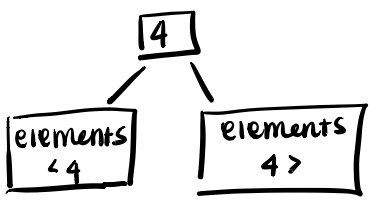
- A 2-3 tree is a B-tree with the order 3
  2-3-4 tree is a B-tree with the order 4

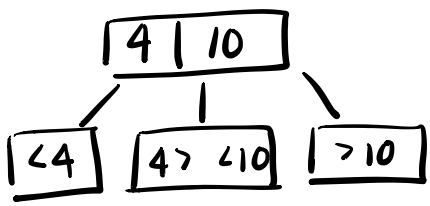
- They must be perfectly balanced: all paths from the root to a null node must have same length

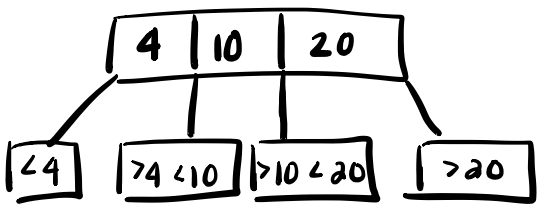
- insertions result in a split rather than a rotation

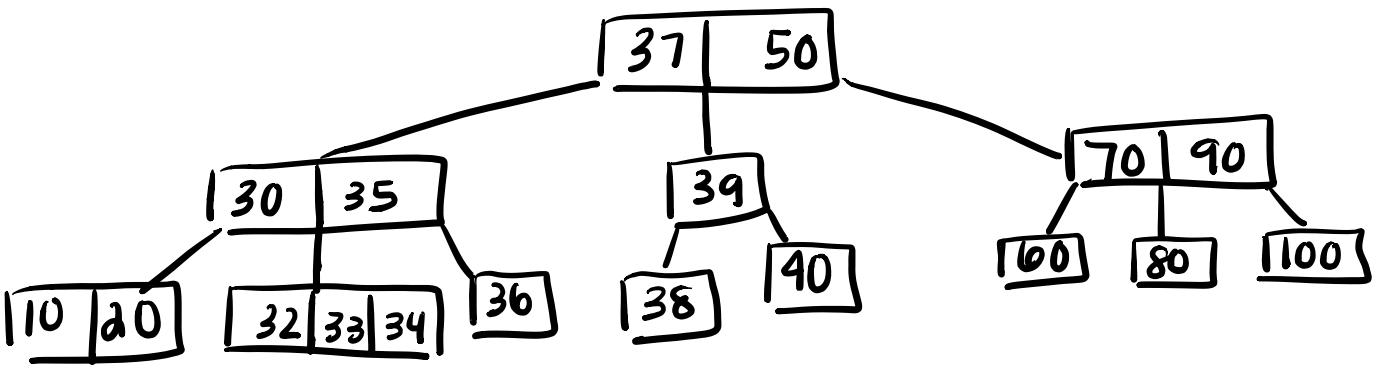
and RBT are a binary implementation of 2-3-4 trees

Quick example:









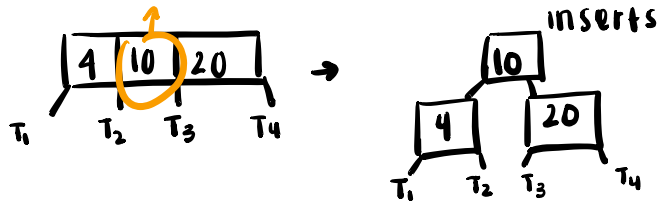
Insertion:

- can split a node when it is full or has overflowed

- splitting on insertion can be bottom-up which means put the node on the bottom of the tree and if over-flows, split on the way up (more common)

- top-down: when looking for insertion point, if full node seen, split

Splitting the node:



Specifications

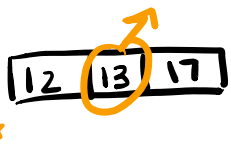
- fixed parameter t, called minimum degree
- nodes between $t-1$ and $2t-1$ keys
- therefore between t and $2t$ children
- root is exception: may have as few as 1 key (2 children)
- all NULL pointers have the same depth (distance from root
- 2-3-4 is a B-tree with minimum degree $t=2$

12, 13, 17, 10, 4, 6, 9, 15, 30, 25, 20, 40

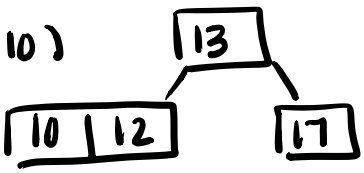
insert 12) → root

insert 13)

insert 17)



insert 10)



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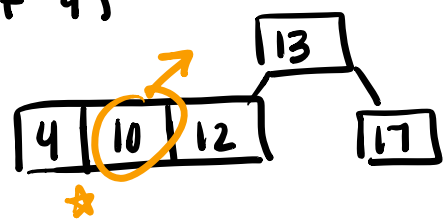

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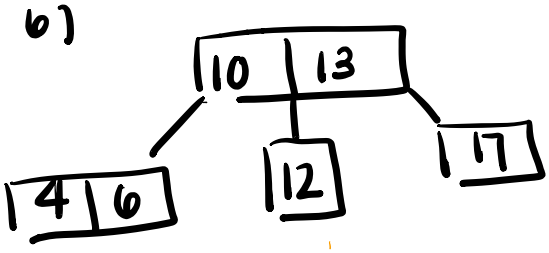

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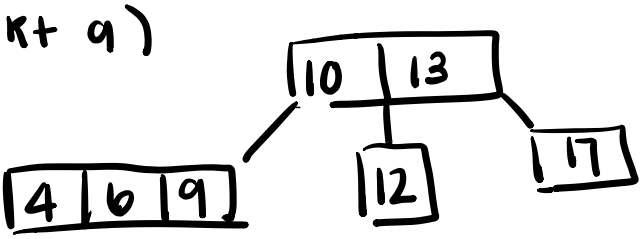

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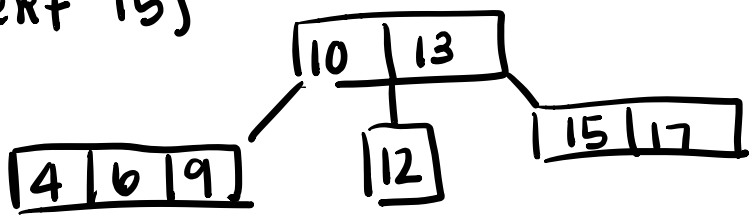
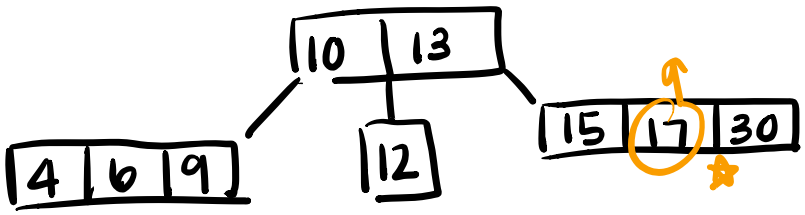
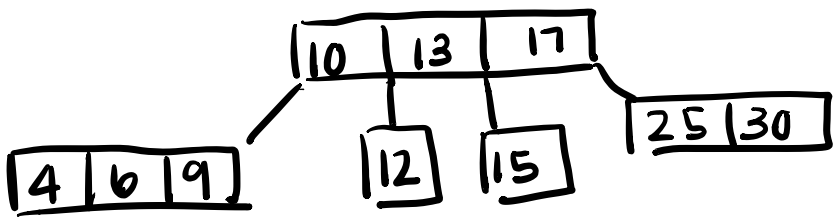


