

20181 CSE0621

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ASSIGNMENT-1

7-CSE-10

Q.2] $C_1 = X_1 = \{(4,1), (2,4), (2,3), (3,6), (4,4)\}$
 $C_2 = X_2 = \{(9,10), (6,8), (9,5), (8,7), (10,8)\}$

Step1: Finding mean of C_1, C_2

$C_1: \mu_1 = \left\{ \frac{4+2+2+3+4}{5}, \frac{1+4+3+6+4}{5} \right\} = [3, 3.6]$

$\mu_2 = \left\{ \frac{9+6+9+8+10}{5}, \frac{10+8+5+7+8}{5} \right\} = [8.4, 7.6]$

Step2: Within Class Scatter Matrix

$$S_w = S_1 + S_2$$

[$S_i \rightarrow$ Covariance Matrix]

$$(x - \mu_1) = \begin{bmatrix} 1 & -1 & -1 & 0 & 1 \\ -2.6 & 0.4 & 0.6 & 2.4 & 0.4 \end{bmatrix}$$

$$S_1 = \sum (x - \mu_1) (x - \mu_1)^T$$

$$\bullet \begin{bmatrix} 1 \\ -2.6 \end{bmatrix} \begin{bmatrix} 1 & -2.6 \end{bmatrix} = \begin{bmatrix} 1 & -2.6 \\ -2.6 & 6.76 \end{bmatrix} - \textcircled{1}$$

$$\bullet \begin{bmatrix} -1 \\ 0.4 \end{bmatrix} \begin{bmatrix} -1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & -0.4 \\ -0.4 & 0.16 \end{bmatrix} - \textcircled{2}$$

$$\bullet \begin{bmatrix} -1 \\ -0.6 \end{bmatrix} \begin{bmatrix} -1 & -0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 0.36 \end{bmatrix} - \textcircled{3}$$

$$\bullet \begin{bmatrix} 0 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0 & 2.4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5.76 \end{bmatrix} - \textcircled{4}$$

$$\bullet \begin{bmatrix} 1 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{bmatrix} - \textcircled{5}$$

Adding $\textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5}$ $S_1 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.64 \end{bmatrix}$

$$(X - \mu_2) = \begin{bmatrix} 0.6 & -2.4 & 0.6 & -0.4 & 1.4 \\ 2.4 & 0.4 & -2.6 & -0.6 & 0.4 \end{bmatrix}$$

$$\bullet \begin{bmatrix} 0.6 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0.6 & 2.4 \end{bmatrix} = \begin{bmatrix} 0.36 & 1.44 \\ 1.44 & 5.76 \end{bmatrix} - (1)$$

$$\bullet \begin{bmatrix} -2.4 \\ 0.4 \end{bmatrix} \begin{bmatrix} -2.4 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.24 \\ 0.24 & 0.36 \end{bmatrix} - (2)$$

$$\bullet \begin{bmatrix} 0.6 \\ -2.6 \end{bmatrix} \begin{bmatrix} 0.6 & -2.6 \end{bmatrix} = \begin{bmatrix} 0.36 & -1.56 \\ -1.56 & 0.76 \end{bmatrix} - (3)$$

$$\bullet \begin{bmatrix} -0.4 \\ -0.6 \end{bmatrix} \begin{bmatrix} -0.4 & -0.6 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.24 \\ 0.24 & 0.36 \end{bmatrix} - (4)$$

$$\bullet \begin{bmatrix} 1.6 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1.6 & 0.4 \end{bmatrix} = \begin{bmatrix} 2.56 & 0.64 \\ 0.64 & 0.16 \end{bmatrix} - (5)$$

$$\text{Adding } \frac{(1)+(2)+(3)+(4)+(5)}{5} \Rightarrow S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

Substituting S_1 & S_2 in S_w

$$S_w = S_1 + S_2 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.64 \end{bmatrix} + \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$\therefore S_w = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$$

