CSE512 Fall 2018 - Machine Learning - Homework 6

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Q1.1 Given,

$$C = \frac{1}{n} X X^T$$

$$v_i^T v_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$$C v_1 = \lambda_1 v_1$$

$$\bar{X} = (I - v_1 v_1^T) X$$

<u>To show:</u> $\bar{C} = \frac{1}{n} X X^T - \lambda_1 v_1 v_1^T$

$$\bar{C} = \frac{1}{n} \bar{X} \bar{X}^T$$

$$= \frac{1}{n} \left[(I - v_1 v_1^T) X \cdot ((I - v_1 v_1^T) X)^T \right]$$

$$= \frac{1}{n} \left[(IX - v_1 v_1^T X)(X^T I - X^T v_1 v_1^T) \right]$$

$\left(\because (I - v_1 v_1^T)^T = I - v_1 v_1^T \right.$
$\left. \because \text{it's symmetric} \right)$

$$= \frac{1}{n} \left[X X^T - X X^T v_1 v_1^T - v_1 v_1^T X X^T + v_1 v_1^T X X^T v_1 v_1^T \right]$$

Now, $X X^T v_1 = n \lambda_1 v_1 \Rightarrow v_1^T X X^T = n \lambda_1 v_1^T$

$$\Rightarrow \bar{C} = \frac{1}{n} \left[X X^T - n\lambda_1 v_1 v_1^T - n\lambda_1 v_1 v_1^T + n\lambda_1 v_1 v_1^T v_1 v_1^T \right]$$

Now, $v_1^T v = 1$

$$\Rightarrow \bar{C} = \frac{1}{n} \left[X X^T - n\lambda_1 v_1 v_1^T \right]$$

$$\Rightarrow \bar{C} = \frac{1}{n} X X^T - \lambda_1 v_1 v_1^T$$

Q1.2 Given $j \neq 1$, v_j is principal eigen vector of C with eigen value λ_j

$$\Rightarrow C v_j = \lambda_j v_j$$

<u>To show</u>: $\bar{C} v_j = \lambda_j v_j$

$$\bar{C} v_j = \frac{1}{n} X X^T v_j - \lambda_1 v_1 v_1^T v_j$$

$$(v_1^T v_j = 0 \because j \neq 1)$$

$$\Rightarrow \bar{C} v_j = \frac{1}{n} X X^T v_j$$

$$= C v_j$$

$$= \lambda_j v_j$$

$$\Rightarrow \bar{C} v_j = \lambda_j v_j$$

Hence, v_j is also a principal eigen vector of $\bar{C}$ with eigen value λ_j

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Q1.3 u is the first principal eigen vector of $\bar{C}$. To show $u = v_2$

We have shown in 1.2 that for $j \neq 1$ v_j's are eigen vectors of $\bar{C}$

If $j = 1$

then $\bar{C} v_1 = C v_1 - \lambda_1 v_1 v_1^T v_1$

$$= \lambda_1 v_1 - \lambda_1 v_1 = 0$$

Hence v_1 is not an eigen vector of $\bar{C}$

since ith eigen value becomes 0

Now eigen vectors

$v_2, v_3, \ldots, v_n$ are sorted in decrea

order of their eigen values $\lambda_2, \lambda_3, \ldots, \lambda_n$

Hence, λ_2 has highest value and hence

v_2 is the first principal eigen vector

of $\bar{C}$

$\Rightarrow \quad u = v_2$

Q 1.4 $\quad [\lambda, u] = f(c)$

pseudo code:

```
C = 1/n X X^T, basisVectors = [ ]
for j in range(K):
    (λ, u) = f(c)
    basisVectors.append(u)
    C = C - λ u u^T
end
```

basis vectors will contain the first

k principal basis vectors