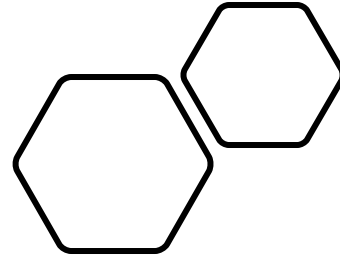# GAME OF NIM

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Game Description

1. There is a fixed number of stone piles with some number of stones in each.

2. Player takes turn alternately.

3. First turn is given randomly to either of the player.

4. In each turn a player selects a pile and removes at least one stone from that pile.

5. The player who removes last stone loses.

NIM SUM

Bitwise XOR ($\oplus$) of all numbers.

e.g: Let a, b and c be three numbers then

NIM-SUM(a, b, c) = a $\oplus$ b $\oplus$ c

Properties of XOR Operator.

$$a \oplus a = 0$$

$$0 \oplus a = a$$

Winning Strategy

- Never Take a move which leaves only even number of single stone.

- Always Take a move which Makes Nim-Sum of stones zero.

Is it always Possible?

- Lemma 1: It is always possible to make the Nim-Sum zero on your turn if it was not already zero at the beginning of your turn.

- Lemma 2 : If the Nim-Sum is zero after a player's turn, then the next player to make a move must change it to non-zero.