Opt HW2

Alyson Brown

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R Markdown

Q1

Part A

Let x_1 = the number of tortes that Max eats

Let x_2 = the number of pies that Max eats

MAX $4x_1 + 5x_2$

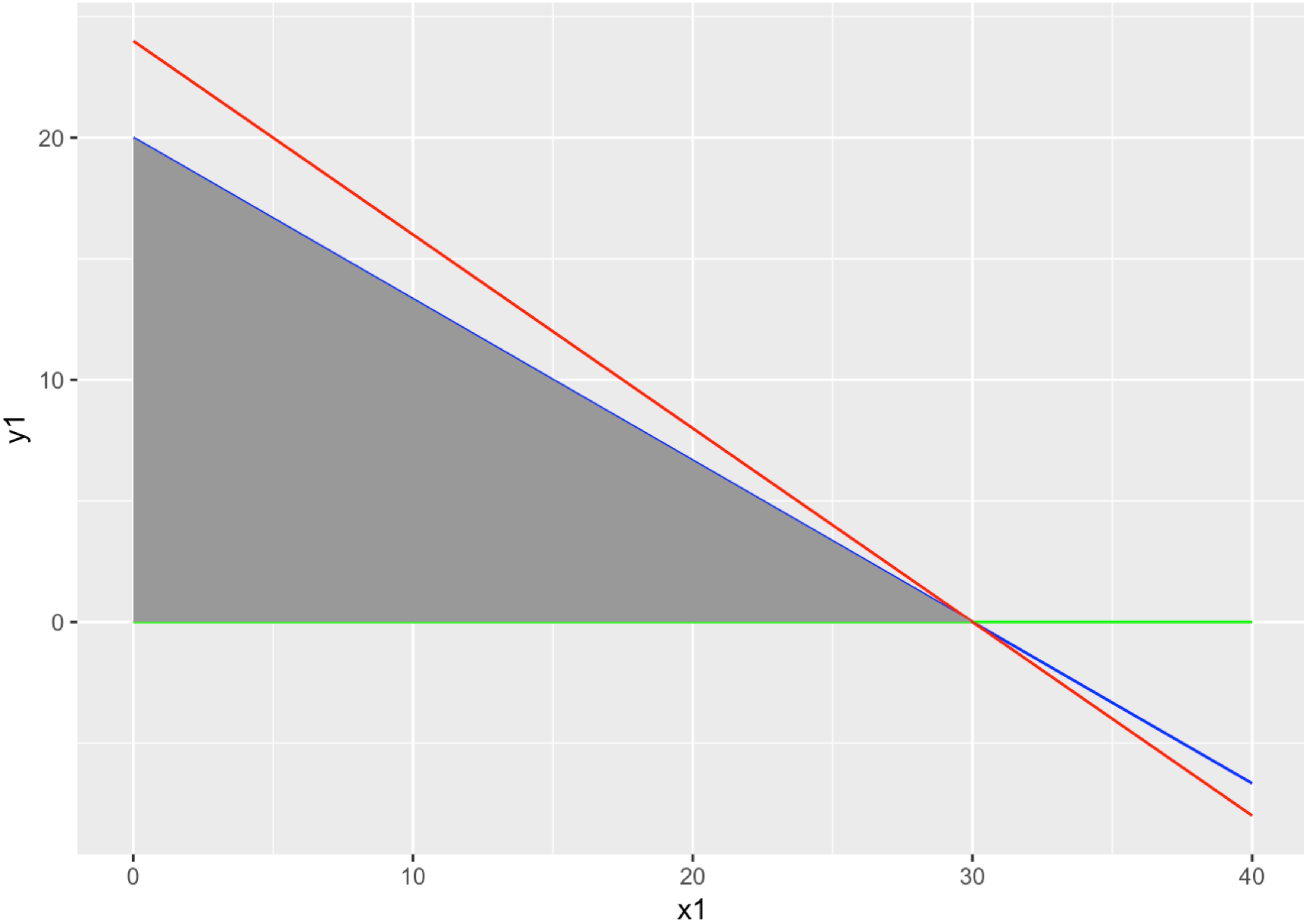
ST:

$2x_1 + 3x_2 <= 60$

$x_1, x_2 >= 0$

Solve using the graphical method:

(Red line is the objectove function.)



