

BINARY SEARCH TREE

RULES:

Left Child — the lesser value than the root node (which is less than any of the right nodes).

Right Child — bigger value than the parent

* BST are beneficial because finding a node is fast $\log(n)$ since you can keep cutting the search in half

Example) put this array in a BST

[10, 4, 12, 3, 2, 5, 6, 7, 8, 9]

Step 1) 10 is the first element so automatically becomes the root.

```
graph TD; 10 --- 4; 10 --- 12;
```

Step 2) 4 is less than 10 so it is a left child

```
graph TD; 10 --- 4; 10 --- 12;
```

Step 3) 12 is larger than 10 so becomes a right child

```
graph TD; 10 --- 4; 10 --- 12;
```

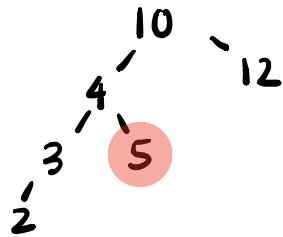
Step 4) 3 is less than 10 and less than 4 so becomes a left child to 4.

```
graph TD; 10 --- 4; 10 --- 12; 4 --- 3;
```

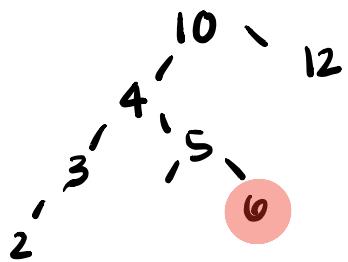
Step 5) 2 is less than 10, less than 4, less than 3 so 2 is a left child of 3.

```
graph TD; 10 --- 4; 10 --- 12; 4 --- 3; 4 --- 2;
```

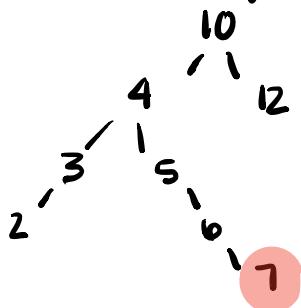
Step 6) 5 is less than 10 and greater than 4 so 5 becomes a Right child of 4



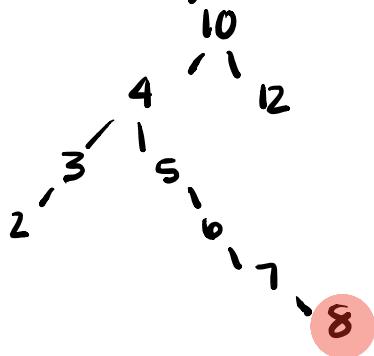
Step 7) 6 is less than 10, and greater than 4 and 5 so becomes a Right child of 5



Step 8) 7 is less than 10, greater than 4, 5, and 6 so Right child to 6.



Step 9) 8 is less than 10, greater than 4, 5, 6, 7 so R.C.

