# PSTAT 122 Homework 1

January 2022

NOTE:
In the 10th Edition that most of you might be using, HW 1 consists of Problems: 2.3, 2.6, 2.7, 2.15, 2. 16, and 2.30

If you are using the 9th Edition, these are the Problems: 2.3, 2.7, 2.8, 2.17, 2.18, 2.35 there.

If you are using the 8th Edition, these are the Problems: 2.3, 2.7, 2.8, 2.15, 2.16, 2.33 there.

PROBLEMS:
2.3. Suppose that we are testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$. Calculate the $P$-value for the following observed values of the test statistic: (a) $Z_0 = 2.25$ (b) $Z_0 = 1.55$ (c) $Z_0 = 2.10$ (d) $Z_0 = 1.95$ (e) $Z_0 = -0.10$

2 sided test  z test
pnorm(2.25)

one sided t test

2.6. Suppose that we are testing $H_0 : \mu_1 = \mu_2$ versus $H_0 : \mu_1 > \mu_2$ where the two sample sizes are $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the $P$-value for the following observed values of the test statistic.

(a) $t_0 = 2.31$ (b) $t_0 = 3.60$ (c) $t_0 = 1.95$ (d) $t_0 = 2.19$  $\boxed{df = 18}$  $P(t > t_0)$

2.7. Consider the following sample data:

$$9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, 7.75$$

Is it reasonable to assume that this data is a sample from a normal distribution? → qqplot (qqnorm/qqline)
Is there evidence to support a claim that the mean of the population is 10? → t test  $H_0: \mu = 10$  $H_A: \mu \neq 10$

2.15

```
Two-Sample T-Test and CI: Y1, Y2

Two-sample T for Y1 vs Y2

      N    Mean    Std. Dev.    SE Mean
Y1   20   50.19      1.71         0.38
Y2   20   52.52      2.48         0.55

Difference = mu (X1) - mu (X2)
Estimate for difference: -2.33341
95% CI for difference: (-3.69547, -0.97135)
T-Test of difference = 0 (vs not =) : T-Value = -3.47
P-Value = 0.001  DF = 38
Both use Pooled Std. Dev. = 2.1277
```

(a) Can the null hypothesis be rejected at the 0.05 level? Why?
(b) Is this a one-sided or a two-sided test?
(c) If the hypotheses had been $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 \neq 2$ would you reject the null hypothesis at the 0.05 level?
(d) If the hypotheses had been $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 < 2$ would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?
(e) Use the output and the $t$ table to find a 95 percent upper confidence bound on the difference in means. → one sided so lower bound is -infinity
(f) What is the $P$-value if the hypotheses are $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 \neq 2$?

2.16
The breaking strength of a fiber is required to be at least 150psi. Past experience has indicated that the standard deviation of breaking strength is $\sigma = 3$psi. A random sample of four specimens is tested, and the results are $y_1 = 145, y_2 = 153, y_3 = 150$, and $y_4 = 147$.
(a) State the hypotheses that you think should be tested in this experiment.
(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?
(c) Find the $P$-value for the test in part (b). $P(Z >$ something$)$
(d) Construct a 95 percent confidence interval on the mean breaking strength.

just assume normal

$\bar{x} \pm \frac{s}{\sqrt{n}} t_{df, 1-\alpha/2}$

$\bar{x} \pm \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} \} z$

$H_0: \mu \geq 150$ Sigma known
$H_A: \mu < 150$

use z test & find z score

2.30
An article in the journal Neurology (1998, Vol. 50 , pp. 1246-1252) observed that monozygotic twins share numerous physical, psychological, and pathological traits. The investigators measured an intelligence score of 10 pairs of twins. The data obtained are as follows:

paired 2 sample t test

| $Pair$ | $BirthOrder$ :1 | $BirthOrder$ :2 |
|---|---|---|
| 1 | 6.08 | 5.73 |
| 2 | 6.22 | 5.80 |
| 3 | 7.99 | 8.42 |
| 4 | 7.44 | 6.84 |
| 5 | 6.48 | 6.43 |
| 6 | 7.99 | 8.76 |
| 7 | 6.32 | 6.32 |
| 8 | 7.60 | 7.62 |
| 9 | 6.03 | 6.59 |
| 10 | 7.52 | 7.67 |

calculate difference & qqplot

(a) Is the assumption that the difference in score is normally distributed reasonable?

(b) Find a 95% confidence interval on the difference in mean score. Is there any evidence that mean score depends on birth order? is 0 included ? → yes no evidence

(c) Test an appropriate set of hypotheses indicating that the mean score does not depend on birth order.

difference can make it one sample t test

c) $d = 0$ where $d = X_i - Y_i$

two sided

# PSTAT 122 HW 1

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2.3. Suppose that we are testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$. Calculate the $P$-value for the following observed values of the test statistic: (a) $Z_0 = 2.25$ (b) $Z_0 = 1.55$ (c) $Z_0 = 2.10$ (d) $Z_0 = 1.95$ (e) $Z_0 = -0.10$

