

Fakultet elektrotehnike i računarstva
Zavod za elektroniku, mikroelektroniku,
računalne i inteligentne sustave

Elektronika 2

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4. Frekvencijski odziv pojačala

Frekvencijski odziv

Pod utjecajem reaktancija \rightarrow prijenosne funkcije pojačala su kompleksni brojevi

Frekvencijski odziv – odziv pojačala na sinusne signale različitih frekvencija

Uz linearni rad pojačala

$$u_{ul} = U_{ulm} \sin \omega t \qquad u_{iz} = U_{izm} \sin(\omega t + \phi)$$

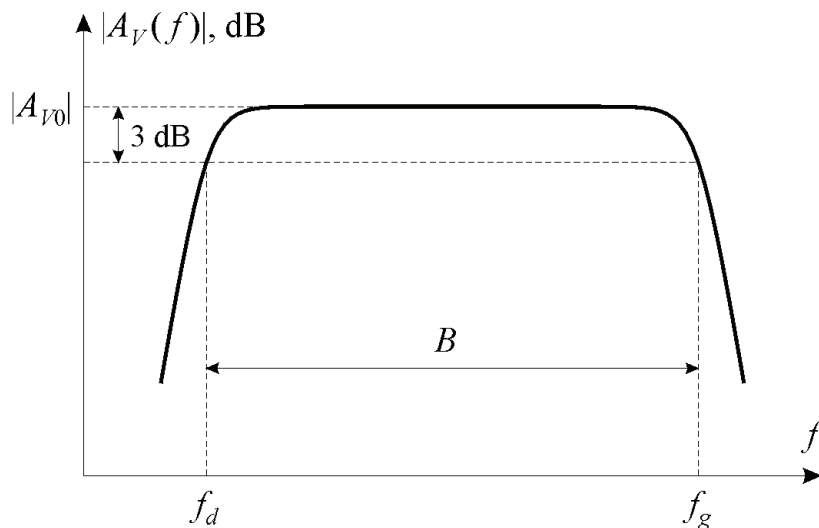
za svaku frekvenciju f određuje se amplituda i faza

$$|A_V(f)| = \frac{U_{izm}}{U_{ulm}} \quad \angle A_V(f) = \phi$$

naponsko pojačanje \rightarrow kompleksni broj $A_V(f) = |A_V(f)| \exp(j\phi)$

Grafički prikaz frekvencijskog odziva \rightarrow **frekvencijska karakteristika**

Amplitudna frekvencijska karakteristika izmjeničnog pojačala



granične frekvencije – pad pojačanja na

$$A_{V0} / \sqrt{2} \quad \text{ili za 3 dB}$$

$f_d \rightarrow$ donja granična frekvencija

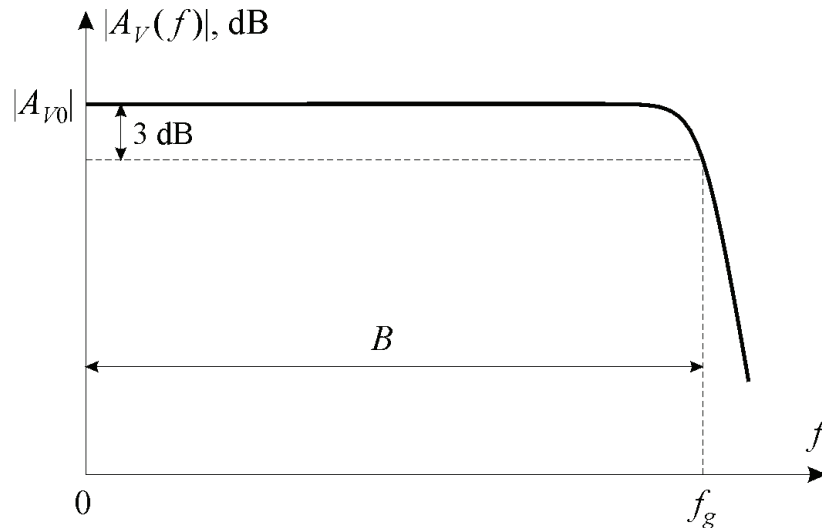
$f_g \rightarrow$ gornja granična frekvencija

$f_d < f < f_g \rightarrow$ područje srednjih frekvencija

pojačanje se smanjuje u području niskih frekvencija i u području visokih frekvencija

$B = f_g - f_d \rightarrow$ širina frekvencijskog pojasa

Amplitudna frekvencijska karakteristika istosmjernog pojačala



nema donje granične frekvencije – $f_d \rightarrow 0$

Polovi i nule prijenosne funkcije

Opći oblik prijenosne funkcije

$$A(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_0}$$

$a_i, b_i \rightarrow$ realni brojevi, $m \leq n$

Određivanjem korijena

$$A(s) = \frac{a_0 (1 + s / \omega_{z1}) (1 + s / \omega_{z2}) \dots (1 + s / \omega_{zm})}{b_0 (1 + s / \omega_{p1}) (1 + s / \omega_{p2}) \dots (1 + s / \omega_{pn})}$$

korijeni polinoma u brojniku $s = -\omega_{zi} \rightarrow$ **nule prijenosne funkcije**

korijeni polinoma u nazivniku $s = -\omega_{pi} \rightarrow$ **polovi prijenosne funkcije**

broj polova $n \rightarrow$ **red prijenosne funkcije**

Bodeov dijagram

Bodeov dijagram - dva grafa: amplitudna i fazna karakteristika u ovisnosti o frekvenciji

Crta se:

- ☐ amplituda - u decibelima,
- ☐ faza - u stupnjevima,
- ☐ frekvencija - u logaritamskom mjerilu.

Prijenosna funkcija \rightarrow produkti i kvocijenti osnovnih članova

Bodeov dijagram:

- ☐ grafičko zbrajanje osnovnih članova
- ☐ frekvencijske karakteristike realnih pojačala dobro se aproksimiraju izlomljenim pravcima

Osnovni član – konstanta

Dio prijenosne neovisan o frekvenciji

$$A(j\omega) = \pm A_0$$

Amplituda

$$|A| = 20 \log A_0, \text{ dB}$$

Faza

$$\phi = 0^\circ \quad \text{za} \quad A(j\omega) = + A_0$$

$$\phi = -180^\circ \quad \text{za} \quad A(j\omega) = - A_0$$

Osnovni član – nula na frekvenciji 0

$$A(j\omega) = j\omega / \omega_1$$

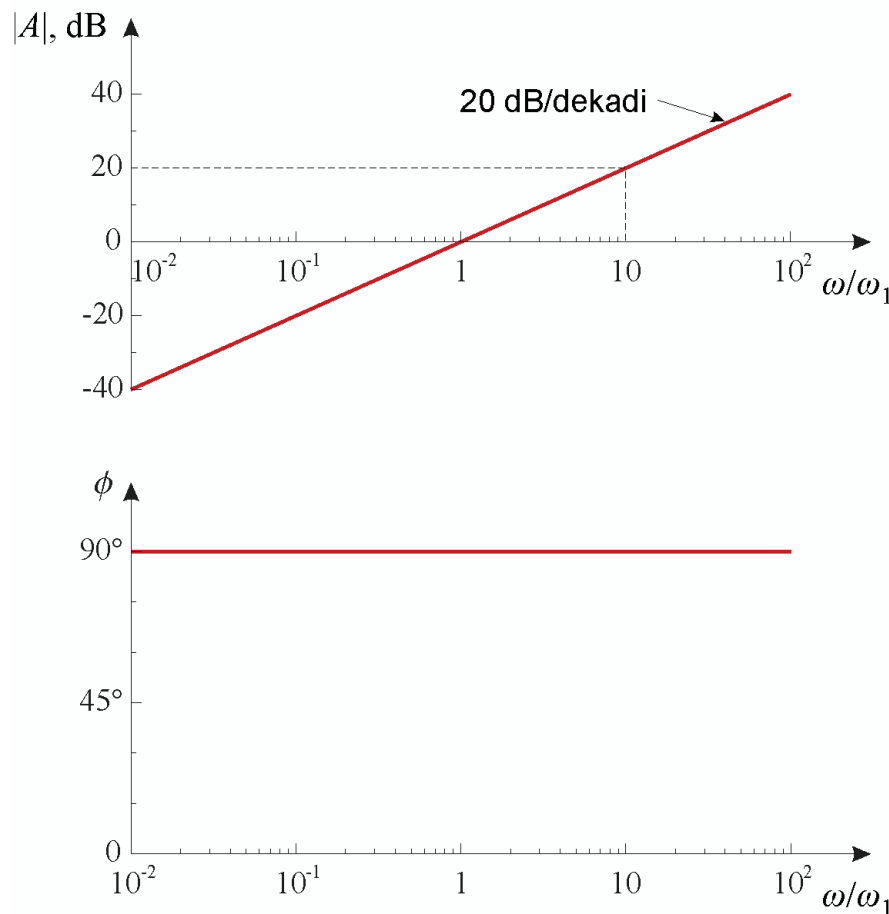
Amplituda

$$|A(j\omega)| = 20 \log(\omega / \omega_1), \text{ dB}$$

porast $\rightarrow +20 \text{ dB/dekadi}$

Faza

$$\phi = \arctg\left(\frac{\omega / \omega_1}{0}\right) = \arctg(\infty) = 90^\circ$$



Osnovni član – pol na frekvenciji 0

$$A(j\omega) = \frac{1}{j\omega / \omega_1}$$

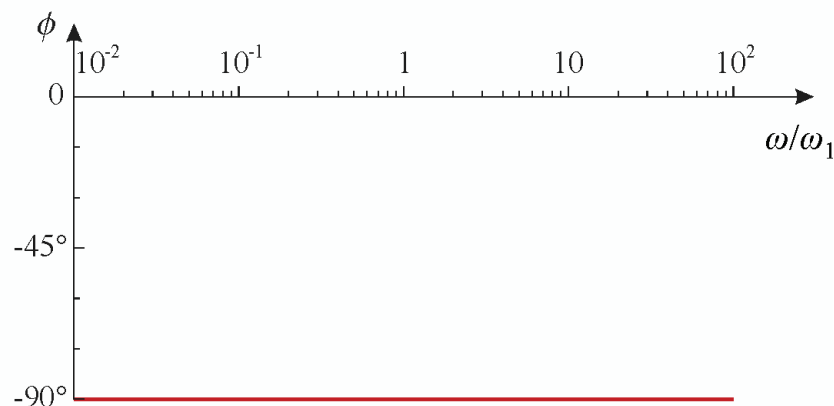
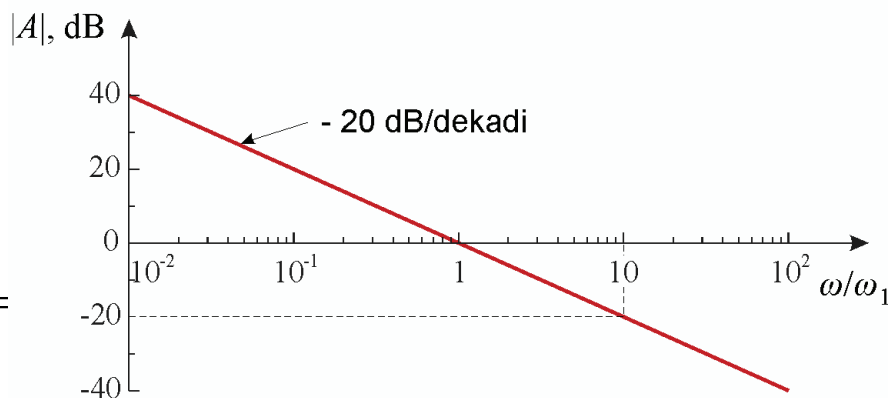
Amplituda

$$\begin{aligned}|A(j\omega)| &= 20\log(1) - 20\log(\omega / \omega_1) = \\ &= -20\log(\omega / \omega_1), \text{ dB}\end{aligned}$$

pad $\rightarrow -20 \text{ dB/dekadi}$

Faza

$$\begin{aligned}\phi &= 0^\circ - \arctg\left(\frac{\omega / \omega_1}{0}\right) = \\ &= -\arctg(\infty) = -90^\circ\end{aligned}$$



Osnovni član – nula na frekvenciji ω_1 (1)

$$A(j\omega) = 1 + j\omega / \omega_1$$

Amplituda

$$|A(j\omega)| = \sqrt{1 + (\omega / \omega_1)^2}$$

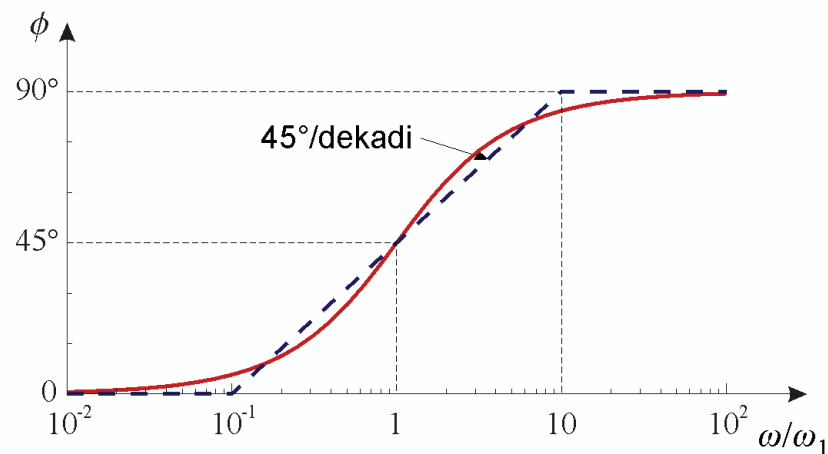
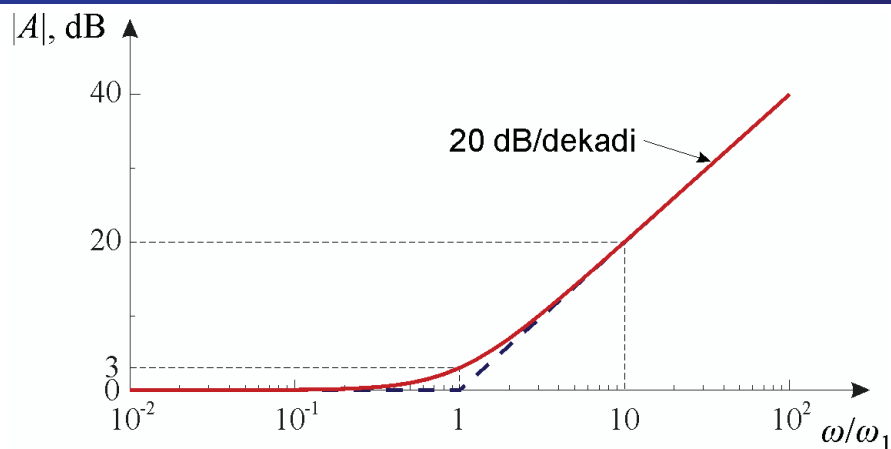
u decibelima

$$|A(j\omega)| = 20 \log \sqrt{1 + (\omega / \omega_1)^2}, \text{ dB}$$

za $\omega \ll \omega_1$ $|A| \approx 20 \log(1) = 0, \text{ dB}$

za $\omega \gg \omega_1$ $|A| \approx 20 \log(\omega / \omega_1), \text{ dB}$

za $\omega = \omega_1$ $|A| = 20 \log \sqrt{2} = 3 \text{ dB}$



Osnovni član – nula na frekvenciji ω_1 (2)

$$A(j\omega) = 1 + j\omega / \omega_1$$

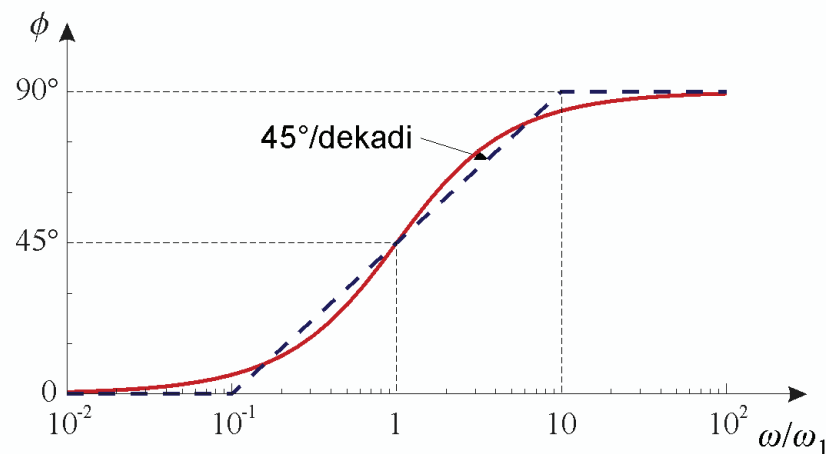
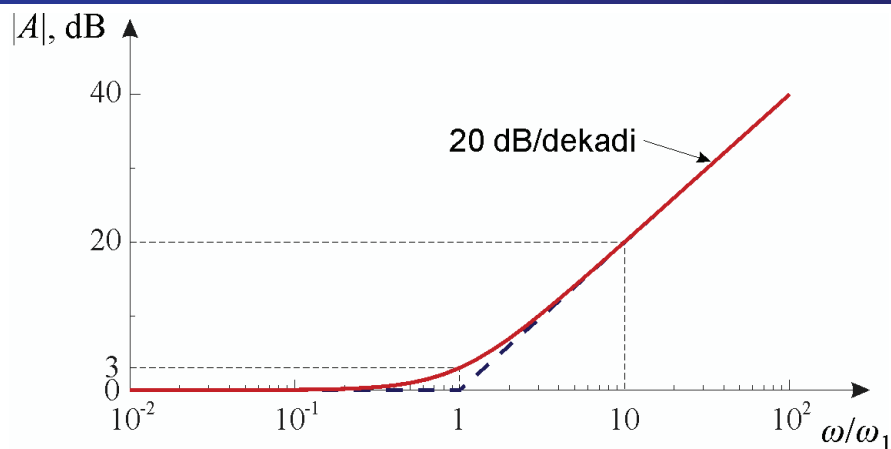
Faza

$$\phi = \arctan(\omega / \omega_1)$$

$$\text{za } \omega \ll \omega_1 \quad \phi \approx 0^\circ$$

$$\text{za } \omega \gg \omega_1 \quad \phi \approx 90^\circ$$

$$\text{za } \omega = \omega_1 \quad \phi = 45^\circ$$



Osnovni član – pol na frekvenciji ω_1 (1)

$$A(j\omega) = \frac{1}{1 + j\omega/\omega_1}$$

Amplituda

$$|A(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_1)^2}}$$

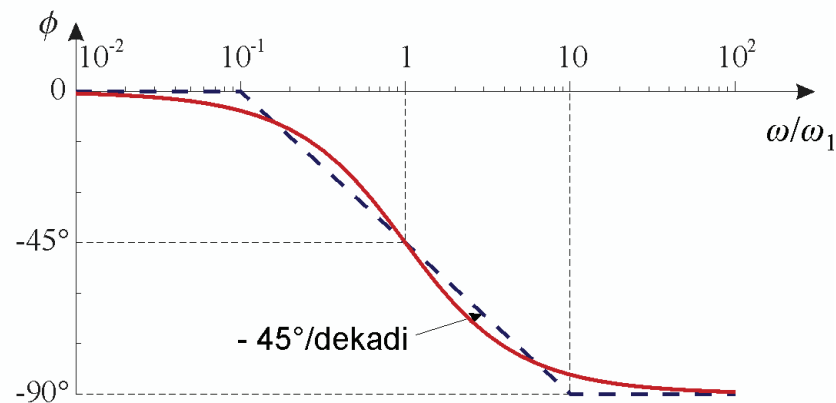
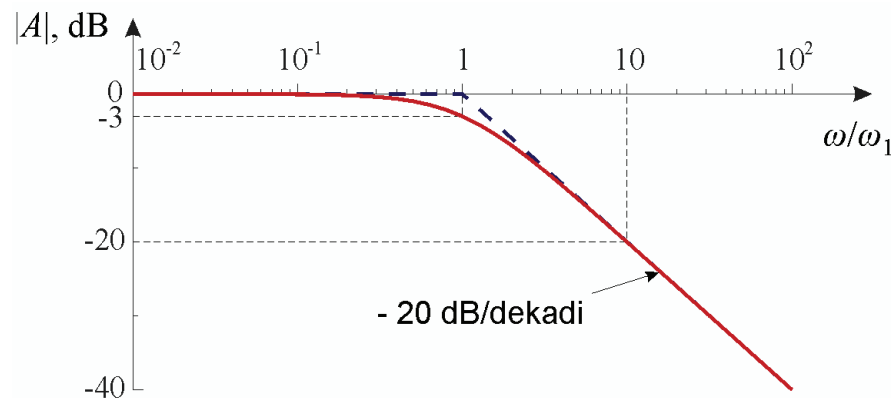
u decibelima

