University of Burgundy

Master of Science in Computer Vision – 2nd Year

Advanced Image Analysis Module

Report on Wavelet Transform and Application to Image Denoising

by

Gopikrishna Erabati

Under the supervision of

Dr. Philippe Carre

CONTENTS

1. Introduction

Mathematical transformations are applied to signals to obtain further information from that signal that is not readily available in the raw signal. There are a number of transformations that can be applied, among which the Fourier transforms are probably by far the most popular.

1.1 Why Wavelet Transform ?

The Fourier transform gives the frequency components (spectral) exists in the signal. But, when the **time-localization** of spectral components is needed, a transform giving the TIME-FREQUENCY representation of signal is needed. This is in short, if we take the Fourier transform over the whole time axis, we cannot tell at what instant a particular frequency rises. The wavelet transform is a transform which gives this sort of information. There are other transforms which give this information too, like Short-time Fourier transform (STFT) uses a sliding window to find spectrogram, which gives the information of both time and frequency. But still another problem exists: The length of window limits the resolution in frequency. Wavelet transform seems to be a solution to the problem above.

2. Discrete Wavelet Transform

The discrete wavelet transform (DWT), provides sufficient information both for analysis and synthesis of the original signal, with a significant reduction in the computation time. The DWT is considerably easier to implement when compared to the Continuous Wavelet Transform (CWT).

The main idea is the same as it is in the Continuous Wavelet Transform. A time-scale representation of a digital signal is obtained using digital filtering techniques. The CWT is a **correlation between a wavelet at different scales and the signal with the scale (or the frequency) being used as a measure of similarity**. The continuous wavelet transform is computed by changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating over all times. In the discrete case, filters of different cutoff frequencies are used to analyze the signal at different scales. The signal is passed through a series of high pass filters to analyze the high frequencies, and it is passed through a series of low pass filters to analyze the low frequencies.

The **resolution of the signal**, which is a measure of the amount of detail information in the signal, **is changed by the filtering operations**, and the **scale is changed by up sampling and down sampling operations**. Down sampling a signal corresponds to reducing the sampling rate, or removing some of the samples of the signal. Up sampling a signal corresponds to increasing the sampling rate of a signal by adding new samples to the signal.

2.1 Wavelet Analysis (Decomposition)

The procedure starts with passing this signal through a **half band digital low pass filter** with impulse response h[n]. A half band low pass filter removes all frequencies that are above half of the highest frequency in the signal. For example, if a signal has a maximum of 500 Hz component, then half band low pass filtering removes all the frequencies above 250 Hz.

After passing the signal through a half band low pass filter, half of the samples can be eliminated according to the Nyquist's rule, since the signal now has a highest frequency of p/2 radians instead of p radians (if p is original highest frequency of signal). Simply discarding every other sample will **down sample** the signal by two, and the signal will then have half the number of points. **The scale of the signal is now doubled**. Here, the low pass filtering removes the high frequency information, but leaves the scale unchanged. Only the down sampling process changes the scale.

The DWT analyzes the signal at different frequency bands with different resolutions by decomposing the signal into a **coarse approximation** and **detail information**. DWT employs two sets of functions, called **scaling functions** and **wavelet functions**, which are associated with **low pass** and **high pass** filters, respectively. The decomposition of the signal into different frequency bands is simply obtained by successive high pass and low pass filtering of the time domain signal. The original signal x[n] is first passed through a half band high pass filter g[n] and a low pass filter h[n]. After the filtering, half of the samples can be eliminated. This constitutes one level of decomposition. This decomposition can be continued to multi level using the coarse approximation as input to next level.

The decomposition process can be seen in a block diagram as in fig.1.

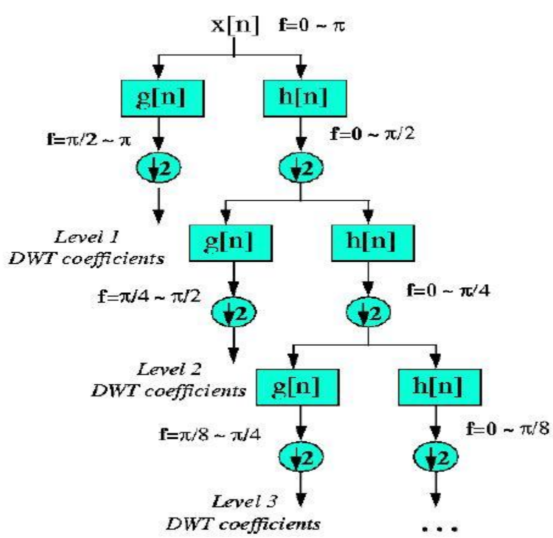


Fig. 1 Wavelet Decomposition of 1D signal

2.2 Wavelet Synthesis (Reconstruction)

The reconstruction of original signal from the wavelet coefficients is the reverse process of the decomposition. As we get the wavelet coefficients, we need to up sample the coefficients which halves the scale of the signal. After the signal is up sampled we apply low pass and high pass filters to the up sampled signal to get the perfect reconstructed signal.

2.3 2D DWT

For images, an algorithm similar to the one-dimensional case is possible for two-dimensional wavelets.

This kind of two-dimensional DWT leads to a decomposition of approximation coefficients at level j in four components: the approximation at level j+1, and the details in three orientations (horizontal, vertical, and diagonal).

