

Edwards-Venn diagrams

Jonathan Swinton

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This vignette is mainly for the use of the package developer.

Edwards CITE starts by constructing Venn sets on a sphere and then projecting onto the plane. On the sphere we use polar coordinates with longitude θ and latitude ϕ . Arc distance from the equator is s and height above the equatorial plane is h :

$$\begin{aligned}s &= r \frac{\phi}{2\pi} \\ h &= r \sin \phi\end{aligned}$$

Project down onto the equatorial plane (a polar stereographic projection).

$$\begin{aligned}\rho &= \frac{\cos \phi}{1 - \sin \phi} 2r \\ x &= \rho \cos \theta \\ y &= \rho \sin \theta\end{aligned}$$

For the purposes of visualisation in this document, we also use a Mercator projection onto the equatorial cylinder

$$\begin{aligned}x &= r \cos \phi \cos \theta \\ y &= r \cos \phi \sin \theta \\ x &= r \theta \\ y &= h\end{aligned}$$

Edwards suggests using the Smith functions. In a Mercator projection the Smith functions are

$$h_n(\theta) = \frac{\cos(2^{n-2}\theta)}{2^{n-2}}$$

So $h_n(\theta) = T_{n-2}\theta$ where

$$\begin{aligned}T_n(x) &= \frac{1}{2^n} \cos 2^n x \\ &= \frac{1}{2^n} \cos \frac{1}{2} 2^{n+1} x \\ 2^{2n} T_n^2 &= \frac{1}{2} (1 + 2^{n+1} T_{n+1}) \\ T_{n+1} &= 2^n T_n^2 - \frac{1}{2^{n+1}}\end{aligned}$$

So $T_{n+1} = 0$ when $T_n = \pm 2^{-n}1/\sqrt{2}$; $2^n x = \pi/4 + (p/2)2\pi$.

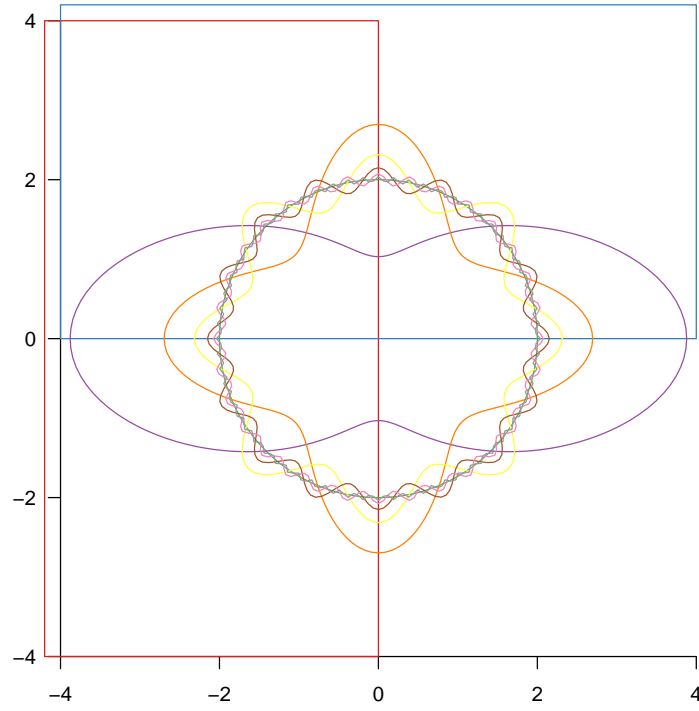


Figure 1: AWFE sets based on Smith functions projected from a sphere

Edwards (cite) suggests using Smith functions as the set boundaries, as shown in Figure 1.

