RESEARCH STATEMENT - SUMMARY

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A central problem of geometric representation theory and enumerative geometry is understanding structures attached to *Calabi-Yau threefolds*: Donaldson-Thomas invariants, mirror symmetry, quantum groups, ... Most of my research is in this area, making physics-inspired rigorous geometric constructions of such structures and proving new relations between them.

For a full version, including detailed Theorems, Conjectures and their proof plans, see
https://alyoshalatyntsev.github.io/plan/plan.pdf.

Critical CY3 vertex quantum groups. For instance, cohomological Hall algebras (categorified DT invariants) are a lynchpin of this area, relating to wall crossing, 3d mirror symmetry, instantons, skeins, ... Thus the following question is important and useful: what are all the algebraic structures on CoHAs?

We construct[CV, CY3] a Joyce vertex coproduct when the CY3 moduli stack is a global critical locus, prove that together with the CoHA structure it forms[CY3] a vertex quantum group, and show[CY3] this recovers Drinfeld/Yang-Zhao coproducts on Yangians, via a comparison functor[CY3] from Davisons' localised bialgebras (we prove are configuration space factorisation algebras) to vertex quantum groups.

Plans. Generalise away from the critical case: show the above construction glues[FJ] to a sheaf of braided factorisation categories over any surface, and relate to Mellit et al's W-algebras. Give a factorisable quiver moduli stack, geometrically inducing[FJ] the nilpotent CoHA vertex quantum group; use this to give[Stab] a Tannakian description of the stable envelope construction of Yangians.

Factorisation techniques. The Yangians, (affine) quantum groups, (affine) W-algebras, . . . appearing as CoHAs are crucial in this subject, yet usually have inspired but inscrutable generators-and-relations definitions: we lacked a common framework to understand their representation categories' structure.

We built[FQG] the theory of factorisation quantum groups using modern operadic techniques, showed[FQG] that this recovers the type of spectral Yang-Baxter matrices appearing in the above algebras, recovers previous definitions of vertex quantum groups, and give[FQG] examples by twisting constructions. We define[Bos] factorisable Tannakian reconstruction, giving a uniform way to Cartan-extend CoHAs. This makes rigorous the physics definition of category of line operators for hol.-top. QFTs.

Ongoing work. A factorisation Drinfeld centre functor[CD], giving doubles of Yangians/CoHAs.

Orthosymplectic CoHAs and Dynkin spacetimes. What happens if we apply the above techniques to orbifolds? What do boundary Chern-Simons, outer automorphisms of Lie algebras, twisted Yangians, boundary KZ equations, orthosymplectic quiver varieties, . . . have in common?

We define[OSp] sympletic/orthogonal cousins of moduli stacks, and extend the structures of[CY3] to a factorisation algebra on orbifolds/on symplectic configuration spaces: they form[OSp] symplectic vertex quantum groups, get solutions to Cherednik's reflection equation,[OSp] and give[OSp] an action on the compactification of ortho/symplectic bundles on a surface. We prove[OSp] shuffle formulas in the quiver with potential case. Give a folding construction[OSp] from ordianry to symplectic vertex algebras.

Ongoing work. Define[SA] analogues of (vertex) algebras, shuffle structures, quiver varieties, . . . factorising over arbitrary system of Kac-Moody groups. Build[SA] G_2 and boundary KZ equations by folding BD Grassmannians. Generalise[SA] Chen's Theorem and relate to q/ABCD multiple zeta values.

Plans. Show we obtain twisted Yangians via folding the type A quiver CoHA. Define[AGT] an action of an orthosymplectic W-algebra on the homology of orthosymplectic instantons on surfaces.

Atiyah-Bott localisation. Graber-Panharipande torus localisation formulas are one of the main techniques in enumerative geometry. To use them e.g. in[CV,CY3,OSp,SA], we strengthened[Loc,Con] these formulas to the Artin moduli stacks appearing in modern algebraic geometry, and arbitrary sheaf coefficients[Eu]. We deduced[Eu] a universal way to get CoHA shuffle formulas and a universal way to prove compatibility with coproducts.

q-vertex algebras. Our understanding of double affine quantum groups or Kazhdan Lusztig equivalences, qKZ equations, . . . is seriously hampered by the abscence of a good definition of q-affine vertex algebra. Our goal is to build this, inspired by Costello's physics work on deformed spacetimes for 5d Chern-Simons in physics and using techniques of[FQG]

Ongoing work. We will develop[qVA] the theory of D-modules on noncommutative spaces enough to show[qVA] that q-vertex algebras are factorisation algebras on the q-affine plane, then construct[qVA] q-affine vertex algebras geometrically via BD Grassmannians.

Plans. Show[qVA] that taking conformal blocks gives qKZ, construct[KL] a q-Zhu algebra functor and use the above to give a filtered version of Chen-Fu's proof of the Kazhdan-Lusztig equivalence.

Side project: LQG. Probabilists are beginning to understand CFT rigorously by construcing Feynman measures, e.g. Liouville Quantum Gravity, and have access to objects/methods we do not, e.g. SLE.

Ongoing work. Build a bridge to that subject, by constructing[LQG] a chiralisation functor from probabilists' Segal CFTs to vertex algebras, show[LQG] that Liouville Quantum Gravity is sent to the Virasoro.

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Each superscript* CY3, FQG, *... refers to a Theorem or Conjecture therein.