The schematic diagram of 2D wavelet analysis (decomposition) is as shown in fig.2.

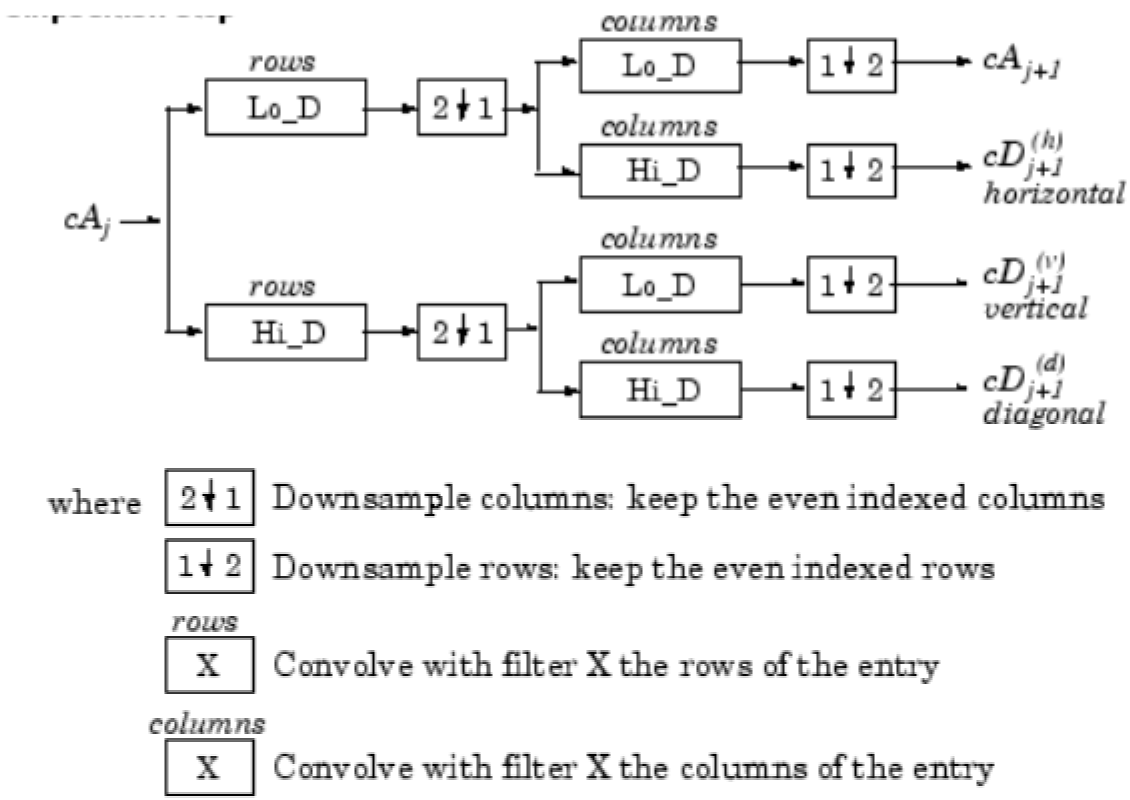


Fig. 2 Schematic diagram of 2D DWT decomposition

We apply low pass filter on rows of an image and then down sample the columns by half and similarly we apply high pass filter and down sample the columns by half. Further, we apply low pass filter on columns of result obtained by down sampling columns, and then we down sample the rows. Similarly high pass filter is also applied on the result after down sampling the columns, and then we down sample the rows. So here the 2D signal is divided into four bands LL, HL, LH, HH , where HL band indicated the variation along the horizontal while the LH band shows the vertical variation.

LL	HL
LH	HH

Fig. 3 Four bands in 2D signal after decomposition

We have used Daubechies D4 filter as low pass filter. the high pass filter is computed by,

$$H(z) = G(-z)$$

This is achieved by making the even terms of D4 filter as negative.

The code for decomposition is developed and it is available as a function

```
[C, S, wc] = discreteWavletTrans(I, J, lpfCoeff)
```

(details are given in function)

The 2D wavelet synthesis (reconstruction) is as reverse process to analysis process. We take the four band coefficients of decomposition and up sample the rows of four bands and apply low pass filter on columns of LL and HL bands and high pass filter on columns of LH and HH bands. We then add the results as shown in fig. 4, and then up sample the columns and apply low pass and high pass filters along rows and add the result to get the final reconstructed result.