* for the following problems, I'm going to solve using the z table below. we can do this because it is a two-sided test and the null distribution is normal and symmetrical

| z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0 | .50000 | .49601 | .49202 | .48803 | .48405 | .48006 | .47608 | .47210 | .46812 | .46414 |
| -0.1 | .46017 | .45620 | .45224 | .44828 | .44433 | .44034 | .43640 | .43251 | .42858 | .42465 |
| -0.2 | .42074 | .41683 | .41294 | .40905 | .40517 | .40129 | .39743 | .39358 | .38974 | .38591 |
| -0.3 | .38209 | .37828 | .37448 | .37070 | .36693 | .36317 | .35942 | .35569 | .35197 | .34827 |
| -0.4 | .34458 | .34090 | .33724 | .33360 | .32997 | .32636 | .32276 | .31918 | .31561 | .31207 |
| -0.5 | .30854 | .30503 | .30153 | .29806 | .29460 | .29116 | .28774 | .28434 | .28096 | .27760 |
| -0.6 | .27425 | .27093 | .26763 | .26435 | .26109 | .25785 | .25463 | .25143 | .24825 | .24510 |
| -0.7 | .24196 | .23885 | .23576 | .23270 | .22965 | .22663 | .22363 | .22065 | .21770 | .21476 |
| -0.8 | .21186 | .20897 | .20611 | .20327 | .20045 | .19766 | .19489 | .19215 | .18943 | .18673 |
| -0.9 | .18406 | .18141 | .17879 | .17619 | .17361 | .17106 | .16853 | .16602 | .16354 | .16109 |
| -1 | .15866 | .15625 | .15386 | .15151 | .14917 | .14686 | .14457 | .14231 | .14007 | .13786 |
| -1.1 | .13567 | .13350 | .13136 | .12924 | .12714 | .12507 | .12302 | .12100 | .11900 | .11702 |
| -1.2 | .11507 | .11314 | .11123 | .10935 | .10749 | .10565 | .10383 | .10204 | .10027 | .09853 |
| -1.3 | .09680 | .09510 | .09342 | .09176 | .09012 | .08851 | .08692 | .08534 | .08379 | .08226 |
| -1.4 | .08076 | .07927 | .07780 | .07636 | .07493 | .07353 | .07215 | .07078 | .06944 | .06811 |
| -1.5 | .06681 | .06552 | .06426 | .06301 | .06178 | .06057 | .05938 | .05821 | .05705 | .05592 |
| -1.6 | .05480 | .05370 | .05262 | .05155 | .05050 | .04947 | .04846 | .04746 | .04648 | .04551 |
| -1.7 | .04457 | .04363 | .04272 | .04182 | .04093 | .04006 | .03920 | .03836 | .03754 | .03673 |
| -1.8 | .03593 | .03515 | .03438 | .03362 | .03288 | .03216 | .03144 | .03074 | .03005 | .02938 |
| -1.9 | .02872 | .02807 | .02743 | .02680 | .02619 | .02559 | .02500 | .02442 | .02385 | .02330 |
| -2 | .02275 | .02222 | .02169 | .02118 | .02068 | .02018 | .01970 | .01923 | .01876 | .01831 |
| -2.1 | .01786 | .01743 | .01700 | .01659 | .01618 | .01578 | .01539 | .01500 | .01463 | .01426 |
| -2.2 | .01390 | .01355 | .01321 | .01287 | .01255 | .01222 | .01191 | .01160 | .01130 | .01101 |
| -2.3 | .01072 | .01044 | .01017 | .00990 | .00964 | .00939 | .00914 | .00889 | .00866 | .00842 |
| -2.4 | .00820 | .00798 | .00776 | .00755 | .00734 | .00714 | .00695 | .00676 | .00657 | .00639 |
| -2.5 | .00621 | .00604 | .00587 | .00570 | .00554 | .00539 | .00523 | .00508 | .00494 | .00480 |
| -2.6 | .00466 | .00453 | .00440 | .00427 | .00415 | .00402 | .00391 | .00379 | .00368 | .00357 |
| -2.7 | .00347 | .00336 | .00326 | .00317 | .00307 | .00298 | .00289 | .00280 | .00272 | .00264 |
| -2.8 | .00256 | .00248 | .00240 | .00233 | .00226 | .00219 | .00212 | .00205 | .00199 | .00193 |
| -2.9 | .00187 | .00181 | .00175 | .00169 | .00164 | .00159 | .00154 | .00149 | .00144 | .00139 |
| -3 | .00135 | .00131 | .00126 | .00122 | .00118 | .00114 | .00111 | .00107 | .00104 | .00100 |
| -3.1 | .00097 | .00094 | .00090 | .00087 | .00084 | .00082 | .00079 | .00076 | .00074 | .00071 |
| -3.2 | .00069 | .00066 | .00064 | .00062 | .00060 | .00058 | .00056 | .00054 | .00052 | .00050 |
| -3.3 | .00048 | .00047 | .00045 | .00043 | .00042 | .00040 | .00039 | .00038 | .00036 | .00035 |
| -3.4 | .00034 | .00032 | .00031 | .00030 | .00029 | .00028 | .00027 | .00026 | .00025 | .00024 |
| -3.5 | .00023 | .00022 | .00022 | .00021 | .00020 | .00019 | .00019 | .00018 | .00017 | .00017 |
| -3.6 | .00016 | .00015 | .00015 | .00014 | .00014 | .00013 | .00013 | .00012 | .00012 | .00011 |
| -3.7 | .00011 | .00010 | .00010 | .00010 | .00009 | .00009 | .00008 | .00008 | .00008 | .00008 |
| -3.8 | .00007 | .00007 | .00007 | .00006 | .00006 | .00006 | .00006 | .00005 | .00005 | .00005 |
| -3.9 | .00005 | .00005 | .00004 | .00004 | .00004 | .00004 | .00004 | .00004 | .00003 | .00003 |
| -4 | .00003 | .00003 | .00003 | .00003 | .00003 | .00003 | .00002 | .00002 | .00002 | .00002 |

