Perturbation Theory and Mixture Models:
Application to Particle Physics

ACAS

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Outline

- Review of score + formula for asymptotic distribution – do we need additional parameters to describe the data?

- Applications to particle physics
  - Ex: search for new particle resonances
  - Ex: energy spectrum of highest energy cosmic rays

Mixture models, score statistic and its asymptotic distribution

- Mixture models:

$$p(x; \tilde{n}; \grave{o}) = (1 - \tilde{n})f(x) + \tilde{n}(x; \grave{o})$$

- Score statistic

$$S^?(x; \grave{o}) := \frac{S(x; \grave{o})}{\sqrt{n\, C(\grave{o}; \grave{o}^0)}}$$

$$= \frac{1}{\sqrt{n\, C(\grave{o}; \grave{o}^0)}} \; \sum_{i=1}^{n} \frac{(x_i; \grave{o})}{f(x_i)} - 1$$

- where $\quad C(\grave{o}; \grave{o}^0) = \int \frac{f(x; \grave{o})g^2}{f(x)}\, dx - 1$

Asymptotic distribution

$$\Pr\left\{\sup_{\grave{o}\in\hat{E}} Z(\grave{o}) \geq c\right\}$$

$$\approx \frac{\hat{o}_0}{A_{d+1}}\Pr(\chi^2_{d+1} \geq c^2) + \frac{I_0}{2A_d}\Pr(\chi^2_{d} \geq c^2)$$

- $\bar{o}_0$ described by d-dimensional volume of manifold expressible through covariance function
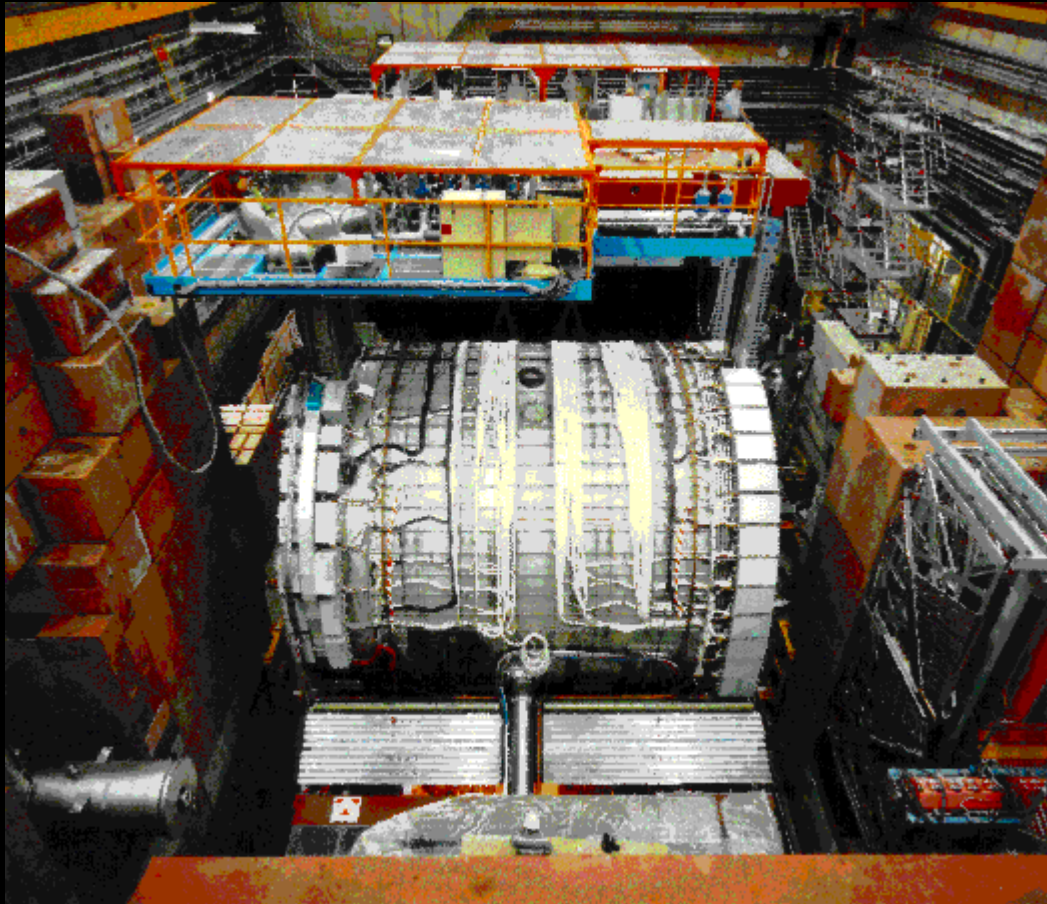
Ex: search for new resonances

- What are physicists searching for?

