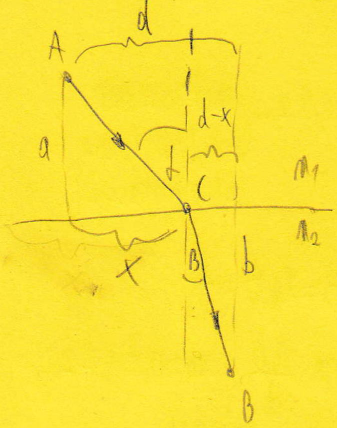
① Izvedi zakon loma svjetlosti iz Fermatovog principa.



$$t_{AB} = t_{AC} + t_{CB} = \frac{\overline{AC}}{v_1} + \frac{\overline{CB}}{v_2} = \frac{\sqrt{a^2+x^2}}{v_1} + \frac{\sqrt{b^2+(d-x)^2}}{v_2}$$

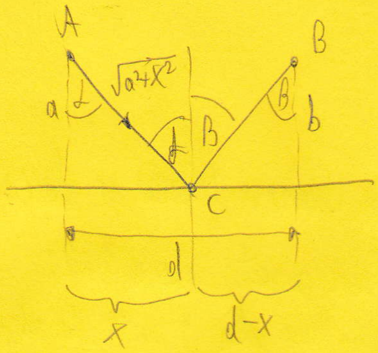
$$\frac{dt_{AB}}{dx} = 0$$

$$\frac{1}{v_1}\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{a^2+x^2}} + \frac{1}{v_2}\cdot\frac{\cancel{2}(d-x)\cdot(-1)}{\cancel{2}\sqrt{b^2+(d-x)^2}} = 0$$

$$\frac{n_1}{c}\cdot\sin\alpha = \frac{n_2}{c}\cdot\sin\beta$$

$$\boxed{\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}}$$

② Izvedi zakon refleksije svjetlosti iz Fermatovog principa.



$$t_{AB} = \int_A^B \frac{n\,dl}{c} = \frac{n}{c}\left(\overline{AC}+\overline{CB}\right) = \frac{n}{c}\left(\sqrt{a^2+x^2}+\sqrt{b^2+(d-x)^2}\right)$$

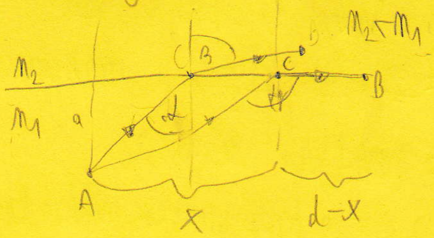
$$\frac{dt_{AB}}{dx} = 0 \Rightarrow \frac{n}{c}\left(\frac{\cancel{2}x}{\cancel{2}\sqrt{a^2+x^2}} + \frac{\cancel{2}(d-x)\cdot(-1)}{\cancel{2}\sqrt{b^2+(d-x)^2}}\right) = 0$$

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$\sin\alpha = \sin\beta$$

$$\boxed{\alpha = \beta}$$

③ Pronadi uvjet za totalnu refleksiju svjetlosti.



$$t_{AB} = t_{AC} + t_{CB} = \frac{\overline{AC}}{v_1} + \frac{\overline{CB}}{v_2} = \frac{\sqrt{a^2+x^2}}{v_1} + \frac{d-x}{v_2}$$

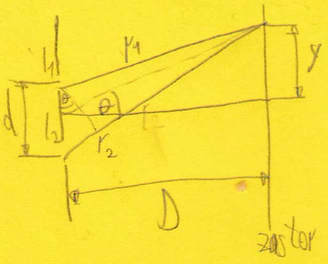
$$\frac{dt_{AB}}{dx} = 0 \Rightarrow \frac{1}{v_1}\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{a^2+x^2}}\left(-\frac{1}{v_2}\right) = 0$$

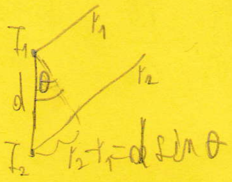
vrijedi ako je $\beta = 90°$

$$n_1 \sin\alpha_g = n_2 \underbrace{\sin(90°)}_{1}$$

$$\boxed{\sin(\alpha_g) = \frac{n_2}{n_1}}$$

④ Izvedi izraz za položaje maksimuma intenziteta na zastor u Youngovom pokusu.





