CSE512 Fall 2018 - Machine Learning - Homework 1

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(1.1) $x = \max(x_1, x_2) - x_1$

$$= \begin{cases} 0 & \text{if } x_1 \geq x_2 \\ x_2 - x_1 & \text{if } x_2 > x_1 \end{cases}$$

x can take values $0, 1, \ldots, i, \ldots N-1$ with probabilities $\left(\frac{N-i}{N^2}\right)$

This is because sample space has N^2 elements and each value i appears $N-i$ times

$$E(x) = \sum_{i=1}^{N-1} x_i \cdot p_i$$

$$= \sum_{i=1}^{N-1} i \cdot \left(\frac{N-i}{N^2}\right)$$

$$= \frac{1}{N^2} \sum_{i=1}^{N-1} (iN - i^2)$$

$$= \frac{1}{N^2} \left(\sum_{i=1}^{N-1} iN - \sum_{i=1}^{N-1} i^2 \right)$$

$$= \frac{1}{N^2} \left(\frac{N(N-1)(N-1+1)}{2} - \frac{(N-1)(N-1+1)(2(N-1)+1)}{6} \right)$$

$$\left(\because \sum_{i=1}^{N} i = \frac{i(i+1)}{2} \ \& \ \sum_{i=1}^{N} i^2 = \frac{i(i+1)(2i+1)}{6} \right)$$

$$= \frac{1}{N^{\cancel{2}}} \left(\frac{N^{\cancel{2}}(N-1)}{2} - \frac{\cancel{N}(N-1)(2N-1)}{6} \right)$$

$$= \frac{1}{6N} \left(3N^2 - 3\cancel{N} - 2N^2 + 3\cancel{N} - 1 \right)$$

$$= \frac{N^2 - 1}{6N}$$

$$\boxed{\therefore E(x) = \frac{N^2 - 1}{6N}}$$

1.2 $x_i = i$

$$P_i = \frac{N-i}{N^2} \qquad E(x) = \frac{N^2-1}{6N}$$

$$\mathrm{Var}(x) = E(x^2) - (E(x))^2$$

$$= \sum_{i=1}^{N-1} x_i^2 p_i - (E(x))^2$$

$$= \sum_{i=1}^{N-1} i^2 \left(\frac{N-i}{N^2}\right) - \left(\frac{N^2-1}{6N}\right)^2$$

$$= \frac{1}{N^2}\left(N\sum_{i=1}^{N-1} i^2 - \sum_{i=1}^{N-1} i^3 - \frac{(N^4-2N^2+1)}{36}\right)$$

$$= \frac{1}{N^2}\left(N\frac{(N-1)(N)(2N-1)}{6} - \frac{(N-1)^2(N^2)}{4} - \left(\frac{N^4-2N^2+1}{36}\right)\right)$$

$$= \frac{1}{N^2}\left(\frac{N^2(2N^2-3N+1)}{6} - \frac{N^2(N^2-2N+1)}{4} - \left(\frac{N^4-2N^2+1}{36}\right)\right)$$

$$= \frac{1}{N^2}\left(\frac{12N^4 - 9N^4 - N^4 - \cancel{18}N^3 + \cancel{18}N^3 + 6N^2 - 9N^2 + 2N^2 - 1}{36}\right)$$

$$= \frac{1}{36N^2}\left(2N^4 - N^2 - 1\right)$$

$$= \frac{1}{36N^2}(2N^2+1)(N^2-1)$$

$$\boxed{\therefore \mathrm{Var}(x) = \frac{(2N^2+1)(N^2-1)}{36N^2}}$$

(1.3) $\mathrm{Cov}(x, x_1) = E(x x_1) - E(x)E(x_1)$

joint prob of x, x_1 is $\frac{1}{N^2}$ & after simplifying we get the value of $E(x_1 x_2)$ as:

$$\sum_{i=1}^{N-1} i \cdot \sum_{j=1}^{N-i} j \cdot \frac{1}{N^2}$$

$$= \frac{1}{N^2} \sum_{i=1}^{N-1} i \cdot \frac{(N-i)(N-i+1)}{2}$$

$$= \frac{1}{2N^2} \sum_{i=1}^{N-1} i^3 - i^2(2N+1) + i(N)(N+1)$$

$$= \frac{1}{2N^2}\left[\frac{N^2(N-1)^2}{4} - \frac{N(N-1)(2N-1)(2N+1)}{6} + \frac{N(N+1)(N)(N-1)}{2}\right]$$

