# CMPS 102 — Fall 2018 – Homework 3

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I have read and agree to the collaboration policy.

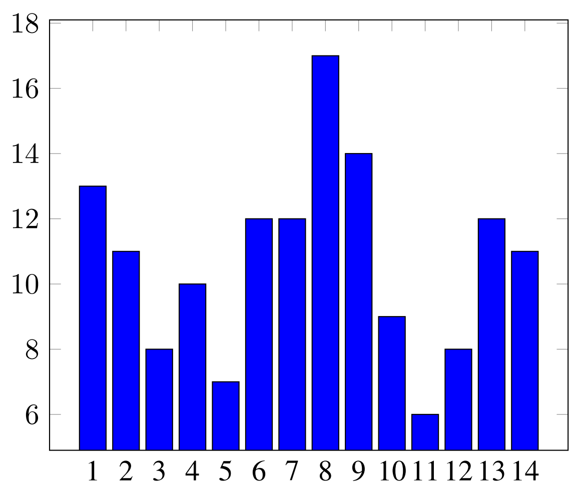
Collaborators: none

## Solution to Problem 1

We are given a set of days, 1 to $n$. For each corresponding day, we are given the price of the crop. We want to find the maximum amount of profit that Phillip could have made in these $n$ days.

While thinking through different things that might help to solve this problem, I graphed prices of crop over $n$ days that were given as an example, namely:

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price | 13 | 11 | 8 | 10 | 7 | 12 | 12 | 17 | 14 | 9 | 6 | 8 | 12 | 11 |




We are given that the most profit is gained, for this partiuclar example, is by doing the following:
Buy a cart on day 3, sell the cart on day 4, with a margin of 2;
buy a cart on day 5, sell a cart on day 8, with a margin of 10;
buy a cart on day 11, sell a cart on day 13, with a margin of 6.
Now, comparing these values to the graph above, I noticed that these directly correspond with the peaks and valleys of the graph.

Defintions:
A day's crop price, $price[x]$, is a peak if $price[x] \leq price[x-1]$ and $price[x] \geq A[x+1]$
A day's crop price, $price[x]$, is a valley if $price[x] \geq price[x-1]$ and $price[x] \leq A[x+1]$

Let us assume that the most profit is gained by buying crop at a price valley, and selling crop at a price peak. Now, consider the following algorithm:

$profit = 0.$

$purchaseprice = null.$
$profitmargin = null.price[0] = null.$
$price[n + 1] = null.$

for each day *i*, 1 to *n*:
    if $price[i - 1] \geq price[i]$ and $price[i] \leq price[i + 1]$
        if $purchaseprice = null$
            buy crop on day *i*.
            purchase price = $price[i]$
    else if $price[i - 1] \leq price[i]$ and $price[i] \geq price[i + 1]$
        if $purchaseprice \neq null$
            sell crop on day *i*
            $profitmargin = price[i] - purchaseprice$
            $profit = profit + profitmargin$

**Claim 1.** *The most profit is gained by buying crop at a price valley, and selling crop at a price peak.*

*Proof.* If the above algorithm, call it, *Greedy*, is not optimal, then there must be some other algorithm, call it *O*, that is optimal. Say that *O* is the same as *Greedy* up until $tie_{k+1}$, and $tie_k$ was the last tie that was the same as the *Greedy*. Then, this tie must have tied two ropes that were not both the smallest ropes in the list, otherwise it would be the same as *greedy*. Call the two smallest ropes $r_1$ and $r_2$. Any other ropes, $r_i$ or $r_j$ where $i, j \in \{1, 2, ...n\}$, is longer than $r_1$ and $r_2$. Then it must be that $r_i + r_j > r_1 + r_2$. Because the cost of adding $r_i$ and $r_j$ is greater than or equal to $r_1 + r_2$, *Greedy* takes no harm in adding $r_1$ and $r_2$ instead. Thus, the *Cost* of *Greedy* must be at most the *Cost* of *O*, thus *Greedy* must be, if not also, optimal. □

**Claim 2.** *The above algorithm is* $O(nlog\ n)$.

*Proof.* The algorithm takes $O(nlog\ n)$ to sort the list (or build the min-heap). It then extracts the top two smallest ropes in $O(log\ n)$ time, and does this $n$ times, so we get $2 * n * logn$. It then inserts the new rope back in in O(logn) time and does this $n$ times. We ultimately get $nlog\ n + 2nlog\ n + nlog\ n$ which evaluates to $O(nlog\ n)$ □