

Name:

Quiz 5
November 13, 2013

1. **(3 pts)** In the general formulation of the hierarchical model, assign the correct label to each line: hyperparameter model, observational model, structural model.

- $y_i | \theta_i \sim f_i(y_i | \theta_i)$
- $\theta_i | \alpha \sim g(\theta_i | \alpha)$
- $\alpha \sim h(\alpha)$

2. Suppose that we are modeling the probability of developing a tumor in groups of rats. We have observed three groups of rats, and 1/10, 2/10 and 0/10 of the rats have developed tumors. Assume a hierarchical model so that each set of rats has its own probability of developing a tumor, and these probabilities are assumed to come from a $\text{Beta}(\alpha, \beta)$ distribution, with $\alpha \sim \text{Gamma}(0.5, 1)$ and $\beta \sim \text{Gamma}(0.5, 1)$.

(a) **(2 pts)** Write down an expression for the observational model.

(b) **(2 pts)** Write down an expression for the posterior distribution.

- (c) **(3 pts)** Describe how you would compute the predictive distribution for the probability of a rat in the next group developing a tumor.

The p.d.f. of a beta distribution is

$$\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

The p.d.f. of a gamma distribution is

$$\frac{\beta^\alpha}{\Gamma(\alpha)} \exp(-\beta x) x^{\alpha-1}$$