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A solution for the cooperative formation-tracking problem in a network of completely unknown nonlinear dynamic systems without relative position information

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ABSTRACT

In this paper, a solution of the formation-tracking problem is provided for a network that contains nonlinear agents with completely unknown dynamics and working under unknown disturbances. By the combination of a cooperative observer and an adaptive model-free controller, the requirement of inter-agent relative position information in the network is eliminated. Here, a cooperative observer is designed to estimate the time-varying reference trajectory and the time-varying parameters of the desired formation topology at each agent in the network. The stability of the proposed cooperative observer is analysed using Lyapunov analysis. Utilising the cooperative observer, the formation-tracking problem in the network of dynamic agents is transformed to a tracking problem in a single agent system. Moreover, an adaptive model-free control policy is applied to each agent for providing the tracking objective. Utilising the algebraic connectivity originating from graph theory, this model-free control algorithm is formulated to scale-up for a network of multi-agents. The proposed decentralised controller includes two model-free adaptive laws for online estimating of the completely unknown dynamics at each agent in the network. The application of the proposed solution is simulated for a network of four quadrotors with unknown internal dynamics and unknown external disturbances.

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Formation-tracking problem; cooperative observer; adaptive model-free control; unknown nonlinear dynamic system

1. Introduction

Great attention has been paid to the problems of controlling a multi-agent network ranging from consensus to flocking movements, formation control and leader-following (Das & Lewis, 2011; Mahyuddin, Herrmann, & Lewis, 2013; Mahyuddin, Herrmann, & Na, 2012; Mahyuddin & Tiang, 2017; Peng, Wang, Sun, & Wang, 2014; Zhang & Lewis, 2012). The formation control problem is an interesting issue in biology, automatic control and robotics, which requires each agent in the network to move according to the reference trajectory, while attaining a prescribed formation topology in cooperation with the other agents (Cui, Zhuang, & Lu, 2016). Two major issues concerning formation control problem are dynamic uncertainties of each agent and information about the inter-agent relative position.

The dynamic system of agents in the network usually have unknown nonlinear terms due to unpredictable environmental disturbances and unmodelled dynamics as well as other uncertainties (Mahyuddin & Safaei, 2017; Safaei & Mahyuddin, 2017b). Work by Wang, Wang, and H (2015) reported the design procedure of a distributed state-output feedback cooperative control approach for multiple uncertain agents in a network with directed communication graphs. Other works such as in Meng, Yang, Jagannathan, and Sun (2014) and Cui, Xu, Lewis, Zhang, and Ma (2016) employ a direct adaptive approach using an artificial neural network (ANN) to approximate an ideal controller to deal with unknown dynamics of agents. Similarly, Peng, Wang, Zhang, Sun, and Wang (2013) and Peng, Wang, Zhang, and Lin (2015) used ANNs to estimate the unknown nonlinear terms at each agent, whilst solving for the cooperative tracking problem. However, the controller gain matrix is defined off-line by the solution of a continuous-time algebraic Riccati equation (CARE). There, the system matrix for each agent is assumed known. Similar technique can be found in Meng, Yang, Sarangapani, and Sun (2017), whereby the asymptotical convergence in consensus error is proven. An ANN is used for online estimation of nonlinear terms in Wang, Wang, and Peng (2017) and further an observer is proposed to predict the unmeasured states of the dynamic system locally at each agent. Similarly, in Wang, Wen, and Huang (2017), the

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nonlinear functions are estimated with a set of ANNs and consensus tracking problem is solved for a network of nonlinear agents. That work considers that the information of the reference trajectory is not available for all agents. Instead, the reference trajectory is estimated using the knowledge on the upper bound of its rate in order to track the designated path. Shi and Shen (2017) and Wen, Yu, Li, Yu, and Cao (2016) proposed a solution to solve a leader-following consensus problem for a network of single-input single-output (SISO) and multi-input multi-output (MIMO) nonlinear agents. While ANNs are used to estimate the unknown dynamics of agents in the network, the linear matrix inequality (LMI) is employed to find the values of the controller gain matrix from the Schur complement.

There is a wide range of nonlinear basis functions that can be utilised for online estimation of the unknown nonlinear terms. In ANN, mostly sigmoid or hyperbolic tangent is used as the basis functions. A fuzzy inference system (FIS) also can be incorporated for nonlinear estimation, where the fuzzy membership functions work as nonlinear basis functions. In Shen, Shi, and Shi (2016), the synchronisation problem is solved for a network of nonlinear agents using a FIS for online estimation of the unmodelled nonlinear dynamics at each agent. In Sakhre, Pratap Singh, and Jain (2017), a fuzzy competitive learning algorithm is used for online training of ANN and an online algorithm is proposed for online estimation of the nonlinearities in the dynamic systems. A solution using radial basis function (RBF)-ANN is proposed in Yang, Cao, Peng, Wen, and Guo (2017) for distributed formation control of non-holonomic autonomous vehicles. Similarly, in Chen, Wen, Liu, and Liu (2016) and Wang, Shi, Li, and Zhou (2017), the RBF-ANN is utilised to estimate the nonlinear dynamic terms at the agents. The Fourier series are another nonlinear basis functions which are considered for online estimation of the nonlinearities of the agents' dynamics in a network (Wang, Wang, Yan, Li, & Cai, 2017).

In Cui (2016), it is mentioned for the first time that the use of ANN (and also FIS) as a tool for approximating the nonlinearities in the agents' dynamic systems can lead to the computational complexity problem. In other words, estimation computational complexity increases in proportion to the number of ANN nodes being employed. Such computation overheads can cause further implication on the inter-agent data communication which is primarily important for the cooperative control framework. The authors in Cui (2016) have proposed a solution for the problem of distributed synchronisation, using the dynamic-surface control (DSC) technique. Despite decreasing the complexity, the proposed algorithm still relies on the use of ANNs. In Bechlioulis and Rovithakis (2017), a decentralised cooperative protocol is proposed for synchronisation problem in a network of nonlinear agents. No online estimation of the agents nonlinear dynamic is evident in the aforementioned work. Instead, the authors proposed an exponential dynamics for the upper bound of the absolute consensus errors in the network. In that work, the characteristics of the proposed dynamic bound are defined according to the desired transient and steady-state performance of the system. By this virtue, neither the prior knowledge of the nonlinearities in the agents' dynamic systems nor common estimation tools (ANN, FIS, etc.) are incorporated in the proposed control algorithm. This promotes lite computation suitable for practical implementation especially when on-board power supply is limited on each agent. In the proposed algorithm in Bechlioulis and Rovithakis (2017), there is not any online process for determining the main controller gains. Thus, they should be chosen manually by some experts in an off-line manner. The idea of using prescribed performance as dynamic bound for the consensus error is used in Hashim, El-Ferik, and Lewis (2017), as well. Adaptive projection algorithm is considered instead of ANN for the online approximation of the unknown nonlinear terms in the agents' system dynamics. For the implementation of the proposed projection algorithm, the upper bounds for the agents' states are required. Moreover, the adaptive law is driven by a transformed consensus error, which is a nonlinear function (exponential function is employed) of the ratio of consensus error to the prescribed performance of the system. The use of nonlinear basis functions is still evident for online estimation of the nonlinearities in that algorithm. Recent work by Li, Du, Yang, and Gui (2018) and Du, Yang, Li, and Gui (2018) showcase a novel sliding-mode control algorithm to compensate for the unknown nonlinear terms in a single agent. Such work can be further extended to accommodate for a multi-agent setup in a network.

Another issue in the formation control problem for a team of dynamic systems is the need for inter-agent relative position information (in 2D or 3D environments). Such requirement inherently exists in almost all cooperative control schemes. While the radar sensors can be used for massive agents, it is not efficient to implement them on drones, since the drones are comparatively small agents that cannot carry heavy sensors like radar or Lidar (Kang, Park, & Ahn, 2017). In addition, the energy consumption of the batteries will be increased by adding some extra weights. Although there are some available investigations for measuring the relative positions among the mobile agents using the on-board equipments like Zigbee, WiFi communication and ultra-wide band (UWB) modules (Guo, Qiu, Meng, Xie, & Teo, 2017;

Han, Guo, Xie, & Lin, 2018), relative localisation task requires a set of reliable tool. It remains an open problem (Kang et al., 2017).

