Date	
Page	

2018ICSE0621
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ASSIGNMENT-1

Q.1]

X	4	8	13	7
Y	11	4	5	14

Step1: No. of features, n = 4
No. of samples, N = 4

Step2: Computing Mean

$$\bar{x} = \frac{4+8+13+7}{4} = 8 \; ; \; \bar{y} = \frac{11+4+5+14}{4} = 8.5$$

Step3: Computing Co-Variance Matrix
Ordered pairs => $(x,x), (x,y), (y,x), (y,y)$

(i) Covariance of (x,x), ~~cov(x,x) =~~

$$\text{Cov}(x,x) = \frac{1}{N-1}\sum_{k=1}^{N}(x_{ik}-\bar{x}_i)(x_{jk}-\bar{x}_j)$$

$$= \frac{1}{4-1}\left[(4-8)^2+(8-8)^2+(13-8)^2+(7-8)^2\right]$$

$$= 14$$

(ii) $$\text{Cov}(x,y) = \frac{1}{4-1}\left[(4-8)(11-8.5)+(8-8)(4-8.5)+(13-8)(5-8.5)+(7-8)(14-8.5)\right]$$

$$= \frac{1}{4-1}\left[(-4)(2.5)+0+5(-3.5)+(5.5)\right]$$

$$= -11$$

(iii) $\text{Cov}(x,y) = \text{Cov}(y,x) = -11$

2018ICSE0621

(iv) $Cov(y,y) = \frac{1}{4-1}\left[(11-8.5)^2 + (4-8.5)^2 + (5-8.5)^2 + (14-8.5)^2\right]$

$= 23$

→ <u>Covariance Matrix</u>

$$S = \begin{bmatrix} cov(x,x) & cov(x,y) \\ Cov(y,x) & Cov(y,y) \end{bmatrix}$$

$$S = \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}$$

Step 4: <u>Eigen value & Eigen vector</u>

1 Eigen Value:

$\det(S - \lambda I) = 0$ $\quad \because \lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$\Rightarrow \det\left(\begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$

$= \det\left(\begin{bmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{bmatrix}\right) = 0$

$= (14-\lambda)(23-\lambda) - (122) = 0$

$\lambda^2 - 37\lambda + 201 = 0$

$\lambda = 30.3849, 6.6151$

$$\boxed{\begin{aligned} \therefore \lambda_1 &= 30.3849 \\ \lambda_2 &= 6.6151 \end{aligned}}$$

2018ICSE0621

Date
Page

(2) Eigen Vector of λ_1

$(S - \lambda_1 I) U_1 = 0$

$\therefore \begin{bmatrix} 14-\lambda_1 & -11 \\ -11 & 23-\lambda_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$

$\begin{bmatrix} (14-\lambda_1)u_1 & -11u_2 \\ -11u_1 & (23-\lambda_1)u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(14-\lambda_1)u_1 - 11u_2 = 0$ --- (1)

$(-11u_1) + (23-\lambda_1)u_2 = 0$ --- (2)

From eqn (1) $\frac{u_1}{11} = \frac{u_2}{14-\lambda_1} = t$

When $t = 1$, $u_1 = 11$, $u_2 = 14 - \lambda_1$

Eigen vector u_1 of $\lambda_1 = \begin{bmatrix} 11 \\ 14-\lambda_1 \end{bmatrix}$

$= \begin{bmatrix} 11 \\ 14 - 30.3841 \end{bmatrix} = \begin{bmatrix} 11 \\ -16.3849 \end{bmatrix}$

→ Normalizing u_1

$e_1 = \begin{bmatrix} 11/\sqrt{11^2 - 16.3849^2} \\ -16.3849/\sqrt{11^2 - 16.3849^2} \end{bmatrix} = \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix}$

$e_2 = \begin{bmatrix} 0.8303 \\ 0.5574 \end{bmatrix}$

Step5: Deriving new Dataset

- Consider the reduced dimension values as P_{11}, P_{12}, P_{13} & P_{14}

2018ICSE0621

We get, $P_{11} = e_1^T \begin{bmatrix} 4-8 \\ 11-8.5 \end{bmatrix}$.

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} -4 \\ 2.5 \end{bmatrix} = -4.3052$$

$$P_{12} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 8-8 \\ 4-8.5 \end{bmatrix} = 3.7361$$

$$P_{13} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 13-8 \\ 5-8.5 \end{bmatrix} = 5.6928$$

$$P_{14} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 7-8 \\ 14-8.5 \end{bmatrix} = -5.1238$$

→ Hence, Our new dataset is :-

PC1	-4.3052	3.7361	5.6928	-5.1238.