$$|A(j\omega)| = -20 \log \sqrt{1 + (\omega/\omega_1)^2}, \text{ dB}$$

$$\text{za } \omega \ll \omega_1 \quad |A| \approx -20 \log(1) = 0, \text{ dB}$$

$$\text{za } \omega \gg \omega_1 \quad |A| \approx -20 \log(\omega/\omega_1), \text{ dB}$$

$$\text{za } \omega = \omega_1 \quad |A| = -20 \log \sqrt{2} = -3 \text{ dB}$$



Osnovni član – pol na frekvenciji ω_1 (2)

$$A(j\omega) = \frac{1}{1 + j\omega/\omega_1}$$

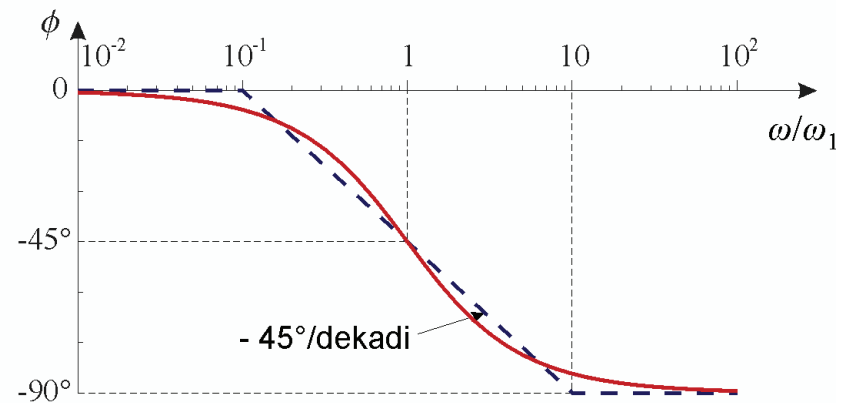
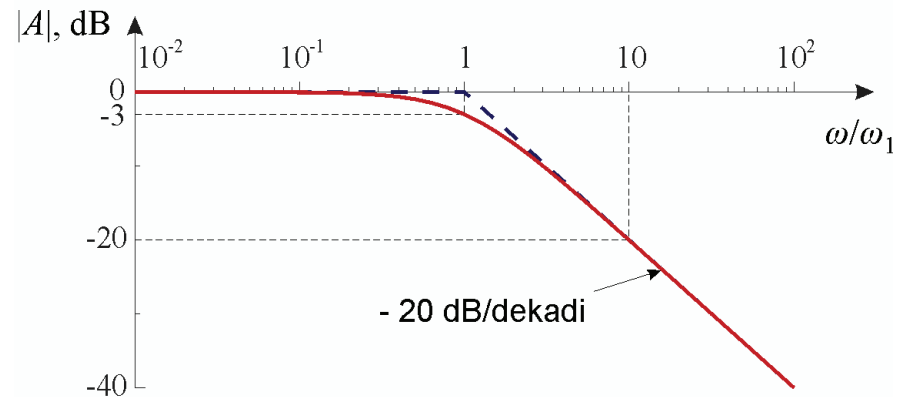
Faza

$$\phi = -\arctg(\omega/\omega_1)$$

za $\omega \ll \omega_1$ $\phi \approx 0^\circ$

za $\omega \gg \omega_1$ $\phi \approx -90^\circ$

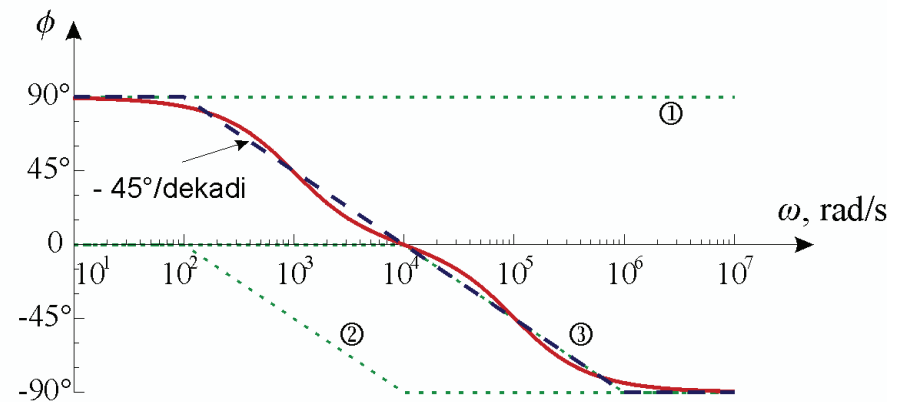
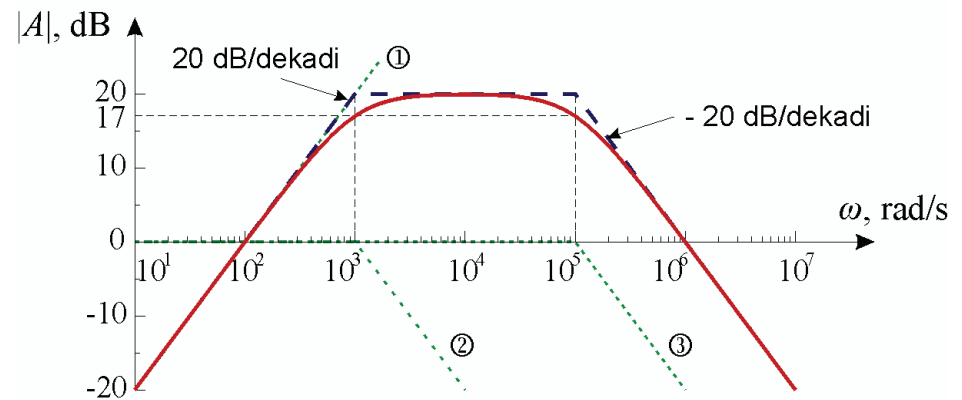
za $\omega = \omega_1$ $\phi = -45^\circ$



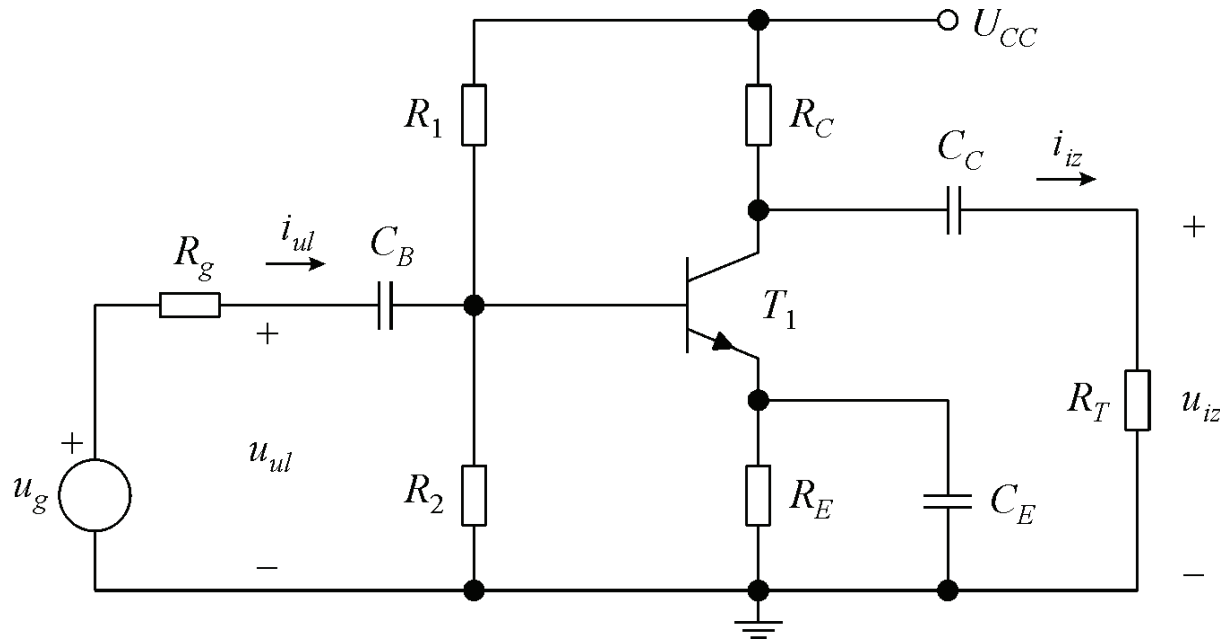
Primjer 4.1

Nacrtati Bodeov dijagram
prijenosne funkcije

$$A(j\omega) = 10^6 \frac{j\omega}{(10^3 + j\omega)(10^5 + j\omega)}$$



Spoj zajedničkog emitera na niskim frekvencijama



Uključenje kapaciteta u dinamičku analizu

Dinamička analiza pojačala → za područja srednjih i niskih frekvencija

Impedancije kondenzatora $Z_C = 1/(j\omega C)$

Za svaki od kondenzatora treba ustanoviti na kojim frekvencijama smanjuje pojačanje

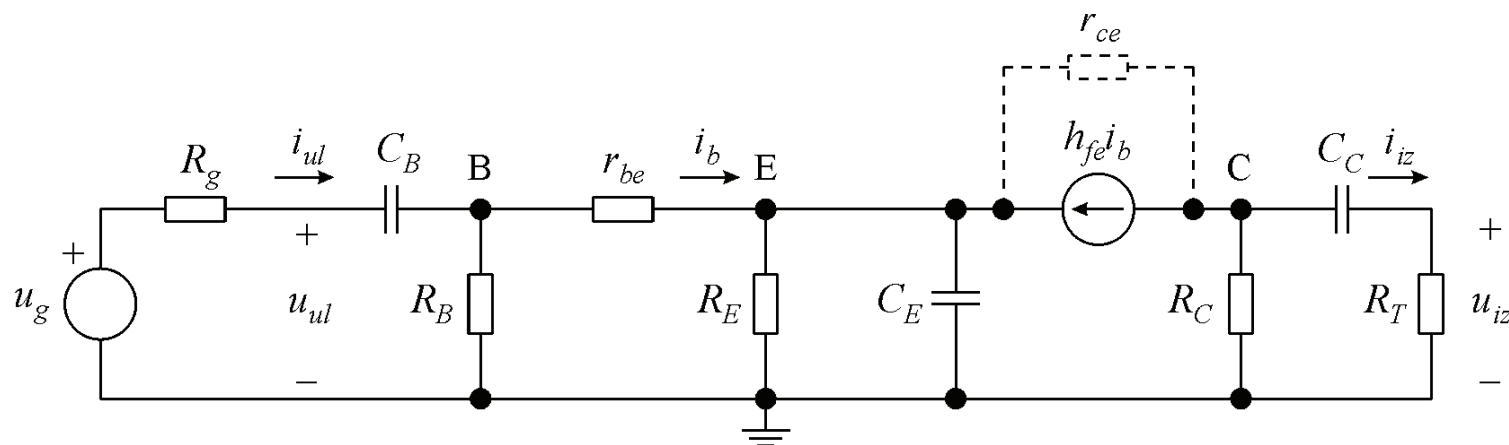
□ C_B i C_C → vezni kondenzatori

- smanjenjem frekvencije predstavljaju sve veći otpor prolazu signala
- djeluju na niskim frekvencijama

□ C_E → premosni kondenzator

- na višim frekvencijama kratko spaja emiter na masu → pojačalo radi bez degeneracije
- na niskim frekvencijama uključuje degeneraciju
- djeluje na niskim frekvencijama

Nadomjesna shema za niskofrekvencijsku analizu

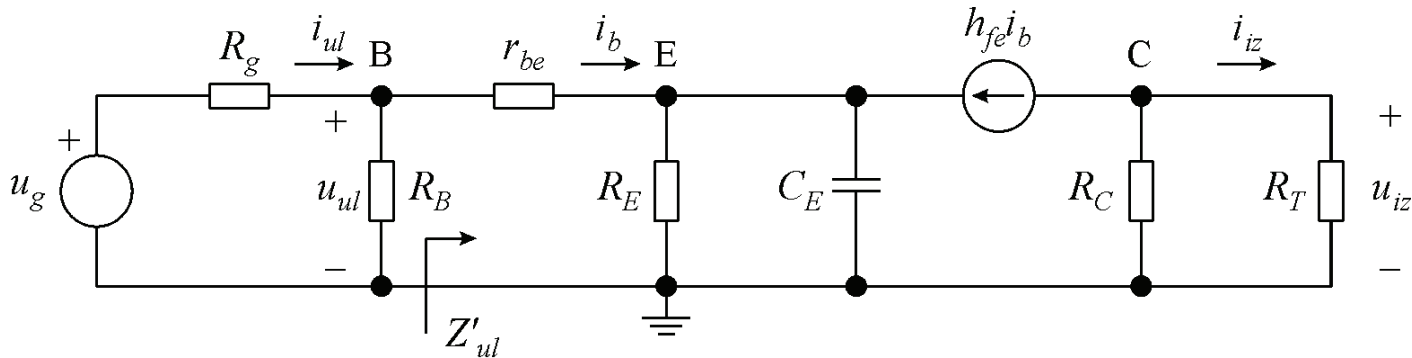


Istovremena analiza kompletne sheme → previše složena

Zbog tri kondenzatora → prijenosna funkcije pojačanja A_V ili A_I funkcija s tri pola → kompleksna jednačba trećeg reda u nazivniku

Analiza pojačanja provodi se u dva koraka → posebno se određuje utjecaj C_E , a posebno utjecaj C_B i C_C

Utjecaj kondenzatora C_E (1)



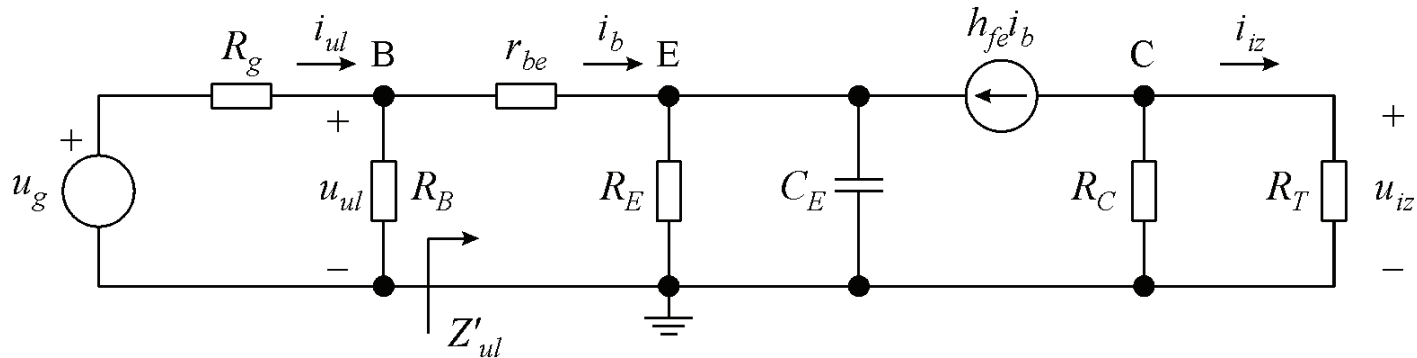
$$A_V = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{I_b} \frac{I_b}{U_{ul}}$$

$$\frac{U_{iz}}{I_b} = -h_{fe} (R_C \parallel R_T)$$

$$\frac{I_b}{U_{ul}} = \frac{1}{r_{be} + (1 + h_{fe}) Z_E}$$

$$Z_E = R_E \parallel \frac{1}{j\omega C_E} = \frac{R_E (1/j\omega C_E)}{R_E + (1/j\omega C_E)} = \frac{R_E}{1 + j\omega R_E C_E}$$

Utjecaj kondenzatora C_E (2)



$$\begin{aligned} \frac{I_b}{U_{ul}} &= \frac{1}{r_{be} + (1 + h_{fe}) \frac{R_E}{1 + j\omega R_E C_E}} = \frac{1 + j\omega R_E C_E}{r_{be} + (1 + h_{fe}) R_E + j\omega r_{be} R_E C_E} = \\ &= \frac{1}{r_{be} + (1 + h_{fe}) R_E} \frac{1 + j\omega R_E C_E}{1 + j\omega \left(\frac{r_{be}}{1 + h_{fe}} \parallel R_E \right) C_E} \end{aligned}$$

Naponsko pojačanje

$$A_V(j\omega) = \frac{U_{iz}}{U_{ul}} = A_{Vn} \frac{1 + j\omega / \omega_1}{1 + j\omega / \omega_2}$$

$$A_{Vn} = -h_{fe} \frac{R_C \parallel R_T}{r_{be} + (1 + h_{fe}) R_E}$$

Vremenske konstante

$$\tau_1 = \frac{1}{\omega_1} = R_E C_E$$

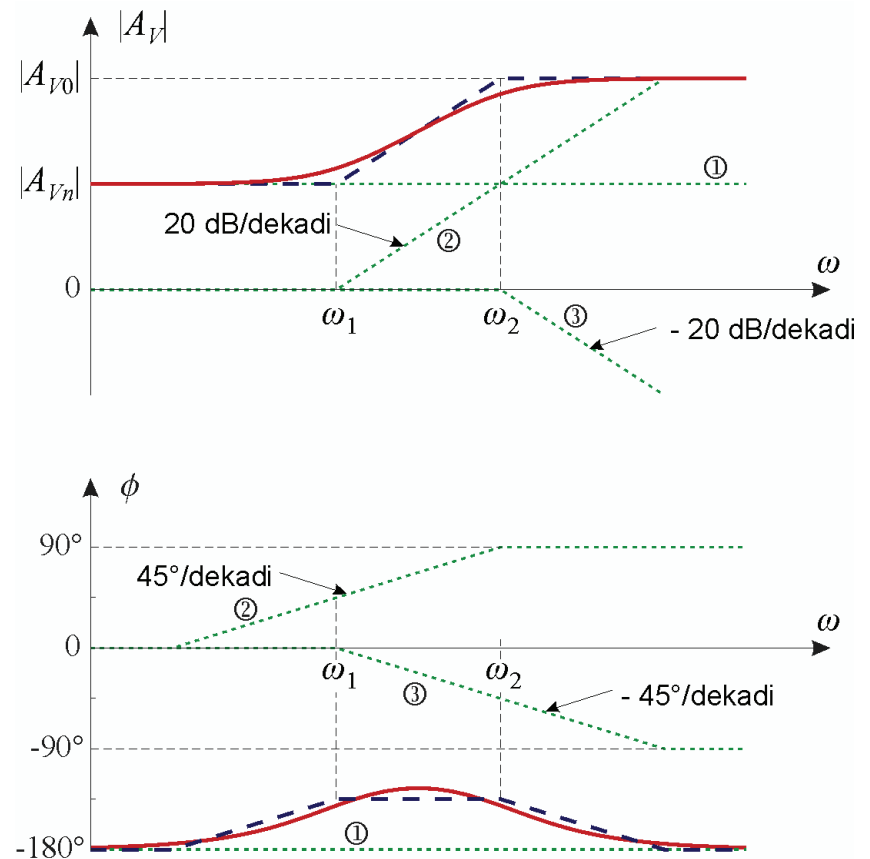
$$\tau_2 = \tau_E = \frac{1}{\omega_2} = \left(\frac{r_{be}}{1 + h_{fe}} \parallel R_E \right) C_E$$

Pojačanje na srednjim frekvencijama

$$A_{V0} = \lim_{\omega \rightarrow \infty} A_V(j\omega) = A_{Vn} \frac{\omega_2}{\omega_1} = A_{Vn} \frac{\tau_1}{\tau_2} = -h_{fe} \frac{R_C \parallel R_T}{r_{be}}$$

Bodeov dijagram

$$A_V(j\omega) = A_{Vn} \cdot (1 + j\omega/\omega_1) \cdot \frac{1}{1 + j\omega/\omega_2}$$



Donja granična frekvencija

Definicija za $\omega = \omega_d \rightarrow |A_V| = |A_{V0}|/\sqrt{2}$

Za pojačanje $A_V(j\omega) = A_{Vn} \frac{1 + j\omega/\omega_1}{1 + j\omega/\omega_2}$

$$|A_V| = |A_{Vn}| \frac{\sqrt{1 + (\omega_d/\omega_1)^2}}{\sqrt{1 + (\omega_d/\omega_2)^2}} = \frac{|A_{V0}|}{\sqrt{2}} = \frac{|A_{Vn}|}{\sqrt{2}} \frac{\omega_2}{\omega_1} \rightarrow \omega_d = \sqrt{\omega_2^2 - 2\omega_1^2}$$