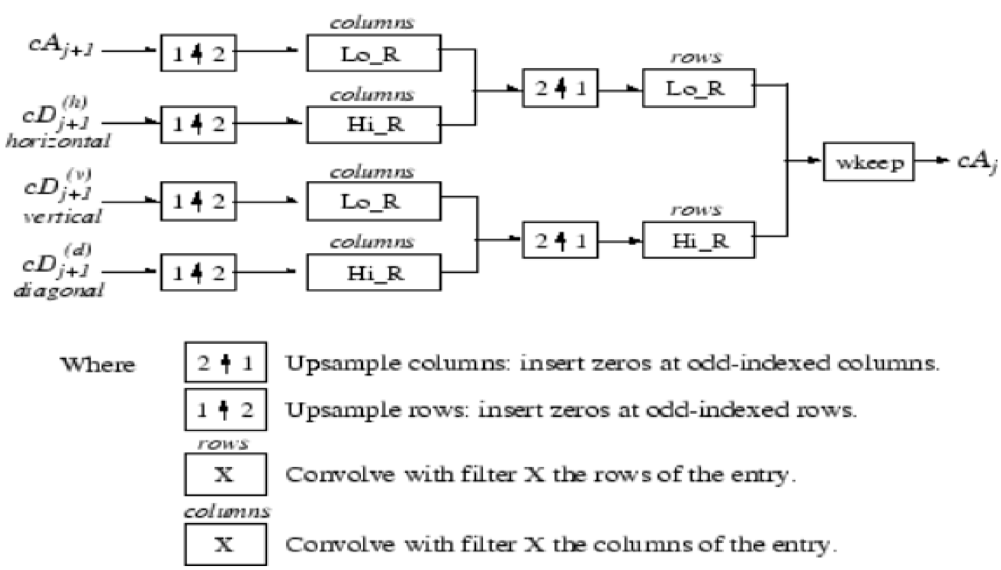


Fig. 4 Schematic diagram of 2D DWT reconstruction

The code for reconstruction is developed and it is available as a function

```
imageReconst = InvdiscreteWavletTrans(C, S, J, lpfCoeff)
```

(details are given in function)

The multi level wavelet transform is also similar to single level, but we consider the approximation as input to the next level in decomposition.

2.4 Results of wavelet transform on Images

I tried to input, synthetic input of random values as a 2D input. I decomposed the input using the developed function and reconstructed the wavelet coefficients and calculated the error between the original input an reconstructed result, which **results in 10^--5.**

The results of wavelet decomposition and reconstruction of Lena image are as given in figs.5 - 7..

