Image Denoising Based on Singular Value Decomposition

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Image denoising based on singular value decomposition is one of my favorites. Actually, I believe that singular value decomposition (SVD) is one of the most powerful tools in signal processing society. In this paper, I want to indicate how we can use SVD for the image denoising purpose. Suppose we have a noisy image $A_{m\times n}$ with the rank p and we apply SVD to it:

$$A = USV^T \tag{1}$$

where $U_{m\times m}$ and $V_{n\times n}$ are orthogonal matrices and $S_{m\times n}$ is a diagonal matrix whose diameter values are the singular values of A.

$$\begin{gathered} S_{m\times n} = diag(\sigma_1, ..., \sigma_\mu) \\ \sigma_1 \geq \sigma_2 \geq ... \geq \sigma_p > 0 \\ \mu = min\{m, n\} \\ \sigma_{p+1} = \sigma_{p+2} = ... = \sigma_\mu = 0 \end{gathered} \tag{2}$$

where σ_1 and σ_p are the largest and smallest nonzero singular values of the matrix A, respectively. It can be proved that if we keep only the first k singular values ($k < \mu$), then we can improve the quality of the noisy image respect to mean square error (MSE) cost function. Actually, finding the optimal solution for k to minimize the cost function is very interesting, however, I do not want to mention the procedure in this paper. Now I show you the results of image denoising based on SVD. I used different values of k and for each k, I calculated the MSE and then plot the curve. The image that I used is

Figure1: Test image

and here are the results for different values of standard deviation of the noise (the noise is additive gaussian)

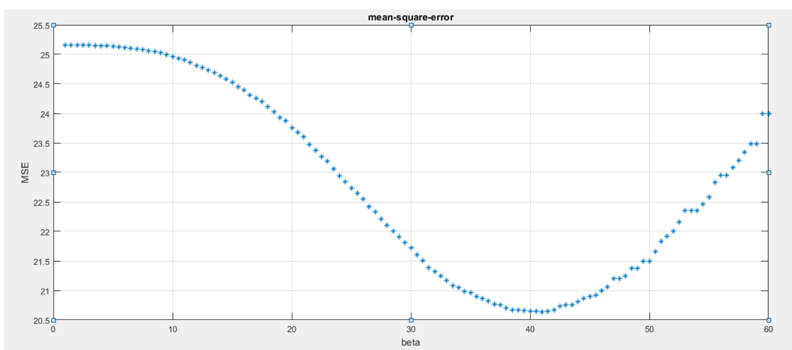

Figure1: Standard deviation = 5.

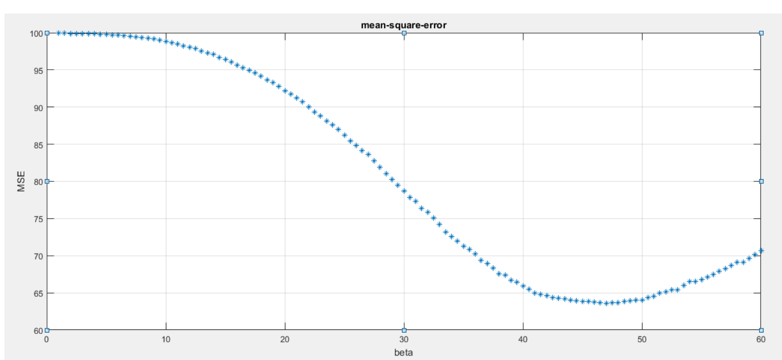

Figure2: Standard deviation = 10.

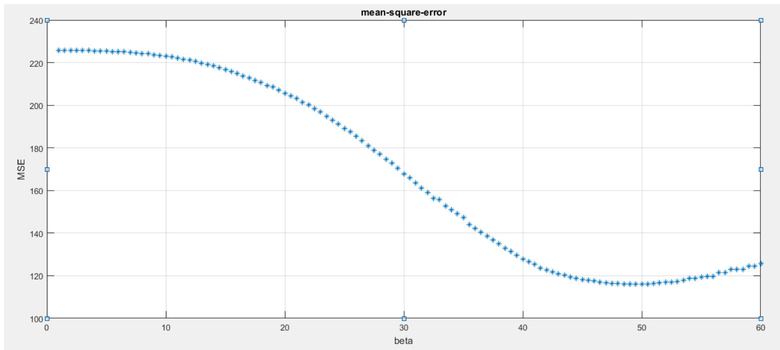

Figure3: Standard deviation = 15.

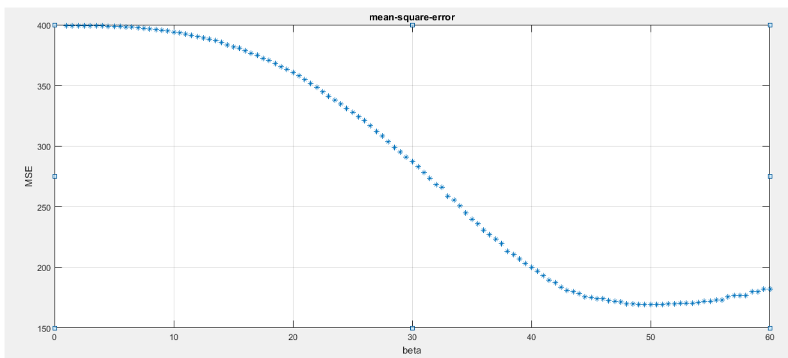

Figure4: Standard deviation = 20.

Now I should explain that what the beta is. In order to find the value k in each step, I used a special thresholding function like below. However, in this paper, I do not want to prove why this thresholding function is true since it is behind the scope of the paper.

$$ThresholdingFunction = \beta\sigma_n \tag{3}$$

where σ_n is the standard deviation of the noise. k is the index of the first singular value that is greater or equal to the thresholding function.

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