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## Part - B

- Q.1] • Bias Variance Decomposition is a way of analyzing a learning algorithm's expected generalized error with respect to 3 terms: Bias, Variance and Noise.

→ Assuming  $g(x)$  as generalization error which is taken from for predicting new target 't' for an input 'x'.

$$E[(t - g(x))^2]$$

Error in estimations can be written as

$$E[(t - g(x))^2] = E[(t - (y(x)))^2] = E[(t - g(x))^2] \\ = E\left[\underbrace{(t - g(x))}_a + \underbrace{g(x) - y(x)}_b\right]^2$$

$$= E[(t - g(x))^2] + E[(g(x) - y(x))^2] \\ + 2E[(t - g(x))(g(x) - y(x))].$$

\* It is of the form  $a^2 + b^2 + 2ab$ .

$$\Rightarrow E[(y(x) - g(x))^2] \text{ --- (1)}$$

Eqn (1) can be written as

$$E_D[(g(x) - y(x, D))^2]$$

$$= E_D\left[\underbrace{(y(x, D) - E_D[y(x, D)])}_P + \underbrace{E_D[y(x, D)] - g(x)}_q\right]^2$$

$$= E_D[P^2 + q^2 + 2Pq]$$

$$= E_D[(y(x, D) - E_D[y(x, D)])^2] + E_D[E_D[y(x, D)] - g(x)]^2 \\ + 2E_D[(y(x, D) - E_D[y(x, D)])(E_D[y(x, D)] - g(x))]$$

⇒ The above eq<sup>n</sup> can be written as in terms of Variance, bias & noise as:-

$$E(\text{loss}) = \text{Variance} + \text{Bias}^2 + \text{Noise}$$

- Thus we can represent the help of overfitting loss with help of the above equation as the total loss will be sum of variance, noise & square of bias.

→ Graphical proof:-

\* Let  $g(x)$  be expectation

Let  $y(x)$  be parametric approach.

