- data ko description ...
un-model ; to model it.

Measures of Centre and Dispersion for ungrouped data

mean, median, mode, range, variance, sd, coefficient of variation

- data ko organize karne ke liye class groups banane ki need na h. to - ungrouped data

- mean, median, mode, range, variance, sd, coefficient of variation, agar population me karke hain tou population parameter kehlayega, agar sample per karen " " " statistics.

$\mu = \frac{\Sigma x}{N}$ (mean for population)

N → total # of data points in population

$\bar{x} = \frac{\Sigma x}{n}$ (mean for sample)

n → " " " " " " sample

- Mean is affected by outliers.

Solution: calculated trimmed mean (excluding outlier) or calculate median instead.

Median: Centre data point of data set, isme pure data ki representation nhi ati.

- not affected by outliers

-

MIGHTY PAPER PRODUCT

Date: _____

Mode. Can be calculated for both qualitative and quantitative
- mean and median only for quantitative data
- hoskta ha data me kar mode na ho ya kar bhi na ho

If distribution of frequency distribution graph is symmetrical then mean = median = mode

→ Right Skewed curve par mean > median > mode
→ left // // // mode > median > mean

Population: σ^2 → variance , σ → Sd
Sample : s^2 → // , s → Sd

- variation defines data ka hisab se !
- variance is highly affected by outliers
- range and coeff of variance are another measures of central tendency

Topic: _____ Date: _____

Binomial Distribution (Discrete)

Conditions:
* There are fixed number of trials.
* Only two (2) possible outcomes. (Success and Failure)

Success = P , Failure = Q

1, 2, 3, 4, 5, 6

Success = P = 1/6

Failure = Q = 5/6

$P(\text{Success}) + P(\text{Failure}) = 1$

$p + q = 1$

* Probability of success is constant
* Trials are independent of each other.

Parameters:

$X \sim B(n, p)$ (~ : Follows)

X = Random Variable , B = Binomial distribution

n = no. of trials , p = prob. of success

Formula:

$$P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

Expectation (Mean) =

$E(x) = np$

Variance [Var(x)] = npq

* Jis ki probability dhoondhe hogi vo = SUCCESS

Topic: _____ P(H) ≠ P(T) no. of trials Date: _____

Example 1: A biased coin is tossed five times. Probability of obtaining a head is $2/3$. Let random variable represents no. of heads obtained.

a) Find the prob. that exactly 2 heads are obtained

Method 1: (2 heads, 3 tails)

$$P(HHTTT) = \left(\frac{2}{3} \times \frac{2}{3}\right) \times \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) \times \frac{5!}{2! \times 3!} = \frac{40}{243}$$

Method 2: $n = 5$ $P = 2/3$ $q = 1/3$

$$X \sim B(5, 2/3)$$

$$P(x = 2) : {}^5C_2 \left(\frac{2}{3}\right)^{②} p \left(\frac{1}{3}\right)^{③} q = \frac{40}{243}$$

5 me se 2 head

Outcomes = No. of trials + 1

b) Draw Prob. distribution table for x.

$P(x = 0) = q^n$
$P(x = n)$ p^n

x	Outcomes	Working	$P(X = x)$
0	T T T T T	$q^n = (1/3)^5$	$1/243$
1	H T T T T	${}^5C_1 (2/3)^1 (1/3)^2$	$10/243$
2	H H T T T	${}^5C_2 (2/3)^2 (1/3)^3$	$40/243$
3	H H H T T	${}^5C_3 (2/3)^3 (1/3)^2$	$80/243$
4	H H H H T	${}^5C_4 (2/3)^4 (1/3)^1$	$80/243$
5	H H H H H	$p^n = (2/3)^5$	$32/243$
			1 → Sum

% wale sawalon me ameeman (mostly) = Enemies

Topic: Date:

P(atleast 1 head) = $P(x \geq 1)$
$= P(1, 2, 3, 4, 5)$
$= 1 - P(x = 0)$

P(more than one head) = $P(x > 1)$
$= P(2, 3, 4, 5)$
$= 1 - P(x = 0, 1)$

Q2: On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars. A random sample ($n = 11$) of 11 vehicles is taken. Find the probability that fewer than 3 are buses.

