}$ with its meromorphic and ordinary tensor products, which *generalises* to $\mathcal{A}\text{-Mod}$ for \mathcal{A} any affine W-algebra, W-algebra attached to a surface, or double of a CoHA. The idea: take the factorisable definition [Ga] of $U_q(\mathfrak{g})\text{-Mod}$ and replace

$$U_q(\mathfrak{n}) = \text{co/free}(\mathfrak{n}) \in \text{Perv}^{\text{fact}}(\text{Conf}_{\Lambda}\mathbf{R}^2) \quad \text{by} \quad H^{\bullet}(\mathcal{M}, \varphi) \in \text{QCoh}^{\text{fact}}(\text{Conf}_{\mathcal{M}}\mathbf{A}^1).$$

Then apply the bosonisation Bos and doubling CD to get analogues of $U_q(\mathfrak{b})$ and $U_q(\mathfrak{g})$. This may also give a conceptual proof of the PBW generation of CoHAs by BPS Lie algebras $\mathfrak{g}_{\text{BPS}}$, by analogy with the above definition of $U_q(\mathfrak{n})$.

Factorisation of moduli stacks. If Q is a quiver with torus T_d , we may define the *factorisable stack* parametris-ing Q -representations and a torus Lie algebra element fixing it:

$$\mathcal{M}^f = \{(V, \lambda) : \lambda \in \mathfrak{t}_{\infty}, V \in \mathcal{M}^{\lambda}\} \xrightarrow{p} \mathfrak{t}_{\infty} \simeq \text{Ran}_{Q_0}\mathbf{A}^1 \quad (1)$$

where $T_{\infty} = \cup T_d$ is the union over diagonal maps and \mathfrak{t}_{∞} is its Lie algebra.

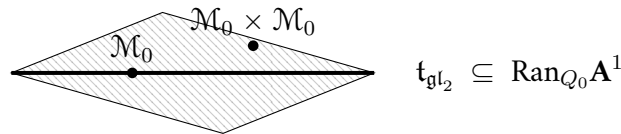
Conjecture D. [FJ] *This is a factorisation space over the Q_0 -coloured Ran space. Thus, the relative critical cohomology $\mathcal{A} = \int_p \varphi_W$ defines a coloured vertex bialgebra. It agrees with the **nilpotent CoHA** $H_{\bullet}^{\text{BM}}(\mathcal{M}_0)$ of [SV] and its structures of Theorem A.*

Conjecture E. [FJ] *For S an appropriate algebraic surface, the moduli stack of $\text{Coh}_0(S)$ is the total space of a factorisation stack $p : \mathcal{M}^f \rightarrow \text{Ran}_S K_S$. The factorisation algebra $\mathcal{B} = \int_p \varphi_W$ recovers **Mellit et al's** W -algebra $W^+(S)\text{-Mod}$ of [MMSV].*

Note that \mathcal{B} is only a vertex algebra on opens with $K_U \simeq U \times \mathbf{A}^1$ and $\mathcal{M}|_U$ a critical locus. On the intersection $U_1 \cap U_2$ the factorisation algebras \mathcal{B}_{U_1} and \mathcal{B}_{U_2} are only Morita-equivalent, so $\mathcal{B}\text{-FactMod}_S$ is a sheaf of categories on S which is locally *meromorphic braided monoidal*.

Pinning down the above structure may allow one to generalise to other Calabi-Yau-threefolds Y .

A general theory of stable envelopes. Conjecture D's factorisation structure on the stack \mathcal{M}^f



looks a lot like Maulik-Okounkov's structures [MO] on quiver varieties $M_{\mathbf{w}}$. Second, [MO]'s construction of the Yangian clearly looks like a Tannakian reconstruction. Thus,

Conjecture F. [Stab] $H_{\bullet}^{\text{BM}}(M_{\mathbf{w}})$ is the global sections of a sheaf $\int_p \omega_{M_{\mathbf{w}}^f}$ on the coloured Ran space. These sheaves together form a meromorphic tensor category \mathcal{C}_Q over $\text{Ran}_{Q_0}\mathbf{A}^1$, with braiding $R_{\text{MO}}(z)$. Applying factorisation Tannakian reconstruction of Bos recovers $Y_{\text{MO}}(\mathfrak{g}_Q)$.