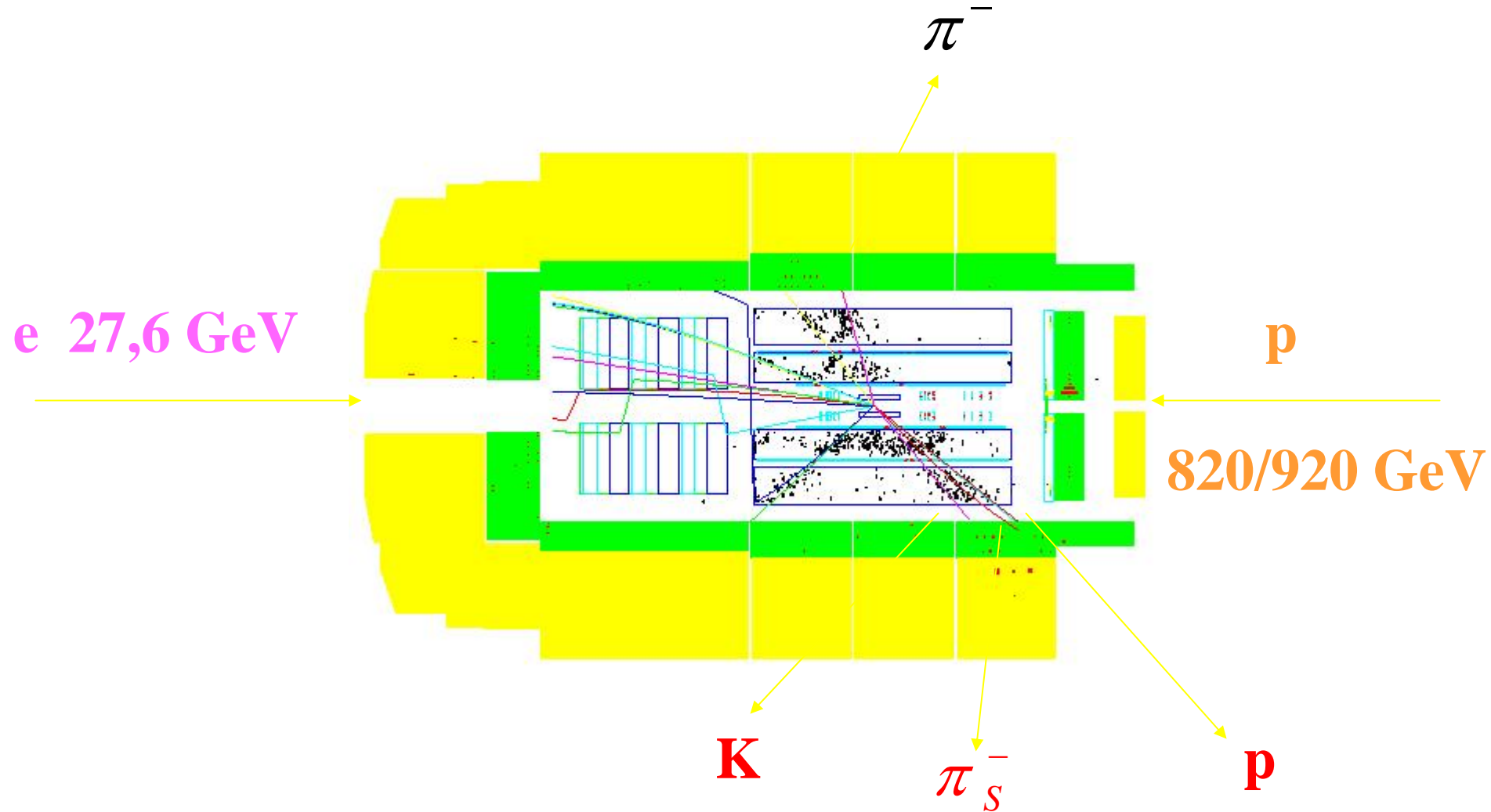- Why are we searching for it?

- How do we search for it?

