

Homework 7
Due Monday, November 11

Think of the question as a “mini-project.” This means that everything must be typed, labeled, and referenced, as appropriate. Your answers for each question should discuss the problem, data, model, method, conclusions.

British Coal-Mining Disasters. The file `coal.dat` contains the number of coal-mining explosions in Great Britain where 10 or more miners were killed (“disasters”) for the years 1851-1962. Analyze this data using the following hierarchical changepoint model, where Y_i is the number of disasters in year i .

$$Y_i = \begin{cases} \text{Poisson}(\theta) & i = 1, \dots, k \\ \text{Poisson}(\lambda) & i = k+1, \dots, n \end{cases}$$

$$k \sim \text{DiscreteUniform}(1, \dots, n), \quad \theta \sim \text{Gamma}(a_1, b_1), \quad \lambda \sim \text{Gamma}(a_2, b_2)$$

k, θ, λ independent

$$b_1 \sim \text{Gamma}(c_1, d_1), \quad b_2 \sim \text{Gamma}(c_2, d_2), \quad b_1, b_2 \text{ independent}$$

Set $a_1 = a_2 = 0.5$, $c_1 = c_2 = 1$, $d_1 = d_2 = 0.1$. Parameterize the $\text{Gamma}(\alpha, \beta)$ distribution as

$$\pi(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \exp(-\beta\theta) \theta^{\alpha-1}$$

so that the full conditional distributions for b_1 and b_2 have a conjugate structure. Focus your analysis and conclusions on k , θ , λ , and $R = \theta/\lambda$.