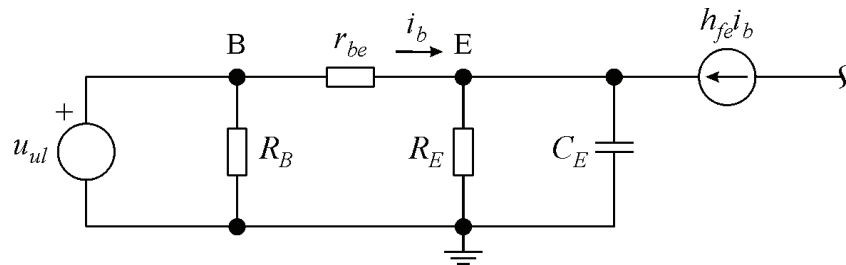
Za $\omega_2 \gg \omega_1 \rightarrow \omega_d = \omega_2$

Uvjet za postojanje ω_d $\omega_2 > \sqrt{2}\omega_1 \rightarrow R_E > (\sqrt{2} - 1) \frac{r_{be}}{1 + h_{fe}}$

Vremenska konstanta τ_E

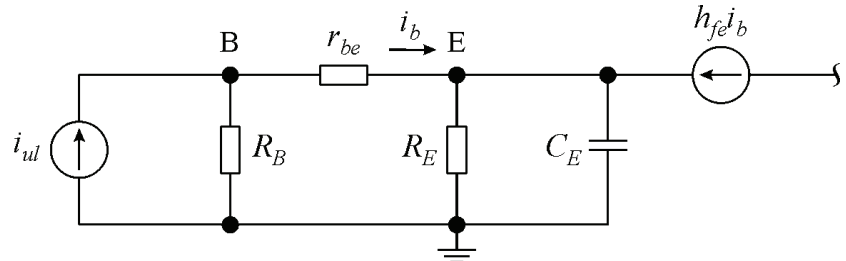
za naponsko pojačanje

$$\tau_E = \frac{1}{\omega_2} = \left(\frac{r_{be}}{1 + h_{fe}} \parallel R_E \right) C_E$$



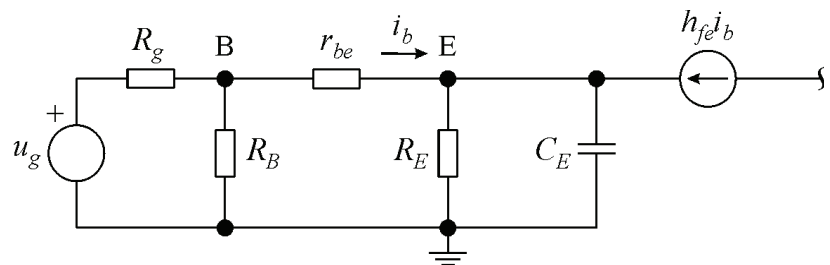
za strujno pojačanje

$$\tau_E = \frac{1}{\omega_2} = \left(\frac{r_{be} + R_B}{1 + h_{fe}} \parallel R_E \right) C_E$$

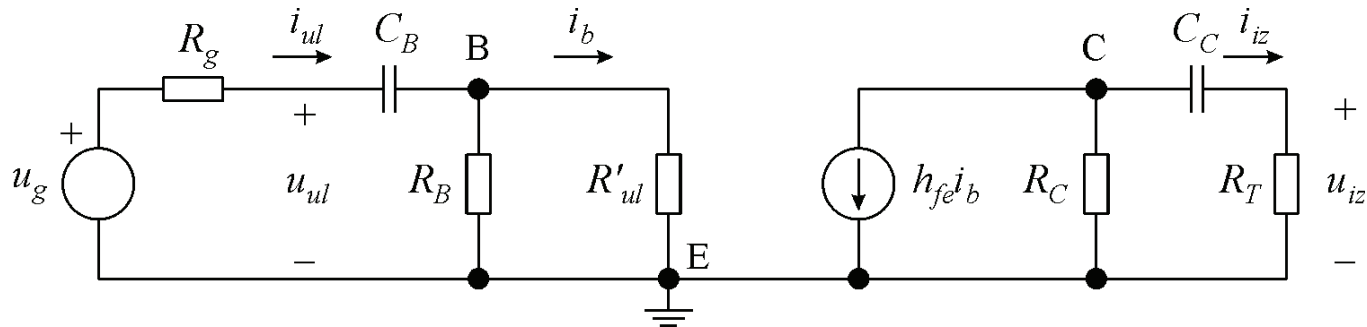


za konačni R_g

$$\tau_E = \frac{1}{\omega_2} = \left(\frac{r_{be} + R_g \parallel R_B}{1 + h_{fe}} \parallel R_E \right) C_E$$



Utjecaj kondenzatora C_B i C_C (1)

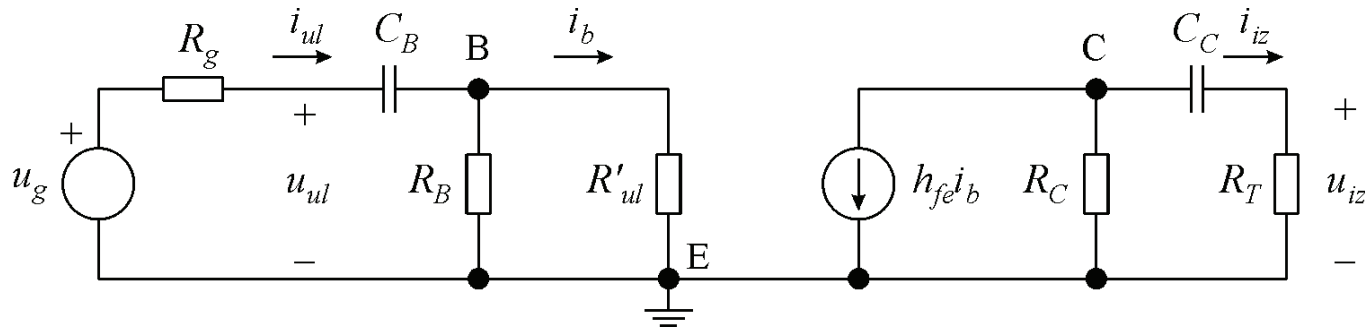


$$\text{za } \omega \gg \omega_2 \rightarrow R'_{ul} = \frac{U_{ul}}{I_b} = r_{be}$$

$$\text{za } \omega \ll \omega_1 \rightarrow R'_{ul} = \frac{U_{ul}}{I_b} = r_{be} + (1 + h_{fe})R_E$$

$$A_V = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{I_b} \frac{I_b}{U_{ul}}$$

Utjecaj kondenzatora C_B i C_C (2)



$$\frac{U_{iz}}{I_b} = \frac{U_{iz}}{I_{iz}} \frac{I_{iz}}{I_b} = R_T (-h_{fe}) \frac{R_C}{R_C + \frac{1}{j\omega C_C} + R_T} = -h_{fe} (R_C \parallel R_T) \frac{j\omega (R_C + R_T) C_C}{1 + j\omega (R_C + R_T) C_C}$$

$$\frac{I_b}{U_{ul}} = \frac{I_b}{I_{ul}} \frac{I_{ul}}{U_{ul}} = \frac{R_B}{R_B + R'_{ul}} \frac{1}{\frac{1}{j\omega C_B} + R_B \parallel R'_{ul}} = \frac{1}{R'_{ul}} \frac{j\omega (R_B \parallel R'_{ul}) C_B}{1 + j\omega (R_B \parallel R'_{ul}) C_B}$$

Naponsko pojačanje

$$A_V(j\omega) = \frac{U_{iz}}{U_{ul}} = A_{V1} \frac{j\omega / \omega_3}{1 + j\omega / \omega_3} \frac{j\omega / \omega_4}{1 + j\omega / \omega_4}$$

Pojačanje $A_{V1} = -h_{fe} \frac{R_C \parallel R_T}{R'_{ul}}$

za $\omega \gg \omega_2 \rightarrow A_{V1} = A_{V0} = -h_{fe} \frac{R_C \parallel R_T}{r_{be}}$

za $\omega \ll \omega_1 \rightarrow A_{V1} = A_{Vn} = -h_{fe} \frac{R_C \parallel R_T}{r_{be} + (1 + h_{fe})R_E}$

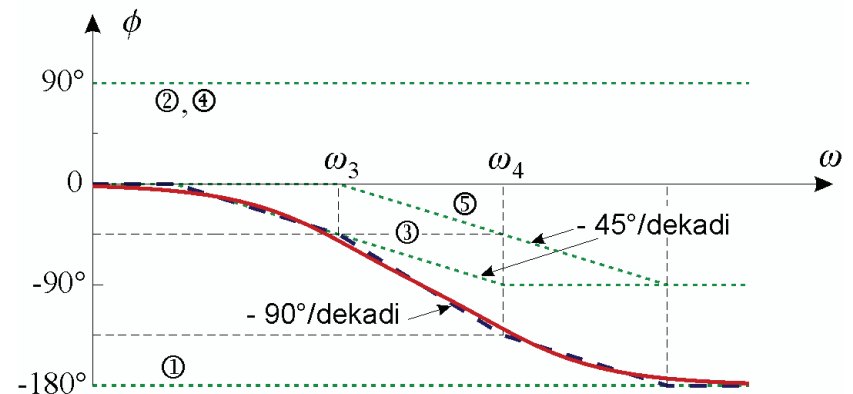
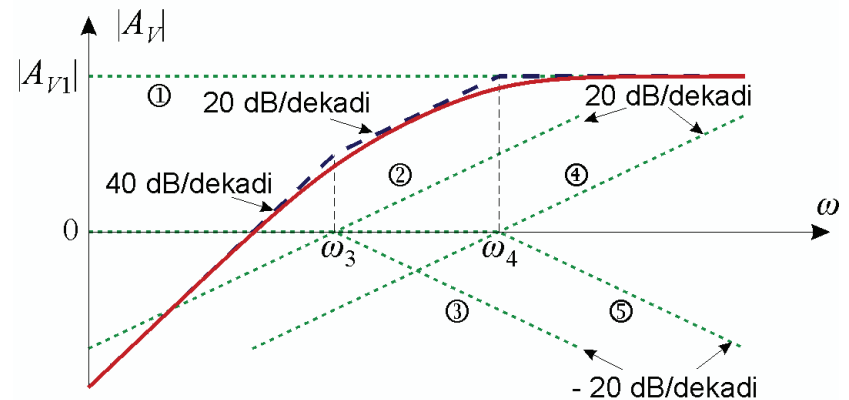
Vremenske konstante

$$\tau_3 = \tau_C = \frac{1}{\omega_3} = (R_C + R_T)C_C$$

$$\tau_4 = \tau_B = \frac{1}{\omega_4} = (R_B \parallel R'_{ul})C_B$$

Bodeov dijagram

$$A_V(j\omega) = A_{V1} \cdot \frac{j\omega}{\omega_3} \cdot \frac{1}{1 + j\omega/\omega_3} \cdot \frac{j\omega}{\omega_4} \cdot \frac{1}{1 + j\omega/\omega_4}$$



Donja granična frekvencija

Prijenosna funkcija $A_V(j\omega) = \frac{U_{iz}}{U_{ul}} = A_{V1} \frac{j\omega / \omega_3}{1 + j\omega / \omega_3} \frac{j\omega / \omega_4}{1 + j\omega / \omega_4}$

Uz $A_{V1} = A_{V0}$

$$|A_V| = |A_{V0}| \frac{\omega_d / \omega_3}{\sqrt{1 + (\omega_d / \omega_3)^2}} \frac{\omega_d / \omega_4}{\sqrt{1 + (\omega_d / \omega_4)^2}} = \frac{|A_{V0}|}{\sqrt{2}}$$

$$\omega_d^2 = \frac{\omega_3^2 + \omega_4^2}{2} + \sqrt{\left(\frac{\omega_3^2 + \omega_4^2}{2}\right)^2 + \omega_3^2 \omega_4^2}$$

Za $\omega_4 \gg \omega_3 \rightarrow \omega_d = \omega_4 \rightarrow \omega_4$ je **dominantni pol**

Za $\omega_4 = \omega_3 \rightarrow \omega_d = \sqrt{1 + \sqrt{2}} \omega_4 = 1,55 \omega_4$

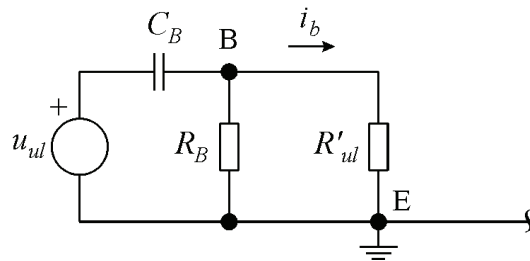
ω_d je viša od frekvencija ω_3 ili ω_4 ako je barem 4 puta viša od druge frekvencije \rightarrow za $\omega_4 = 4 \omega_3 \rightarrow \omega_d = 1,06 \omega_4 \approx \omega_4$

Vremenske konstante τ_B i τ_C

$\tau_C = (R_C + R_T) C_C \rightarrow$ određuje se iz izlaznog kruga

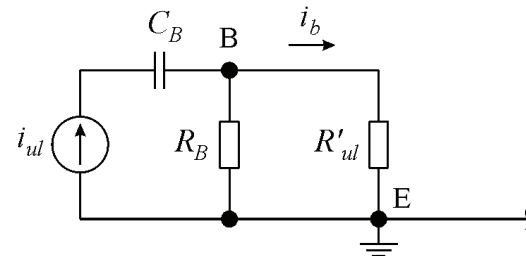
$\tau_B \rightarrow$ ovisi o pojačanju

za naponsko pojačanje



$$\tau_B = (R_B \parallel R'_{ul}) C_B$$

za strujno pojačanje



$$\tau_B = (R_B \parallel R'_{ul} + \infty) C_B \rightarrow \infty$$

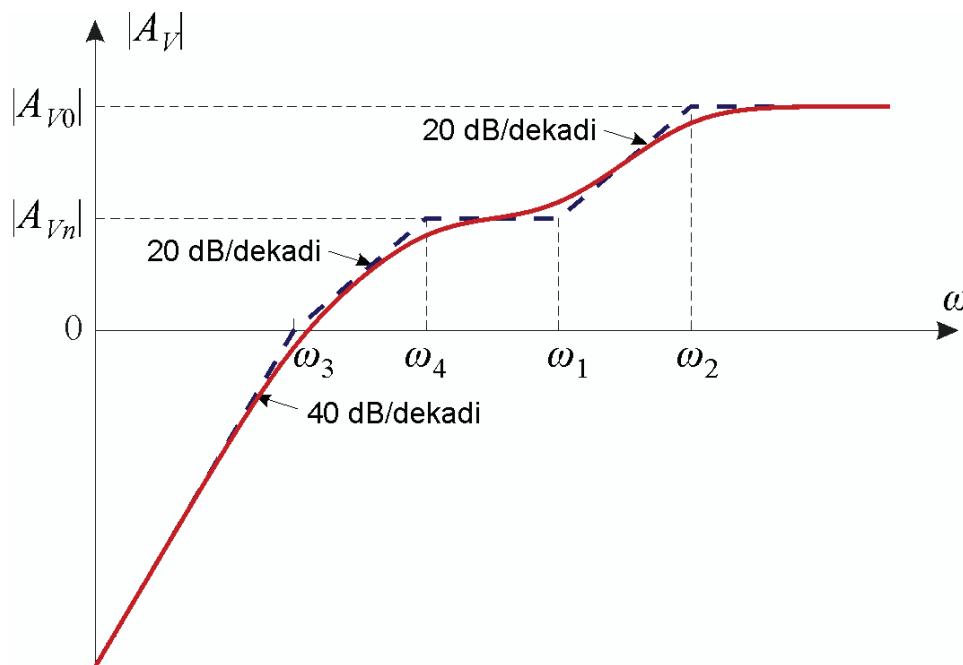
za konačni R_g

$$\tau_B = (R_B \parallel R'_{ul} + R_g) C_B$$

Ukupno djelovanje kondenzatora C_E , C_B i C_C (1)

za ω_1 i $\omega_2 > \omega_3$ i ω_4

$$A_V(j\omega) = \frac{U_{iz}}{U_{ul}} = A_{Vn} \frac{1 + j\omega/\omega_1}{1 + j\omega/\omega_2} \frac{j\omega/\omega_3}{1 + j\omega/\omega_3} \frac{j\omega/\omega_4}{1 + j\omega/\omega_4}$$



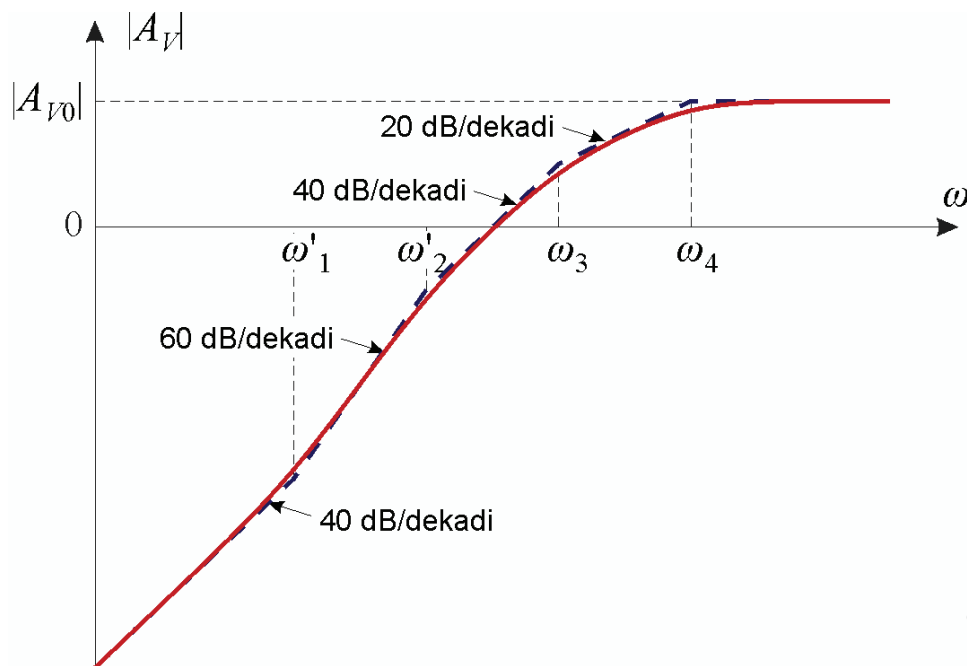
$$R'_{ul} = \frac{U_{ul}}{I_b} = r_{be} + (1 + h_{fe})R_E$$

$$A_{V1} = A_{Vn}$$

$$\omega_d = \sqrt{\omega_2^2 - 2\omega_1^2}$$

Ukupno djelovanje kondenzatora C_E , C_B i C_C (2)

za ω_3 i $\omega_4 > \omega_1$ i ω_2



$$R'_{ul} = \frac{U_{ul}}{I_b} = r_{be}$$

$$A_{V1} = A_{V0}$$

$$\omega_d^2 = \frac{\omega_3^2 + \omega_4^2}{2} + \sqrt{\left(\frac{\omega_3^2 + \omega_4^2}{2}\right)^2 + \omega_3^2 \omega_4^2}$$

Određivanje donje granične frekvencije

Na srednjim frekvencijama (svi se kondenzatori kratko spajaju)

$$A_{V0} = -h_{fe} \frac{R_C \parallel R_T}{r_{be}}$$

Vremenske konstante koje određuju pojedini kondenzatori (ostali se kondenzatori kratko spajaju)

$$\tau_B = (R_B \parallel r_{be}) C_B \quad \tau_E = \left(\frac{r_{be}}{1 + h_{fe}} \parallel R_E \right) C_E \quad \tau_C = (R_C + R_T) C_C$$

Donja granična frekvencija

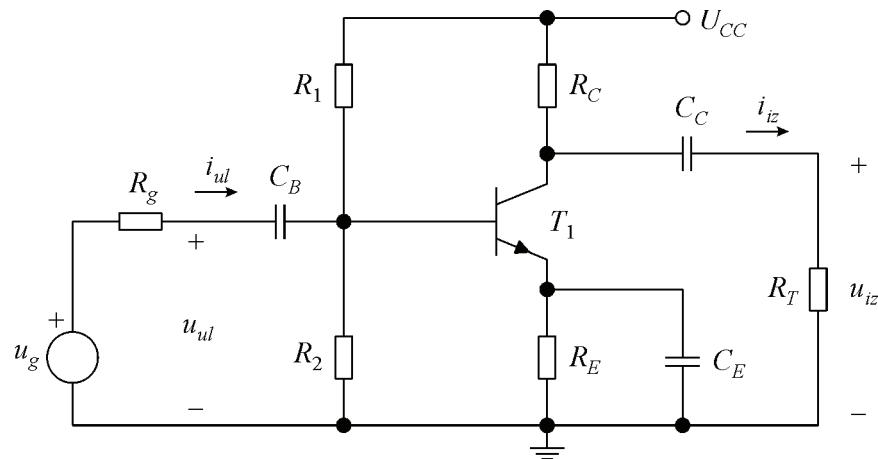
$$\omega_d = \max \{ \omega_B, \omega_E, \omega_C \} = \max \left\{ \frac{1}{\tau_B}, \frac{1}{\tau_E}, \frac{1}{\tau_C} \right\}$$

