Reducing Regret in Q-Learning with Ensemble Mechanics

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Introduction

Q-learning:

$$Q(s,a) \leftarrow Q(s,a) - \alpha\big(Q(s,a) - (r + \gamma \max_{a' \in \mathcal{A}(s')} Q(s',a'))\big)$$

ε-greedy:

$$\pi_{\text{act}}(s) = \begin{cases} \arg\max_{a \in \mathcal{A}(s)} Q(s,a) & \text{with probability } 1-\epsilon \\ \text{uniformly from } \mathcal{A}(s) & \text{with probability } \epsilon \end{cases}$$

Softmax:

$$\pi_{\text{act}}(a \mid s) = \begin{cases} \frac{\exp(Q(s,a)/\tau)}{\sum_b \exp(Q(s,b)/\tau)} & \text{probability } \epsilon \\ \mathbf{1}[a = \arg\max_{a' \in \mathcal{A}(s)} Q(s,a')] & \text{probability } 1-\epsilon \end{cases}$$

VDBE: adaptive π_{act} automatically decrease ε in response to environment

$$\delta = r + \gamma \max_{a' \in \mathcal{A}(s')} Q(s',a') - Q(s,a)$$

$$\epsilon \leftarrow \lambda \frac{1-\exp(-|\alpha\delta|/\sigma)}{1+\exp(-|\alpha\delta|/\sigma)} + (1-\lambda)\epsilon$$

Goal: improve VDBE to reduce cumulative regret:

$$R(N) = N \times \mathbb{E}_{\tau \sim \pi_{\text{opt}}}\Big[\sum_{t \in \tau} r(\hat{s}_t, \hat{a}_t)\Big] - \sum_{i=1}^{N} \sum_{t \in \tau_i} r(s_t, a_t)$$

Methodology

Let

$$Q(s,a) = f(\phi(s,a); \mathbf{w})$$

$$\delta_{\mathbf{w}} = \nabla_{\mathbf{w}} \tfrac{1}{2}\big(Q(s,a) - (r + \gamma \max_{a' \in \mathcal{A}(s')} Q(s',a'))\big)^2$$

Fluctuation energy, where $\hat{a}$ is action from previous state

$$H(s,\hat{a}) = |Q(s,\hat{a}) - Q'(s,\hat{a})|$$

Then

$$\epsilon \leftarrow \lambda \frac{1-\exp(-\sum_{\hat{a}} H(s,\hat{a})/|\mathcal{A}(s)|\sigma)}{1+\exp(-\sum_{\hat{a}} H(s,\hat{a})/|\mathcal{A}(s)|\sigma)} + (1-\lambda)\epsilon$$

- Baselines: ε-greedy, decaying ε, VDBE, and **ensemble-mechanics ε-greedy** (aka "**Stat Mech**")
  - Python 3.6.8
- Tests: **Blackjack** (stochastic) from homework; **Minefield** (nonstochastic) and **FrozenLake** (stochastic) from Ref. 9

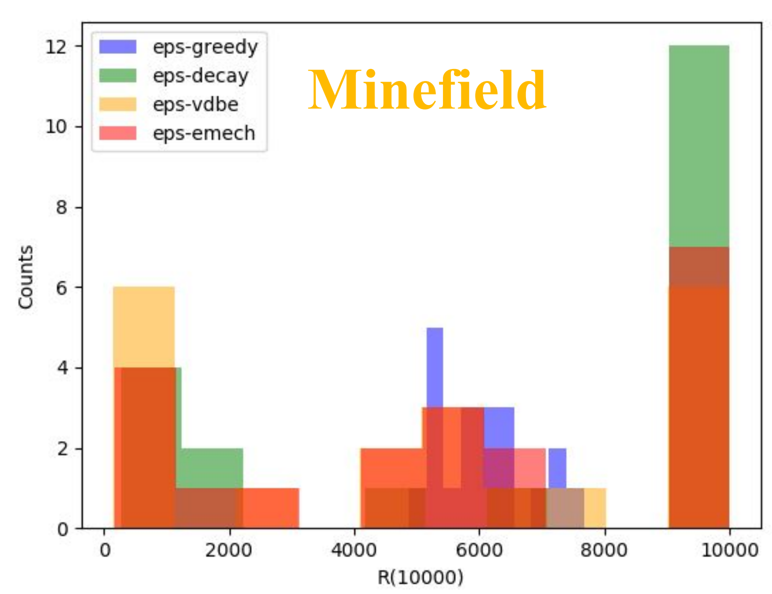
```
SLLL    S = start
LHLH    L = land, safe
LLLH    H = hole/mine
HLLE    E = end
```

