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Part-C

Q.3 $C_1 = X_1 = \{(4,1), (2,4), (2,3), (3,6), (4,4)\}$
$C_2 = X_2 = \{(9,10), (6,8), (9,5), (8,7), (10,8)\}$

Step1: Finding mean of C_1, C_2

C_1: $\mu_1 = \left\{\frac{4+2+2+3+4}{5}, \frac{1+4+3+6+4}{5}\right\} = [3, 3.6]$

$\mu_2 = \left\{\frac{9+6+9+8+10}{5}, \frac{10+8+5+7+8}{5}\right\} = [8.4, 7.6]$

Step2: Within Class Scatter Matrix

$S_W = S_1 + S_2$ [S_1 -> Covariance Matrix]

$$(x-\mu_1) = \begin{bmatrix} 1 & -1 & -1 & 0 & 1 \\ -2.6 & 0.4 & 0.6 & 2.4 & 0.4 \end{bmatrix}$$

$S_1 = \Sigma (x-\mu_1)(x-\mu_1)^T$

- $\begin{bmatrix} 1 \\ -2.6 \end{bmatrix} \begin{bmatrix} 1 & -2.6 \end{bmatrix} = \begin{bmatrix} 1 & -2.6 \\ -2.6 & 6.76 \end{bmatrix}$ — (1)
- $\begin{bmatrix} -1 \\ 0.4 \end{bmatrix} \begin{bmatrix} -1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & -0.4 \\ -0.4 & 0.16 \end{bmatrix}$ — (2)
- $\begin{bmatrix} -1 \\ -0.6 \end{bmatrix} \begin{bmatrix} -1 & -0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 0.36 \end{bmatrix}$ — (3)
- $\begin{bmatrix} 0 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0, & 2.4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5.76 \end{bmatrix}$ — (4)
- $\begin{bmatrix} 1 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{bmatrix}$ — (5)

Adding $\frac{(1)+(2)+(3)+(4)+(5)}{5}$, $S_1 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.64 \end{bmatrix}$

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$$(x-\mu_2) = \begin{bmatrix} 0.6 & -2.4 & 0.6 & -0.4 & 1.4 \\ 2.4 & 0.4 & -2.6 & -0.6 & 0.4 \end{bmatrix}$$

- $\begin{bmatrix} 0.6 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0.6 & 2.4 \end{bmatrix} = \begin{bmatrix} 0.36 & 1.44 \\ 1.44 & 5.76 \end{bmatrix}$ — (1)
- $\begin{bmatrix} -2.4 \\ 0.4 \end{bmatrix} \begin{bmatrix} -2.4 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.24 \\ 0.24 & 0.36 \end{bmatrix}$ — (2)
- $\begin{bmatrix} 0.6 \\ -2.6 \end{bmatrix} \begin{bmatrix} 0.6 & -2.6 \end{bmatrix} = \begin{bmatrix} 0.36 & -1.56 \\ -1.56 & 0.76 \end{bmatrix}$ — (3)
- $\begin{bmatrix} -0.4 \\ -0.6 \end{bmatrix} \begin{bmatrix} -0.4 & -0.6 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.24 \\ 0.24 & 0.36 \end{bmatrix}$ — (4)
- $\begin{bmatrix} 1.6 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1.6 & 0.4 \end{bmatrix} = \begin{bmatrix} 2.56 & 0.64 \\ 0.64 & 0.16 \end{bmatrix}$ — (5)

Adding $\dfrac{(1)+(2)+(3)+(4)+(5)}{5} \Rightarrow S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$

Substituting S_1 & S_2 in S_w

$$S_w = S_1 + S_2 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.64 \end{bmatrix} + \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$\therefore S_w = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$$

Step3: Between Class Scatter Matrix:

$$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

$\mu_1 = \begin{bmatrix} 3 & 3.6 \end{bmatrix}$ $\quad \mu_2 = \begin{bmatrix} 8.4 & 7.6 \end{bmatrix}$

$$S_B = \begin{bmatrix} -5.4 & -4 \end{bmatrix} \begin{bmatrix} -5.4 \\ -4 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}$$

Step 4: Finding Eigen values & Eigen Vector

(i) Eigen value: $|S_\omega^{-1} S_B - \lambda I| = 0$

$$S_\omega^{-1} = \begin{bmatrix} 3.64 & -0.44 \\ -0.44 & 5.24 \end{bmatrix}^{-1} = \begin{bmatrix} 0.38 & 0.032 \\ 0.032 & 0.143 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}$$

$$S_\omega^{-1} \cdot S_B = \begin{bmatrix} 0.38 & 0.032 \\ 0.032 & 0.143 \end{bmatrix} \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 11.77 & 8.72 \\ 5.10 & 3.77 \end{bmatrix}$$

$$\lambda \cdot I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\therefore \left| \begin{bmatrix} 11.77 & 8.72 \\ 5.10 & 3.77 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 11.77-\lambda & 8.72 \\ 5.10 & 3.77-\lambda \end{vmatrix} = 0.$$

$$(11.77-\lambda)(3.77-\lambda) - 5.10 \times 8.72 = 0$$

$$\lambda_1 = 15.54 \ ; \ \lambda_2 = 6.4 \times 10^{-3}$$

Choosing 'λ_1' as it has highest value.

(ii) Eigen Vector: $S_\omega^{-1} \cdot S_B \omega = \lambda \omega$.

$$\begin{bmatrix} 11.77 & 8.72 \\ 5.10 & 3.77 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = 15.54 \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

$$11.77\omega_1 + 8.72\omega_2 = 15.54\omega_1 \quad \text{—— (1)}$$

$$5.10\,\omega_1 + 3.77\,\omega_2 = 15.54\,\omega_2 \quad \text{—— (2)}$$

Eqn (1) $\Rightarrow -3.77\omega_1 + 8.72\,\omega_2 = 0.$

Eqn (2) $\Rightarrow 5.10\omega_1 - 11.77\omega_2 = 0.$

$$\therefore \begin{bmatrix} -3.77 & 8.72 \\ 5.10 & -11.77 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = 0$$

$$\omega_1 = \frac{8.72}{3.77}\,\omega_2 \Rightarrow \omega_1 = 2.31\,\omega_2$$

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$5.10\omega_1 - 11.77\omega_2 = 0$

$\omega_1 = \frac{11.77}{5.10}\omega_2$, let $\omega_2 = 1$

$\Rightarrow \omega_1 = 2.30, \omega_2 = 1$

$\therefore \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} 2.3 \\ 1 \end{bmatrix}$

$\longrightarrow$ Normalizing:

$$\begin{bmatrix} 2.3 / \sqrt{1^2 + 2.3^2} \\ 1 / \sqrt{1^2 + 2.3^2} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 0.91 \\ 0.39 \end{bmatrix}$$