Uvjet → najviša frekvencija barem 4 puta viša od ostalih → **dominantni pol**

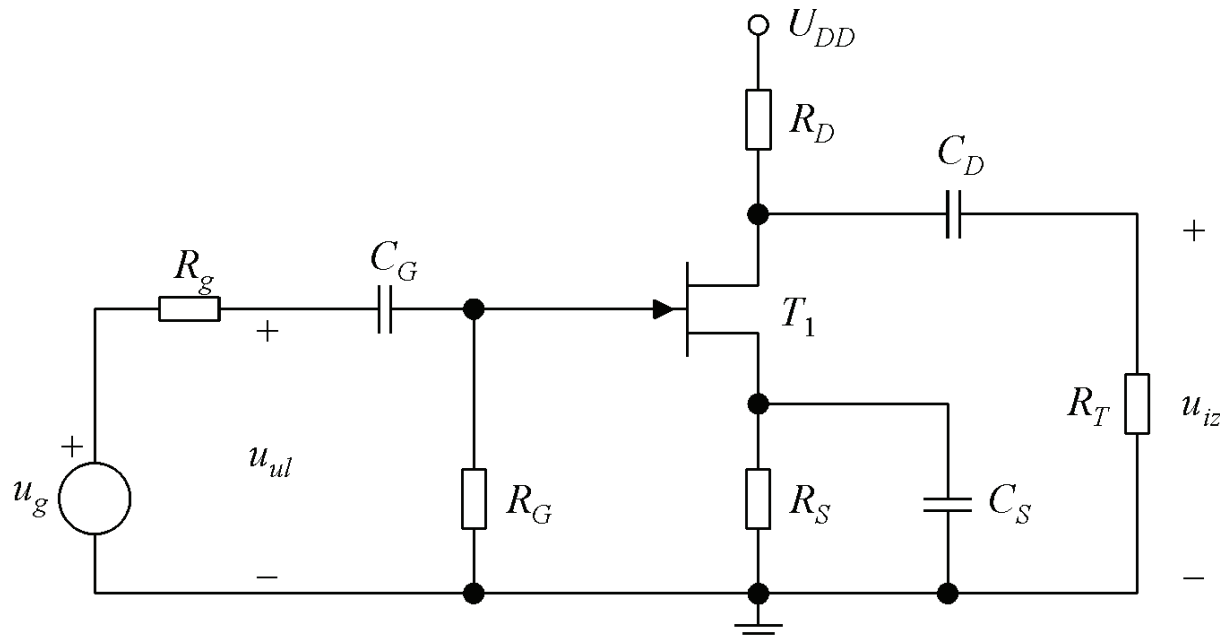
Ako ne → procjena $\omega_d \approx \frac{1}{\tau_B} + \frac{1}{\tau_E} + \frac{1}{\tau_C}$

Primjer 4.2

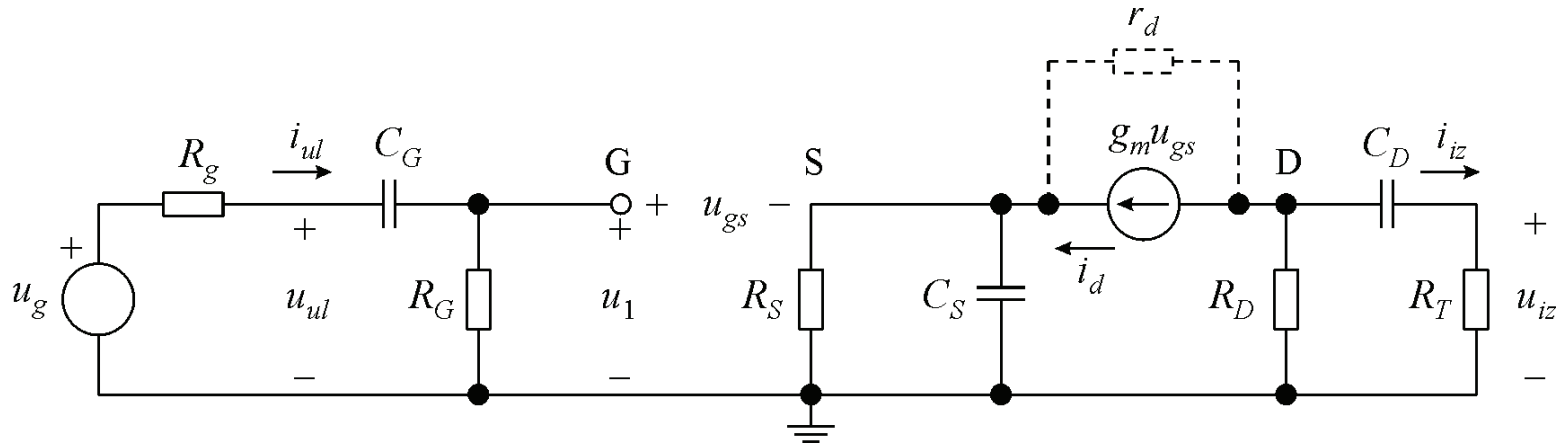
U pojačalu sa slike zadano je: $U_{CC} = 15 \text{ V}$, $R_g = 500 \Omega$, $C_B = 10 \mu\text{F}$, $R_1 = 30 \text{ k}\Omega$, $R_2 = 11 \text{ k}\Omega$, $R_C = 2 \text{ k}\Omega$, $R_E = 1 \text{ k}\Omega$, $C_E = 100 \mu\text{F}$, $C_C = 3 \mu\text{F}$ i $R_T = 1,2 \text{ k}\Omega$. Parametri bipolarnog tranzistora su $\beta \approx h_{fe} = 100$ i $U_\gamma = 0,7 \text{ V}$. Zanemariti porast struje kolektora s naponom u_{CE} u normalnom aktivnom području. Naponski ekvivalent temperature $U_T = 25 \text{ mV}$. Odrediti naponsko pojačanje $A_V = U_{iz}/U_{ul}$ na srednjim frekvencijama, te donju graničnu frekvenciju tog pojačanja.



Spoj zajedničkog uvoda na niskim frekvencijama



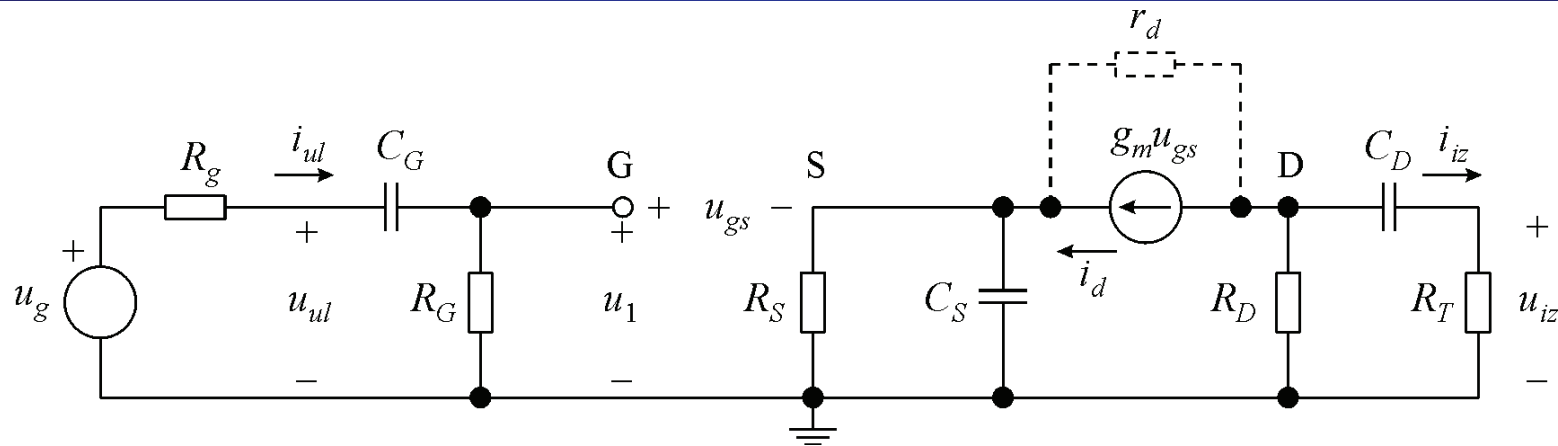
Niskofrekvencijska analiza (1)



$$A_V = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{I_d} \frac{I_d}{U_1} \frac{U_1}{U_{ul}}$$

$$\frac{U_{iz}}{I_d} = \frac{U_{iz}}{I_{iz}} \frac{I_{iz}}{I_d} = -R_T \frac{R_D}{R_D + \frac{1}{j\omega C_D} + R_T} = -(R_D \parallel R_T) \frac{j\omega(R_D + R_T)C_D}{1 + j\omega(R_D + R_T)C_D}$$

Niskofrekvencijska analiza (2)

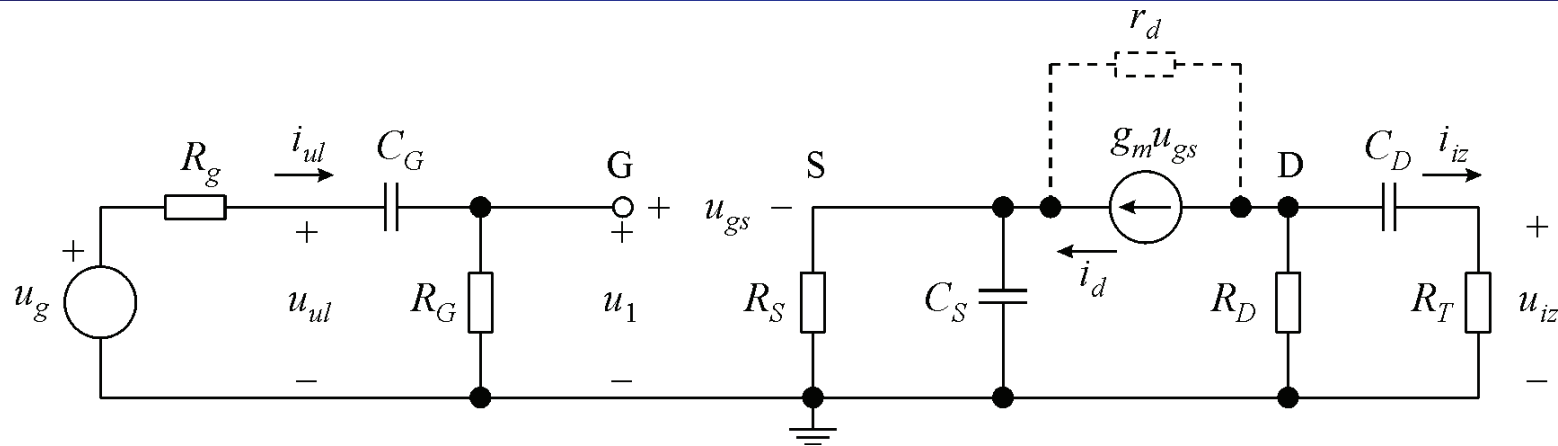


$$Z_S = R_S \parallel \frac{1}{j\omega C_S} = \frac{R_S (1/j\omega C_S)}{R_S + (1/j\omega C_S)} = \frac{R_S}{1 + j\omega R_S C_S}$$

$$I_d = g_m U_{gs} = g_m (U_1 - I_d Z_S)$$

$$\frac{I_d}{U_1} = \frac{g_m}{1 + g_m Z_S} = \frac{1}{1/g_m + \frac{R_S}{1 + j\omega R_S C_S}} = \frac{g_m}{1 + g_m R_S} \frac{1 + j\omega R_S C_S}{1 + j\omega (R_S \parallel 1/g_m) C_S}.$$

Niskofrekvencijska analiza (2)

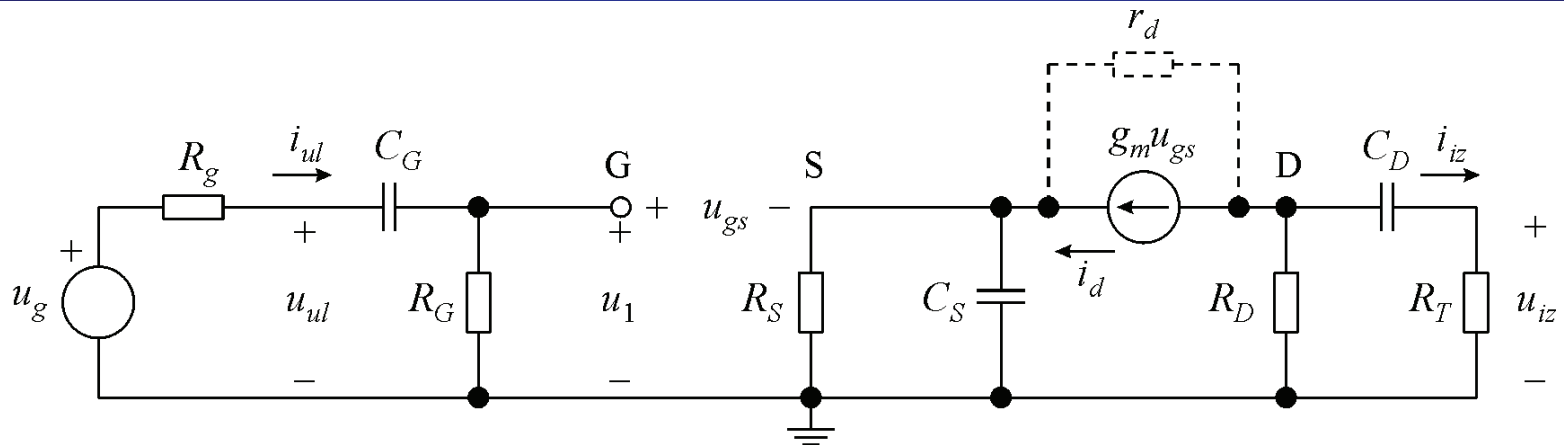


$$Z_S = R_S \parallel \frac{1}{j\omega C_S} = \frac{R_S (1/j\omega C_S)}{R_S + (1/j\omega C_S)} = \frac{R_S}{1 + j\omega R_S C_S}$$

$$I_d = g_m U_{gs} = g_m (U_1 - I_d Z_S)$$

$$\frac{I_d}{U_1} = \frac{g_m}{1 + g_m Z_S} = \frac{1}{1/g_m + \frac{R_S}{1 + j\omega R_S C_S}} = \frac{g_m}{1 + g_m R_S} \frac{1 + j\omega R_S C_S}{1 + j\omega (R_S \parallel 1/g_m) C_S}.$$

Niskofrekvencijska analiza (2)



$$\frac{U_1}{U_{ul}} = \frac{R_G}{(1/j\omega C_G) + R_G} = \frac{j\omega R_G C_G}{1 + j\omega R_G C_G}$$

Naponsko pojačanje (1)

$$A_V(j\omega) = \frac{U_{iz}}{U_{ul}} = A_{Vn} \frac{1 + j\omega/\omega_1}{1 + j\omega/\omega_2} \frac{j\omega/\omega_3}{1 + j\omega/\omega_3} \frac{j\omega/\omega_4}{1 + j\omega/\omega_4}$$

$$A_{Vn} = - \frac{g_m (R_D \parallel R_T)}{1 + g_m R_S}$$

Vremenske konstante

$$\tau_1 = \frac{1}{\omega_1} = R_S C_S$$

$$\tau_2 = \tau_S = \frac{1}{\omega_2} = \left(R_S \parallel \frac{1}{g_m} \right) C_S$$

$$\tau_3 = \tau_D = \frac{1}{\omega_3} = (R_D + R_T) C_D$$

$$\tau_4 = \tau_G = \frac{1}{\omega_4} = R_G C_G$$

Naponsko pojačanje (2)

Za ulazni izvor s konačnim unutarnjim otporom R_g

$$\tau_G = (R_G + R_g) C_G$$

Pojačanje na srednjim frekvencijama

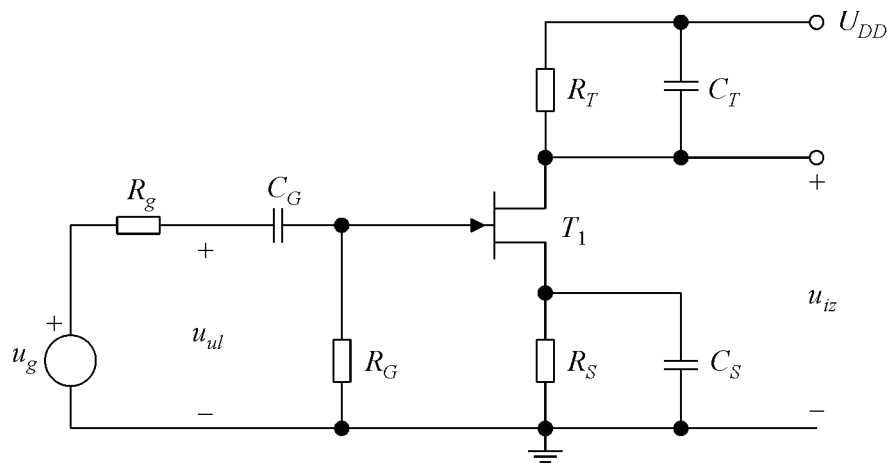
$$A_{V0} = A_{Vn} \frac{\omega_2}{\omega_1} = A_{Vn} \frac{\tau_1}{\tau_2} = A_{Vn} \frac{\omega_2}{\omega_1} = -g_m (R_D \parallel R_T)$$

Donja granična frekvencija

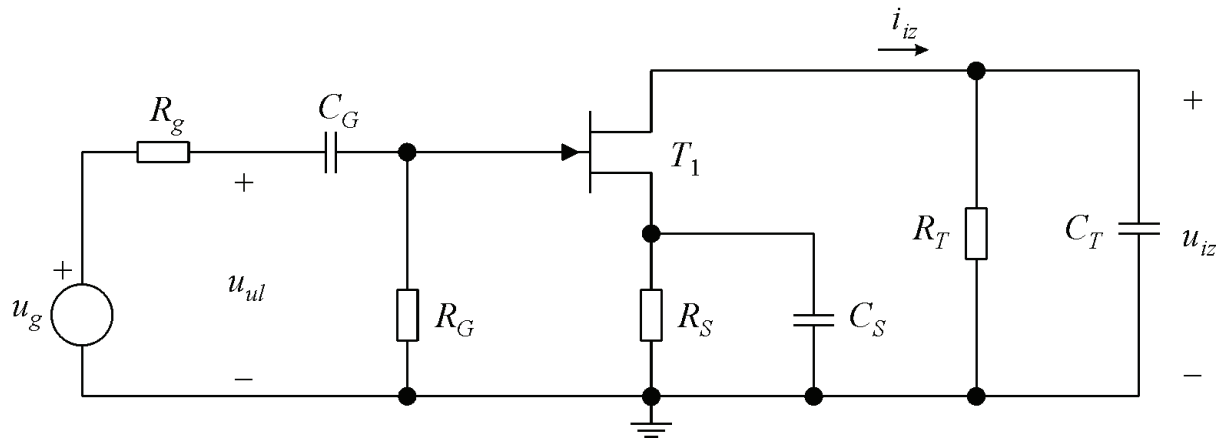
$$\omega_d = \max \{ \omega_G, \omega_S, \omega_D \} = \max \left\{ \frac{1}{\tau_G}, \frac{1}{\tau_S}, \frac{1}{\tau_D} \right\}$$

Primjer 4.3 (1)

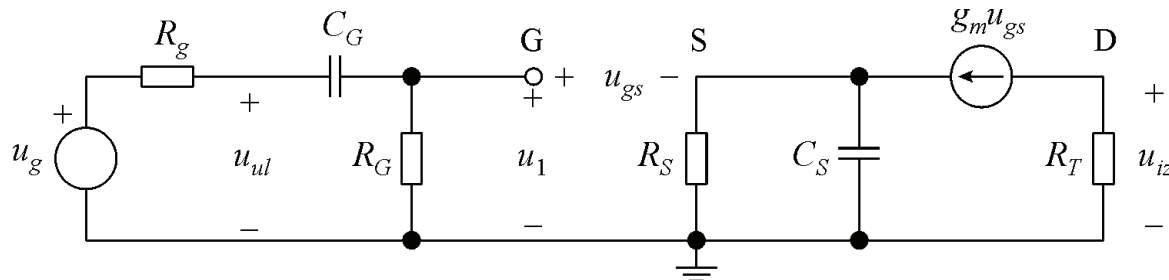
Za pojačalo na slici odrediti otpor otpornika R_S tako da struja odvoda spojnog FET-a u statičkoj radnoj točki bude $I_{DQ} = 1 \text{ mA}$. Odrediti naponsko pojačanje $A_V = U_{iz}/U_{ul}$ na srednjim frekvencijama, te donju graničnu frekvenciju tog pojačanja. Zadano je: $U_{DD} = 12 \text{ V}$, $R_g = 100 \Omega$, $C_G = 5 \text{ nF}$, $R_G = 1 \text{ M}\Omega$, $R_T = 3 \text{ k}\Omega$, $C_T = 10 \text{ pF}$ i $C_S = 20 \text{ }\mu\text{F}$. Parametri spojnog FET-a su $I_{DSS} = 16 \text{ mA}$ i $U_P = -2 \text{ V}$. Zanemariti porast struje odvoda s naponom u_{DS} u području zasićenja.



