Industrial Robot: An International Journal

A distance error based industrial robot kinematic calibration method

Wang Zhenhua Xu Hui Chen Guodong Sun Rongchuan Lining Sun

A distance error based industrial robot kinematic calibration method

Wang Zhenhua and Xu Hui
Robotics and Microsystem Center, Soochow University, Suzhou, China
Chen Guodong
Robotics and Microsystem Center, Soochow University, Suzhou, China and
School of Electronic and Computer Engineering, Peking University, Beijing, China, and
Sun Rongchuan and Lining Sun
Robotics and Microsystem Center, Soochow University, Suzhou, China

Abstract

Purpose – The purpose of this paper is to present a distance accuracy-based industrial robot kinematic calibration model. Nowadays, the repeatability of the industrial robot is high, while the absolute positioning accuracy and distance accuracy are low. Many factors affect the absolute positioning accuracy and distance accuracy, and the calibration method of the industrial robot is an important factor. When the traditional calibration methods are applied on the industrial robot, the accumulative error will be involved according to the transformation between the measurement coordinate and the robot base coordinate.

Design/methodology/approach – In this manuscript, a distance accuracy-based industrial robot kinematic calibration model is proposed. First, a simplified kinematic model of the robot by using the modified Denavit–Hartenberg (MDH) method is introduced, then the proposed distance error-based calibration model is presented; the experiment is set up in the next section.

Findings – The experimental results show that the proposed calibration model based on MDH and distance error can improve the distance accuracy and absolute position accuracy dramatically.

Originality/value – The proposed calibration model based on MDH and distance error can improve the distance accuracy and absolute position accuracy dramatically.

Keywords Industrial robot, Calibration, MDH model, Distance accuracy, Laser tracker

Paper type Research paper

Introduction

Nowadays, the industrial robots are widely used in the fields of manufacturing industry to assist human beings in doing more and more tasks, and the applications of industrial robots are gradually getting more sophisticated (Craig, 2005; Wang *et al.*, 2013). However, in the more challenging domain, such as assembly and robotic surgery, the higher performances of the industrial robots are required (Gatla *et al.*, 2007). The performances of the robot mainly depend on two important evaluation criteria: repeatability and accuracy. Repeatability is defined as the ability how precisely a robot can move to a point that has been taught previously. Accuracy of the industrial robot is represented by the precision with which the end-effector can move to a command point (Slamani *et al.*, 2012). Most industrial robots currently perform the tasks by applying the teach pendant methods, in which repeatability of the robot is a major factor affecting the robot performance. However, since the offline programming was introduced into the industry, the advantages of offline programming have become popular (Young and Pickin, 2000). As the robots work in the offline programming mode, which improves the work efficiency of the robot and is more suitable for complicated operations than teach mode, the position accuracy becomes critical (Motta *et al.*, 2001).

Many researchers have shown that the repeatability of the industrial robots is much better than the position accuracy, and that the position accuracy is bounded by the robot repeatability (Craig, 2005). Then, in the premise of ensuring good repeatability, calibration techniques can be applied to improve the accuracy of the robot (Veitschegger and Wu, 1988). Due to some influencing factors (such as vibration, manufacturing and assembly errors), there are some differences between the nominal value and the actual value of the robot kinematic parameters (Karan and Vukobratovic, 1994). Clearly, these differences are the errors of the robot kinematic parameters. By analysis, even though the parameter errors are very small, they can also cause a large deviation for the robot end-effector. Therefore, the goal of the robot calibration is to apply the precise measurement devices and model-based parameter identification method to identify the exact parameters of the robot model.

The current issue and full text archive of this journal is available at
www.emeraldinsight.com/0143-991X.htm

Industrial Robot: An International Journal
41/5 (2014) 439–446

[DOI 10.1108/IR-04-2014-0319]

This research is supported by National High Technology Research and Development Program of China (Grant No. SS2013AA041001).

Since the calibration techniques were first applied into the robotics field, these methods have been a cost-effective way to promote the accuracy of the industrial robots. As summarized by Roth *et al.* (1987), robot calibration can be classified into three levels. Level-1 calibration only takes the joint displacement, which is the rotation angle for the revolute joint and is linear displacement for the prismatic joint, into account. The main task of the level-1 calibration is to identify the errors between the actual joint displacement. Level-2, including geometric parameters only, is known as the entire kinematic parameters' calibration. The calibration not only identifies the kinematic parameters of the robot but also the joint displacement in level-1, i.e. level-2 includes the level-1. Compared to level-1 and level-2 calibration, although level-3 calibration named non-kinematic calibration is sufficiently accurate, due to the difficulty in identifying non-geometric parameters, such as end-effector acceleration, deflection of the robot links and gear backlash, etc., it is difficult to perform in practice.

