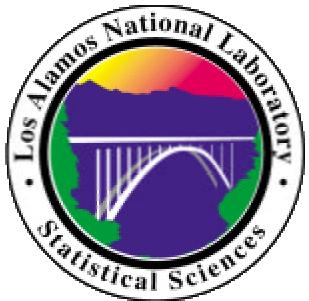


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# ESTIMATION, PLANNING, AND DECISION MAKING



# INTRODUCTION

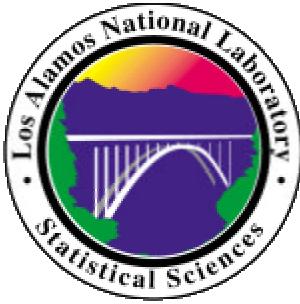
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## Purpose

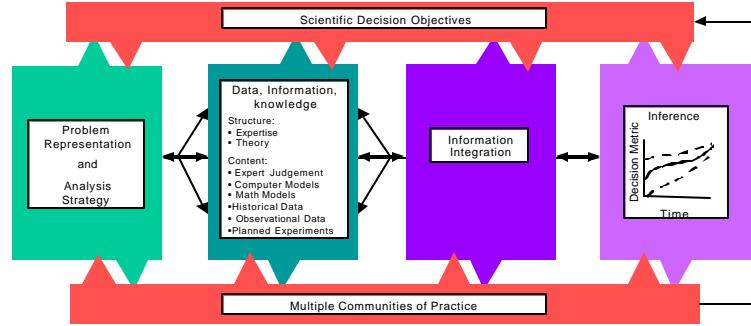
Introduce concepts of decision theory into the IIT Framework

## Overview

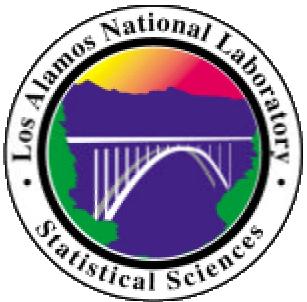
- Review where we are in the IIT process
- Set up the decision theory context
- Make a decision



# WHERE ARE WE?

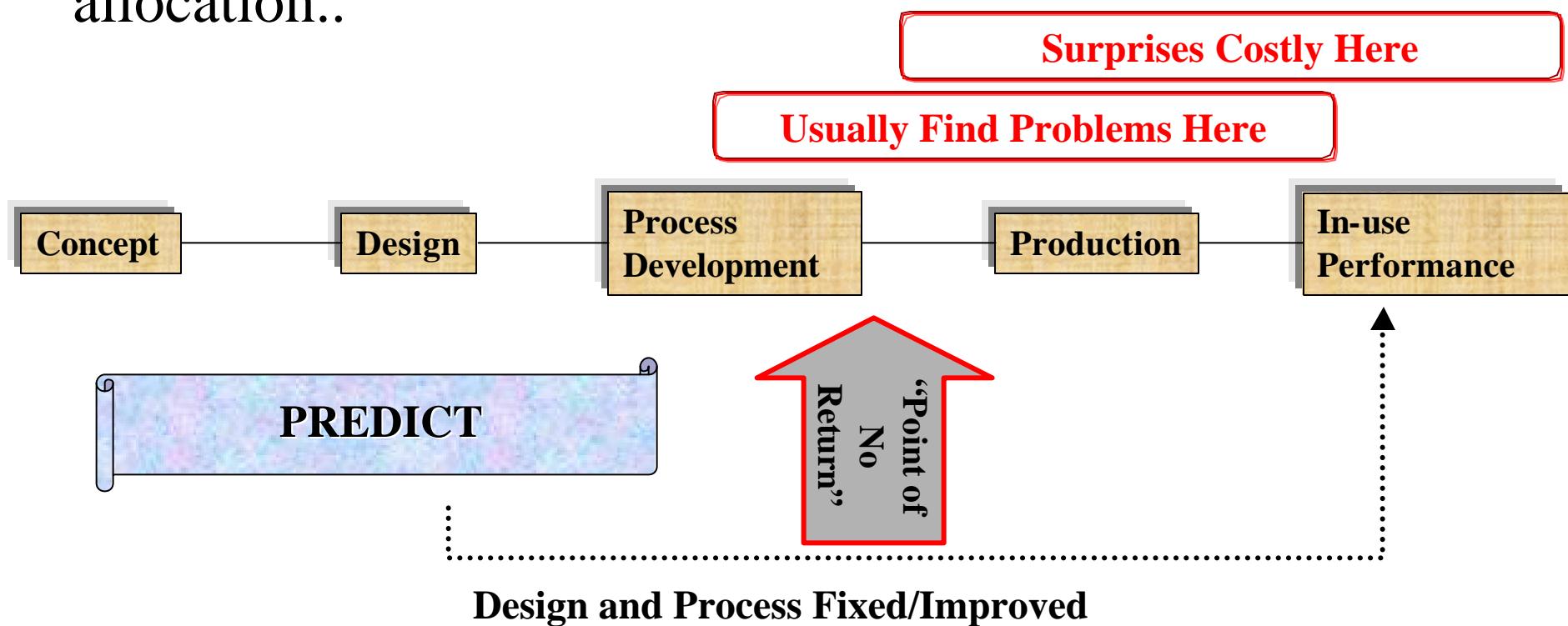


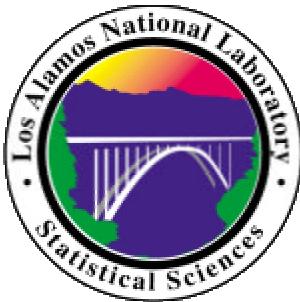
- We have learned how to build an explicit system representation based upon multiple points-of-view
- We have learned how to develop and estimate metrics based on diverse information
- We have learned how to propagate information through the system and address “what-if” questions
- We have learned the need for, and the development of, knowledge systems to keep track and evolve the problem-solving process



