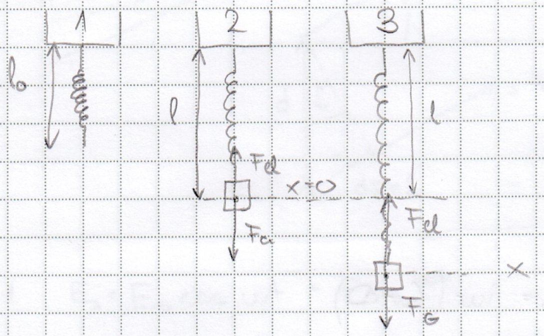
FIZIKA 2 - NARANČIĆ

1. Harmonički oscilator



Ravnotežni položaj (2)

$$\vec{F}_G = -\vec{F}_{el}$$

$$mg = -k\Delta l \rightarrow \text{Hooke}$$

$$mg = -k(l-l_0)$$

Produljenje x (3)

$$ma = F$$

$$m\frac{d^2x}{dt^2} = F_G + F_{el}$$

$$m\frac{d^2x}{dt^2} = mg - k(l+x-l_0)$$

$$m\frac{d^2x}{dt^2} = \underbrace{mg - k(l-l_0)}_{0} - kx$$

$$m\frac{d^2x}{dt^2} + kx = 0 \rightarrow \text{harmonički oscilator u diferencijalnom obliku}$$

$x(t) = ?$

Početni uvjeti: $t=0$, $x(t=0)=x_0$, $v(t=0)=v_0$

$$x = Ce^{\Omega t}$$

$$m\Omega^2 Ce^{\Omega t} + kCe^{\Omega t} = 0$$

$$Ce^{\Omega t}(m\Omega^2 + k) = 0 \Rightarrow m\Omega^2 + k = 0$$

$$\Omega^2 = -\frac{k}{m} = -\omega^2$$

$$\Omega = \pm i\omega$$

Rješenje linearne dif. jednadžbe:

$$x = Ce^{i\omega t} + De^{-i\omega t}$$

$v(t) = ?$

$$v(t) = \frac{dx}{dt} = i\omega Ce^{i\omega t} - i\omega De^{-i\omega t}$$

Početni uvjeti:

$$t = 0, \quad x_0 = C + D, \quad v_0 = i\omega(C - D)$$

$$C + D = \mathrm{Re}C + \mathrm{Re}D + i\mathrm{Im}C + i\mathrm{Im}D$$

$$x_0 \in R \Rightarrow C + D \in R \Rightarrow \mathrm{Im}C = -\mathrm{Im}D$$

$$i\omega(C - D) = i\omega(\mathrm{Re}C + i\mathrm{Im}C - \mathrm{Re}D - i\mathrm{Im}D)$$

$$v_0 \in R \Rightarrow i\omega(C - D) \in R \Rightarrow \mathrm{Re}C = \mathrm{Re}D$$

$C = D^*$ (kon. kompl. broj)

$C = Ee^{i\varphi}$

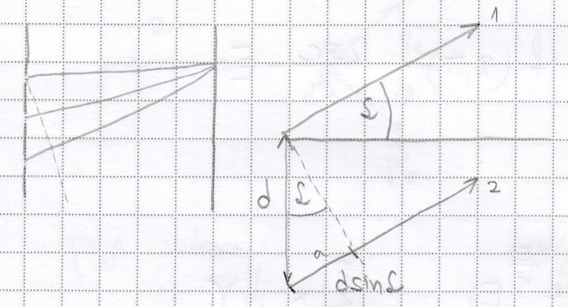
$D = Ee^{-i\varphi}$

$$x = Ee^{i(\omega t + \varphi_0)} + Ee^{-i(\omega t + \varphi_0)} = 2E\cos(\omega t + \varphi_0)$$

$$2E = \text{const} = A$$

$$x = A\sin(\omega t + \varphi_0) \qquad \varphi_0 = \varphi + \frac{2}{\pi}$$

2. Interferencija za N koherentnih izvora



$\sin\alpha = \frac{a}{d}$ $\quad a = d\sin\alpha$

1. $E_1 = E_0 \cos\omega t = E_0 e^{i\omega t}$
2. $E_2 = E_0 \cos(\omega t + \varphi) = E_0 e^{i(\omega t + \varphi)}$
3. $E_3 = \dots$
4. $E_n = \dots = E_0 e^{i(\omega t + (n-1)\varphi)}$

