

HOMEWORK #1□ WRITTEN

1.) Use Boolean Algebra to prove that

$$(\bar{A} * B * \bar{C}) + (\bar{A} * B * C) + (A * \bar{B} * C) + (A * B * \bar{C}) + (A * B * C) = (A+B) * (B+C)$$

$$B(\bar{A}\bar{C} + \bar{A}C + A\bar{C} + AC) + (A\bar{B}C) = (A+B)(B+C)$$

$$= (A+B)(B+C)$$

$$B + (A\bar{B}C)$$

$$= (A+B)(B+C)$$

$$B + \bar{B}(AC)$$

$$= B + (AC)$$

B

$$B + AC$$

$$= B + AC$$

$$\text{LHS} = \text{RHS} \checkmark$$

 \therefore equivalent2.) Prove that $A \text{ XOR } B = A * \bar{B} + \bar{A} * B$ \rightarrow Truth Table

| A | B | \bar{A} | \bar{B} | $A * \bar{B}$ | $\bar{A} * B$ | $A * \bar{B} + \bar{A} * B$ | $A \oplus B$ |
|---|---|-----------|-----------|---------------|---------------|-----------------------------|--------------|
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

\nwarrow tautology \approx equivalent

XOR (\oplus) = cannot be same

$$\text{i.e.: } 0 \oplus 0 = 0 \quad 1 \oplus 0 = 1$$

$$0 \oplus 1 = 1 \quad 1 \oplus 1 = 0$$

$$\therefore A \text{ XOR } B = A * \bar{B} + \bar{A} * B$$

via truth table \checkmark

3.) Write function that represents following circuit. Do not simplify

$$(AB\bar{C}) + [\bar{D}(AB\bar{C})] + ((AB\bar{C})' D)] * (\bar{A}D) + (\bar{D}+B) \text{ (simplified)}$$

$$f(x) = [(AB\bar{C}) \oplus D * (\bar{A}D)] + (\bar{D}+B) = F$$

$$AB\bar{C} \text{ XOR } D * \text{NOT}(AD) + (\bar{D}+B) = F$$

4.) Given the following truth table.

4.1) Write function in SOP form. Do not simplify.

SOP = sum of products, minterms

• want all 1's output

$$= m_0 + m_1 + m_2 + m_5 + m_6$$

where: $m_0 = (\bar{A}\bar{B}\bar{C})$

$m_5 = (A\bar{B}C)$

$m_1 = (\bar{A}\bar{B}C)$

$m_6 = (A\bar{B}\bar{C})$

$m_2 = (\bar{A}B\bar{C})$

so:

$$f(x) = (\bar{A}\bar{B}\bar{C}) + (\bar{A}\bar{B}C) + (\bar{A}B\bar{C}) + (A\bar{B}C) + (A\bar{B}\bar{C})$$

4.2) Write function in POS form. Do not simplify.

POS = product of sums, maxterms

• want all 0's output

$$= M_3 + M_4 + M_7$$

where: $M_3 = (A + \bar{B} + \bar{C})$

$M_4 = (\bar{A} + B + C)$

$M_7 = (\bar{A} + \bar{B} + \bar{C})$

so:

$$f(x) = (A + \bar{B} + \bar{C}) * (\bar{A} + B + C) * (\bar{A} + \bar{B} + \bar{C})$$

5.) most simplified SOP & POS form for ea. of the functions.