# DELPHI: SHORTEN THE LEAD TIME

**CLAIM:** IIT avoids costly “surprises,” saving time and money. It allows better planning and resource allocation..

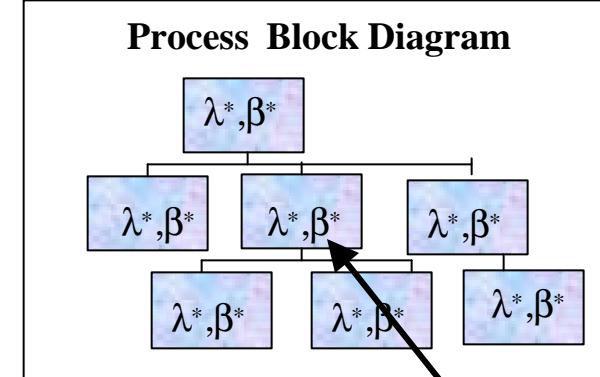
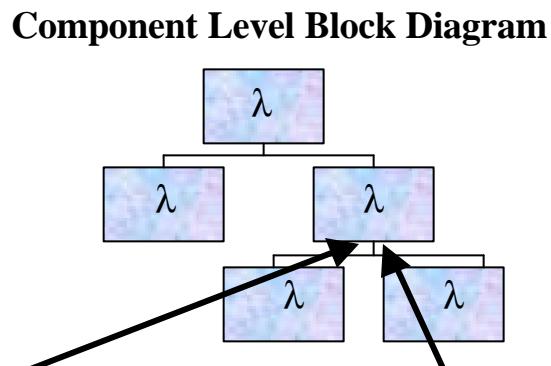
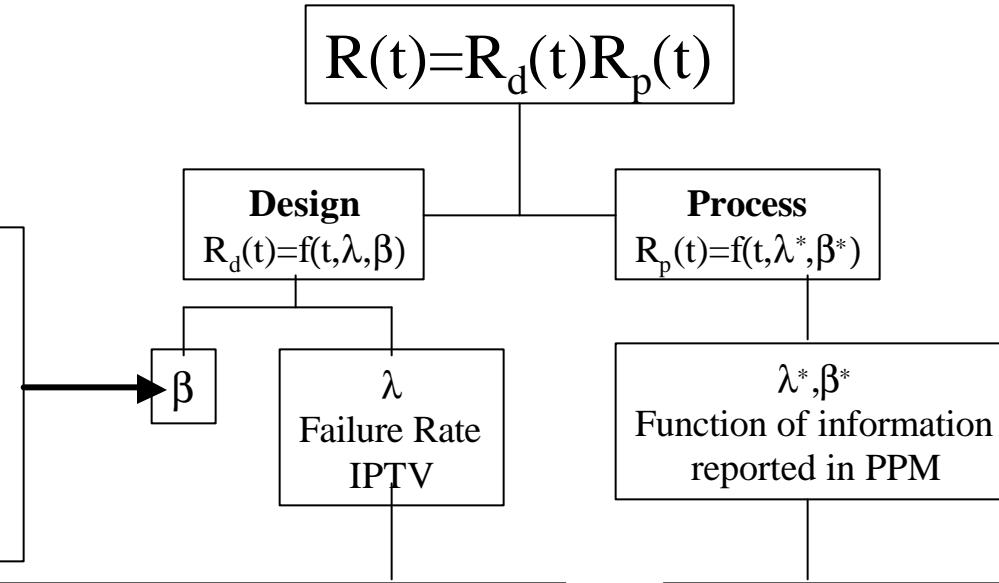




# EXAMPLE: PREDICT - DELPHI

**Full System Model**

- hard data on similar system (warranty data)
- maximum likelihood estimation
- expert's confidence check/adjustment



$T_0$

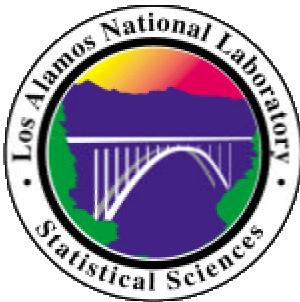
- E.J.: (min, most likely, max)
- quantify into 2-part gamma

$T_i$

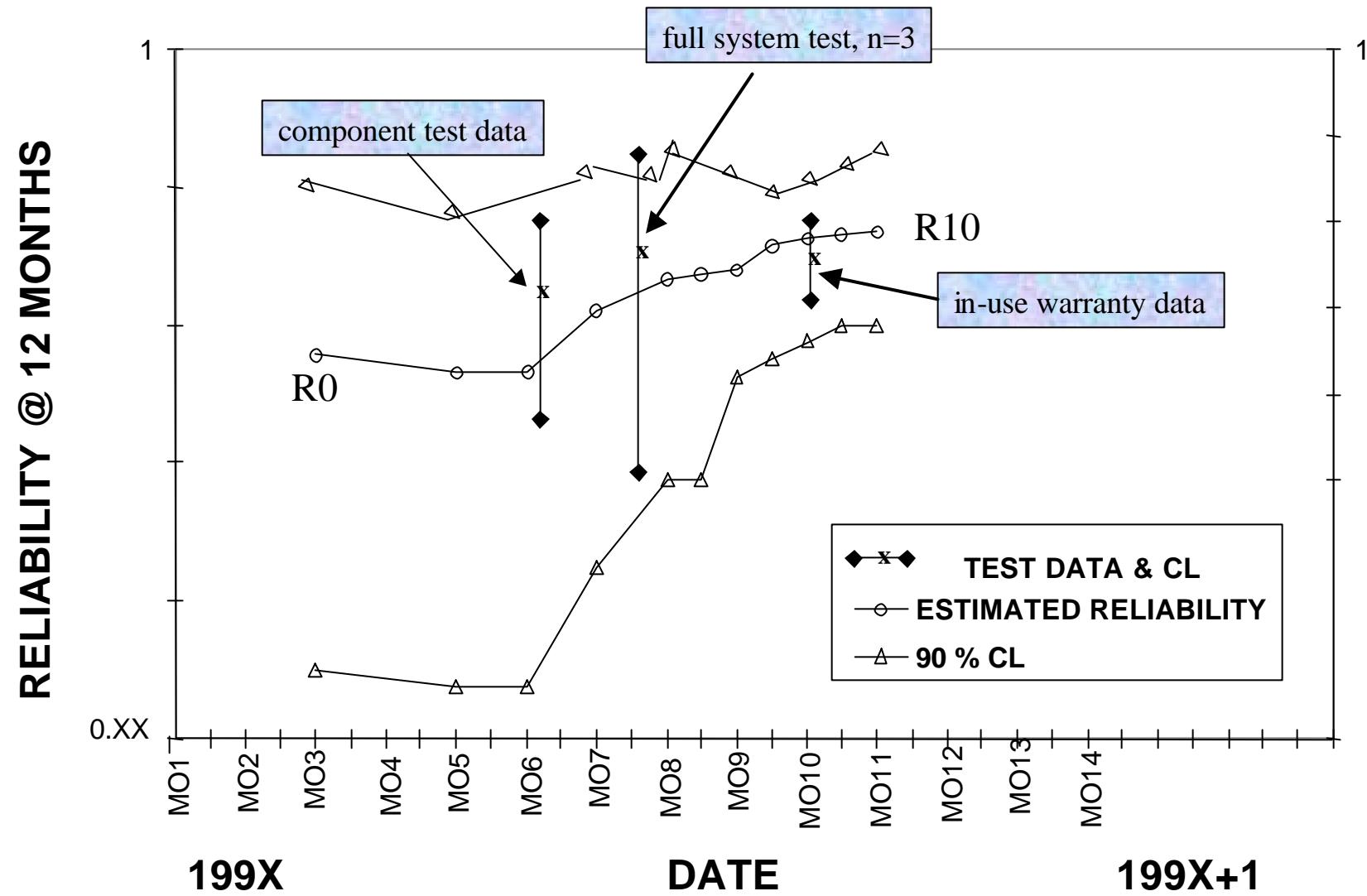
- E.J. + test data
- quantify w/Bayesian update