Ex: Pentaquarks

- QCD – nobel prize
- Q q-bar, 3 q states
- Pentaquark discovered
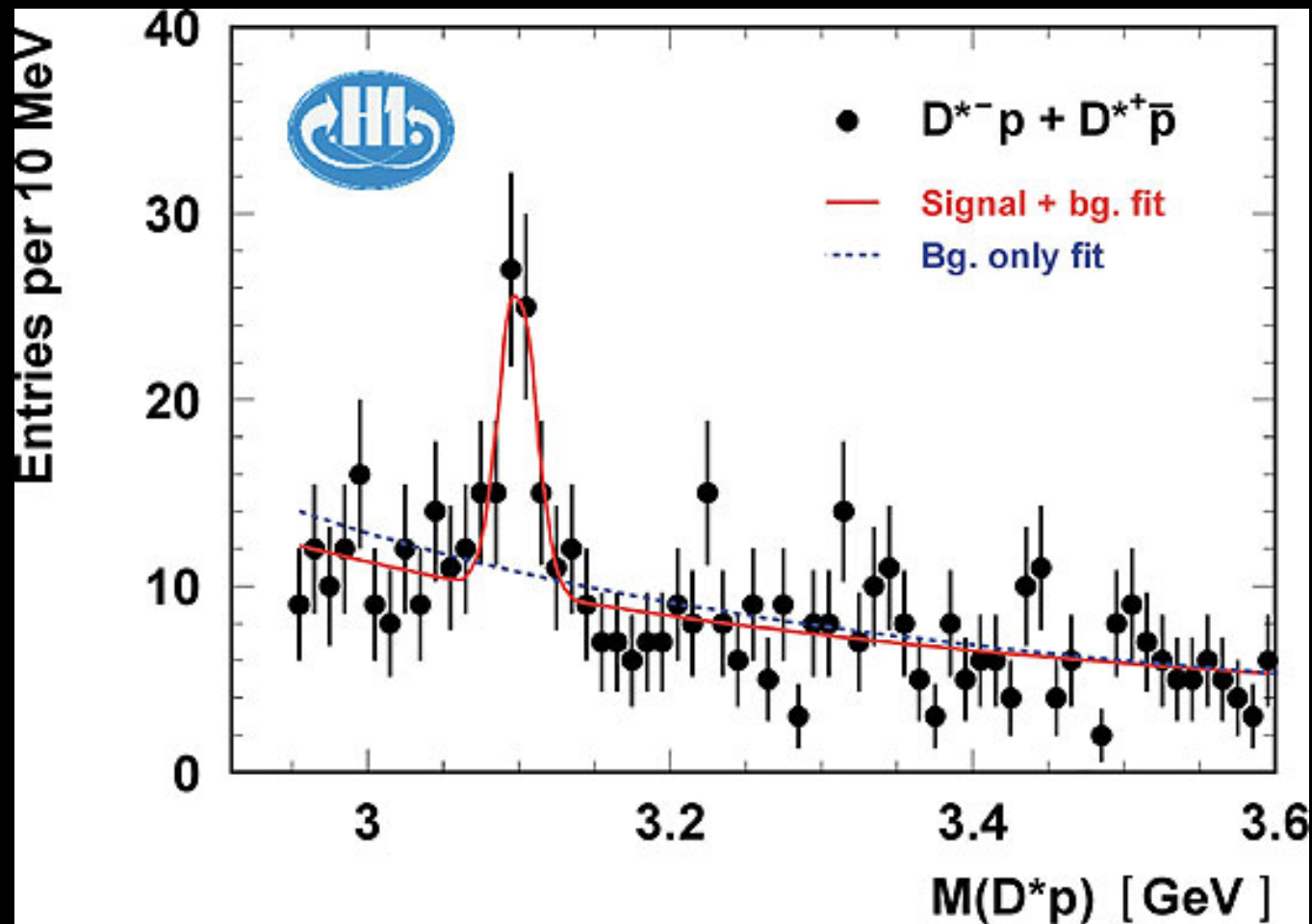- Charmed pentaquarks
- H1 claims discovery; Zeus doesn't see it

H1 detector
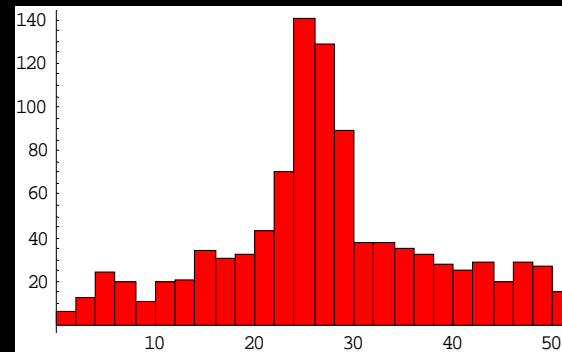
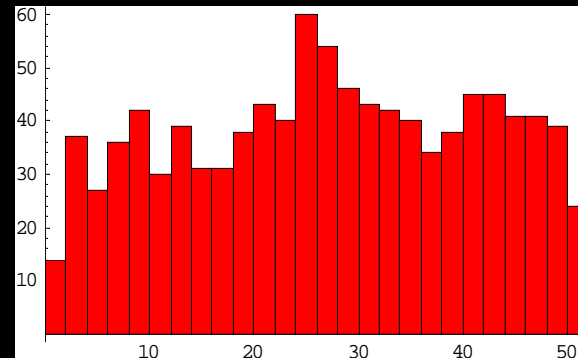# Pentaquark in H1 setup

