

Sai Ram.K.

5 - CSE-1D.

2018|CSE0621

08/01/2021

Part-B

Q.1) Quicksort :-

Quicksort is a divide and conquer algorithm. It picks an element as pivot and partitions the given array around the pivot.

→ ALGORITHM:-

Step 1:- Partitioning the array into 2.

partition(a, low, high)

 pivot = a[low], i = low + 1, j = high

 while (pivot > a[i])

 i++

 while (pivot < a[j])

 j--

 if (i < j)

 swap(a[i], a[j])

 else

 swap(a[low], a[j])

 return j

}

Step 2: Sorting the array

qsort(a, low, high)

{

 if (low < high)

{

 j = partition(a, low, high)

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Quicksort

q.sort(a, low, j-1)
q.sort(a, j+1, high)

→ Efficiency of Quicksort :-

* Best & average case $\Rightarrow T(n)$

$$T(n) = \begin{cases} T(1) = 0, & \text{if } n=1 \\ T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n, & \text{otherwise} \end{cases}$$

Time taken for Time for Time for
left pivot right pivot partitioning

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

Using Master's theorem,

$$a = 2 \quad b = 2^1$$

$$\boxed{a = b^k}, \quad \boxed{k = 1}$$

$$\therefore T(n) = \Theta(n^k \log n) \\ = \Theta(n^1 \log n)$$

$$\boxed{\therefore T(n) = \Theta(n \log n)}$$

Average & Best case have same complexity

$$\boxed{\therefore T(n) = \Theta(n \log n)}$$

* Worst case :-

$$T(n) = T(0) + T(n-1) + n$$

$$T(n) = \begin{cases} T(1) = 0, & \text{if } n=1 \\ T(n-1) + n, & \text{otherwise} \end{cases}$$

We proceed further,

$$T(n) = T(n-1) + n \quad \text{--- (1)}$$

Replace n by n-1 in (1)

$$\begin{aligned}
 &= T(n-1-1) + n-1 + n \\
 &= T(n-2) + (n-1) + n \\
 &= T(n-3) + (n-2) + (n-1) + n \\
 &\vdots \\
 &= T(n-n) + (n-(n-1)) + (n-(n-2)) + n \\
 &= T(0) + (n-n+1) + (n-n+2) + n \\
 &= 0 + 1 + 2 + \dots + n \\
 &= \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \approx n^2
 \end{aligned}$$

We consider the highest power,

Hence, $T(n) = O(n^2)$

→ Differences between Merge Sort & Quick Sort.

Quick Sort	Merge Sort
(i) The splitting of array is in any ratio not exactly into half.	Splitting of array is also not necessarily half but uses different approach.
(ii) Worst case complexity is $O(n^2)$	Worst case complexity is $O(n \log n)$
(iii) Works well on smaller arrays	Works good with any size.
(iv) Works faster than other algorithms for small dataset.	It has <u>consistent</u> speed on any data.
(v) It is <u>not stable</u>	It is <u>stable</u>
(vi) Preferred for arrays	Preferred for Linked lists
(vii) Method of sorting is <u>internal</u>	Method of sorting is <u>external</u> .