$$E_p = E_1 + E_2 + \dots + E_n$$
$$= E_0 e^{i\omega t}\left(1 + e^{i\varphi} + e^{i2\varphi} + \dots + e^{i(n-1)\varphi}\right)$$

$$\left[\begin{array}{l} S_N = a_1 \frac{q^n - 1}{q - 1} \\ S_N = \frac{e^{in\varphi} - 1}{e^{i\varphi} - 1} \end{array}\right]$$

$$E_p = E_0 e^{i\omega t} \cdot \frac{e^{in\varphi} - 1}{e^{i\varphi} - 1}$$

$$= E_0 e^{i\omega t} \frac{e^{in\frac{\varphi}{2}}\left(e^{in\frac{\varphi}{2}} - e^{-in\frac{\varphi}{2}}\right)}{e^{i\frac{\varphi}{2}}\left(e^{i\frac{\varphi}{2}} - e^{-i\frac{\varphi}{2}}\right)} = E_0 e^{i\omega t} e^{i(N-1)\frac{\varphi}{2}} \frac{\operatorname{sh} in\frac{\varphi}{2}}{\operatorname{sh} i\frac{\varphi}{2}}$$

$$= E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}} e^{i\left(\omega t + (N-1)\frac{\varphi}{2}\right)}$$

$$= E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}} \cos\left(\omega t + (N-1)\frac{\varphi}{2}\right)$$

- amplituda $\rightarrow$ $E_{op} = E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}$

$$I = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_{op}^2$$

$$I(\alpha) = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0^2 \frac{\sin^2 N\frac{\varphi}{2}}{\sin^2\frac{\varphi}{2}}$$

- max I :

$\varphi = 0$

$$\lim_{\varphi\to 0}\left(\frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}\right)^2 = \lim_{\varphi\to 0}\left(\frac{N\cos N\frac{\varphi}{2}}{\cos\frac{\varphi}{2}}\right)^2 = N^2$$

$$I_p(\varphi = 0) = N^2 I_0$$

3. Planckov zakon zračenja crnog tijela

Rayleigh-Jeansov zakon: $f(\lambda, T) = \frac{2\pi c}{\lambda^4} \bar{E}$

$\bar{E} = \frac{E}{N}$

$N = N_0 e^{-\frac{E_N}{kT}}$ $\qquad x = e^{-\frac{h\nu}{kT}}$

$$\bar{E} = \frac{\sum_{i=0}^{\infty} N_i E_i}{\sum_{i=0}^{\infty} N_i} = \frac{\sum_{i=0}^{\infty} \cancel{N_0} e^{-\frac{E_i}{kT}} E_i}{\sum_{i=0}^{\infty} \cancel{N_0} e^{-\frac{E_i}{kT}}} = \frac{\sum_{i=0}^{\infty} x^i \, i h\nu}{\sum_{i=0}^{\infty} x^i} =$$

$$= h\nu \frac{\sum_{i=0}^{\infty} i x^i}{\sum_{i=0}^{\infty} x^i} = h\nu \frac{x + 2x^2 + \ldots}{1 + x + \ldots} = h\nu x \frac{1 + 2x + \ldots}{1 + x + \ldots}$$

$$= h\nu x \frac{\frac{d}{dx}\left(\frac{1}{1-x}\right)}{\frac{1}{1-x}} = h\nu x \frac{\frac{1}{(1-x)^2}}{\frac{1}{(1-x)}} = h\nu x \frac{1}{(1-x)}$$

$$\bar{E} = \frac{h\nu x}{1-x} = \frac{h\nu x}{x(x-1)} = \frac{h\nu}{x-1} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \qquad \nu = \frac{c}{\lambda}$$