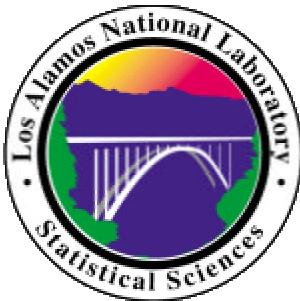
$T_i$

- E.J. + test data
- 2-part Beta w/Bayesian update



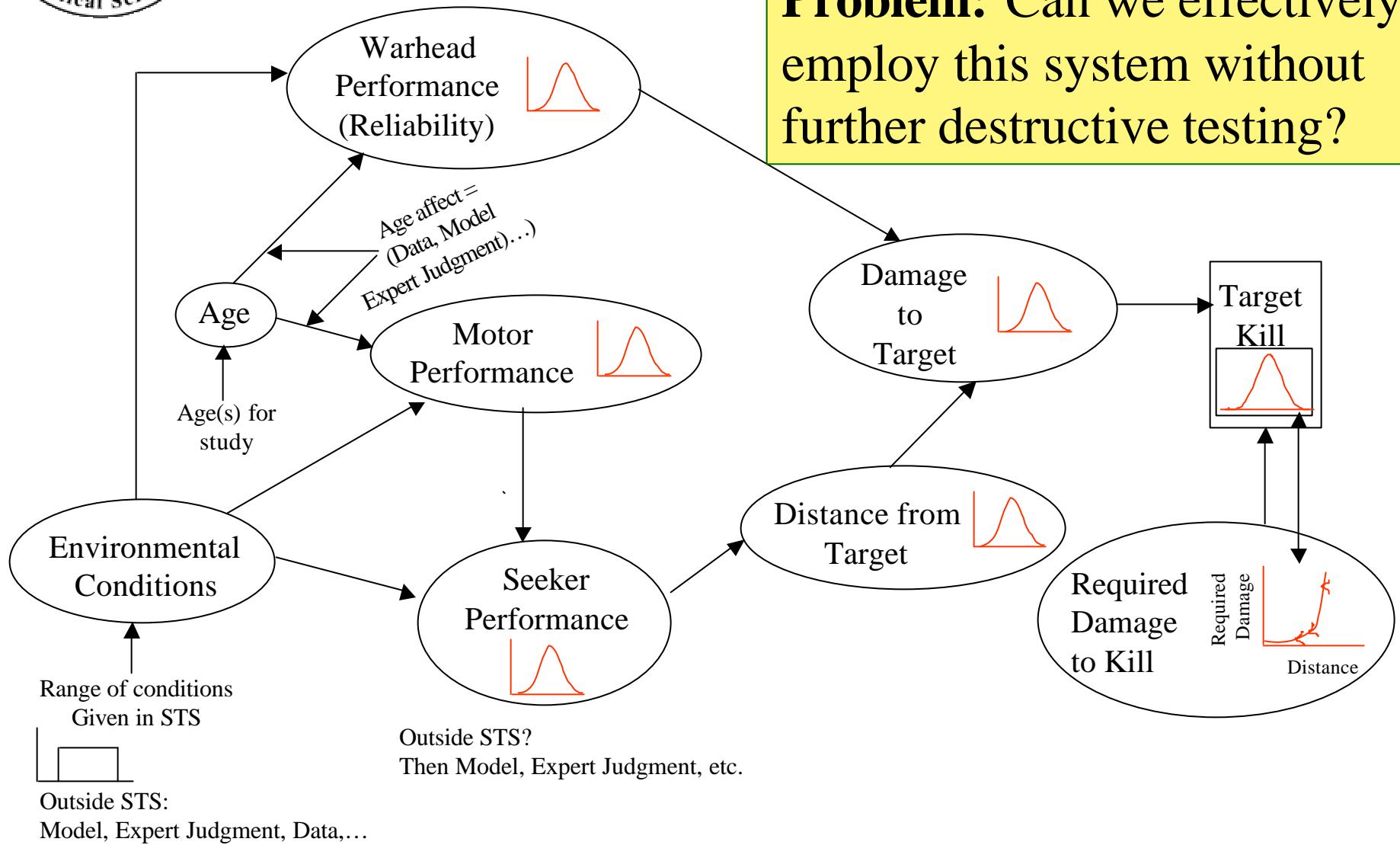
# PREDICT RELIABILITY ESTIMATES OVER TIME

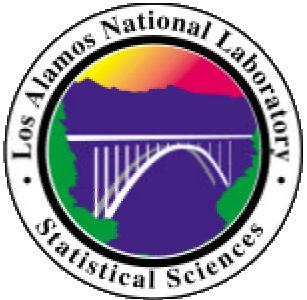




# RDMS EXAMPLE

**Problem:** Can we effectively employ this system without further destructive testing?

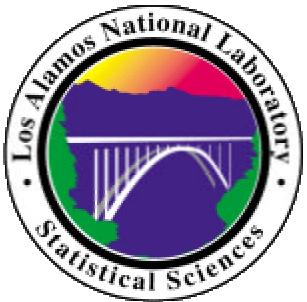




# THE REAL DECISION PROBLEM

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RDMS is an aging system that is scheduled to be replaced in 10 years. There will be no additional production of the system. You are responsible for interim system maintenance and operate with a fixed annual budget. How should you allocate your budget to best maintain RDMS?

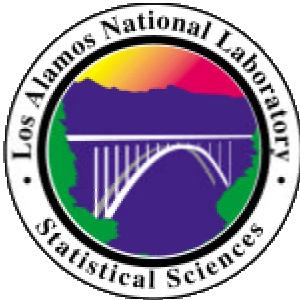


# CRITICAL SYSTEM METRICS

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**The performance of RDMS is characterized by the following metrics:**

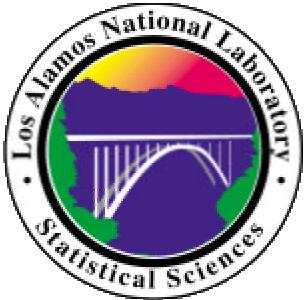
- Probability of Kill (PKill) - the probability that a given missile will successfully destroy its target
- Standard Deviation of Pkill ( $STD_{PKill}$ ) - a metric for the accuracy of the predicted Pkill value.
- Availability (Avail)- the number of deployed, fully operational units.



# OPTIONS

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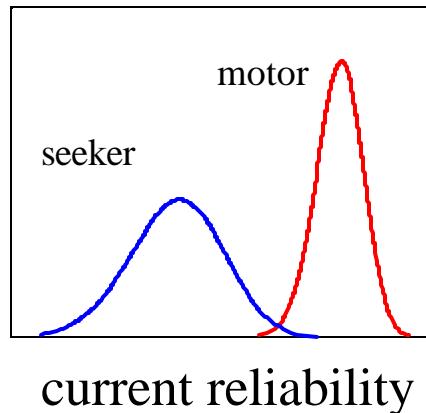
- Replace components from inventory
  - execute in the field OR bring into depot, depending on which component(s) are replaced
- Test, then replace components from inventory
- Perform development, then replace components with upgrades
- Test, then develop, then replace components with upgrades
  - 
  - 
  -



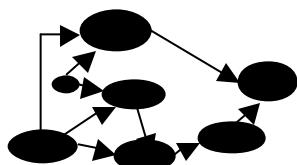
# RDMS DATA FOR REPLACE OPTION

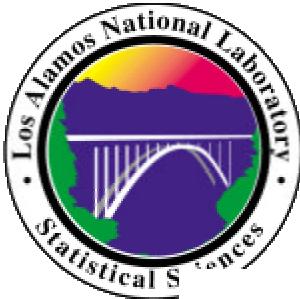
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- What is the optimal component to replace?