You can see from the above graph that the optimal solution is the point [30,0], meaning that Max should eat 30 tortes and 0 pies for a total of 120 points. This conclusion is confirmed in the code below:

```
library('lpSolve')
#objective coefficients
c<-c(4,5)

#LHS of constraints
# define a 4x2 matrixn of zeros
A<-matrix(0,3,2)

# minutes constraint
A[1,]<-c(2,3)
# Non-negativity constraint
A[2,]<-c(1,0)
#Non-negativity constraints
A[3,]<-c(0,1)

#RHS of constraints
b<-c(60,0,0)

#All constraints have a <=
#dir<-rep("<=",4)
dir<-c("<=",">=",">=")
#solve the LP and assign the returned strcture to variable s
s=lp("max",c,A,dir,b,compute.sens = 1)
```

```
## Status: 0
```

```
## Solution: 30 0
```

```
## Points: 120
```

Part B

Let x_1 = the number of tortes that Max eats

Let x_2 = the number of pies that Max eats

MAX $4x_1 + 5x_2$

ST:

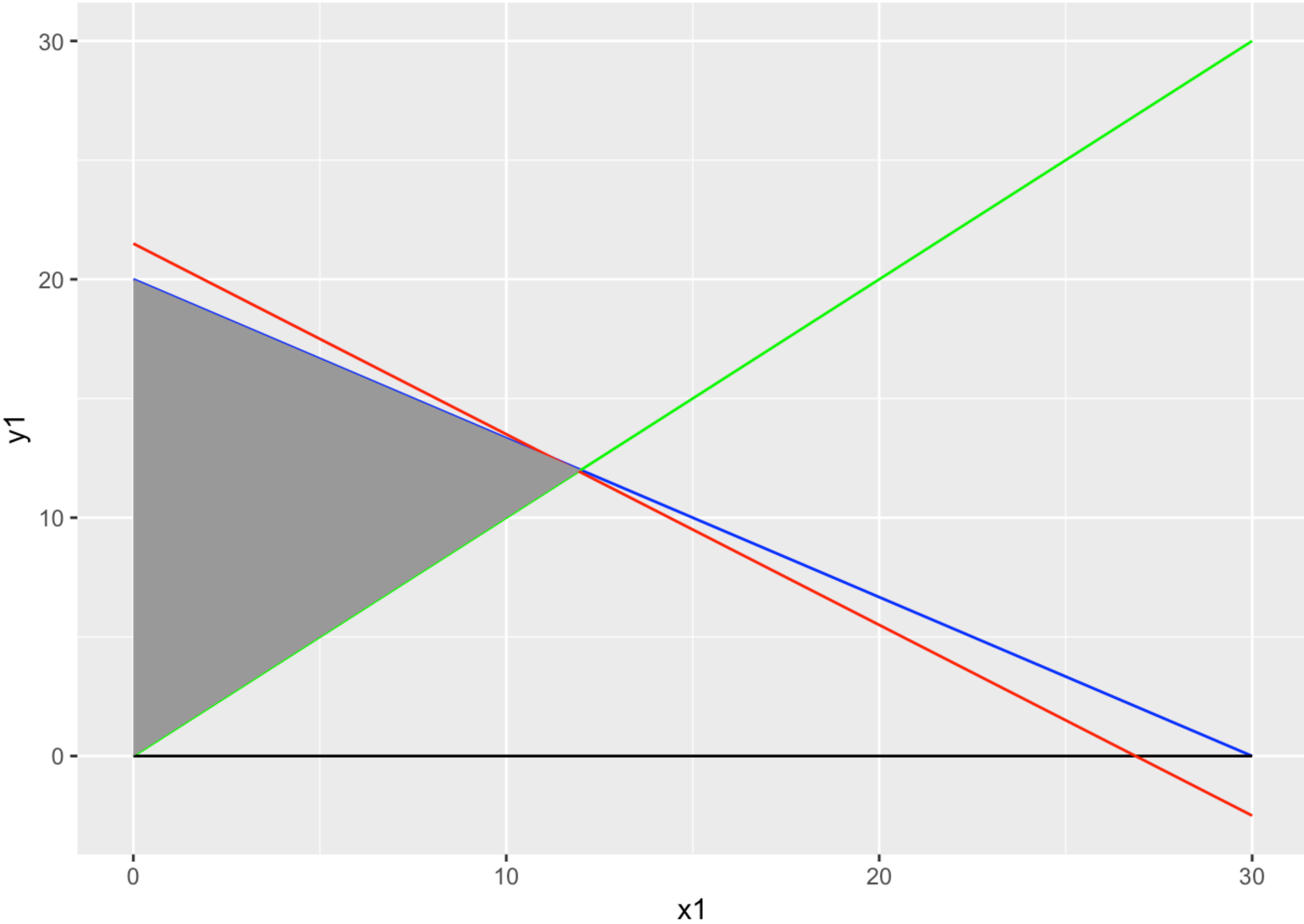
$2x_1 + 3x_2 <= 60$

$x_1 - x_2 <= 0$

$x_1, x_2 >= 0$

Solve using the graphical method:

(Red line is the objectove function.)