a) $z_0 = 2.25$

$P(|Z| > |z_0|) = 2P(|Z| > |z_0|)$
$= 2[0.01222] = 0.02444$

b) $z_0 = 1.55$

$P(|Z| > |z_0|) = 2P(|Z| > |z_0|)$
$= 2[0.06057] = 0.12114$

c) $z_0 = 2.10$

$P(|Z| > |z_0|) = 2P(|Z| > |z_0|)$
$= 2[0.01786] = 0.03572$

d) $z_0 = 1.95$

$P(|Z| > |z_0|) = 2P(|Z| > |z_0|)$
$= 2[0.02559] = 0.05118$

e) $z_0 = -0.10$

$P(|Z| > |z_0|) = 2P(|Z| > |z_0|)$
$= 2[0.46017] = 0.92034$

2.6. Suppose that we are testing $H_0 : \mu_1 = \mu_2$ versus $H_0 : \mu_1 > \mu_2$ where the two sample sizes are $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the $P$-value for the following observed values of the test statistic.

(a) $t_0 = 2.31$ (b) $t_0 = 3.60$ (c) $t_0 = 1.95$ (d) $t_0 = 2.19$

for this question, I am using a t-table but only looking at the row for degrees of freedom $10-1=9$

I am searching for where the given t statistic lays in that row.

| df/p | 0.40 | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.324920 | 1.000000 | 3.077684 | 6.313752 | 12.70620 | 31.82052 | 63.65674 | 636.6192 |
| 2 | 0.288675 | 0.816497 | 1.885618 | 2.919986 | 4.30265 | 6.96456 | 9.92484 | 31.5991 |
| 3 | 0.276671 | 0.764892 | 1.637744 | 2.353363 | 3.18245 | 4.54070 | 5.84091 | 12.9240 |
| 4 | 0270722 | 0.740697 | 1.533206 | 2.131847 | 2.77645 | 3.74695 | 4.60409 | 8.6103 |
| 5 | 0.267181 | 0.726687 | 1.475884 | 2.015048 | 2.57058 | 3.36493 | 4.03214 | 6.8688 |
| 6 | 0.264835 | 0.717558 | 1.439756 | 1.943180 | 2.44691 | 3.14267 | 3.70743 | 5.9588 |
| 7 | 0.263167 | 0.711142 | 1.414924 | 1.894579 | 2.36462 | 2.99795 | 3.49948 | 5.4079 |
| 8 | 0.261921 | 0.706387 | 1.396815 | 1.859548 | 2.30600 | 2.89646 | 3.35539 | 5.0413 |
| 9 | 0.260955 | 0.702722 | 1.383029 | 1.833113 | 2.26216 | 2.82144 | 3.24984 | 4.7809 |
| 10 | 0260185 | 0.699812 | 1.372184 | 1.812461 | 2.22814 | 2.76377 | 3.16927 | 4.5869 |
| 11 | 0259556 | 0.697445 | 1.363430 | 1.795885 | 2.20099 | 2.71808 | 3.10581 | 4.4370 |
| 12 | 0259033 | 0.695483 | 1.356217 | 1.782288 | 2.17881 | 2.68100 | 3.05454 | 43178 |
| 13 | 0.258591 | 0.693829 | 1.350171 | 1.770933 | 2.16037 | 2.65031 | 3.01228 | 4.2208 |
| 14 | 0.258213 | 0.692417 | 1.345030 | 1.761310 | 2.14479 | 2.62449 | 2.97684 | 4.1405 |
| 15 | 0.257885 | 0.691197 | 1.340606 | 1.753050 | 2.13145 | 2.60248 | 2.94671 | 4.0728 |
| 16 | 0257599 | 0.690132 | 1.336757 | 1.745884 | 2.11991 | 2.58349 | 2.92078 | 4.0150 |
| 17 | 0.257347 | 0.689195 | 1.333379 | 1.739607 | 2.10982 | 2.56693 | 2.89823 | 3.9651 |
| 18 | 0.257123 | 0.688364 | 1.330391 | 1.734064 | 2.10092 | 2.55238 | 2.87844 | 3.9216 |
| 19 | 0.256923 | 0.687621 | 1.327728 | 1.729133 | 2.09302 | 2.53948 | 2.86093 | 3.8834 |
| 20 | 0.256743 | 0.686954 | 1.325341 | 1.724718 | 2.08596 | 2.52798 | 2.84534 | 3.8495 |
| 21 | 0.256580 | 0.686352 | 1.323188 | 1.720743 | 2.07961 | 2.51765 | 2.83136 | 3.8193 |
| 22 | 0256432 | 0.685805 | 1.321237 | 1.717144 | 2.07387 | 2.50832 | 2.81876 | 3.7921 |
| 23 | 0256297 | 0.685306 | 1.319460 | 1.713872 | 2.06866 | 2.49987 | 2.80734 | 3.7676 |
| 24 | 0.256173 | 0.684850 | 1.317836 | 1.710882 | 2.06390 | 2.49216 | 2.79694 | 3.7454 |
| 25 | 0.256060 | 0.684430 | 1.316345 | 1.708141 | 2.05954 | 2.48511 | 2.78744 | 3.7251 |
| 26 | 0.255955 | 0.684043 | 1.314972 | 1.705618 | 2.05553 | 2.47863 | 2.77871 | 3.7066 |
| 27 | 0.255858 | 0.683685 | 1.313703 | 1.703288 | 2.05183 | 2.47266 | 2.77068 | 3.6896 |
| 28 | 0.255768 | 0.683353 | 1.312527 | 1.701131 | 2.04841 | 2.46714 | 2.76326 | 3.6739 |
| 29 | 0.255684 | 0.683044 | 1.311434 | 1.699127 | 2.04523 | 2.46202 | 2.75639 | 3.6594 |
| 30 | 0.255605 | 0.682756 | 1.310415 | 1.697261 | 2.04227 | 2.45726 | 2.75000 | 3.6460 |
| z | 0.253347 | 0.674490 | 1.281552 | 1.644854 | 1.95996 | 2.32635 | 2.57583 | 3.2905 |
| CI | ——— | ——— | 80% | 90% | 95% | 98% | 99% | 99.9% |