Insert 30)



Insert 25)



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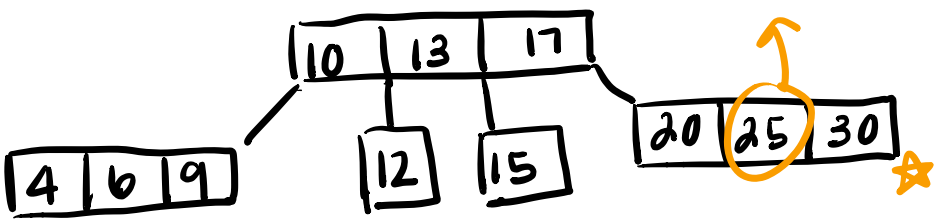
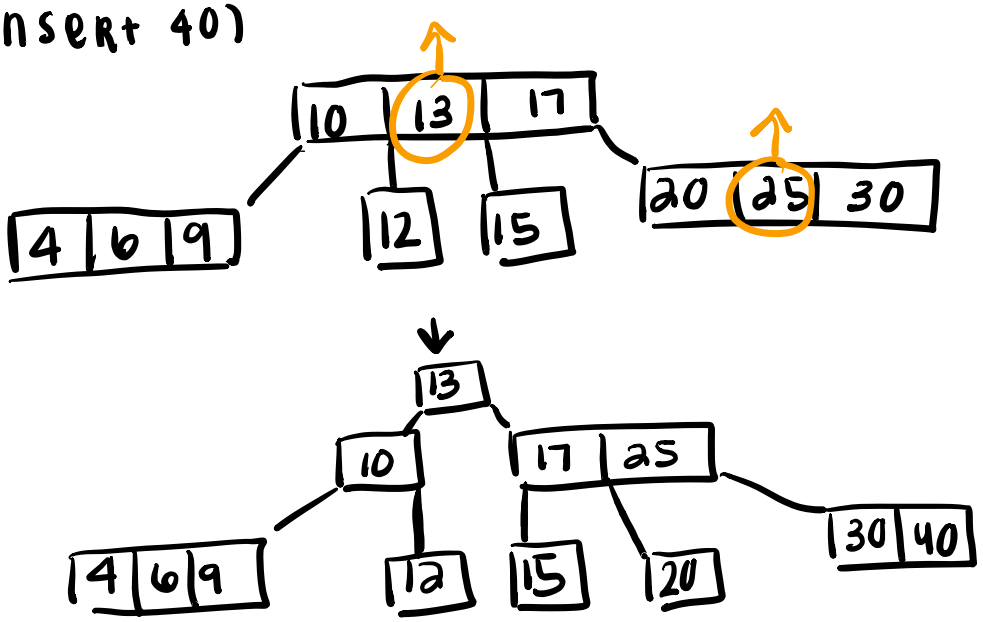


Insert 40)



TOP DOWN

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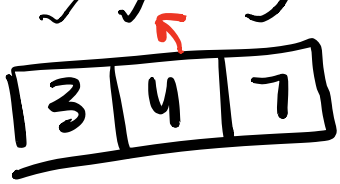