You can see in the above graph that the new optimal solution is [12,12], meaning that Max should eat 12 tortes and 12 pies for a total of 108 points, decreasing his total points by 12 from the previous scenario. This solution is confirmed in the code below:

```
c<-c(4,5)
# define a 4x2 matrixn of zeros
A<-matrix(0,4,2)

# minutes constraint
A[1,]<-c(2,3)
# minutes constraint
A[2,]<-c(1,-1)
# Non-negativity constraint
A[3,]<-c(1,0)
#Non-negativity constraints
A[4,]<-c(0,1)

#RHS of constraints
b<-c(60,0,0,0)

#All constraints have a <=
#dir<-rep("<=",4)
dir<-c("<=","<=",">=",">=")
#solve the LP and assign the returned strcture to variable s
s=lp("max",c,A,dir,b,compute.sens = 1)
```

```
## Status: 0
```

```
## Solution: 12 12
```

```
## Points: 108
```

Q2A

Let x_1 = the number of acres of wheat

Let x_2 = the number of acres of corn

MAX $2000x_1 + 3000x_2$

ST:

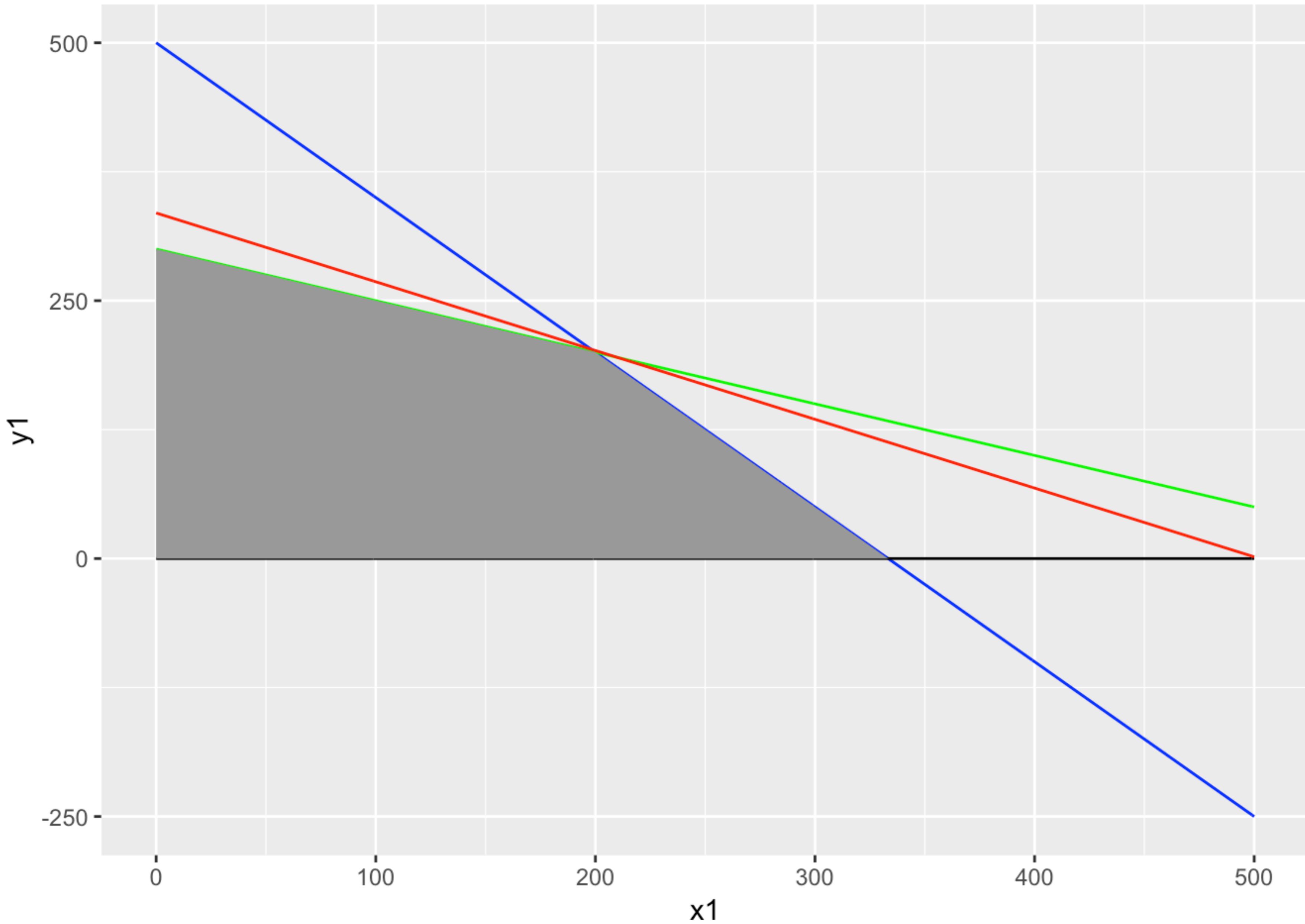
$3x_1 + 2x_2 <= 1000$

$2x_1 + 4x_2 <= 1200$

$x_1, x_2 >= 0$

Solve using the graphical method:

(Red line is the objectove function.)



You can see in the above graph that the new optimal solution is [200,200], meaning that the farmer should plant 200 acres of wheat and 200 acres of corn for a total profit of $1 million. This solution is confirmed in the code below:

Q2B

```
c<-c(2000,3000)
# define a 4x2 matrixn of zeros
A<-matrix(0,4,2)

# minutes constraint
A[1,]<-c(3,2)
# minutes constraint
A[2,]<-c(2,4)
# Non-negativity constraint
A[3,]<-c(1,0)
#Non-negativity constraints
A[4,]<-c(0,1)

#RHS of constraints
b<-c(1000,1200,0,0)

#All constraints have a <=
#dir<-rep("<=",4)
dir<-c("<=","<=",">=",">=")
#solve the LP and assign the returned strcture to variable s
s=lp("max",c,A,dir,b,compute.sens = 1)
```

```
## Status: 0
```

```
## Solution: 200 200
```

```
## Profit: 1e+06
```

Q2C