Regarding the above two mentioned issues, here in this paper, a formation-tracking problem is solved for a network of completely unknown dynamic agents without the need for relative position information among the agents. Here, it is assumed that both the time-varying reference trajectory and the time-varying desired formation topology are not available to all agents. Instead, the agents which are pinned to a virtual leader can access to the mentioned data. Hence, a cooperative observer is designed to estimate the reference trajectory and the formation topology by considering the communication graph among the agents. The proposed cooperative observer is completely decoupled from the agents' dynamics and independent of the control signals. Owing to the robustification term in the cooperative observer, the parameters can be estimated in a finite time. By use of the observed values for the desired trajectory and the formation topology, the formation-tracking problem among the agents in a network is converted to a simple tracking problem at a single agent. In fact, the desired path at each agent can be determined according to the observed reference trajectory and the formation topology parameters. In addition, a general structure is proposed for unknown nonlinear dynamics of agents in a network and then an adaptive model-free control (AMFC) policy is applied for tracking problem at each agent. The AMFC does not use model-based tools like ANN or FIS for online estimations. Instead, it includes two model-free adaptive laws for online estimation of unknown linear and nonlinear terms, locally at each agent. These adaptive laws are derived without requiring the linear-in-parameter (LIP) condition, i.e. the dependence on any model regressors or nonlinear basis functions and consequently the persistently excitation (PE) requirement is eliminated for the adaptive laws. Furthermore, by updating the unknown system matrix (linear term) at each agent, a differential Ricatti equation (DRE) is utilised for online computing of the controller gains. The proposed cooperative formation control based on the novel AMFC algorithm is formulated to be incorporated locally at each agent without requiring *a priori* graph communication information. The additional salient feature exhibited by the proposed algorithm is that it will still work even under the condition of a simply-connected communication graph. Utilising the cooperative observer and the AMFC policy, there is not any need for the relative position information among the agents in the network. Only, each agent should measure its local states (including position and velocity). The stability analysis for the cooperative observer is provided based on Lyapunov stability theorem, while the stability proof of the AMFC algorithm is referred to a recently published work. Finally, the proposed solution is exemplified in a form of simulation on a network of aerial mobile robots and appropriate results are achieved. In this regard, the contributions of the current work are as follows:

- a cooperative observer is proposed for estimating the time-varying reference trajectory and time-varying formation topology parameters at each agent in the network. The proposed cooperative observer is completely decoupled from the agents' dynamics as well as the decentralised controllers at the agent;
- the information on inter-agent relative position among the agents in the network is not required by the use of the proposed solution for the formation-tracking problem. Moreover, the formation-tracking problem among the agents in a network is transformed to a tracking problem at each of the agents by utilising the observed data;
- an AMFC policy is applied for tracking problem at each agent in the network without use of any ANN or FIS for online estimation. Instead, the online estimations are performed using two model-free adaptive laws. The proposed AMFC does not depend on the properties of the communication graph. In addition, the main controller gains are updated online locally at each agent by use of a DRE.

Based on the above contributions, the advantages of the proposed solution over the similar algorithms in the literature are reviewed as follows:

- here, the formation-tracking objective is achieved without the need for inter-agent relative position information among the agents in the network, while this information is required at all of the cooperative formation control policies proposed in the literature (Bechlioulis & Rovithakis, 2017; Cui, 2016; Hashim et al., 2017; Safaei & Mahyuddin, 2018d). Since the relative position estimation (or the relative localisation) problem still remains open, it can provide less practical concerns regarding the implementations of the solution in real applications;
- here, the online estimation for the unknown nonlinear dynamics of the agents as well as the unknown external disturbances is performed without using tools like ANN or FIS, which are used in almost all of the algorithms in the literature (Cui et al., 2016; Meng et al., 2017; Sakhre et al., 2017; Wang et al., 2017; Yang et al., 2017). Instead, two model-free adaptive laws are used in the current solution for online estimation task. This leads to less computation complexity and

more energy efficiency of the solution. In addition, the requirement of the PE condition and consequently the sufficiently rich property of the input signal is revoked in the proposed solution. This lessens the practical concerns of the solution.

In the following sections, first some preliminary definitions are provided and a general structure is proposed for unknown dynamics of a nonlinear system. Then, the design procedure for a cooperative observer is presented in Section 3. The AMFC policy along with the adaptive laws for online estimation are presented in Section 4. Finally, the simulation results for a case with constant formation topology parameters and for a case with time-varying formation topology parameters are provided in Section 5 for a network of quadrotors.

2. Problem definition

2.1. Communication network

Definition 2.1: Consider a network consisting of N heterogeneous agents. Let $\mathcal{G}(\digamma, \mathcal{E}, \mathcal{A})$ be a graph with the set of N *nodes* $\digamma = (v_1, v_2, \ldots, v_N)$, a set of *edges* $\mathcal{E} = (e_{ij}) \in \mathbb{R}^{N\times N}$ and associated adjacency matrix $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$. An edge e_{ij} in $\mathcal{G}$ is a link between a pair of nodes (v_i, v_j), representing the flow of information from v_j to v_i. The e_{ij} is in existence if and only if $a_{ij} > 0$. The graph is undirected, i.e. the e_{ij} and e_{ji} in $\mathcal{G}$ are considered to be the same. We name v_i and v_j as neighbours, if $e_{ij} \in \mathcal{E}$. The communication graph is considered to be connected, meaning that there is a path between each pair of agents in the network. The in-degree matrix is defined as $\mathcal{D} = \mathrm{diag}(d_1, d_2, \ldots, d_N) \in \mathbb{R}^{N\times N}$, where each d_i is the input degree to each node, i.e. $d_i = \Sigma_{j=1}^{N} a_{ij}$. Hence, we can define Laplacian matrix $\mathcal{L}$ as below (Lewis, Zhang, Hengster-Movric, & Das, 2014; Li & Duan, 2015):

$$\mathcal{L} = \mathcal{D} - \mathcal{A}. \tag{1}$$

Furthermore, by considering a virtual leader with access to the desired trajectory to be tracked by the whole network and also the formation parameters, one can define the *leader pinning gain matrix* as follows:

$$\mathcal{B} = \mathrm{diag}(b_1, b_2, \ldots, b_N) \in \mathbb{R}^{N\times N}, \tag{2}$$

in which b_i indicates the existence of a communication link between the virtual leader and the ith agent in the network (Lewis et al., 2014; Li & Duan, 2015; Mahyuddin, 2013; Mahyuddin et al., 2012; Mahyuddin & Tiang, 2017). Then, we denote

$$\mathcal{H} = \mathcal{L} + \mathcal{B}. \tag{3}$$

Assumption 2.1: There is at least one communication connection between one of the agents and the virtual leader. In other words, at least one of the diagonal elements in $\mathcal{B}$ is non-zero.

Definition 2.2: We define $\eta^i \in \mathbb{R}^{n\times 1}$ as the time-varying formation parameter corresponding to agent i (0 is for the virtual leader). The values for elements of η^i are defined regarding the desired formation (i.e. separation) among the agents in the network in a mutual reference frame.

2.2. General formulation for an unknown nonlinear MIMO dynamic system

Definition 2.3: Consider the dynamic system of ith agent in the proposed network as follows:

$$\begin{aligned} \dot{x}^i &= f_0^i(x^i, u_0^i), \\ y^i &= C_0^i x^i, \end{aligned} \tag{4}$$

where $x^i \in \mathbb{R}^{n\times 1}$ is a vector including system states, $u_0^i \in \mathbb{R}^{m\times 1}$ is the control input, $f_0^i(.) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a vector including unknown bounded Lipschitz continuous nonlinear functions depending on both x^i and u_0^i (refer to Safaei & Mahyuddin, 2018a for further discussions on the properties of $f_0^i(.)$); $C_0^i \in \mathbb{R}^{r\times n}$ is the output matrix and $y^i \in \mathbb{R}^{r\times 1}$ is the system output. Referring to Definition 2.1, it is assumed that the network includes heterogenous agents. Hence, the dynamics of different agents in the network are not identical and we should use f_0^i and C_0^i corresponding to the dynamics of agent i. But, all of the agents have n states, m control inputs and r outputs. Assuming that all the states are reachable (either locally or by using a distributed algorithm), and the nonlinear system at each agent is *output controllable*, one can have a state-feedback controller $u_0^i = u_0^i(x^i)$ to make the system stable (Hou & Jin, 2014). Then, we can have

$$\dot{x}^i = f_0^i(x^i, u_0^i(x^i)) - B_0 u_0^i(x^i) + B_0 u_0^i(x^i), \tag{5}$$

where $B_0 = [b_{0jk}] \in \mathbb{R}^{n\times m}$ is a gain matrix defined as (for $j = [1, n]$ and $k = [1, m]$)

$$b_{0jk} = \begin{cases} 0 & \text{if } \dot{x}_j^i \text{ does not depend on } u_{0k}^i, \\ 1 & \text{if } \dot{x}_j^i \text{ depends on } u_{0k}^i. \end{cases} \tag{6}$$

The minimal information about the dynamic system at agent i is required to determine whether the rate for each of the states at agent i depends on each of the elements in control input u_0^i or not. According to (6), the matrix B_0 which is identical for all heterogeneous agents in the network, can be constructed by this information. Besides, one can partition f_0^i into a linear-in-states part and another nonlinear function (Safaei & Mahyuddin, 2018a), i.e.

$$f_0^i(x^i, u_0^i(x^i)) = A^i x^i + f^i(x^i, u_0^i(x)), \tag{7}$$

where $A^i = (A^i)^{\mathrm{T}} \in \mathbb{R}^{n\times n}$ is a diagonal matrix including unknown time-varying elements on the main diameter corresponding to the dynamics of agent i. Consequently, we reach to

$$\dot{x}^i = A^i x^i + f^i(x^i, u_0^i(x^i)) - B_0 u_0^i(x^i) + B_0 u_0^i(x^i). \tag{8}$$

Finally, by considering

$$g^i(x^i) = f^i(x^i, u_0^i(x^i)) - B_0 u_0^i(x^i), \tag{9}$$

the unknown nonlinear dynamic system in (4) can be represented as

$$\dot{x}^i = A^i x^i + B_0 u_0^i + g^i, \tag{10}$$

where $g^i = g^i(x^i) \in \mathbb{R}^{n\times 1}$ is a vector of unknown bounded Lipschitz continuous nonlinear functions (refer to Safaei & Mahyuddin, 2018a for further discussions on the properties of g^i). It should be noted that the off-diagonal elements of A^i are considered to be zero (Horowitz & Tomizuka, 1986). In other words, the coupling terms are assumed to be included in the lumped nonlinear function f^i. Utilising the structure defined in (10), a technique can be proposed for online estimation of the controller gains.