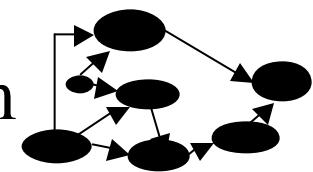
- It might appear that the seeker should be replaced. The correct answer also depends upon reliability time-paths and inventory reliability. The entire system must be considered.



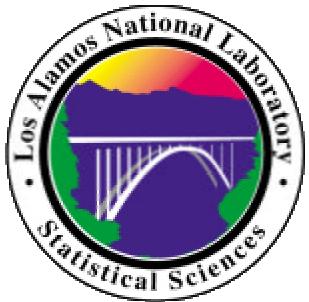


# RDMS DATA FOR TEST AND REPLACE OPTION

- Are tests needed?
  - If so, how many?
  - What kind?
  - What is the anticipated outcome?
- 
- It is necessary to study the entire system
  - Once you answer these questions, you are in the “replace” option mode.

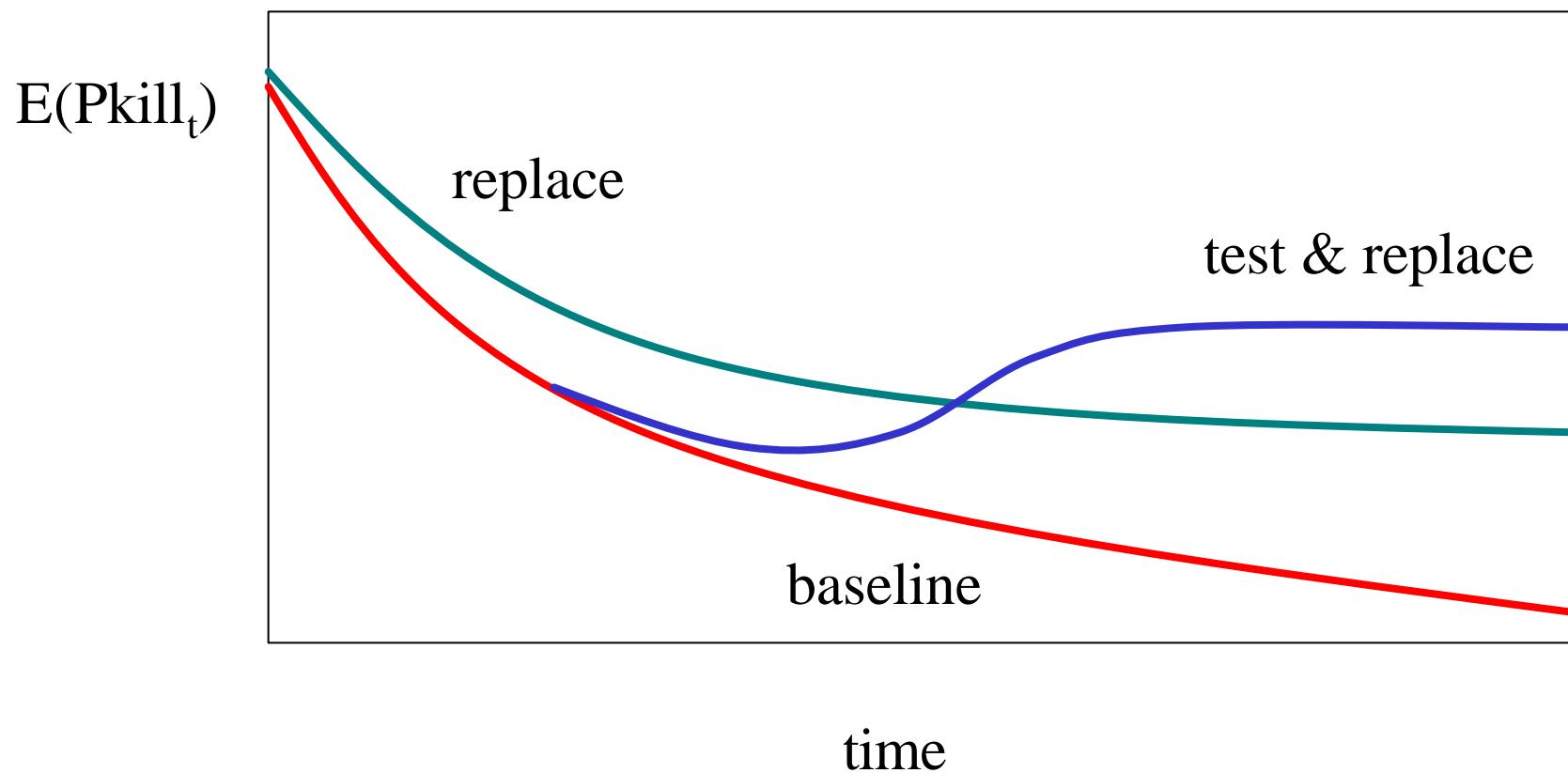


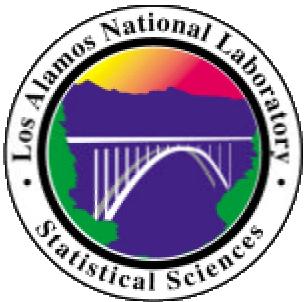
HOW DO WE MAKE A CHOICE IN A  
NON-AD HOC MANNER??



