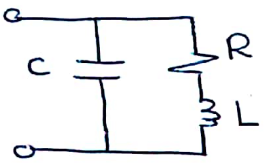
Sheet 8

1] Measurements on a practical inductor at 10 MHZ give $L = 8\,mH$ and $Q_{ind} = 40$

(a) Find the ideal capacitance C for parallel resonance at 10 MHZ and calculate the corresponding bandwidth β.

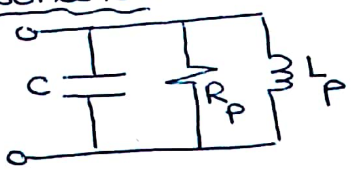
Sol.



$$\because Q_{ind} = \frac{\omega_o L}{R}$$

$$\therefore R = \frac{\omega_o L}{Q_{ind}} = \frac{2\pi(10\times10^{6})(8\times10^{-3})}{40} = 12566.37061\,\Omega$$

For parallel resonance



$$Y_{series} = Y_{parallel}$$

$$\therefore \frac{1}{R + j\omega l} = \frac{1}{R_p} + \frac{1}{j\omega l_p}$$

$$\therefore \frac{(R - j\omega l)}{(R + j\omega l)(R - j\omega l)} = \frac{1}{R_p} + \frac{1}{j\omega l_p}$$

$$\therefore \frac{R - j\omega l}{R^2 + (\omega l)^2} = \frac{1}{R_p} + \frac{1}{j\omega l_p}$$

$$\therefore \frac{R}{R^2 + (\omega l)^2} - \frac{j\omega l}{R^2 + (\omega l)^2} = \frac{1}{R_p} - \frac{j}{\omega l_p}$$

$$\therefore R_p = \boxed{\frac{R^2 + (\omega l)^2}{R}}$$

$$\omega l_p = \boxed{\frac{R^2 + (\omega l)^2}{\omega l}}$$

$$\boxed{X_c = X_{L_p} = \omega_o l_p}$$

$$\therefore X_c = 502968.9838$$

$$\therefore \frac{1}{\omega_o C} = 502968.9838$$

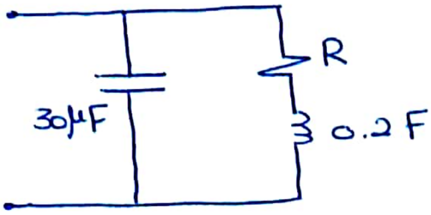
$$\therefore C = 3.164\times10^{-14}\,F$$

$$Bw = \frac{1}{R_p C}$$

$$R_p = 20118759.36$$

$$\therefore Bw = \boxed{1.571\,M \text{ rad/sec}}$$

2] Compare the resonant frequency of the circuit shown in Fig. 1 for R = 0 to that for R = 50 ohm



Sol.

ⓐ $R = 0$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 \times 30 \times 10^{-6}}} = 408.248 \text{ rad/sec}$$

ⓐ $R = 50\,\Omega$

For resonance: $imag(Y_{eq}) = 0$

$$\therefore \frac{1}{R + j\omega \ell} + j\omega c = Y_{eq}$$

$$\therefore Y_{eq} = \frac{R - j\omega \ell}{(R + j\omega \ell)(R - j\omega \ell)} + j\omega c$$

$$= \frac{R - j\omega \ell}{R^2 + (\omega \ell)^2} + j\omega c$$

$$imag(Y_{eq}) = \frac{-\cancel{\omega_o}\ell}{R^2 + (\omega_o \ell)^2} + \cancel{\omega_o} c = 0$$

$$\therefore c = \frac{\ell}{R^2 + \omega_o^2 \ell^2}$$

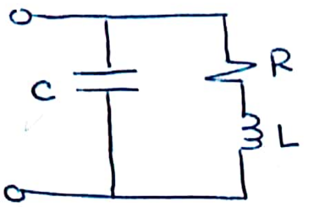
$$\ell = R^2 c + \omega_o^2 \ell^2 c$$

$$\therefore \ell - R^2 c = \omega_o^2 \ell^2 c$$

$$\therefore \omega_o = \sqrt{\frac{\ell - R^2 c}{\ell^2 c}} = \sqrt{\frac{0.2 - (50)^2 (30 \times 10^{-6})}{(0.2)^2 (30 \times 10^{-6})}} = 322.75 \text{ rad/sec}$$