$\delta = r_2 - r_1 = d\sin(\theta)$

$d\sin\theta_m = m\lambda$ $m = 0, \pm1, \pm2, \dots$ maksimum (konstruktivna interferencija)

pretpostavka:

za mali θ: $\theta \approx \sin\theta = tg\,\theta = \frac{y}{D}$

$d \cdot \frac{y}{D} = m\lambda \Rightarrow y_{max} = m \cdot \frac{\lambda D}{d}$ $m = 0, \pm1, \pm2, \dots$

⑤ Izvedi izraz za položaje minimuma intenziteta na zastoru u Youngovom pokusu (slika ista)

$\delta = r_2 - r_1 = d\sin(\theta)$

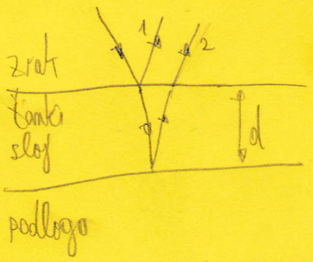
$d\sin\theta_m = (m + \frac{1}{2})\lambda$ $m = 0, \pm1, \pm2, \dots$ minimum (destruktivna interferencija)

pretpostavka:

za mali θ: $\theta \approx \sin\theta = tg\,\theta = \frac{y}{D}$

$d \cdot \frac{y}{D} = m\lambda \Rightarrow y_{min} = (m + \frac{1}{2}) \cdot \frac{\lambda D}{d}$ $m = 0, \pm1, \pm2, \dots$ minimum

⑥ Izvedi uvjete maksimuma za interferenciju pri refleksiji na tankom filmu u slučaju kada je $n_{sloj} > n_{podloga}$



① promjena faze za π

② nema promjene u fazi

$n_{sloj} > n_{zrak}$ $n_{sloj} > n_{podloga}$

Razlika faza između valova 1 i 2:

- promjena faze vala 1 za π, što odgovara $\frac{\lambda_n}{2}$
- geometrijska razlika putova je $2d$
- da bi valovi 1 i 2 bili u fazi:

$2d = \frac{\lambda_n}{2}, \frac{3\lambda_n}{2}, \frac{5\lambda_n}{2}, \dots$ maximum

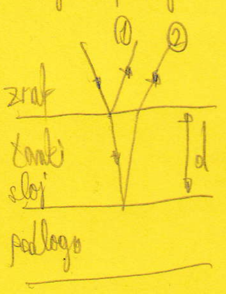
$2d = (m + \frac{1}{2})\lambda_n$ $m = 0, 1, 2, \dots$

- val 2 širi se kroz sredstvo indeksa loma n

$\lambda_n = \frac{\lambda}{n}$

$2nd = (m + \frac{1}{2})\lambda$ $m = 0, 1, 2, \dots$

⑦ Izvedi uvjete maksimuma za interferenciju pri refleksiji na tankom filmu u slučaju kada je $n_{sloj} < n_{podloga}$



$n_{sloj} > n_{zrak}$ $n_{sloj} < n_{podloga}$

① promjena faze za π

② —— || ——

Razlika faza između valova 1 i 2:

- promjena faze valova 1 i 2 za π, razlika je 0
- geometrijska razlika putova (za okomite zrake) je $2d$
- da bi valovi 1 i 2 bili u fazi:

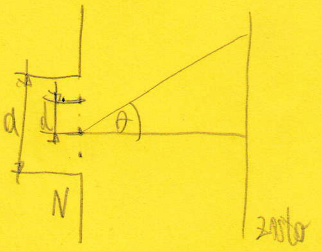
$2d = 0, \lambda_n, 2\lambda_n, \dots$ maksimum

$2d = m \cdot \lambda_n \quad m = 0,1,2,\dots$

- val 2 širi se kroz sredstvo indeksa loma n ($\lambda_n = \frac{\lambda}{n}$)

$2nd = m\lambda \quad m = 0,1,2,\dots$

⑧ Počevši od izraza za interferenciju N točkastih izvora $I(\alpha) = I_0 \cdot \frac{\sin^2\left(N \cdot \frac{\pi d}{\lambda} \sin\alpha\right)}{\sin^2\left(\frac{\pi d}{\lambda} \sin\alpha\right)}$ izvedi izraz za intenzitet pri difrakciji na pukotini.



$$I(\theta) = I_0 \cdot \frac{\sin^2\left(N \cdot \frac{\pi d}{\lambda} \sin\theta\right)}{\sin^2\left(\frac{\pi d}{\lambda} \sin\theta\right)}$$

$Nd = a$

$N \to \infty \Rightarrow d \to 0$

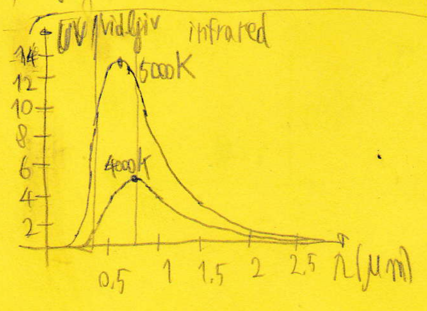
$$\sin\left(\frac{\pi d}{\lambda} \sin\theta\right) \approx \frac{\pi d}{\lambda} \cdot \sin\theta \approx \frac{\pi \cdot a}{N \cdot \lambda} \sin\theta$$

$$I(\theta) = I_0' \cdot \frac{\sin^2\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\left(\frac{\pi}{\lambda} a \sin\theta\right)^2} \qquad \boxed{I_0' = I_0 \cdot N^2}$$

minimumi: $\frac{\pi}{\lambda} a \sin\theta = m\pi$

$\sin\theta = m\frac{\lambda}{a} \quad m = 0, \pm 1, \pm 2, \dots$

⑨ Skiciraj Planckovu distribuciju zračenja jednog crnog tijela na dvije različite temp. Diskutiraj položaj maksimuma. Diskutiraj zračenje površine ispod krivulje.



- što je veća temperatura, to je veći intezitet (veći maksimum)
- $\lambda_{max} \cdot T = konst.$
- Površina ispod krivulje ≅ ukupni intezitet

(10) Napišite Schrödingerovu jedn. za općeniti potencijal $V(x,t)$, a zatim riješite jedn. za slobodnu česticu $V(x,t)=0$.

$$-\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\cdot\Psi(x,t)+V(x,t)\Psi(x,t)=i\hbar\cdot\frac{\partial}{\partial t}\Psi(x,t)$$

$V(x,t)=0$

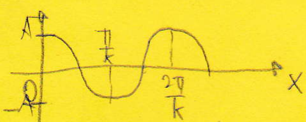
$$-\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\cdot\Psi(x,t)=i\hbar\frac{\partial}{\partial t}\cdot\Psi(x,t)$$

$$\Psi(x,t)=Ae^{i(kx-\omega t)}$$

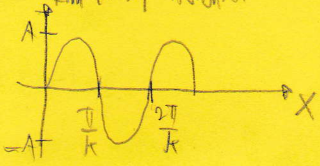
– uvrštavanjem u jedn.:

$$\boxed{\frac{\hbar^2k^2}{2m}=E}\qquad \text{jer je } p=\hbar k$$

$Re\,\Psi(x,0)=A\cos kx$



$Im\,\Psi(x,0)=A\sin kx$



(11) Za danu fizikalnu veličinu f (npr. položaj čestice x), napiši izraz za njenu neodređenost Δf, ako je zadana valna f-ja $\Psi(x)$ koja opisuje česticu.