a) $t_0 = 2.31$

since $2.101 < t_0 = 2.31 < 2.552$

$0.01 < p < 0.025$

b) $t_0 = 3.60$

since $2.878 < t_0 = 3.60 < 3.922$

$0.0005 < p < 0.001 < 0.005$

added because table above only has 0.005 & 0.0005 and not 0.001.

c) $t_0 = 1.95$

since $1.734 < t_0 = 1.95 < 2.101$

$0.025 < p < 0.05$

d) $t_0 = 2.19$

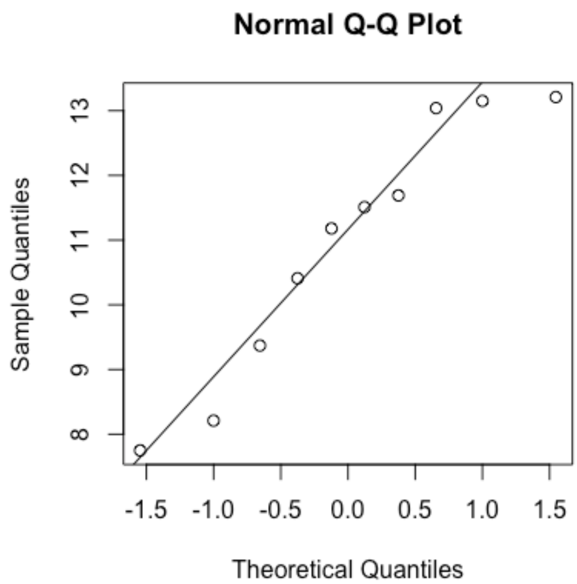
since $2.101 < t_0 = 2.19 < 2.552$

$0.01 < p < 0.025$

2.7. Consider the following sample data:

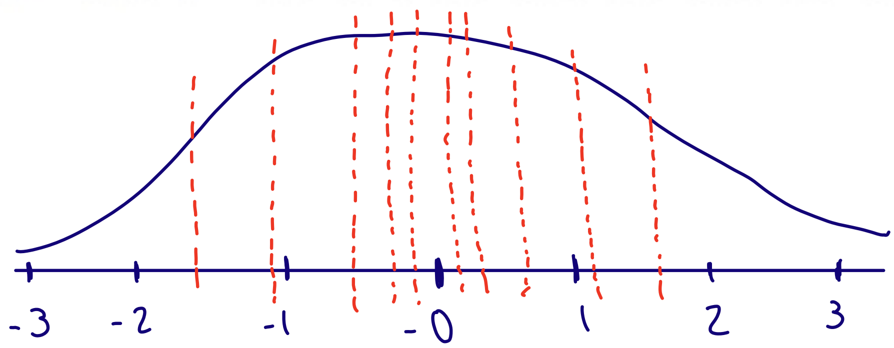
$$9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, 7.75$$

Is it reasonable to assume that this data is a sample from a normal distribution?
Is there evidence to support a claim that the mean of the population is 10?