3] A 20 μF capacitor is in Parallel with a practical inductor represented by L = 1mH in Series with R = 7 Ω . Find the resonant freq. in rad/sec and in HZ of the Parallel circuit.

Sol.



For resonance

imag. (Y_{eq}) = 0

$$\therefore Y_{eq} = j\omega c + \frac{1}{R + j\omega l}$$

$$\therefore Y_{eq} = j\omega c + \frac{(R - j\omega l)}{(R + j\omega l)(R - j\omega l)}$$

$$\therefore Y_{eq} = j\omega c + \frac{R - j\omega l}{R^2 + (\omega l)^2}$$

$$\therefore imag(Y_{eq}) = \omega_o c - \frac{\omega_o l}{R^2 + (\omega_o l)^2} = 0 .$$

$$\therefore c = \frac{l}{R^2 + (\omega_o l)^2}$$

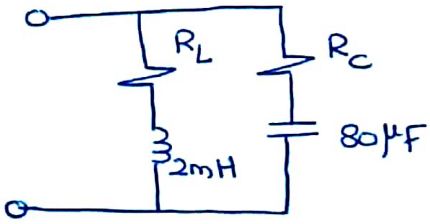
$$\therefore cR^2 + \omega_o^2 l^2 c = l$$

$$\therefore \omega_o = \sqrt{\frac{l - cR^2}{l^2 c}} = \sqrt{\frac{(1 \times 10^{-3}) - (20 \times 10^{-6} \times 7^2)}{(1 \times 10^{-3})^2 (20 \times 10^{-6})}}$$

$$\omega_o = 1000 \text{ rad/sec}$$

$$f_o = \frac{\omega_o}{2\pi} = 159.155 \text{ HZ}$$

4] what must be the relationship between the values of R_L and R_C if the network shown in Fig. 2 is to be resonant at all frequencies?



Sol.

@ resonance : imag $(Y_{eq}) = 0$

$$\because Y_{eq} = \frac{1}{R_L + j\omega l} + \frac{1}{R_c + \frac{1}{j\omega c}}$$

$$\therefore Y_{eq} = \frac{1}{R_L + j\omega l} + \frac{j\omega c}{j\omega c R_c + 1}$$

$$\therefore Y_{eq} = \frac{R_L - j\omega l}{R_L^2 + (\omega l)^2} + \frac{j\omega c(1 - j\omega c R_c)}{1 + (\omega c R_c)^2}$$

$$\therefore Y_{eq} = \frac{R_L - j\omega L}{R_L^2 + (\omega l)^2} + \frac{j\omega c + \omega^2 c^2 R_c}{1 + (\omega c R_c)^2}$$

$$\therefore \text{imag}(Y_{eq}) = \frac{-\omega_o l}{R_L^2 + (\omega_o l)^2} + \frac{\omega_o c}{1 + (\omega_o c R_c)^2} = 0$$

$$\therefore \frac{L}{R_L^2 + (\omega_o l)^2} = \frac{c}{1 + (\omega_o c R_c)^2}$$

$$\therefore L + L\omega_o^2 c^2 R_c^2 = c R_L^2 + c\,\omega_o^2 L^2$$

$$\therefore L c^2 R_c^2 = c L^2 \rightarrow (1) \quad \& \quad L = c R_L^2 \rightarrow (2)$$