$P(T) = 0.2$ $\quad P(x < 3) = ?$ → all failure

P $P(B) = 0.16$ $\quad P(x = 0) = (0.84)^{11}$

$P(C) = 0.64$

$n = 11$ $\quad P(x = 1) = {}^{11}C_1 (0.16)^1 (0.84)^{10}$

$P = 0.16$

$q = 1 - 0.16 = 0.84$ $\quad P(x = 2) = {}^{11}C_2 (0.16)^2 (0.84)^9$

$X \sim B(11, 0.16)$

Q3 A survey of adults in certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% didn't wear a watch.

A random sample of 14 adults was taken

Find the probability that more than 2 adults did not wear a watch.

Topic: _____ Date: _____

$P(L) = 0.76$

$P(R) = 0.15$

$P(W') = 0.09 \rightarrow X \sim B(14, 0.09)$

$n = 14$

$P(x > 2) = P(x = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)$

$= 1 - P(x = 0, 1, 2)$

$P(x = 0) = (0.91)^{14} = 0.2670$

$P(x = 1) = {}^{14}C_1 (0.09)^1 (0.91)^{13} = 0.3697$

$P(x = 2) = {}^{14}C_2 (0.09)^2 (0.91)^{12} = 0.2376$

0.87439

$P(x > 2) = 1 - 0.87439 = 0.126$

Q A manufacturer makes two sizes of elastic bands; large and small. 40% of the bands produced are large and 60% are small bands. Assuming that each of these elastic bands contains a random selection, calculate the probability that, is a pack containing 20 bands, there are

i) equal number of large and small bands.

Large = 0.4

Small band = 0.6

$n = 20$

$= {}^{20}C_{10} (0.4)^{10} (0.6)^{10}$

$= 0.117$

Topic: Date:

Mean (Expectation) and Variance of a Binomial Distribution

Q1: A garden shop sells polyanthus plants in boxes, each box containing the same no. of plants. The no. of plants per box which produces yellow flower has a binomial distr. with mean 11 and variance 4.95. Find the number of plants per hour.

$X \sim B(n, p)$

$E(X) = 11$

$np = 11 \quad (i)$

$n = 11/0.55$

$\boxed{n = 20}$

$Var(X) = 4.95$

$npq = 4.95 \quad (ii)$

$\downarrow$

$11q = 4.95$

$\boxed{q = 0.45}$

$\boxed{p = 1 - 0.45 = 0.55}$

Q2: The mean number of defectives batteries in pack of 20 is 0.6. Use binomial distr. to calculate the prob. that a randomly chosen pack of 20 will have more than 2 defective batteries.

$X \sim B(20, p)$

$E(X) = 1.6$

$np = 1.6$

$20p = 1.6$

$\boxed{p = 0.08}$

$q = 1 - p$

$\boxed{q = 0.92}$

$P(x > 2) = P(x = 3, 4, 5, 6 \ldots 20)$

$1 - P(x = 0, 1, 2)$

$P(x = 0) = (0.92)^{20} = 0.188$

$P(x = 1) = {}^{20}C_1 (0.08)^1 (0.92)^{19} = 0.328$

$P(x = 2) = {}^{20}C_2 (0.08)^2 (0.92)^{18} = 0.271$

1 - = Ans

* jab tak success na ajaye outcomes pattern kro change → Discrete

Date: 12/3/21

Topic: # Geometric Distribution

Definition : A distribution that represents number of failures in a series of tests before you get a success

Conditions ✓ ① Probability of success is constant.
✓ ② Repeated trials are independent.
✓ ③ Only two possible outcomes i.e: Success and Failure
p = success q = failure

difference than Binomial ④ Repeated trials can be infinite in number

Eg 1 : Tossing a coin until head is obtained
X = Random Variable = No. of heads

x	Outcome	
1	H	p
2	TH	qp
3	TTH	q^2p
4	TTTH	q^3p
⋮	⋮	⋮
n		$q^{n-1}p$

* Success on 1st attempt } is tarah pichle hain
" " 2nd attempt } geometric

fixed trials — binomial
infinite " — geometric

* Geometric distribution me? (X) can not be zero!