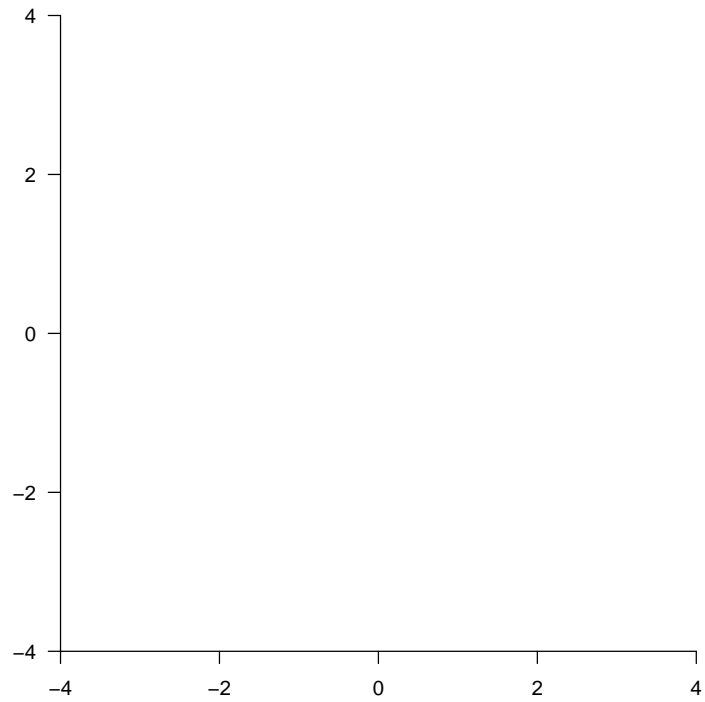


Figure 2: 8th order AWFE diagram

0.1 Rescaled Smith functions

The amplitude of those functions tends geometrically to zero. If we know in advance how many sets we need, we can try rescaling so the amplitudes decreases linearly. These sets are shown in Figure 3.