From eq. 1

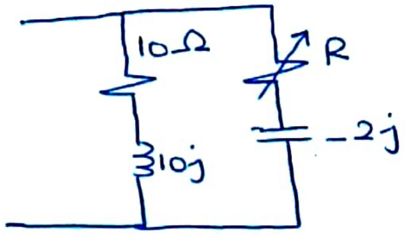
$$\therefore R_c = \sqrt{\frac{L}{c}} = 5\Omega$$

From eq. 2

$$R_L = \sqrt{\frac{L}{c}} = 5\Omega$$

$$\therefore R_L = R_c = 5\Omega$$

5] For the parallel network shown in Fig. 3, find the value of R for resonance, what is the value of X_c at resonance.



Sol.

@ resonance:

$$imag.(Y_{eq}) = 0$$

$$\because \quad Y_{eq} = \frac{1}{10+10j} + \frac{1}{R-2j}$$

$$\therefore \quad Y_{eq} = \frac{10-10j}{100+100} + \frac{R+2j}{R^2+4}$$

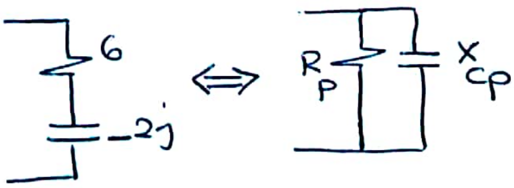
$$\therefore \quad imag(Y_{eq}) = \frac{-10}{200} + \frac{2}{R^2+4} = 0$$

$$\therefore \quad \frac{10}{200} = \frac{2}{R^2+4}$$

$$\therefore \quad R^2+4 = \frac{2(200)}{10} = 40$$

$$\therefore \quad R^2 = 36 \Rightarrow \therefore \boxed{R = 6\,\Omega}$$

To get X_c at resonance



$$Y_{series} = Y_{parallel}$$

$$\therefore \quad \frac{1}{6-2j} = \frac{1}{R_p} + j\omega C_p \Rightarrow \therefore \frac{6+2j}{36+4} = \frac{1}{R_p} + j\omega C_p$$

$$\therefore \quad \frac{1}{R_p} = \frac{6}{40} \Rightarrow \therefore \boxed{R_p = 6.67\,\Omega}$$

$$\omega C_p = \frac{2}{40}$$

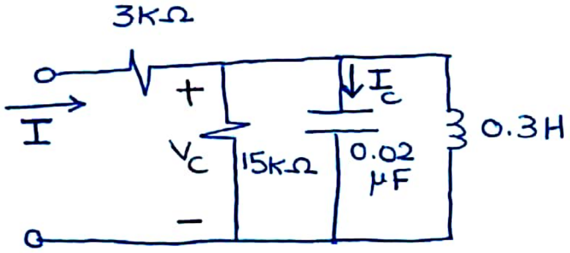
$$\therefore \quad X_{cp} = \frac{1}{\omega C_p} \Rightarrow \boxed{X_{cp} = 20\,\Omega}$$

6] Assume that a sinusoidal voltage source with a variable frequency and $V_{max} = 50\,V$ is applied to the circuit shown in Fig. 4

(a) At what frequency f is $|I|$ a minimum ?

(b) Calculate this minimum current

(c) What is $|I_c|$ at this frequency ?



Sol.

(a) $\omega_o = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.3\times 0.02\times 10^{-6}}} = 12909.94 \text{ rad/Sec}$

$\therefore f_o = \dfrac{\omega_o}{2\pi} = 2054.68 \text{ HZ}$

(b) $|I|_{min} = \dfrac{V_{max}}{(3K+15K)} = \dfrac{50}{18K} = 2.78\times 10^{-3}\,A$