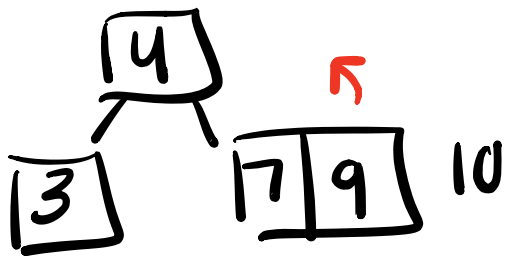

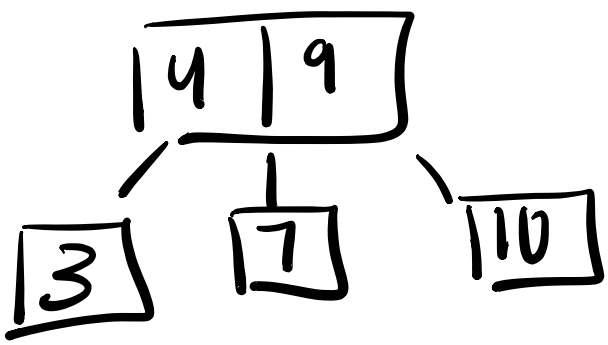

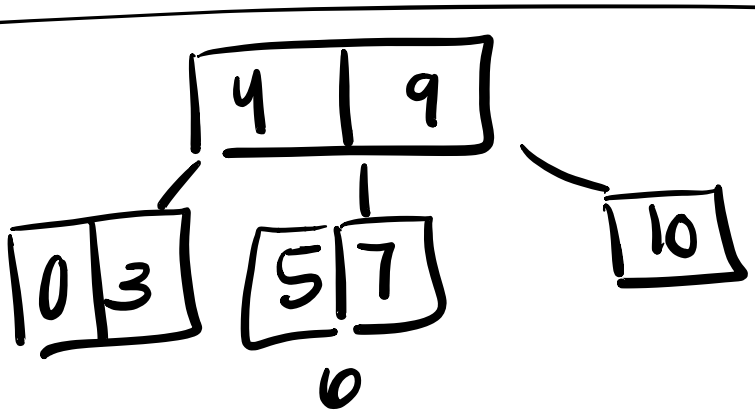

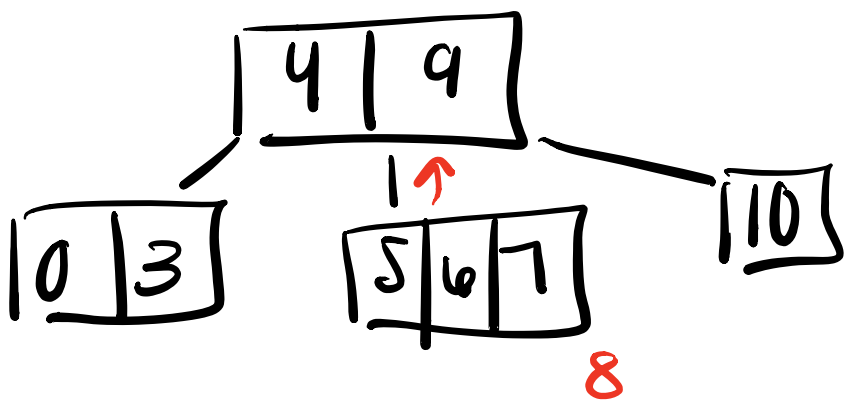

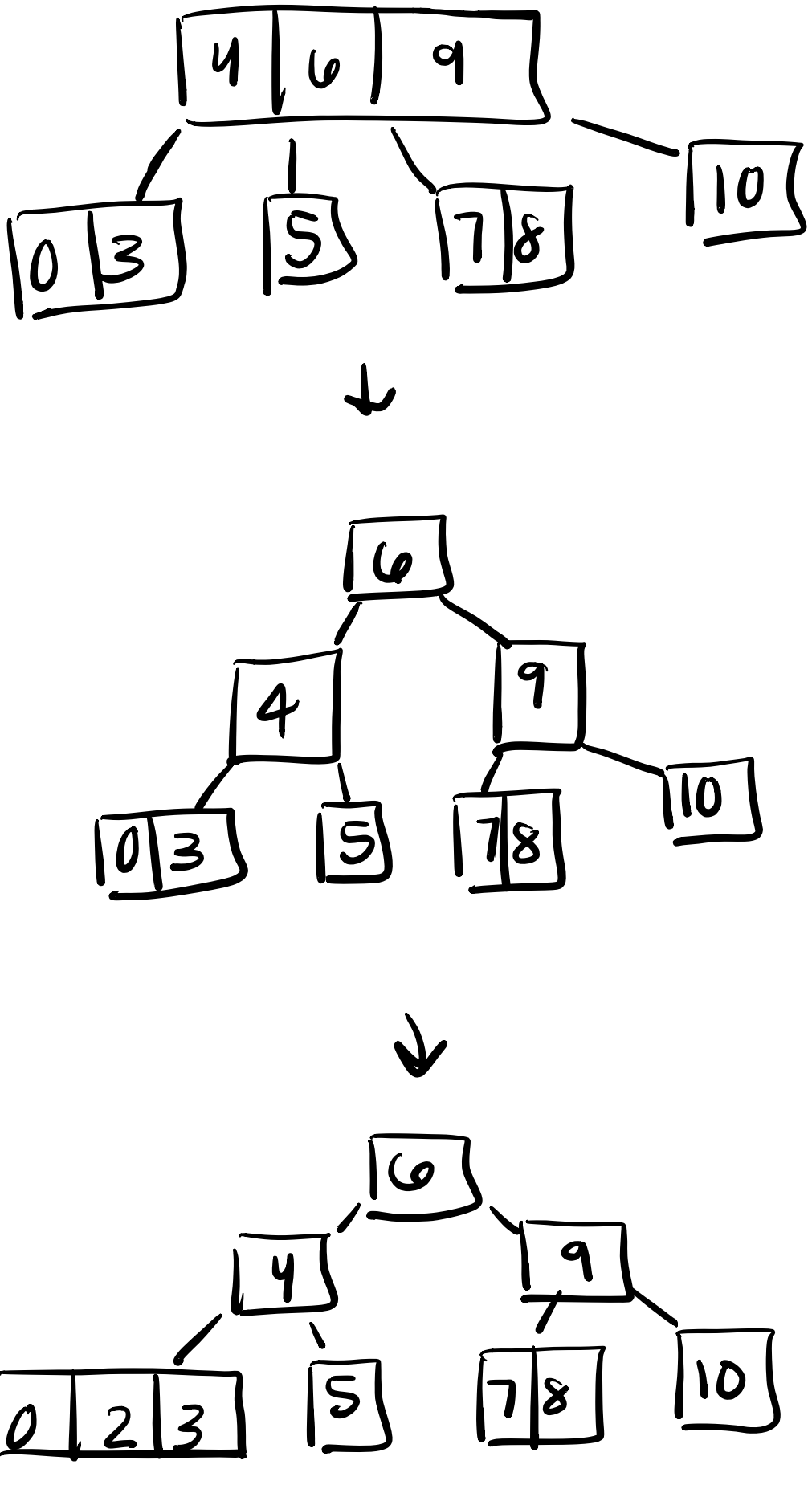
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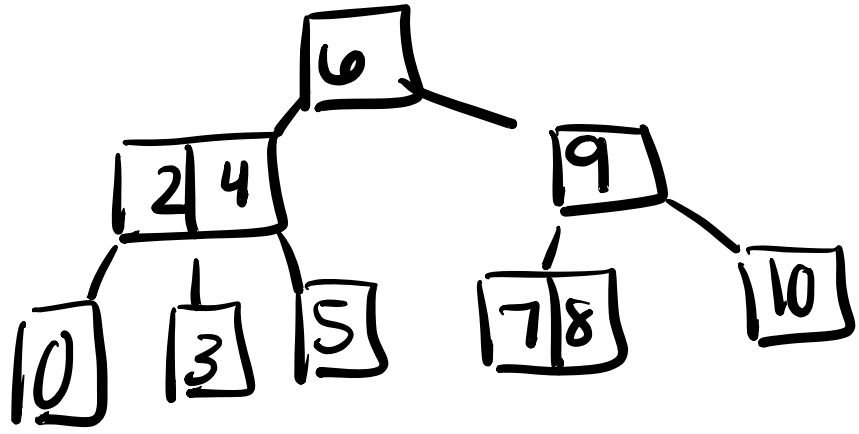
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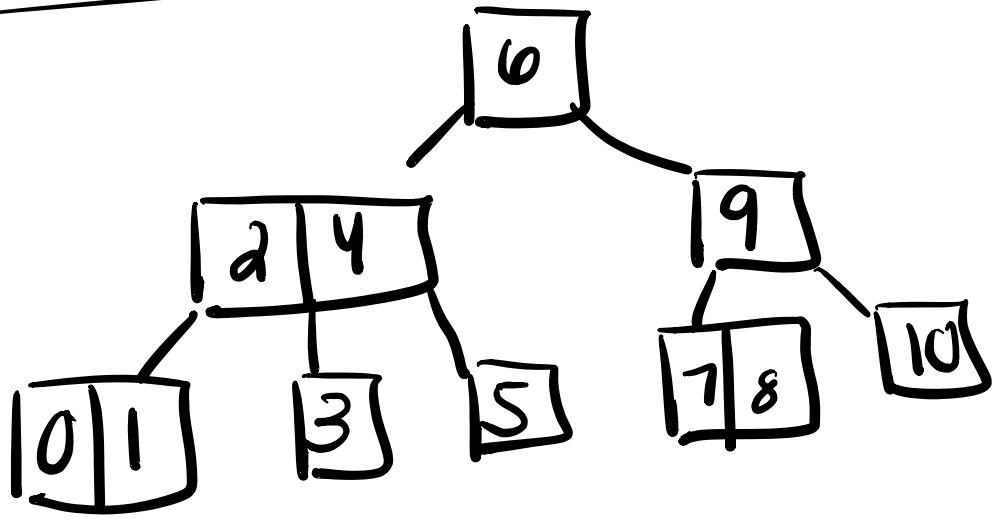
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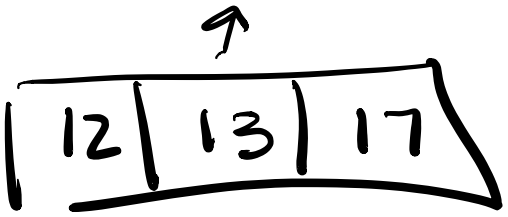
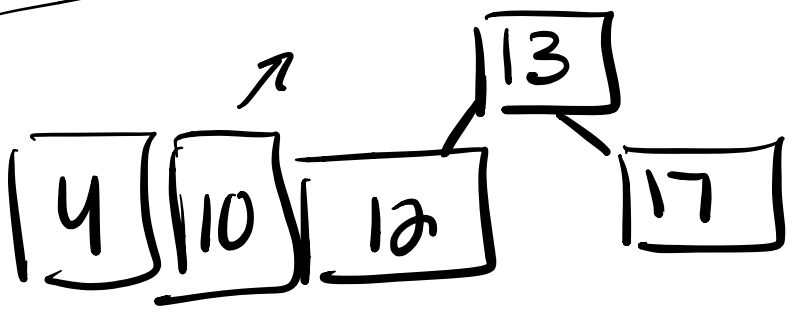
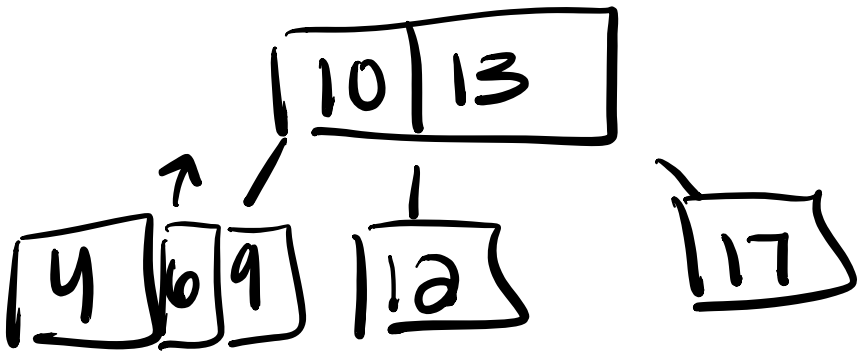
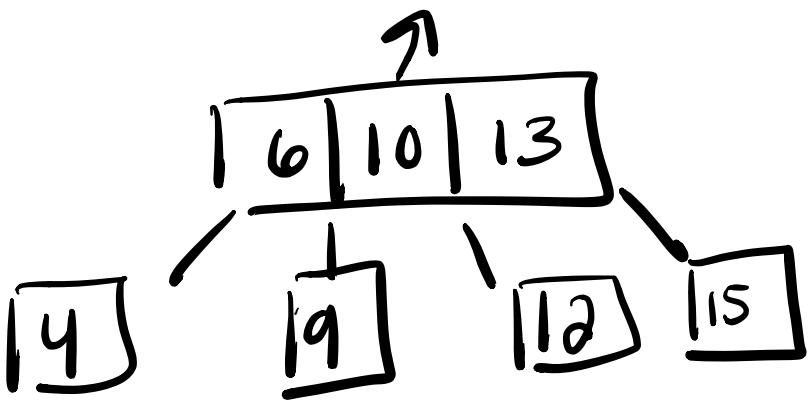
12, 13, 17, 10, 4, 6, 9, 15, 30, 25, 20, 40