$$= \frac{\cancel{N}(N-1)}{24N^{\cancel{2}}}\left[3N^2 - 3N - 8N^2 + 2 + 6N^2 + 6N\right]$$

$$= \left(\frac{N-1}{24N}\right)\left(N^2 + 3N + 2\right)$$

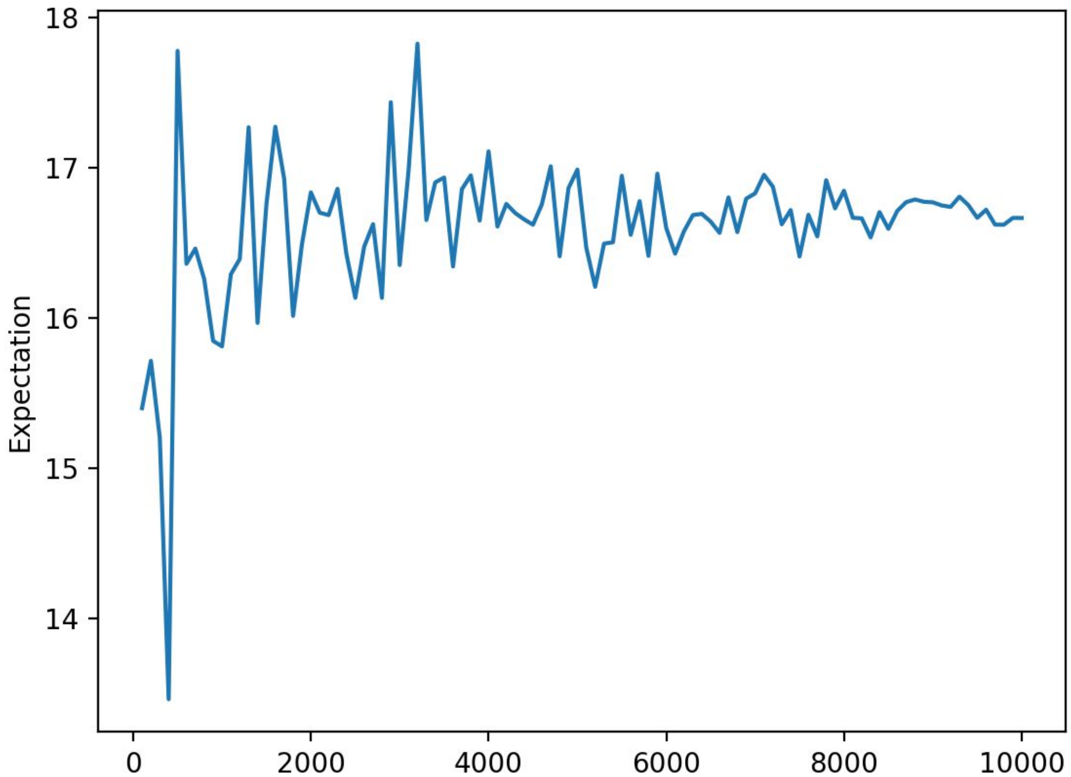
$$= \frac{(N-1)(N+1)(N+2)}{24N}$$

$$\mathrm{Cov}(x, x_1) = \frac{(N-1)(N+1)(N+2)}{24N} - \left(\frac{N+1}{2}\right) \times \frac{N^2-1}{6N}$$