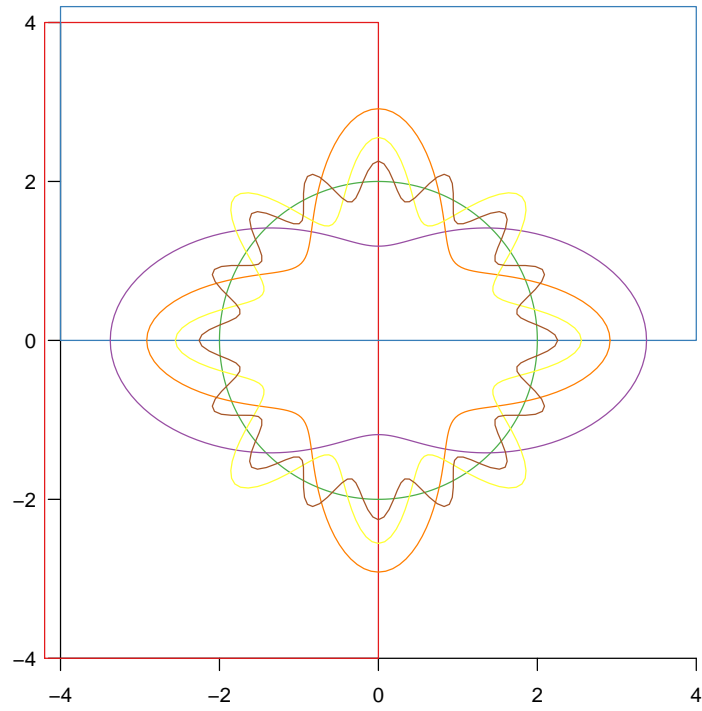


Figure 3: AWFE sets based on rescaled Smith functions

0.2 Cog sets

Alternatively we could replace each sine curve by a toothed cog

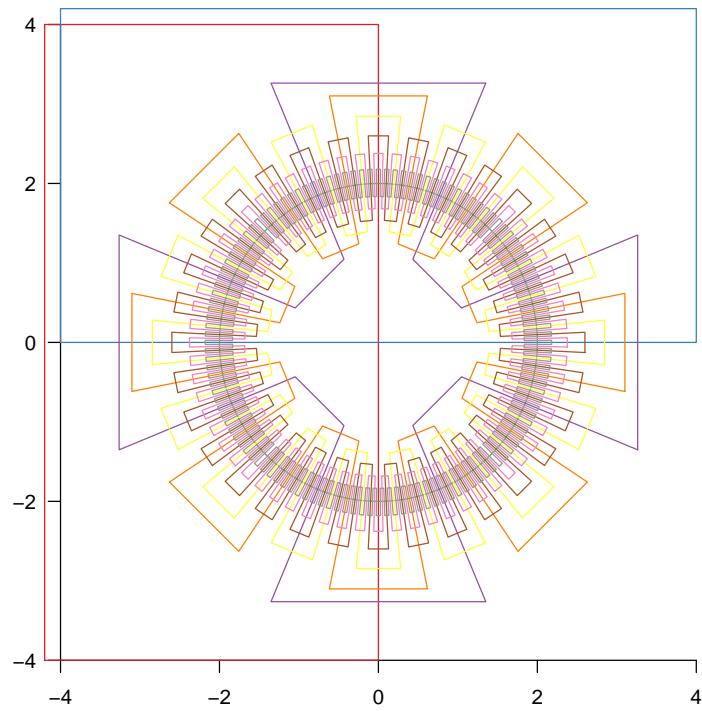


Figure 4: AWFE sets based on cogs

We don't offer these as TissueDiagrams

0.3 Battlement sets

Alternatively we can square the circle

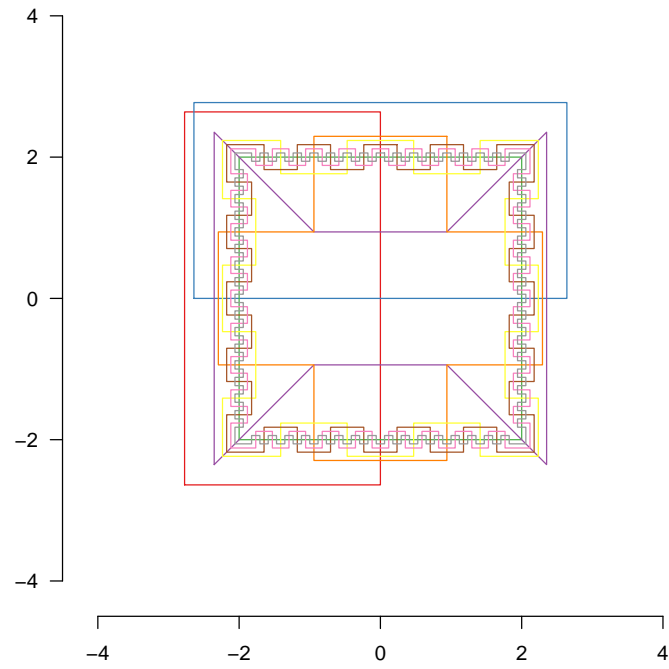


Figure 5: AWFE sets based on battlements

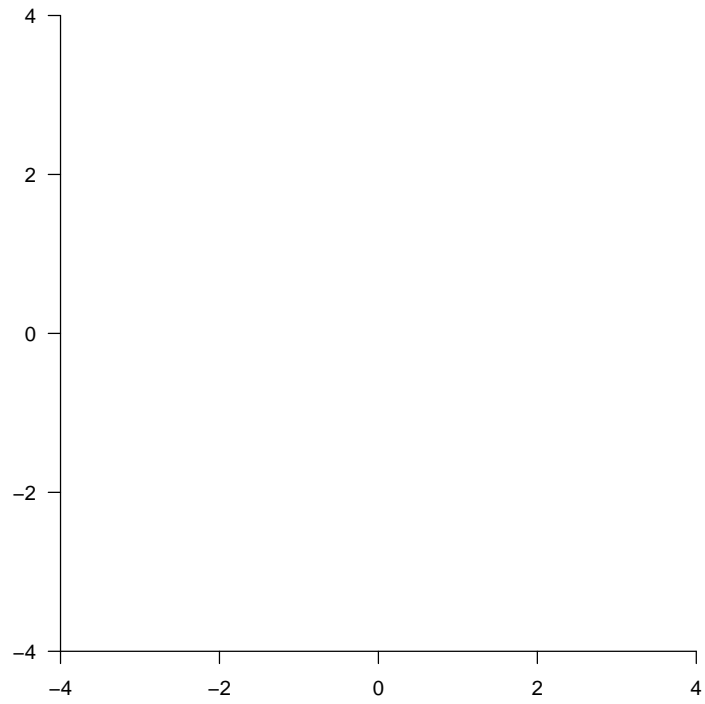


Figure 6: 8th order AWFE diagram based on battlements

References