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Kelas : TI-A

1.) (a) $y'' - \frac{1}{4}y = 0$ $y(0) = 4$, $y'(0) = 0$

$$L(y'') + \frac{1}{4}L(y) = 0$$

$$s^2 L(y) - sy(0) - y'(0) + \frac{1}{4}L(y) = 0$$

$$s^2 Y - sy(0) - y'(0) - \frac{1}{4}Y = 0$$

$$s^2 Y - 4s - \frac{1}{4}Y = 0$$

$$Y(s^2 - \frac{1}{4}) - 4s = 0$$

$$Y(s^2 - \frac{1}{4}) = 4s$$

$$Y = \frac{4s}{s^2 - \frac{1}{4}}$$

$$Y = \frac{4s}{(s - \frac{1}{2})(s + \frac{1}{2})}$$

$$Y = \frac{2}{s - \frac{1}{2}} + \frac{2}{s + \frac{1}{2}}$$

$$y(s) = L^{-1}\left(\frac{2}{s - \frac{1}{2}}\right) + L^{-1}\left(\frac{2}{s + \frac{1}{2}}\right)$$

$$y(t) = 2e^{\frac{1}{2}t} + 2e^{-\frac{1}{2}t}$$

$$y(t) = 2(e^{\frac{1}{2}t} + e^{-\frac{1}{2}t})$$

> kosinus hiperbolik

$$\cosh\left(\frac{1}{2}t\right) = \frac{e^{\frac{1}{2}t} + e^{-\frac{1}{2}t}}{2}$$

$$y(t) = 2(e^{\frac{1}{2}t} + e^{-\frac{1}{2}t}) = 4 \cosh\left(\frac{1}{2}t\right)$$

$$y(t) = 4 \cosh\left(\frac{1}{2}t\right)$$

Menggunakan transformasi Laplace

$$L^{-1}\left(\frac{2}{s + \frac{1}{2}}\right) = 2e^{-\frac{1}{2}t}$$

Jdi Penyebut diganti dgn $\frac{1}{2}$

→ Pecahan Parsial

$$\frac{4s}{(s - \frac{1}{2})(s + \frac{1}{2})} = \frac{A}{s - \frac{1}{2}} + \frac{B}{s + \frac{1}{2}}$$

dikalikan $(s - \frac{1}{2})(s + \frac{1}{2})$

$$4s = A(s + \frac{1}{2}) + B(s - \frac{1}{2})$$

$$4s = As + \frac{A}{2} + Bs - \frac{B}{2}$$

$$4s = (A+B)s + \frac{A-B}{2}$$

disetarakan koefisiennya

$$A+B = 4 \text{ dan } \frac{A-B}{2} = 0$$

$$A = B$$

$$2A = 4 \Rightarrow A = 2$$

$$B = 2$$

$$\text{Jadi } \frac{4s}{(s - \frac{1}{2})(s + \frac{1}{2})} = \frac{2}{s - \frac{1}{2}} + \frac{2}{s + \frac{1}{2}}$$

$$b. y'' + y' - 6y = 0 \quad y(0) = 6, y'(0) = 13$$

$$L(y'') - L(y') - 6L(y) = L(0)$$

$$s^2 L(y) - s y(0) - y'(0) - [s L(y) - y(0)] - 6L(y) = 0$$

$$(s^2 + s - 6)Y - 6s - 13 - 6 = 0$$

$$(s^2 + s - 6)Y = 6s + 19$$

$$Y = \frac{6s + 19}{s^2 + s - 6} = \frac{6s + 19}{(s-2)(s+3)}$$

→ Pecahan Parsial

$$\frac{6s + 19}{(s-2)(s+3)} = \frac{A}{s-2} + \frac{B}{s+3}$$

$$Y = \frac{31/s}{s-2} - \frac{1/s}{s+3}$$

$$6s + 19 = A(s+3) + B(s-2)$$

$$6s + 19 = As + 3A + Bs - 2B$$

$$6s + 19 = (A+B)s + (3A-2B)$$

disetarakan koefisiennya :