1.2. The structure of factorisation quantum groups (FQG, Bos, CD)

There is an incongruity in the literature:

- (1) There is by now a powerful general theory [EGNO; Lu; Maa] of all finite quantum groups A (quasitriangular bialgebras) and their representations.
- (2) Meromorphic quantum groups like CoHAs, $U_q(\hat{\mathfrak{g}})$, $Y_h(\mathfrak{g})$, $\mathcal{E}_{h,\tau}(\mathfrak{g})$, $Y_h(\hat{\mathfrak{g}})$, $\mathcal{W}_{1+\infty}(\mathfrak{g})$, \dots still rely solely on ingenious but laborious and hard-to-generalise explicit (shuffle) formulae [Dr; MO; MMSV; SV; YZ].

The purpose of this project is to take Lurie's definition of $A\text{-Mod}$ as a topological factorisation category over \mathbf{R}^2 , and generalise it to a theory of *factorisation categories* over arbitrary factorisation spaces X .

Factorisation quantum groups. In FQG we define \mathbf{E}_n -factorisation spaces, categories, and algebras.

Theorem G. [FQG] *A braided factorisation structure on $\mathcal{A}\text{-FactMod}_X(\mathcal{C})$ is equivalent to a factorisation bialgebra structure on \mathcal{A} with a **factorisation R -matrix** $R : \mathcal{A} \otimes_{\mathcal{C},X} \mathcal{A} \xrightarrow{\sim} \mathcal{A} \otimes_{\mathcal{C},X} \mathcal{A}$.*

When $X = \text{Ran } \mathbf{C}$ this gives $R(z)$ satisfying the *spectral Yang-Baxter equation* [Dr; CWY], and we expect that a factorisation category \mathcal{C} generalises Soibelman's meromorphic tensor categories.

Theorem H. [FQG] *When $X = \text{Ran } \mathbf{A}^1$ the data of Theorem G has the structure of an Etingof-Kazhdan and Frenkel-Reshetikhin **quantum vertex algebra**.*

An easy consequence² is for semisimple complex \mathfrak{g} ,

Theorem I. *The standard and meromorphic coproduct on $Y_h(\mathfrak{g})$ and the spectral R -matrices*

$$R^\pm(s), R^{0,\epsilon}(s), R^\epsilon(s)$$

*relating them [GLW] makes $Y_h(\mathfrak{g})^\vee$ into a lax quasitriangular factorisation bialgebra over $X = \text{Ran } \mathbf{A}^1$.*³

Finally, in [FQG] we generalise **Borcherd's twist construction** to arbitrary factorisation algebras. This twists a factorisation algebra by an “interaction term” to give more interesting examples.

Relevance. To understand *affine Yangians* [GRZ] or *q -vertex algebras* (see qVA) we need a good definition of factorisation algebra on the noncommutative affine plane, and we expect this definition is best made categorically. In turn this may help us understand the *new KL equivalences* [BCDN].

Factorisation bosonisation. A lot of algebraic effort [Da; RSYZ; YZ], [CY3; OSp] is spent each time one extends a CoHA by adding in a “Cartan piece”, e.g. to obtain Yangians of Borels $Y_h(\mathfrak{b}_Q)$.

But in the finite case this is easy: one just applies Tannakian reconstruction to $U_q(\mathfrak{n})\text{-Mod}(\text{Rep}_q T)$ to recover $U_q(\mathfrak{b})$, and this can be done for any finite quantum group [Ga; Maa; Mab].

²Not written in the current version of [FQG].

³Here we use the equivalence between vertex bialgebras and *ch*-* factorisation bialgebras on $\text{Ran } \mathbf{A}^1$.

Theorem J. [Bos, in preparation] *There is a factorisation Tannakian reconstruction functor*

$F : (\text{braided}) \text{ factorisation categories } \mathcal{C} \rightarrow (\text{quasitriangular}) \text{ factorisation quantum groups } \mathcal{A}.$

Applying this to $\mathcal{C} = Y_h(\mathfrak{n}_Q)\text{-Mod}(Y_h(\mathfrak{t}_Q)\text{-Mod})$ or $H^\bullet(\mathcal{M}, \varphi)\text{-Mod}(H^\bullet(\mathcal{M})\text{-Mod})$ gives the Yangian of the Borel and the extended CoHAs with its vertex bialgebra structure.