# FORMALIZATION OF THE DECISION OPTIONS

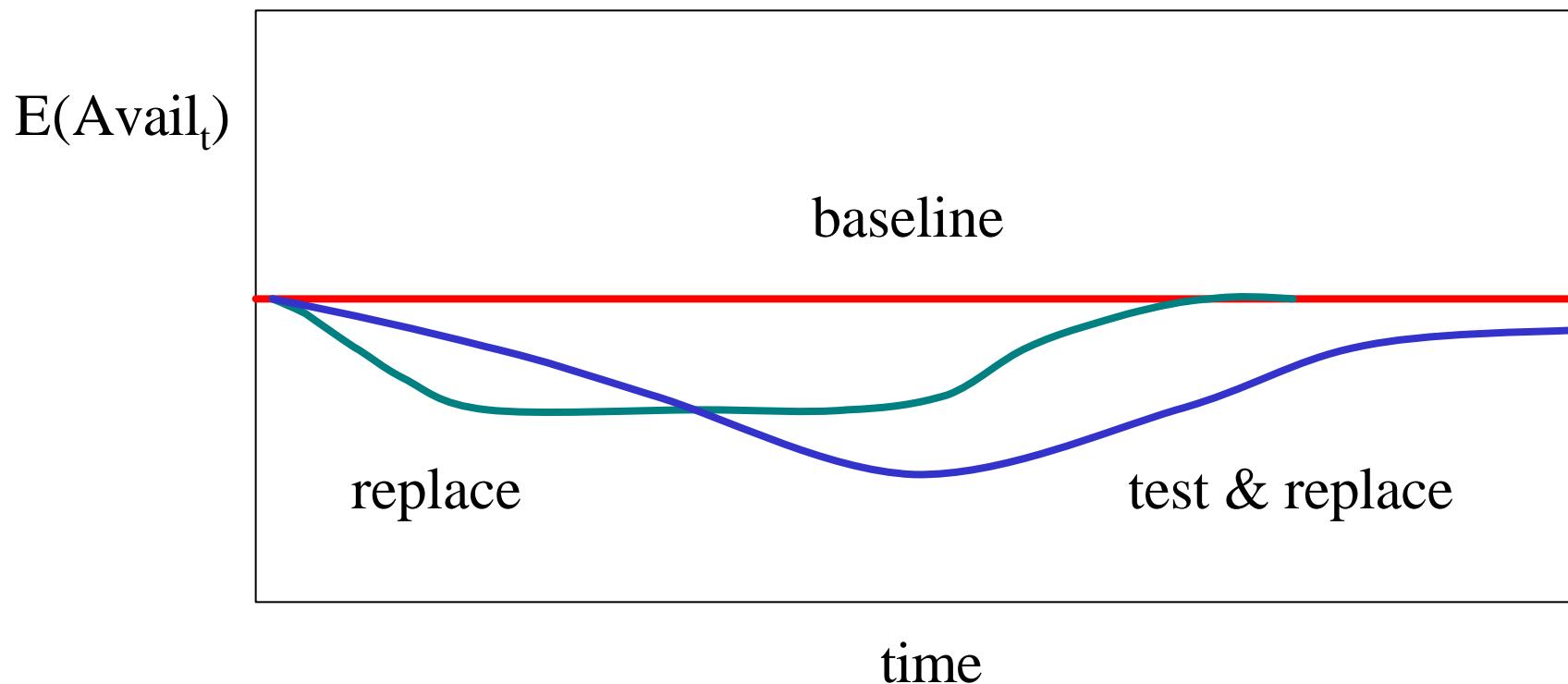
## Notional option time paths

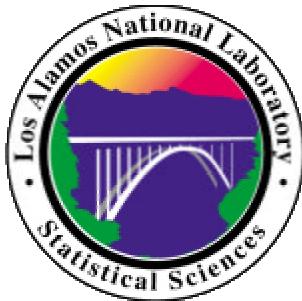




# FORMALIZATION OF THE DECISION OPTIONS

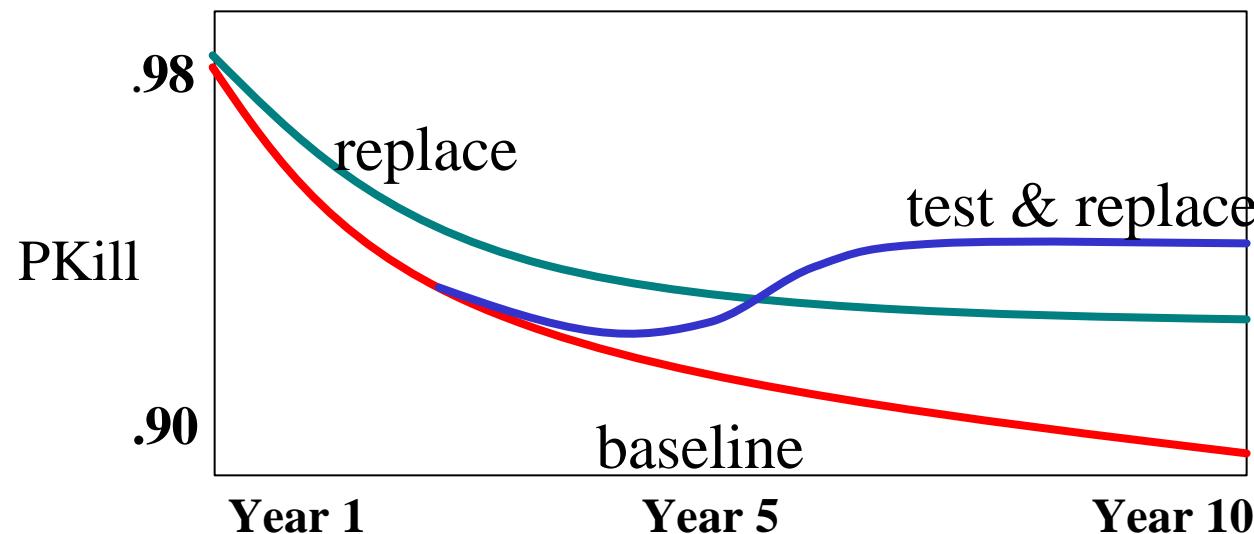
## Notional option time paths



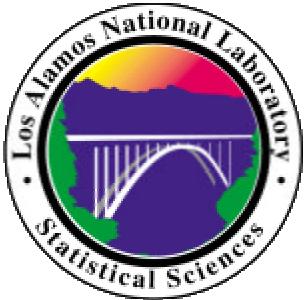


## FORMALIZATION OF THE DECISION OPTIONS

- Suppose you have fully utilized the IIT framework to develop “real” time paths for your options



- Which option should you choose???

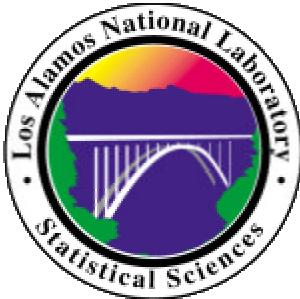


# GENERAL DECISION FRAMEWORK

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Decision making is really a problem of **resource allocation**

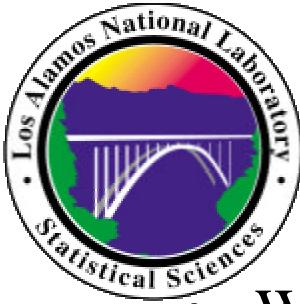
- All of the system variables, parameters, relationships, etc, can be classified as either **State**, **Control**, or **Response** components.
- State components the parts of the system the decision maker does not control
  - 10 year replacement date, budget allocation, system specifications, technology, ...