Topic: _____ Date: _____

Eg 2: Throwing a Dice until 4 is obtained

X = Random Variable = Obtaining a 4

$p = 1/6$

$q = 5/6 \longrightarrow$ 4' = Not 4 = 1, 2, 3, 5, 6

x	Outcomes	Working	
1	4	$1/6$	p
2	4' 4	$5/6 * 1/6$	qp
3	4' 4' 4	$(5/6)^2 \times 1/6$	q^2p
4	4' 4' 4' 4	$(5/6)^3 \times 1/6$	q^3p
5	4' 4' 4' 4' 4	$(5/6)^4 \times 1/6$	q^4p
⋮			⋮
x		$(5/6)^{x-1} \times 1/6$	$q^{x-1}p$

Q Why this distribution is geometric?

$$1/6 + (5/6)(1/6) + (5/6)(5/6)(1/6) + (5/6)(5/6)(5/6)(5/6)(1/6) \ldots$$

$a = 1/6$

$r = 5/6$

$$S_\infty = \frac{a}{1-r} = \frac{1/6}{1-5/6} = \frac{1/6}{1/6} = 1$$

$$S_\infty = \frac{p}{1-q} = \frac{p}{p} = 1$$

*Continuous ke liye inclusive / exclusive ek hee baat

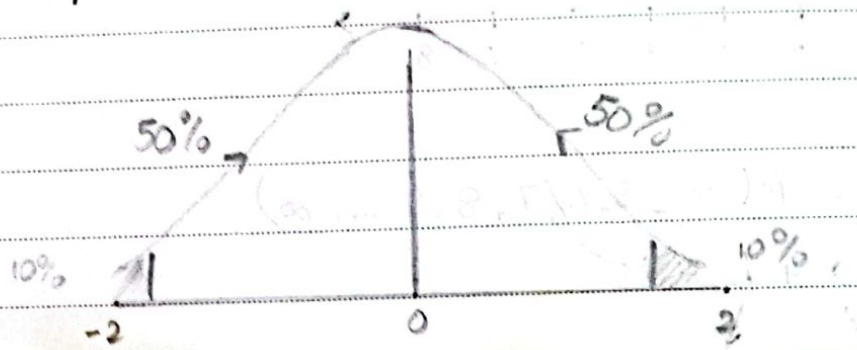
Topic: Normal Distribution Date: 13/3/21

Properties:

① Continuous Data
inclusive / exclusive is the same.

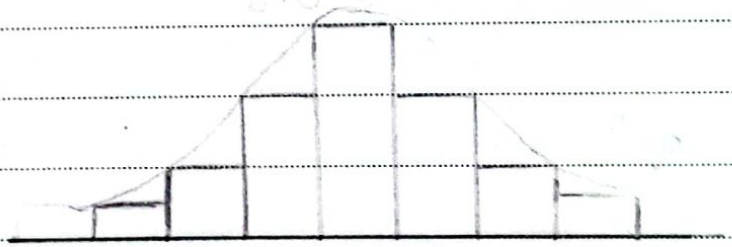
② Symmetrical

③ Bell-shaped Curve.



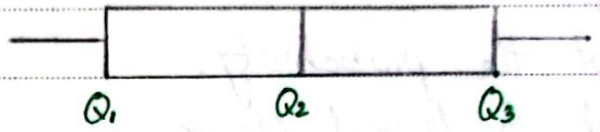
④ Area under graph represents Probability (Total Area = 1)

⑤ Histogram



Topic: _____ Date: _____

⑥ Box and Whisker Plot.



$$Q_2 - Q_1 = Q_3 - Q_2$$

$$Q_2 = \frac{Q_3 + Q_1}{2}$$

⑦ Mean = Median = Mode

⑧ Parameters.

$$X \sim N(\text{Mean}, \text{Variance})$$

Mean = μ

Variance = σ^2

SD = σ $\quad X \sim N(\mu, \sigma^2)$.

