2018ICSE0621
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6-CSE-10

Part-C

Q.1] Given, $X \equiv 3 \bmod 5$
$x \equiv 1 \bmod 7$
$X \equiv 6 \bmod 8$

General form => $X \equiv a_i \bmod m_i$

We have, $a_1 = 3$; $a_2 = 1$; $a_3 = 6$
$m_1 = 5$; $m_2 = 7$; $m_3 = 8$

(i) $\gcd(m_1, m_2) = \gcd(m_2, m_3) = \gcd(m_3, m_1) = 1$
Therefore they are co prime.

(ii) To compute M :-
$M = m_1 \times m_2 \times m_3 = 5 \times 7 \times 8$
$M = 280$

~~M_1 m_1~~ Hence, $M_1 = \dfrac{M}{m_1} = \dfrac{280}{5} = 56$

$M_2 = \dfrac{M}{m_2} = \dfrac{280}{7} = 40$

$M_3 = \dfrac{M}{m_3} = \dfrac{280}{8} = 35$

(iii) We know that,

$Z = [M_1 x_1 a_1 + M_2 x_2 a_2 + M_3 x_3 a_3] \bmod M$

We need to compute the value of x_1, x_2 & x_3 to proceed.

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(iv) To find x_i values,

We know $M_1 X_1 = 1 \bmod (m_1)$ (or)
$M_1 x_1 \pmod{m_1} = 1.$

- $56\, x_1 \pmod 5 = 1$
  $1\, x_1 \pmod 5 = 1$
  $\boxed{\therefore x_1 = 1}$
- $40\, x_2 \pmod 7 = 1$
  $5\, x_2 \pmod 7 = 1$
  $\boxed{\therefore x_2 = 5}$
- $35\, x_3 \pmod 8 = 1$
  $3\, x_3 \pmod 8 = 1$
  $\boxed{\therefore x_3 = 3}$

(v) $Z = [M_1 X_1 a_1 + M_2 X_2 a_2 + M_3 X_3 a_3] \bmod 280$

Substituting the values we get

$Z = [56 \cdot 1 \cdot 3 + 40 \cdot 5 \cdot 1 + 35 \cdot 3 \cdot 6] \bmod 280$

$= [168 + 200 + 630] \bmod 280$

(PTO)

$Z = 998 \bmod 280$

Solving for Z,
$998 - 280 = 718$
$718 - 280 = 438$
$438 - 280 = \underline{158}$

And hence $\boxed{Z = 158}$

$158 \equiv 3 \bmod 5$
$158 \equiv 1 \bmod 7$
$158 \equiv 6 \bmod 8$

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$$Z = \left[168 + 120 + 630\right] \bmod 280.$$

$$Z = 918 \bmod 280.$$

Solving for Z we get,

$918 - 280 = 638$
$638 - 280 = 358$
$350 - 280 = 78.$

$$\boxed{\begin{aligned} \therefore\ 78 &\equiv 3 \bmod 5 \\ 78 &\equiv 1 \bmod 7 \\ 78 &\equiv 6 \bmod 8 \end{aligned}}$$