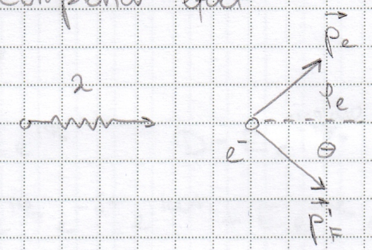
$$f(\lambda, T) = \frac{2\pi c}{\lambda^4} \frac{h \frac{c}{\lambda}}{e^{\frac{h\nu}{kT}} - 1}$$

$$= \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}$$

$$\frac{hc}{\lambda} \ll kT \Rightarrow \frac{\frac{hc}{\lambda}}{kT} \ll 1$$

$$f(\lambda, T) = \frac{2\pi}{\lambda^5} hc^2 \frac{1}{\frac{hc}{\lambda kT}} = \frac{2\pi}{\lambda^5} hc^2 \frac{\lambda kT}{hc} = \frac{2\pi}{\lambda^4} ckT$$

4. Comptonov efekt



prije sudara:

$$E = h\frac{c}{\lambda}$$

$$p_F = \frac{h}{\lambda}$$

$$E_0 = m_e c^2$$

poslije sudara:

$$E' = h\frac{c}{\lambda'}$$

$$p_F' = \frac{h}{\lambda'}$$

$$p_e, E_e$$

ZOE: $E + E_0 = E' + E_e$

$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E_e \quad /:c$$

$$\frac{h}{\lambda} + m_e c = \frac{h}{\lambda'} + \frac{E_e}{c}$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'} + m_e c = \frac{E_e}{c} \quad |^2 \qquad E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 + 2\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right) m_e c + \cancel{(m_e c)^2} = p_e^2 + \cancel{m_e^2 c^2} \qquad (*)$$

ZOKG: $\vec{p}_F = \vec{p}_F' + \vec{p}_e \quad |^2$

$$\left(\vec{p}_F - \vec{p}_F'\right)^2 = \vec{p}_e^{\,2}$$

$$\left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h^2}{\lambda\lambda'}\cos\theta_f = p_e^2 \qquad (**)$$

$$(*) = (**) \Rightarrow \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 + 2 m_e c\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right) = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h^2}{\lambda\lambda'}\cos\theta_f$$

$$\ldots \quad m_e c \Delta\lambda = h(1 - \cos\theta) = h \cdot 2\sin^2\frac{\theta}{2}$$

$$\Delta\lambda = \frac{2h}{m_e c}\sin^2\frac{\theta}{2}$$

5. Prigušeno titranje + slabo prigušenje

- imamo silu koja djeluje u suprotnom smjeru

$F = -b\vec{v}$

$m\frac{d^2x}{dt^2} = -kx - bv$

$m\frac{d^2x}{dt^2} + kx + bv = 0 \quad / : m$

$\frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}\frac{dx}{dt} = 0$

$2\delta = \frac{b}{m}$

$\delta = \frac{b}{2m}$

$\frac{k}{m} = \omega_0^2$

P.U. $t=0 \quad x = x_0 \quad v = v_0$

$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = 0 \qquad x = Be^{\lambda t}$

$\frac{dx}{dt} = B\lambda e^{\lambda t} \qquad \frac{d^2x}{dt^2} = B\lambda^2 e^{\lambda t}$

$B\lambda^2 e^{\lambda t} + 2\delta B\lambda e^{\lambda t} + \omega_0^2 Be^{\lambda t} = 0$

$Be^{\lambda t}(\lambda^2 + 2\delta\lambda + \omega_0^2) = 0$

$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$

... slabo prigušenje $\quad \omega_0^2 > \delta^2$

$\omega^2 = \omega_0^2 - \delta^2$

$\lambda_{1,2} = -\delta \pm i\omega$

Rj: $x = Ce^{-\delta t}e^{i\omega t} + De^{-\delta t}e^{-i\omega t}$

P.U. $x_0 = C + D$

- deriviramo po vremenu

$\frac{dx}{dt} = v = C(i\omega - \delta)e^{-\delta t}e^{i\omega t} + D(-i\omega - \delta)e^{-\delta t}e^{-i\omega t}$

$t = 0, \quad v = v_0$

$v_0 = C(i\omega - \delta) + D(-i\omega - \delta) = -\delta(C+D) + i\omega(C-D)$

$\mathrm{Re}\,C = \mathrm{Re}\,D$

$D = C^*$

$C = E e^{i\varphi_0} \qquad D = E e^{-i\varphi_0}$

$x = E e^{i\varphi_0} e^{-\delta t} e^{i\omega t} + E e^{-i\varphi_0} e^{-\delta t} e^{-i\omega t}$