$$A + B = 6 \quad | \times 2 \quad | 2A + 2B = 12$$

$$3A - 2B = 19 \quad | \times 1 \quad | 3A - 2B = 19$$

$$5A = 31$$

$$A = \frac{31}{5}$$

$$B = 6 - A$$

$$B = 6 - \frac{31}{5}$$

$$B = \frac{30-31}{5} = -\frac{1}{5}$$

$$y = L^{-1}\left(\frac{31/s}{s-2}\right) - L^{-1}\left(\frac{1/s}{s+3}\right)$$

$$y = \frac{31}{s} e^{2t} - \frac{1}{s} e^{-3t}$$

menggunakan tabel transformasi laplace

$$L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$$

$$\text{Jadi } \frac{6s + 19}{(s-2)(s+3)} = \frac{31/s}{s-2} - \frac{1/s}{s+3}$$

$$C. y'' - 2y' = 12e^{3x} - 8e^{-2x} \quad y(0) = -2 \quad y'(0) = 12$$

$$y(x) = y_n + y_{p1} + y_{p2}$$

$$* y_n(x)$$

$$y'' - 2y' = 0$$

$$r^2 - 2r = 0$$

$$r(r-2) = 0$$

$$r = 0 \text{ dan } r = 2$$

$$y_n = C_1 e^{0x} + C_2 e^{2x}$$

$$= C_1 + C_2 e^{2x}$$

$$* y_p$$

$$> 12e^{3x} \Rightarrow y_{p1}$$

$$y_{p1} = Ae^{3x}$$

$$y'_{p1} = 3Ae^{3x}$$

$$y''_{p1} = 9Ae^{3x}$$

$$\Rightarrow 9Ae^{3x} - 2(3Ae^{3x}) = 12e^{3x}$$

$$9Ae^{3x} - 6Ae^{3x} = 12e^{3x}$$

$$3Ae^{3x} = 12e^{3x}$$

$$A = \frac{12}{3} = 4$$

$$y_{p1} = 4e^{3x}$$

$$> -8e^{-2x} \Rightarrow y_{p2}$$

$$y_{p2} = Ke^{-2x}$$

$$y'_{p2} = -2Ke^{-2x}$$

$$y''_{p2} = 4Ke^{-2x}$$

$$\Rightarrow 4Ke^{-2x} - 2(-2Ke^{-2x}) = -8e^{-2x}$$

$$4Ke^{-2x} + 4Ke^{-2x} = -8e^{-2x}$$

$$8Ke^{-2x} = -8e^{-2x}$$

$$K = \frac{-8}{8} = -1$$

$$y_{p2} = -e^{-2x}$$

$$y_{p2} = y_n + y_{p1} + y_{p2}$$

$$= C_1 + C_2 e^{2x} + 4e^{3x} - e^{-2x}$$

$$\text{Menentukan konstanta } C_1 \text{ dan } C_2 \Rightarrow y(0) = -2 \quad y'(0) = 12$$

$$y = C_1 + C_2 e^{2x} + 4e^{3x} - e^{-2x}$$

$$y(0) = C_1 + C_2 e^0 + 4e^0 - e^0 = -2$$

$$C_1 + C_2 + 4 - 1 = -2$$

$$C_1 + C_2 + 3 = -2$$

$$C_1 + C_2 = -5$$

$$y'(x) = 2C_2 e^{2x} + 12e^{3x} + 2e^{-2x}$$

$$y'(0) = 2C_2 e^0 + 12e^0 + 2(-1)e^0 = 12$$

$$2C_2 + 12 - 2 = 12$$

$$2C_2 + 10 = 12$$

$$2C_2 = 2$$

$$C_2 = 1$$

$$C_1 + C_2 = -5$$

$$C_1 + 1 = -5$$

$$C_1 = -5 - 1$$

$$C_1 = -6$$

$$y = C_1 + C_2 e^{2x} + 4e^{3x} - e^{-2x}$$

$$= -6 + 1e^{2x} + 4e^{3x} - e^{-2x}$$

$$= -6 + e^{2x} + 4e^{3x} - e^{-2x}$$

2.) Perbandingan Penyelesaian PDB dengan menggunakan metode Undetermined Coefficient dengan Laplace Transform

> Undetermined Coefficient

- Cocok untuk menyelesaikan Persamaan diferensial dgn suku non-homogen yang sederhana.
- Metode ini membutuhkan dugaan bentuk solusi khusus yang yang kemudian disubstitusikan kedalam Persamaan untuk menemukan koefisien yg tidak diketahui
- Proses ini lebih manual dan sering digunakan untuk Persamaan dengan suku non homogen yang Standar.

> Transformasi Laplace

- Mengubah Persamaan diferensial menjadi Persamaan aljabar yg lebih mudah diselesaikan.
- Sangat efektif untuk kondisi awal yg jelas
- Berguna untuk Suku non-homogen yg lebih kompleks
- Memerlukan tabel tentang transformasi Laplace