Monte Carlo Simulation of the 2D XY Model

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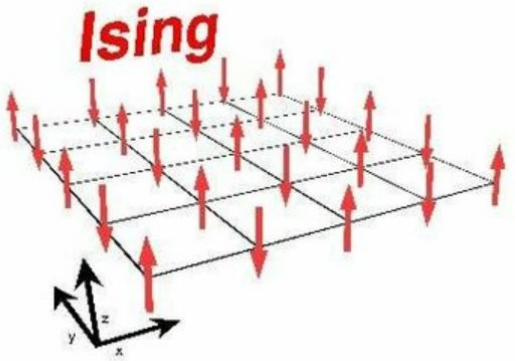
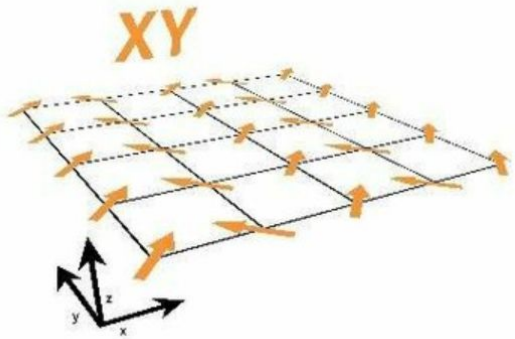
2D XY Model

- Similar to Ising model
- Each site has a unit vector, characterized by an angle
- Can be used to model effects in thin films of helium and superconductors





XY Model Hamiltonian

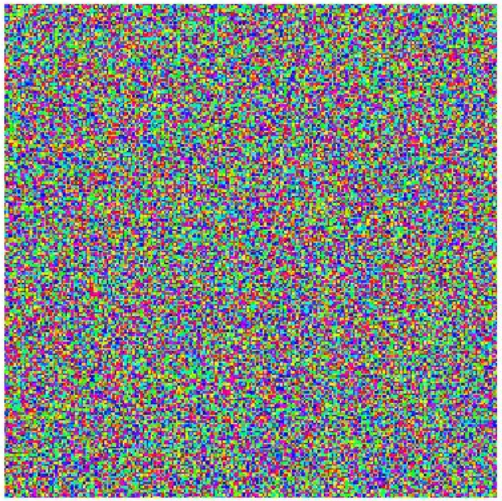
$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) - h \sum_i \cos(\theta_i)$$

- Restrict to interactions between adjacent spins
- Alignment is energetically favorable
- Omit the Zeeman term

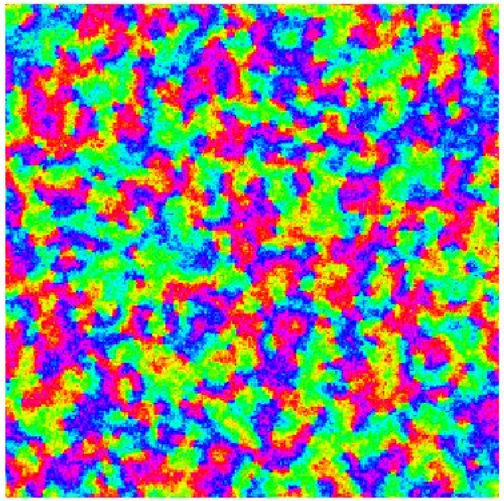
Monte Carlo Simulation

- Propose new spins on each site
- Calculate ΔE for each change
- Accept changes based on Metropolis algorithm

Visualization of States



High Temperature

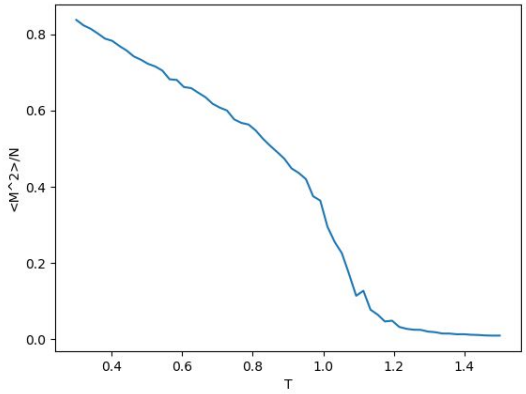


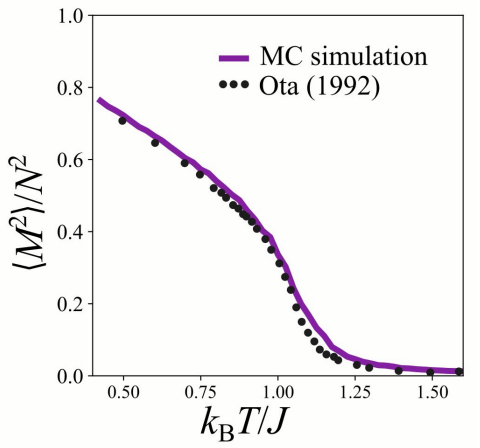
Low Temperature

Squared Magnetization

- Largest at low temperature
  - Ordered state
- Falls to 0 at high temperature
- Kosterlitz-Thouless phase transition at T~.9
- Similar to known results

$$\frac{\langle M^2 \rangle}{N^2} = \frac{1}{N^2} \langle s_x^2 + s_y^2 \rangle = \frac{1}{N^2} \left\langle \left(\sum_{i=1}^{N} \cos\theta_i \right)^2 + \left(\sum_{i=1}^{N} \sin\theta_i \right)^2 \right\rangle$$

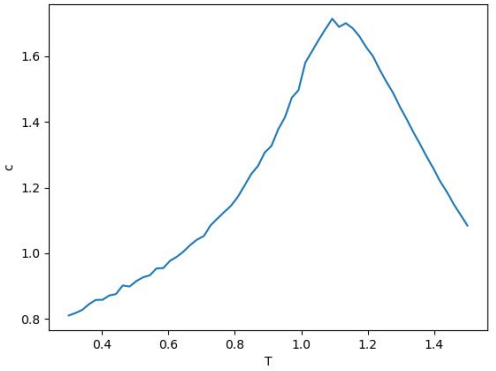


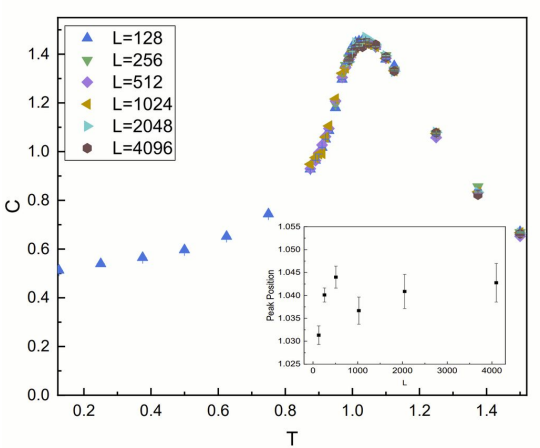


Specific Heat

- Peak at about T=1.1
- Agrees with known results

$$c/k_{\mathrm{B}} = \frac{\langle E^2 \rangle - \langle E \rangle^2}{N(k_{\mathrm{B}}T)^2}$$





Future Things

- Create animation of MC steps
- Further investigate phase transition
- Look at other physical quantities
- Think through the justification for the MC algorithm