Cases

① $P(z < a) = P(a)$

② $P(z > a) = 1 - P(a)$

③ $P(z > -a) = P(a)$

④ $P(z < -a) = 1 - P(a)$

⑤ $P(a < z < b) = P(b) - P(a)$

⑥ $P(-a < z < -b) = P(a) - P(b)$

⑦ $P(-a < z < b) = P(b) + P(a) - 1$

2 (+) → right pe mark hoga
2 (-) → left pe mark hoga

Topic: * 50% se kam region shade ha ; 1 se minus hoga. Date:
* // // zyada // // // ; 1 se minus nhi hoga

Table

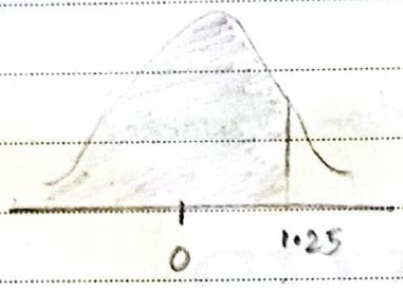
* Only applied on positive z values.
* Only left side of the probability.
* Can be applied till 3 decimal places
* Only applied when Mean = 0 and Variance = 1

$X \sim N(0, 1)$

* Shading depends upon sign of inequality ; (> → Right), (< → Left)

Case 1 : $P(z < 1.25)$

$P(1.25) = 0.8944$

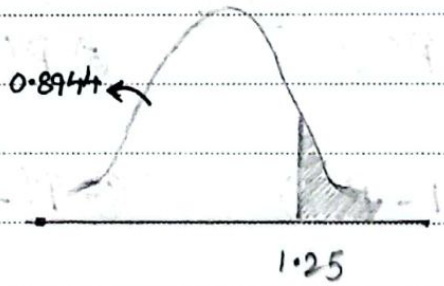


Case 2: $P(Z > 1.25)$

$1 - P(1.25)$

$1 - 0.8944$

$= 0.1056$



Q (z) left pe ya right pe mark hoga ?
Jahan (z) +ve ha wahan (right), jahan (z) -ve ha wahan (left)

Q shading right pe hogi ya left pe ?
Depends upon inequality
> → greater → right | < → less → left

Topic: _____ Case 3: Date: _____

iii) $P(Z > -0.89)$

$P(0.89) = 0.8133$

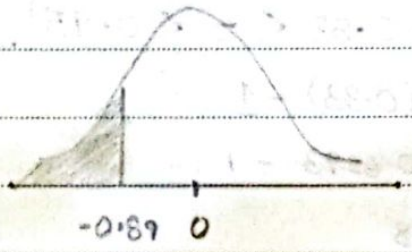
Case 4:

$P(Z < -0.89)$

$1 - P(0.89)$

$= 1 - 0.8133$

$= 0.1867$

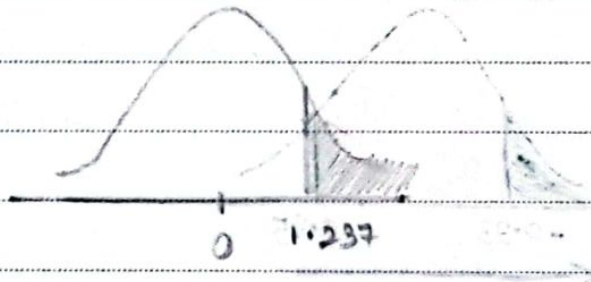


E.g: $P(Z > 1.237)$

$1 - P(1.237)$

$1 - 0.892$

0.108

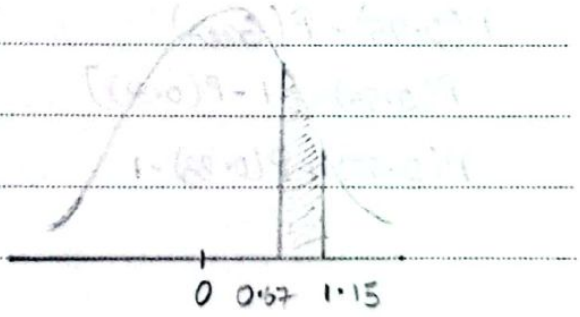


Case 5: $P(0.67 < z < 1.15)$

$P(1.15) - P(0.67)$

$0.8749 - 0.7486$

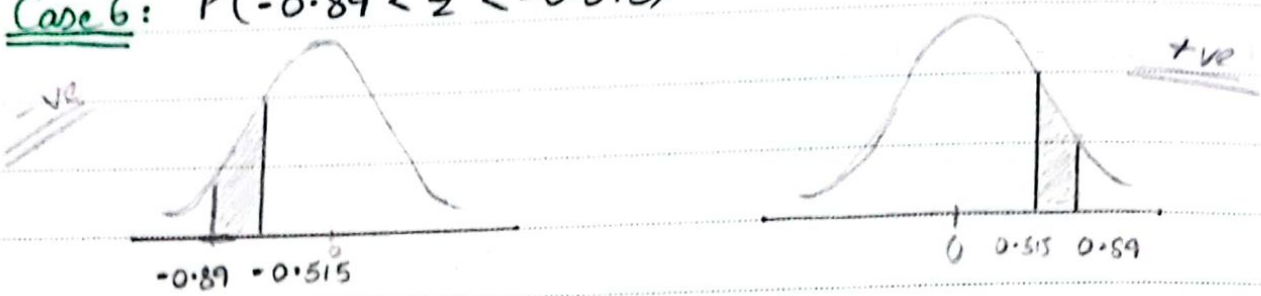
$= 0.1263$



* Probability hamesha belt ki batata ha!

Topic: _____ → hum -ve ke liye symmetrical property, read kiye hain, i.e positive Date:

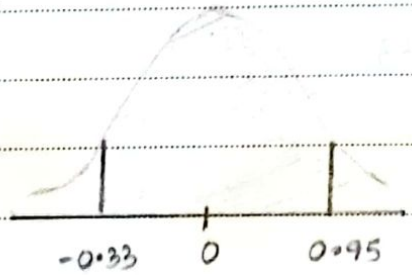
Case 6: $P(-0.89 < z < -0.515)$



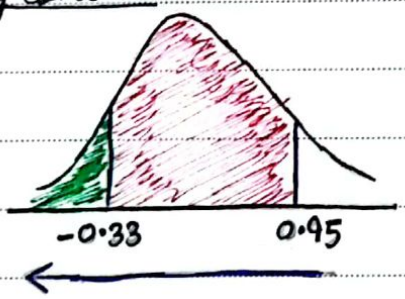
$$P(0.89) - P(0.515)$$
$$= 0.8133 - 0.6967$$
$$= 0.1166$$

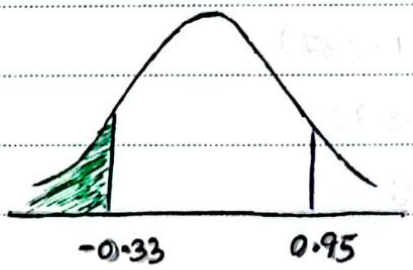
Case 7: $P(-0.33 < z < 0.95)$

$$P(0.95) + P(0.33) - 1$$
$$0.8289 + 0.6293 - 1$$
$$= 0.458$$



Explanation

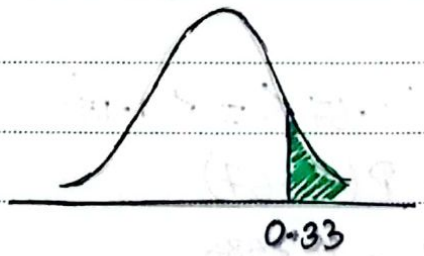




$P(0.95) - P(\text{Green})$ →

$P(0.95) - [1 - P(0.33)]$

$P(0.95) + P(0.33) - 1$



$P(\text{Green}) = 1 - P(0.33$

agar mean se agay wali value pe probability ho tou vo hamesha (+z) banaye ge, agar mean se peeche wale value ho to (-z) banaye ge.

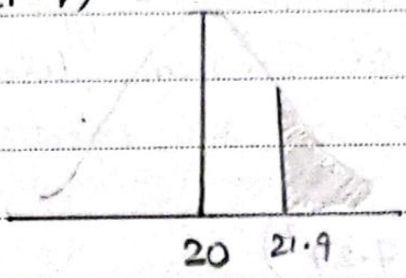
Topic: Date:

$X \sim N(20, 12)$ $P(x > 21.9)$

Mean = 20

Var = 12

SD = $\sqrt{12}$



$$z = \frac{x - \mu}{\sigma}$$

x → Random variable which is Normally Distributed

μ → Mean

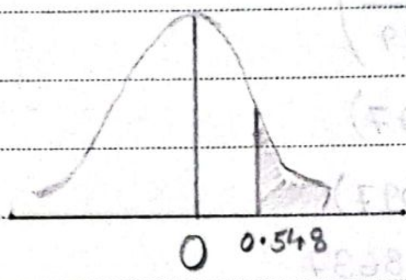
σ → SD

$$P\left(Z > \frac{21.9 - 20}{\sqrt{12}}\right)$$

$P(Z > 0.548)$

$= 1 - P(0.548)$

$= 1 - 0.7681 = \boxed{0.292}$



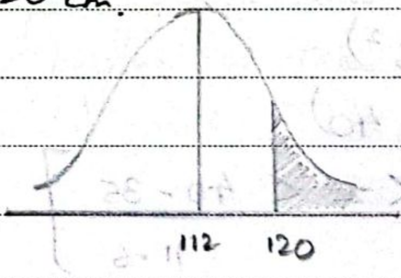
formula → vo value jo standardise krne ke baad ati ha

* humne yahan z banaya taake table apply hoske formula lagake

Q The height of sunflowers follows a normal distr. with mean 112 cm, sd = 17.2 cm, Find the prob. that the height of the randomly chosen sunflower is greater than 120 cm.

$X \sim N(112, 17.2^2)$

$P(x > 120)$



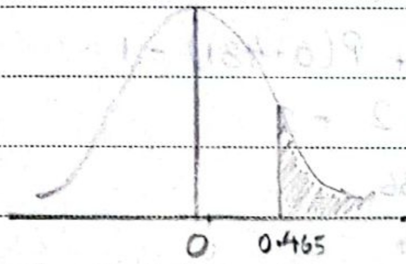
$$P\left[Z > \frac{120 - 112}{17.2}\right]$$

$P(Z > 0.465)$

$1 - P(0.465)$

$1 - 0.679$

0.321



Topic: _____ Date: _____

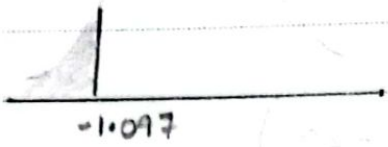
Q2 The length of fish of certain type have a normal distribution with mean 38 cm, and sd 7.29 cm. When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proption of fish rejected.

↳ Mean se pehle wale value ho (-ve)

$X \sim N(38, 7.29^2)$

$P(x < 30)$

$= P\left(z < \frac{30-38}{7.29}\right)$



$= P(z < -1.097)$

$= 1 - P(1.097)$

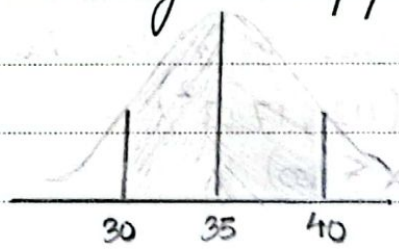
$= 1 - 0.8637$

$= \boxed{0.136}$

Q3 The distance in metres that a ball can be thrown by pupils at particular school follows a normal distr., with Mean = 35.0m and sd = 11.6m. Find the probability that a randomly chosen pupil can throw a ball b/w 30m and 40m.

$X \sim N(35, 11.6^2)$

$P(30 < x < 40)$



$P\left[\frac{30-35}{11.6} < z < \frac{40-35}{11.6}\right]$

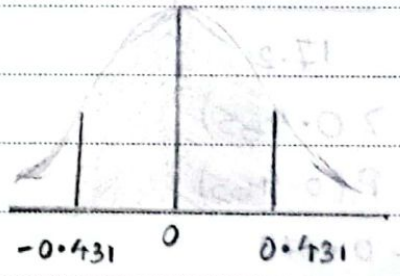
$P(-0.431 < z < 0.431)$

$P(0.431) + P(0.431) - 1$

$0.6668 \times 2 - 1$

$= 0.336$

≈ 0.334



* random sample = Binomial
* Success normal se aayega

Topic: _____ Date: _____

Binomial with Normal Distribution

Q1 The lengths of a fish of certain type have a normal distr., with mean 38 cm. It is found that 5% of the fish are longer than 50 cm. 9 fish are chosen at random. Find the prob; that atleast one of them is longer than 50 cm.

