Homework 4
Due Monday, September 23, 2013

Think of each question as a "mini-project." This means that everything must be typed, labeled, and referenced, as appropriate. Your answers for each question should discuss the problem, data, model, method, conclusions. If you develop an interesting computational way to solve a problem, feel free to include pseudo (or actual) code, but, in general, code is not required.

1. Consider the data in *test_scores.dat*. These data are 29 test scores from the first exam in Intro Stats (110 points possible). Use the sampling distribution
$$f(y_i \mid \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(y_i - \mu)^2}{2\sigma^2}\},$$
where $\mu = 87$.

   (a) Derive the Jeffreys' prior for σ.

   (b) Using the Jeffrey's prior, plot the posterior distribution and find the posterior mean, posterior median, posterior mode, posterior variance, posterior standard deviation, and 95% central credible interval for the parameter of interest.

   (c) Find the (posterior) predictive distribution for the test score of a future Intro Stats student.

2. Suppose you drive on a particular interstate roadway and typically drive at a constant speed of 70 mph. One day, you pass one car and get passed by 17 cars. Suppose that the speeds are normally distributed with unknown mean μ and standard deviation $\sigma = 10$. If you pass s cars, and f cars pass you, the likelihood of μ is given by
$$L(\mu \mid s, f, \sigma) = \Phi(70, \mu, \sigma)^s (1 - \Phi(70, \mu, \sigma))^f,$$
where $\Phi(y, \mu, \sigma)$ is the cdf of the normal distribution with mean μ and standard deviation σ. Assign the unknown mean μ a flat (improper) prior density.

   (a) If $s = 1$ and $f = 17$, plot the posterior density of μ.

(b) Using the density found in (2a), find the posterior mean of μ.

(c) Find the probability that the average speed of the cars on this interstate roadway exceeds 80 mph.

3. Suppose we are interested in learning about the sleeping habits of undergraduate students at NCSU. We collect the sleeping times (in hours) for 20 randomly selected students in a statistics course. These are the observations:

9.0, 8.5, 7.0, 8.5, 6.0, 12.5, 6.0, 9.0, 8.5, 7.5,
8.0, 6.0, 9.0, 8.0, 7.0, 10.0, 9.0, 7.5, 5.0, 6.5

(a) Use a Normal(μ, σ^2) sampling distribution to model the data, and assign the noninformative prior $f(\mu, \sigma) = 1/\sigma^2$. Describe how to and draw a random sample from the posterior distribution for (μ, σ^2).

(b) Plot the marginal posterior distributions for μ and σ^2 and calculate a mean and 90% credible interval.

(c) Plot the posterior distribution for the upper 75th quantile, $p_{75} = \mu + 0.674\sigma$ and calculate its mean and a 90% credible interval.