The goal of robot modelling is to establish a corresponding kinematic model in accordance with the specific mechanical structure of the robot calibrated (Roning and Kormn, 1997). The Denavit–Hartenberg model (DH model) is the most popular kinematic notation for the robot mechanisms (Denavit, 1955). However, the singularity problem occurs when two adjacent joints are parallel or nearly parallel. To solve this problem, Hayati *et al.* (1988) proposed a modified DH model (MDH) by introducing a new rotation parameter. In recent years, many researchers have proposed the kinematic calibration model based on the standard DH and MDH model (In Won *et al.*, 2012; Ha, 2008; Ruibo *et al.*, 2010). In Won *et al.* (2012) propose a calibration technique to identifying entire kinematic parameter errors of robot based on the standard DH model; deviation between actual and measured positions are denoted by the Jacobian matrices derived from differential kinematics. With assistance of a structured laser module and a stationary camera, effectiveness of the technique is verified with 4 degrees of freedom manipulator by actual test. Ha (2008) presents a new relative position measurement calibration method based on the MDH model, and the effects of the models used in laboratory and industrial environments are discussed. Meanwhile, some new models are also proposed by the researchers. Ruibo *et al.* (2010) propose a error model that is based on the product of exponentials formula; the authors analyze the parameters' identification in this error model, and simulation results show that this error model is effective for serial-robot calibration.

Nubiola and Bonev (2013) point out that the improvement of robot position accuracy is affected by the accuracy of the measurement system. With the development of science and technology, some new and effective measuring techniques are applied in the robot calibration, such as vision-based measurement and laser-based measurement systems, and that the measurement devices used in these systems are very accurate (Nubiola and Bonev, 2013; Sun *et al.*, 2004; Newman *et al.*, 2000; Andreff *et al.*, 2004; Santolaria *et al.*, 2012). Andreff *et al.* (2004) propose a kinematic calibration for an H4 parallel prototype robot with a vision-based measurement device; the final result is that the position error of the end-effector has reduced from more than 10 mm to less than 0.5 mm. Santolaria *et al.* (2012) introduce a new technique called improved circle point analysis (CPA) method to identify the kinematic parameters by using a laser tracker with an active target, which simplifies the screw-axis measurement process. The experiment shows that CPA is a good method in understanding the operating principle of the robot. Nubiola and Bonev (2013) present a error model that takes all possible geometric errors (a 29-parameter calibration model) into account; with the assistance of a laser tracker, the authors use the least-squares optimization method to find the 29 error parameters. The results show that the mean/maximum position errors of arbitrary eight different points on the end-effector are reduced from 0.968 mm/2.158 mm to 0.364 mm/0.696 mm.

In general, when we use the measurement systems to calibrate the robot position accuracy directly, the transformation matrix between the measurement system frame and the robot base frame is involved inevitably, which is very difficult to measure and calculate accurately (Ren *et al.*, 2008; Veitschegger and Wu, 1986; Kuu-Young *et al.*, 1996; Xuecai *et al.*, 1994). So the accuracy of the measurement system will be decreased due to the errors caused by the transformation matrix. To avoid the drawback of frame transformation, the distance error of any two positions in robot workspace is applied to calibrate the robot position accuracy indirectly.

Therefore, in this paper, we propose a novel and simple calibration method based on the distance errors, and to avoid the singularity problem, the actual kinematic model used in this paper is a hybrid model with the standard DH model being used for generic joints and MDH model being used for parallel joints. In practice, this new calibration model simplifies the procedure of robot calibration compared to conventional methods greatly.

This paper is organized into the following five sections. Section 2 proposes a kinematic model of the industrial robot using MDH model. Section 3 presents a calibration model based on distance errors. To demonstrate the effectiveness of the presented calibration method, we carry out an experiment that calibrates a 6 degree of freedoms industrial robot with a laser tracker in Section 4. Sections 5 and 6 give the results of our experiments and concludes this paper, respectively.

Kinematic model

MDH kinematic model

Robot calibration is a process of applying precise measurement devices and model-based parameter identification method to identify the exact parameters of the robot model. Therefore, to calibrate the robot, the kinematic model of the robot is determined first, and then, the corresponding kinematic parameters of the model can be found. The DH model is the most popular kinematic representation in robotics literature, in which Cartesian coordinate frame is attached to each joint of robot in accordance with certain rules: *Z*-axis is coincident with joint axis and *X*-axis is coincident with the common normal between two adjacent *Z*-axes. The relationship of two adjacent joints can be expressed as a homogeneous transformation matrix, which is represented by using four independent parameters calledα_i, a_i, d_i and θ_i; these four parameters include one joint variable and three fixed link parameters. For the robots with all rotary joints, α_i, a_i and d_i are the fixed parameters, and θ_i represents the rotation angle of joint$i$. For joint $i-1$ and i, the homogeneous transformation matrix ${}^{i-1}_{i}T$ can be expressed as equation (1):

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i\, c\alpha_i & s\theta_i\, s\alpha_i & a_i\, c\theta_i \\ s\theta_i & c\theta_i\, c\alpha_i & -c\theta_i\, s\alpha_i & a_i\, s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

Where, joint number $i = 1,2$ [. . .] N; $c\theta_i$ and $c\alpha_i$ represent $\cos\theta_i$ and $\cos\alpha_i$, and $s\theta_i$ and $s\alpha_i$ represent $\sin\theta_i$ and $\sin\alpha_i$, respectively.