(c) $I_c = \dfrac{V_c}{\frac{1}{j\omega C}} = V_c\, j\omega_o C$

$\therefore |I_c| = |V_c|\,\omega_o C$

$\because V_c = I_{min} \times 15000$

$\therefore |V_c| = |I_{min}|\ 15000$

$= 41.67\,V$

$\therefore |I_c| = 41.67 \times 12909.94 \times 0.02\times 10^{-6} = 0.0108\,A$

$= 10.8\,mA$

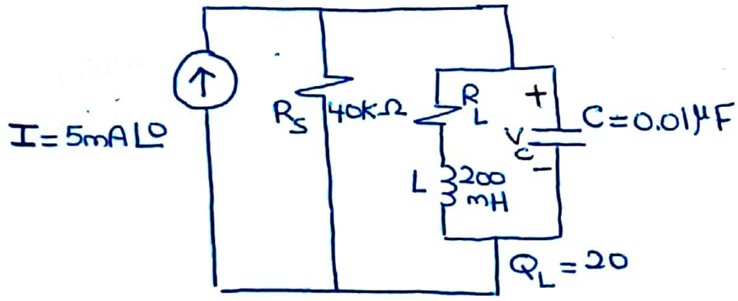
7] For the network of fig. 5

(a) Find the resonance frequency

(b) Calculate the magnitude of V_c at resonance

(c) Determine the power absorbed at resonance

(d) Find the BW.

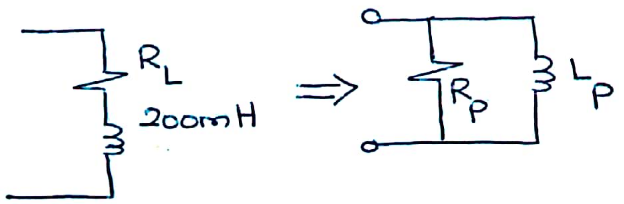


Sol.

(a) $\because \; Q_L = \dfrac{\omega_0 L}{R_L}$

$$\therefore \; 20 = \frac{\omega_0 \times 200\times10^{-3}}{R_L}$$

$$\therefore \; \omega_0 = \frac{20 R_L}{200\times10^{-3}} = 100 R_L \rightarrow ③$$



$$(X_{L_p}) = \frac{R_L^2 + (X_L)^2}{X_L} \rightarrow ①$$ "as prooved in question 1"

$\because \; X_{L_p} = X_c$ (@ resonance)

$$\therefore \; X_{L_p} = \frac{1}{\omega_0 C} = \frac{1}{\omega_0 \times 0.01\times10^{-6}} \rightarrow ②$$

From 1 & 2

$$\therefore \; \frac{1}{\cancel{\omega_0}\times0.01\times10^{-6}} = \frac{(R_L)^2 + (\omega_0 L)^2}{\cancel{\omega_0} L}$$

using eq. 3

$$\therefore \; \frac{1}{0.01\times10^{-6}} = \frac{R_L^2 + (100R_L\times200\times10^{-3})^2}{200\times10^{-3}}$$

$$\therefore \; R_L^2 + 400R_L^2 = 2\times10^7$$

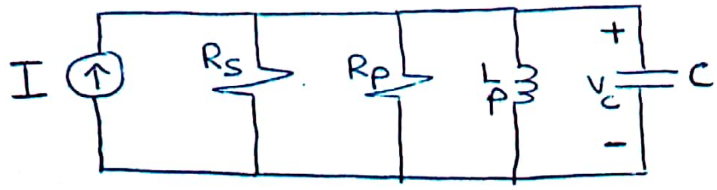
$$\therefore \; R_L^2(1+400) = 2\times10^7$$

$$\therefore \; R_L = 223.328\,\Omega$$

From eq. 3: $\omega_0 = 22332.8$ rad/Sec

$f_0 = 3554.376$ HZ

(b)



$$\because R_p = \frac{R_L^2 + (\omega_0 L)^2}{R_L}$$

$$\therefore R_p = \frac{(223.328)^2 + (22332.8 \times 200 \times 10^{-3})^2}{223.328}$$

$$R_p = 89554.528\ \Omega$$

$$R_S \,//\, R_p \Rightarrow R_{eq} = \frac{R_S R_p}{R_S + R_p} = 27649.988\ \Omega$$