Step 3: Between Class Scatter Matrix:

$$S_B = (\mu_1 - \mu_2) (\mu_1 - \mu_2)^T$$

$$\mu_1 = \begin{bmatrix} 3 & 3.6 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 8.4 & 7.6 \end{bmatrix}$$

$$S_B = \begin{bmatrix} -5.4 & -4 \end{bmatrix} \begin{bmatrix} -5.4 \\ -4 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}$$

Step 4: Finding Eigen values & Eigen Vector

(i) Eigen value: $|S_{\omega}^{-1} S_B - \lambda I| = 0$

$$S_{\omega}^{-1} = \begin{bmatrix} 3.64 & -0.44 \\ -0.44 & 5.24 \end{bmatrix}^{-1} = \begin{bmatrix} 0.38 & 0.032 \\ 0.032 & 0.143 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}$$

$$\begin{aligned} S_{\omega}^{-1} \cdot S_B &= \begin{bmatrix} 0.38 & 0.032 \\ 0.032 & 0.143 \end{bmatrix} \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix} \\ &= \begin{bmatrix} 11.77 & 8.72 \\ 5.10 & 3.77 \end{bmatrix} \end{aligned}$$

$$\lambda \cdot I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\therefore \left| \begin{bmatrix} 11.77 & 8.72 \\ 5.10 & 3.77 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 11.77 - \lambda & 8.72 \\ 5.10 & 3.77 - \lambda \end{vmatrix} = 0$$

$$(11.77 - \lambda)(3.77 - \lambda) - 5.10 \times 8.72 = 0$$

$$\lambda_1 = 15.54 ; \lambda_2 = 6.4 \times 10^{-3}$$

Choosing ' λ_1 ' as it has highest value.

(ii) Eigen Vector: $S_{\omega}^{-1} \cdot S_B \omega = \lambda \omega$

$$\begin{bmatrix} 11.77 & 8.72 \\ 5.10 & 3.77 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = 15.54 \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

$$11.77\omega_1 + 8.72\omega_2 = 15.54\omega_1 \quad \text{--- (1)}$$

$$5.10\omega_1 + 3.77\omega_2 = 15.54\omega_2 \quad \text{--- (2)}$$

$$\text{Eqn (1)} \Rightarrow -3.77\omega_1 + 8.72\omega_2 = 0$$

$$\text{Eqn (2)} \Rightarrow 5.10\omega_1 - 11.77\omega_2 = 0$$

$$\therefore \begin{bmatrix} -3.77 & 8.72 \\ 5.10 & -11.77 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = 0$$

$$\omega_1 = \frac{8.72}{3.77} \omega_2 \Rightarrow \omega_1 = 2.31\omega_2$$

$$5.10w_1 - 11.77w_2 = 0$$

$$w_1 = \frac{11.77}{5.10} w_2, \text{ let } w_2 = 1$$

$$\Rightarrow w_1 = 2.30, w_2 = 1$$

$$\therefore \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 2.3 \\ 1 \end{bmatrix}$$

$$\rightarrow \text{Normalizing: } \begin{bmatrix} 2.3 / \sqrt{1^2 + 2.3^2} \\ 1 / \sqrt{1^2 + 2.3^2} \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 0.91 \\ 0.39 \end{bmatrix}$$

Steps: Dataset: (4,1), (2,4), (2,3), (3,6), (4,4)

we know $y = wx$

$$\text{For } C_1: \therefore \text{Output of LDA} = (3.64, 0.39), (1.82, 1.56) \\ (1.82, 1.17), (2.78, 2.34) \\ (3.64, 4.56)$$

For C_2 :-

$$(9,16), (6,8), (9,5), (8,7), (10,8)$$

$$\therefore \text{Output of LDA} = (8.14, 3.9), (3.46, 3.12) \\ (8.19, 1.95), (7.28, 2.73) \\ (9.1, 3.12)$$