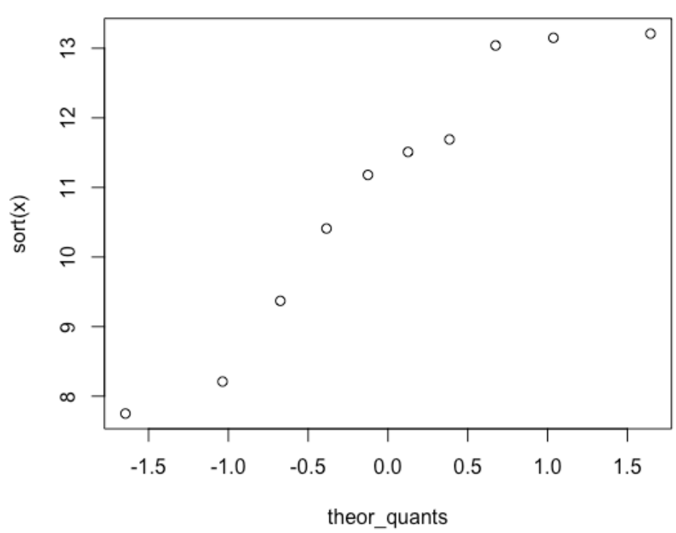

To check if the data is normal, I plotted the data using qqnorm(x) & qqline(x). I think it follows the line very well can can be said to follow a normal distribution

```
# using the equation to compute the percentiles
p_j <- ((1:10) - 0.5)/10 # 0.05 0.15 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95
```




split into 10 percentiles

Using a Z table, I can approximate the theoretical quantiles. Z=[-1.64, -1.04, -0.67, -0.39, -0.13, 0.13, 0.39, 0.67, 1.04, 1.64]




When we plot the sorted values against the theoretical quantiles, we get our qqplot.

$H_0: \mu = 10$

$H_A: \mu \neq 10$

TS: $\dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{10.952 - 10}{\frac{1.993472}{\sqrt{10}}} = 1.5102$ p-value *calculated in R*

↳ 0.1653

```
> t.test(x, mu = 10)

        One Sample t-test

data:  x
t = 1.5102, df = 9, p-value = 0.1653
alternative hypothesis: true mean is not equal to 10
95 percent confidence interval:
  9.525956 12.378044
sample estimates:
mean of x
   10.952

> 2*(1-pt(ts,9))
[1] 0.165281
```

With a p-value of 0.1653 > $\alpha$, we fail to reject the null hypothesis. There is not enough statistically significant evidence to state that the true mean of the population is not 10.

2.15

**Two-Sample T-Test and CI: Y1, Y2**

```
Two-sample T for Y1 vs Y2

      N     Mean    Std. Dev.   SE Mean
Y1   20    50.19       1.71       0.38
Y2   20    52.52       2.48       0.55

Difference = mu (X1) - mu (X2)
Estimate for difference: -2.33341
95% CI for difference: (-3.69547, -0.97135)
T-Test of difference = 0 (vs not =) : T-Value = -3.47
P-Value = 0.001 DF = 38
Both use Pooled Std. Dev. = 2.1277
```

(a) Can the null hypothesis be rejected at the 0.05 level? Why?

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$

TS: $-3.47$

p-value: $0.001$

With a p-value of $0.001 < \alpha = 0.05$, we reject the null hypothesis. We have statistically significant evidence to conclude a difference in the means for population 1 and population 2.

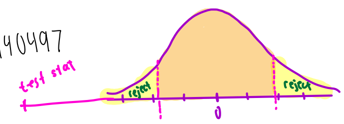
(b) Is this a one-sided or a two-sided test?

Since we are testing for a difference and not whether one population mean is greater than another, this is a two-sided test.

(c) If the hypotheses had been $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 \neq 2$ would you reject the null hypothesis at the 0.05 level?

Since the sample difference is 2.33341 and the pooled standard deviation is 2.1277, we can calculate the new test statistic:

$$t = \frac{-2.33341 - 2}{2.1277\sqrt{\frac{1}{20} + \frac{1}{20}}} = -6.440497$$




Using the same t-table from question 2, the critical value for a t-distribution of df=38 follows very close to the z-score for alpha = 0.05 (2) (since it is 2-sided) is approximately 1.96.

With a test statistic of $|-6.440497| >$ the critical value 1.96, we reject the null hypothesis. There is statistically significant evidence to conclude that the difference in the two population means does not equal 2.

(d) If the hypotheses had been $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 < 2$ would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

You can answer this question by looking at the one sided t critical value on the t-table for df=38. You do not need to make any more calculations. The t value for $\alpha = 0.05$ for a one-sided test is 1.684. Since the t-statistic we calculated in the previous part is still > the new critical value 1.684, we still reject the null hypothesis. There is enough statistically significant evidence to conclude that the difference between population means is less than 2.

(e) Use the output and the $t$ table to find a 95 percent upper confidence bound on the difference in means.

Confidence interval equation for two-sample t-test: $(\bar{x}_1 - \bar{x}_2) \pm t_{df=n_1+n_2-2} \times SE$

$SE = S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 2.1277\sqrt{\frac{1}{10}} = 0.6728378178$ $\quad t_{df=n_1+n_2-2} = 1.684$

$\bar{x}_1 - \bar{x}_2 = -2.33341$ $\quad -2.33341 + 1.684 \cdot 0.6728 \Rightarrow (-\infty, -1.1969)$

The upper bound on the 95% confidence interval on the difference of means is -1.1969.

(f) What is the $P$-value if the hypotheses are $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 \neq 2$?

$p = 2 * p(t > |t_0|) = 2 \cdot (1 - p(t \le |t_0|)) = 2 * (1 - P(t \le 6.440497))$

Using R, I calculated the p-value with the new test statistic 0.4955 and df=38.

$H_0: \mu_1 - \mu_2 = 2$

$H_A: \mu_1 - \mu_2 \neq 2$

TS: from previous part $= -6.440497$

p-value:
```
> 2*(1-pt(6.440497,38))
[1] 1.418836e-07
```

With a p-value of $1.418e^{-07} < \alpha = 0.05$, we reject $H_0$. There is statistically significant evidence to conclude that the difference between the population means is not equal to 2.