Remark 2.1: A system is said to be *output controllable* at a given input–output data point in the current time, if the output of the system can be driven to a feasible specified setting point within a finite time by a sequence of control inputs (Hou & Jin, 2014).

Definition 2.4: We define a technique to make the matrix B_0 to be a full-rank square matrix. In this sense, we define a vector of virtual control parameters $\pi^i \in \mathbb{R}^{(n-m)\times 1}$ for the ith agent. The idea is based on the well-known Backstepping method (Khalil, 2002). Then, we define

$$u^i = [\pi^i \; u_0^i]^{\mathrm{T}}, \tag{11}$$

where $u^i \in \mathbb{R}^{n\times 1}$ is a new vector of control inputs including the virtual control parameters.

Definition 2.5: Utilising Definition 2.4, the dynamic system proposed in (4) and (10) can be represented as

$$\begin{aligned} \dot{x}^i &= A^i x^i + B u^i + g^i, \\ y^i &= x^i, \end{aligned} \tag{12}$$

where $B = [b_{jk}] \in \mathbb{R}^{n\times n}$ is a gain matrix defined at agent i as

$$b_{jk} = \begin{cases} 0 & \text{if } \dot{x}_j^i \text{ does not depend on } u_k^i, \\ 1 & \text{if } \dot{x}_j^i \text{ depends on } u_k^i. \end{cases} \tag{13}$$

It can be seen that the matrix B is full-rank. Moreover, all of the states are incorporated as the system outputs in (12), based on the Backstepping method and the use of virtual control parameters π^i.

Definition 2.6: We define a vectorising function $\mathcal{V}(.)$ for generating a vector $l \in \mathbb{R}^{n_0\times 1}$ constructed by diagonal elements of a matrix $M \in \mathbb{R}^{n_0\times n_0}$ as follows:

$$l = \mathcal{V}(M) = v_M, \tag{14}$$

where $l[j_0] = M[j_0, j_0]$ for $j_0 \in [1, n_0]$.

Definition 2.7: We define a function $\mathcal{M}(.)$ for generating a diagonal matrix with zero off-diagonal elements $M \in \mathbb{R}^{n_0\times n_0}$, by the elements of a vector $l \in \mathbb{R}^{n_0\times 1}$ as follows:

$$M = \mathcal{M}(l) = \mathcal{M}_l, \tag{15}$$

where $M[j_0, j_0] = l[j_0]$ for $j_0 \in [1, n_0]$.

3. Cooperative observer

There is a set of data (reference trajectory and desired formation) in the network that is only available at the virtual leader which is connected to a portion of the agents (not to all of the agents). The need for a cooperative observer arises to provide each agent in the network with the mentioned data, in a distributed manner. Here, first the cooperative observer algorithm is proposed for estimating the reference trajectory, which is the state of virtual leader x_0. This is later extended to estimate the values for formation variables η^is at each agent in the network.

Proposition 3.1: *The estimation error for observing x_0 at agent i can be represented as follows:*

$$\tilde{x}_0^i = \sum_{j=1}^{N} a_{ij}(\hat{x}_0^i - \hat{x}_0^j) + b_i(\hat{x}_0^i - x_0), \tag{16}$$

where $\hat{x}_0^i \in \mathbb{R}^{n\times 1}$ includes the estimated values of x_0 at ith agent. The objective of reaching consensus on x_0 among the *agents in the network is presented by*

$$\lim_{t\to\infty} \bar{\tilde{x}}_0 = \mathbf{0}, \tag{17}$$

where $\mathbf{0}$ *is a vector in* $\mathbb{R}^{Nn\times 1}$ *with zero elements and* $\bar{\tilde{x}}_0 = [\tilde{x}_0^1; \tilde{x}_0^2; \cdots; \tilde{x}_0^N] \in \mathbb{R}^{Nn\times 1}$ *is defined as follows:*

$$\bar{\tilde{x}}_0 = (\mathcal{H} \otimes I_n)\bar{\hat{x}}_0 - (\mathcal{B} \otimes x_0)\mathbf{1}. \tag{18}$$

Here, $\bar{\hat{x}}_0 = [\hat{x}_0^1; \hat{x}_0^2; \cdots; \hat{x}_0^N] \in \mathbb{R}^{Nn\times 1}$ *and* $\mathbf{1}$ *is a vector in* $\mathbb{R}^{N\times 1}$ *with all elements equal to one.*

Theorem 3.1: *If one uses the following equation as the rate for observing* x_0 *at agent i:*

$$\dot{\hat{x}}_0^i = -\lambda\hat{x}_0^i - [\mathcal{M}(sgn\{\sum_{j=1}^{N}(\mathcal{H}(i,j)\tilde{x}_0^j\})\ X_M], \tag{19}$$

where $\lambda > 0$ *is a scalar,* $\mathcal{M}(sgn\{\Sigma_{j=1}^N(\mathcal{H}(i,j)\tilde{x}_0^j)\}) \in \mathbb{R}^{n\times n}$ *is a diagonal matrix whose elements on the main diameter are the sign of elements in* $\Sigma_{j=1}^N(\mathcal{H}(i,j)\tilde{x}_0^j) \in \mathbb{R}^{n\times 1}$ *(referring to Definition 2.7) and* $X_M \in \mathbb{R}^{n\times 1}$ *includes the maximum absolute values for the elements of* $\dot{x}_0$*; then the consensus objective proposed in Proposition 3.1 will be achieved.*

Proof: Considering the following Lyapunov function:

$$V_1 = \frac{1}{2}\bar{\tilde{x}}_0^{\mathrm{T}}\bar{\tilde{x}}_0, \tag{20}$$

we have

$$\dot{V}_1 = \bar{\tilde{x}}_0{}^{\mathrm{T}}[(\mathcal{H} \otimes I_n)\dot{\bar{\hat{x}}}_0 - (\mathcal{B} \otimes \dot{x}_0)\mathbf{1}]. \tag{21}$$

Since the summation of all elements in each row of the Laplacian matrix is zero (Lewis et al., 2014; Li & Duan, 2015), we can say that

$$(\mathcal{L} \otimes \dot{x}_0)\mathbf{1} = \mathbf{0}. \tag{22}$$

Hence, (21) can be represented as

$$\dot{V}_1 = \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\dot{\bar{\hat{x}}}_0 - \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes \dot{x}_0)\mathbf{1}. \tag{23}$$

Considering $\dot{\bar{\hat{x}}}_0 = -\lambda\bar{\hat{x}}_0 + \hat{s}$ where $\hat{s}$ is defined as

$$\hat{s} = -\mathcal{M}(sgn\{\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\})(I_N \otimes X_M)\mathbf{1}, \tag{24}$$

we have

$$\begin{aligned}\dot{V}_1 = &-\lambda\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\bar{\hat{x}}_0 + \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} \\ &- \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes \dot{x}_0)\mathbf{1}.\end{aligned} \tag{25}$$

Recalling Definition 2.1 and Assumption 2.1, $(\mathcal{H} \otimes I_n)$ is a symmetric matrix with positive diagonal and non-positive off-diagonal elements. This means that, $(\mathcal{H} \otimes I_n)$ has positive determinant and positive eigenvalues. Hence, it is a nonsingular M-matrix (Lewis et al., 2014; Li & Duan, 2015). As a result, we can say that $(\mathcal{H} \otimes I_n) > 0$. Besides, let $\dot{V}_1 = \dot{V}_{11} + \dot{V}_{12}$, where

$$\dot{V}_{11} = -\lambda\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\bar{\tilde{x}}_0 < 0 \tag{26}$$

and

$$\dot{V}_{12} = \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} - \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes \dot{x}_0)\mathbf{1}. \tag{27}$$

To achieve $\dot{V}_1 < 0$, we should show that

$$\dot{V}_{12} = \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} - \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes \dot{x}_0)\mathbf{1} \le 0. \tag{28}$$

Recalling the mixed-product property of Kronecker product (Lewis et al., 2014; Li & Duan, 2015), we have

$$(\mathcal{H} \otimes \dot{x}_0) = (\mathcal{H} \otimes I_n)(I_N \otimes \dot{x}_0). \tag{29}$$

Hence, (28) can be written as follows:

$$\dot{V}_{12} = \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} - \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)(I_N \otimes \dot{x}_0)\mathbf{1}. \tag{30}$$

Then, we have

$$\dot{V}_{12} \le \bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} + ABS(\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n))(I_N \otimes X_M)\mathbf{1}, \tag{31}$$

where for $\upsilon \in \mathbb{R}^{1\times Nn}$, we define

$$ABS(\upsilon) = [|\upsilon(1)|, |\upsilon(2)|, \ldots, |\upsilon(Nn)|]. \tag{32}$$

Now for satisfying (28), we should only show that

$$\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} + ABS(\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n))(I_N \otimes X_M)\mathbf{1} = 0. \tag{33}$$

By replacing $\hat{s}$ from (24), the first term on the left-hand side of (33) will be as follows:

$$\begin{aligned}&\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\hat{s} \\ &= -\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\mathcal{M}(sgn\{\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\})(I_N \otimes X_M)\mathbf{1},\end{aligned} \tag{34}$$

where by recalling the definition in (32), we have

$$\begin{aligned}&\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\mathcal{M}(sgn\{\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\}) \\ &= ABS(\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)).\end{aligned} \tag{35}$$

Thus, by replacing (34) and (35) in (33), the equation in (33) is satisfied and then the rates for observed parameters are defined as

$$\dot{\bar{\hat{x}}}_0 = -\lambda\bar{\hat{x}}_0 - \mathcal{M}(sgn\{\bar{\tilde{x}}_0{}^{\mathrm{T}}(\mathcal{H} \otimes I_n)\})(I_N \otimes X_M)\mathbf{1}. \tag{36}$$

By using $\dot{\bar{\hat{x}}}_0$ from (36), we can have $\dot{V}_1 < 0$, which in turn shows that the consensus error on observation of x_0 (i.e. $\bar{\tilde{x}}_0$) is asymptotically stable and converges to zero, recalling the Lyapunov stability theorem. Hence, the

Proposition 1 is achieved. Moreover, the observer for agent i can be represented as proposed in (19).