Primjer 4.3 (2)



Shema pojačala u dinamičkim prilikama



Nadomjesna shema za niskofrekvencijsku analizu

Ostali spojevi pojačala na niskim frekvencijama

Donju graničnu frekvenciju određuju vezni kondenzatori. Svaki u prijenosnu funkciju pojačala unosi član oblika

$$\frac{j\omega / \omega_i}{1 + j\omega / \omega_i}$$

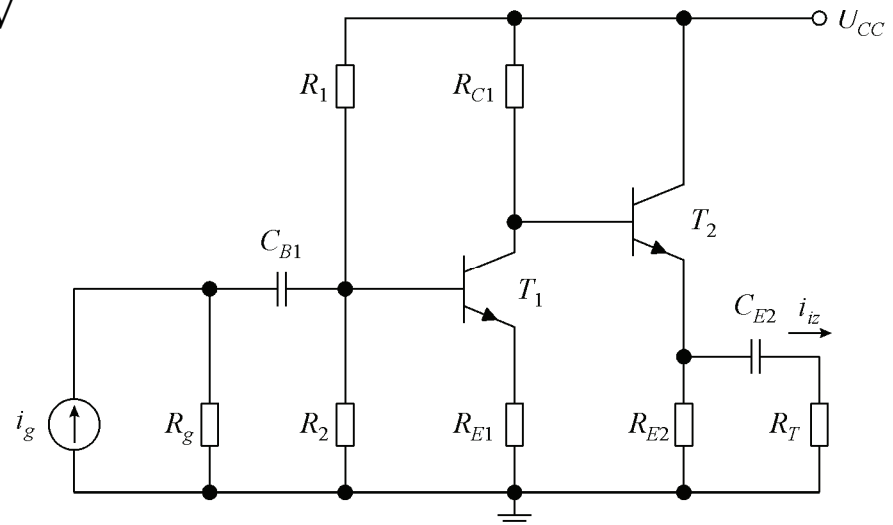
Karakteristične frekvencije ω_i određuju vremenske konstante $\tau_i = 1 / \omega_i$ uzrokovane pojedinim veznim kondenzatorom.

Za određivanje donje granične frekvencije → proračun svih vremenskih konstanti → donja granična frekvencija **najviša** frekvencija ako je barem 4 puta veća od ostalih → ako nije može se procijeniti

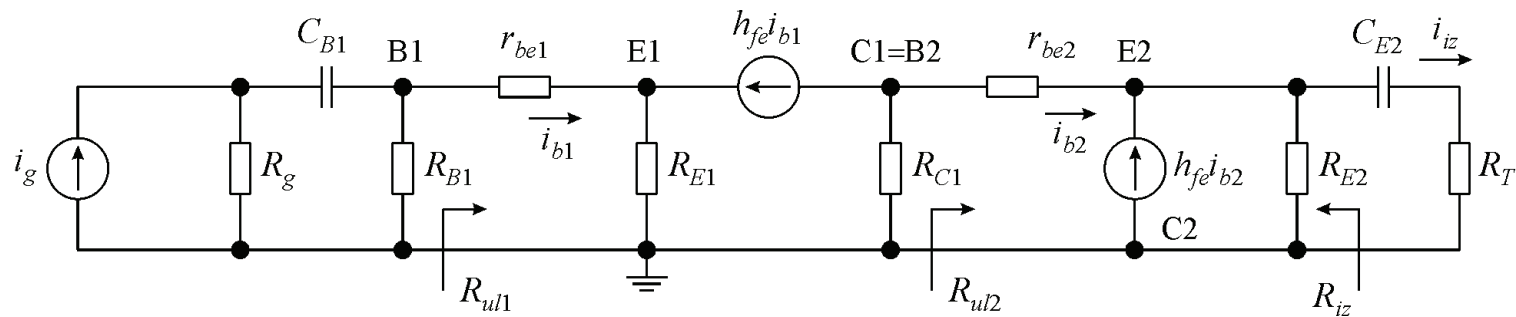
$$\omega_d \approx \sum_i \frac{1}{\tau_i}$$

Primjer 4.4 (1)

Za pojačalo na slici odrediti strujno pojačanje $A_{I_g} = I_{iz}/I_g$ na srednjim frekvencijama, te donju graničnu frekvenciju tog pojačanja. Zadano je: $U_{CC} = 15\text{ V}$, $R_g = 50\text{ k}\Omega$, $C_{B1} = 0,2\text{ }\mu\text{F}$, $R_1 = 40\text{ k}\Omega$, $R_2 = 10\text{ k}\Omega$, $R_{C1} = 2\text{ k}\Omega$, $R_{E1} = R_{E2} = 500\text{ }\Omega$, $C_{E2} = 10\text{ }\mu\text{F}$ i $R_T = 100\text{ }\Omega$. Tranzistori imaju jednake parametre $\beta \approx h_{fe} = 100$ i $U_\gamma = 0,7\text{ V}$. Za oba tranzistora zanemariti porast struje kolektora s naponom u_{CE} normalnom aktivnom području. Naponski ekvivalent temperature $U_T = 25\text{ mV}$

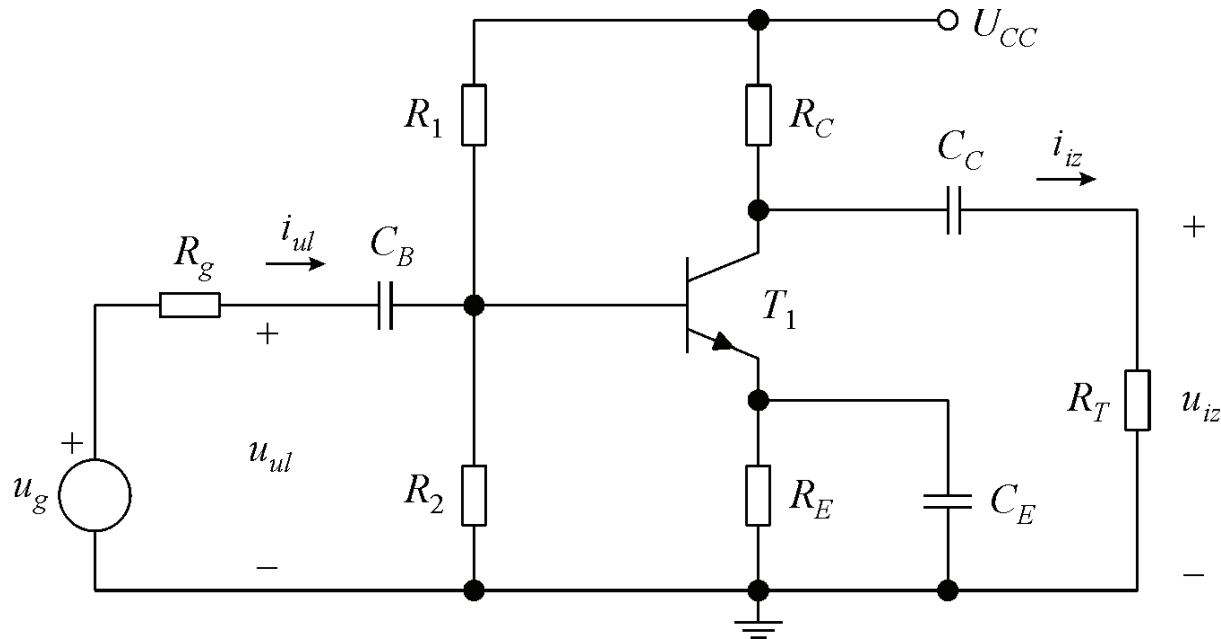


Primjer 4.4 (2)



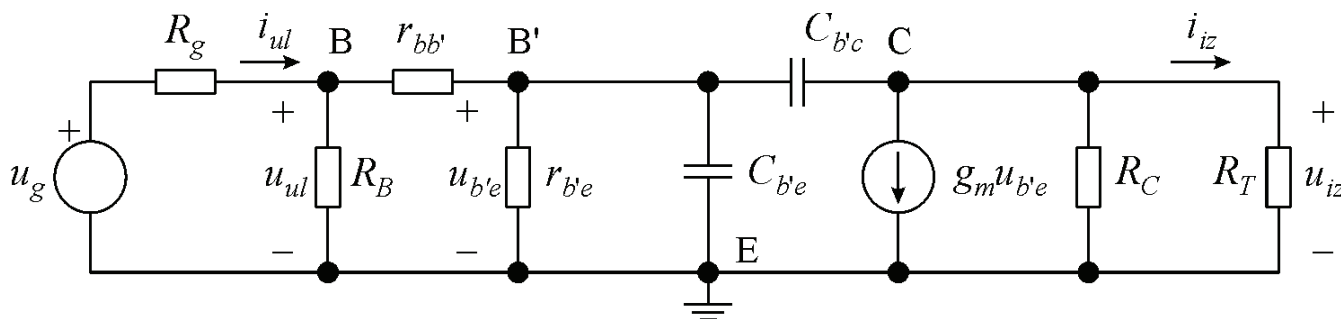
Nadomjesna shema za niskofrekvencijsku analizu

Spoj zajedničkog emitera na visokim frekvencijama



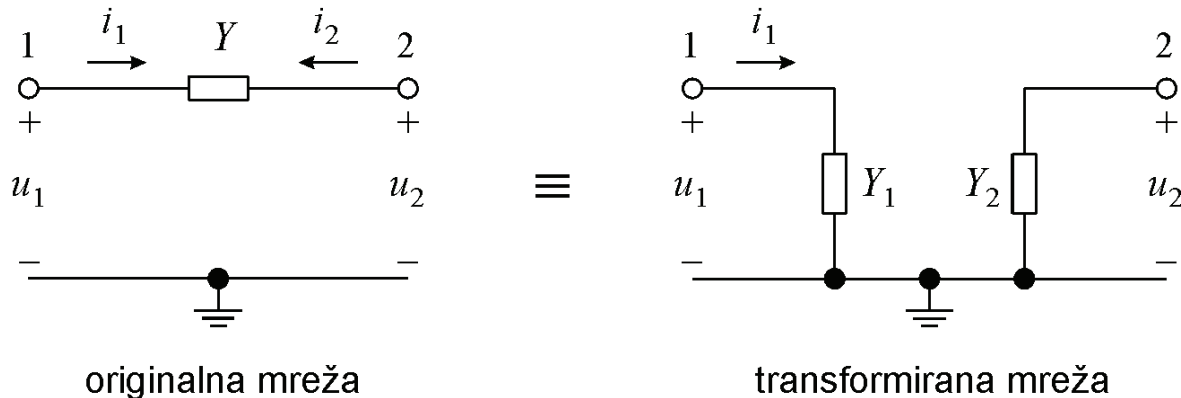
Nadomjesna shema za visokofrekvencijsku analizu

Na visokim frekvencijama djeluju kapaciteti tranzistora



Složenost analize uzrokuje kapacitet $C_{b'c}$ koji povezuje ulaznu i izlazni krug → pojednostavljenje analize primjenom Millerovog teorema

Millerov teorem

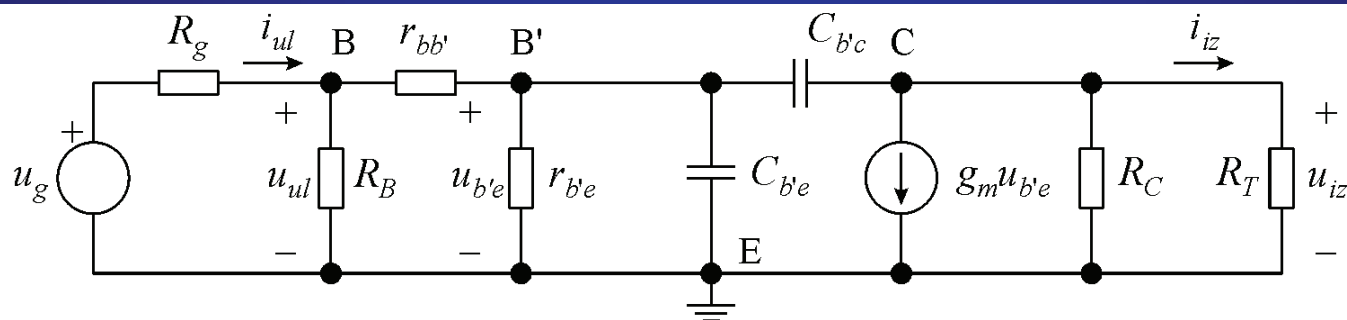


$$I_1 = Y(U_1 - U_2) = YU_1(1 - K) = Y_1 U_1 \quad \rightarrow \quad Y_1 = Y(1 - K) \quad K \equiv \frac{U_2}{U_1}$$

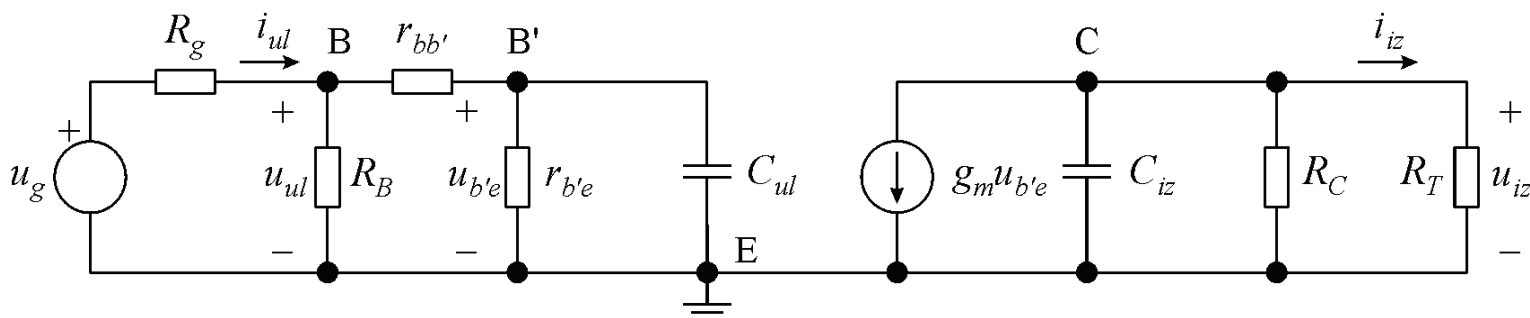
$$I_2 = Y(U_2 - U_1) = YU_2\left(1 - \frac{1}{K}\right) = Y_2 U_2 \quad \rightarrow \quad Y_2 = Y\left(1 - \frac{1}{K}\right) = Y \frac{K - 1}{K}$$

Za impedancije $\rightarrow Z_1 = \frac{Z}{1 - K} \quad Z_2 = Z \frac{K}{K - 1}$

Primjena Millerovog teorema na kapacitet $C_{b'c}$

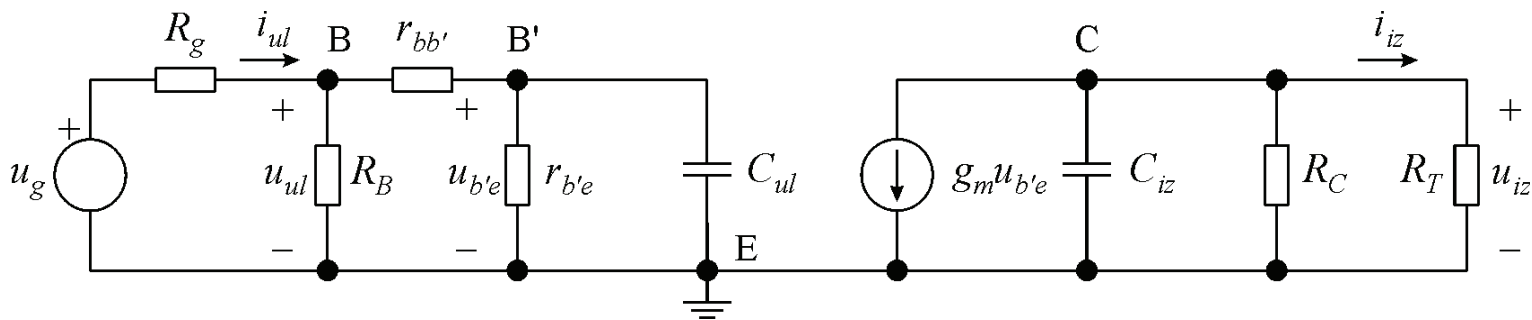


Preslikani kapacitet $C_{b'c} \rightarrow C_{b'c,ul} = C_{b'c}(1 - K) \quad C_{b'c,iz} = C_{b'c} \frac{K - 1}{K} \quad K = \frac{U_{iz}}{U_{b'e}}$



$$C_{ul} = C_{b'e} + C_{b'c}(1 - K) \quad C_{iz} = C_{b'c} \frac{K - 1}{K}$$

Aproksimacija prijenosne funkcije jednim polom (1)

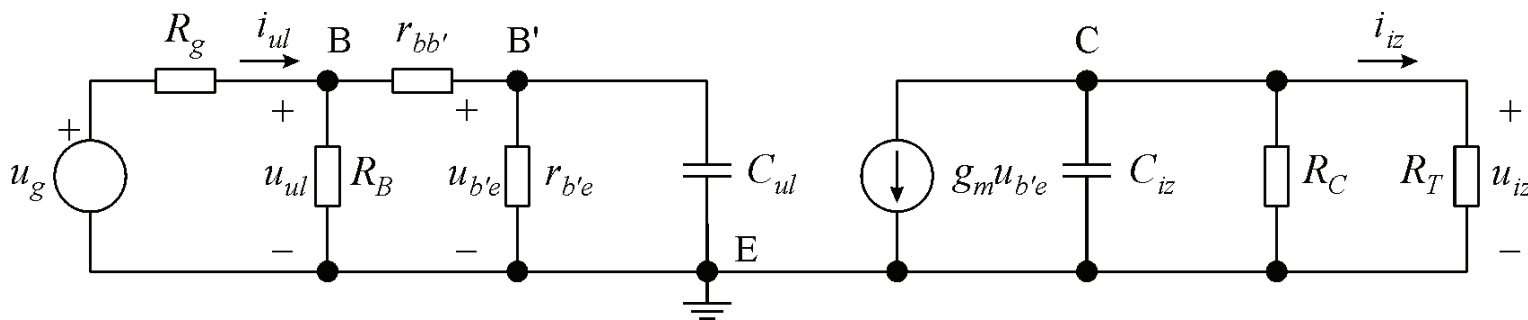


$C_{ul} \gg C_{iz} \rightarrow \tau_{ul} \gg \tau_{iz} \rightarrow \omega_{ul} \ll \omega_{iz} \rightarrow$ za proračun gornje granične frekvencije odspaja se C_{iz}

$$A_V = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{U_{b'e}} \frac{U_{b'e}}{U_{ul}}$$

$$\frac{U_{iz}}{U_{b'e}} = -g_m (R_C \parallel R_T) = K$$

Aproksimacija prijenosne funkcije jednim polom (2)



$$Z_{b'e} = r_{b'e} \parallel \frac{1}{j\omega C_{ul}} = \frac{r_{b'e} (1/j\omega C_{ul})}{r_{b'e} + (1/j\omega C_{ul})} = \frac{r_{b'e}}{1 + j\omega r_{b'e} C_{ul}}$$

$$\begin{aligned} \frac{U_{b'e}}{U_{ul}} &= \frac{Z_{b'e}}{r_{bb'} + Z_{b'e}} = \frac{\frac{r_{b'e}}{1 + j\omega r_{b'e} C_{ul}}}{r_{bb'} + \frac{r_{b'e}}{1 + j\omega r_{b'e} C_{ul}}} = \frac{r_{b'e}}{r_{bb'} + r_{b'e} + j\omega r_{bb'} r_{b'e} C_{ul}} = \\ &= \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \frac{1}{1 + j\omega (r_{bb'} \parallel r_{b'e}) C_{ul}} \end{aligned}$$