Factorisation Drinfeld doubling. The purpose of the project [CD] is to define a factorisation analogue of the **Drinfeld centre** construction, and as a result, a functor from vertex bialgebras to vertex quantum groups. We aim to show this sends $Y_h(\mathfrak{b}_Q)$ to $Y_h(\mathfrak{g}_Q)$, the affine Grassmannian to the bubble affine Grassmannian, and recovers Takiff algebra double construction of [AN].

This may help understand the *doubling* of CoHAs [RSYZ; YZ] and the relation of this double to the derived category.

1.3. Orthosymplectic structures (OSp, SA, AGT)

When $G = \text{SO}_k, \text{Sp}_{2n}$, an (ortho)symplectic object of \mathcal{M} a moduli stack as in CY3 is an object $c \in \mathcal{C}$ with

$$\kappa : c \otimes_{\mathcal{C}} c \rightarrow 1$$

nondegenerate and (anti)symmetric.

Theorem K. [OSP] *The moduli of orthosymplectic objects is a fixed point stack \mathcal{M}^τ for an involution τ . There is a τ -equivariant CoHA action*

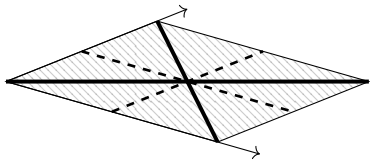
$$H^\bullet(\mathcal{M}, \varphi) \otimes H^\bullet(\mathcal{M}^\tau, \varphi^\tau) \rightarrow H^\bullet(\mathcal{M}^\tau, \varphi^\tau) \quad (2)$$

and a τ -equivariant vertex coaction

$$H^\bullet(\mathcal{M}^\tau, \varphi^\tau) \rightarrow H^\bullet(\mathcal{M}, \varphi) \otimes H^\bullet(\mathcal{M}^\tau, \varphi^\tau)((z^{-1})). \quad (3)$$

These together form a τ -twisted vertex Yetter-Drinfeld module.

More than this, $H^\bullet(\mathcal{M}^\tau, \varphi^\tau)$ forms an *orthosymplectic localised coalgebra*: it factorises over

$$\begin{array}{c} p^* H^\bullet(\mathcal{M}^\tau, \varphi^\tau) \\ \swarrow \quad \searrow \\ \text{Conf} \mathbf{A}^1 \xrightarrow{p} \text{Conf}_G \mathbf{A}^1 = \text{Spec} H^\bullet(BG) \end{array}$$


We prove Theorem K by defining an *orthosymplectic localised to vertex* functor. Thus

$$H^\bullet(\mathcal{M}^\tau, \varphi^\tau) \rightarrow H^\bullet(\mathcal{M}^\tau, \varphi^\tau) \otimes H^\bullet(\mathcal{M}^\tau, \varphi^\tau)((w_1 \pm w_2)^{-1})$$

in addition has the structure of a **orthogonal vertex algebra** (which we develop the theory of), e.g. OPEs have poles along the SO_k root hyperplanes.

Theorem L. [OSP] *In the quiver with potential case, (2) and (3) are given by W_G -shuffle (co)products for the Weyl group W_G .*

Conjecture M. *When Q is a ADE, the subalgebra of $\text{End}(\mathbf{H}^\bullet(\mathcal{M}^\tau, \varphi^\tau))$ generated by the CoHA is the positive part of the **twisted Yangian** [BR].*

Kac-Moody shuffle structures and “Dynkin spacetimes”. What happens if we let G in OSp vary over any Kac-Moody group?

Conjecture N. [SA] *When we restrict to the category of groups with Coxeter diagram*

a vertex/localised algebra is equivalent to a vertex/localised algebra equivariant for the action of

a square root of (231) generating W_{G_2} acting on \mathbf{C}^3 .

We define a *shuffle algebra* to be a monoidal functor $\mathcal{A} : \mathbf{KM}_{\text{Par}} \rightarrow \mathbf{Vect}$ from the category of Kac-Moody groups [Ku] with morphisms parabolics induction diagrams. Restricting to products of GL_n 's gives the usual definition [KS; Gr]. Generalising Chen's Theorem [Ch],

We expect this relates to Dynkin and q -analogues of multiple zeta values [KMT; Mi].

Conjecture P. [AGT] *The equivariant intersection homology of the invariant locus $\mathcal{U}_{\mathbf{P}^2, \mathrm{GL}_n}^{\mathbf{Z}/2}$ in the Uhlenbeck space is a **Verma module** for an orthosymplectic analogue $\mathcal{W}^k(\mathfrak{gl}_n)^{\mathrm{OSp}}$ of a \mathfrak{gl}_n W -algebra.*

We expect the proof to proceed in much the same way as in [BFN], but with free field realisations and parabolic induction replaced by their OSp -analogue; see the shuffle discussion in SA.