Nonetheless, a defect of DH model is that it results in singularities with respect to two adjacent parallel joints or nearly parallel joints; even a minor deviation of parallelism can cause a large deviation between the actual and theoretical common normal line. This important problem is pointed out by Hayti, who proposed a model that modifies the DH model (MDH model) by adding a rotation parameter β_i which represents a small rotation angle around the Y_i-axis in the coordinate frame of joint i, $Rot\ (y,\ \beta_i)$ in equation (2). Therefore, the homogeneous transformation matrix A_i between joint $i-1$ and i can be rewritten as equation (3):

$$ Rot\ (y_i, \beta_i) = \begin{bmatrix} c\beta_i & 0 & s\beta_i & 0 \\ 0 & 1 & 0 & 0 \\ -s\beta_i & 0 & c\beta_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ \begin{aligned} A_i &= {}^{i-1}_{i}T \cdot Rot(y_i, \beta_i) \\ &= \begin{bmatrix} c\theta_i c\beta_i - s\theta_i s\alpha_i s\beta_i & -s\theta_i c\alpha_i & c\theta_i s\beta_i + s\theta_i s\alpha_i c\beta_i & a_i c\theta_i \\ s\theta_i c\beta_i + c\theta_i s\alpha_i s\beta_i & c\theta_i c\alpha_i & s\theta_i s\beta_i - c\theta_i s\alpha_i c\beta_i & a_i s\theta_i \\ -c\alpha_i s\beta_i & s\alpha_i & c\alpha_i c\beta_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{3} $$

Where $c\beta_i$ and $s\beta_i$ represent $\cos\beta_i$ and $\sin\beta_i$, respectively.

For the generic industrial robot with N degrees of freedom, the individual link transformation matrices can be computed by using the MDH model. Then, the Cartesian position and orientation of the coordinate frame N, which is attached to the end-effector, with respect to the robot base coordinate frame can be expressed as:

$$ {}^{0}_{N}T = A_1 \cdot A_2 \dots A_N \tag{4} $$

Error model of homogeneous transformation matrix

Because there are some systematic errors between the actual structure and theoretical model of the robot in the process of manufacturing and assembly, the tool centre point (TCP) of robot cannot reach command position and orientation with respect to the robot base coordinate frame. Thus, these systematic errors affect the position accuracy of the robot to a great extent, in which the errors of link geometric parameters are the main factor. In the MDH model, the errors of link geometric parameters can be expressed as $\Delta\alpha_i$, Δa_i, Δd_i, $\Delta\theta_i$ and $\Delta\beta_i$; thus, the actual transformation matrix of two adjacent links can be represented by:

$$ A_i^* = A_i + dA_i \tag{5} $$

Where dA_i represents the differential change.

Because $\Delta\alpha_i$, Δa_i, Δd_i, $\Delta\theta_i$ and $\Delta\beta_i$ are very small, dA_i can be expressed as:

$$ dA_i = \frac{\partial A_i}{\partial \alpha_i}\Delta\alpha_i + \frac{\partial A_i}{\partial a_i}\Delta a_i + \frac{\partial A_i}{\partial d_i}\Delta d_i + \frac{\partial A_i}{\partial \theta_i}\Delta\theta_i + \frac{\partial A_i}{\partial \beta_i}\Delta\beta_i $$

Where,

$$ \frac{\partial A_i}{\partial \lambda_i} = A_{\lambda i},\ \ \lambda = \{a, \alpha, d, \theta, \beta\} $$

Therefore,

$$ dA_i = A_{\alpha i}\,\Delta\alpha_i + A_{ai}\,\Delta a_i + A_{di}\,\Delta d_i + A_{\theta i}\,\Delta\theta_i + A_{\beta i}\,\Delta\beta_i \tag{6} $$

Then, the Cartesian position and orientation of the end-effector coordinate frame with respect to the robot base coordinate frame can be rewritten as:

$$ {}^{0}_{N}T^* = {}^{0}_{N}T + d{}^{0}_{N}T = A_1^* \cdot A_2^* \dots A_N^* \tag{7} $$

Where, $d{}^{0}_{N}T$ is the differential change of matrix ${}^{0}_{N}T$.

Assuming that the point P_N is the TCP of the robot, $P_N = (x_N, y_N, z_N)^T$ is the Cartesian coordinate values of point P_N with respect to the coordinate frame N. For convenience of calculations, P_N can be rewritten as:

$$ P_N = [x_N, y_N, z_N, 1]^T \tag{8} $$

By analysis, $dP_N = (\Delta x, \Delta y, \Delta z)^T$, which is the position error vector of the TCP in the robot base frame, can be denoted as equation (8). Because the values of link geometric errors ($\Delta\alpha_i$, Δa_i, Δd_i, $\Delta\theta_i$ and $\Delta\beta_i$) are very small, the higher-order terms are negligible.

$$ \begin{aligned} \begin{bmatrix} dP_N \\ 0 \end{bmatrix} &= ({}^{0}_{N}T^* - {}^{0}_{N}T)\cdot[x_N, y_N, z_N, 1]^T \\ &= \left(A_1^* \cdot A_2^* \dots A_N^* - {}^{0}_{N}T\right)P_N \\ &\approx \sum_{i=1}^{N} A_1 A_2 \dots dA_i \dots A_N P_N \\ &= \sum_{i=1}^{N} A_1 A_2 \dots (A_{\alpha i}\Delta\alpha_i + A_{ai}\Delta a_i + A_{di}\Delta d_i \\ &\quad + A_{\theta i}\Delta\theta_i + A_{\beta i}\Delta\beta_i) \dots A_N P_N \\ &= \sum_{i=1}^{N} (A_1 A_2 \dots A_{\alpha i} \dots A_N P_N \Delta\alpha_i \\ &\quad + A_1 A_2 \dots A_{ai} \dots A_N P_N \Delta a_i \\ &\quad + A_1 A_2 \dots A_{di} \dots A_N P_N \Delta d_i \\ &\quad + A_1 A_2 \dots A_{\theta i} \dots A_N P_N \Delta\theta_i \\ &\quad + A_1 A_2 \dots A_{\beta i} \dots A_N P_N \Delta\beta_i) \end{aligned} \tag{9} $$

