Pitzer MATH194
Experimental Mathematics
Fall 2025

Alyssa Sawyer, HMC'26
Collaborators : N/A

Written (LaTeX) Assignment 0

Due Friday, September 12 on Canvas

Solutions/Proofs.

1. We will exhibit and verify a homeomorphism between $S^1 - \{(-1, 0)\}$ and $\mathbb{R}$.

   a. Our homeomorphism is as follows: $f(x) = \frac{y}{x+1}$.

   b. This is a homeomorphism. We will show that by first showing it is continuous and then proving that it has a continuous inverse.

   i. It is clearly continuous since $x \neq -1$ is in $S^1 - \{(-1, 0)\}$ and that is the only possible value that could cause the function to not be continuous.

   ii. Next, we can solve for the inverse by switching the variables and solving for y, which gives us $f^{-1}(x) = (-\frac{1-\sqrt{1+4x}}{2}, -\frac{1-\sqrt{1+4x}}{2})$. This function is also continuous, as desired.

2. Now, we will prove that $\{(x, y, z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1\} - \{(0, 0, -1)\}$ is homeomorphic to R^2.

3. §2 #4. Let $A \subset X$; suppose $r\colon X \to A$ is a continuous map such that $r(a) = a$ for each $a \in A$. (The map r is called a *retraction* of X onto A.) If $a_0 \in A$, show that $r_*\colon \pi_1(X, a_0) \to \pi_1(A, a_0)$ is surjective.