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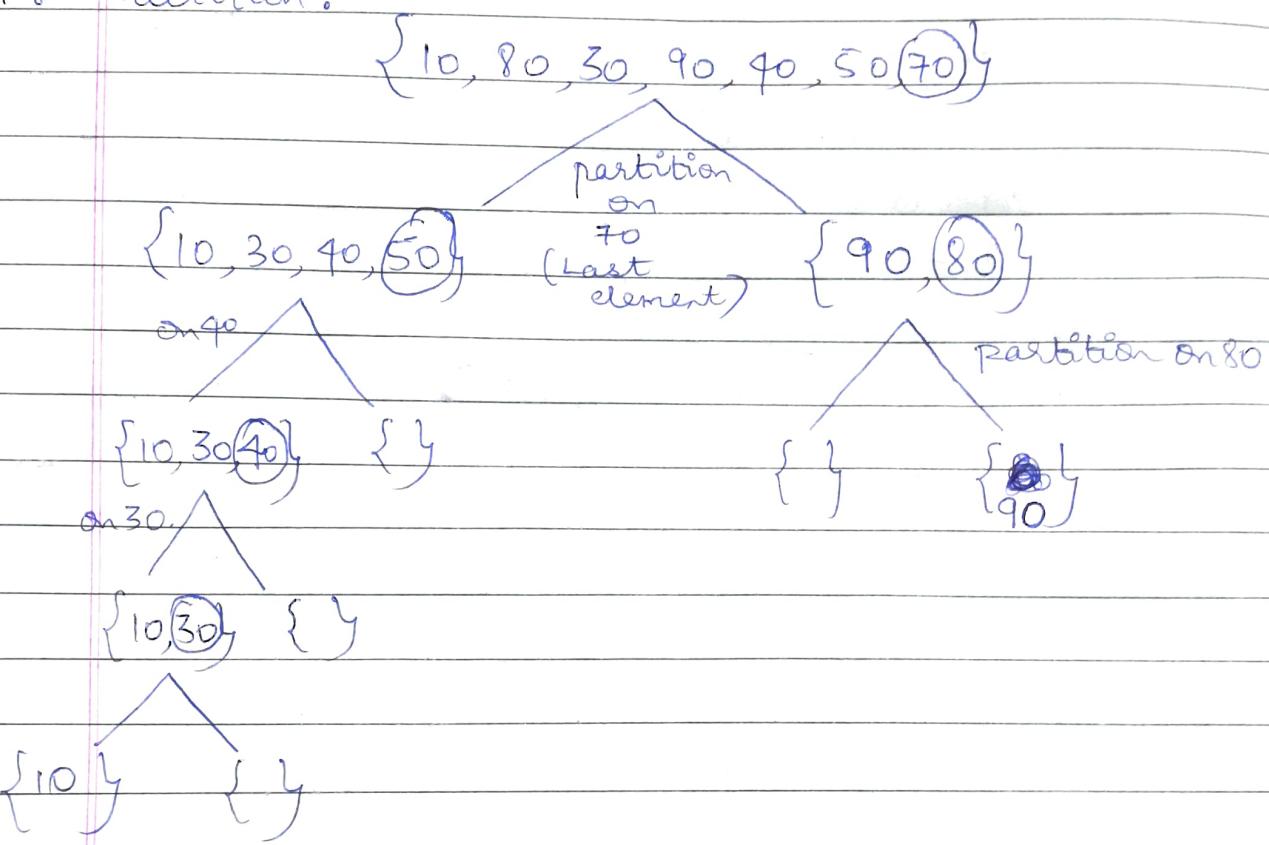
Solve :-



Example for quicksort :

Consider $\{10, 80, 30, 90, 40, 50, 70\}$

Step 1 :- Partition :



Step 2 :- Merging Back we get :-

$\{10, 30, 40, 50, 70, 80, 90\}$

[Merging is done by traversing.]

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Part-B.

Q.2] Backtracking :-

Backtracking can be defined as an algorithmic technique for solving problems recursively by trying to build a solution one piece at a time, removing those solutions that fail to satisfy the constraints of the problem at any point of time.

→ ALGORITHM OF N-QUEENS :-Backtrack ($x[l \dots i]$)

// Gives a template of generic backtracking algorithm.

// Input : $x[l:i]$ specifies first i promising components of a solution.

// Output : All tuples representing the - problem's solution.

if $x[l:i]$ is a solution write $x[l \dots i]$

else

for each element $x \in S; + j$ consistent with $x[l:i]$ and the constraints do $x[i+1] * - x$ Backtrack ($x[1 \dots i+1]j$)