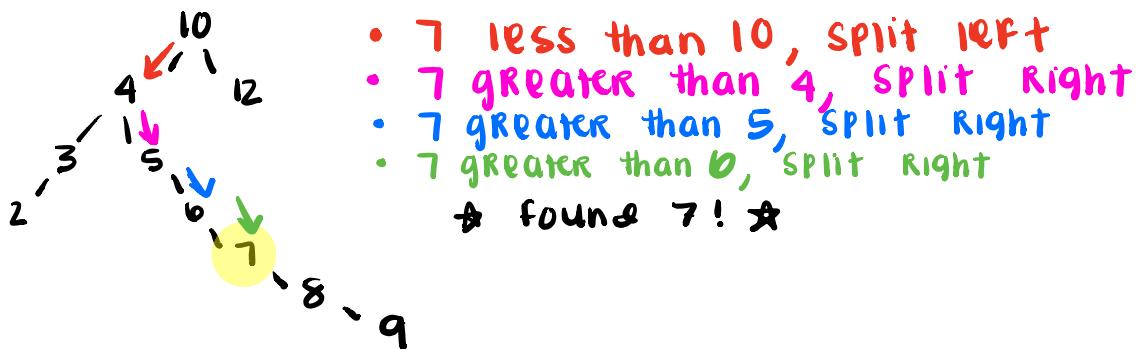


Step 10) 9 is less than 10, greater than 4, 5, 6, 7, 8

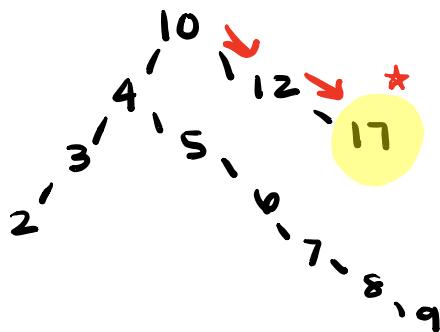


Final tree

Example: TRY to find 7 is this tree



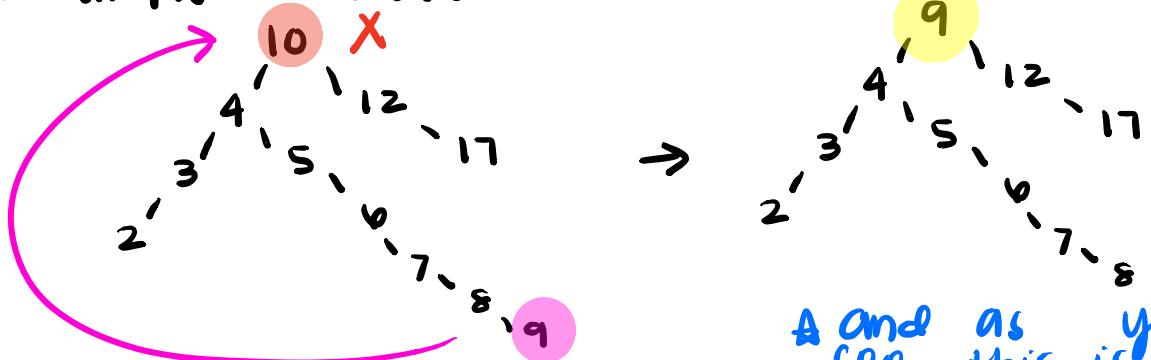
Example: INSERT 17 (see algorithm)



Deletion:

ROOT - Remove 5 and traverse down the left side of the tree and pick the largest value there. (so in the example above this would be 9)

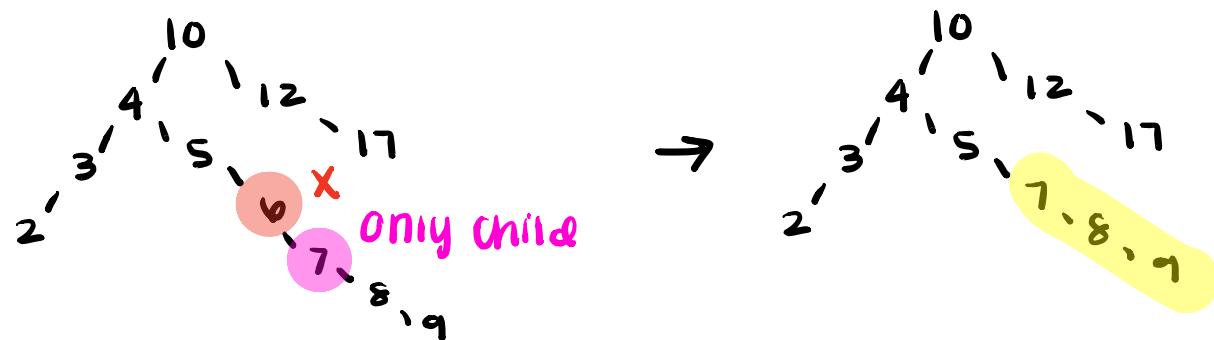
Example: REMOVE root (10)



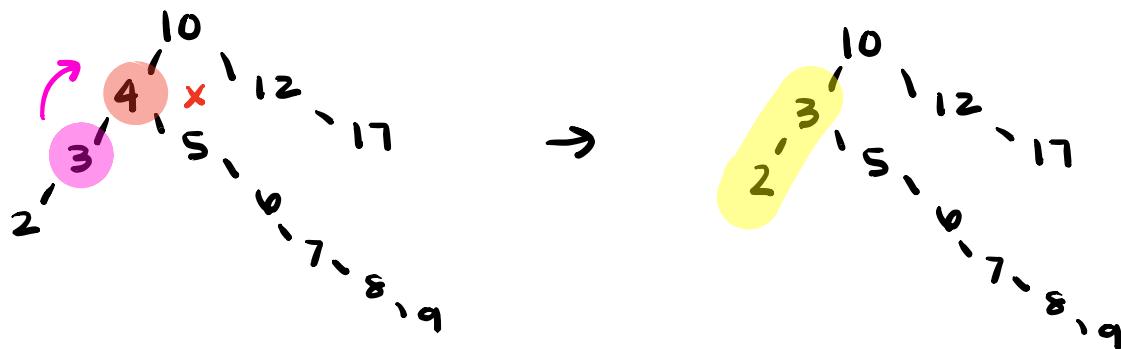
And as you can see, this is still a valid BST.

Node w/ one child - traverse to the node you want to delete and replace it with its child.