Ex: pentaquarks

Mixture models in particle physics

- Background : power law
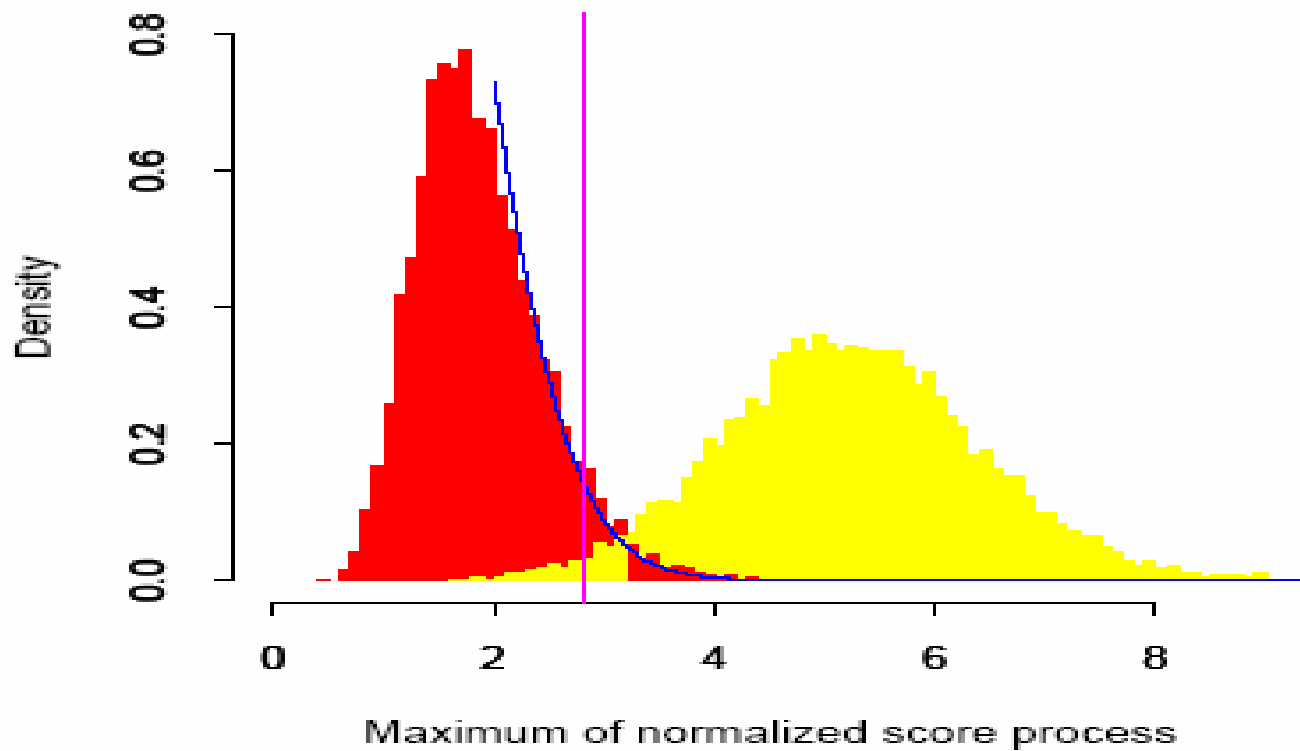
- Perturbation (resonance): Breit-Wigner (Cauchy):

