Name:

Quiz 3
November 4, 2013

1. Suppose that I have data $x_1, \ldots, x_n$, from a Beta(α, β) distribution. I use independent Gamma$(\theta_\alpha, \gamma_\alpha)$ and Gamma$(\theta_\beta, \gamma_\beta)$ prior distributions for α and β.

   The p.d.f. of a beta distribution is

$$f(x \mid \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

   The p.d.f. of a gamma distribution is

$$f(x \mid \theta, \gamma) = \frac{\gamma^\theta}{\Gamma(\theta)} \exp(-\gamma x) x^{\theta-1}$$

   (a) **(2 pts)** Write the expression for the likelihood. (Be sure to include the normalizing constant.)

   $L(\alpha, \beta \mid x_1, \ldots, x_n) =$

   (b) **(2 pts)** Write the expression for the prior.

   $\pi(\alpha, \beta) \propto$

   (c) **(1 pt)** Write the expression for the posterior distribution.

   $\pi(\alpha, \beta \mid x_1, \ldots, x_n) \propto$

(d) **(2 pts)** Write the expressions for the full conditionals (full conditional distributions) for α and β.

(e) **(3 pts)** Describe how you would implement a Gibbs sampler to draw a sample from the posterior distribution. There's no need to describe the details of any Metropolis steps (e.g., you don't need to specify a proposal distribution or write out the expression for the acceptance ratio).