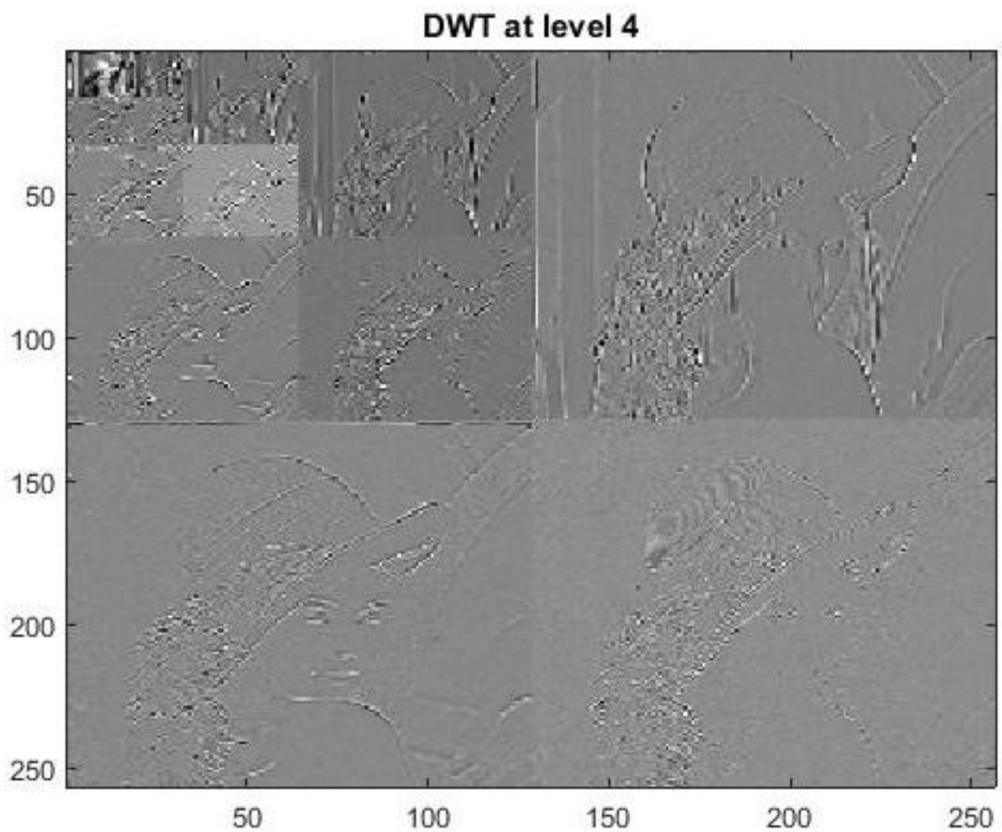


Fig. 5 Discrete wavelet decomposition at level 4 of Lena image

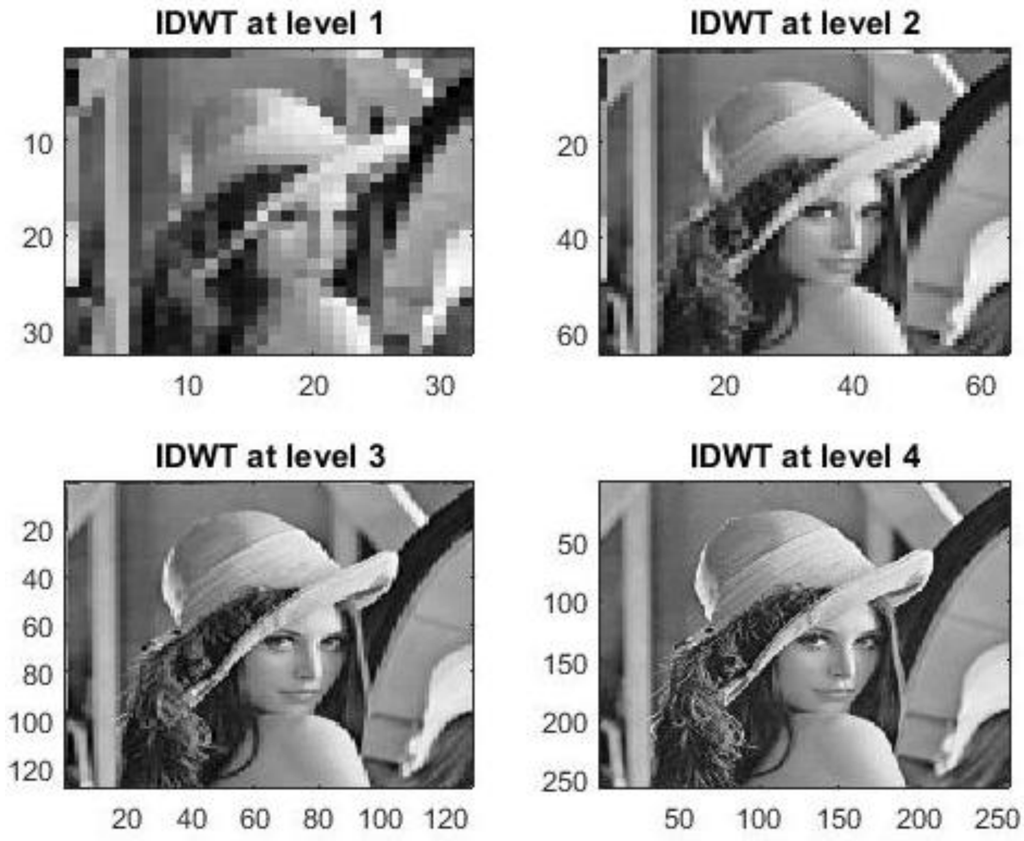


Fig. 6 DW reconstruction of Lena at 4 levels

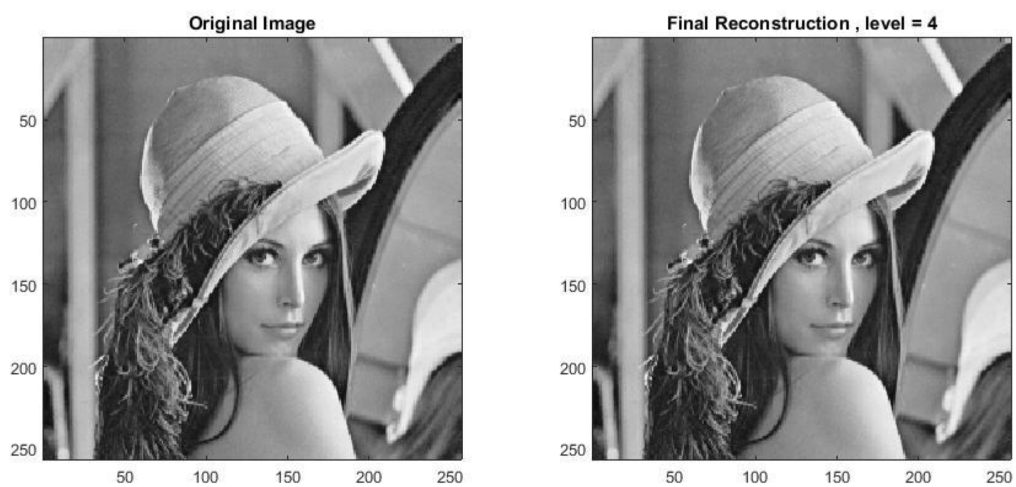


Fig. 7 Original and reconstructed image using DWT at level 4

The results of wavelet decomposition and reconstruction of cameraman image are as given in fig.8 and 9.