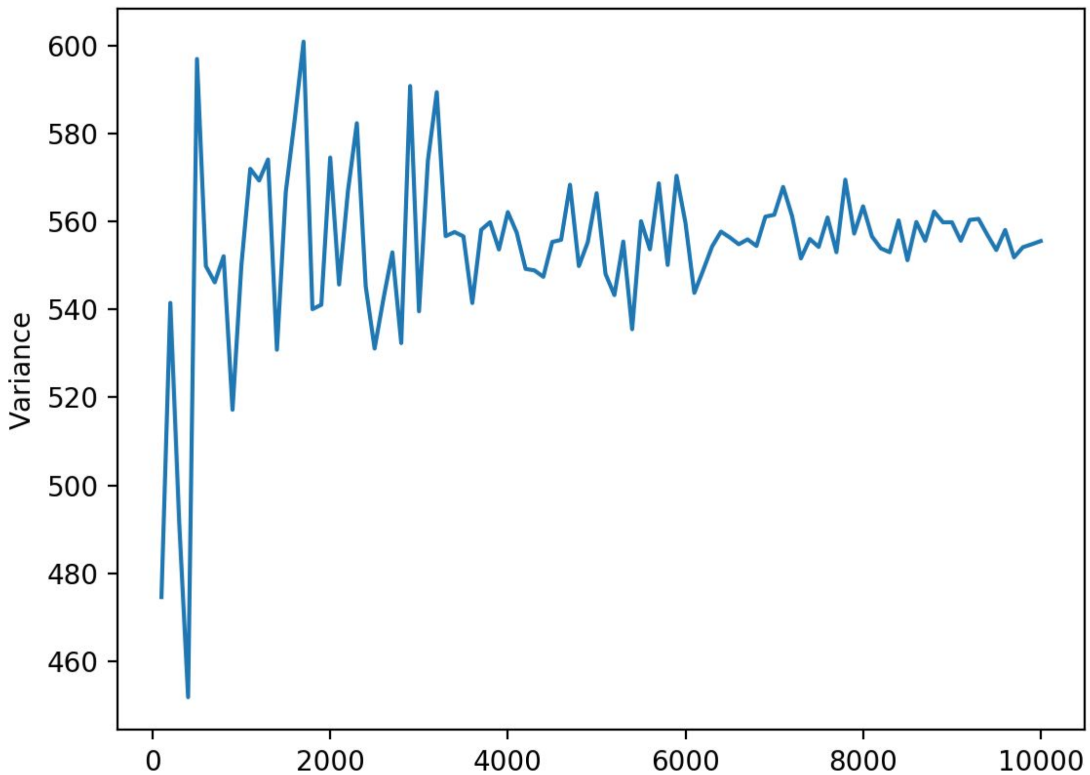
$$= \frac{N^2-1}{12N}\left(\frac{N+2}{2} - (N+1)\right)$$

$$= \frac{N^2-1}{12\cancel{N}}\left(\frac{-\cancel{N}}{2}\right) = \frac{1-N^2}{24}$$

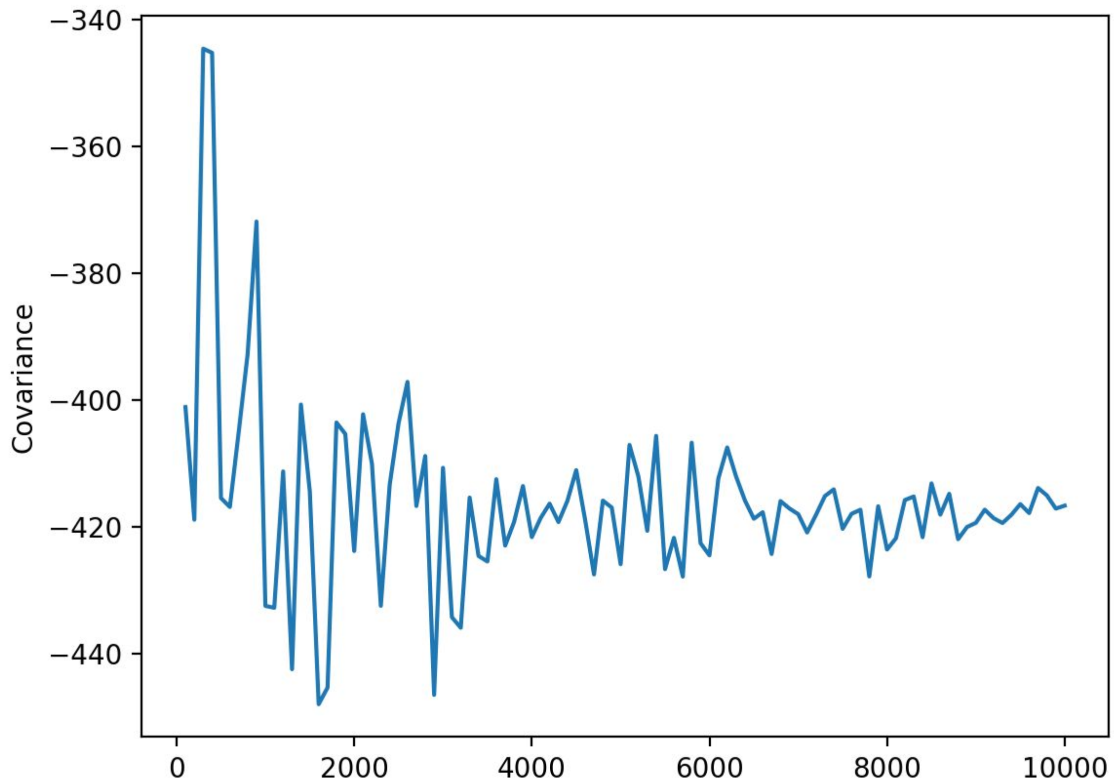
$$\boxed{\therefore \mathrm{Cov}(x, x_1) = \frac{1-N^2}{24}}$$

