FAIR PROBLEMS: given a problem, outline a greedy strategy to solve it, describe the time of your approach, prove an exchange property.

activity selection problem

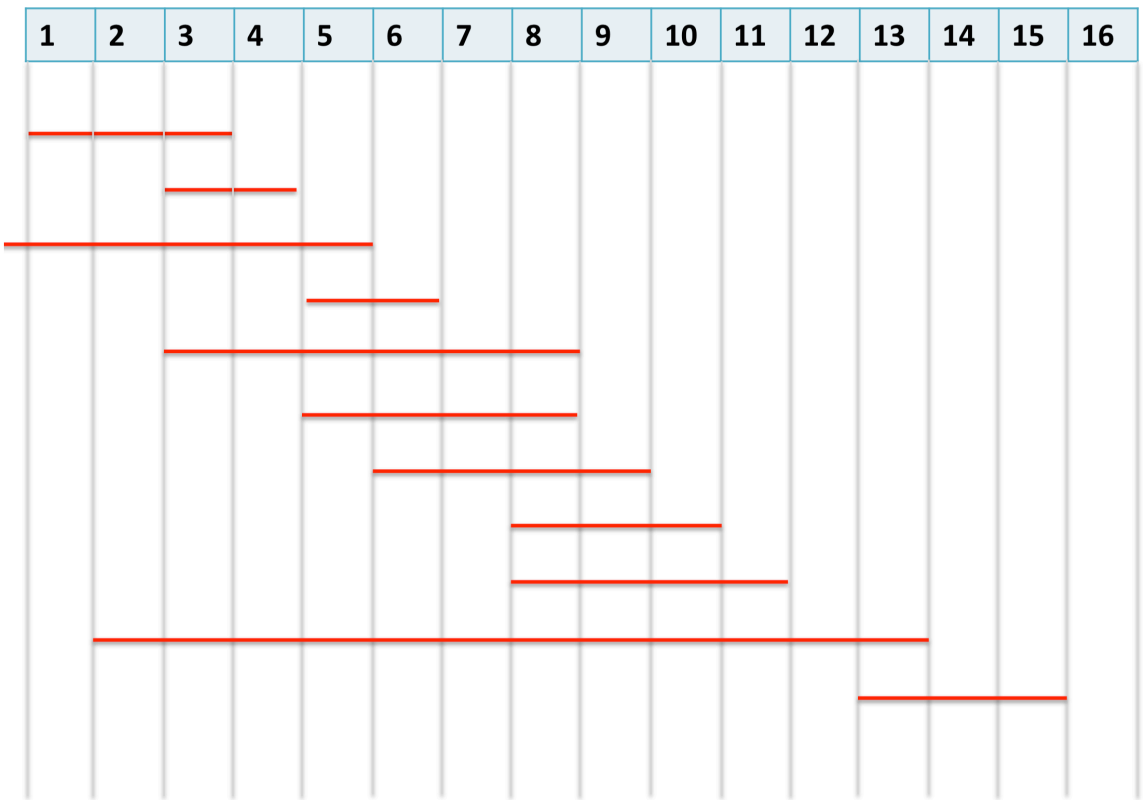
- given set $S=\{a_1, a_2, \ldots, a_n\}$ of possible activities
- each a_i has a **start** time s_i and **finish** time f_i
- $0 \leq s_i < f_i < \infty$
- a_i takes place during $[s_i,fi)$ (half-open interval)
- activities a_i and a_j are **compatible** if their intervals $[s_i,f_j)$ and $[s_j,f_j)$ do not overlap
- **problem**: select maximum size subset of compatible activities

example

i	1	2	3	4	5	6	7	8	9	10	11
s	1	3	0	5	3	5	6	8	8	2	12
f	4	5	6	7	9	9	10	11	12	14	16

- $\{a_3, a_9, a_{11}\}$ is a mutually compatible set
- but is not maximum
- both $\{a_1, a_4, a_8, a_{11}\}$ and $\{a_2, a_4, a_9, a_{11}\}$ are larger

- this is sorted by finish time (useful for greedy method, but we're not there yet)



dynamic programming approach

define $S_{ij} = \{ a_k \mid f_i \le s_k \text{ and } f_k \le s_j \}$

the set of activities that start after a_i finishes and finish before a_j starts – that is, they can be scheduled between a_i and a_j

subproblem: let c[i,j] be the size of the largest solution for S_{ij}

try scheduling all possible k

recurrence
- c[i,j] =0 if $S_{ij}=\emptyset$
- otherwise $c[i,j]=MAX \{ c[i,k]+c[k,j]+1 \mid a_k \in S_{ij} \}$

greedy choice

always pick the earliest finishing activity

define $S_k = \{ a_i \mid f_k \le s_i \}$
- activities starting after a_k is done

theorem 16.1 (p 418) Consider any non-empty subproblem S_k, and let a_m be an activity in S_k with the earliest finish time. Then a_m is included in some maximum-size subset of mutually compatible activities of S_k.

proof in text

Every Greedy algorithm needs:

- subproblems
- optimal substructure (like dynamic programming)
- greedy choice algorithm ★

Recursive Algo

RAS on inputs s (start times), f (finish times), k (subproblem), and n

```
RAS(s,f,k,n)
  m=k+1
  while m≤n and s[m]<f[k]
     m=m+1
  if m≤n
    return {a_m}∪RAS(s,f,m,n)
  else return ∅
```

find first a_m in S_k to finish

finish times already sorted, time now looks like O(n)

Iterative Algo

```
A={a_1}
k=1
for m=2 to n
   if s[m]≥f[k]
      A=A∪{a_m}
      k=m
return A
```

time is pretty clearly O(n), but don't forget the O(n*lg*n) time used to sort f

other similar problems

- (ex 16.1-4) interval coloring: given activities as above, assign the fewest number of colors to each activity so that no two overlapping activities have the same color
- (ex 16.1-5) same as activity selection, but in addition give each activity a_i a value v_i, and maximize the total value of selected activities

Greedy Strategy

1) determine optimal substructure of the problem
2) develop a recursive solution
3) show that if greedy choice made, only one subproblem remains
4) prove it is always safe to make greedy choice
5) develop recursive algorithm implementing greedy
6) convert recursive version to iterative

↳ THEORY: Matroids = ordered pair $M = (S, I)$ satisfying:

1) S is a finite set
2) I is a family of subsets of S, called the independent sets. It must have the hereditary property:
   if $B \in I$ and $A \subseteq B$ then $A \in I$
3) M has the exchange property:
   if $A \in I$, $B \in I$ and $|A| < |B|$, then $x \in B - A$ s.t. $A \cup \{x\} \in I$

applied to graphs

Given a graph G=(V,E), the graphic matroid of G is the structure $M_G=(S_G,I_G)$ where

- S_G is the set of edges E
- If A is a set of edges, then $A \in I_G$ iff A is acyclic. That is, the independent sets of edges of G are those that form a forest.

we can also add a weight function w:S→N to any matroid

generic greedy algorithm on matroid

```
greedy(M,w)
  A=∅
  sort the elements S of M by weight w
  for each x∈S taken in order
     if AU{x}∈I
       then A=AU{x}
return A
```

looks like MST

also, the dual of a matroid is a matroid
for graphs, the dual I are sets of edges containing a cycle
the combinatorics gets complicated