$$(E\,;E_0) = \{\grave{E}/(2\grave{u})\}\{(E\,\grave{a}\,E_0)^2 + (\grave{E}/2)^2\}^{\grave{a}1}$$
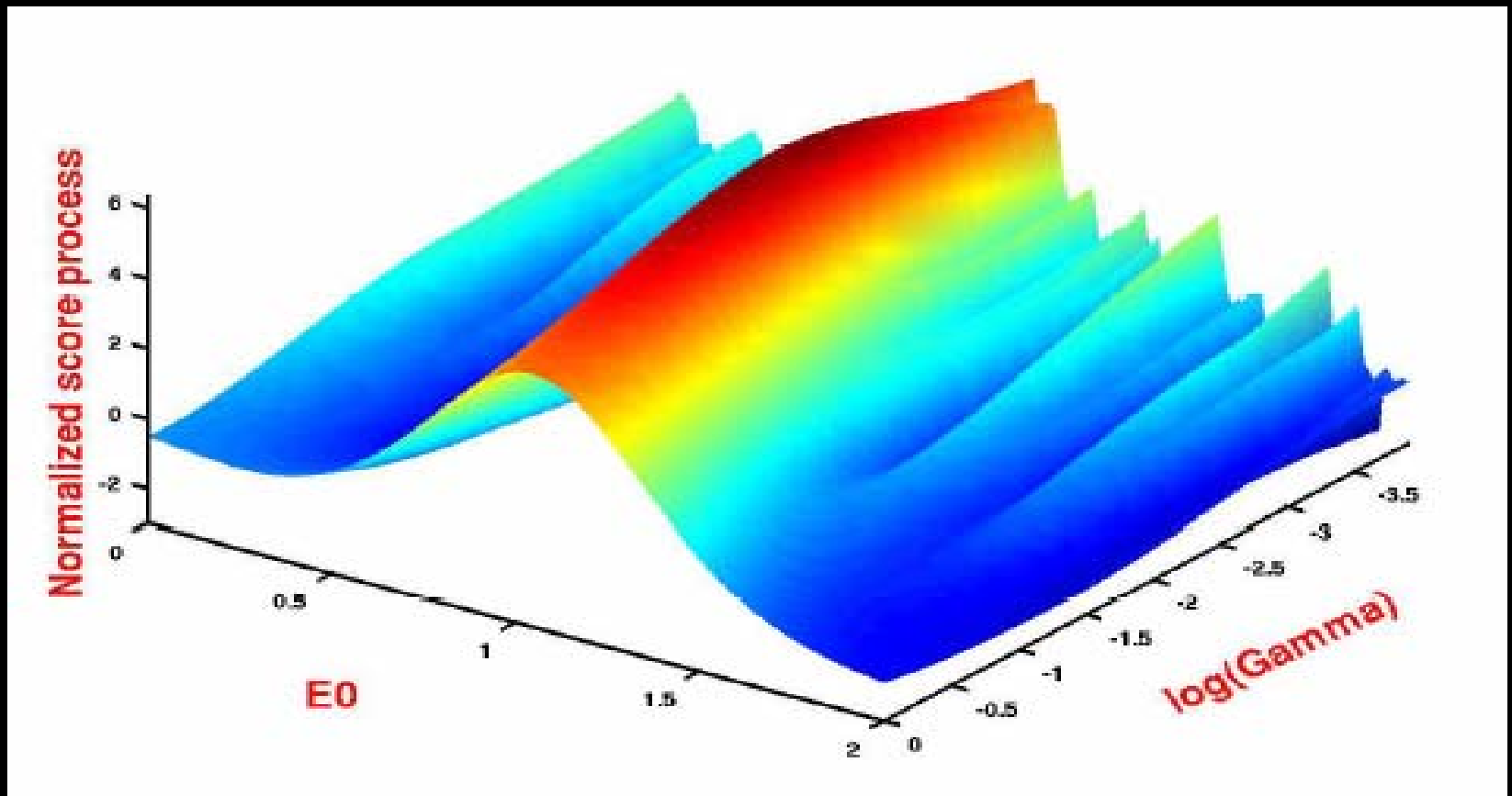
Application of score analysis (MC)
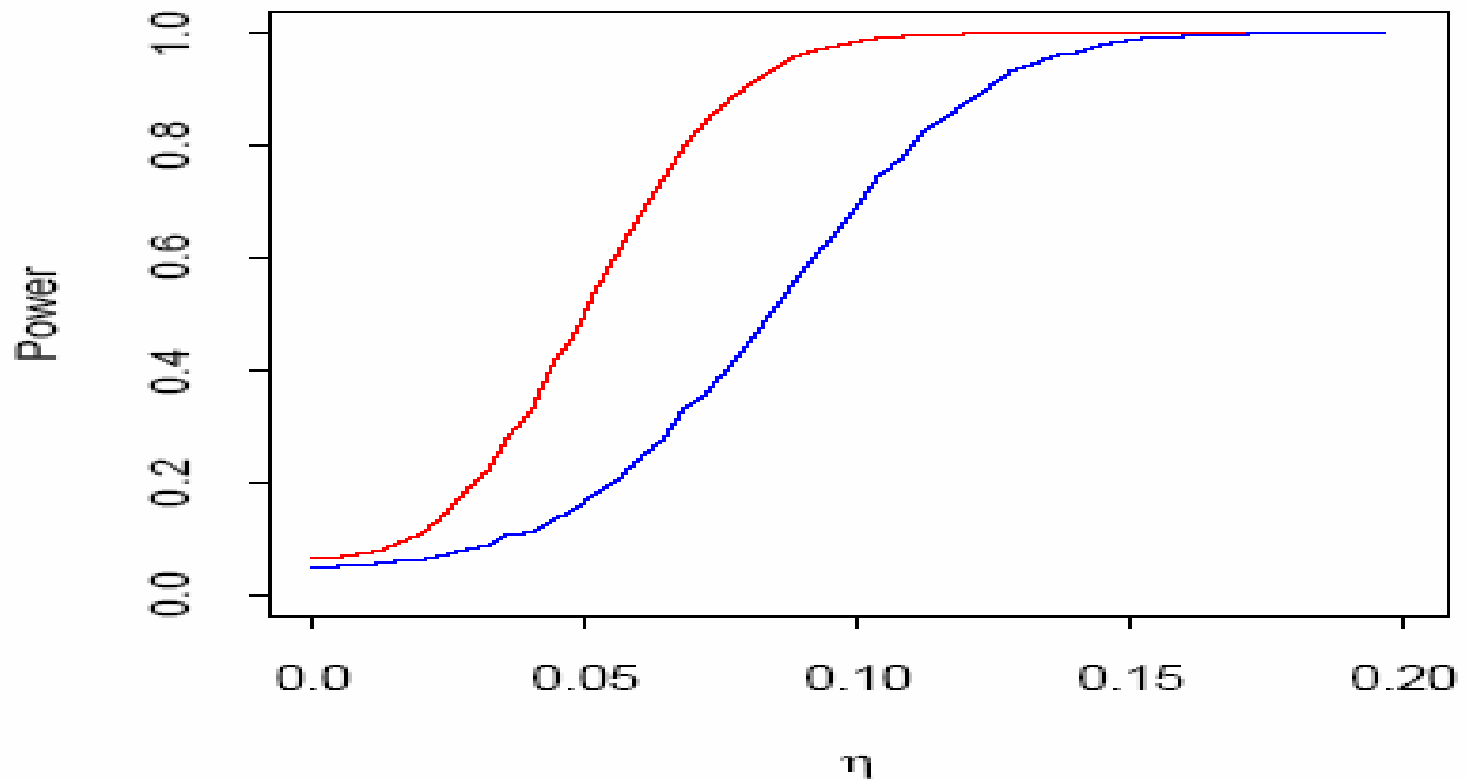
- 10% mixture

- 50% mixture

- Model parameters:

T: ñ = 0:1

Surface of normalized score process

Power of $\ddot{y}^2$ and normalized score
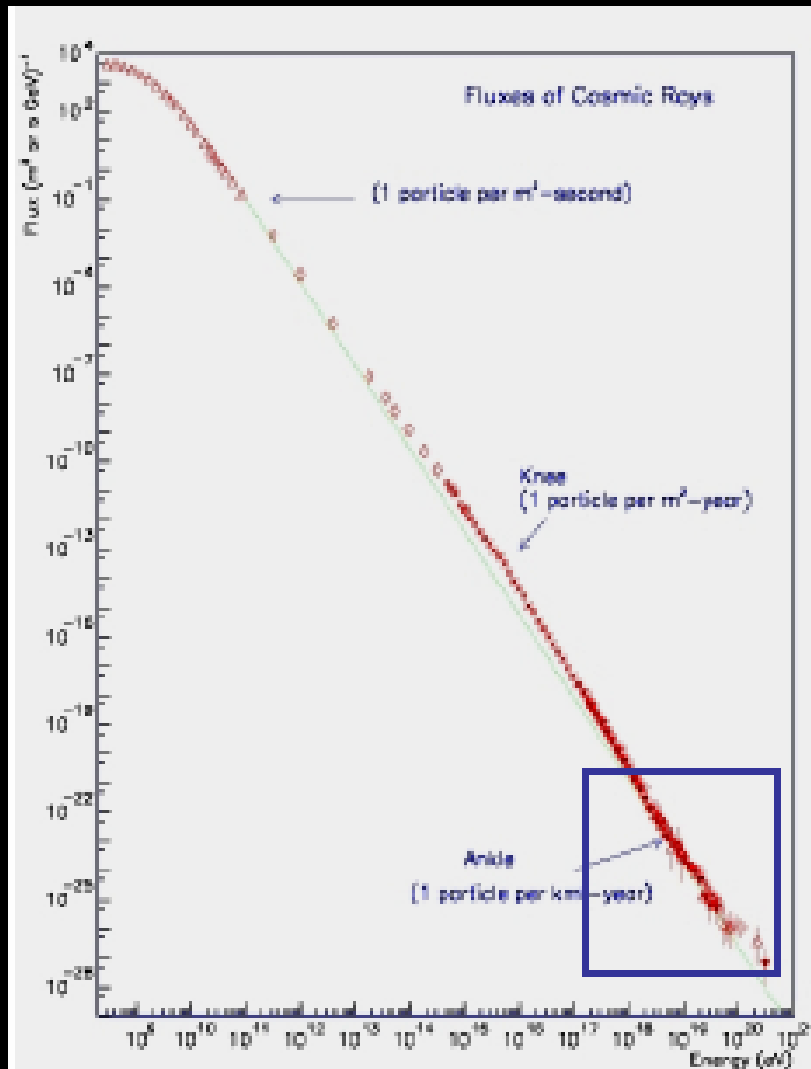
Next: LHC

- Search for Higgs
- Search for SUSY
- Search for the unexpected

Cosmic Rays

- spectrum

AGASA
Akeno Giant Air Shower Array

- **Detectors**
  - **111 surface detectors (2.2m^2)**
    - 5cm thinck scintillator
    - Optical fibre cable to observatory
      - Delay time monitored @100ps accuracy
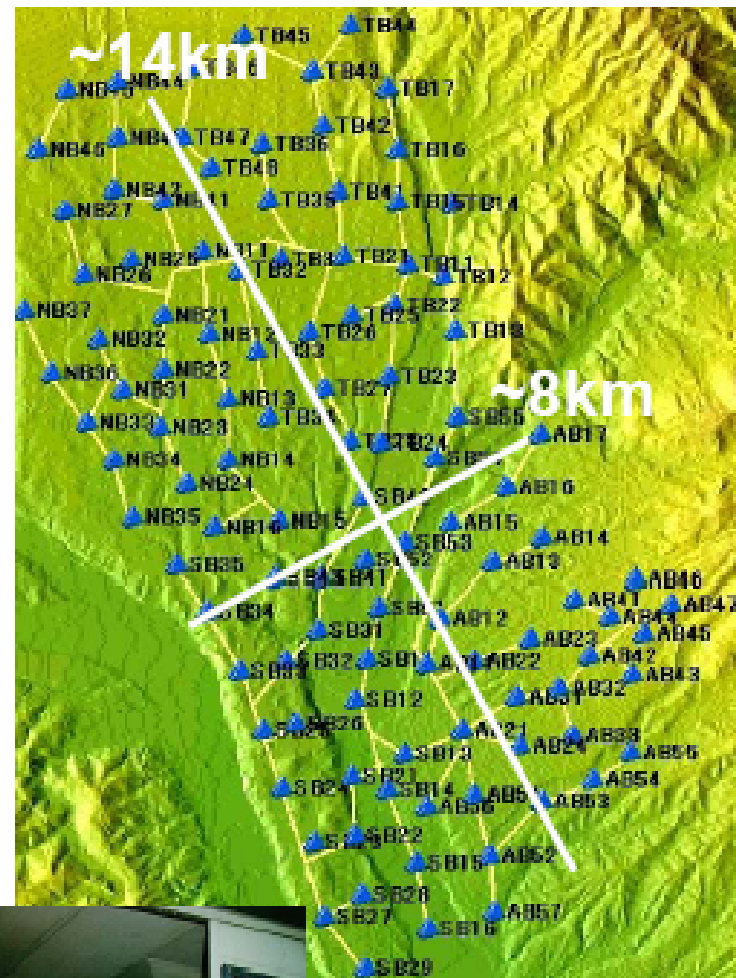   - Location suveyed: $\Delta x,y$=0.1m; Δz=0.3m
  - **27 muon detectors (2.8–10m^2)**
    - Fe / concrete absorber +proportional counters
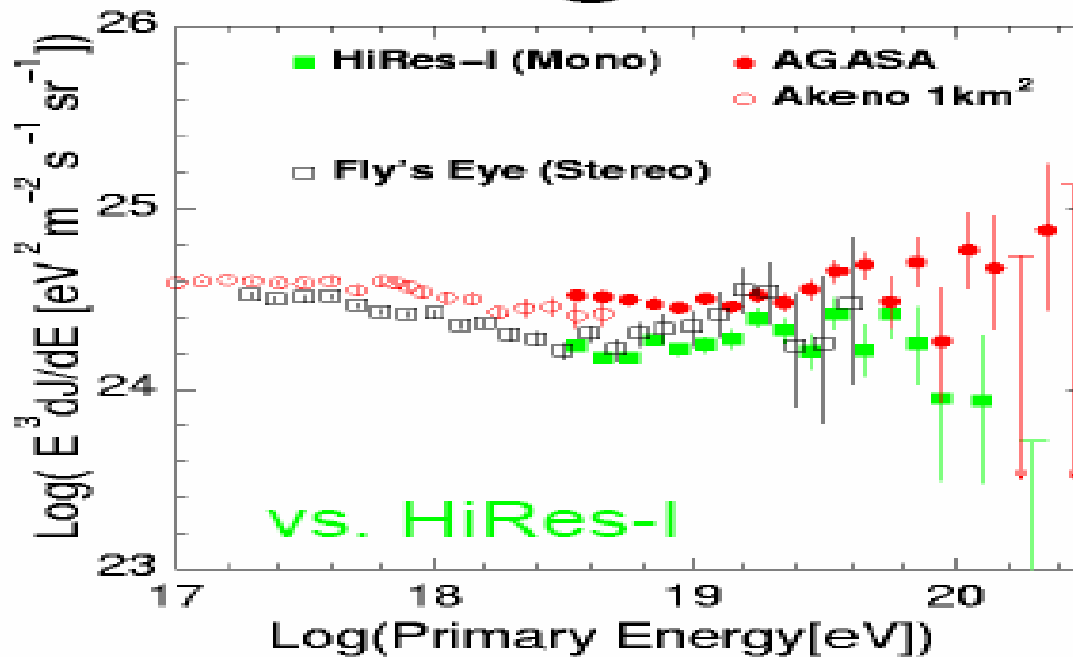   - E_{th}>0.5GeV