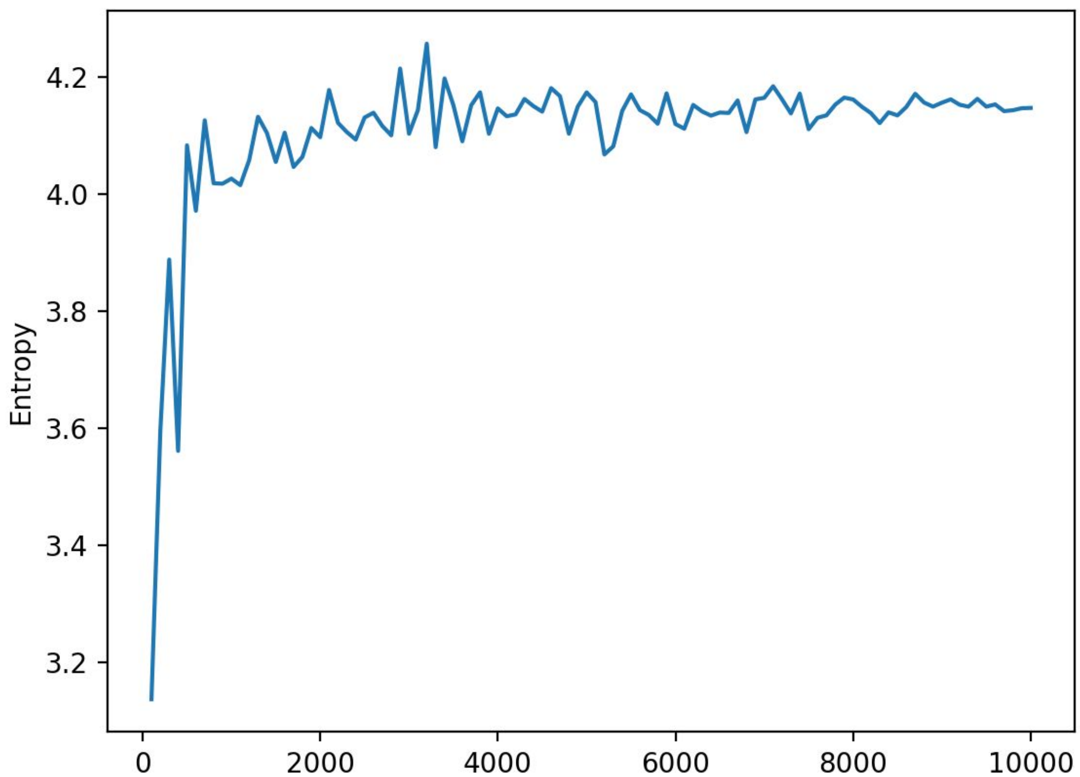


18
17
16
15
14
Expectation
0
2000
4000
6000
8000
10000



Variance
600
580
560
540
520
500
480
460
0
2000
4000
6000
8000
10000


Covariance
−340
−360
−380
−400
−420
−440
0
2000
4000
6000
8000
10000



Entropy
4.2
4.0
3.8
3.6
3.4
3.2
0
2000
4000
6000
8000
10000