2.16

The breaking strength of a fiber is required to be at least 150psi. Past experience has indicated that the standard deviation of breaking strength is $\sigma = 3$psi. A random sample of four specimens is tested, and the results are $y_1 = 145, y_2 = 153, y_3 = 150$, and $y_4 = 147$.

(a) State the hypotheses that you think should be tested in this experiment.

$H_0: \mu \geq 150$ psi (The mean breaking strength is at least 150 psi)

$H_A: \mu < 150$ psi (The mean breaking strength is less than 150 psi).

Left tailed

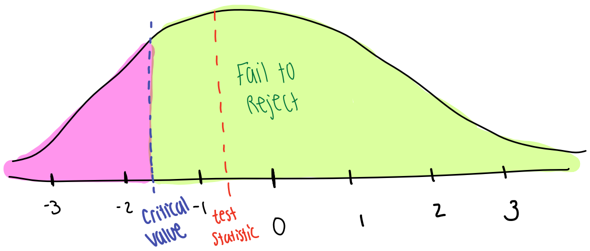
$\therefore$ reject when

$Z_0 \leq -Z_{1-\alpha}$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

TS: $\frac{\bar{y} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{148.75 - 150}{\frac{3}{\sqrt{4}}} = -0.833$

Critical value: $Z_{0.05} = -1.645$




With a test statistic $-0.833 > -1.645$, we fail to reject the null hypothesis. There is not enough statistically significant evidence to conclude that the mean breaking strength is less than 150 psi.

(c) Find the $P$-value for the test in part (b).

```
> pnorm(-0.833)
[1] 0.2024224
```

$P(Z \leq Z_0) = P(Z \leq -0.833)$

Following strongly with the conclusion we obtained in the previous part, we failed to reject the null hypothesis because our p-value of $0.2024 > \alpha = 0.05$.

(d) Construct a 95 percent confidence interval on the mean breaking strength.

since $z_{0.05} = 1.645$ and $SE = \frac{3}{\sqrt{4}} = 1.5$, then the 95% confidence interval is

$(-\infty\ ,\ 148.75 + 1.645 * 1.5) = (-\infty, 151.22]$

I am 95% confidence that the true mean breaking strength has a psi of up to about 151.22.

2.30

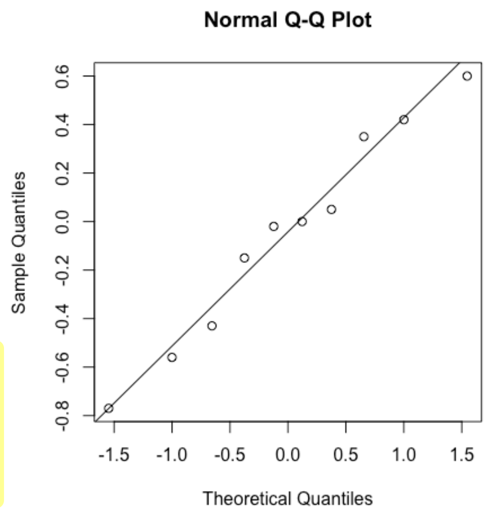
An article in the journal Neurology (1998, Vol. 50 , pp. 1246-1252) observed that monozygotic twins share numerous physical, psychological, and pathological traits. The investigators measured an intelligence score of 10 pairs of twins. The data obtained are as follows:

| *Pair* | *BirthOrder* :1 | *BirthOrder* :2 |
|---|---|---|
| 1 | 6.08 | 5.73 |
| 2 | 6.22 | 5.80 |
| 3 | 7.99 | 8.42 |
| 4 | 7.44 | 6.84 |
| 5 | 6.48 | 6.43 |
| 6 | 7.99 | 8.76 |
| 7 | 6.32 | 6.32 |
| 8 | 7.60 | 7.62 |
| 9 | 6.03 | 6.59 |
| 10 | 7.52 | 7.67 |

(a) Is the assumption that the difference in score is normally distributed reasonable?

for this question, I used R to calculate the differences in score and plot the QQ plot.

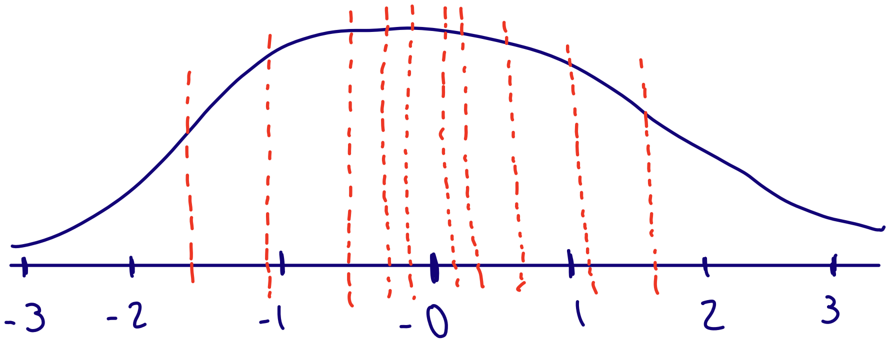
```
bo1 <- c(6.08,6.22,7.99,7.44,6.48,7.99,6.32,7.6,6.03,7.52)
bo2 <- c(5.73,5.8,8.42,6.84,6.43,8.76,6.32,7.62,6.59,7.67)
diff <- bo1 - bo2
qqnorm(diff)
qqline(diff)
```




Since the observations follow the line really well, it is reasonable to assume the differences in scores is normally distributed.