$$\langle f(x)\rangle=\int_{-\infty}^{+\infty}\Psi^*(x)\,f(x)\,\Psi(x)\,dx$$

$$\langle f(x)^2\rangle=\int_{-\infty}^{+\infty}\Psi^*(x)\,(f(x))^2\,\Psi(x)\,dx$$

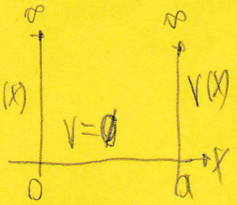
$$(\Delta f)^2=\sigma^2=\langle(f(x)-\langle f(x)\rangle)^2\rangle=\int_{-\infty}^{+\infty}\Psi^*(x)\,(f(x)-\langle f(x)\rangle)^2\,\Psi(x)\,dx$$

$$=\int_{-\infty}^{+\infty}\Psi^*(x)(f(x))^2\Psi(x)dx-2\langle f(x)\rangle\int_{-\infty}^{+\infty}\Psi^*(x)f(x)\Psi(x)dx+\langle f(x)\rangle^2\int_{-\infty}^{+\infty}\Psi^*(x)\Psi(x)dx$$

$$=\langle f(x)^2\rangle-\langle f(x)\rangle^2$$

$$\boxed{\Delta f=\sqrt{\langle f(x)^2\rangle-\langle f(x)\rangle^2}}$$

(12) Krenuvši od vremenski nezavisne Schrödingerove jedn., nađite rješenja za česticu u 1D potencijalnoj jami, tj. odredi valne f-je i energije čestice.



$$-\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\Psi(x)+V(x)\Psi(x)=E\cdot\Psi(x)$$

$$V(x)=\begin{cases}0 & ,\text{za } 0<x<a\\ \infty & ,\text{za } x<0 \text{ i } x>a\end{cases}$$

$\Psi(x)=A_1e^{ikx}+A_2e^{-ikx}$

rubni uvjeti:

$\Psi(0)=0\qquad \Psi(a)=0$

unutar jame:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x)=E\,\Psi(x)$$

$ka=n\pi\;,\;n=1,2,3\ldots\qquad p_n=\frac{h}{\lambda}=\frac{n\cdot h}{2a}\quad n=1,2,3\ldots$

$\Psi(0)=A_1+A_2=0\;\Rightarrow\;\boxed{A_2=-A_1}$

$\boxed{\Psi(x)}=i\cdot 2A_1\sin kx=\boxed{C\sin kx}\;\Rightarrow\;\Psi(a)=C\sin(ka)=0$

$\boxed{E}=\frac{p_n^2}{2m}=\boxed{\frac{n^2h^2}{8ma^2}}$ ✓

$$\int_0^a|\Psi(x)|^2dx=1=C^2\cdot\int_0^a\sin^2\left(\frac{n\pi x}{a}\right)dx=C^2\cdot\frac{1}{2}\int_0^a\left(1-\cos\left(\frac{2n\pi x}{a}\right)\right)dx=C^2\cdot\frac{1}{2}\int_0^a dx-C^2\cdot\frac{1}{2}\int_0^a\cos\left(\frac{2n\pi x}{a}\right)dx$$

$$=\frac{C^2}{2}x\Big|_0^a=\frac{C^2a}{2}=1\;\Rightarrow\;\boxed{C=\sqrt{\frac{2}{a}}}\;\Rightarrow\;\boxed{\Psi_n=\sqrt{\frac{2}{a}}\cdot\sin\left(\frac{n\pi x}{a}\right)}$$ ✓