This completes the proof. ■

Remark 3.1: According to (19), only the values on the ith row of $\mathcal{H}$ are required to compute the rates for the cooperative observer parameter. These values are available locally at each agent based on the existing communication links between the agent and the neighbouring ones. In other words, only the information from the neighbouring agents are required at the ith agent (no need for global information of the whole network). Refer to Li and Duan (2015), Lewis et al. (2014), Mahyuddin and Tiang (2017), Mahyuddin and Herrmann (2013b), Mahyuddin et al. (2012), Mahyuddin (2013) and Mahyuddinand Herrmann (2013a) for further details.

Remark 3.2: The values for X_M can be determined according to the actuators specifications. For example, if x_0 is the reference positions of a mobile robot, then X_M is the maximum absolute values for the mobile robot speed, which can be defined according to the actuators specifications and some data from previous experiments.

Remark 3.3: The value of λ should be large enough to increase the convergence rate of the proposed distributed observer in (19).

Lemma 3.1: *Referring to Definition 2.2, let us define*

$$\Omega = \begin{bmatrix} (\eta^0)^{\mathrm{T}} \\ (\eta^1)^{\mathrm{T}} \\ \cdots \\ (\eta^N)^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{(N+1)\times n}. \tag{37}$$

Since the formation variables η^is are only available at the virtual leader and there is at least one communication link between one of the agents and the virtual leader, one can propose an observer similar to (19) as follows:

$$\dot{\hat{\Omega}}^i = -\mu\varepsilon^i - \left[\left(sgn\left\{\sum_{j=1}^{N}(\mathcal{H}(i,j)\varepsilon^j\right\}\right)\mathcal{M}(\Upsilon^M\Bigg)\right], \tag{38}$$

to observe the formation variables Ω at agent i. In (38), $\mu > 0$ is a constant scalar, $\hat{\Omega}^i \in \mathbb{R}^{(N+1)\times n}$ is the observed formation variables at the ith agent, $|\dot{\eta}^0| \le \Upsilon^M \in \mathbb{R}^{n\times 1}$ and

$$\varepsilon^i = \sum_{j=1}^{N} a_{ij}(\hat{\Omega}^i - \hat{\Omega}^j) + b_i(\hat{\Omega}^i - \Omega). \tag{39}$$

The proof procedure for this lemma is similar to the procedure presented in the proof of Theorem 3.1.

4. Adaptive model-free control

Lemma 4.1: *Combination of a stable observer and a stable controller within a dynamic system will lead to a stable system as stated in Separation Principle (Atassi & Khalil, 1999).*

Proposition 4.1: *Utilising Theorem 3.1 and Lemma 3.1, the reference trajectory and the desired formation topology among the agents are available at each agent in the network. One can represent the formation-tracking problem in form of a simple tracking problem to be solved locally at each agent. Recalling Lemma 4.1, the desired path to be followed by each agent is defined as follows:*

$$y_d^i = \hat{x}_0^i + \hat{\Omega}^i(1,:)^{\mathrm{T}} + \hat{\Omega}^i(i,:)^{\mathrm{T}}, \tag{40}$$

where $\hat{x}_0^i$ and $\hat{\Omega}^i$ are determined using the cooperative observers defined in (19) *and* (38). *In this regard, the following tracking error is defined:*

$$e^i = y_d^i - y^i. \tag{41}$$

Also, the time-integral of this error is considered to eliminate the steady-state error and thus the following joint error variable is proposed

$$\sigma^i = e^i + \zeta^i, \tag{42}$$

where $\zeta^i = \int e^i \mathrm{d}t$. The tracking objective at agent i is achieved when σ^i reaches to zero as time goes to infinity. As a secondary result of achieving the tracking objective, the formation topology among the agents is satisfied according to (40). *In other words, the formation topology in the network will be satisfied if there is a local controller at agent i that can track y_d^i proposed in* (40).

Assumption 4.1: It is assumed that agent i can measure its states x^i with on-board sensor modules, locally without any need for communication with other agents.

Theorem 4.1: *For the dynamic system proposed in* (12) *and by considering Assumption 4.1, if we define the controller $u^i = u_1^i + u_2^i$ as*

$$\begin{aligned} u_1^i &= \frac{1}{2}RB^{\mathrm{T}}P^i\sigma^i \\ u_2^i &= B^{-1}[\dot{y}_d^i - \hat{A}^i x^i - \hat{g}^i - \zeta^i \\ &\quad + (I_n + 2(P^i)^{-1}Q + \hat{A}^i)\sigma^i] - \frac{3}{4}RB^{\mathrm{T}}P^i\sigma^i, \end{aligned} \tag{43}$$

where I_n is the identity matrix with dimensions of n, $R = R^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $Q = Q^{\mathrm{T}} \in \mathbb{R}^{n\times n}$ are positive definite matrices, and $P^i = (P^i)^{\mathrm{T}} > 0 \in \mathbb{R}^{n\times n}$ is defined using

$$\dot{P}^i = (\hat{A}^i)^{\mathrm{T}}P^i + P^i\hat{A}^i - P^iBRB^{\mathrm{T}}P^i + 2Q, \tag{44}$$

with incorporating the following adaptive laws:

$$\begin{aligned} \dot{\hat{g}}^i &= -\Gamma_1 P^i \sigma^i - \rho_1 \Gamma_1 \hat{g}^i, \\ \upsilon_{\hat{A}^i} &= -\Gamma_2 P^i \mathcal{M}_{\sigma^i}(x^i - \sigma^i) - \rho_2 \Gamma_2 \upsilon_{\hat{A}^i}, \end{aligned} \tag{45}$$

where $\Gamma_1 \in \mathbb{R}^{n\times n}$ and $\Gamma_2 \in \mathbb{R}^{n\times n}$ are two positive definite matrices as adaptive gains, ρ_1 and ρ_2 are two positive scalar leakage gains and $\upsilon_{\hat{A}^i}$ and $\mathcal{M}_{\sigma^i}$ are defined based on Definitions 2.6 and 2.7; then the tracking objective in Proposition 4.1 will be achieved.

Proof: Detailed proof can be found in Safaei and Mahyuddin (2018a) for an individual dynamic system. ■

Remark 4.1: The values for $\dot{y}_d^i$ can be computed using the sliding-mode differentiators proposed in Levant (2003). Refer to Safaei and Mahyuddin (2018a, 2018b) for further details.

Remark 4.2: Reminding that the off-diagonal elements of P^i are zero, the diagonal values in P^i as the main gains for the proposed AMFC in Theorem 4.1, are updated online utilising (44). Hence, minimal effort is needed for off-line tuning of other constants in the proposed controller. Moreover, it can be seen that the updating law for P^i is independent of the communication graph properties. In addition, since the values of elements in P^i are computed using the DRE proposed in (44), it is confirmed that $P > 0$, which is a requirement for the stability analysis of Theorem 4.1. For further discussions, refer to Safaei and Mahyuddin (2018a), Lewis, Xie, and Popa (2008) and Deshpande (2011). Furthermore, the simulation results for application of the proposed AMFC in Theorem 4.1 with online updating of P can be found in Safaei, Koo, and Mahyuddin (2017) for a robotic manipulator, in Safaei and Mahyuddin (2017) for a chaotic resonator, in Safaei and Mahyuddin (2018a) for ground and aerial autonomous mobile robots and in Safaei and Mahyuddin (2018c) for an autonomous underwater vehicle.

5. Simulation study

Recently, several investigations are dedicated to the formation-tracking problem in a network of quadrotors (Kang et al., 2017; Lee, 2018; Mahmood, 2017). In this section, the performance of the proposed cooperative observer as well as the AMFC policy is studied on a 3D motion of a network including four quadrotors. Since the quadrotor is an under-actuated system, the reference trajectory for translational motion is specified as the main goal in tracking problem and then the reference trajectory for rotational motion is defined according to the transformation of the body frame to the inertial frame (Safaei & Mahyuddin, 2016, 2017a).

