Spherical Polyhedron

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MATH 128A: Classical Geometry

December 14, 2018

Abstract

We will explore properties and differences of tessellations in different geometries. In particular, looking at spherical geometry, tiling over a sphere, and the relation to polyhedra.

1 Tessellations and Tiling

A tessellation refers to a uniform tiling of a plane with polygons, such that an equal number of identical polygons meet at each vertex. We can denote tessellations in the following form: $\{a, b\}$ where a is the number of sides each polygon has, and b is the number of polygons that share a vertex. This is called a Schlafli symbol.

Euclidean Plane There are precisely three possible tessellations of regular polygons in Euclidean Space, namely $\{3, 6\}$, $\{6, 3\}$ and $\{4, 4\}$. We recognize these as the tiling of regular triangles, squares, and hexagons.

Hyperbolic Plane The hyperbolic plane however has an infinite number of tessellations available for creation. We find that tessellations in hyperbolic space hold the following: $\frac{a}{a} + \frac{1}{b} < \frac{1}{2}$. This is true for infinitely many a and b.

Surface of a Sphere However, I am far more interested in tessellations over the surface of a sphere. Rather than speaking of tessellations, the tiling of a sphere is more commonly reffered to as spherical tiling, and the flattening of these tiles create spherical polyhedra. We say a spherical polyhedron is the partition of the surface of the sphere by *great arcs*. Some examples that we recognize are the hosohedron, which resembles the a blow up beach ball, and the truncated icoshedron which resembles the common soccer ball.

Programming While attempting to display these visually pleasing patterns, I ran into a lot of difficulty finding ways to recursively draw the same tiling pattern on a circle, disk, sphere or euclidean plane. In particular, attempting to plot tessellations in the Poincare Disk was difficult because of the hurdles I had to address with plotting lines. Because Processing takes cartesian coordinates, I had to first find a mapping of the upper half plane to a disk, and map the polar coordinates to cartesian coordinates. I found issues accounting for infinity and looping around the circle ensuring a clean match between the beginning of the recursive function and when it ties back to the beginning.

Conclusion Unfortunately, instead of learning an extensive amount on one topic in geometry for this project, I learned a little about many things while testing the waters of a potential visual program idea, and running into barriers in implimentation. These topics include the Farey Ford Sequence / Circle Packing, Tessellations in hyperbolic space, tessellations in Euclidean Space, Spherical Geometry, Spherical Polyhedra, and regular polyhedra. Ultimately I found regular polyhedra and their construction incredibly interesting and wish I knew I'd wanted to pursue that topic from the beginning.