Distance error calibration model

For two different positions of the TCP in three-dimensional space, although their coordinate values in the robot base frame and in the measurement device (e.g. laser tracker) frame are different, the distances between two positions with respect to the two frames are equal. However, as there are some systematic errors, the TCP of robot cannot reach the command positions; the distance, which is calculated according to the command coordinate values, between two positions in robot base frame is a theoretical value. Meanwhile, the accuracy of the external measurement device is higher than the required calibration accuracy, so the distance with respect to the measurement device frame is an actual value. Therefore, there is a distance error between the theoretical value and the actual value. Based on this distance error, we propose the distance error calibration model.

Description of distance error

According to ISO 9283 norm: Manipulating industrial robots – Performance criteria and related test methods, the distance accuracy is represented by the position and orientation deviation between the command distance (i.e. theoretical distance) and the average value of actual distance. In this paper, we use distance error to represent the position deviation between the command distance and the actual distance. As shown in Figure 1, the Euclidean distance between the positions j and k on the command path is calculated with the following formulation:

$$\begin{aligned} & l_R(j,k) \\ &= \sqrt{[x_R(k)-x_R(j)]^2+[y_R(k)-y_R(j)]^2+[z_R(k)-z_R(j)]^2}, \\ & \qquad j,k=1,2,\cdots,M. \qquad j\neq k. \end{aligned} \tag{10}$$

Where, $l_c(j', k')$ is the command distance between the positions j and k, M is the number of measured points, the suffix R indicates coordinate values with respect to robot base frame.

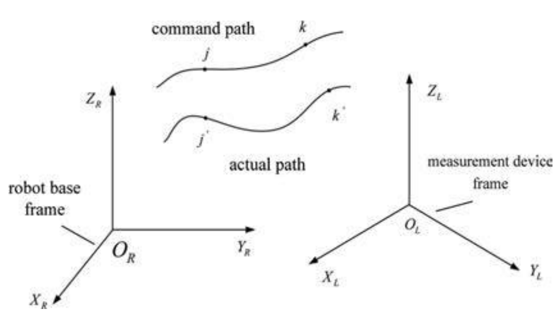
Similarly, the actual distance between the corresponding positions j' and k' on the actual path in robot base frame is expressed as the following formulation:

$$\begin{aligned} & l_C(j',k') \\ &= \sqrt{\begin{aligned}&[x_C(k')-x_C(j')]^2+[y_C(k')-y_C(j')]^2 \\ &+[z_C(k')-z_C(j')]^2\end{aligned}}, \\ & \qquad j',k'=1,2,\cdots,M. \qquad j'\neq k'. \end{aligned} \tag{11}$$

Where $l_c(j', k')$ is the actual distance between the corresponding positions j' and k' on the actual path, M is the number of measured points, the suffix C denotes coordinate values of measured points with respect to the robot base frame.

However, the exact values of x_C, y_C and z_C are very difficult to obtain due to the systematic errors, so $l_C(j', k')$ cannot be calculated directly. By analysis of the calculation, the value of $l_C(j', k')$ is equal to that of $l_L(j', k')$ which can be computed directly by substituting measured values into the following formulation:

Figure 1 Distance error between two frames



$$\begin{aligned} & l_L(j',k') \\ &= \sqrt{[x_L(k')-x_L(j')]^2+[y_L(k')-y_L(j')]^2+[z_L(k')-z_L(j')]^2} \end{aligned} \tag{12}$$

Where $l_L(j', k')$ is the actual distance between the corresponding positions j' and k', and the suffix L denotes coordinate values of measured points with respect to the measurement device frame.

Therefore, the distance error between the actual distance and the corresponding command distance is represented by:

$$\begin{aligned} \Delta l(j,k) &= l_C(j',k') - l_R(j,k) \\ &= l_L(j',k') - l_R(j,k) \end{aligned} \tag{13}$$

The relationship of accuracy and distance error

From equation (9), Δx_R, Δy_R and Δz_R, which can be expressed as equation (14), are used to denote the errors of position vector components between an arbitrary point k on command path and the corresponding point k' on the actual path.

$$\begin{aligned} \Delta x\,(k) &= x_C\,(k') - x_R\,(k); \\ \Delta y\,(k) &= y_C\,(k') - y_R\,(k); \\ \Delta z\,(k) &= z_C\,(k') - z_R\,(k). \\ \mathrm{d}P_k &= (\Delta x(k), \Delta y(k), \Delta z(k))^T \end{aligned} \tag{14}$$

Where, $\mathrm{d}P_k$ is a position error vector.