5.1. Quadrotor model

The model considered for simulation of quadrotors (Figure 1) is as follows (Safaei & Mahyuddin, 2016):

$$\begin{aligned} \dot{\vec{p}}_q &= \vec{v}_q, \\ \dot{\vec{\Phi}}_q &= R_{qt}^{-1} \vec{w}_q, \\ \dot{\vec{v}}_q &= \frac{1}{M_q}[R_q \vec{F}_q - K_d \vec{v}_q - M_q \vec{F}_g] + \vec{f}_q, \\ \dot{\vec{w}}_q &= J_q^{-1}[\vec{\tau}_q - K_a \vec{w}_q - \vec{w}_q \times J_q \vec{w}_q] + \vec{t}_q, \end{aligned} \tag{46}$$

where $\vec{p}_q = [x_q; y_q; z_q]$ and $\vec{\Phi}_q = [\phi_q; \theta_q; \psi_q]$ are the absolute positions and Euler angles (roll, pitch and yaw) of a quadrotor, respectively. Also, $\vec{v}_q$ and $\vec{w}_q$ are the vectors of linear and angular velocities. M_q is the quadrotor mass and $J_q \in \mathbb{R}^{3\times 3}$ is its inertia matrix. K_d and K_a

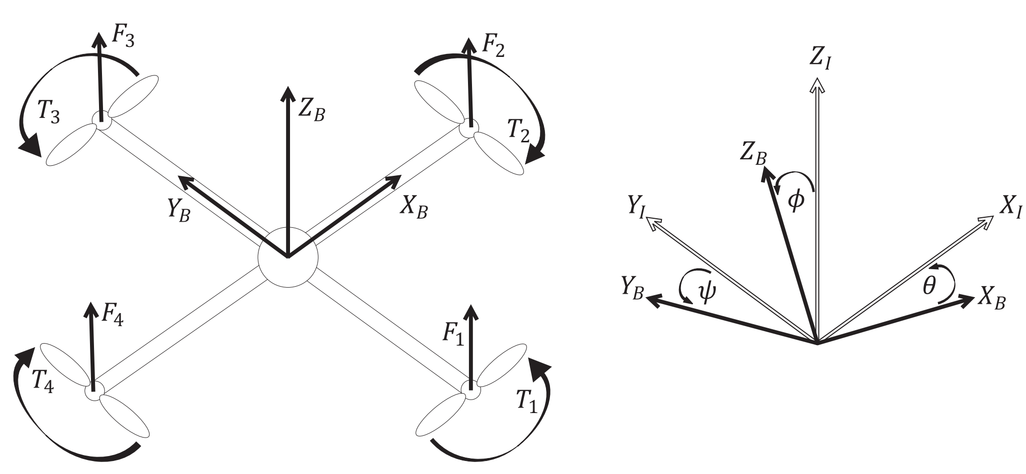

Figure 1. Schematic of a quadrotor.

are two constants coefficients for the friction forces and torques. $\vec{F}_g = [0; 0; g_{earth}]$ is the vector of gravity force and $\vec{f}_q$ and $\vec{t}_q$ are the vectors of unknown disturbances. The generated force and torques by the electric motors are represented by $\vec{F}_q = [0; 0; F_T]$ and $\vec{\tau}_q = [\tau_x; \tau_y; \tau_x]$, where

$$\begin{pmatrix} F_T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} k_l & k_l & k_l & k_l \\ -k_l L_q & 0 & k_l L_q & 0 \\ 0 & -k_l L_q & 0 & k_l L_q \\ k_t & -k_t & k_t & -k_t \end{pmatrix} \begin{pmatrix} w_{1q}^2 \\ w_{2q}^2 \\ w_{3q}^2 \\ w_{4q}^2 \end{pmatrix}. \tag{47}$$

Here, k_l and k_t are the constants of electric motors for generating lift forces and torques, respectively. L_q is the length for each arm of the quadrotor and w_{iq} for $i = \{1, 2, 3, 4\}$ is the angular speed for the ith electric motor. In the quadrotor model, there are two matrices R_q and R_{qt} defined in $\mathbb{R}^{3\times3}$ to transform the coordinates between the body frame and the inertial frame. Refer to Safaei and Mahyuddin (2016, 2018a) for detailed definition of these matrices. The values of parameters for case of simulation study are $M_q = 2$, $J_q = 1.24e - 3 \times \mathrm{diag}(1, 1, 2)$, $L_q = 0.2$, $K_d = K_a = 0.01$, $k_l = 1e - 5$, $k_t = 1e - 7$, $g = 9.81$, $\vec{f}_q = 1\sin(t)\vec{v}_3$ and $\vec{t}_q = 1\sin(t)\vec{v}_3$, where $\vec{v}_3 = [1; 1; 1]$.

5.1.1. Implementation of AMFC on each quadrotor

The proposed AMFC in (43) can be implemented directly on the dynamic model of a quadrotor in (46), without any prior knowledge about the values of parameters in the model. The system has 12 system states as $[\vec{p}_q; \vec{\Phi}_q; \vec{v}_q; \vec{w}_q]$ and 12 control inputs as $[u_{iq}]$ for $i \in [1, 12]$, including 6 virtual control variables (i.e. u_{jq} for $j \in [1, 6]$). When the values for control inputs are determined using the AMFC, the values for angular speeds of electric motors are determined using (47), where (Safaei & Mahyuddin, 2016)

$$F_T = \sqrt{u_{7q}^2 + u_{8q}^2 + u_{9q}^2}, \qquad \tau_x = u_{10q}\ \tau_y = u_{11q}\ \tau_z = u_{12q}. \tag{48}$$

Here, we assume no yaw angle involved; i.e. $\psi_{dq} = 0$. The desired values for roll and pitch angles are also defined as (Safaei & Mahyuddin, 2016)

$$\phi_{dq} = \sin^{-1}\left(\frac{\sin\psi_{dq}u_{7q} - \cos\psi_{dq}u_{8q}}{F_T}\right), \qquad \theta_{dq} = \tan^{-1}\left(\frac{\cos\psi_{dq}u_{7q} + \sin\psi_{dq}u_{8q}}{u_{9q}}\right). \tag{49}$$

Table 1. Tuning parameters of the cooperative observer.

Parameter	Value
λ	100
μ	10
χ^M	$10 \times [1; 1; 1]$
Υ	$1 \times [1; 1; 1]$

5.2. Communication graph

In this simulation study, only one of the quadrotors is connected to the virtual leader. Hence, the adjacency matrix, the pinning gain matrix and the in-degree matrix for this communication graph are defined as follows:

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathcal{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Table 2. Tuning parameters of the AMFC policy.

Parameter	Value
$R = B$	I_{12}
Q	$(1e-3) \times I_{12}$
ρ_1	$1e{-}1$
ρ_2	$1e{+}1$
Γ_1	$[I_6; (1e+3) \times I_6]$
Γ_2	$[(1e-2) \times I_6; (1e+1) \times I_6]$

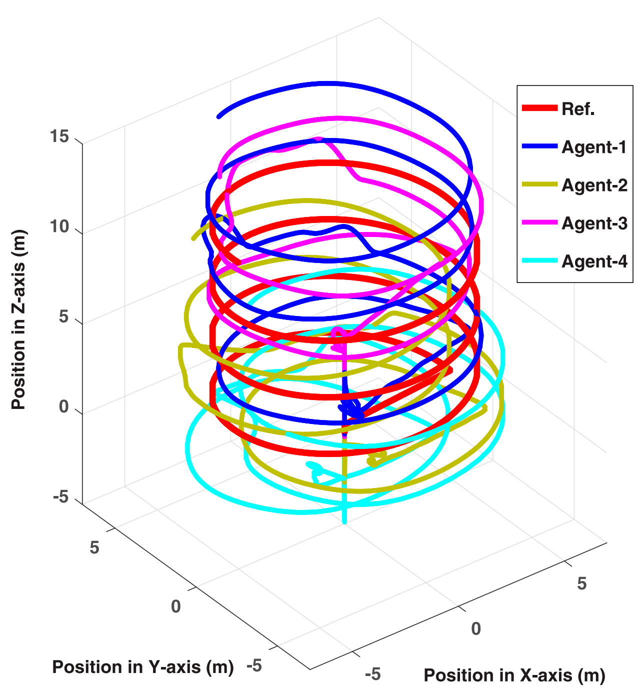

Figure 2. Positions in 3D space.

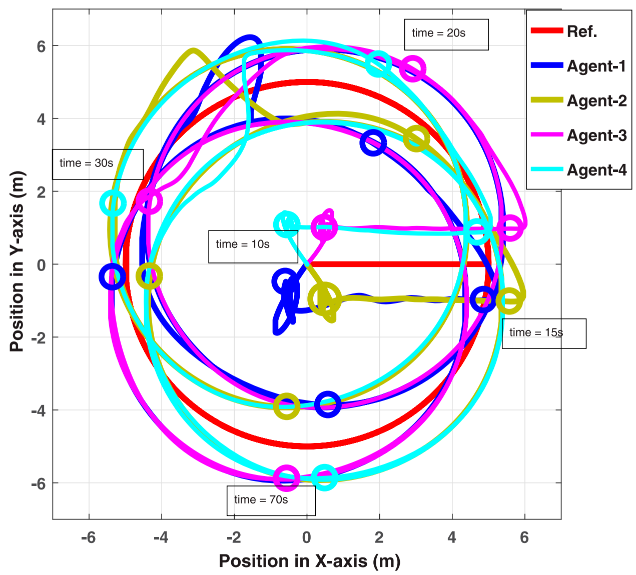

Figure 3. Positions in 2D space.

$$\mathcal{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{50}$$

5.3. Reference trajectory and desired formation topology

Here, the reference trajectory is a helix in the 3D space. Also, the parameter Ω for formation topology is considered as follows:

$$\Omega = \begin{bmatrix} 0 & 0 & 0 \\ +r_x & +r_y & +r_z \\ -r_x & +r_y & -r_z \\ -r_x & -r_y & +2 \times r_z \\ +r_x & -r_y & -2 \times r_z \end{bmatrix}. \tag{51}$$

The values of formation parameters are $r_x = -0.5$, $r_y = -1$ and $r_z = 2$ for simulation time under 50 s. These values are changed to $r_x = 0.5$, $r_y = 1$ and $r_z = 3$ for the rest of simulation time till 120 s.