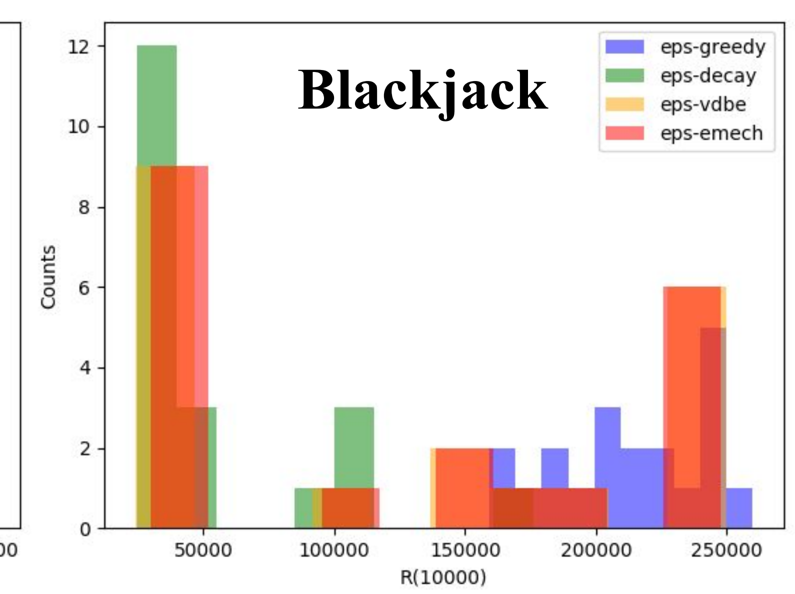
Abstract

- Reduce speed of convergence (regret) in RL algorithms
- Our algorithm: ε-greedy, but update ε as agent moves through environment according to statistical mechanics
- Compare with three existing RL algorithms
- No improvement but a better *a priori* hyperparameter distribution could help

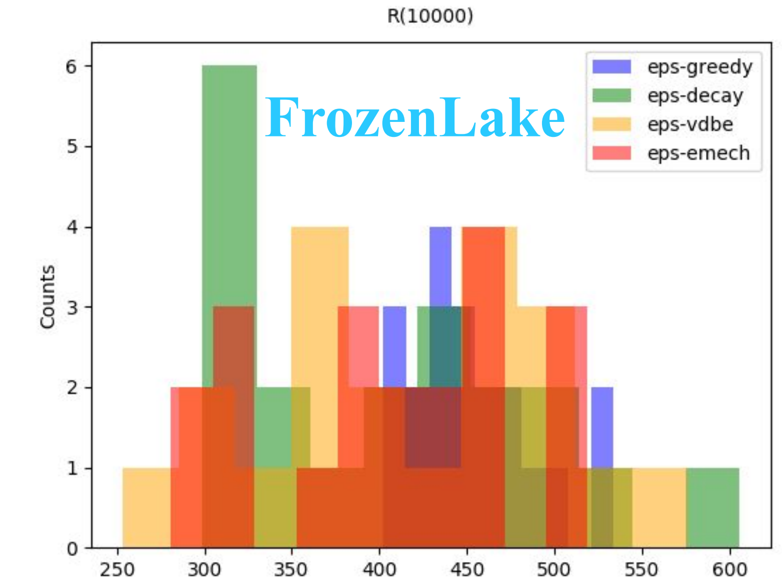
Results and Analysis

		ε-greedy	ε-decay	ε-VDBE	Stat Mech
Blackjack	Mean	213744	55601	127089	128443
$R(10^4)$	Std Error	6572	8898	20313	20115
Minefield	Mean	6089	6816	5350	5840
$R(10^4)$	Std Error	161	920	840	809
Frozen Lake	Mean	453	407	414	403
$W(10^4)$	Std Error	8.1	18.6	18.5	15.7