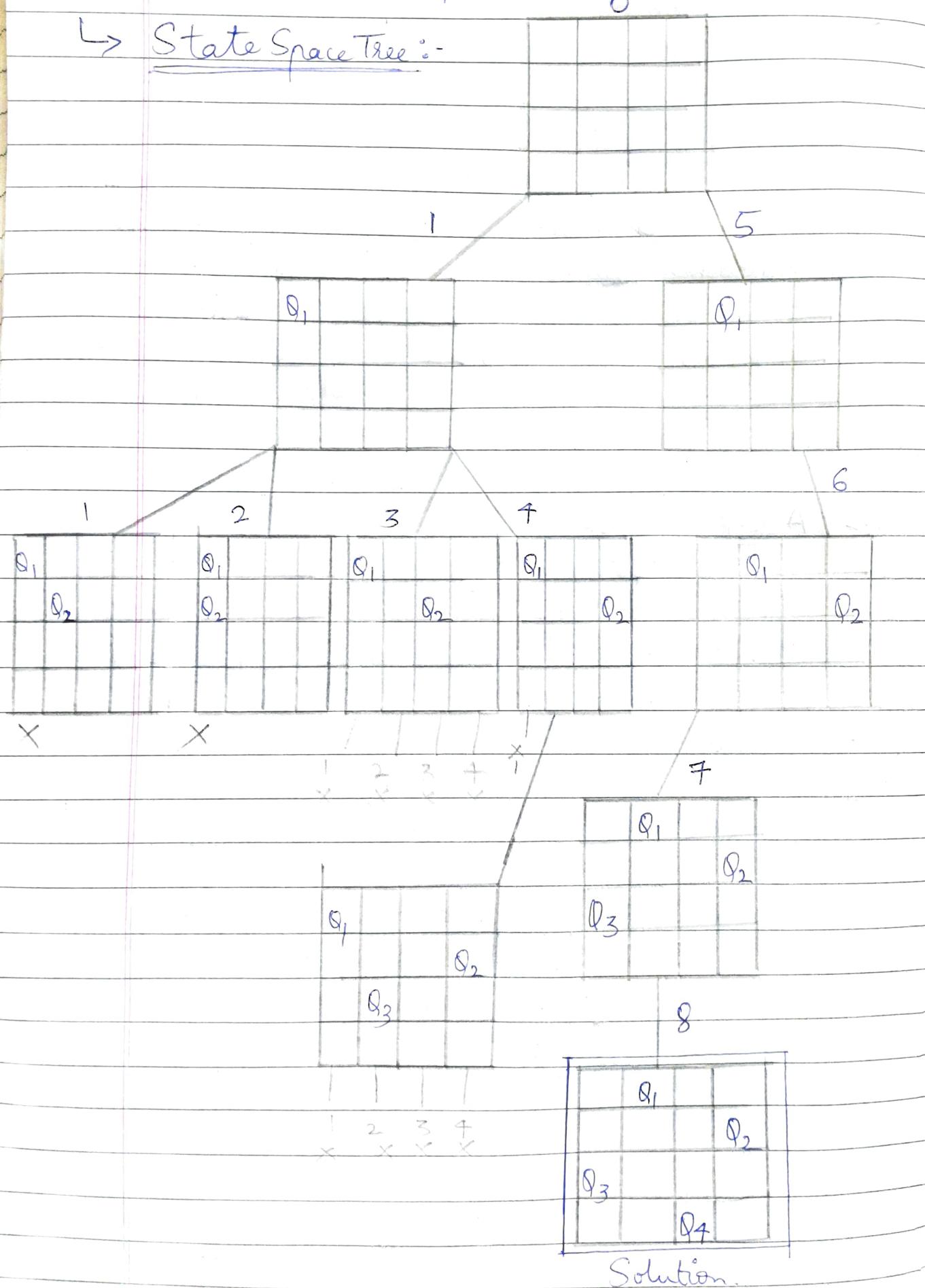
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Sister

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4 * 4 Queens problem :

↳ State Space Tree :-



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Part - B

Q.3) Knapsack :-

↳ Algorithm :-

// Input : n is total number of items, W is the capacity w[] stores weights of each item & v[] stores values.

// Output : Returns total value of selected items from knapsack.

```
for (i ← 0 to n) do {
    for (j ← 0 to w) do {
        table[i, 0] = 0
        table[0, j] = 0 }}
```

```
for (i ← 0 to n) do {
    for (j ← 0 to w) do {
        if (j < w[i]) then {
            table[i, j] ← table[i-1, j]
        } else if (j ≥ w[i]) then
            table[i, j] ← max[table(i-1, j), (v[i+j] +
                table[i-1, j-1])]}
    }
}
return table[n, w]
```

Time efficiency = O(nw)

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Given, $n=4$, $\omega = [1, 2, 2, 3]$ $P = [18, 16, 6, 4]$
 capacity = 4.

i	P	ω	0	1	2	3	4
0	18	1	0	0	0	0	0
1	16	2	1	0	18	18	18
2	6	2	2	0	18	18	34
3	4	3	3	0	18	18	34
			4	0	18	18	34

Formula: $v[i, \omega] = \max \{ v[i-1, \omega], v[i-1, \omega - \omega[i]] + P[i] \}$