As seen from the output below, the farmer would not produce wheat when the amount of fertilizer available >=2000 tons, and would not plant corn when the amount of fertilizer <= 600 tons.

```r
c<-c(2000,3000)
# define a 4x2 matrixn of zeros
A<-matrix(0,4,2)

# minutes constraint
A[1,]<-c(3,2)
# minutes constraint
A[2,]<-c(2,4)
# Non-negativity constraint
A[3,]<-c(1,0)
#Non-negativity constraints
A[4,]<-c(0,1)

dir<-c("<=","<=",">=",">=")
fert = c(200,300,400,500,600,700,800,900,1000,1100,1200,1300,1400,1500,1600,1700,1800
,1900,2000,2100, 2200)

for (i in fert) {
  b<-c(1000,i,0,0)
  s=lp("max",c,A,dir,b,compute.sens = 1)
  print(i)
  cat("Solution:",s$solution,"\n")
  cat("Profit:",s$objval,"\n")
}
```

```
## [1] 200
## Solution: 100 0
## Profit: 2e+05
## [1] 300
## Solution: 150 0
## Profit: 3e+05
## [1] 400
## Solution: 200 0
## Profit: 4e+05
## [1] 500
## Solution: 250 0
## Profit: 5e+05
## [1] 600
## Solution: 300 0
## Profit: 6e+05
## [1] 700
## Solution: 325 12.5
## Profit: 687500
## [1] 800
## Solution: 300 50
## Profit: 750000
## [1] 900
## Solution: 275 87.5
## Profit: 812500
```

```
## [1] 1000
## Solution: 250 125
## Profit: 875000
## [1] 1100
## Solution: 225 162.5
## Profit: 937500
## [1] 1200
## Solution: 200 200
## Profit: 1e+06
## [1] 1300
## Solution: 175 237.5
## Profit: 1062500
## [1] 1400
## Solution: 150 275
## Profit: 1125000
## [1] 1500
## Solution: 125 312.5
## Profit: 1187500
## [1] 1600
## Solution: 100 350
## Profit: 1250000
## [1] 1700
## Solution: 75 387.5
## Profit: 1312500
## [1] 1800
## Solution: 50 425
## Profit: 1375000
## [1] 1900
## Solution: 25 462.5
## Profit: 1437500
## [1] 2000
## Solution: 0 500
## Profit: 1500000
## [1] 2100
## Solution: 0 500
## Profit: 1500000
## [1] 2200
## Solution: 0 500
## Profit: 1500000
```

Q3

(3) Let x_{ij} = amt of investment in i @ time j

$$\text{Max NPV} = \sum_{j=0}^{1} 13x_{1j} + 16x_{2j} + 16x_{3j} + 14x_{4j} + 39x_{5j}$$