5.4. Results

The values of tuning parameters for the proposed cooperative observer and the AMFC policy used on each of the quadrotors are provided in Tables 1 and 2, respectively. The simulation results are presented in Figures 2– 11. As can be seen, the finite-time convergence is provided for estimating the reference trajectory and the formation

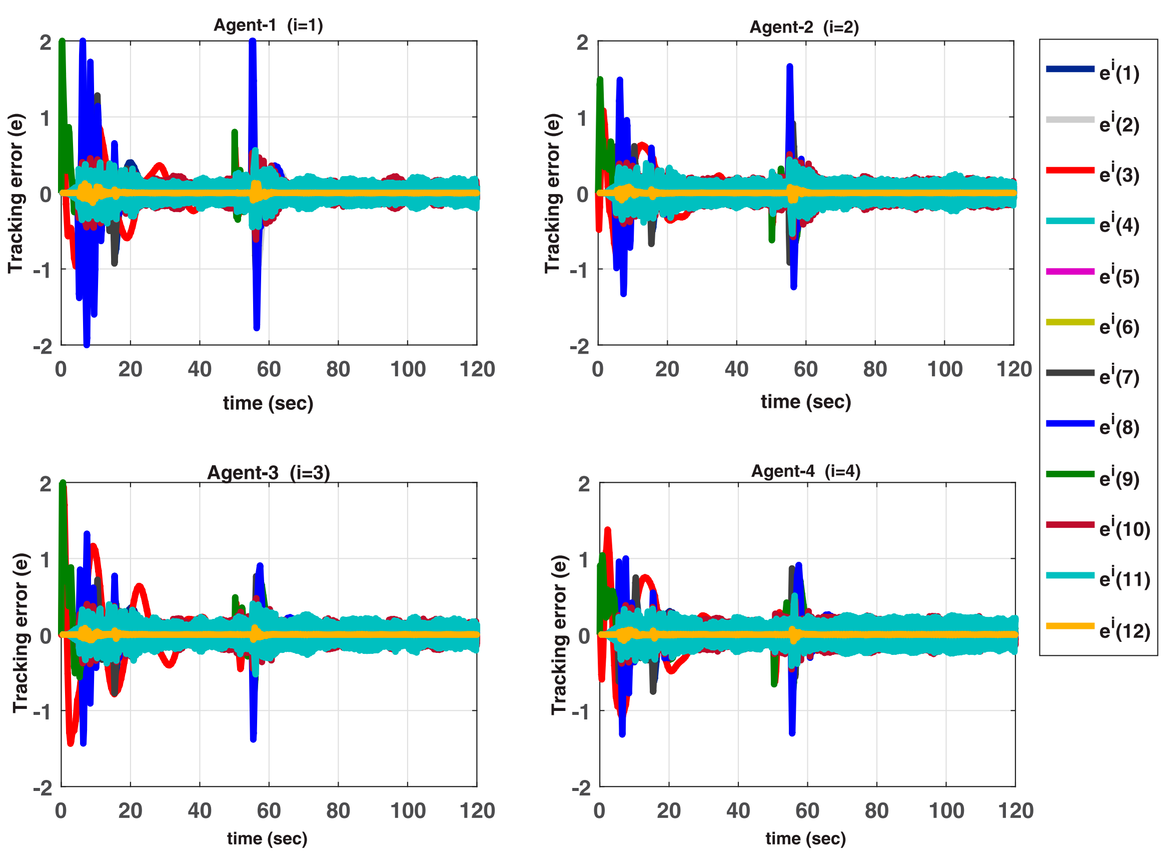

Figure 4. Tracking errors.

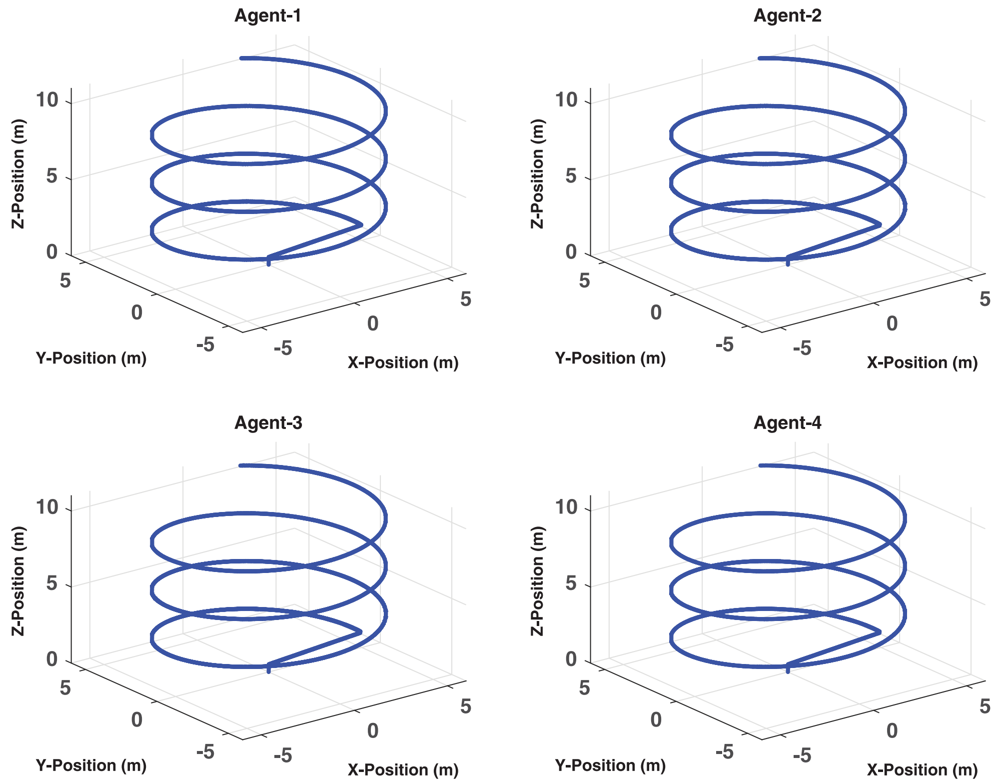

Figure 5. Estimated values for reference trajectory.

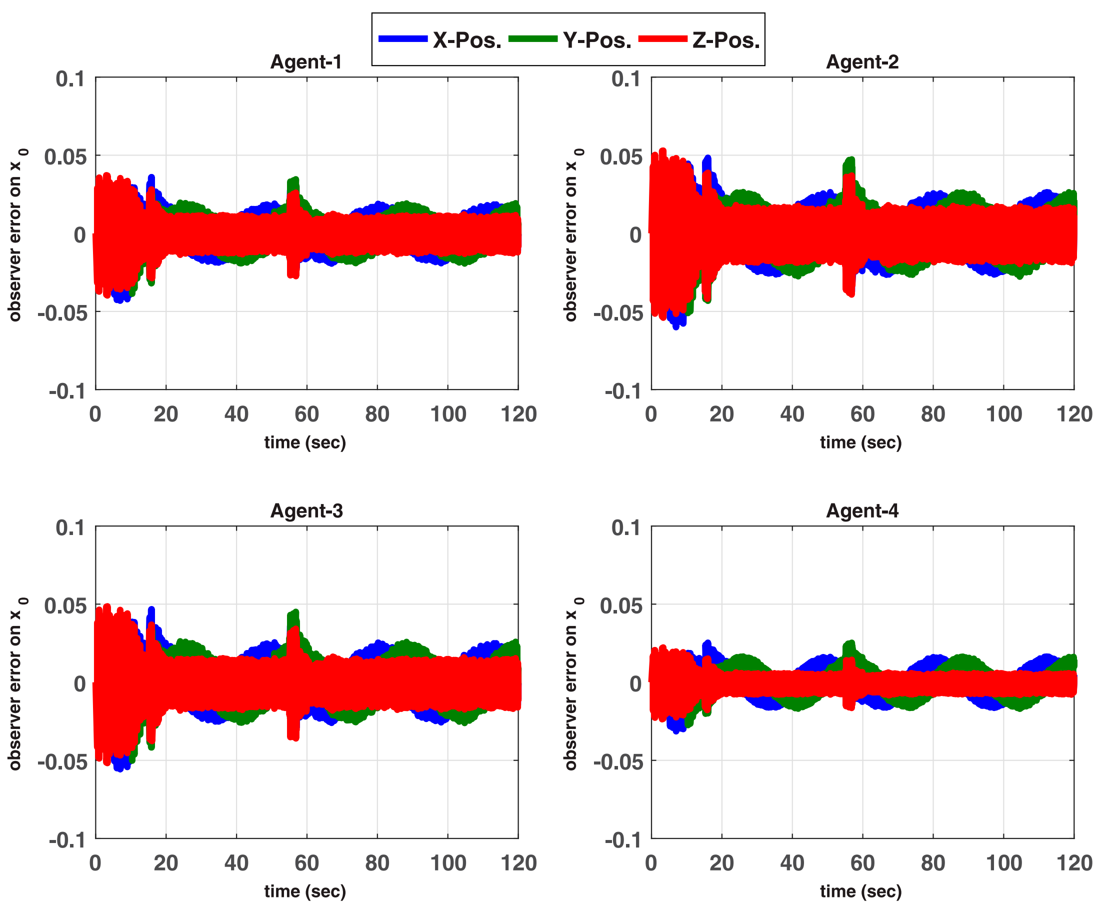

Figure 6. Consensus errors for observing the reference trajectory.