$$v[1, 1] = \max \{ v[0, 1], v[0, 1-1] + 18 \} = 18$$

$$v[1, 2] = \max \{ v[0, 2], v[0, 2-1] + 16 \} = 18$$

$$v[1, 3] = \max \{ v[0, 3], v[0, 3-1] + 18 \} = 18$$

$$v[1, 4] = \max \{ v[0, 4], v[0, 4-1] + 18 \} = 18$$

$$v[2, 1] = \max \{ v[1, 1], v[1, 1-2] + 16 \} = 18$$

$$v[2, 2] = \max \{ v[1, 2], v[1, 2-2] + 16 \} = 18$$

$$v[2, 3] = \max \{ v[1, 3], v[1, 3-2] + 16 \} = 34$$

$$v[2, 4] = \max \{ v[1, 4], v[1, 4-2] + 16 \} = 34$$

$$v[3, 1] = \max \{ v[2, 1], v[2, 1-2] + 6 \} = 18$$

$$v[3, 2] = \max \{ v[2, 2], v[2, 2-2] + 6 \} \\ = \max \{ 18, 6 \} = 18$$

$$v[3, 3] = \max \{ v[2, 3], v[2, 3-2] + 6 \} = \max \{ 34, 24 \} = 34$$

$$v[3, 4] = \max \{ v[2, 4], v[2, 4-2] + 6 \} = \max \{ 34, 18+6 \} = 34$$

$$v[4, 1] = \max \{ v[3, 1], v[3, 1-2] + 4 \} = \max \{ 18, - \} = 18$$

$$v[4, 2] = \max \{ v[3, 2], v[3, 2-2] + 4 \} = \max \{ 18, 4 \} = 18$$

$$v[4, 3] = \max \{ v[3, 3], v[3, 3-2] + 4 \} = \max \{ 34, 22 \} = 34$$

$$v[4, 4] = \max \{ v[3, 4], v[3, 4-2] + 4 \} \\ = \max \{ 34, 22 \} = 34$$

Says

Hence Max value is 34

As table [4, 4] = table [3, 4]

& table [3, 4] = table [2, 4]

We don't select 4th & 3rd item.

Selecting item 1 & 2 :-

∴ Solution will be (1, 1, 0, 0)

Profits : $34 - 16 = 18 \rightarrow$ 2nd item

$18 - 18 = 0 \rightarrow$ 1st item.

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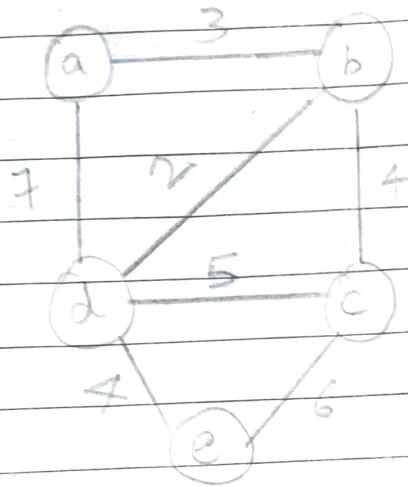
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Part-C

Q.1] Graph:-



→ Dijkstra's Algorithm:-

Let start node & node n 's distance be ∞ .

Step 1: Assign to every node a distance value & set it to 0 for initial node & ∞ to other nodes.

Step 2: Set initial node as current & mark other nodes as unvisited.

Step 3: Consider current node & calculate distance of neighbors.

Step 4: Set unvisited nodes with smallest distance from initial node as next & continue.

Step 5: When all nodes are visited the algorithm ends.

Algorithm :-

Let s be source node
 $\text{dist}[s] \leftarrow 0$
for all $v \in V - \{s\}$ do
 $\text{dist}[v] \leftarrow \infty$
 $S \leftarrow \emptyset$
 $Q \leftarrow V$
while ($Q \neq \emptyset$) do
 $u \leftarrow \min \text{dist}(Q, \text{dist})$
 $S \leftarrow S \cup \{u\}$
for all $v \in \text{neighbors}[u]$ do
if ($\text{dist}[v] > \text{dist}[u] + \text{cost}[u][v]$)
then $\text{dist}[v] = \text{dist}[u] + \text{cost}[u][v]$
return dist .

→ Finding shortest path :-

Tree Vertices	Remaining Vertices	Graph Diagram
$a(-, 0)$	$b(a, 3), c(-\infty)$ $d(a, 7), e(-, \infty)$	
$b(a, 3)$	$c(b, 3+4), d(b, 3+2)$ $e(-, \infty)$	
$d(b, 5)$	$c(b, 7)$ $c(d, 5+4)$	
$e(d, 9)$		

Hence, Shortest path is $a \rightarrow b \rightarrow d \rightarrow c$

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Part - C.

Q.2] Pseudocode for Backtracking:

It's of 2 types:-

- (1) Recursive Backtracking solution
- (2) Finding whether solution exists or not.