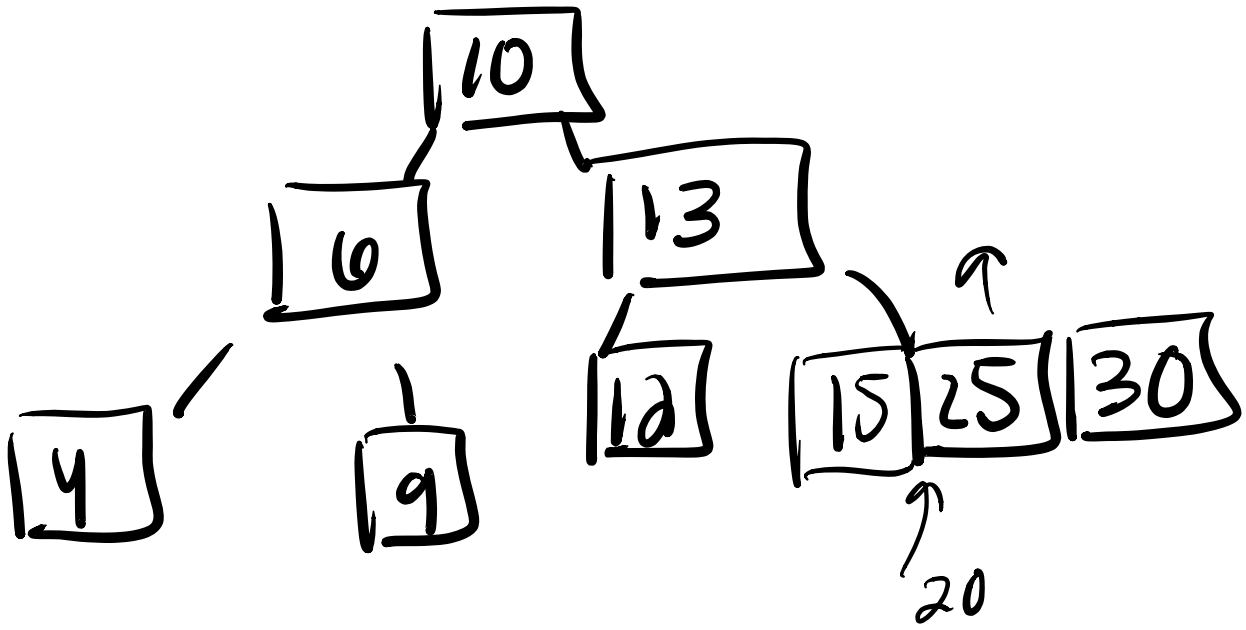
top-down

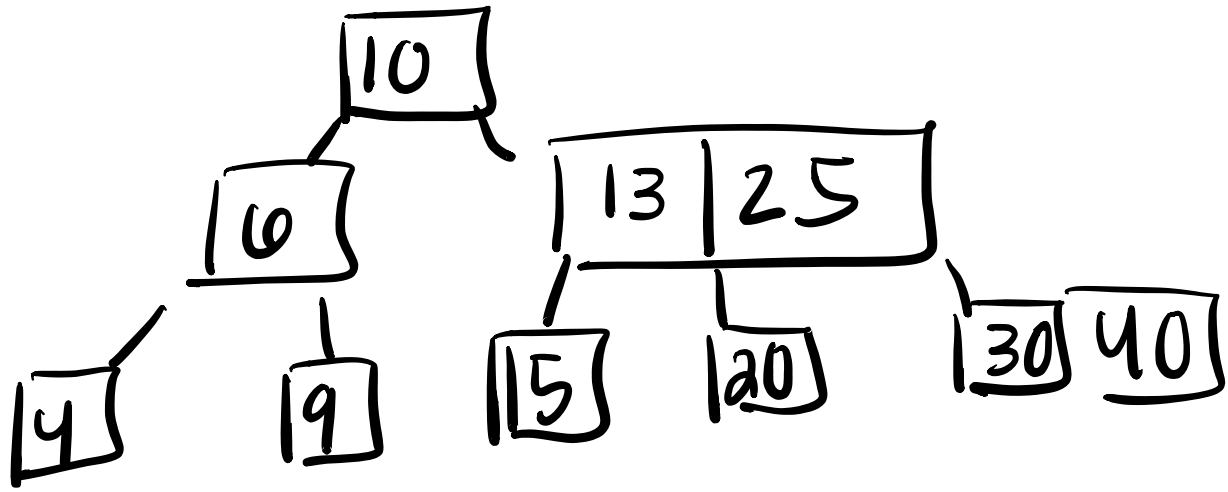






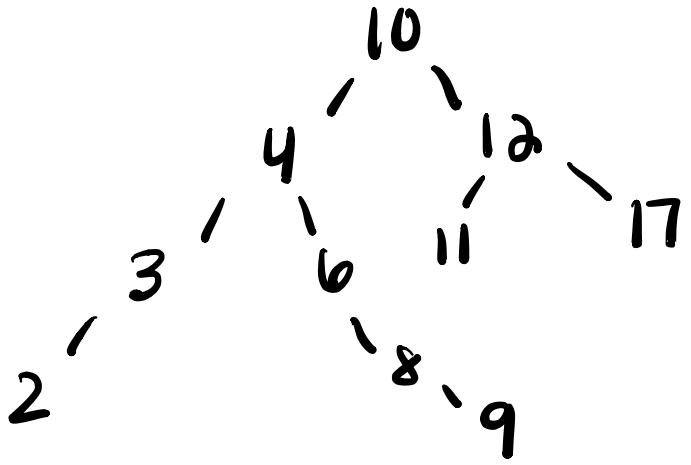






done!