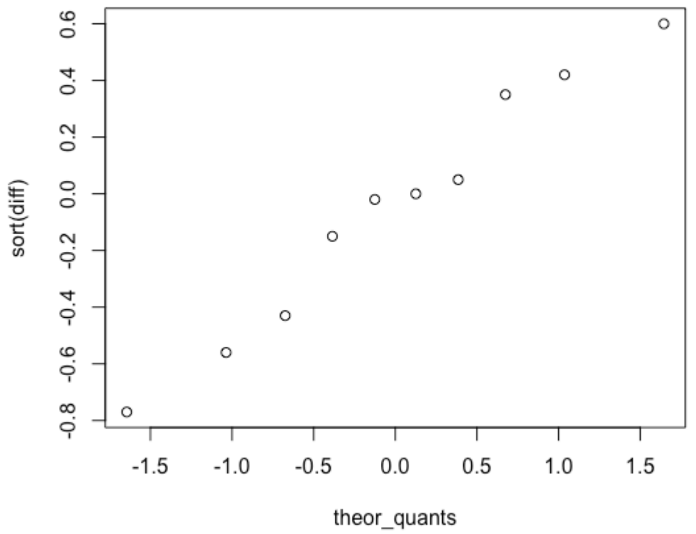
# Q-Q plot by hand

Since this data also has $n=10$ samples, we can refer back to the division/split of the quantiles from question 2.7.




split into 10 percentiles

Using a Z table, I can approximate the theoretical quantiles. $Z=[-1.64, -1.04, -0.67, -0.39, -0.13, 0.13, 0.39, 0.67, 1.04, 1.64]$




When we plot the sorted values against the theoretical quantiles, we get our qqplot.

(b) Find a 95% confidence interval on the difference in mean score. Is there any evidence that mean score depends on birth order?

With critical value $t_{0.05,9} = 2.2616$, $se = \frac{sd(diff)}{\sqrt{10}} = 0.139431$ and $\bar{x} = -0.051$, we can compute the confidence interval with the following formula:

$$-0.051 \pm 2.2616 * 0.139431 \Rightarrow [-0.366, 0.264]$$

We are 95% confident that the true difference in mean score is between -0.366 and 0.264. Since 0 falls within the confidence interval, we cannot conclude that there is a difference in mean scores based on birth order.

(c) Test an appropriate set of hypotheses indicating that the mean score does not depend on birth order.

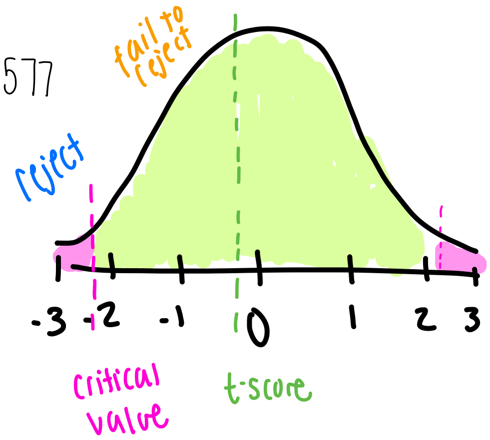
$H_0$: $\mu_1 - \mu_2$ or $\bar{d}$ $= 0$ (The mean difference in intelligence scores of twins is 0 and not dependent on birth order.)

$H_A$: $\mu_1 - \mu_2$ or $\bar{d}$ $\neq 0$ (The mean difference in intelligence scores of twins is not 0 and is dependent on birth order.)

TS: $\dfrac{\bar{d} - 0}{\frac{s}{\sqrt{n}}}$ where $\bar{d}$ is average difference in scores.

$$\frac{-0.051}{\frac{0.4409}{\sqrt{10}}} = \frac{-0.051}{1} \cdot \frac{\sqrt{10}}{0.4409} = -0.36577$$

Critical Value: $t_{0.05(2),9} = 2.2616$




With a test statistic $|-0.36577| <$ our critical value 2.2616, we fail to reject the null hypothesis. There is not enough statistically significant evidence to conclude a difference in mean scores based on twin birth order.