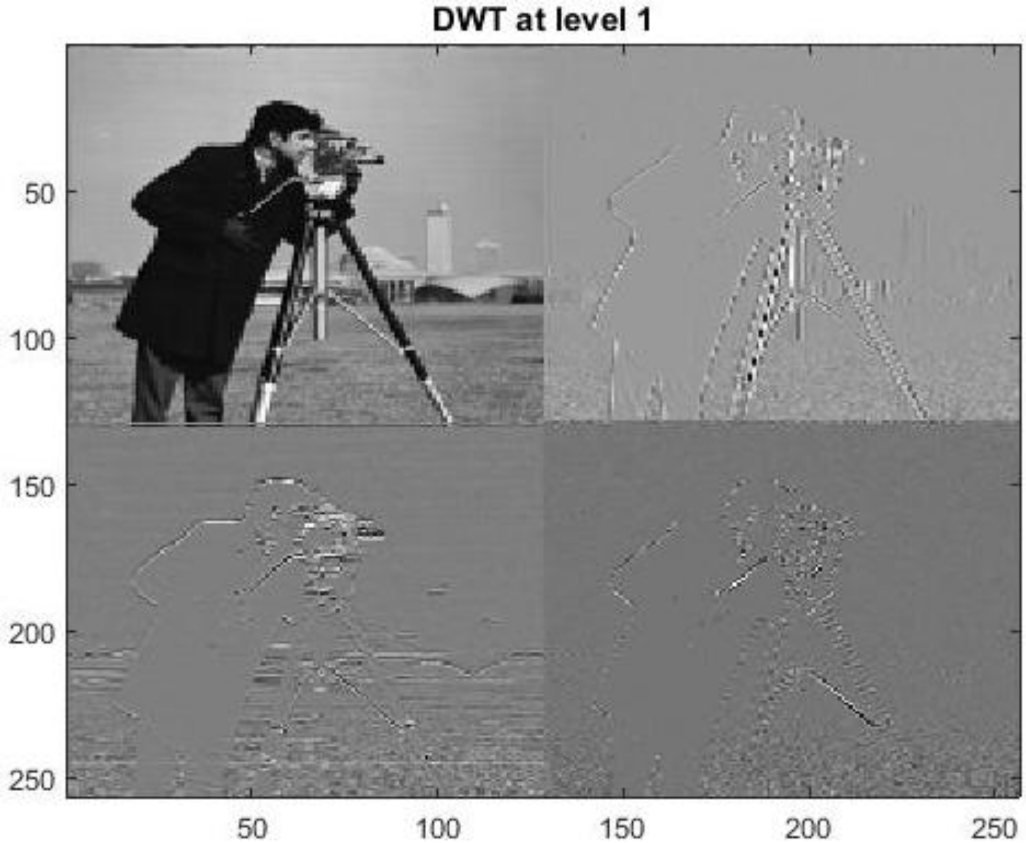


Fig. 8 Discrete wavelet decomposition at level 1 of cameraman image

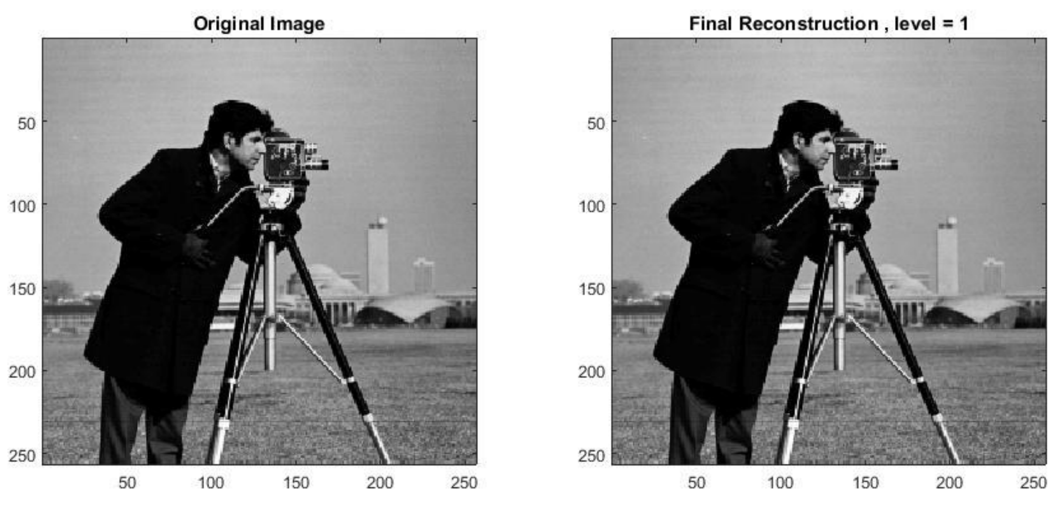


Fig. 9 Original and reconstructed image using DWT at level 1

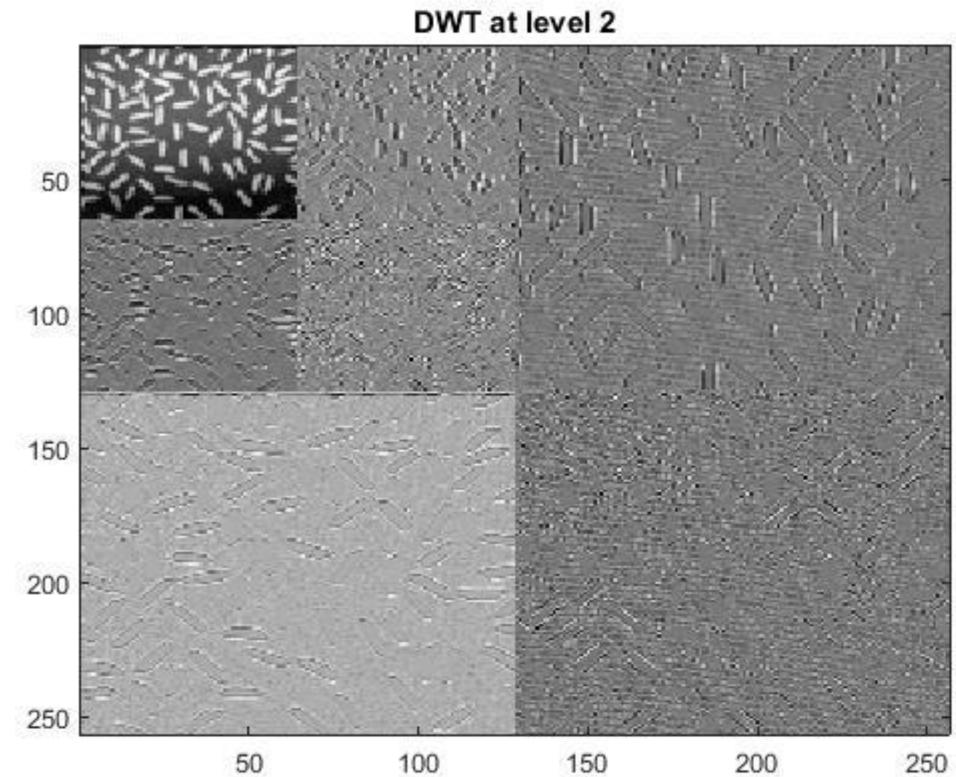


Fig. 10 Discrete wavelet decomposition at level 2 of rice image

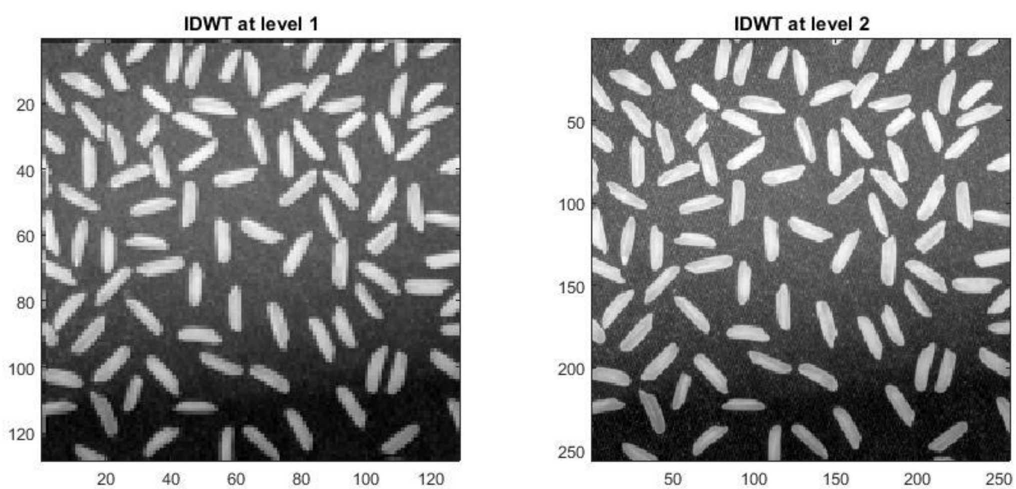


Fig. 11 DW reconstruction of rice image at 2 levels

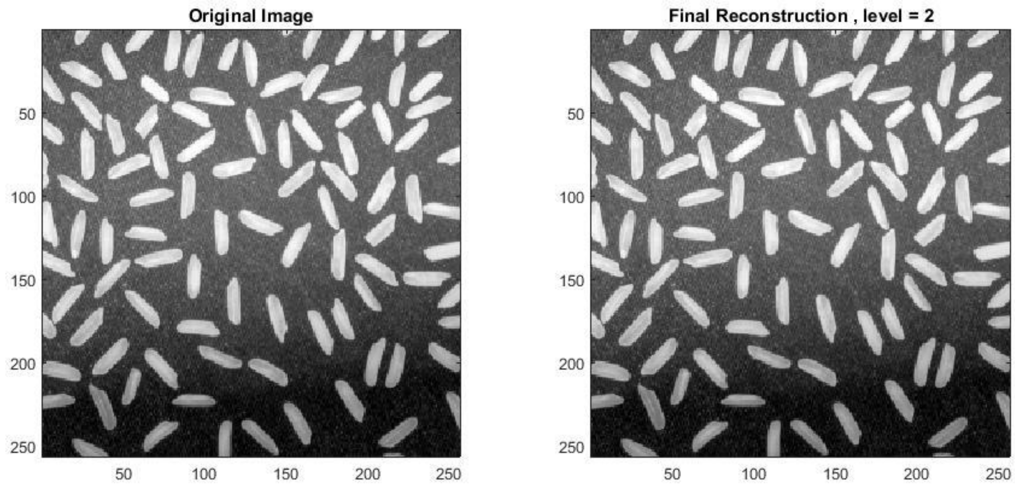


Fig. 12 Original and reconstructed image using DWT at level 2

3. Denoising

The data sets of many scientific experiments are corrupted with noise, either because of the data acquisition process, or because of environmental effects. A first pre-processing step in analyzing such datasets is denoising, that is, estimating the unknown signal of interest from the available noisy data. There are several different approaches to denoise signals and images. Generally smoothing removes high frequency and retains low frequency (with blurring). De-blurring increases the sharpness signal features by boosting the high frequencies, whereas denoising tries to remove whatever noise is present regardless of the spectral content of a noisy signal.

Wavelet transforms enable us to represent signals with a high degree of sparsity. This is the principle behind a non-linear wavelet based signal estimation technique known as wavelet denoising. Wavelet denoising attempts to remove the noise present in the signal while preserving the signal characteristics, regardless of its frequency content.

The principal work on denoising is is based on thresholding the DWT of the signal. The method relies on the fact that noise commonly manifests itself as fine-grained structure in the signal, and WT provides a scale-based decomposition. Thus, most of the noise tends to be represented by the wavelet coefficients at finer scales. Discarding these coefficients would result in a natural filtering out of noise on the basis of scale. Because the coefficients at such scale also tend to be the primary carriers of edge information, the method of Donoho, thresholds the wavelet coefficients to zero if their values are below a threshold. These coefficients are mostly those corresponding to the noise. The edge related coefficients of the signal on the other hand, are usually above the threshold.