BST [10, 4, 12, 3, 2, 11, 6, 17, 8, 9]



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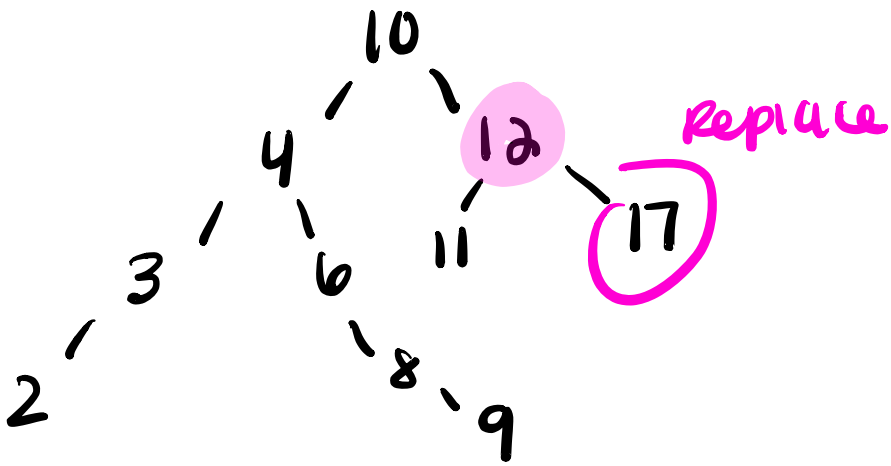

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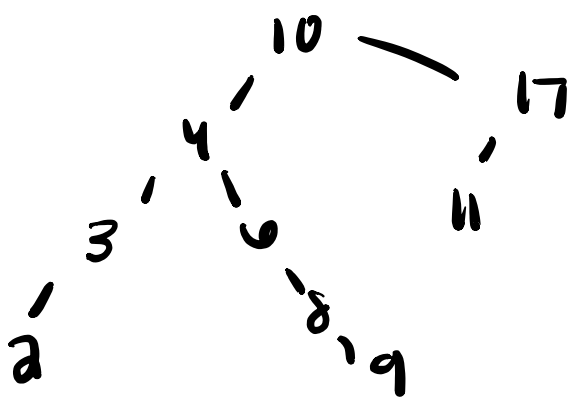

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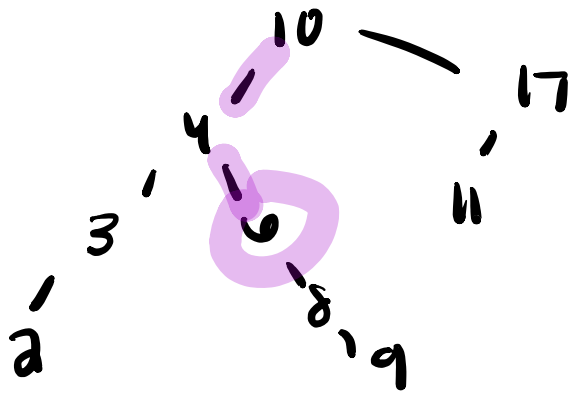

RBT:

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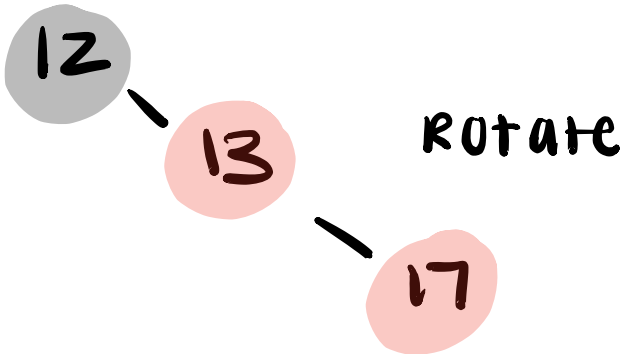