- **Operation**
  - Started in February 1990 up to now ~95% live ratio
  - **We will shut down at the end of this year...;_;**

High End of spectrum

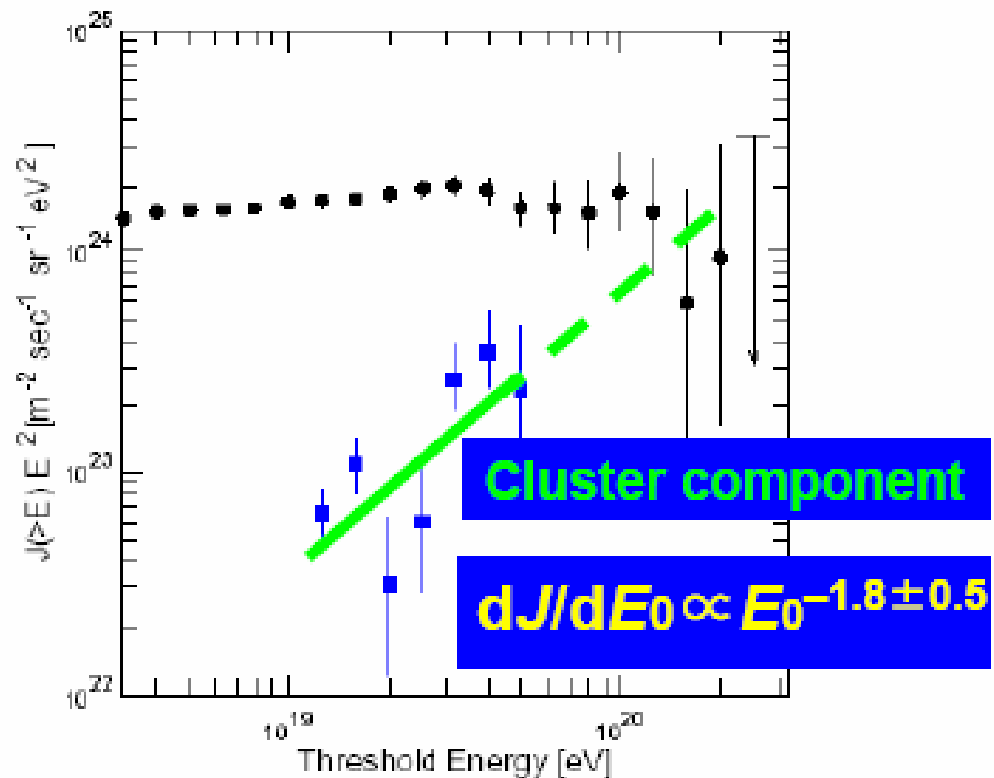**Recent spectra**
(AGASA vs. HiRes@Tsukuba ICRC)

- ~2.5 sigma discrepancy between AGASA & HiRes

Greissen cutoff

- Shouldn't be any very high energy cosmic rays – interactions with microwave background radiation

- Where are they coming from? New sources?