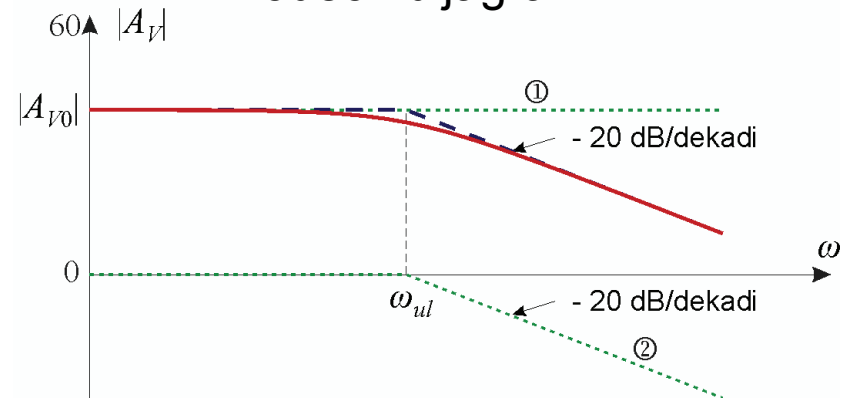
Naponsko pojačanje

$$A_V(j\omega) = \frac{A_{V0}}{1 + j\omega/\omega_{ul}}$$

$$A_{V0} = -g_m(R_C \parallel R_T) \frac{r_{b'e}}{r_{bb'} + r_{b'e}}$$

$$\tau_{ul} = \frac{1}{\omega_{ul}} = (r_{bb'} \parallel r_{b'e})C_{ul}$$

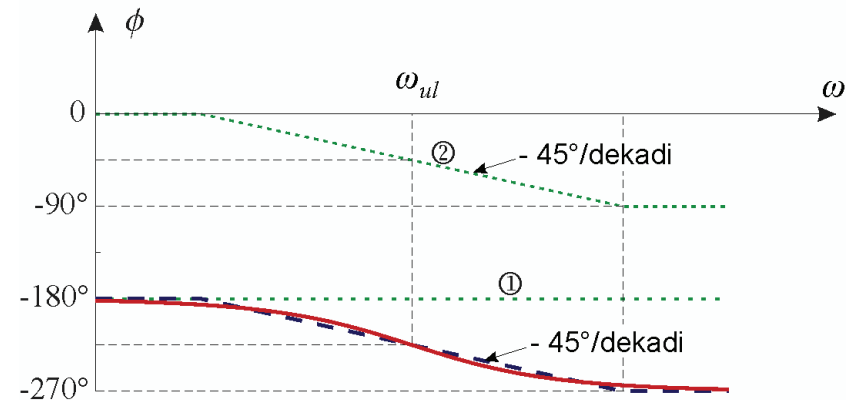
Bodeov dijagram



Gornja granična frekvencija

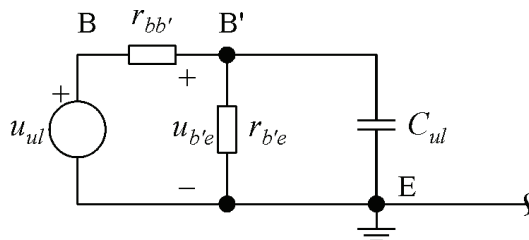
$$|A_V| = \frac{|A_{V0}|}{\sqrt{1 + (\omega_g/\omega_{ul})^2}} = \frac{|A_{V0}|}{\sqrt{2}}$$

$$\omega_g = \omega_{ul}$$



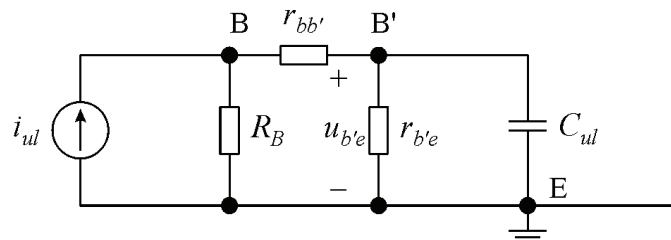
Vremenska konstanta τ_{ul}

za naponsko pojačanje



$$\tau_{ul} = (r_{bb'} \parallel r_{b'e}) C_{ul}$$

za strujno pojačanje



$$\tau_{ul} = [(R_B + r_{bb'}) \parallel r_{b'e}] C_{ul}$$

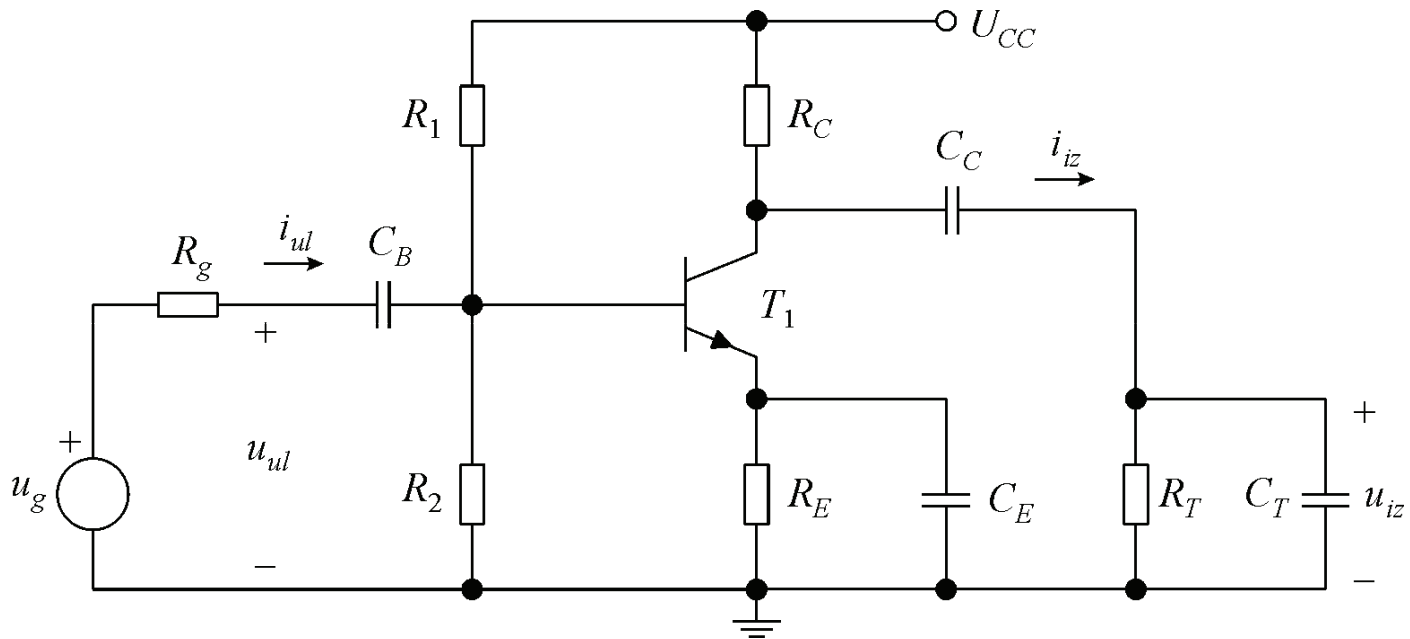
uz konačni R_g $\tau_{ul} = [(R_g \parallel R_B + r_{bb'}) \parallel r_{b'e}] C_{ul}$

Millerov efekt

$$C_{ul} = C_{b'e} + C_{b'c} [1 + g_m (R_C \parallel R_T)]$$

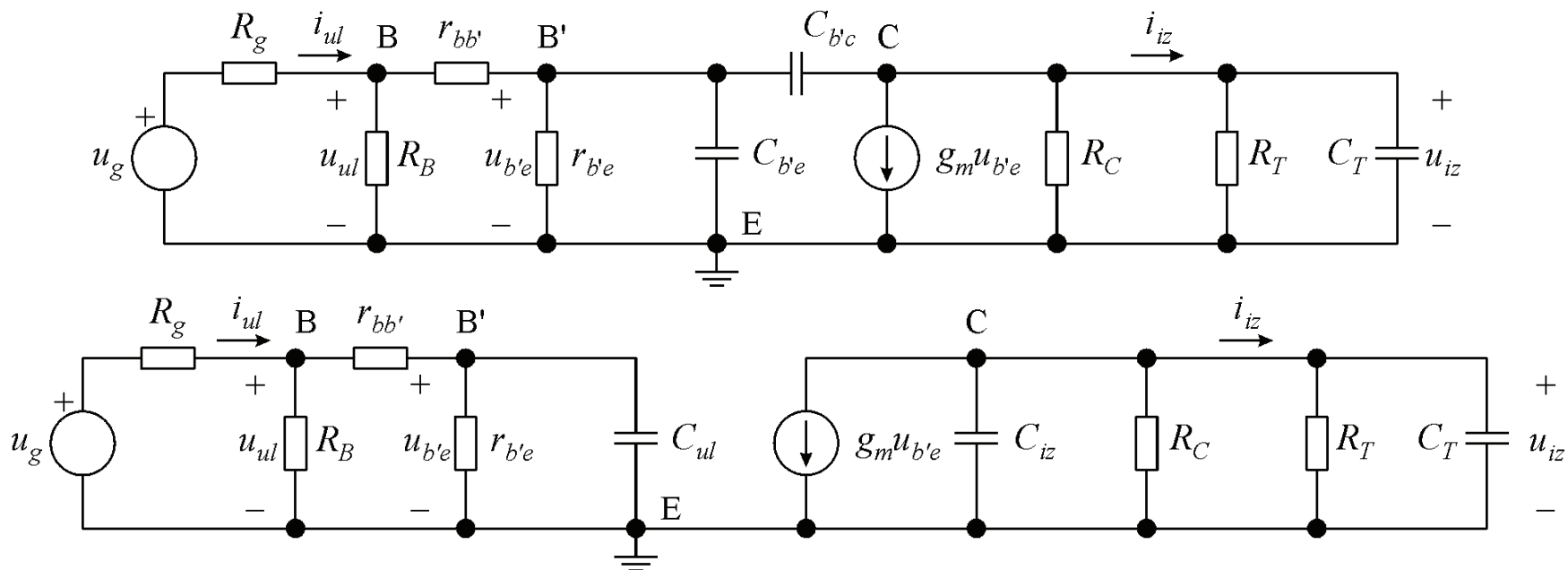
kapacitet $C_{b'c}$ spojen između izlaza i ulaza preslikava se u ulazni krug pomnožen s iznosom naponskog pojačanja → smanjenje gornje granične frekvencije

Utjecaj oba pola



$C_T \rightarrow$ kapacitivno opterećenje

Nadomjesna shema za visokofrekvencijsku analizu



kapacitet izlaznog kuga $C'_{iz} = C_{iz} + C_T = C_{b'c} \frac{K-1}{K} + C_T \approx C_{b'c} + C_T$

vremenska konstanta izlaznog kuga $\tau_{iz} = \frac{1}{\omega_{iz}} = (R_C \parallel R_T) C'_{iz}$

Određivanje gornje granične frekvencije

Na srednjim frekvencijama (svi se kondenzatori kratko odspajaju)

$$A_{V0} = -g_m (R_C \parallel R_T) \frac{r_{b'e}}{r_{bb'} + r_{b'e}}$$

Vremenske konstante koje određuju pojedini kapaciteti (drugi se kapacitet odspaja)

$$\tau_{ul} = (r_{bb'} \parallel r_{b'e}) C_{ul} \qquad \tau_{iz} = \frac{1}{\omega_{iz}} = (R_C \parallel R_T) C'_{iz}$$

Gornja granična frekvencija

$$\omega_g = \min \{ \omega_{ul}, \omega_{iz} \} = \min \left\{ \frac{1}{\tau_{ul}}, \frac{1}{\tau_{iz}} \right\}$$

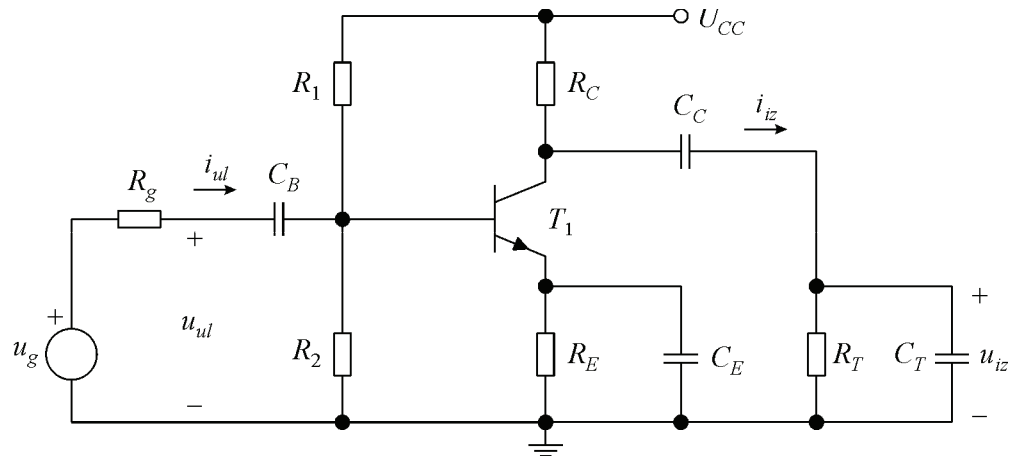
Uvjet → niža frekvencija barem 4 puta niža od više → **dominantni pol**

Ako ne → procjena $\omega_g \approx \frac{1}{\tau_{ul} + \tau_{iz}}$ općenito $\omega_g \approx \frac{1}{\sum_i \tau_i}$

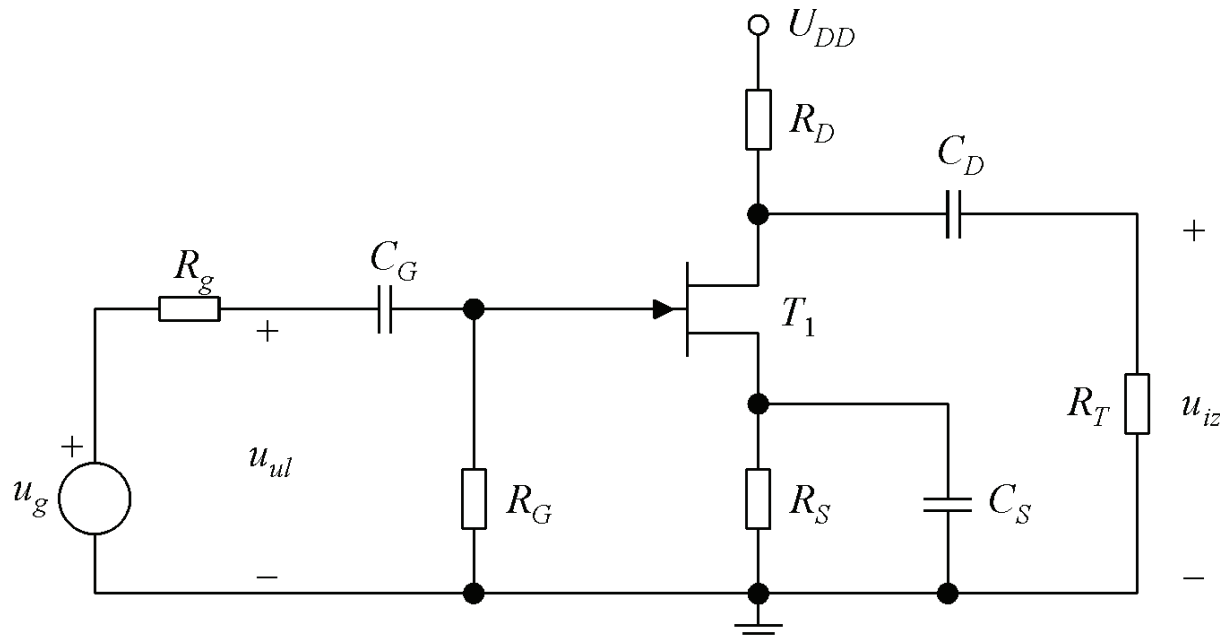
Primjer 4.5

U pojačalu sa slike zadano je: $U_{CC} = 15 \text{ V}$, $R_g = 500 \Omega$, $C_B = 10 \mu\text{F}$, $R_1 = 30 \text{ k}\Omega$, $R_2 = 11 \text{ k}\Omega$, $R_C = 2 \text{ k}\Omega$, $R_E = 1 \text{ k}\Omega$, $C_E = 100 \mu\text{F}$, $C_C = 3 \mu\text{F}$ i $R_T = 1,2 \text{ k}\Omega$. Parametri bipolarnog tranzistora su $\beta \approx h_{fe} = 100$, $U_\gamma = 0,7 \text{ V}$, $r_{bb'} = 50 \Omega$, $C_{b'e} = 50 \text{ pF}$ i $C_{b'c} = 2 \text{ pF}$. Zanemariti porast struje kolektora s naponom u_{CE} u normalnom aktivnom području. Naponski ekvivalent temperature $U_T = 25 \text{ mV}$. Odrediti naponsko pojačanje $A_V = U_{iz}/U_{ul}$ na srednjim frekvencijama, te gornju graničnu frekvenciju tog pojačanja uz:

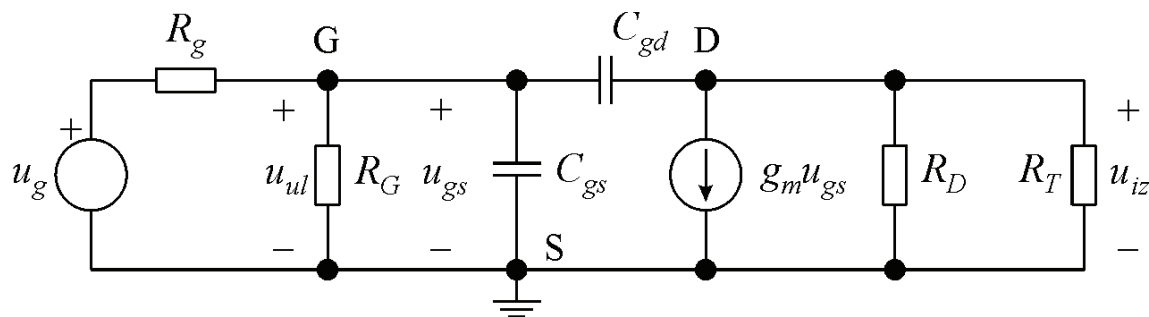
- a) $C_T = 0$ i
- b) $C_T = 100 \text{ pF}$.