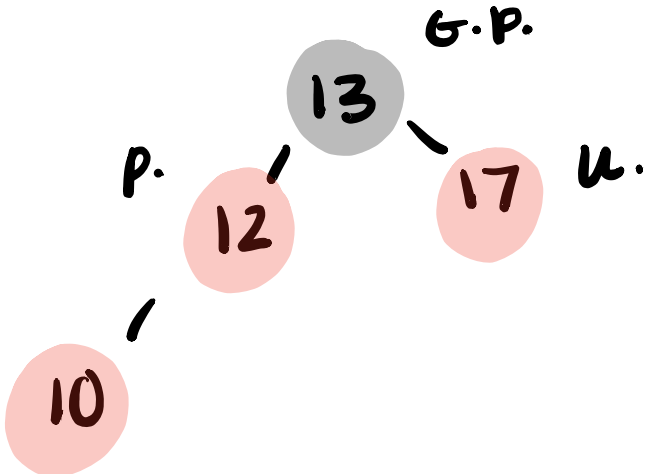

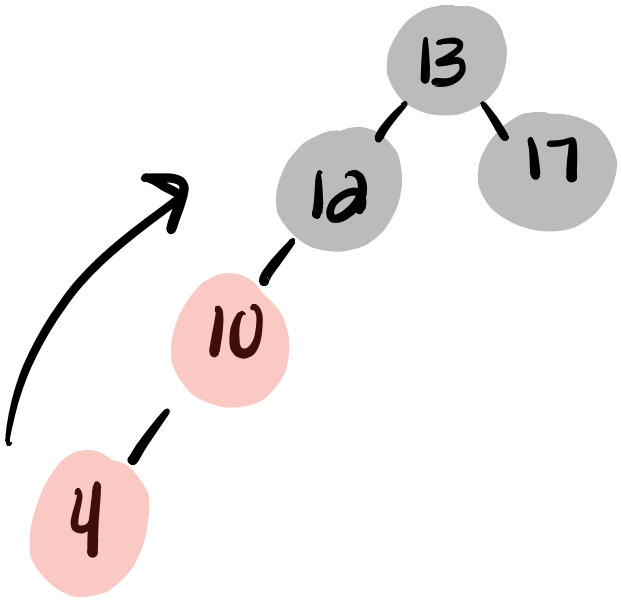
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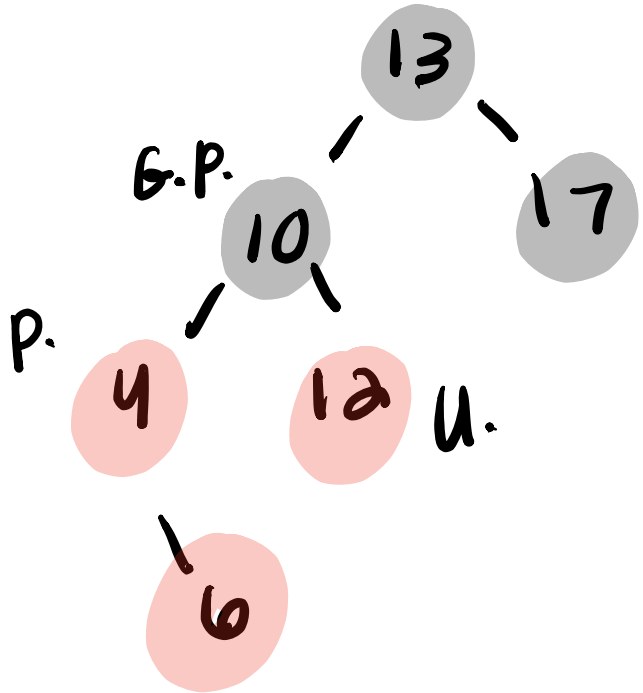
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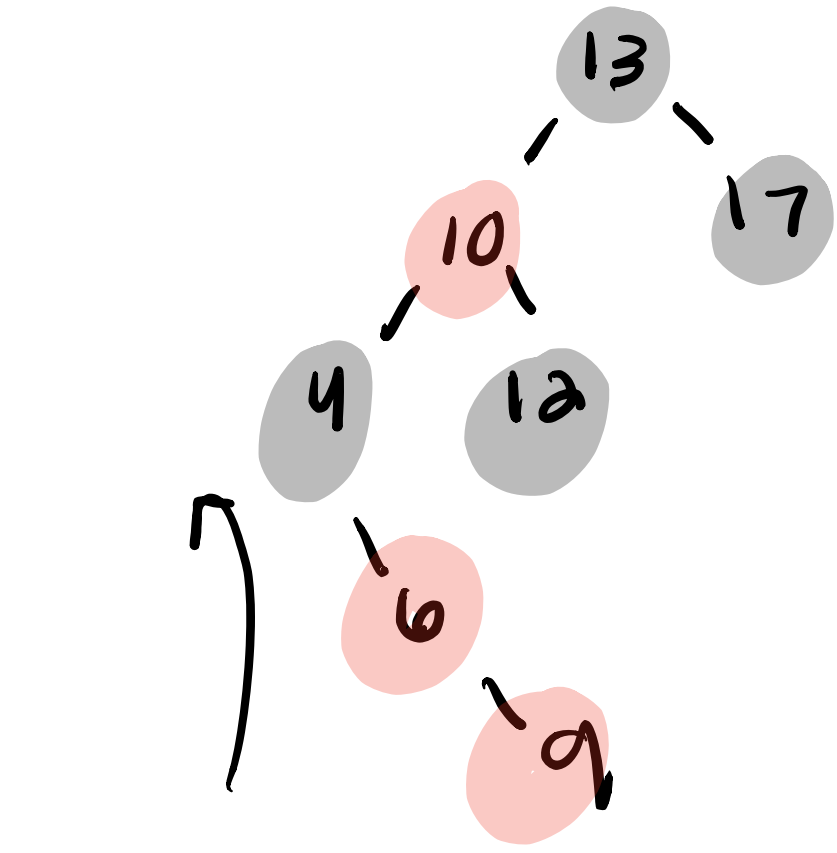
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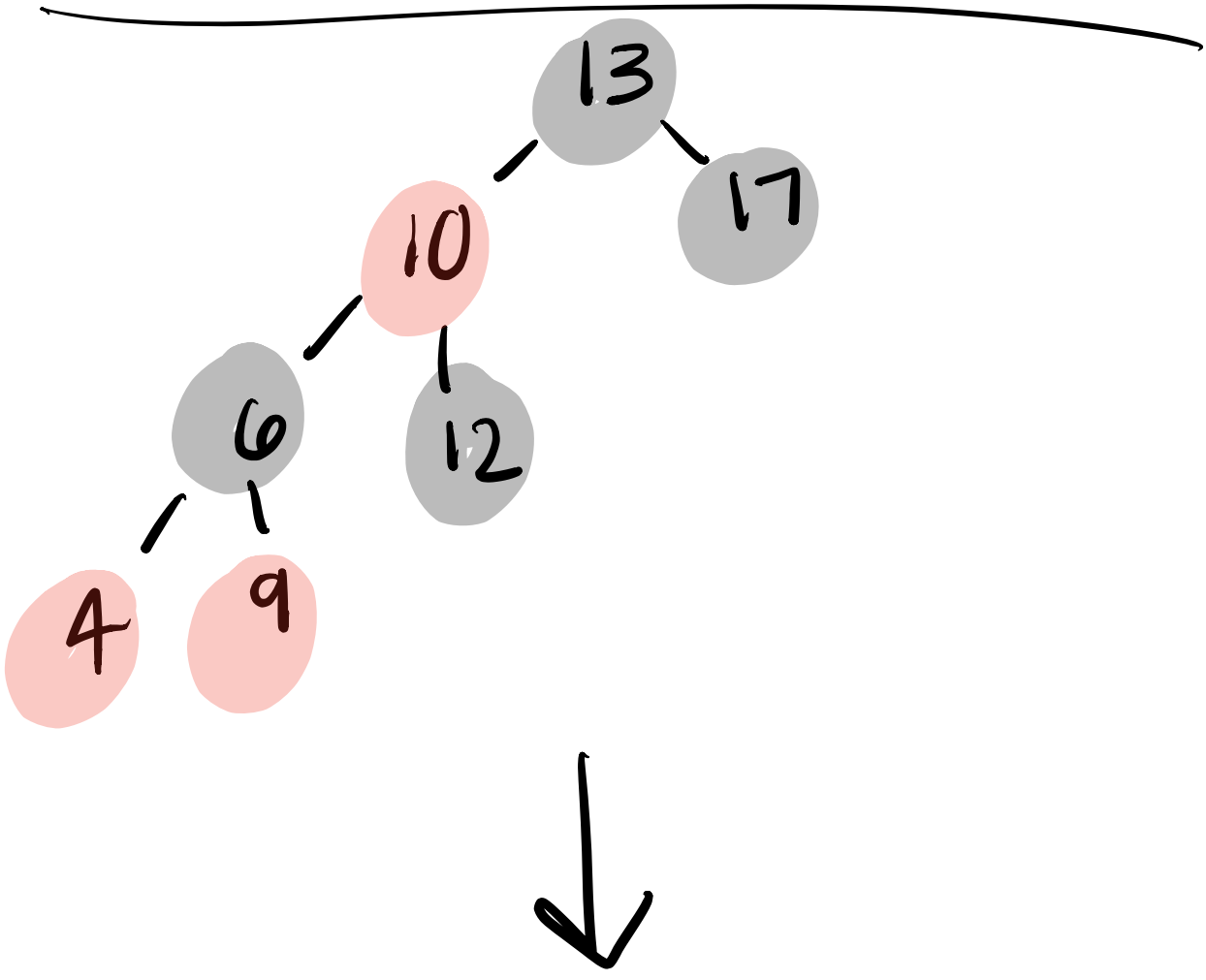