Several approaches have been suggested for setting the threshold for each band of the wavelet decomposition. A common approach is to compute the sample variance of the coefficients in a band and set the threshold to some multiple of the deviation, which is given by the universal thresholding method **'sqtwolog'** method [1].

$$\boldsymbol{\lambda} = \boldsymbol{\sigma}\sqrt{\mathbf{2}\ \mathbf{log}\,\boldsymbol{n}}, where\ n\ is\ no.of\ data\ points$$

$$\lambda = 3\sigma$$

$$where, \sigma = \frac{median(|w|)}{0.6745}$$

$$where, w\ are\ wavelet\ coefficients$$

There are two types of thresholding [2]:

1. Hard thresholding

$$T^{hard}(d_{j,k}) = \begin{cases} d_{j,k}\, if\ |d_{j,k}| \geq \lambda \\ 0\ if\ |d_{j,k}| < \lambda \end{cases}$$

2. Soft Thresholding

$$T^{soft}(d_{j,k}) = \begin{cases} sign(d_{j,k})(|d_{j,k}| - \lambda) if\ |d_{j,k}| \geq \lambda \\ 0\ if\ |d_{j,k}| < \lambda \end{cases}$$

3.1 Results and Discussions

The results of denoising the images are as shown in figs. 13 -14.