Conjecture Q. [AGT] *The dimension zero CoHA $U_h(\mathcal{D}(\mathbf{C}/\pm))$ of \mathbf{A}^3 , admits $\mathcal{W}^{k_n}(\mathfrak{gl}_n)^{\mathrm{OSp}}$ as quotients, and acts on the equivariant intersection cohomology $\mathrm{IH}_T^\bullet(M^{\mathbf{Z}/2})$ of \mathbf{A}^3 -instantons in [RSYZ].*

1.4. q -vertex algebras (qVA, KL)

q -vertex algebras. Our understanding of double affine quantum groups or Kazhdan Lusztig equivalences, $q\mathrm{KZ}$ equations, . . . is hampered by the absence of a good definition of q -affine vertex algebra. Note that many attempts at defining q -affine vertex algebras used q -difference modules, and:

- (1) From physics, Costello [Co] suggests these should factorise over the q -affine plane \mathbf{A}_q^2 .
- (2) The derivation $y\partial_x$ defines a q -difference operator on $\mathbf{A}_{y=1}^1 \subseteq \mathbf{A}_q^2$.

Conjecture R. [qVA] *Using the theory [MS] of jet spaces on noncommutative schemes X , one may define $\mathcal{D}\text{-Mod}(X)$ and functors $f^!, f_*$ for any map f satisfying the usual properties. There is a functor from $\mathcal{D}\text{-Mod}(\mathbf{A}_q^2)$ to the category of q -difference modules on \mathbf{A}^1 .*

We then may define $\mathcal{D}\text{-Mod}(\mathrm{Ran}\mathbf{A}_q^2)$ as a (co)limit of $\mathcal{D}\text{-Mod}((\mathbf{A}_q^2)^n)$. We expect this carries a q -**chiral** monoidal product, allowing us to define q -vertex algebras as usual, and prove they are equivalent to concrete definitions [EK; FR] in terms of field maps with q -OPEs.

Conjecture S. [qVA] *For G semisimple, there is a q -deformation of the BD Grassmannian $\mathrm{Gr}_{G,q} \rightarrow \mathrm{Ran}\mathbf{A}_q^2$, defining the q -affine vertex algebra $V_q^k(\mathfrak{g})$. Their conformal blocks⁴ give the $q\mathrm{KZ}$ connection.*

Kazhdan-Lusztig equivalences. The goal of this project is to relate q -vertex algebras to the geometric proof [CF] of Iwahori Kazhdan-Lusztig.

Conjecture T. [KL] *There is a **Zhu algebra** functor from q -affine VOAs to q -deformed associative algebras, which sends $V_q^k(\mathfrak{g}) \mapsto U_q(\mathfrak{g})$.*

Having this would imply $V_q^k(\mathfrak{g})$ is thus an object which interpolates between $U_q(\mathfrak{g})$ and $V^k(\mathfrak{g})$. Then, to make this precise, we aim to construct a factorisation category $\mathcal{C} = V_q^k(\mathfrak{g})\text{-Mod}$ on $\mathbf{C} \times \mathbf{R}_{\geq 0}$ with a connection ∇ , whose fibres over give $V^k(\mathfrak{g})\text{-Mod}$ and $U_q(\mathfrak{g})\text{-Mod}$, and ∇ -parallel transport gives $\mathrm{RH} : V^k(\mathfrak{g})\text{-Mod} \mapsto U_q(\mathfrak{g})\text{-Mod}$ in the proof [CF].

Relevance. Constructing analogues of \mathcal{C} may give proofs of new Kazhdan-Lusztig equivalences [BCDN] from $3d$ mirror symmetry, or affine analogues of Kazhdan-Lusztig.

1.5. Sheaf methods (Con, Loc, Eu)

Localisation methods. One of the main techniques in enumerative geometry are the *torus localisation* and *Graber-Panharipande formulas* [GP]. We generalise these to the Artin moduli stacks appearing in modern enumerative/algebraic geometry.

⁴i.e. homology of the factorisation algebra over $(\mathbf{A}_q^2)^n$.