Then, equation (11) can be rewritten as:

$$\begin{aligned} (\Delta l(j,k)+l_R(j,k))^2 &= (x_R(k)-x_R(j)\Delta x(k)-\Delta x(j))^2 \\ &\quad +(y_R(k)-y_R(j)+\Delta y(k)-\Delta y(j))^2 \\ &\quad +(z_R(k)-z_R(j)+\Delta z(k)-\Delta z(j))^2 \end{aligned} \tag{15}$$

We assume that the errors in equation (15) are very small, so the higher-order terms are negligible. Equation (15) can be given by the following vector notation:

$$\begin{aligned} \Delta l(j,k) &= \left[\frac{x_R(k)-x_R(j)}{l_R(j,k)}, \frac{y_R(k)-y_R(j)}{l_R(j,k)}, \frac{z_R(k)-z_R(j)}{l_R(j,k)}\right] \\ &\quad \times \begin{bmatrix} \Delta x(k)-\Delta x(j) \\ \Delta y(k)-\Delta y(j) \\ \Delta z(k)-\Delta z(j) \end{bmatrix} \\ &= \left[\frac{x_R(k)-x_R(j)}{l_R(j,k)}, \frac{y_R(k)-y_R(j)}{l_R(j,k)}, \frac{y_R(k)-y_R(j)}{l_R(j,k)}\right] \\ &\quad \times [\mathrm{d}P_k - \mathrm{d}P_j] \end{aligned} \tag{16}$$

Where, $\mathrm{d}P_j$ and $\mathrm{d}P_k$ are position error vectors corresponding with point j and point k, respectively.

From equations (8) and (9), the position error vector of an arbitrary point k can be expressed as the following compact form:

$$\mathrm{d}P_k = B_k \cdot \Delta q_k \tag{17}$$

Where, B_k represents a $3 \times 5N$ (N is the number of the robot's degrees of freedom) coefficient matrix given by equation (18), and Δq_k, which can be represented by equation (19), is a $5N \times 1$ error vector of the link geometric parameters in the MDH model.

$$B_k = \begin{bmatrix} x_{\alpha 1} & x_{a1} & x_{d1} & x_{\theta 1} & x_{\beta 1} & x_{\alpha 2} & \cdots\cdots & x_{\beta N} \\ y_{\alpha 1} & y_{a1} & y_{d1} & y_{\theta 1} & y_{\beta 1} & y_{\alpha 2} & \cdots\cdots & y_{\beta N} \\ z_{\alpha 1} & z_{a1} & z_{d1} & z_{\theta 1} & z_{\beta 1} & z_{\alpha 2} & \cdots\cdots & z_{\beta N} \end{bmatrix} \quad (18)$$

$$\Delta q_k = \begin{bmatrix} \Delta\alpha_1 & \Delta a_1 & \Delta d_1 & \Delta\theta_1 & \Delta\beta_1 & \Delta\alpha_2 & \cdots & \Delta\beta_N \end{bmatrix}^T \quad (19)$$

Where $x_{\lambda m}$, $y_{\lambda m}$ and $z_{\lambda m}$ represent the corresponding coefficients of $\Delta\lambda_m$ ($\lambda = \{a, \alpha, d, \theta, \beta\}$, $m = 1, 2 [\ldots] N$).

From equations (9) and (18), we can obtain the following expression:

$$\begin{bmatrix} x_{\lambda m} \\ y_{\lambda m} \\ z_{\lambda m} \end{bmatrix} = \begin{bmatrix} (A_1 A_2 \cdots A_{\lambda m} \cdots A_6 P_N)_{\langle 1,1 \rangle} \\ (A_1 A_2 \cdots A_{\lambda m} \cdots A_6 P_N)_{\langle 2,1 \rangle} \\ (A_1 A_2 \cdots A_{\lambda m} \cdots A_6 P_N)_{\langle 3,1 \rangle} \end{bmatrix} \quad (20)$$

Assuming that the robot errors contain only the inherent systematic errors of link geometric parameters, or other errors (such as vibration and temperature) are very small. Then, we can conclude that:

$$\Delta q = \Delta q_j = \Delta q_k$$

$$dP_k - dP_j = (B_k - B_j)\Delta q$$

Therefore,

$$\Delta l(j,k) = \left[\frac{x_R(k) - x_R(j)}{l_R(j,k)}, \frac{y_R(k) - y_R(j)}{l_R(j,k)}, \frac{y_R(k) - y_R(j)}{l_R(j,k)} \right] \times (B_k - B_j)\Delta q \quad (21)$$

Equation (21) is the relationship of position errors and distance errors. As shown in equation (21), the robot position errors and distance errors are one-to-one relationship. Thus, the robot distance errors can be used to calculate the corresponding position errors. Based on equation (21), we propose the error compensation algorithm to improve the robot position accuracy and distance accuracy.

From the above equations, $l_R(j,k)$, $x_R(k) - x_R(j)$, $y_R(k) - y_R(j)$, $z_R(k) - z_R(j)$ and $\Delta l(j,k)$ can be computed by applying equations (10) and (13), respectively; B_j and B_k, which can be calculated from equations (18) and (20), are coefficient matrices related to the kinematic parameters of corresponding position j and k on the command path. Δq is an unknown variable, which can be calculated from equation (23).