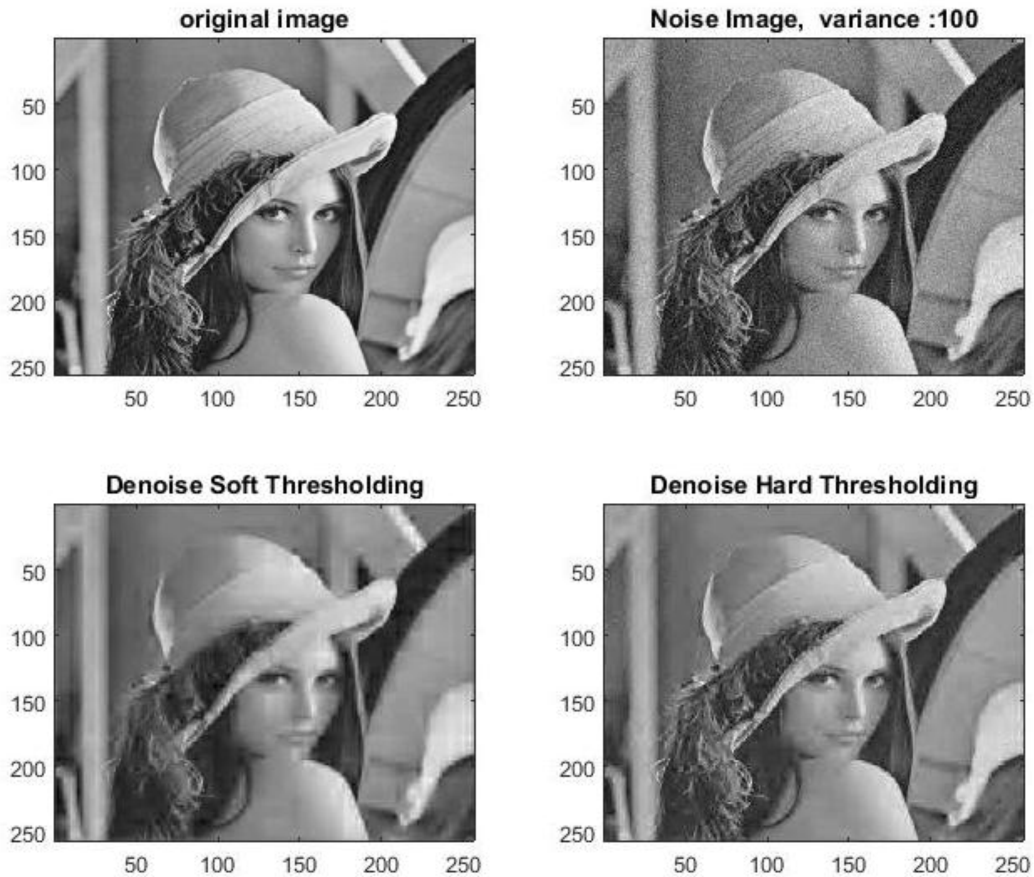


Fig. 13 Denoising image with a variance 100

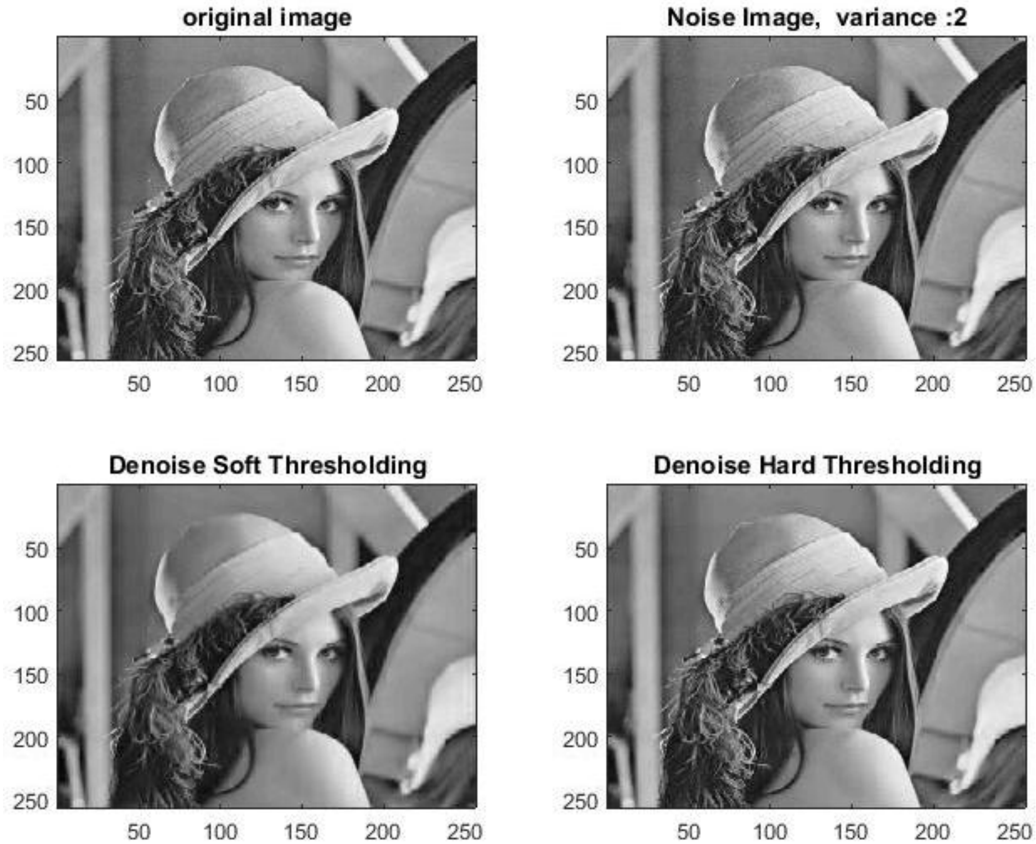
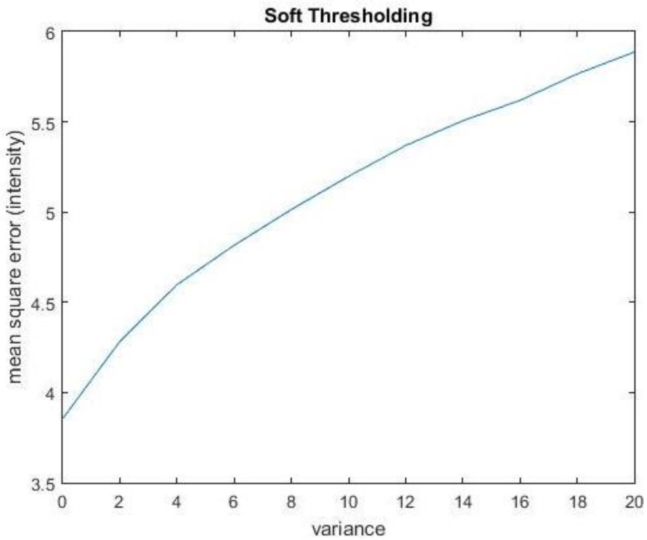


Fig. 14 Denoising image with a variance 2

Soft thresholding, shrinks coefficients above the threshold in absolute value. While at first sight hard thresholding may seem to be natural, the continuity of soft thresholding has some advantages. It makes algorithms mathematically more tractable. Sometimes, pure noise coefficients may pass the hard threshold and appear as annoying "blips" in the output. Soft thresholding shrinks these false structures.

Soft thresholding provides smoother results in comparison with the hard thresholding. Hard threshold, however, provides better edge preservation in comparison with the soft one.

I varied the variance from 0 to 20 and plotted the mean square error between the original and denoised images. The plot is as shown in fig. 15 for soft and hard thresholding.



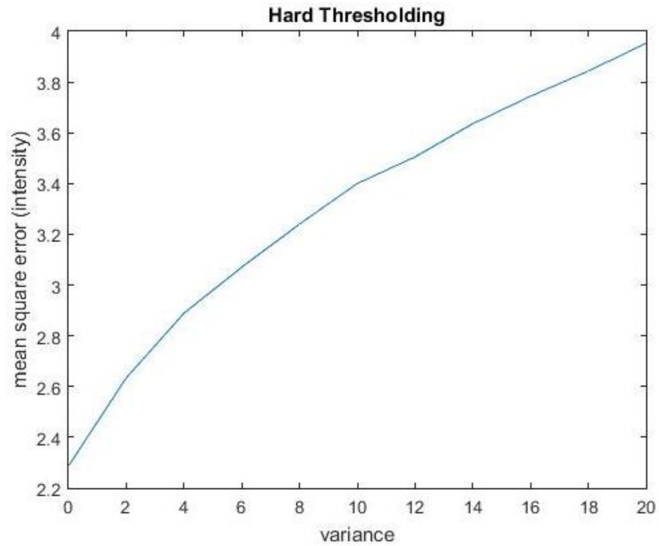


Fig. 15 Mean square error vs variance for soft and hard threshold

TABLE 1 TIME TAKEN BY CODE

Decomposition & Reconstruction	**Time (in sec)**
Level = 1	0.8
Level = 4	1.2

4. Conclusion

Discrete Wavelet transform is computationally competence which makes it very interesting. It has many applications like denoising, compression, security etc. In image denoising the soft threshold gives smooth result but id does not preserve edges whereas hard threshold preserve edges but it does not remove noise as perfectly as soft thresholding.

References

[1] Jeena Roy, Salice Peter and Neetha John, "Denoising using Soft Thresholding", in *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering* Vol. 2, Issue 3, March 2013.

[2] Lecture notes of Dr. Philippe Carre.