5.1) $m_0 + m_1 + m_2$

| $x_0 \backslash x_1$ | 0 | 1 |
|----------------------|----------------|----------------|
| 0 | 1 ₀ | 1 ₂ |
| 1 | 1 ₁ | 0 ₃ |

SOP = $(\bar{x}_1 * \bar{x}_0)$

POS = $(\bar{x}_1 + \bar{x}_0)$

5.2) $M_0 * M_3 * M_4 * M_7$

| $x_0 \backslash x_1$ | 00 | 01 | 11 | 10 |
|----------------------|----------------|----------------|----------------|----------------|
| 0 | 0 ₀ | 1 ₂ | 1 ₆ | 0 ₄ |
| 1 | 1 ₁ | 0 ₃ | 0 ₇ | 1 ₅ |

SOP = $(x_1 * \bar{x}_0) + (\bar{x}_1 * x_0)$

POS = $(x_1 + x_0) * (\bar{x}_1 + \bar{x}_0)$

5.3) $m_4 + m_5 + m_7 + m_{12} + m_{13} + m_{15}$

$2^4 = 16$

| $x_3 \backslash x_2$ $x_1 \backslash x_0$ | 00 | 01 | 11 | 10 |
|--|----|----|----------------|-----------------|
| 00 | 0 | 0 | 1 ₄ | 1 ₂ |
| 01 | 0 | 1 | 1 ₅ | 1 ₃ |
| 11 | 0 | 1 | 1 ₇ | 1 ₁₅ |
| 10 | 0 | 0 | 0 | 0 |

SOP = $(x_0 * x_2) + (\bar{x}_1 * x_2)$

POS = $x_2 * (\bar{x}_1 + x_0)$

• POS is SOP essentially but,
product of sums w/
 x_2 factored out

5.4) $m_0 + m_3 + m_4 + m_8 + D_2 + D_5 + D_7 + D_{10} + D_{13} + D_{15}$

| $x_3 \backslash x_2$ $x_1 \backslash x_0$ | 00 | 01 | 11 | 10 |
|--|-----------------|-----------------|------------------|------------------|
| 00 | 1 ₀ | 1 ₄ | 0 ₁₂ | 1 ₆ |
| 01 | 0 ₁ | DC ₅ | DC ₁₃ | 0 ₉ |
| 11 | 1 ₃ | DC ₇ | DC ₁₅ | 0 ₁₁ |
| 10 | DC ₂ | 0 ₆ | 0 ₁₄ | DC ₁₀ |

SOP = $(\bar{x}_0 * \bar{x}_2) + (\bar{x}_3 * \bar{x}_1 * \bar{x}_0) +$
 $(\bar{x}_3 * \bar{x}_2 * x_1)$

POS = $(\bar{x}_2 + \bar{x}_3) * (\bar{x}_0 + x_1) *$
 $(\bar{x}_3 + \bar{x}_0) * (\bar{x}_2 + \bar{x}_1)$

5.5) $m_1 + m_3 + m_7 + m_9 + m_{11} + m_{15} + m_{17} + m_{19} + m_{25} + m_{27} + D_4 + D_6 +$
 $D_{12} + D_{14} + D_{16} + D_{18} + D_{20} + D_{22} + D_{24} + D_{26} + D_{28} + D_{30}$

| $x_3 \backslash x_2$ $x_1 \backslash x_0$ | 00 | 01 | 11 | 10 |
|--|------------------|------------------|------------------|------------------|
| 00 | 0 ₀ | DC ₄ | DC ₁₂ | 0 ₈ |
| 01 | 1 ₁ | 0 ₅ | 0 ₁₃ | 1 ₉ |
| 11 | 1 ₃ | 1 ₇ | 1 ₁₅ | 1 ₁₁ |
| 10 | 0 ₂ | DC ₆ | DC ₁₄ | 0 ₁₀ |
| 00 | DC ₁₆ | DC ₂₀ | DC ₂₈ | DC ₂₄ |
| 01 | 1 ₁₇ | 0 ₂₁ | 0 ₂₉ | 1 ₂₅ |
| 11 | 1 ₁₉ | 0 ₂₃ | 0 ₃₁ | 1 ₂₇ |
| 10 | DC ₁₈ | DC ₂₂ | DC ₃₀ | DC ₂₆ |

SOP = $(x_0 * \bar{x}_2) + (x_2)$

POS = $(\bar{x}_2 + x_1) * (x_2 + \bar{x}_1)$
 $* (\bar{x}_2 + \bar{x}_0)$