N

$X \sim N(38, \sigma^2)$

$P(X > 50) = 0.05$

Normal k sawal me binomial ✓

B

$X \sim B(9, 0.05)$

$P = 0.05 \quad q = 0.95$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - q^n$

$= 1 - (0.95)^9$

$= 0.369$

$= 0.37$

Q2: The length of new pencils are randomly distributed with mean = and SD = 0.095 cm. Find the probability that in a random samp of 6 pencils, atleast two have lengths less than 10.9 cm

$X \sim N(11, 0.095^2)$

$P(X < 10.9) =$ Success

$P\left(Z < \frac{10.9 - 11}{0.095}\right)$

$P(Z < -1.053)$

$X \sim B(6, 0.1462)$

$p = 0.1462, \quad q = 0.8538$

Normal ke sawal me binomial ✓

Q A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist, and 9% dinot wear a watch.

A random sample of 200 adults was taken. Using a suitable approximation, find the prob. that more than 155 wore a watch on their left wrist.

$X \sim B(200, 0.76)$

$P(x > 155)$

$n = 200$

$p = 0.76$

$q = 0.24$

$\mu = (200)(1.76)$

$= 152$

$\sigma^2 = (200)(0.76)(0.24)$

$= 3648.$

$1 - P(0.$

$= 1 - 0$

$= 0.$

P(z value) → Prob (Forward)

Prob → P(z value) (Backward)

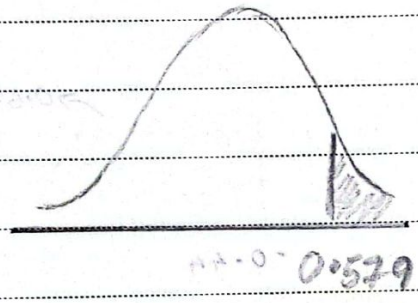
Topic: _____ Date: _____

$X \sim N(152, 36.48)$

$P(X > 155) = P(X > 155.5)$

$= P\left[z > \frac{155.5 - 152}{\sqrt{36.48}} \right]$

$= P(z > 0.579)$



$1 - P(0.579)$

$= 1 - 0.7188$

$= 0.281$

Topic: Date:

Reverse Working

Find

$P(z \text{ value}) = \text{Probability}$

Reverse

$z = \frac{x - \mu}{\sigma}$

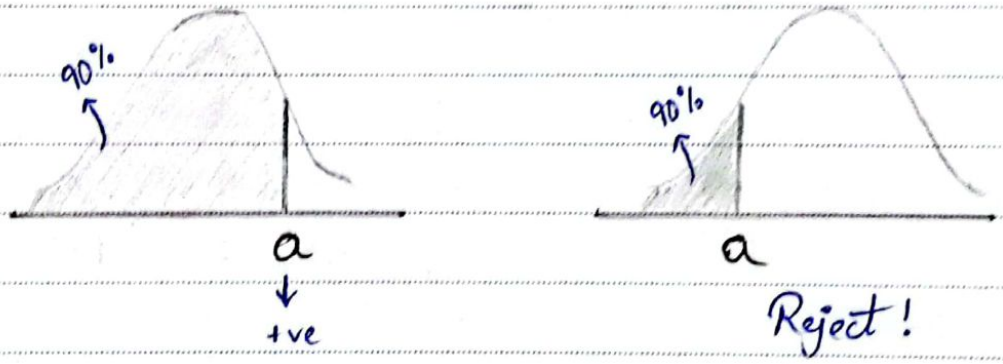
$P(\quad) = 0.6700$

$P(z < 0.44) = 0.6700$ $P(z > -0.44) = 0.6700$

greater than pe bht wala shade

lesser " " right " " "

:) $P(z < a) = 0.9$



$a = 1.282$

Topic: _____ Date: _____

21 : When Saad makes a phone call, the amount of time that the call takes has a normal distribution with mean 6.5 and sd 1.76 mins. 90% of Saad's phone calls takes Large than t mins. Find the value of t.