# GENERAL DECISION FRAMEWORK

---

- Control components are the parts of the system the decision maker does control
  - the decision maker almost invariably controls resources, time, materials, facilities, budget distribution, ...
  - RDMS: how much should be spent on development, testing, and replacement?
- Response components are the system attributes the decision maker is interested in
  - reliability, performance, total cost
  - RDMS:  $P_{Kill}$ ,  $STD_{P_{Kill}}$ ,  $Avail$

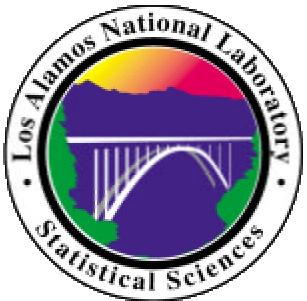


# GENERAL DECISION FRAMEWORK

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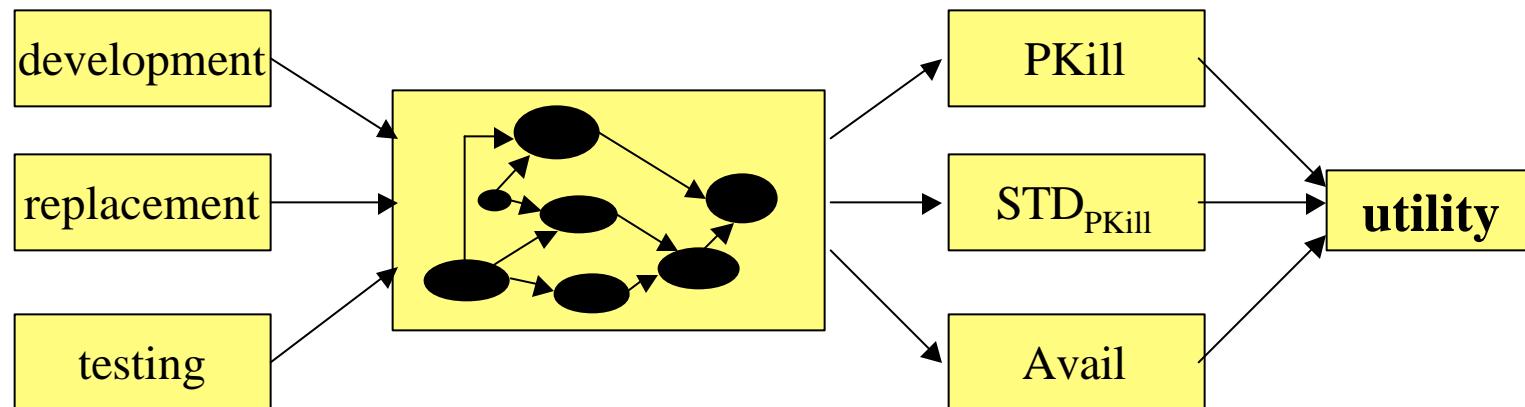
- With few controls and one or two response attributes, analyses that guide decision making can be straightforward
- Decision making is typically not this simple:
  - multiple controls
  - multiple, conflicting responses
  - multiple time horizons
  - uncertainty
- IIT produces a system model that allows the decision maker to predict the distribution of responses given arbitrary settings of the control components.

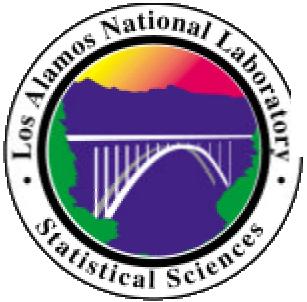
CAN IT DO MORE?



# OPTIMAL DECISION MAKING

- Optimal decision making requires a single metric giving the relative worth to the decision maker of any set of responses.
- This metric is given by the utility function:



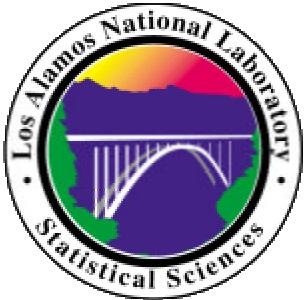


# UTILITY FUNCTIONS YOU HAVE SEEN

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- Cost for spares inventory problems
- Cost for a fixed number and type of modifications with known performance
- Reliability for design problems
- Cost or Reliability for optimal use of redundancy

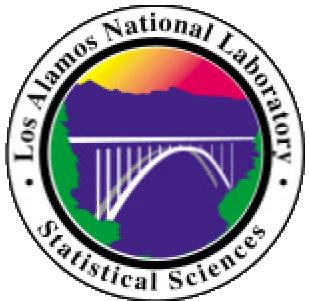
A more general framework than used in the above examples is needed to handle even our simple RDMS example.



# OPTIMAL DECISION MAKING

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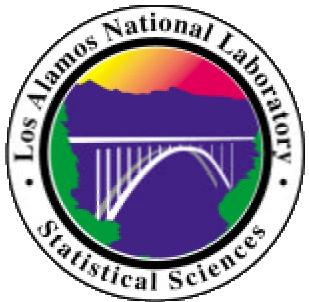
- The optimal set of choices maximizes the decision maker's utility.
- The utility maximization model probably has the most empirical verification of any model in all of the social sciences - you do this every day.
- Utility maximization requires a complete understanding of the entire system - provided by IIT.



# SO WHAT IS A UTILITY FUNCTION?

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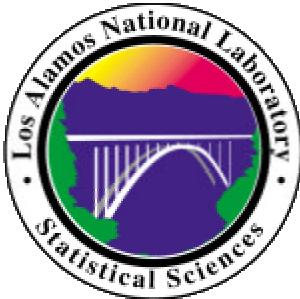
- A utility function returns larger values for preferred response bundles.
- Consider two response bundles:
  - $A = \{PKill = 0.98, STD_{PKill} = 0.03, Avail = 1,000\}$
  - $B = \{PKill = 0.99, STD_{PKill} = 0.03, Avail = 950\}$
  - $C = \{PKill = 0.90, STD_{PKill} = 0.10, Avail = 2,000\}$
- For a given decision maker, if they prefer A to B, then their utility function  $U(Pkill, STD_{PKill}, Avail)$  is such that  $U(0.98, 0.03, 1000) > U(0.99, 0.03, 950)$



# THE UTILITY FUNCTION

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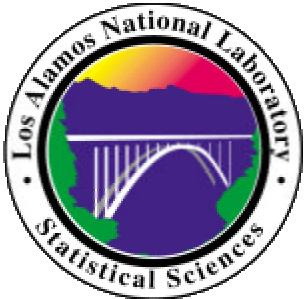
- Utility functions are only ordinal.
- In order to apply quantitative methods, the utility function must be quantified.
- In order to quantify the utility function, the decision maker must behave rationally.



# WHEN IS THE DECISION MAKER RATIONAL?

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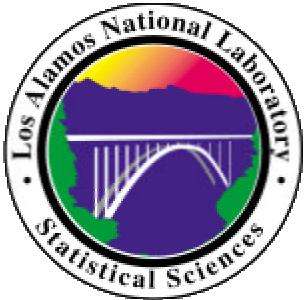
- Consider three response bundles A, B, and C. You are rational if your preferences are:
- Complete:
  - either you prefer A to B, you prefer B to A, or you are indifferent between A and B.
- Reflexive:
  - if A is identical to B then you are indifferent between A and B.
- Continuous:
  - if B is very, very close to A, then you are “nearly” indifferent between A and B



# WHEN IS THE DECISION MAKER RATIONAL?

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- Transitive:
  - if you prefer A to B and you prefer B to C then you prefer A to C
- Monotonic
  - more (or less) is always better



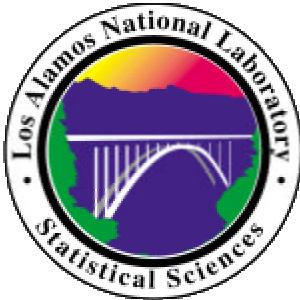
# THE UTILITY FUNCTION

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- The requirements for rationality are rather weak.
- If a decision maker is rational, then a continuous, real-valued utility function representing their preferences does exist.