| z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0 | .50000 | .49601 | .49202 | .48803 | .48405 | .48006 | .47608 | .47210 | .46812 | .46414 |
| -0.1 | .46017 | .45620 | .45224 | .44828 | .44433 | .44034 | .43640 | .43251 | .42858 | .42465 |
| -0.2 | .42074 | .41683 | .41294 | .40905 | .40517 | .40129 | .39743 | .39358 | .38974 | .38591 |
| -0.3 | .38209 | .37828 | .37448 | .37070 | .36693 | .36317 | .35942 | .35569 | .35197 | .34827 |
| -0.4 | .34458 | .34090 | .33724 | .33360 | .32997 | .32636 | .32276 | .31918 | .31561 | .31207 |
| -0.5 | .30854 | .30503 | .30153 | .29806 | .29460 | .29116 | .28774 | .28434 | .28096 | .27760 |
| -0.6 | .27425 | .27093 | .26763 | .26435 | .26109 | .25785 | .25463 | .25143 | .24825 | .24510 |
| -0.7 | .24196 | .23885 | .23576 | .23270 | .22965 | .22663 | .22363 | .22065 | .21770 | .21476 |
| -0.8 | .21186 | .20897 | .20611 | .20327 | .20045 | .19766 | .19489 | .19215 | .18943 | .18673 |
| -0.9 | .18406 | .18141 | .17879 | .17619 | .17361 | .17106 | .16853 | .16602 | .16354 | .16109 |
| -1 | .15866 | .15625 | .15386 | .15151 | .14917 | .14686 | .14457 | .14231 | .14007 | .13786 |
| -1.1 | .13567 | .13350 | .13136 | .12924 | .12714 | .12507 | .12302 | .12100 | .11900 | .11702 |
| -1.2 | .11507 | .11314 | .11123 | .10935 | .10749 | .10565 | .10383 | .10204 | .10027 | .09853 |
| -1.3 | .09680 | .09510 | .09342 | .09176 | .09012 | .08851 | .08692 | .08534 | .08379 | .08226 |
| -1.4 | .08076 | .07927 | .07780 | .07636 | .07493 | .07353 | .07215 | .07078 | .06944 | .06811 |
| -1.5 | .06681 | .06552 | .06426 | .06301 | .06178 | .06057 | .05938 | .05821 | .05705 | .05592 |
| -1.6 | .05480 | .05370 | .05262 | .05155 | .05050 | .04947 | .04846 | .04746 | .04648 | .04551 |
| -1.7 | .04457 | .04363 | .04272 | .04182 | .04093 | .04006 | .03920 | .03836 | .03754 | .03673 |
| -1.8 | .03593 | .03515 | .03438 | .03362 | .03288 | .03216 | .03144 | .03074 | .03005 | .02938 |
| -1.9 | .02872 | .02807 | .02743 | .02680 | .02619 | .02559 | .02500 | .02442 | .02385 | .02330 |
| -2 | .02275 | .02222 | .02169 | .02118 | .02068 | .02018 | .01970 | .01923 | .01876 | .01831 |
| -2.1 | .01786 | .01743 | .01700 | .01659 | .01618 | .01578 | .01539 | .01500 | .01463 | .01426 |
| -2.2 | .01390 | .01355 | .01321 | .01287 | .01255 | .01222 | .01191 | .01160 | .01130 | .01101 |
| -2.3 | .01072 | .01044 | .01017 | .00990 | .00964 | .00939 | .00914 | .00889 | .00866 | .00842 |
| -2.4 | .00820 | .00798 | .00776 | .00755 | .00734 | .00714 | .00695 | .00676 | .00657 | .00639 |
| -2.5 | .00621 | .00604 | .00587 | .00570 | .00554 | .00539 | .00523 | .00508 | .00494 | .00480 |
| -2.6 | .00466 | .00453 | .00440 | .00427 | .00415 | .00402 | .00391 | .00379 | .00368 | .00357 |
| -2.7 | .00347 | .00336 | .00326 | .00317 | .00307 | .00298 | .00289 | .00280 | .00272 | .00264 |
| -2.8 | .00256 | .00248 | .00240 | .00233 | .00226 | .00219 | .00212 | .00205 | .00199 | .00193 |
| -2.9 | .00187 | .00181 | .00175 | .00169 | .00164 | .00159 | .00154 | .00149 | .00144 | .00139 |
| -3 | .00135 | .00131 | .00126 | .00122 | .00118 | .00114 | .00111 | .00107 | .00104 | .00100 |
| -3.1 | .00097 | .00094 | .00090 | .00087 | .00084 | .00082 | .00079 | .00076 | .00074 | .00071 |
| -3.2 | .00069 | .00066 | .00064 | .00062 | .00060 | .00058 | .00056 | .00054 | .00052 | .00050 |
| -3.3 | .00048 | .00047 | .00045 | .00043 | .00042 | .00040 | .00039 | .00038 | .00036 | .00035 |
| -3.4 | .00034 | .00032 | .00031 | .00030 | .00029 | .00028 | .00027 | .00026 | .00025 | .00024 |
| -3.5 | .00023 | .00022 | .00022 | .00021 | .00020 | .00019 | .00019 | .00018 | .00017 | .00017 |
| -3.6 | .00016 | .00015 | .00015 | .00014 | .00014 | .00013 | .00013 | .00012 | .00012 | .00011 |
| -3.7 | .00011 | .00010 | .00010 | .00010 | .00009 | .00009 | .00008 | .00008 | .00008 | .00008 |
| -3.8 | .00007 | .00007 | .00007 | .00006 | .00006 | .00006 | .00006 | .00005 | .00005 | .00005 |
| -3.9 | .00005 | .00005 | .00004 | .00004 | .00004 | .00004 | .00004 | .00004 | .00003 | .00003 |
| -4 | .00003 | .00003 | .00003 | .00003 | .00003 | .00003 | .00002 | .00002 | .00002 | .00002 |

Say the actual percentile is 9%.

↳ theoretical quantile is -1.34