Experiment results and analyses

Experiment set-up

In this paper, we use Leica AT901 B laser tracker to calibrate the 6-DOF industrial robot which is developed by Soochow University robotics laboratory (Figure 2). The measurement range of the laser tracker is 80 m, and the accuracy of absolute distance meter (ADM) is less than 10 μm, so it is sufficient to meet the accuracy requirements in our tests.

Figure 2 Experimental configuration

The industrial robot we used in this manuscript is as shown in Figure 2, the payload of the robot is 10 kg, and the accuracy of the robot is ± 0.01 mm. The robot is driven by the motor integrated with encoders.

The process of robot calibration is as follows:

- *Data acquisition*: the command positions of the sampling points with respect to the robot base frame and the corresponding angles of robot joints are displayed in external industrial computer, and the actual positions with respect to the measurement device frame are captured directly by the measurement device(in this paper, we use laser tracker to measure).
- *Parameter identification*: from equation (21),Δq can be figured out by applying measured data.
- *Error compensation*: the kinematic parameters of the robot are corrected according to the parameter errors Δq.
- *Experimental tests*: compare the position errors of the data measured by laser tracker before and after error compensation, respectively, and check the effect of this compensation algorithm.

As shown in Figure 3, the link coordinate system of the industrial robot is established based on the DH model. Then, the corresponding nominal kinematic parameters of the robot are as given in Table I. Thus, the homogeneous transformation matrix of the TCP with respect to the robot base coordinate frame can be expressed as:

$${}^0_6T = A_1 \cdot A_2 \ldots A_6 P_6 \quad (22)$$

Data acquisition and pre-process

For most of the 6-DOF industrial robots (including the industrial robot in our tests), the axes of joint 2 and joint 3 are parallel and other adjacent joints are not, so according to MDH model and DH model, only the parameter β_2 is added. For the joints in general, the DH model will be used; the corresponding geometric parameters are: α_i, a_i, d_i, θ_i. Then, Δq can be rewritten as:

$$\Delta q = [\Delta\alpha_1, \Delta a_1, \Delta d_1, \Delta\theta_1, \Delta\alpha_2, \cdots, \Delta\theta_6, \Delta\beta_2]^T$$

Where the number of unknown parameters in Δq is 25 ($4 \times 6 + 1$). As a rule, to improve the accuracy of calculation, the number of measured points should be far greater than 25.

In the test process, the spherical reflector (SMR) is attached to the flange plate of the end-effector first. As shown in Figure

Figure 3 Robot model

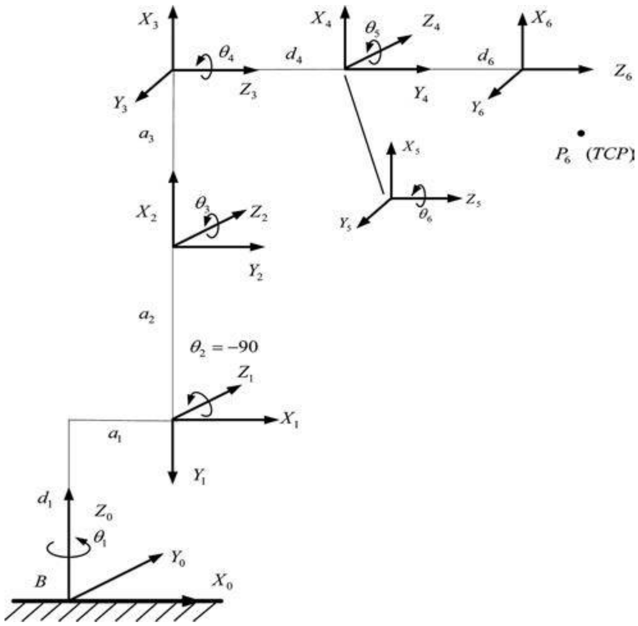


Table I Nominal kinematic parameters

Joint	a_i (deg)	a_i (mm)	d_i (deg)	θ_i (deg)	γ_i (deg)
1	−90	170	504	0	0
2	0	780	0	−90	0
3	−90	140	0	0	0
4	90	0	760	0	0
5	−90	0	0	0	0
6	0	0	125	0	0

2, because the centre of SMR is not coincident with the centre of end-effector, the centre of SMR can be regarded as TCPP_6; thus we should determine the coordinate values of SMR, and the Cartesian coordinate values of point P_6 with respect to the coordinate frame 6 can be measured by laser tracker with CPA. By analysis, we know that the Cartesian coordinate errors (i.e. Δx_6, Δy_6 and Δz_6) of P_6 with respect to the coordinate frame 6 is mainly affected by the errors of robot kinematic parameters, and that the accuracy of laser tracker system is very high, soΔx_6, Δy_6 and Δz_6 are not taken into account in our kinematic calibration model. In this experiment, the coordinate value of point P_6 is $(x_6, y_6, z_6)^T = (-1.473, \ 26.384, \ 71.991)^T$.

Then, we start to collect the experimental data. The robot moves to one specific position first, it pauses for 2s, in the same time, the laser tracker system records the Cartesian coordinate values of the centre of SMR (i.e. TCP) with respect to laser tracker frame. After the position is recorded, the robot moves to the next position until 56 positions of robot are measured.