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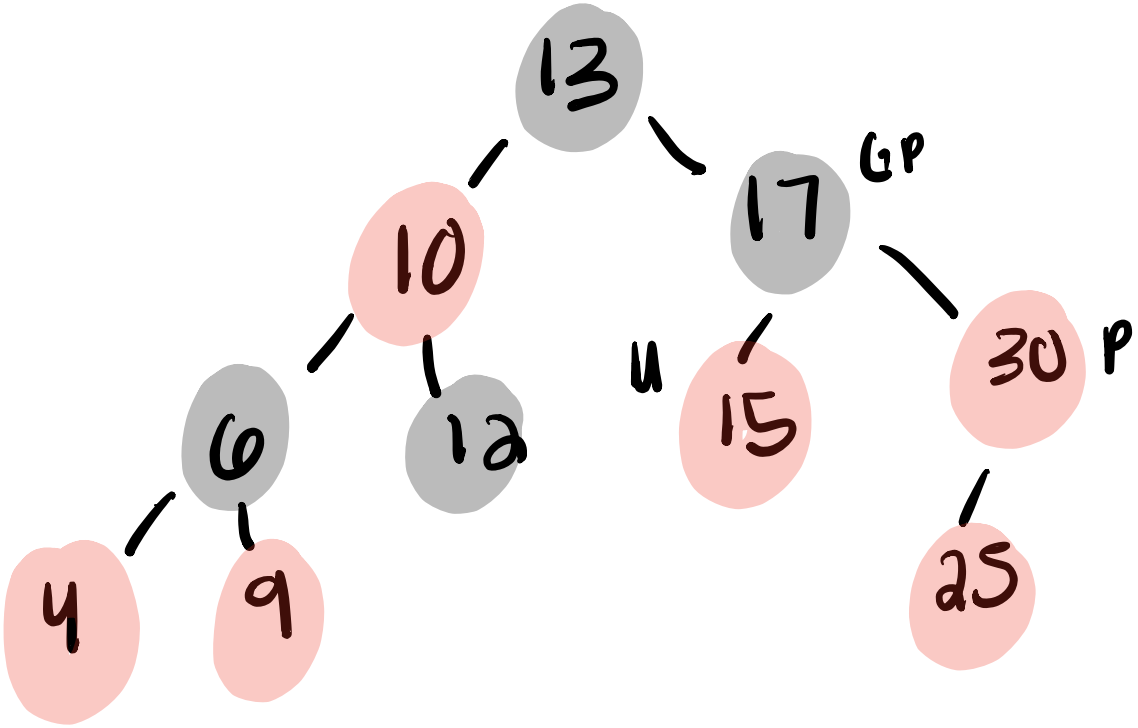
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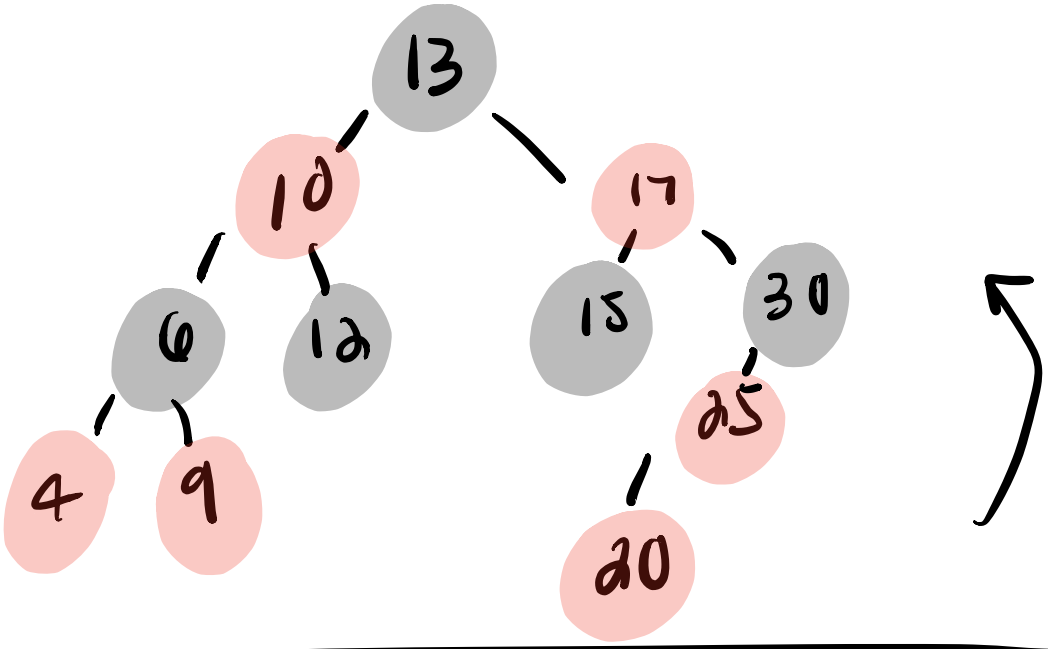
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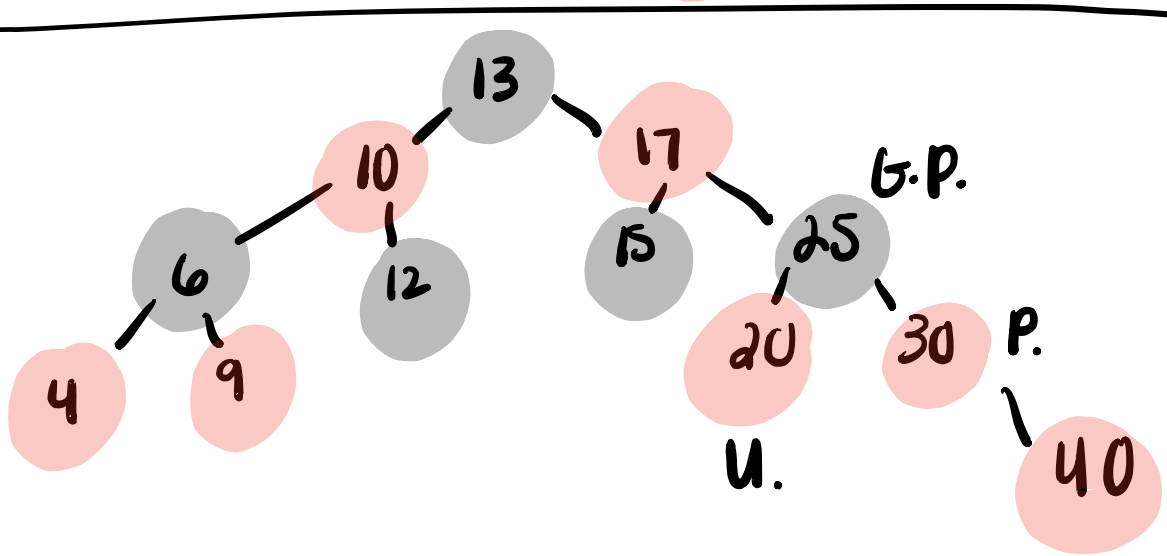
