2018ICSE0621

Part-C

2018ICSE0621

Q.1) We know, $W_{ML} = [\phi^T \phi]^{-1} \phi^T . t$

Given $\phi(x) = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$t = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Step 1 :- $\phi^t = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}^t = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

$\phi^t . \phi = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & 1 & 2 \\ 0 & 2 & -1 & 0 \\ 1 & -1 & 2 & 2 \\ 2 & 0 & 2 & 3 \end{bmatrix}$ ---- (1)

Let eqn (1) be 'Z'. To calculate Z^{-1} we obtain

2018ICSE0621

Date
Page

Step2 :-

2018ICSE0621

$$\phi^t \cdot \phi = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix} \text{ --- } (1).$$

Step2 :- To calculate inverse of eqn (1).
Say we consider eqn (1) as Z.

$$Z^{-1} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & -0.8 & -0.4 \\ -1 & 1.4 & 0.2 \\ 0 & -0.2 & 0.4 \end{bmatrix} \text{ — } (2).$$

Step3 :- Let eqn (2) be 'q'. We need to obtain

$$q \times \phi^t = \begin{bmatrix} 1 & -0.8 & -0.4 \\ -1 & 1.4 & 0.2 \\ 0 & -0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -1.4 & 0.2 & -0.2 \\ -0.8 & 1.2 & 0.4 & 0.6 \\ 0.4 & 0.4 & -0.2 & 0.2 \end{bmatrix}$$

2018ICSE0621

Step 4: Multiply result with 't'. Consider (3) as X

we have $t = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$$X \times t = \begin{bmatrix} 0.6 & -1.4 & 0.2 & -0.2 \\ -0.8 & 1.2 & 0.4 & 0.6 \\ 0.4 & 0.4 & -0.2 & 0.2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.8 \\ 1.4 \\ 0.8 \end{bmatrix}$$

Therefore, $\boxed{W_{ML} = \begin{bmatrix} -0.8 \\ 1.4 \\ 0.8 \end{bmatrix}}$

∴ Corresponding values are, $w_1 = \cancel{0.8}\ -0.8$

$w_2 = 1.4$

$w_3 = 0.8.$