CSE512 Fall 2018 - Machine Learning - Homework 7

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Q1) M step

(1) $Q(\theta, \theta^{old}) =$

$$\sum_i \sum_c R_{ic} \log \pi_c + \sum_i \sum_c R_{ic} \log p(x_i | \theta_c)$$

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(2) $$\pi_c = \frac{1}{N} \sum_{i=1}^{N} R_{ic}$$

$$\therefore \pi_1 = \frac{1}{3} \sum_{i=1}^{3} R_{i1} = \frac{1}{3} (1 + 0.3 + 0) = \frac{1.3}{3}$$

$$= 0.433$$

$$\pi_2 = \frac{1}{3} \sum_{i=1}^{3} R_{i2} = \frac{1}{3} (0 + 0.7 + 1) = \frac{1.7}{3}$$

$$= 0.567$$

$$\therefore \pi_1 = 0.433 \text{ and } \pi_2 = 0.567$$

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(3) $$\mu_c = \frac{\sum_{i=1}^{N} R_{ic} x_i}{\sum_{i=1}^{N} R_{ic}}$$

$$\mu_1 = \frac{(1 \times 1) + (0.3 \times 10) + (0 \times 20)}{1 + 0.3 + 0} = \frac{4}{1.3} = 3.077$$

$$\mu_2 = \frac{(0 \times 1) + (0.7 \times 10) + (1 \times 20)}{0 + 0.7 + 1} = \frac{27}{1.7} = 15.882$$

(4) $\sigma_c = \sqrt{\frac{\sum_{i=1}^{N} Ric\, x_i^2}{\sum_{i=1}^{N} Ric} - (\mu_c)^2}$

$\sigma_1 = \sqrt{\frac{(1\times1)+(0.3\times100)+(0\times400)}{1+0.3+0} - (3.077)^2}$

$= \sqrt{\frac{31}{1.3} - 9.468} = \sqrt{23.846 - 9.468}$

$= \sqrt{14.378}$

$= 3.791$

$\sigma_2 = \sqrt{\frac{(0\times1)+(0.7\times100)+(1\times400)}{0+0.7+1} - (15.882)^2}$

$= \sqrt{\frac{470}{1.7} - 252.238} = \sqrt{276.471 - 252.238}$

$= \sqrt{24.233}$

$= 4.922$

E step

(1) $Ric = \frac{\pi_c \cdot p(x_i \mid \mu_c, \sigma_c)}{\sum_c \pi_c \cdot p(x_i \mid \mu_c, \sigma_c)}$

$= \frac{\pi_c \cdot \frac{1}{\sqrt{2\pi\sigma_c^2}} e^{\frac{-(x_i-\mu_c)^2}{2\sigma_c^2}}}{\sum_c \pi_c \cdot \frac{1}{\sqrt{2\pi\sigma_c^2}} e^{\frac{-(x_i-\mu_c)^2}{2\sigma_c^2}}}$

(2)

$$R_{11} = \frac{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{-(x_1-\mu_1)^2}{2\sigma_1^2}}}{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{-(x_1-\mu_1)^2}{2\sigma_1^2}} + \pi_2 \cdot \frac{1}{\sqrt{2\pi\sigma_2^2}} \cdot e^{\frac{-(x_1-\mu_2)^2}{2\sigma_2^2}}}$$

$$= \frac{0.433}{3.791} \times e^{-(1-3.077)^2/2\times 14.378}$$

$$\frac{\frac{0.433}{3.791} \times e^{\frac{-(1-3.077)^2}{2\times 14.378}}}{\frac{0.433}{3.791} \times e^{\frac{-(1-3.077)^2}{2\times 14.378}} + \frac{0.567}{4.922} \times e^{\frac{-(1-15.882)^2}{2\times 24.233}}}$$

$$= \frac{0.0983}{0.0983 + 0.0012} = \underline{0.988}$$

$$\Rightarrow R_{12} = 1 - 0.988 = \underline{0.012}$$

$$R_{21} = \frac{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{-(x_2-\mu_1)^2}{2\sigma_1^2}}}{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{-(x_2-\mu_1)^2}{2\sigma_1^2}} + \pi_2 \cdot \frac{1}{\sqrt{2\pi\sigma_2^2}} \cdot e^{\frac{-(x_2-\mu_2)^2}{2\sigma_2^2}}}$$

$$= \frac{0.433}{3.791} \times e^{\frac{-(10-3.077)^2}{2\times 14.378}}$$

$$\frac{\frac{0.433}{3.791} \times e^{\frac{-(10-3.077)^2}{2\times 14.378}}}{\frac{0.433}{3.791} \times e^{\frac{-(10-3.077)^2}{2\times 14.378}} + \frac{0.567}{4.922} \times e^{\frac{-(10-15.882)^2}{2\times 24.233}}}$$

$$= \frac{0.0216}{0.0216 + 0.0547} = \underline{0.283}$$

$$\therefore R_{22} = 1 - 0.283$$

$$= 0.717$$

$$R_{31} = \frac{\pi_1 \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{-(x_3-\mu_1)^2}{2\sigma_1^2}}}{\pi_1 \frac{1}{\sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{-(x_3-\mu_1)^2}{2\sigma_1^2}} + \pi_2 \cdot \frac{1}{\sqrt{2\pi\sigma_2^2}} \cdot e^{\frac{-(x_3-\mu_2)^2}{2\sigma_2^2}}}$$

$$= \frac{0.433}{3.791} \times e^{\frac{-(20-3.077)^2}{2\times 14.378}}$$

$$\frac{0.433}{3.791} \times e^{\frac{-(20-3.077)^2}{2\times 14.378}} + \frac{0.567}{4.922} \times e^{\frac{-(20-15.882)^2}{2\times 24.233}}$$

$$= 5.4 \times 10^{-6}$$

$$R_{32} = 1 - 5.4 \times 10^{-6}$$

$$= 0.9999946$$

$$\therefore R = \begin{bmatrix} 0.988 & 0.012 \\ 0.283 & 0.717 \\ 5.4\times10^{-6} & 0.9999946 \end{bmatrix}$$

Q3)
Kaggle Accuracy: 81%
Rank: 9