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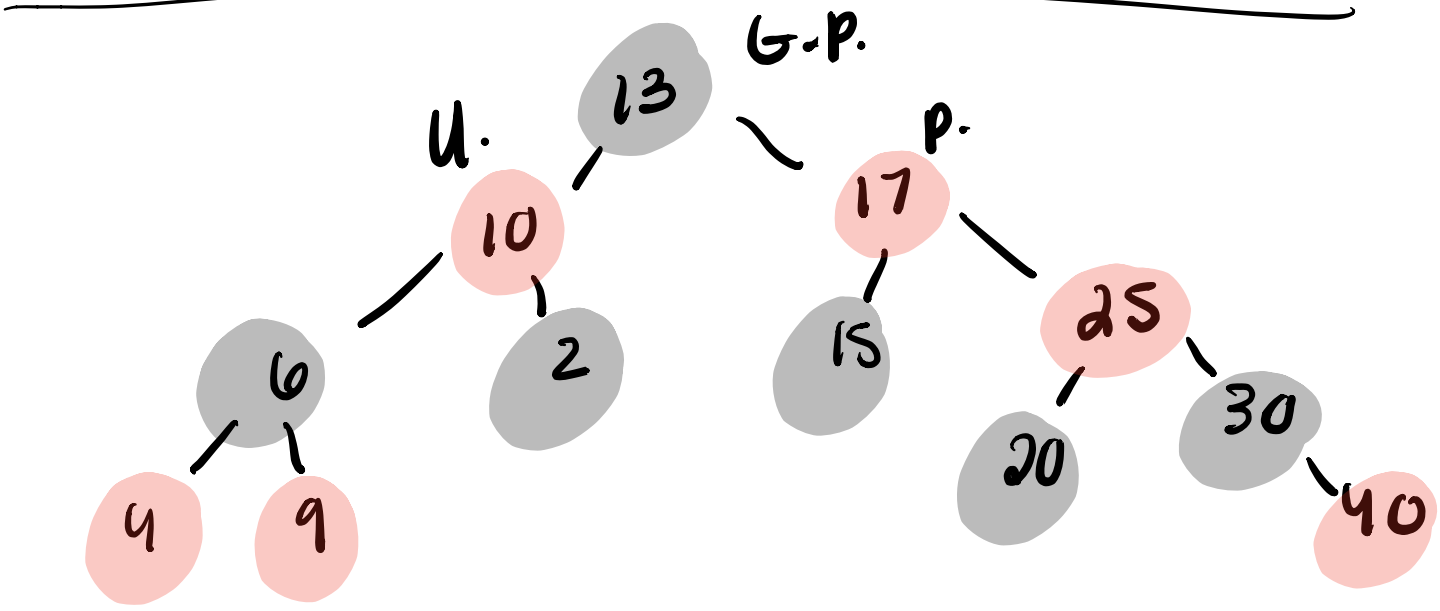
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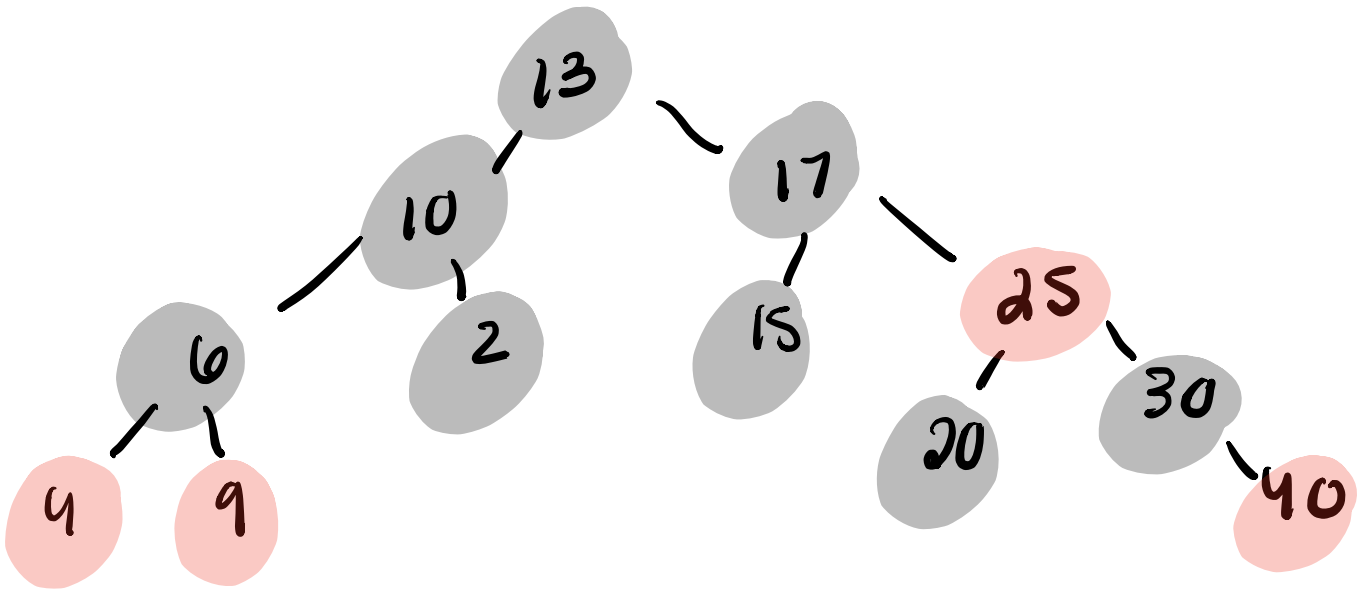
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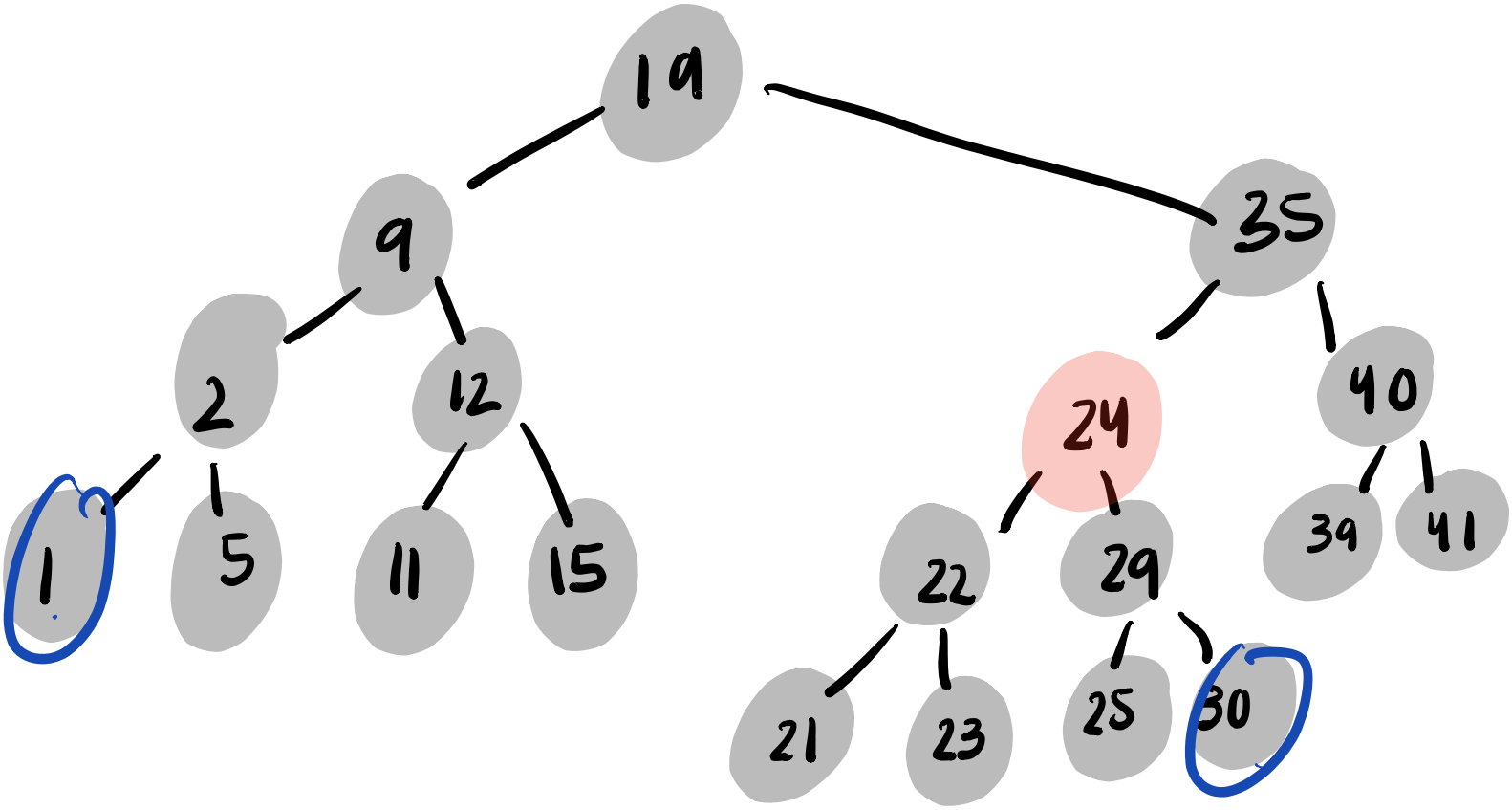
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