$$U = \alpha_1 P_{\text{Kill}} + \alpha_2 \text{STD}_{P_{\text{Kill}}} + \alpha_3 \text{Avail}$$

$$U = P_{\text{Kill}}^{\alpha_1} * \text{STD}_{P_{\text{Kill}}}^{\alpha_2} * \text{Avail}^{\alpha_3}$$



# THE FORMAL PROBLEM

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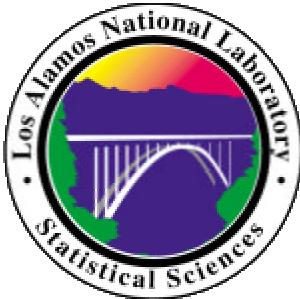
$$\max_{R,T} \int_{t=0}^{10} U(P\text{Kill}_t, \text{STD}_{P\text{Kill}_t}, \text{Avail}_t, t) dt$$

subject to :

$$P\text{Kill}_t = f(R, T, t)$$

$$\text{STD}_{P\text{Kill}_t} = g(R, T, t)$$

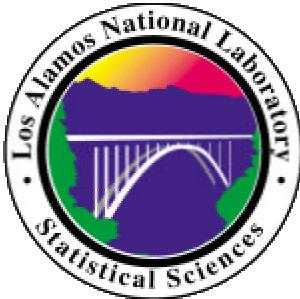
$$\text{Avail}_t = h(R, T, t)$$



# ELICITING UTILITY FUNCTIONS

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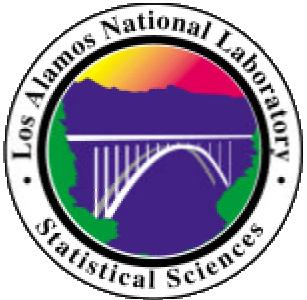
- The required data are preferences for different response bundles:
  - $A = \{PKill = 0.98, STD_{PKill} = 0.03, Avail = 1,000\}$
  - $B = \{PKill = 0.99, STD_{PKill} = 0.03, Avail = 950\}$
- The data may be observed: the decision maker is observed to choose A over B
- The data may be elicited:
  - the decision maker claims to be indifferent between A and B



# THE NATURE OF THE SOLUTION: TRADEOFFS

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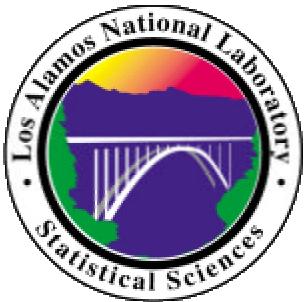
- Because resources are limited, tradeoffs must be made between responses.
- The rate at which the decision maker is **willing** to exchange one response for another is given by the utility function.
- The rate at which the decision maker **can** exchange one response for another is given by the system model.
- The optimal solution is where the two exchange rates are equal.



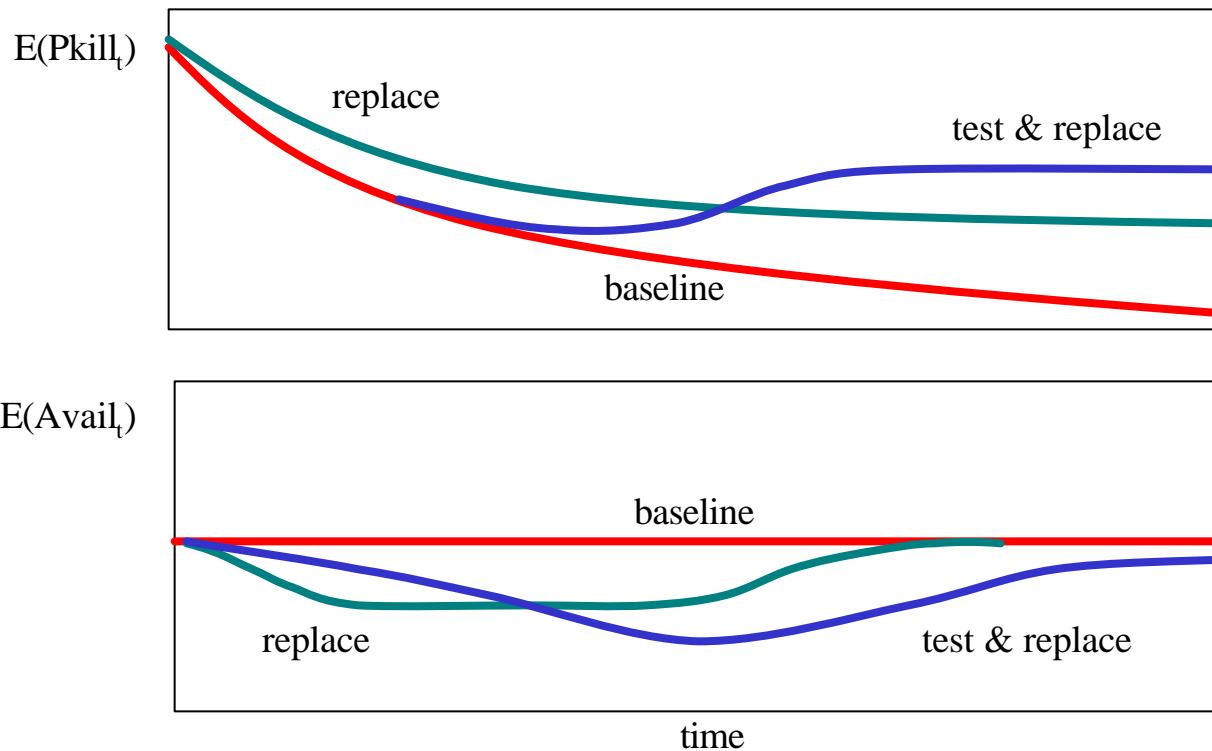
# TRADEOFFS

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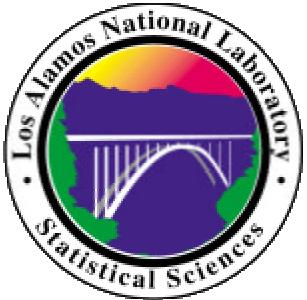
- If the decision maker is indifferent between  $\{PKill = 0.98, Avail = 1000\}$  and  $\{PKill = 0.99, Avail = 950\}$ , the decision maker is willing to exchange 50 missiles for 0.01 unit of PKill.
- Any change that increases PKill by 0.01 and uses fewer than 50 missiles makes the decision maker better off.
- Any change that increases PKill by 0.01 and uses more than 50 missiles makes the decision maker better off.