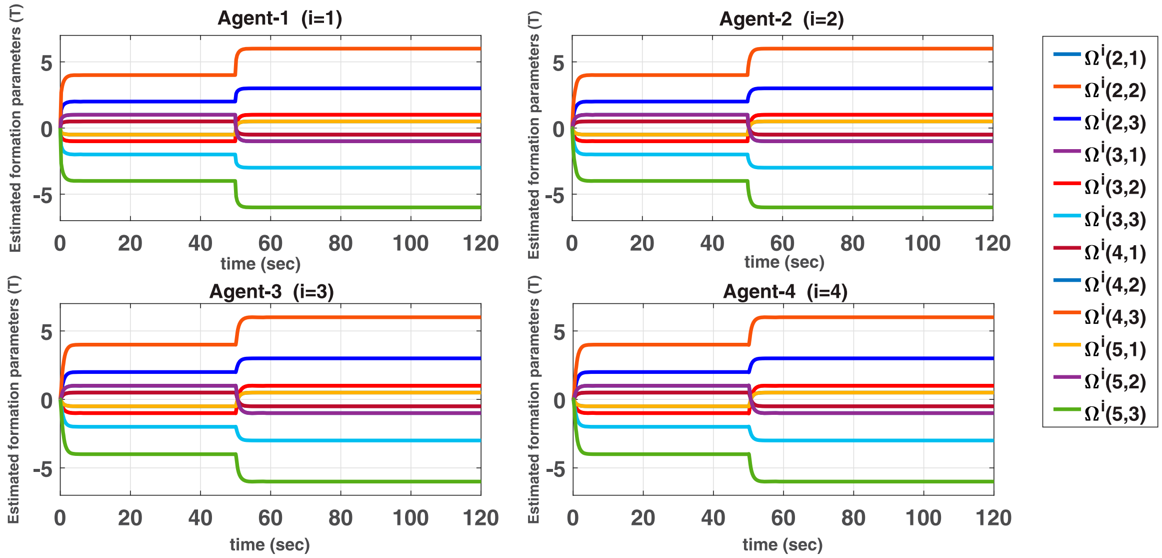

Figure 7. Estimated values for formation parameters.

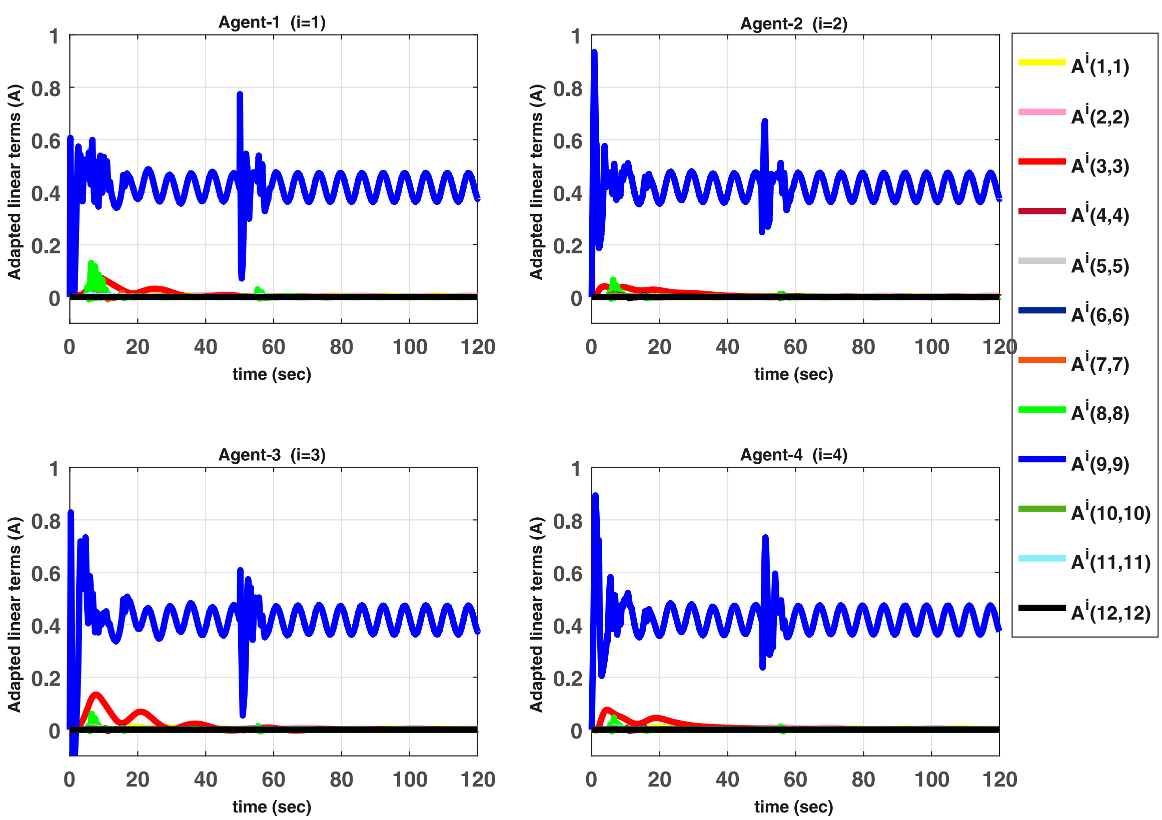

Figure 8. Adapted values for linear terms.

topology parameters by use of the proposed cooperative observer. Moreover, the tracking error for all the agents is bounded in a small region around zero. Refer to Figures 3 and 7, the locations of agents 1 and 3 and similarly the locations of agents 2 and 4 are exchanged after 50 s of simulation time. The desired formation is satisfied before and after this change. Note that the values for parameters $\hat{A}(9,9)$, $\hat{g}(9)$, $\hat{P}(9,9)$ and the other corresponding parameters in the figures are associated with the gravity acceleration g_{earth}.

6. Conclusion

This paper contains the design procedure of a decentralised cooperative observer to estimate a reference trajectory and also a set of formation parameters

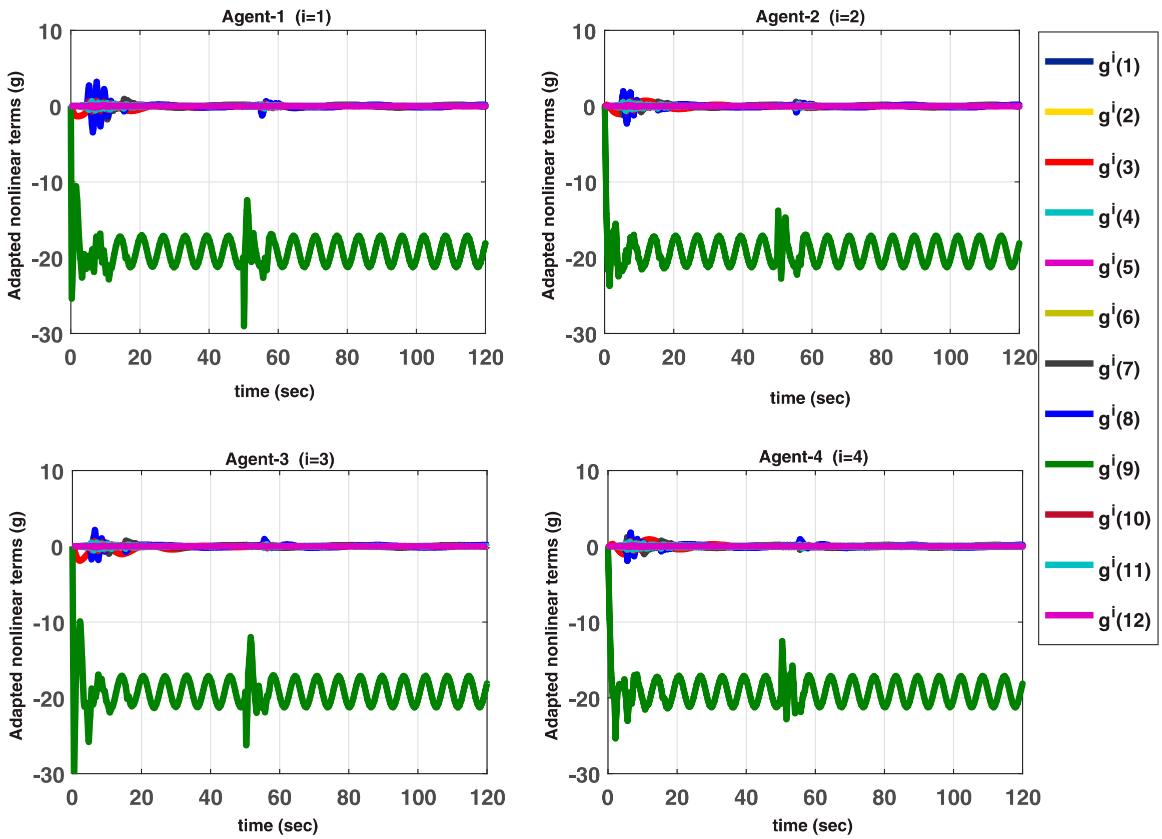

Figure 9. Adapted values for nonlinear terms.