(1) Recursive:

```
void findsln (n, other params):
    if (found a solution):
        solnfound = solnfound + 1;
        displaySoln ();
    if (solnfound >= solntarget):
        system.exit (0)
    return
    for (val = first to last):
        if (isValid (val, n)):
            applyValue (val, n);
            findsln (n+1, otherparams);
            removeValue (val, n);
```

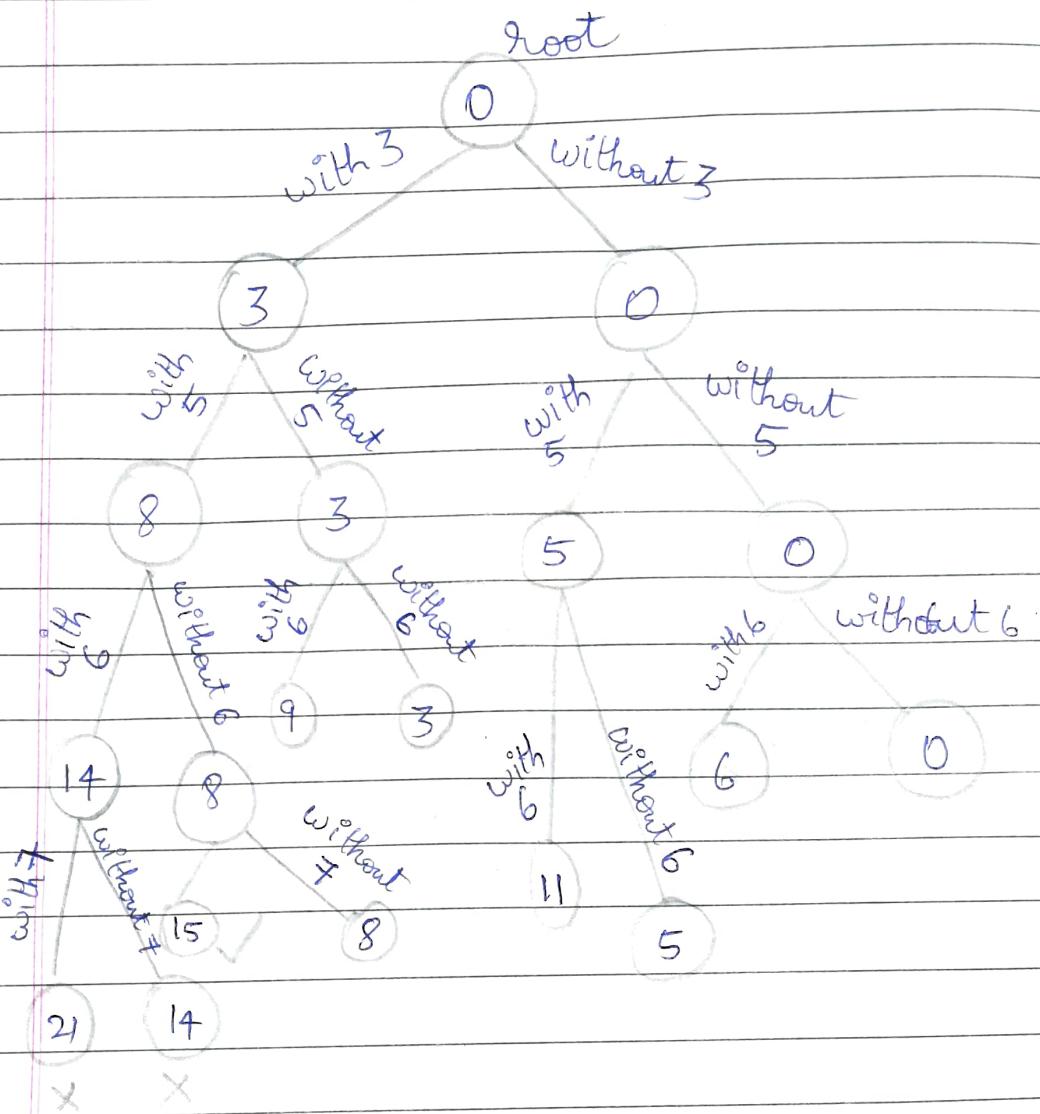
(2) Finding if solution exists:

```
boolean findsln (n, otherparams):
    if (found a solution):
        displaySolution ()
    return True
    for (val = first to last):
        if (isValid (val, n)):
            if (findsln (n+1, otherparams)):
                return True;
                removeValue (val, n);
            return False;
```

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→ Given: $W = \{3, 5, 6, 7\}$
 $m = 15$.



Solution :- With 3 ✓

With 5 ✓

Without 6

With 7 ✓

$$\therefore 3 + 5 + 7 = 15$$