Finally, the measured data are substituted into equation (21). Then, we obtain a set of equations, and these equations can form a system of linear equations. To calculate conveniently, the system of linear equations is rewritten as matrix form:

$$b = A \cdot \Delta q \tag{23}$$

Where, A is coefficient matrix, Δq is an unknown error vector and b is column vector consisting of $\Delta l\,(j, k)$.

According to the calculated results, the condition number of matrix A is very great, and the matrix A is rank-deficient, so Gaussian elimination method is invalid for solving the equation (22) (Shupan, 2009). To solve the least-square solution, we first use singular value decomposition method to figure out the Moore–Penrose generalized inverse matrix A^+ of matrix A. Then, the generalized inverse matrixA^+ is substituted into equation (22), and Δq can be computed as follows:

$$\Delta q = A^+ \cdot b$$

Experimental results and analyses

The result of Δq is shown as Table II. The robot nominal kinematic parameters are corrected based on theΔq. Then, the data of 56 points and corrected kinematic parameters are substituted into equations (13) and (17) to figure out the distance accuracy and absolute position accuracy of the robot calibrated.

Figure 4 and Table III are the distance accuracy before and after calibration. As shown in Table III, the average distance error after calibration is only 13 per cent of that before calibration.

Figure 5 and Table IV are the absolute position accuracy before and after calibration. As shown in Table IV, the average position error after calibration is very small compare to that before calibration.

Conclusions

In this paper, we use the MDH model with five kinematic parameters to establish the kinematic model of a 6-DOF

Table II Errors of kinematic parameters

Joint	Δa_i (deg)	Δa_i (mm)	Δd_i (deg)	$\Delta\theta_i$ (deg)	$\Delta\gamma_i$ (deg)
1	−0.0516	−3.885	0	0	0
2	−0.0459	1.2834	1.3308	−0.671	0.155
3	0.086	1.1404	1.3308	0.0975	0
4	0.0057	−0.1408	−0.3557	−1.04	0
5	0.143	1.6238	−1.4628	1.77	0
6	−0.0115	0.0199	−0.9708	−0.0287	0

Figure 4 Distance errors before and after calibration

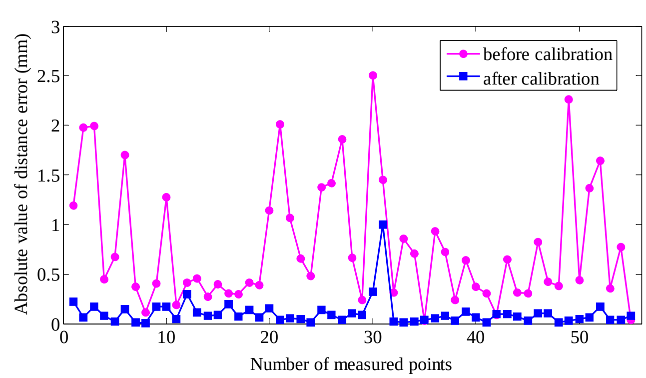


Table III Comparison of distance errors

Distance errors	Max absolute values (mm)	Average absolute values (mm)
Before calibration	2.5018	0.78355
After calibration	0.99	0.10323

Figure 5 X, Y and Z components of position errors

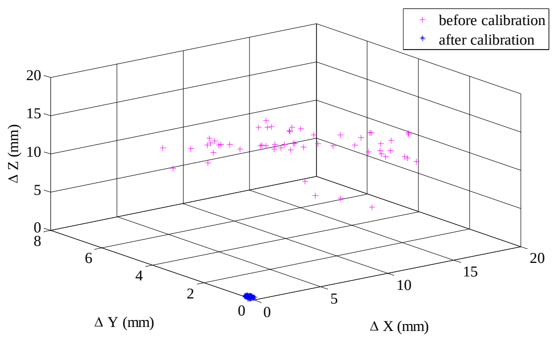


Table IV Comparison of position errors

Position error	Before calibration ΔX	ΔY	ΔZ	After calibration ΔX	ΔY	ΔZ
Max absolute values (mm)	15.534	6.325	18.551	0.193	0.395	0.277
Average absolute values (mm)	9.667	3.229	12.975	0.095	0.270	0.160

industrial robot. The MDH model overcomes the shortcoming of the DH model, which will result in singularities when two adjacent joints are parallel or nearly parallel. At the same time, combining with the definition of distance accuracy, a new distance error model of the robot is proposed. This model can avoid the coordinate transformation error between robot base frame and measurement device frame, which makes robot calibration more convenient and effective. In the experiment, with the help of a high-precision laser tracker system, we measure the TCP Cartesian coordinate values. Based on these measured data, the errors of robot kinematic parameters are figured out, and then we corrected the nominal kinematic parameters according to the errors. The experimental results show that the robot calibration model based on MDH and distance error can improve the distance accuracy and absolute position accuracy dramatically.

References

Andreff, N., Renaud, P., Martinet, P. and Pierrot, F. (2004), “Vision-based kinematic calibration of an H4 parallel mechanism: practical accuracies”, *Industrial Robot: an International Journal*, Vol. 31 No. 3, pp. 273-283.

