

problem set 2

Problem 1 - Chapter 6 (Stats Mods), Exercise set C, 2-4 (p.90)

2. We need to compute the path coefficients in figure 2. If we look at Freedman p. 84, we see that we can calculate these coefficients using the standard OLS regression coefficient algorithm: $(M'M)^{-1}M'U$, utilizing the fact that the variables of interest are standardized. Particularly,

$$M'M = n \begin{bmatrix} 1 & r_{VX} \\ r_{VX} & 1 \end{bmatrix} = n \begin{bmatrix} 1 & 0.52 \\ 0.52 & 1 \end{bmatrix}$$

and

$$M'U = n \begin{bmatrix} r_{VU} \\ r_{XU} \end{bmatrix} = n \begin{bmatrix} -0.26 \\ -0.42 \end{bmatrix}$$

So

$$(M'M)^{-1} = \frac{1}{n(1 - (0.52)^2)} \begin{bmatrix} 1 & -0.52 \\ -0.52 & 1 \end{bmatrix}$$

and further

$$\begin{aligned} (M'M)^{-1}M'U &= \frac{1}{n(1 - (0.52)^2)} \begin{bmatrix} 1 & -0.52 \\ -0.52 & 1 \end{bmatrix} \times n \begin{bmatrix} -0.26 \\ -0.42 \end{bmatrix} \\ &= \begin{bmatrix} -0.06 \\ -0.39 \end{bmatrix} \end{aligned}$$

3. We would like to estimate the standard deviation of δ in equation 10.
4. find standard errors for path coefficients and their difference; and also t-ratios and statistical significance.

Problem 7

- a. total effect is c which we break down into direct and indirect effects. d is direct effect, ab is the indirect effect. the overarching derivation comes from the book (look at slides for this)
- b. why would this break down if it varies across subjects? there's a trick. we can re-arrange covariance as the expectation of the product of those things - product of the expectations. solve for e of xy - > ; then there exists at least one pair for which i and j are not the same. we can't compute the covariance. expectation of a and b we can't compute if we don't have the covariance - it breaks down in the indirect effect. aka its a very strong assumption to make to say that the indirect effect is constant.
- c. look at the mediator equation. if treatment effect is 0 for everyone - a is 0 (for the first equation.) $E[cX] = cE[X]$; $E[aib_i] = aE[b_i] = 0 * e[b_1] = 0$; when we deal with averages, we might have to use expectations.

- d. why does the complex potential outcome defy empirical investigation? you can never observe it. you can't at the same time set and not set z to 1.
- e. y_1 given $d = 1$ - y_0 given $d = 1$; one of these terms i dont observe.the distinction is that the in this equation 2 things are moving (z 0/1 and other one 0/1) whereas in the other, we fix z and manipulate the mediator.