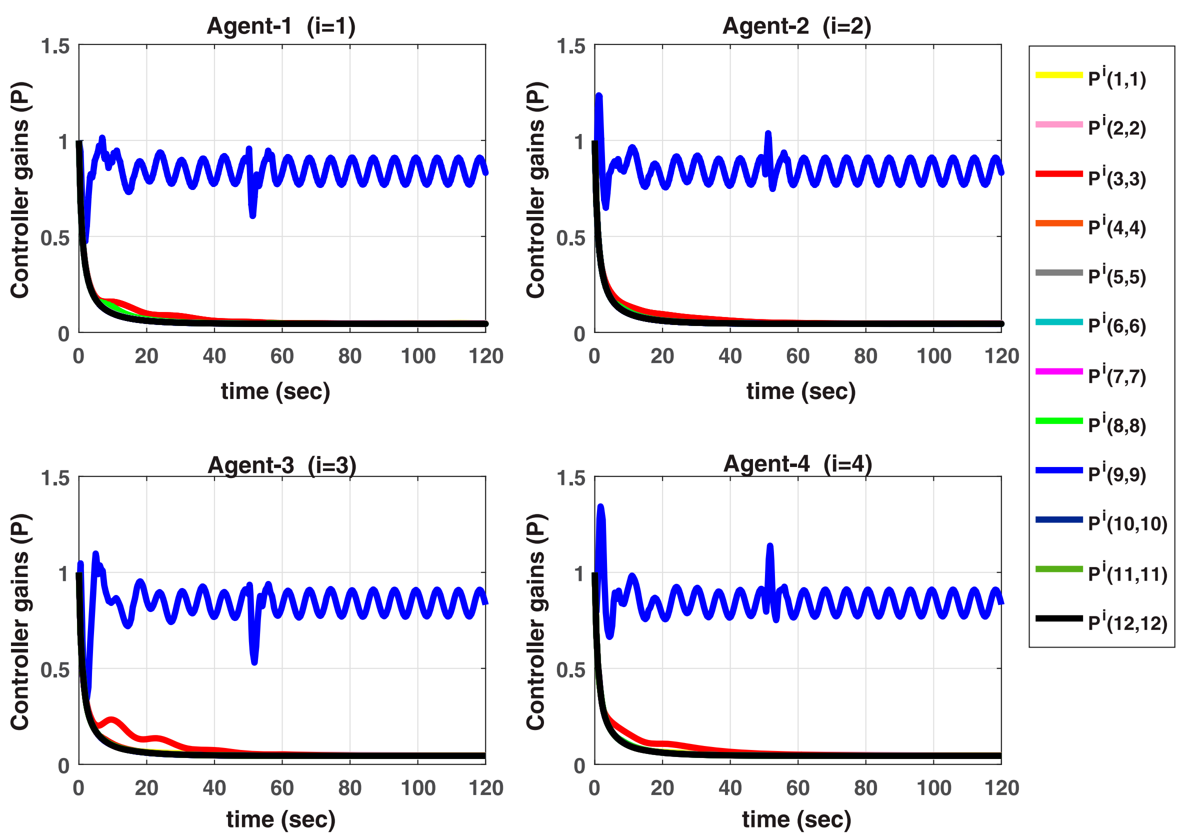

Figure 10. Controller gains.

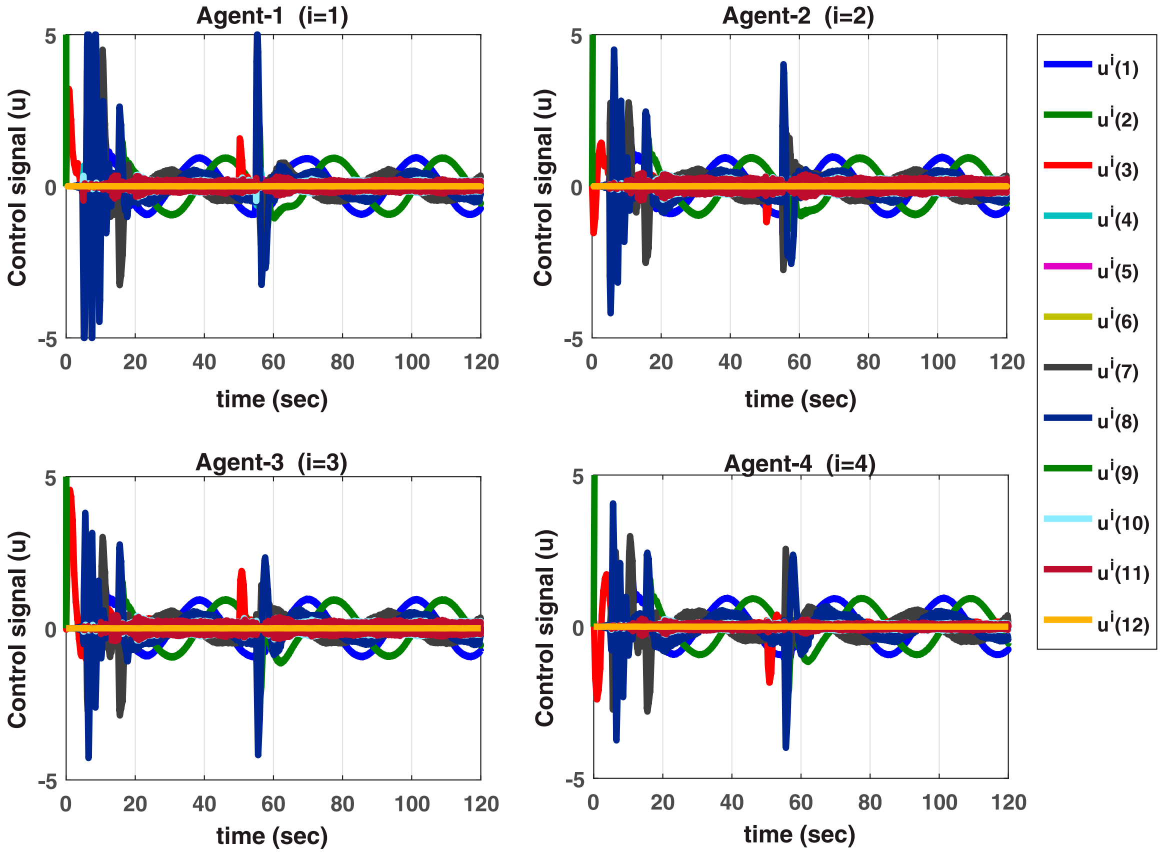

Figure 11. Control signals.

in a network of nonlinear dynamic agents. The proposed cooperative observer includes a term for providing the finite-time convergence of the observer error. This is an essential property to be featured, especially when the reference trajectory and the desired formation parameters are time-varying. Moreover, the cooperative observer is completely decoupled from the agents' dynamics in the network, which leads to more stable performance of the observer algorithm regardless of the controller performance. Based on the estimated parameters by the cooperative observer, the desired path to be followed by each agent in the network is defined locally at the agent, so as to satisfy the objective of formation-tracking problem. Thereby, the formation-tracking problem in a network of multi-agent systems is converted to a tracking problem at each agent. Then, a decentralised adaptive model-free control policy is applied for satisfying the tracking objective at each agent in the network. The proposed controller includes two adaptive laws for online estimation of the unknown terms in the dynamics of each agent. Since the adaptive laws are model-free and do not depend on artificial neural networks, the need for the PE condition is removed. This leads to more convenient implementation of the proposed algorithm with fewer computation complexity. In addition, the main controller gains are updated online based on the adapted values of the unknown linear terms in the agents' dynamics. Minimal tuning procedure of the controller allows more convenient practical implementations. By combination of the proposed cooperative observer and the decentralised adaptive model-free control policy, a solution is provided for the formation-tracking problem without the need for information on the relative positions among the agents in the network. This essentially brings a significant cost saving in terms of hardware development point of view as a number of sensors can be reduced. Moreover, since the relative localisation problem remains open, the solution can be considered for implementations in real applications with fewer practical concerns. Furthermore, our work has proven that the cooperative formation control is formulated and defined locally at each agent, thereby *a priori* global network-wise information is not needed by all agents. Simply-connected communication graph is only required for our proposed solution. The exemplified simulation yields a promising result, showing the preferable formation-tracking performance in a network of four quadrotors. In this simulation study, the dynamics of quadrotors are considered to be completely unknown while unknown disturbances are included, as well.

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