Theorem U. [Conc; Loc] We give conditions for the cohomology of an Artin stack \mathcal{X} to be **concentrated** on a closed substack \mathcal{Z} ; when $\mathcal{Z} = \mathcal{X}^T$ is fixed points of a quasismooth dg scheme we give **Atiyah-Bott** $\text{id} = i_* (i^!(-)/e(N_{\text{vir}}))$ and **Graber-Pandharipande localisation** formulas $[\mathcal{X}]^{\text{vir}} = i_* ([\mathcal{X}^T]^{\text{vir}}/e(N_{\text{vir}}))$.

Theorem V. [Eu] We strengthen both parts of the theorem to **critical cohomology** of **arbitrary** closed embeddings $\mathcal{Z} \hookrightarrow \mathcal{X}$ quasismooth over a common base. As a result, for \mathcal{M} as in CY3, we have the following Theorem:

Theorem W. [Eu] For any “split locus” map $\pi : \mathcal{M}^s \rightarrow \mathcal{M}$, we have $\text{CoHA} = \text{CoHA}^s/e(N_{i,\text{vir}})$.

Taking \mathcal{M}^s a *shuffle space*⁵, this gives a **universal, geometric** way of producing/interpreting shuffle products for CoHAs [Da; SV; YZ]. Taking $\mathcal{M}^s = \mathcal{M} \times \mathcal{M} \xrightarrow{\oplus} \mathcal{M}$ proves **compatibility** [CV,CY3,Li] between **coproducts** and the CoHA (c.f. CY3). This easily **generalises** to the OSp/Dynkin SA cases.

1.6. Liouville quantum gravity to vertex algebras (LQG)

Probabilists are beginning construct Feynman measures (GFF [BPR], LQG [KRV]) **rigorously** to understand 2d CFTs [CRV; DS; Sh]. There is almost no interreaction with the algebraic geometry/functorial school to QFT, and they understand/have access to techniques (e.g. SLE curves [MS; SS], combinatorial approximations) that the latter school does not. We want to build a bridge:

Conjecture X. [LQG] Let $\mathcal{Z} : \text{Cob}_2^{\text{hol}} \rightarrow \text{Vect}$ be a Segal CFT. If its unstraightening

$$\mathcal{C}_{\mathcal{Z}} \rightarrow \text{Cob}_2^{\text{hol}}$$

respects the holomorphic structure on (Teichmuller) Hom spaces, we may take its chiral part \mathcal{Z}^{ch} , which gives a translation-equivariant holomorphic factorisation algebra, hence [CG] vertex algebra.

Conjecture Y. [LQG] The Liouville Quantum Gravity CFT [GKRV] satisfies the condition of Conjecture X, and its chiral part is the Virasoro vertex algebra. Likewise the Gaussian Free Field CFT [BPR] gives the Heisenberg vertex algebra.

Conjecture Y would give an explicit relation between [KRV]’s DOZZ formula to the Virasoro OPE.

⁵i.e. shuffle algebra in the category of spaces, see SA.

2. Paper list

- OSp de Hory, S., Latyntsev, A. *Orthosymplectic instantons and cohomological Hall algebras*. 2025
- Conc Aranha, D., Khan, A.A., Latyntsev, A., Park, H. and Ravi, Charanya, 2024, *The stacky concentration theorem*. arxiv:2407.08747 2024
- FQG Latyntsev, A., 2023. *Factorisation quantum groups*. arXiv:2312.07274 2023
- Loc Aranha, D., Khan, A.A., Latyntsev, A., Park, H. and Ravi, Charanya, 2022. *Virtual localization revisited*. arXiv preprint arXiv:2207.01652. 2022
- CV Latyntsev, A., 2021. *Cohomological Hall algebras and vertex algebras*. arXiv:2110.14356 2021

To appear imminently (~ 1 week):⁶

- CY3 Jindal, S., Kaubrys, S., Latyntsev, A. *Vertex quantum groups for deformed CY3 completions and the Drinfeld coproduct on Yangians*

To appear imminently:

- Bos de Hory, S., Latyntsev, A. *Factorisation bosonisation*.
- Eu Latyntsev, A. *Virtual Euler classes for Artin stacks*.

In preparation:

- CD Latyntsev, A. and Niu, W. *Chiral doubling*.
- LQG Giri, V. and Latyntsev, A. *Louville quantum gravity and vertex algebras*.

⁶See e.g. <https://arxiv.org/search/math?searchtype=author&query=Latyntsev,+A>

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