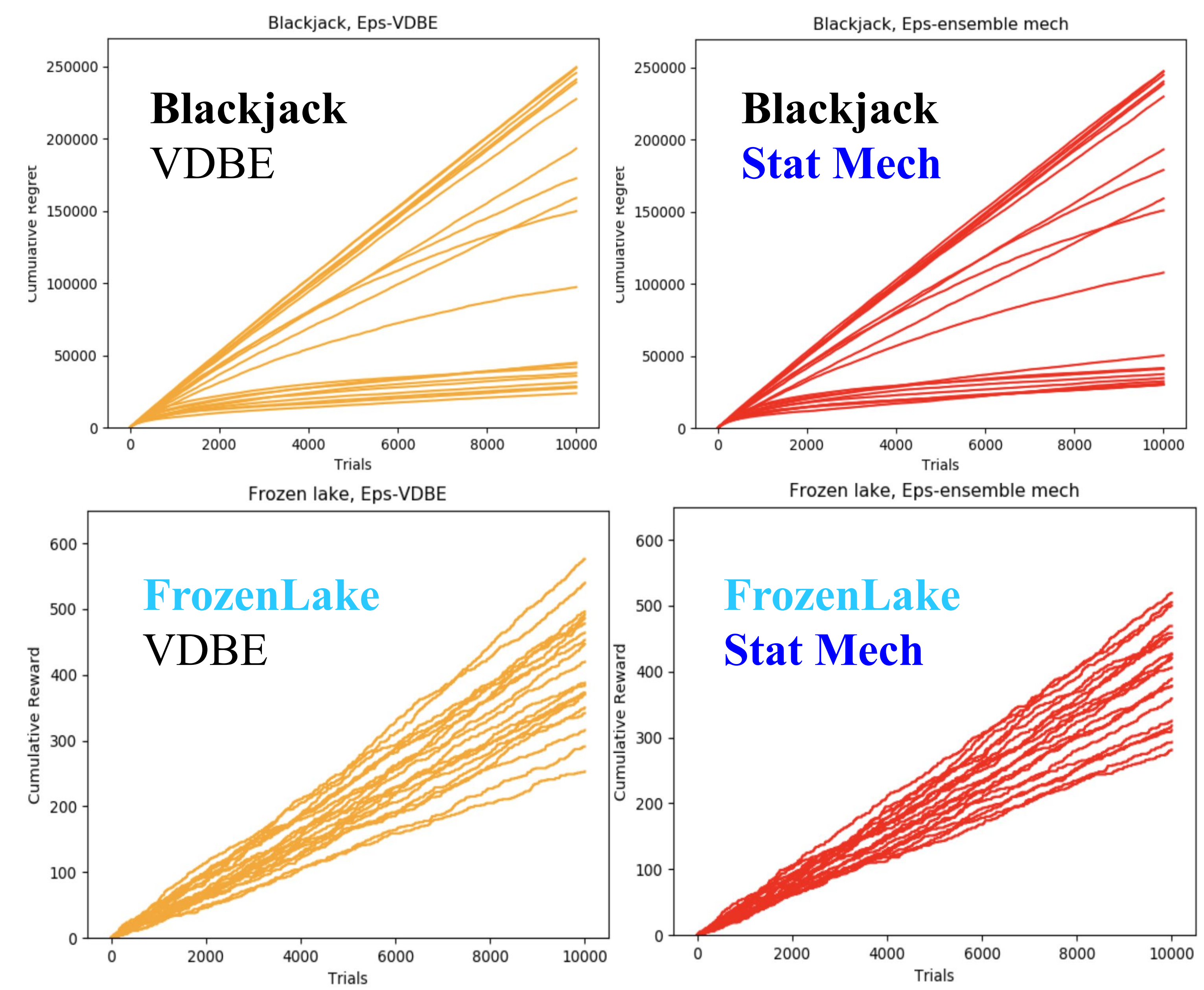




- **Blackjack**: high variance, average performance
- **Minefield**: high variance, decent performance
  - Only nonstochastic environment
- **FrozenLake**: low variance, bad performance
  - Use cumulative reward W (higher is better): $W(N) = \sum_{i=1}^{N} \sum_{t \in \tau_i} r(s_t, a_t)$



Results and Analysis (continued)



Conclusion and Future Work

- Our algorithm ("**Stat Mech**") performs comparable to VDBE in two environments
- VDBE performs slightly better in one environment
- Used same hyperparameter distribution (a convenient assumption that can be improved) for all tests
  - Future: improve hyperparameter distributions
- Adaptive algorithms underperform nonadaptive in 2/3 tests, possibly due to stochasticity, as noted in Ref. 8
  - Future: formalize stochasticity

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