$= E e^{-\delta t} e^{i(\omega t + \varphi_0)} + E e^{-\delta} e^{-i(\omega t + \varphi_0)} = 2E e^{-\delta t} \cos(\omega t + \varphi_0)$

$x = A_0 e^{-\delta t} \sin(\omega t + \varphi_0)$

$A(t) = A_0 e^{-\delta t}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \delta^2}} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$

za malo prigušenje $\omega = \omega_0$

6. Transverzalni val koji se širi tankom žicom

$\mu = \frac{m}{L}$ - linearna gustoća mase užeta

$dF_y = F \sin \mathcal{L}_2 - F \sin \mathcal{L}_1$

$A \ll \lambda$ pa su kuteви mali

$\Rightarrow \sin \mathcal{L}_1 \approx tg\, \mathcal{L}_1 \qquad \sin \mathcal{L}_2 \approx tg\, \mathcal{L}_2$

$dF_y = F\, tg\, \mathcal{L}_2 - F\, tg\, \mathcal{L}_1$

$dF_y = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_x\right]$

$= F\left[\left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial^2 x}\right)_x dx - \left(\frac{\partial y}{\partial x}\right)_x\right]$

$= F \frac{\partial^2 y}{\partial x^2} dx$

$dF_y = \mu \frac{\partial^2 y}{\partial t^2} dx$ zbog $F = ma$ i $m = \mu dx$

$\Downarrow$

$F \frac{\partial^2 y}{\partial x^2} = \mu \frac{\partial^2 y}{\partial t^2}$

$\frac{\partial^2 y}{\partial x^2} - \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2} = 0$ - valna jednadžba

Rj: $u = t - \frac{x}{v} \qquad = f(u)$

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial t} = f' \frac{\partial u}{\partial t} = f'$

$\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial u^2}\frac{\partial u}{\partial t} = f''$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} = f' \frac{-1}{v}$

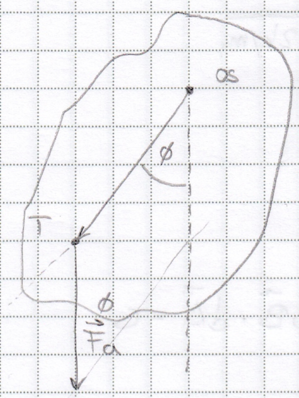
$\frac{\partial^2 f}{\partial x^2} = f''\left(\frac{-1}{v}\right)\frac{\partial u}{\partial x} = \frac{1}{v^2} f''$

$\Rightarrow$

$\frac{1}{v^2} f'' - \frac{\mu}{F} f'' = 0$

$v = \sqrt{\frac{F}{\mu}}$ → fazna brzina

7. Fizičko njihalo



$$M_z = -mgb\sin\phi = I\alpha = I\frac{d^2\phi}{dt^2}$$

deriviramo M po vremenu

$$I\frac{d^2\phi}{dt^2} = -mgb\sin\phi$$

$$\frac{d^2\phi}{dt^2} + \frac{mgb}{I}\sin\phi = 0$$

$$\frac{d^2\phi}{dt^2} + \omega^2\phi = 0$$

$$\omega^2 = \frac{mgb}{I}$$

$$T = 2\pi\sqrt{\frac{I}{mgb}}$$

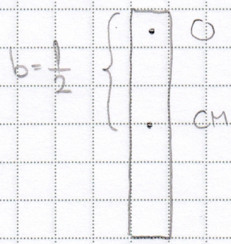
$$\phi = \phi_0 \sin(\omega t + \varphi_0)$$

- mijenjamo fizičko njihalo s matematičkim

$$T_F = T_L \qquad l = l_r$$

$$2\pi\sqrt{\frac{I}{mgb}} = 2\pi\sqrt{\frac{l_r}{g}}$$

$$l_r = \frac{I}{mb}$$ - reducirana duljina



$$I = I_{CM} + md^2 = \frac{1}{12}mL^2 + m\frac{L^2}{4} = \frac{1}{3}mL^2$$

$$l_r = \frac{I}{mb} = \frac{\frac{1}{3}mL^2}{m\frac{L}{2}} = \frac{2}{3}L$$