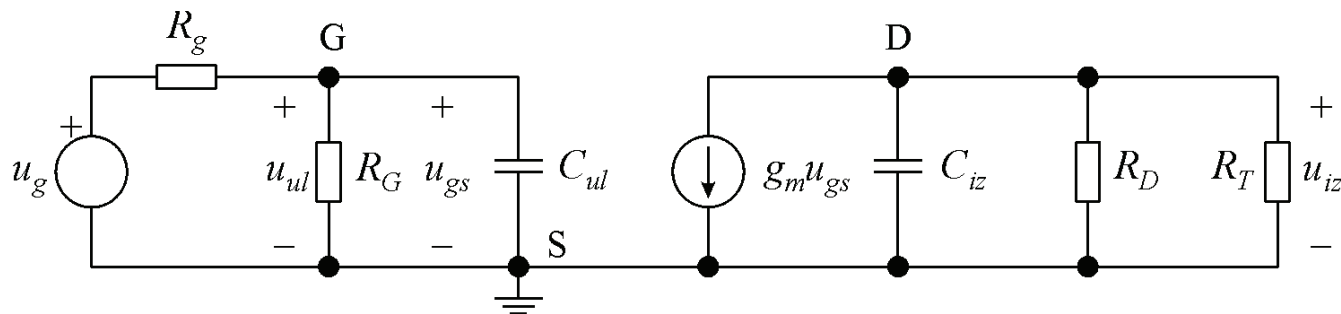
Spoj zajedničkog uvoda na visokim frekvencijama



Nadomjesna shema za visokofrekvencijsku analizu

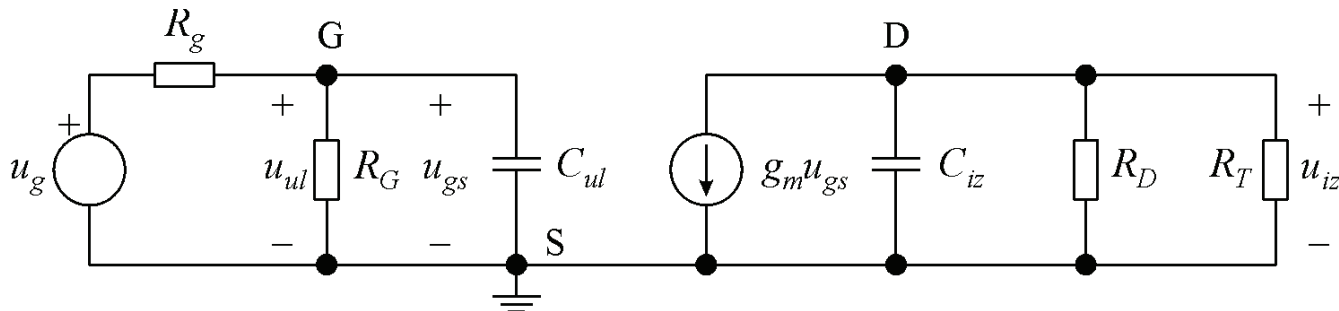


Preslikani kapacitet $C_{gd} \rightarrow C_{gd,ul} = C_{gd}(1 - K) \quad C_{gd,iz} = C_{gd} \frac{K - 1}{K} \quad K = \frac{U_{iz}}{U_{gs}} = \frac{U_{iz}}{U_{ul}} = A_V$



$$C_{ul} = C_{gs} + C_{gd}(1 - K) \quad C_{iz} = C_{gd} \frac{K - 1}{K} \approx C_{gd}$$

Visokofrekvencijska analiza



$$A_V = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{U_{gs}} \frac{U_{gs}}{U_{ul}}$$

$$\frac{U_{iz}}{U_{gs}} = -g_m \frac{\frac{1}{j\omega C_{iz}} (R_D \parallel R_T)}{\frac{1}{j\omega C_{iz}} + (R_D \parallel R_T)} = \frac{-g_m (R_D \parallel R_T)}{1 + j\omega (R_D \parallel R_T) C_{iz}}$$

Naponsko pojačanje

$$A_V(j\omega) = \frac{A_{V0}}{1 + j\omega/\omega_{iz}}$$

$$A_{V0} = -g_m(R_D \parallel R_T) \quad \tau_{iz} = \frac{1}{\omega_{iz}} = (R_D \parallel R_T)C_{iz} \approx (R_D \parallel R_T)C_{gd}$$

Gornja granična frekvencija

$$\omega_g = \omega_{iz}$$

uz konačni $R_g \rightarrow$ utječe i vremenska konstanta ulaznog kruga

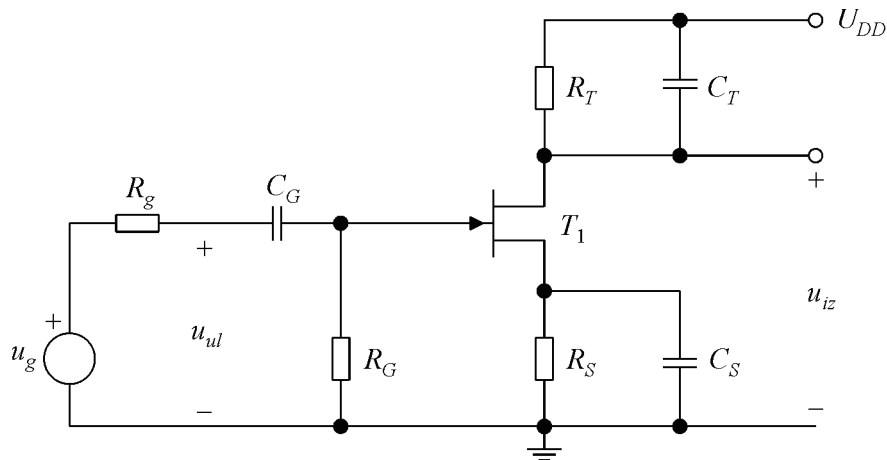
$$\tau_{ul} = \frac{1}{\omega_{ul}} = (R_g \parallel R_G)C_{ul}$$

$$\omega_g = \min\{\omega_{ul}, \omega_{iz}\} = \min\left\{\frac{1}{\tau_{ul}}, \frac{1}{\tau_{iz}}\right\}$$

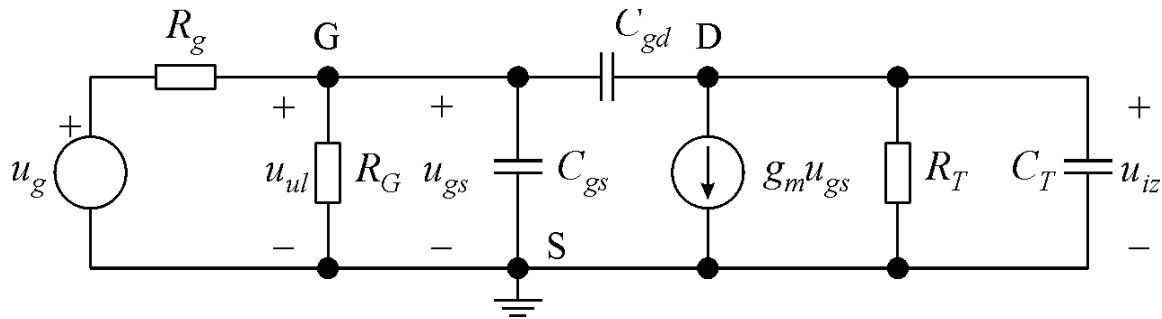
Millerov efekt $C_{ul} = C_{gs} + C_{gd} \left[1 + g_m(R_D \parallel R_T)\right]$

Primjer 4.6 (1)

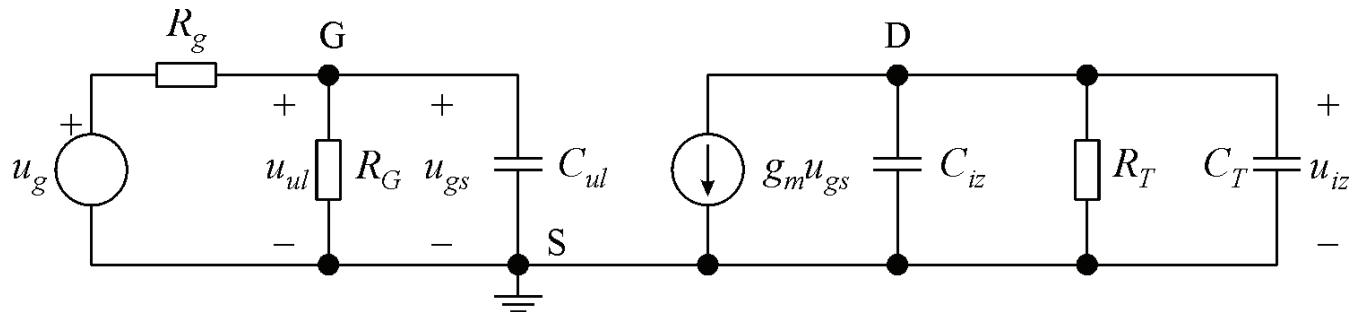
Za pojačalo iz primjera 4.3 odrediti naponsko pojačanje $A_{Vg} = U_{iz}/U_g$ na srednjim frekvencijama, te gornju graničnu frekvenciju tog pojačanja. Kapaciteti FET-a su $C_{gs} = 3 \text{ pF}$ i $C_{gd} = 2 \text{ pF}$.



Primjer 4.6 (2)

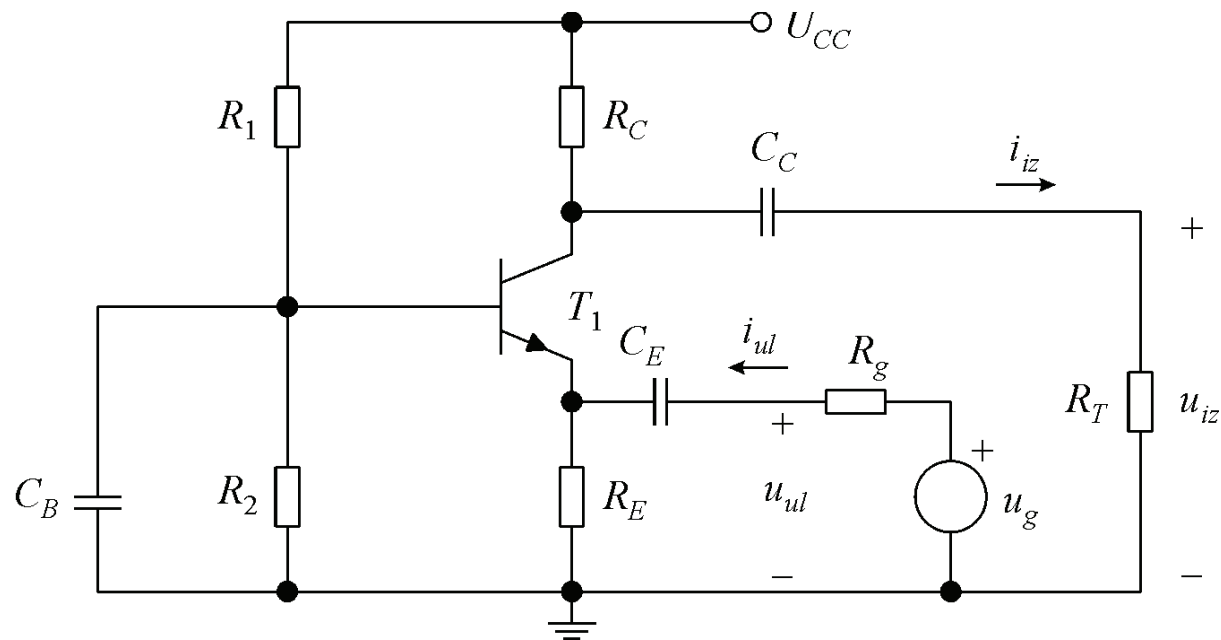


Nadomjesna shema za visokofrekvencijsku analizu

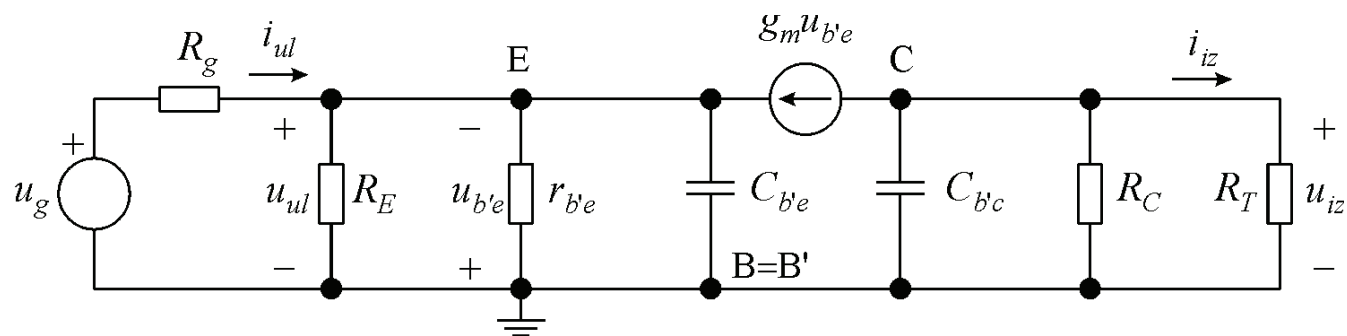


Primjena Millerovog teorema

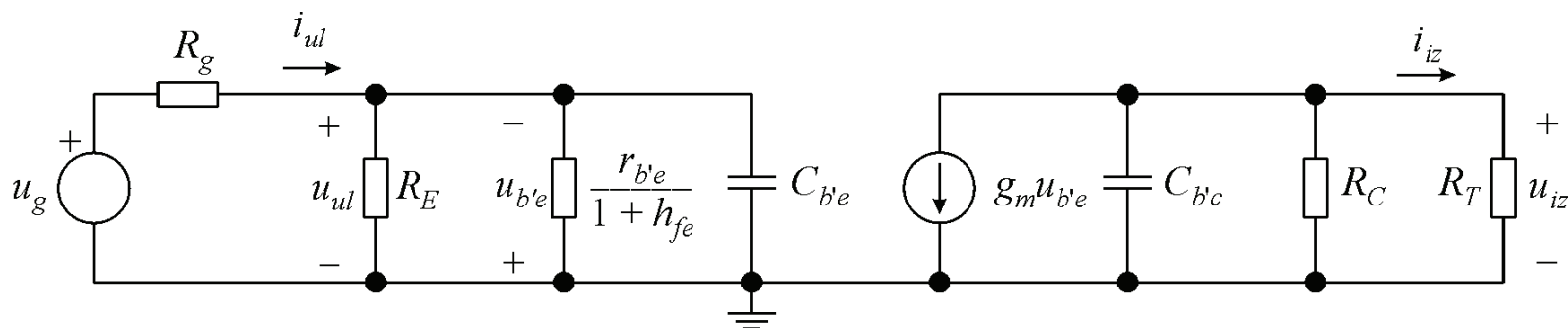
Spoj zajedničke baze na visokim frekvencijama



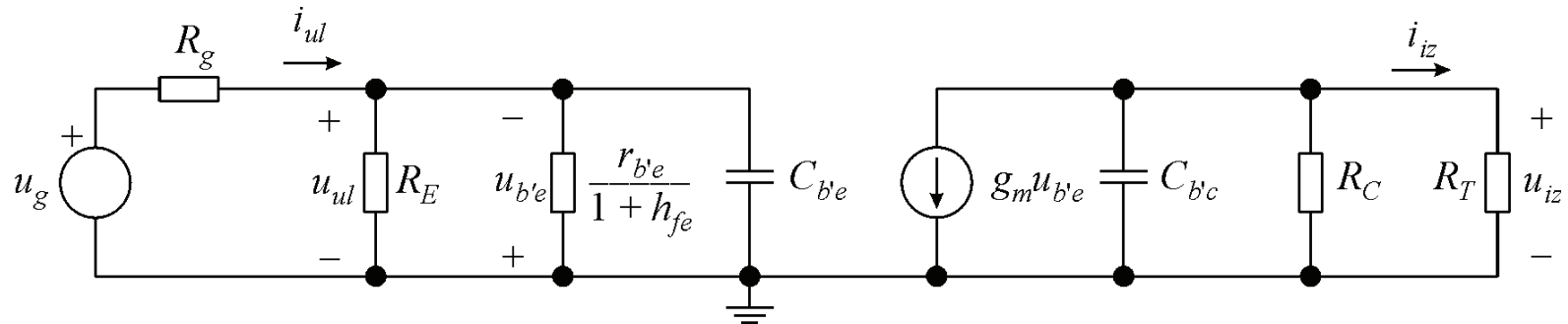
Nadomjesna shema za visokofrekvencijsku analizu



$$I_{ul} = \frac{1}{R_E} U_{ul} - \left(\frac{1}{r_{b'e}} + j\omega C_{b'e} \right) U_{b'e} - g_m U_{b'e} = \left(\frac{1}{R_E} + \frac{1+h_{fe}}{r_{b'e}} + j\omega C_{b'e} \right) U_{ul}$$



Visokofrekvencijska analiza



$$Z_C = \frac{1}{j\omega C_{b'c}} \parallel (R_C \parallel R_T) = \frac{(1/j\omega C_{b'c})(R_C \parallel R_T)}{(1/j\omega C_{b'c}) + (R_C \parallel R_T)} = \frac{(R_C \parallel R_T)}{1 + j\omega(R_C \parallel R_T)C_{b'c}}$$

$$A_V = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{-U_{b'e}} = g_m Z_C = \frac{g_m(R_C \parallel R_T)}{1 + j\omega(R_C \parallel R_T)C_{b'c}}$$

Naponsko pojačanje

$$A_V(j\omega) = \frac{A_{V0}}{1 + j\omega/\omega_{iz}}$$

$$A_{V0} = g_m (R_C \parallel R_T) \quad \tau_{iz} = \frac{1}{\omega_{iz}} = (R_C \parallel R_T) C_{b'c}$$

Gornja granična frekvencija $\omega_g = \omega_{iz}$

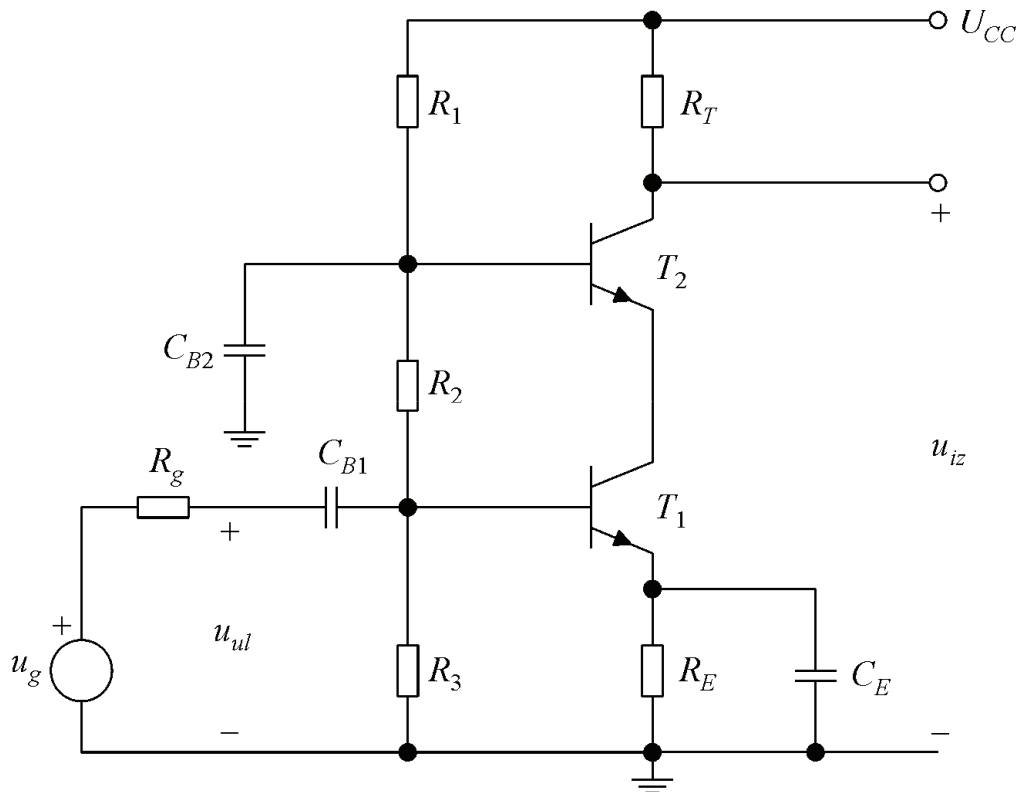
uz konačni $R_g \rightarrow$ utječe i vremenska konstanta ulaznog kruga

$$\tau_{ul} = \frac{1}{\omega_{ul}} = \left(\frac{r_{b'e}}{1 + h_{fe}} \parallel R_E \parallel R_g \right) C_{b'e}$$

$$\omega_g = \min \{ \omega_{ul}, \omega_{iz} \} = \min \left\{ \frac{1}{\tau_{ul}}, \frac{1}{\tau_{iz}} \right\}$$

Nema Millerovog efekta \rightarrow obje su vremenske konstante male

Kaskodno pojačalo na visokim frekvencijama

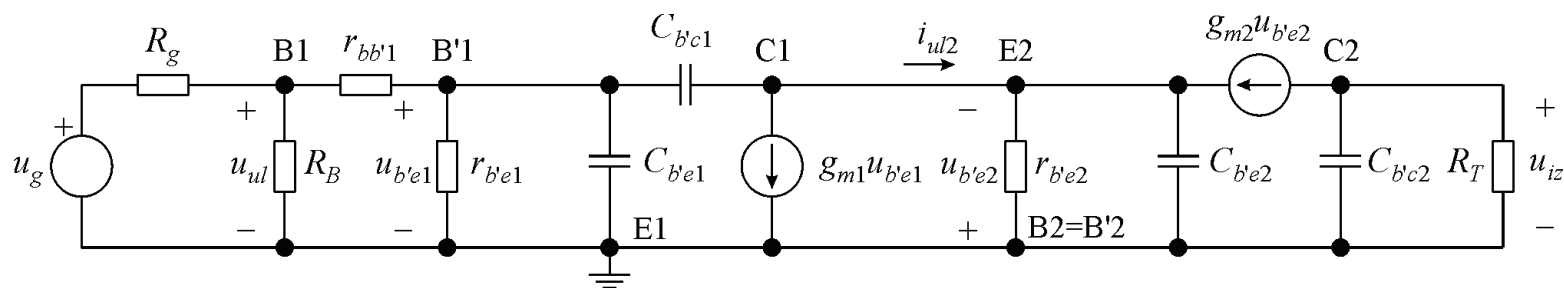


Pojačalo u spoju zajedničkog emitera → dobra svojstva na srednjim i lošija na visokim frekvencijama