ST: $11x_{10} + 53x_{20} + 5x_{30} + 5x_{40} + 29x_{50} \leq 40$

$3x_{11} + 6x_{21} + 5x_{31} + x_{41} + 34x_{51} \leq 20$

$x_{ij} \geq 0$

$x_{ij} \leq 1$

Q4

Let x_1 = the number of servings of corn consumed

Let x_2 = the number of servings of milk consumed

Let x_3 = the number of servings of bread consumed

MIN $0.18x_1 + 0.23x_2 + 0.05x_3$

ST:

$72x_1 + 121x_2 + 65x_3 >= 2000$

$72x_1 + 121x_2 + 65x_3 <= 2500$

$107x_1 + 500x_2 + 0x_3 >= 5000$

$107x_1 + 500x_2 + 0x_3 <= 50,000$

$x_1, x_2, x_3 >= 0$

$x_1, x_2, x_3 <= 10$

```
c<-c(0.18,0.23, 0.05)
# define a 4x2 matrixn of zeros
A<-matrix(0,10,3)

# minutes constraint
A[1,]<-c(72, 121, 65)
A[2,]<-c(72, 121, 65)
A[3,]<-c(107, 500, 0)
A[4,]<-c(107, 500, 0)
A[5,]<-c(1, 0, 0)
A[6,]<-c(1, 0, 0)
A[7,]<-c(0, 1, 0)
A[8,]<-c(0, 1, 0)
A[9,]<-c(0, 0, 1)
A[10,]<-c(0, 0, 1)

#RHS of constraints
b<-c(2000,25000,5000,50000,10,0,10,0,10,0)

#All constraints have a <=
dir<-c(">=","<=",">=","<=","<=",">=","<=",">=","<=",">=")
s=lp("min",c,A,dir,b,compute.sens = 1)
```

```
## Status: 0
```

```
## Solution: 1.944444 10 10
```

```
## Total cost: 3.15
```

Based on the solution above, you should consume 1.95 servings of corn, 10 servings of milk, and 10 servings of bread.

Q4

Let x_1 = the number of acres harvested in year 1 from forest 1

Let x_2 = the number of acres harvested in year 2 from forest 1

Let x_3 = the number of acres harvested in year 3 from forest 1

Let y_1 = the number of acres harvested in year 1 from forest 2

Let y_2 = the number of acres harvested in year 2 from forest 2

Let y_3 = the number of acres harvested in year 3 from forest 2

MAX $x_1 + 1.3x_2 + 1.4x_3 + y_1 + 1.2y_2 + 1.6y_3$

ST:

$x_1 + x_2 + x_3 <= 2$

$y_1 + y_2 + y_3 <= 3$

$x_1 + y_1 >= 1.2$

$x_1 + y_1 <= 2$

$x_2 + y_2 >= 1.5$

$x_2 + y_2 <= 2$

$x_3 + y_3 >= 2$

$x_3 + y_3 <= 3$

$x_1, x_2, x_3, y_1, y_2, y_3 >= 0$

```
c<-c(1,1.3,1.4, 1, 1.2, 1.6)
# define a 4x2 matrixn of zeros
A<-matrix(0,14,6)

# minutes constraint
A[1,]<-c(1,1,1,0,0,0)
A[2,]<-c(0,0,0,1,1,1)
A[3,]<-c(1,1,0,0,0,0)
A[4,]<-c(1,1,0,0,0,0)
A[5,]<-c(0,0,1,1,0,0)
A[6,]<-c(0,0,1,1,0,0)
A[7,]<-c(0,0,0,0,1,1)
A[8,]<-c(0,0,0,0,1,1)
A[9,]<-c(1,0,0,0,0,0)
A[10,]<-c(0,1,0,0,0,0)
A[11,]<-c(0,0,1,0,0,0)
A[12,]<-c(0,0,0,1,0,0)
A[13,]<-c(0,0,0,0,1,0)
A[14,]<-c(0,0,0,0,0,1)

#RHS of constraints
b<-c(2,3,1.2,2,1.5,2,2,3,0,0,0,0,0,0)

#All constraints have a <=
dir<-c("<=","<=",">=","<=",">=","<=",">=","<=",">=",">=",">=",">=",">=",">=")
s=lp("max",c,A,dir,b,compute.sens = 1)
```

```
## Status: 0
```

```
## Solution: 0 1.2 0.8 0.7 0 2.3
```

```
## Total weight (tons): 7.06
```

Based on the solution above, the cutting schedule is to harvest 0, 1.2, & 0.8 acres from forest 1 in years 1, 2, & 3, and to harvest 0.7, 0.3, & 2 acres, respectively, from forest 2 in years 1, 2, & 3.

Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.