Homework 2
Due Wednesday, September 4, 2013

Think of each question as a "mini-project." This means that everything must be typed, labeled, and referenced, as appropriate.

1. For each of the following data sets and likelihoods, do the following:

   - Plot and briefly describe the data set (shape, summary statistics).
   - Find the conjugate prior distribution and write out the functional form of the density. Be sure to demonstrate that the posterior distribution is in the same family as the conjugate prior distribution.
   - Choose parameter values to specify an informative prior for the conjugate prior distribution and *justify* your choices.
   - Plot the prior distribution and the posterior distribution using your parameter choices for the conjugate prior distribution. Add the maximum likelihood estimate to the plot.
   - Propose a nonconjugate prior distribution. Specify the distribution in detail and justify your parameter choices. Add the plots of the non-conjugate prior and posterior distribution to the plot for your conjugate choice.

   (a) Data in test_scores.dat. These data are 29 test scores from the first exam in STAT 101 (110 points possible). Use the sampling distribution

   $$f(y_i \,|\, \mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(y_i - \mu)^2}{2\sigma^2}\},$$

   where $\sigma = 9$.

   (b) Data in test_scores.dat. These data are 29 test scores from the first exam in STAT 101 (110 points possible). Use the sampling distribution

   $$f(y_i \,|\, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(y_i - \mu)^2}{2\sigma^2}\},$$

   where $\mu = 87$.

(c) Data in rivers.dat. These data are the lengths (in miles) of 141 major rivers in North America, as compiled by the U.S. Geological Survey. Use

$$f(y_i \mid \lambda) = \lambda \exp(-\lambda y_i).$$

2. You work for Light Bulbs International. You have developed a new bulb, and you are interested in characterizing it statistically. You test 20 new bulbs to determine their lifetimes, and you observe the following data (in hours), which have been sorted from smallest to largest.

   1, 13, 27, 43, 73, 75, 154, 196, 220, 297,
   344, 610, 734, 783, 796, 845, 859, 992, 1066, 1471

   Based on your experience with light bulbs, you believe that their lifetimes can be modeled using an exponential distribution. You expect the bulb's average lifetime to be around 400 hours, and you'd be surprised if it exceeded 600 hours. Based on this information, what can you say about your new bulb? In particular, your boss would be interested in the probability that the average bulb lifetime exceeds 550 hours.

   Be sure that your answer discusses the problem, data, model, method, conclusions.