Example: delete 6



Node w/ multiple children - traverse to the node, then go to the left and replace it



SEARCHING :

node ↴ key value

```
def tree-search(x, k):
    if x is None OR k is x.key : IF there is no node OR the current node x you are looking at holds that value, return that x node.
        return x

    if k < x.key value k is smaller than the value in node x so you need to go left.
        return Tree-search(x.left, k)

    else the value is bigger than what is in node x so look Right
        return Tree-search(x.right, k)
```

INSERTING :

tree node to insert

```
def tree-insert(T, z):
    y = None
    x = T.root
    while x is not None: while the tree, T, exists
        y = x y now = root
        if z.key < x.key: is value at Node z is less than root value (x)
            x = x.left then go to the left
        else: if value at z is greater than root value (x)
            x = x.right then go to the right
    z.parent = y changing y to be the parent value
    if y == None: if there is no parent to z then z
        T.root = z becomes the root of the tree
    else if z.key < y.key: if z value is less than parent value
        y.left = z z is a left child of its parent y
    else: if z value is bigger than parent, y, value
        y.right = z z is a right child of parent y
```

DELETING :

tree ↴ node

```
def tree-delete(T, z):
    if z.left is None: if there is no left child then transplant with the only other child (right)
        transplant(T, z, z.right)
    else if z.right is None: if there is no right child then transplant with the only other child (left)
        transplant(T, z, z.left)
    else: if there are two children, then
        y = tree-minimum(z.right) find the min value on the right side of the tree
        if y.parent is not z:
            transplant(T, y, y.right)
        y.right = z.right
        y.right.parent = y
        transplant(T, z, y)
        y.left = z.left
        y.left.parent = y
```

transplant (used w/ delete):

```
def transplant (T, u, v):
    if u.parent is None:
        T.root = v
    else if u is u.parent.left:
        u.parent.left = v
    else u.parent.right = v
    if v is not None:
        v.parent = u.parent
```

```
def insert(self, data):  
  
    new_node = Node(data) # This is the node we need to insert so we create an  
    # instance of a node.  
  
    if (self.__root == None): # If there is no root, then it becomes the root.  
        self.__root = new_node  
    else:  
        current_node = self.__root # Since there is a root, we are going to start there,  
        # and move down to find where we can insert the new_node.  
  
        parent = None # Keeping track of the parent of current node (so initially that is root  
        # but this will change)  
  
        while(current_node != None): # Wanting to find the location/position  
            # where we can insert the new_node so we are looping through until the current position  
            # is not null.  
            current_node.setParent(current_node)  
            parent = current_node  
  
            if data < current_node.getData():  
                current_node = current_node.getLeftChild()  
            else:  
                current_node = current_node.getRightChild()  
  
        # We found the right position, and now we need to insert that new_node into  
        # the correct position.  
        if data < parent.getData():  
            new_node.setParent(parent)  
            current_node = parent  
            current_node.setLeftChild(new_node)  
  
        else:  
            new_node.setParent(parent)  
            current_node = parent  
            current_node.setRightChild(new_node)
```

```

def delete(self, data):
    # Find the node to delete.
    # If the value specified by delete does not exist in the tree, then don't change the tree.
    # If you find the node and ...
    #   a) The node has no children, just set it's parent's pointer to Null.
    #   b) The node has one child, make the nodes parent point to its child.
    #   c) The node has two children, replace it with its successor, and remove
    #       successor from its previous location.
    # Recall: The successor of a node is the left-most node in the node's right subtree.
    # Hint: you may want to write a new method, findSuccessor() to find the successor when there are
    #       two children

    root = self.getRoot()

    deleted_node = self.deleteTheNode(root, data)

def deleteTheNode(self, root, data):
    # This is a function I created to assist the delete function so I can take in the current root
    # (which
    # changes as you can see below) to accurately delete the desired node.

    if root == None:
        return root

    if data < root.getData():
        root.setLeftChild(self.deleteTheNode(root.getLeftChild(), data))

    elif data > root.getData():
        root.setRightChild(self.deleteTheNode(root.getRightChild(), data))
    else:
        if root.getLeftChild() == None: # If there is no left child, then that means there are either
            # no children,
            # or it has just one right child.
            temp_node = root.getRightChild()
            root = None
            return temp_node
        elif root.getRightChild() == None: # No children.
            temp_node = root.getLeftChild()
            root = None
            return temp_node

        # This is a node with two children

        temp_node = self.findSuccessor(root.getRightChild())

        root.setData(temp_node.getData())

        # delete the successor node

        root.setRightChild(self.deleteTheNode(root.getRightChild(), temp_node.getData()))

    return root

```

```
def findSuccessor(self, aNode):
    # This is a new method to find the successor when there are two children and it does so
    # by looping through the node when it definitely has two children (aka when there is
    # a left child, you know there is an automatic right child so there are guaranteed
    # two children) and then you keep going to the left child and checking.

    # Successor = smallest value greater than the node aka the next # after the node.

    current_node = aNode

    while(current_node.getLeftChild() != None):
        current_node = current_node.getLeftChild()
    return current_node
```