Gatla, C.S., Lumia, R., Wood, J. and Starr, G. (2007), “An automated method to calibrate industrial robots using a virtual closed kinematic chain”, *IEEE Transactions On Robotics*, Vol. 23 No. 6, pp. 1105-1116.

Ha, I.C. (2008), “Kinematic parameter calibration method for industrial robot manipulator using the relative position”, *Journal of Mechanical Science and Technology*, Vol. 22 No. 6, pp. 1084-1090.

Hayati, S., Tso, K. and Roston, G. (1988), “Robot geometry calibration”, *Proceedings of IEEE International Conference on Robotics and Automation*, *Philadelphia, PA*.

In Won, P., Bum Joo, L., Se Hyoung, C., Young Dae, H. and Jong Hwan, K. (2012), “Laser-based kinematic calibration of robot manipulator using differential kinematics”, *IEEE/ASME Transactions on Mechatronics*, Vol. 17 No. 6, pp. 1059-1067.

Craig, J. (2005), *Introduction to Robotics: Mechanics and Control*, Pearson Education.

Denavit, R.S.H. (1955), “A kinematic notation for lower-pair mechanisms based on matrices”, *Transaction of the ASME: Journal of Applied Mechanics*, Vol. 22, pp. 215-221.

Karan, B. and Vukobratovic, M. (1994), “Calibration and accuracy of manipulation robot models - an overview”, *Mechanism and Machine Theory*, Vol. 29 No. 3, pp. 479-500.

Kuu-Young, Y., Jin-Jou, C. and Chia-Ching, W. (1996), “An automated robot calibration system based on a variable D-H parameter model,” *Proceedings of the 35th IEEE Conference on Decision and Control*, *Kobe*.

Motta, J., De Carvalho, G.C. and Mcmaster, R.S. (2001), “Robot calibration using a 3D vision-based measurement system with a single camera”, *Robotics and Computer-Integrated Manufacturing*, Vol. 17 No. 6, pp. 487-497.

Newman, W.S., Birkhimer, C.E., Horning, R.J. and Wilkey, A.T. (2000), “Calibration of a motoman P8 robot based on laser tracking”, *Proceedings ICRA’00 IEEE International Conference on Robotics and Automation*, *San Francisco, CA*.

Nubiola, A. and Bonev, I.A. (2013), “Absolute calibration of an ABB IRB 1600 robot using a laser tracker”, *Robotics and Computer-Integrated Manufacturing*, Vol. 29 No. 1, pp. 236-245.

Ren, Y., Zhu, J., Yang, X. and Ye, S. (2008), “Measurement robot calibration model and algorithm based on distance accuracy”, *Acta Metrologica Sinica*, Vol. 29 No. 3, pp. 198-202.

Roning, J. and Kormn, A. (1997), “A method for industrial robot calibration”, *1997 IEEE International Conference on Robotics and Automation*, *Albuquerque, NM*.

Roth, Z.S., Mooring, B.W. and Ravani, B. (1987), “An overview of robot calibration”, *IEEE Journal of Robotics and Automation*, Vol. 3 No. 5, pp. 377-385.

Ruibo, H., Yingjun, Z., Shunian, Y. and Shuzi, Y. (2010), “Kinematic-parameter identification for serial-robot calibration based on POE formula”, *IEEE Transactions on Robotics*, Vol. 26 No. 3, pp. 411-423.

Santolaria, J., Conte, J. and Gin, S.M. (2012), “Laser tracker-based kinematic parameter calibration of industrial robots by improved CPA method and active retroreflector”, *The International Journal of Advanced Manufacturing Technology*, Vol. 66 No. 9-12, pp. 2087-2106.

Shupan, W. (2009), “Condition numbers for Moore-Penrose inverse of matrix and linear least squares problem”, *M.S. thesis*, Lanzhou University.

Slamani, M., Nubiola, A. and Bonev, I. (2012), "Assessment of the positioning performance of an industrial robot", *Industrial Robot: An International Journal*, Vol. 39 No. 1, pp. 57-68.

Sun, L., Liu, J., Sun, W., Wu, S. and Huang, X. (2004), "Geometry-based robot calibration method", *IEEE International Conference on Robotics and Automation Proceedings ICRA'04*, *New Orleans, LA*.

Veitschegger, W. and Wu, C.H. (1986), "Robot accuracy analysis based on kinematics", *IEEE Journal of Robotics and Automation*, Vol. 2 No. 3, pp. 171-179.

Veitschegger, W.K. and Wu, C.H. (1988), "Robot calibration and compensation", *IEEE Journal of Robotics and Automation*, Vol. 4 No. 6, pp. 643-656.

Wang, H., Ma, Y., Cheng, S., Li, Y., Lu, X. and Su, X. (2013), "Accurate robot calibration by SAI method through visual measurement", *IEEE International Conference on Robotics and Biomimetics (ROBIO)*, *Shenzhen*.

Xuecai, Z., Qixian, Z., Gruver, W.A. and Gongliang, G. (1994), "Distance and positioning accuracy for robotic manipulators", *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'94)*, IEEE, New York, NY.

Young, K. and Pickin, C.G. (2000), "Accuracy assessment of the modern industrial robot", *Industrial Robot: An International Journal*, Vol. 27 No. 6, pp. 427-436.

Corresponding author

Chen Guodong can be contacted at: guodongxyz@gmail.com