$X \sim N(6.5, 1.76^2)$

$P(X > t) = 0.9$

$$P\left[Z > \frac{t - 6.5}{1.76}\right] = 0.9$$

$$a = \frac{t - 6.5}{1.76}$$

$P(Z > a) = 0.9$

$a = -1.282$

$$\frac{t - 6.5}{1.76} = -1.282$$

$t = 4.24$

Topic: _____ Date: _____

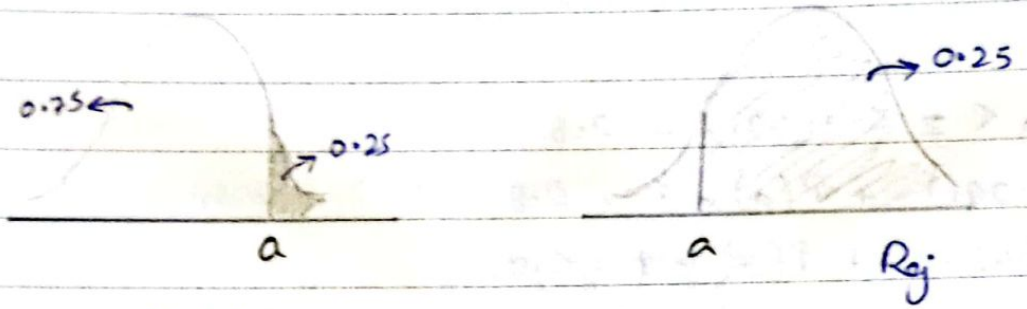
Q2 : A farmer finds that the weights of sheep have a normal distr. with mean u kg, and sd 4.92 kg. 25% of these sheep weigh more than 67.5 kg. Find the value of u.

$$X \sim N(u, 4.92^2)$$

$$P(x > 67.5) = 0.25$$

$$P\left[z > \frac{67.5 - u}{4.92}\right] = 0.25$$

$$P[z > a] = 0.25$$



$$a = 0.674$$

$$\frac{67.5 - u}{4.92} = 0.674$$

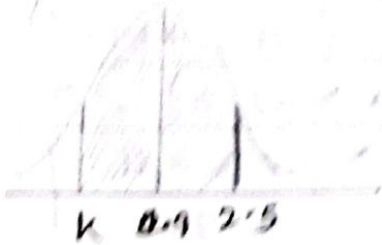
$$u =$$

Topic: Date:

Stones in a carton chemical including have weights x grams, which are normally distributed, with mean 1.9kg and SD 0.55kg. Find the value of k such that $P(k < x < 2.5) = 0.8$

$X \sim N(1.9, 0.55)^2$

$P(k < x < 2.5) = 0.8$



$$P\left[\frac{k-1.9}{0.55} < z < \frac{2.5-1.9}{0.55}\right] = 0.8$$

$P(a < z < 1.091) = 0.8$

$P(1.091) + P(a) - 1 = 0.8$

$0.8623 + P(a) - 1 = 0.8$

$P(a) = 0.9377$

Reverse

$a = -1.536$

$\frac{k-1.9}{0.55} = -1.536$

$k = 1.055$

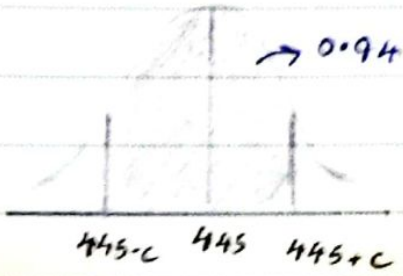
$\boxed{k = 1.06}$

Topic: _____ Date: _____

Q4 Cans of lemon juice are supposed to contain 440ml of juice. It is found that actual volume of juice in a can is normally dist: with mean 445 ml, and a sd 3.6ml. It is found that 94% of the cans contain b/w (445-c) ml and (445+c) ml of juice. Find the value of c.

$$X \sim N(445, 3.6)^2$$

$$P(445-c < x < 445+c) = 0.94$$



$$P\left[\frac{445-c-445}{3.6} < z < \frac{445+c-445}{3.6}\right] = 0.94$$

$$P\left(\frac{-c}{3.6} < z < \frac{c}{3.6}\right) = 0.94$$

$$P\left(\frac{c}{3.6}\right) + P\left(\frac{c}{3.6}\right) - 1 = 0.94$$

$$2P\left(\frac{c}{3.6}\right) = 1.94$$

$$P\left(\frac{c}{3.6}\right) = 0.97$$

$$\frac{c}{3.6} = 1.881$$

$$c = 6.77$$