(13) Napiši izraze pomoću kojih biste izračunali srednju vrijednost položaja i količine gibanja za česticu opisanu valnom f-jom $\psi(x,t)=\psi(x)\,\phi(t)$.

$$\Psi(x,t)=\psi(x)\cdot\phi(t)=\psi(x)\cdot e^{-i\frac{Et}{\hbar}}$$

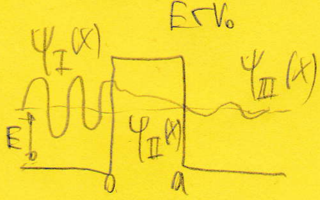
• Srednja vrijednost položaja: $\langle x\rangle=\int_{-\infty}^{+\infty}\Psi^*(x,t)\,x\,\Psi(x,t)\,dx=\int_{-\infty}\psi(x)\cdot e^{i\frac{Et}{\hbar}}\,x\,\psi(x)\cdot e^{-i\frac{Et}{\hbar}}\,dx$

$$=\int_{-\infty}^{+\infty}\psi^*(x)\,x\,\psi(x)\,dx$$

• Srednja vrijednost količine gibanja: $\langle p\rangle=\int_{-\infty}^{+\infty}\Psi^*(x,t)\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi(x,t)\,dx=-i\hbar\cdot\int_{-\infty}^{+\infty}\Psi(x,t)\cdot\frac{\partial\Psi(x,t)}{\partial x}\,dx$

$$=-i\hbar\int_{-\infty}^{+\infty}\psi^*(x)\frac{\partial\psi(x)}{\partial x}\,dx$$

(14) Napiši opća rješenja (neodređene konst.) za valnu f-ju koja opisuje česticu koja nailazi na potencijalnu barijeru. Napiši rubne uvjete koje biste primijenili za određivanje konstanti koje ste napisali. Pomoću tih konstanti napiši def. koeficijenata transmisije i refleksije.



$$\Psi_I(x)=Ae^{ik_1x}+Be^{-ik_1x}\qquad k_1=\frac{\sqrt{2mE}}{\hbar}$$

$$\Psi_{II}(x)=Ce^{k_2x}+De^{-k_2x}\qquad k_2=\frac{\sqrt{2m(V_0-E)}}{\hbar}$$

$$\Psi_{III}(x)=Ee^{ik_1x}$$

rubni uvjeti: $\psi_I(0)=\psi_{II}(0)\qquad\frac{\partial\psi_I(0)}{\partial x}=\frac{\partial\psi_{II}(0)}{\partial x}$

$$\psi_{II}(a)=\psi_{III}(a)\qquad\frac{\partial\psi_{II}(a)}{\partial x}=\frac{\partial\psi_{III}(a)}{\partial x}$$

→ upadni val: $\psi_u(x)=Ae^{ik_1x}$

→ reflektirani val: $\psi_r(x)=Be^{-ik_1x}$

→ transmitirani val: $\psi_t(x)=Ee^{ik_1x}$

$$R=\frac{|\psi_r(x)|^2}{|\psi_u(x)|^2}=\frac{B^*B}{A^*A}\quad i\quad T=\frac{|\psi_t(x)|^2}{|\psi_u(x)|^2}=\frac{E^*E}{A^*A}$$

(15) Komentirajte izraz za transmisijski koef. za česticu energije E koja nailazi na potencijalnu barijeru širine a i visine V_0, $T\simeq\frac{16(V_0/E-1)}{(V_0/E)^2}\cdot e^{-2k_2a}$, gdje je $k_2=\frac{\sqrt{2m(V_0-E)}}{\hbar}$

→ postoji vjerojatnost prolaska čestice kroz potencijalnu barijeru – ta je vjerojatnost veća što je barijera uža (tj. a manji), što je barijera niža i energija čestice E veća (tj. što je razlika V_0-E manja) i što je masa čestice manja

→ ovo vrijedi u skladu s relacijom neodređenosti