# TRADEOFFS



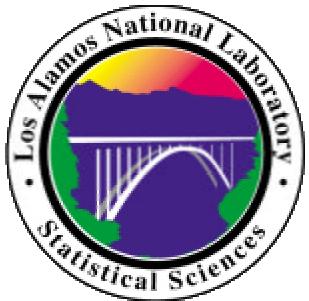
- Tradeoffs between
  - $P_{kill}$ ,  $STD_{P_{Kill}}$ , and  $Avail$
  - current and future values



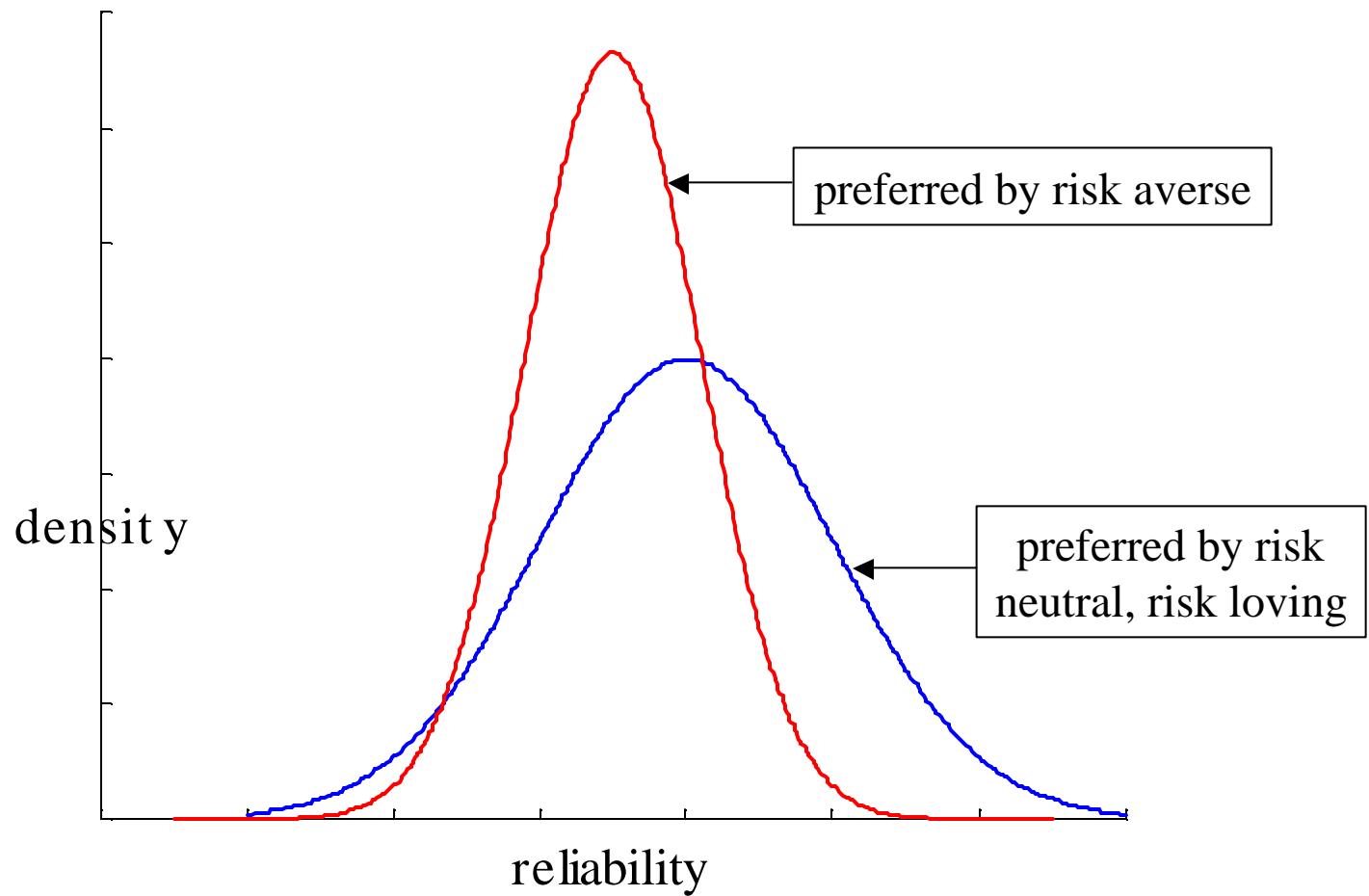
# UNCERTAINTY

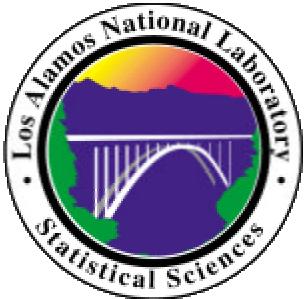
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- When responses are subject to uncertainty, there are numerous possible optimization criteria:
  - maximize minimum utility (ultra-pessimistic)
  - maximize maximum utility (ultra-optimistic)
- With a couple more assumptions, maximizing utility is identical to maximizing expected (average) utility.
- The decision maker may be risk neutral, risk loving, or risk averse.
- Response variability reduces (increases) the utility of risk averse (loving) individuals.



# UNCERTAINTY

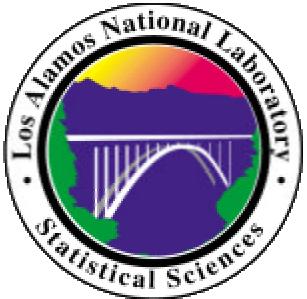




# UNCERTAINTY

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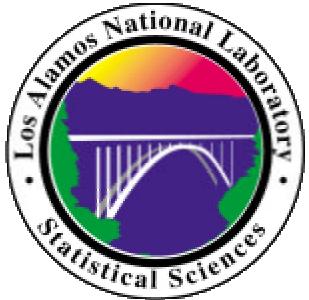
- In RDMS, level of risk aversion is reflected in utility function parameters associated with  $STD_{PKill}$  variable.
- The level of risk aversion has a major impact on the final decision.
- The level of risk aversion is totally and completely subjective.



# SENSITIVITY ANALYSIS

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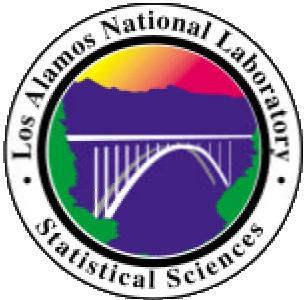
- With a complex problem, the optimal decision will depend upon a large number of specifications and assumptions. These are all documented in the knowledge system.
- Decisions should be robust: small changes in model specifications should not produce major changes in decisions. If they do, critical specifications should be identified and understood.



# SENSITIVITY ANALYSIS

---

- Sensitivity analysis is performed by perturbing model specifications and observing the impact on the optimal decision.
  - Change a prior distribution from gamma to normal.
  - Double the variance of a component's reliability.
  - Halve the level of risk aversion.
- Optimal strategies for sensitivity analysis are the subject of ongoing research.



# CONCLUSIONS

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- The problem: how to allocate maintenance budget?
- The system representation gives relationship between choices (testing, replacement) and responses ( $P_{Kill}$ ,  $STD_{P_{Kill}}$ ,  $Avail$ ).
- The utility function gives the relative worth of responses.
- The optimal decision is determined by equating rates of exchange.
- The Knowledge System provides complete documentation of the decision.
- Sensitivity analyses are critical!!