- Auger project: $50 million air shower array

Auger…New component?

- ## Integral EHECR spectrum
  (Ordinary EHECR vs. cluster comp.)

Cluster component

$$dJ/dE_0 \propto E_0^{-1.8\pm0.5}$$

Score sensitivity for power-law mix

$$p_{\tilde{o}}(x) = \frac{1}{B_{\tilde{o}}} x^{à\tilde{o}}$$

$$B_{\tilde{o}} = \int_a^b x^{à\tilde{o}}\, dx$$

$$H_0 = p_{\ddot{e}}(x)$$

$$H_1 = (1 à \tilde{n})\, p_{\ddot{e}}(x) + \tilde{n}\, p_{\grave{\imath}}(x); \quad \grave{\imath} < \ddot{e}$$

- Ex: slightly softer (~2.7 added in at 20% level); cut off at region where disagreement begins

Sensitivity $\tilde{n} = 0{:}2; N = 1000$
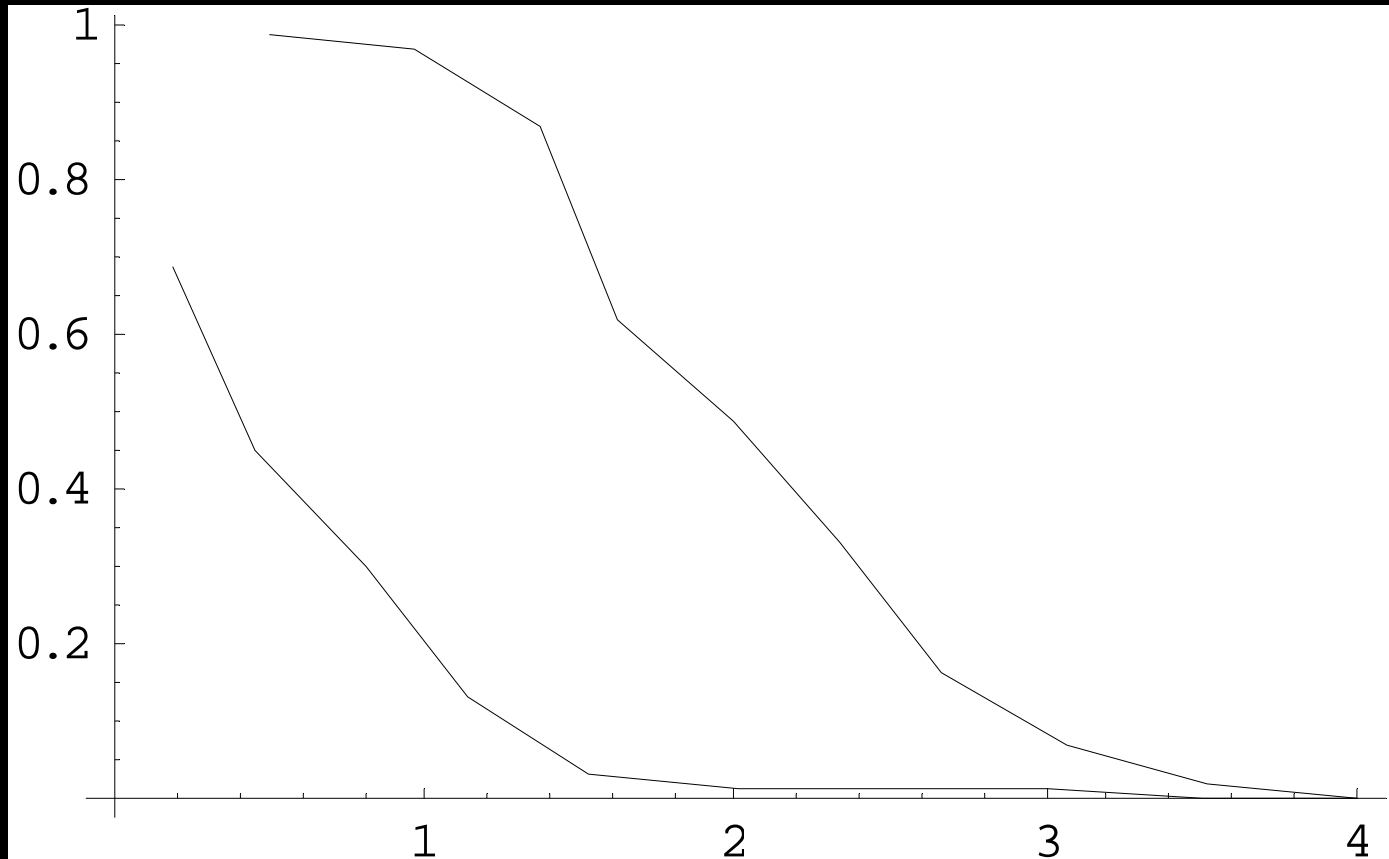
Conclusions

- Score test statistic + asymptotic distribution represents a powerful new tool for the search for new physics in high energy physics and particle astrophysics