@ resonance: $V_c = I R_{eq}$

$$\therefore |V_c| = |I| R_{eq}$$

$$= 5 \times 10^{-3} \times 27649.988$$

$$|V_c| = 138.25\ V$$

(c) $P = |I|^2 R_{eq}$

$$= (5 \times 10^{-3})^2 \times 27649.988$$

$$P = 0.69\ W$$

(d) $BW = \dfrac{1}{R_{eq} C} = \dfrac{1}{27649.988 \times 0.01 \times 10^{-6}} = 3616.638\ \text{rad/sec}$

8] For the network of Fig. 6:

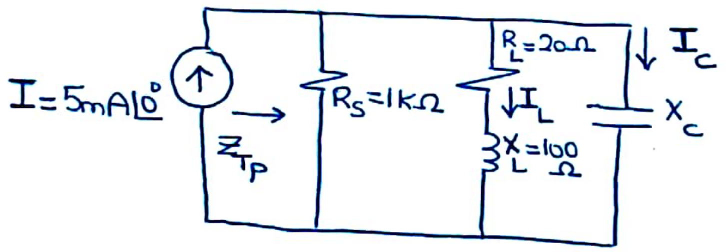
(a) Find the value of X_c at resonance

(b) Find the total impedance Z_{Tp} at resonance

(c) Find the currents I_L and I_C at resonance

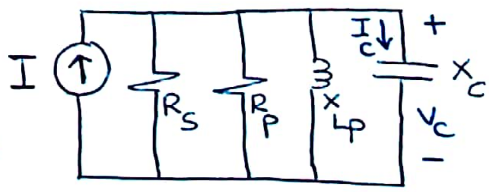
(d) If the resonant frequency is 20,000 HZ, find the value of L and C at resonance

(e) Find the BW.



Sol.

(a) @ resonance : $X_C = X_{Lp}$



$\because X_{Lp} = \dfrac{R_L^2 + (X_L)^2}{X_L}$ "as prooved in question 1"

$\therefore X_{Lp} = \dfrac{(20)^2 + (100)^2}{100} = 104\,\Omega.$

$\therefore X_C = 104\,\Omega$

(b) Z_{Tp} at resonance $\Rightarrow R_S // R_P$

$\because R_P = \dfrac{R_L^2 + X_L^2}{R_L}$

$\therefore R_P = \dfrac{(20)^2 + (100)^2}{20} = 520\,\Omega.$

$Z_{Tp} = R_{eq} = \dfrac{R_S R_P}{R_S + R_P} = \dfrac{1 \times 10^3 \times 520}{10^3 + 520} = 342.11\,\Omega$

(c) @ resonance: $V_c = I R_{eq}$

$\therefore V_c = 5\times10^{-3} \times 342.11$

$= 1.71055\ V.$

$\because I_c = \dfrac{V_c}{-jX_c} \Rightarrow \therefore I_c = \dfrac{1.71055}{-j\,104} \Rightarrow \therefore I_c = 0.016j\ A$

$\therefore I_L = \dfrac{V_c}{20+100j} = \dfrac{1.71055}{20+100j} = 0.0168 \angle -78.69\ A$

(d) $f_0 = 20{,}000 \Rightarrow \omega_0 = 2\pi f_0 = 125663.7061\ rad/Sec$

$\because X_c = \dfrac{1}{\omega_0 C} = 104$

$\therefore C = \dfrac{1}{104\,\omega_0} = 7.652\times10^{-8} F$

$\because X_L = \omega_0 L = 100$

$\therefore L = \dfrac{100}{\omega_0} = 7.958\times10^{-4}\ H$

(e) $BW = \dfrac{1}{R_{eq}C} = \dfrac{1}{(342.11\times 7.652\times10^{-8})}$

$= 38199.64\ rad/Sec$