Pojačalo u zajedničke baze → lošija svojstva na srednjim i dobra na visokim frekvencijama

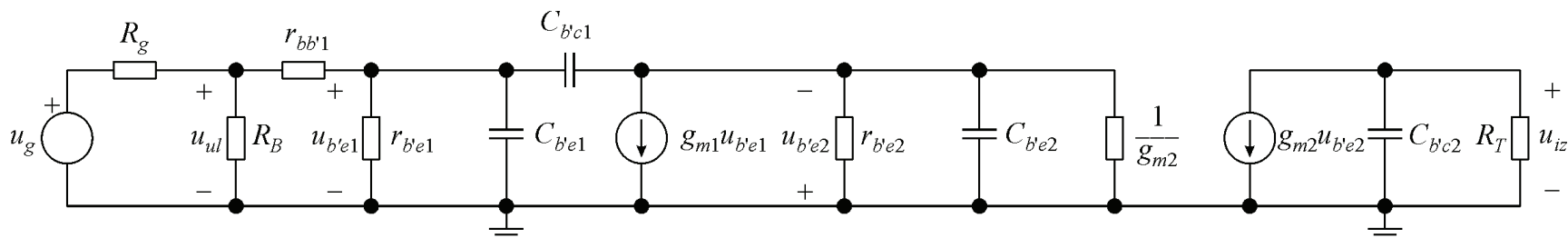
Kaskodno pojačalo → objedinjuje dobra svojstva

Nadomjesna shema za visokofrekvencijsku analizu (1)

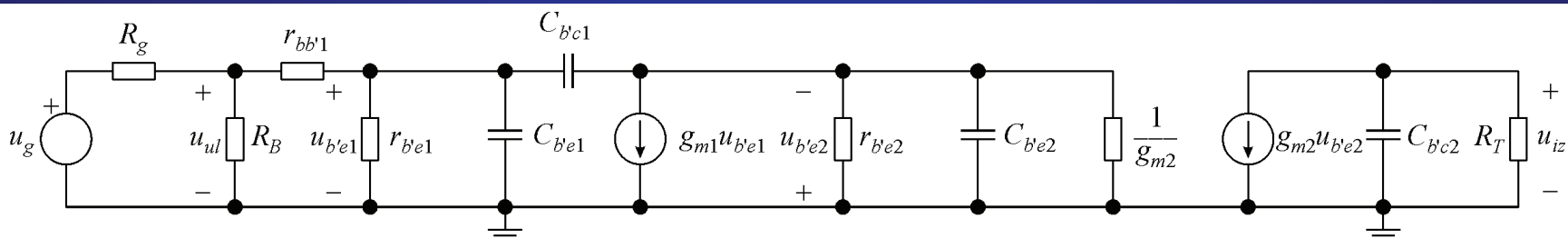


$$R_B = R_2 \parallel R_3$$

$$I_{ul2} = - \left(\frac{1}{r_{b'e2}} + j\omega C_{b'e2} + g_{m2} \right) U_{b'e}$$



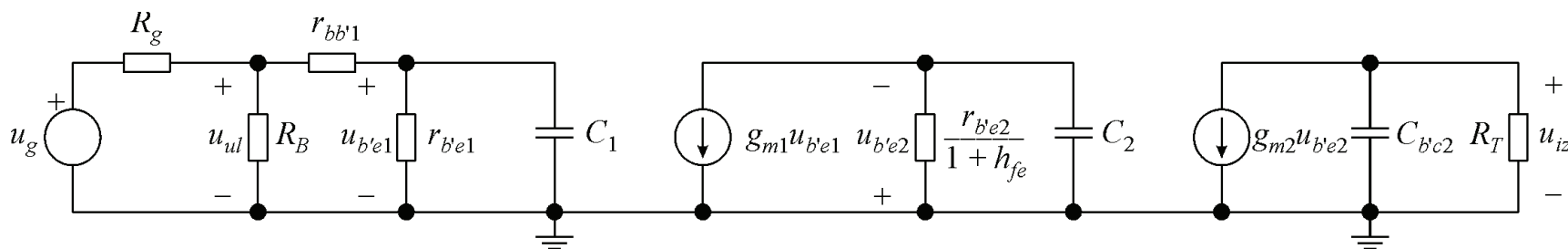
Nadomjesna shema za visokofrekvencijsku analizu (2)



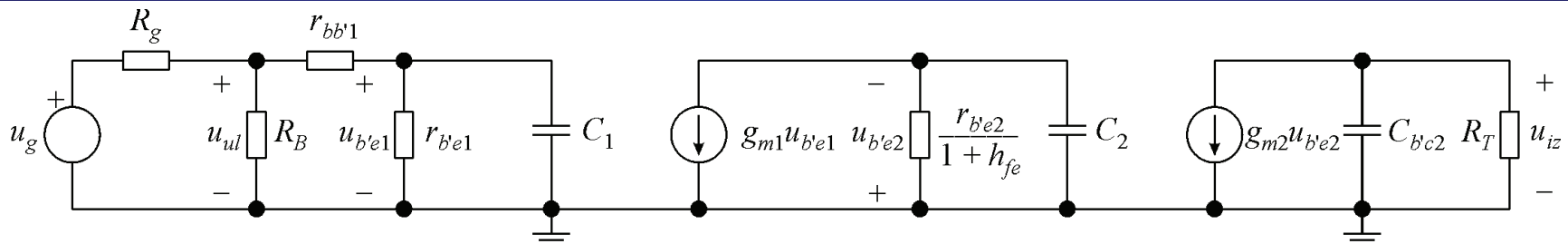
$$K = \frac{-U_{b'e2}}{U_{b'e1}} = -g_{m1} \frac{r_{b'e2}}{1 + h_{fe2}} = -\frac{h_{fe1}}{r_{b'e1}} \frac{r_{b'e2}}{1 + h_{fe2}} \approx -1$$

$$C_{b'c1,ul} = C_{b'c1} (1 - K) \approx 2C_{b'c1} \rightarrow C_1 \approx C_{b'e1} + 2C_{b'c1}$$

$$C_{b'c1,ul} = C_{b'c1} \frac{K - 1}{K} \approx 2C_{b'c1} \rightarrow C_2 \approx 2C_{b'c1} + C_{b'e2}$$



Naponsko pojačanje



$$A_{V0} = \frac{U_{iz}}{U_{ul}} = \frac{U_{iz}}{U_{b'e2}} \frac{U_{b'e2}}{U_{b'e1}} \frac{U_{b'e1}}{U_{ul}} \approx -g_{m2} R_T \frac{r_{b'e1}}{r_{bb'1} + r_{b'e1}}$$

$$\tau_1 = \frac{1}{\omega_1} = (r_{b'e1} \parallel r_{bb'1}) C_1 \quad \tau_2 = \frac{1}{\omega_2} = \frac{r_{b'e2}}{1 + h_{fe2}} C_2 \quad \tau_3 = \frac{1}{\omega_3} = R_T C_{b'e2}$$

1. stupanj → mali iznos naponskog pojačanja zbog malog ulaznog otpora

2. stupnja → smanjen Millerov efekt → smanjena vremenska konstanta τ_1

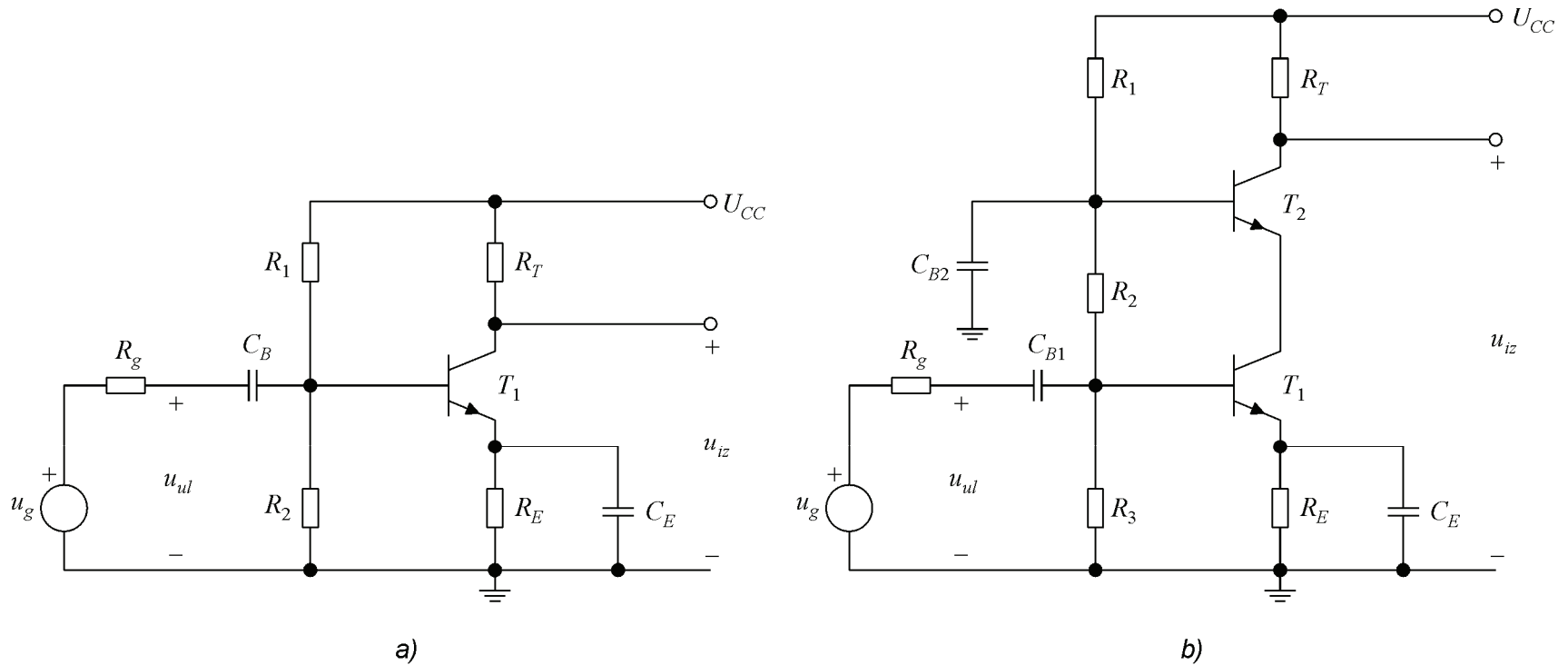
Male vremenske konstante τ_2 i τ_3

Povećana gornja granična frekvencija

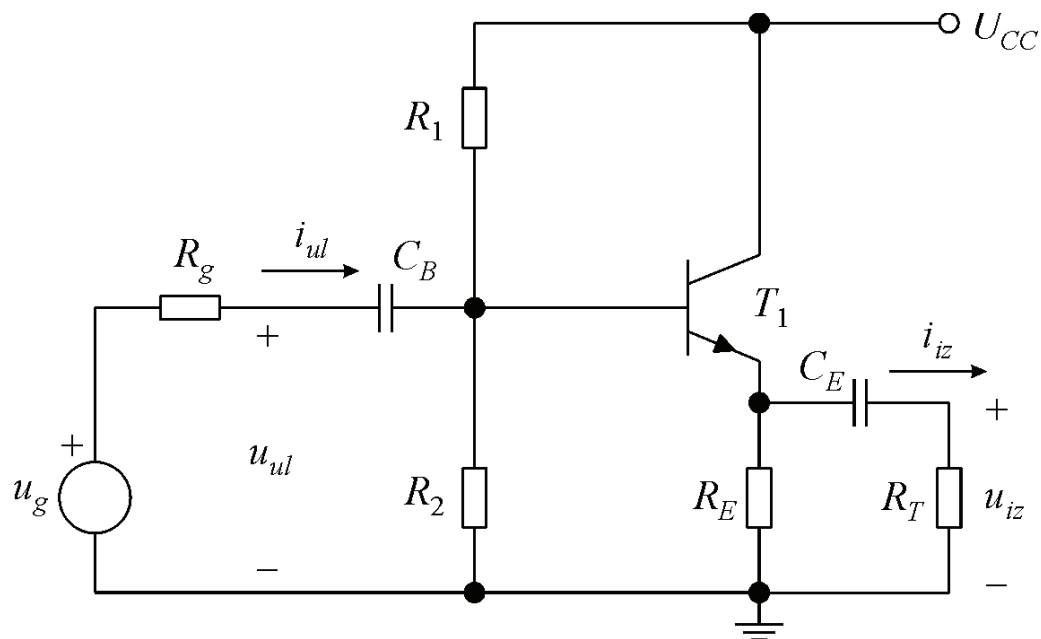
Primjer 4.7 (1)

Odrediti naponsko pojačanje $A_V = U_{iz}/U_{ul}$ na srednjim frekvencijama, te gornju graničnu frekvenciju tog pojačanja za pojačalo u spoju zajedničkog emitera sa slike a i za kaskodno pojačalo sa slike b. Zajednički podaci za oba pojačala su: $U_{CC} = 15 \text{ V}$, $R_g = 500 \Omega$, $R_E = 1,1 \text{ k}\Omega$ i $R_T = 1 \text{ k}\Omega$. Dodatni podaci za pojačalo sa slike a su: $R_1 = 5,5 \text{ k}\Omega$ i $R_2 = 2 \text{ k}\Omega$, a za pojačalo sa slike b: $R_1 = 4 \text{ k}\Omega$, $R_2 = 1,5 \text{ k}\Omega$ i $R_3 = 2 \text{ k}\Omega$. Impedancije kondenzatora C_B , C_E , C_{B1} i C_{B2} su zanemarivo male u području srednjih i visokih frekvencija. Svi tranzistori imaju jednake parametre $\beta \approx h_{fe} = 100$, $U_\gamma = 0,7 \text{ V}$, $r_{bb'} = 100 \Omega$, $C_{b'e} = 50 \text{ pF}$ i $C_{b'c} = 1 \text{ pF}$. Zanemariti porast struje kolektora s naponom u_{CE} u normalnom aktivnom području. Naponski ekvivalent temperature $U_T = 25 \text{ mV}$. Usporediti dobivene rezultate.

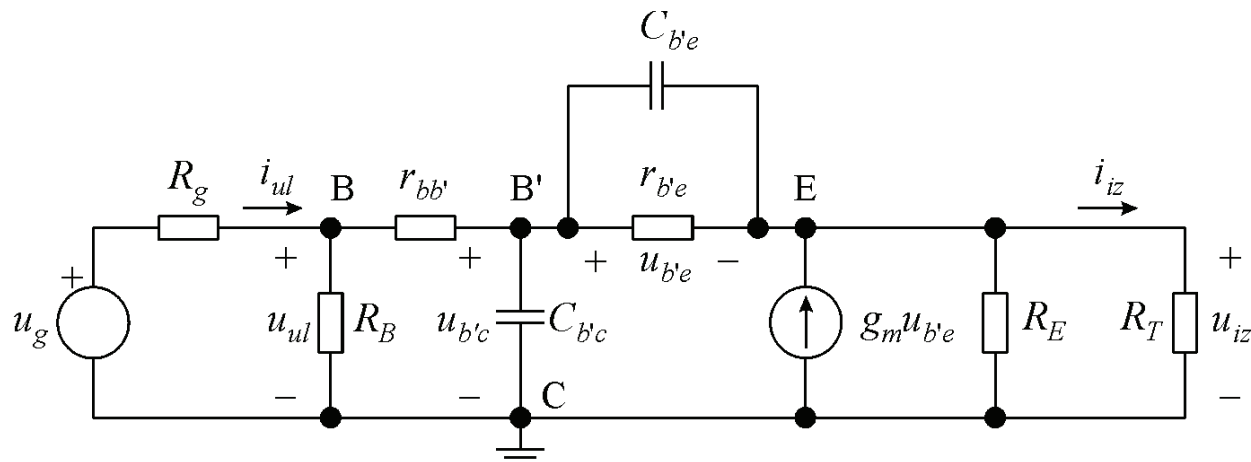
Primjer 4.7 (2)



Emitorsko sljedilo na visokim frekvencijama



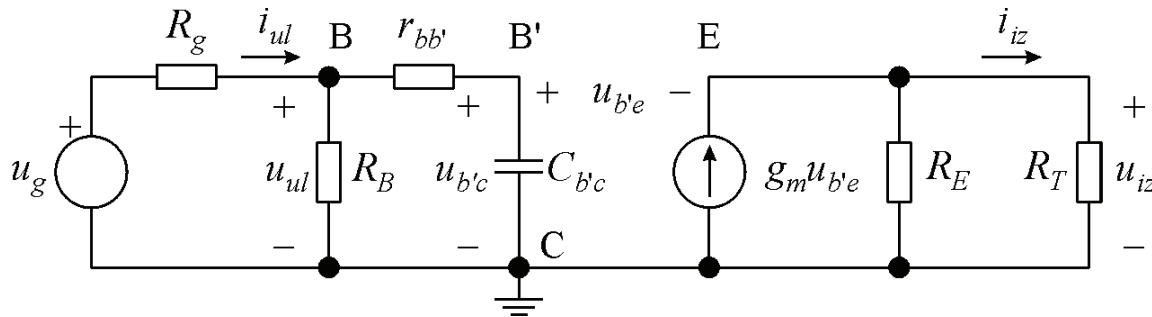
Nadomjesna shema za visokofrekvencijsku analizu



$$A_{V0} = U_{iz}/U_{ul} \approx 1 \rightarrow K = \frac{U_{iz}}{U_{b'c}} \approx \frac{U_{iz}}{U_{ul}} = A_{V0} \approx 1$$

Preslikavanje – $r_{b'e} \rightarrow \infty$, $C_{b'e} \rightarrow 0$ – $r_{b'e}$ i $C_{b'e}$ mogu se izostaviti

Visokofrekvencijska analiza



$$U_{iz} = g_m (R_E \parallel R_T) U_{b'e}$$

$$\frac{U_{b'c}}{U_{ul}} = \frac{1/j\omega C_{b'c}}{r_{bb'} + 1/j\omega C_{b'c}} = \frac{1}{1 + j\omega r_{bb'} C_{b'c}}$$

$$U_{b'e} = U_{b'c} - U_{iz} = \frac{U_{ul}}{1 + j\omega r_{bb'} C_{b'c}} - U_{iz}$$

Naponsko pojačanje

$$A_V(j\omega) = \frac{U_{iz}}{U_{ul}} = \frac{A_{V0}}{1 + j\omega/\omega_{ul}}$$

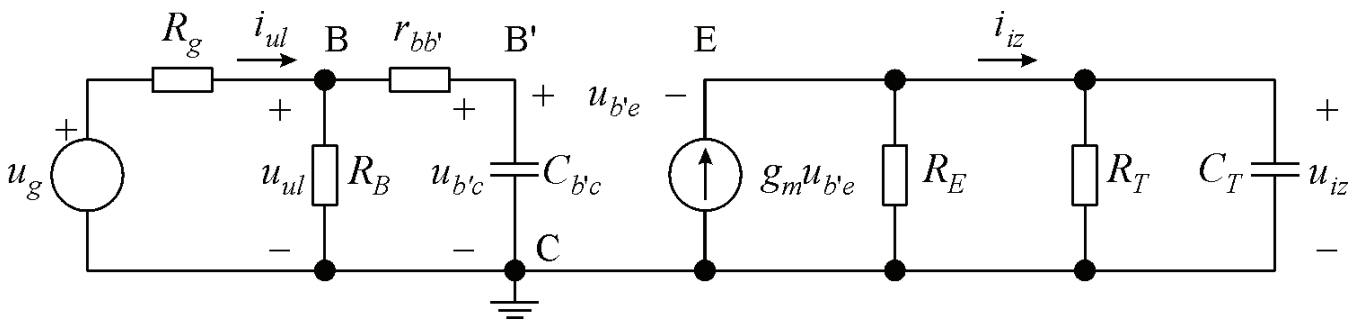
$$A_{V0} = \frac{g_m(R_E \parallel R_T)}{1 + g_m(R_E \parallel R_T)} \quad \tau_{ul} = \frac{1}{\omega_{ul}} = r_{bb'} C_{b'c}$$

Gornja granična frekvencija $\rightarrow \omega_g = \omega_{ul}$

uz konačni R_g

$$\tau_{ul} = (r_{bb'} + R_g \parallel R_B) C_{b'c}$$

Naponsko pojačanje – uz kapacitivno opterećenje



$$\tau_{iz} = \frac{1}{\omega_{iz}} = (R_E \parallel R_T) C_T$$

$$\omega_g = \min \{ \omega_{ul}, \omega_{iz} \} = \min \left\{ \frac{1}{\tau_{ul}}, \frac{1}{\tau_{iz}} \right\}$$