8. KVANTNA PRIRODA SVJETLOSTI

$I = \sigma T^4$ → intenzitet

↓ Stefan-Boltzmannova konstanta

$$f(\lambda, T) = \underbrace{\left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left(\frac{hc}{\lambda}\right)}_{R\text{-J}} \cdot \underbrace{\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}}_{\text{Planckov dio}}$$

$$f(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

8-1

$$\frac{f(\lambda_1, T)}{f(\lambda_2, T)} = 20$$

FOTOELEKTRIČNI EFEKT

$E_0 = h \cdot f \cdot m$

m - broj kvanta (1, 2, 3, ...)

↓ po Plancku - nije moguć m dobar izlazni rad

$$\boxed{E = W_i - E_k}$$ → opći slučaj

8.12

8.2 $f = 1{,}23 \cdot 10^{20}\ Hz$

$m_e = 9{,}11 \cdot 10^{-31}\ kg$

$E_{k_{max}} = h \cdot f$

↓

ako zanemarimo W_i (ne znamo o kojem metalu je riječ)

$$1\,J = 1\,eV \cdot 1{,}602 \cdot 10^{-19}\,C$$

$E_{k_{max}} = 8{,}15 \cdot 10^{-15}\ J$

$E_k = \frac{m v^2}{2}$ [J] → deterministički

$$E_{k_{max}} = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

→ mnogo to komplicirati

relativistički

$v = 0{,}866\ c$

8.5 $\lambda_1 = 0{,}35\ \mu m$

$\lambda_2 = 0{,}54\ \mu m$

$W_i = ?$

$V_{max_1} = 2 V_{max_2}$

$f = \frac{c}{\lambda}$

$hf = W_i + E_{k_{max}}$

$h \cdot \frac{c}{\lambda_1} = W_i + E_{k_{max_1}}$

$h \cdot \frac{c}{\lambda_2} = W_i + E_{k_{max_2}}$

$W_i = W_i$

$h \cdot \frac{1}{\lambda_1} - E_{k_1} = h \cdot \frac{c}{\lambda_2} - E_{k_2}$

$E_k = \frac{m_e v^2}{2}$

W_i → konstanta za metal

$hc \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) = E_{k_1} - E_{k_2}$

$= m \cdot \frac{4 v_2^2}{2} - \frac{m v_2^2}{2}$

$= \frac{3}{2} \cdot m v_2^2$

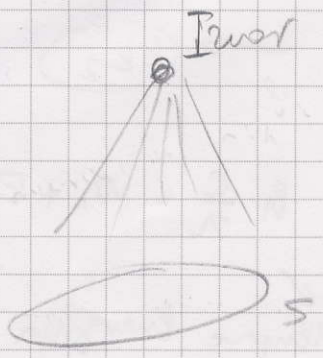
$v_2 = \ldots$

$W_i = \ldots$

8.6 $\lambda = 300\ nm = 300 \cdot 10^{-9}\ m$

$S = 4\ cm^2 = 4 \cdot 10^{-4}\ m^2$

$I = 15 \cdot 10^{-2}\ W/m^2$



$$I = \frac{P}{S}$$

$$P = I \cdot S$$

$$P = \frac{W}{t} = \frac{E}{t}$$

$$E_k = h \cdot \frac{c}{\lambda}$$

$$E = 6,626 \cdot 10^{-19}\ J$$

$$t = \frac{E}{P} \Rightarrow \frac{1}{t} = \frac{P}{E} = 9,055 \cdot 10^{13}\ s^{-1}$$

8.7 = 8.8

$U = 150\ V$

$E_k = U \cdot e$

$e = 1,602 \cdot 10^{-19}\ As$

$E_k = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$

$U \cdot e$

$v = 0,024\ c$

8.9 $\lambda = 325\ nm$

$U = 1{,}91\ V \Rightarrow E_k = U \cdot e$

E_{k1} → ← $U \cdot e$ → zaustavni napon

→ E_{k2}

$h \cdot \frac{c}{\lambda} = W_i + E_k$

↙ Izlazna energija

$W_i = h \cdot \frac{c}{\lambda} - U \cdot e\ [J]$

$W_i^{[eV]} = \frac{W_i^{[J]}}{e}$

8.10 $\lambda_{max} = 545\ nm$

$h \cdot \frac{c}{\lambda} = W_i + E_{k\,max}$

$\lambda = 200\ nm$

1.) $h \cdot \frac{c}{\lambda_{max}} = W_i$

$\frac{c}{\lambda_{max}} = f_{min}$

$W_i = 2{,}276\ eV$

2.) $E_k = h\frac{c}{\lambda} - W_i^{[J]}$

$E_k^{[eV]} = \frac{1}{e}(\ldots)$

$U = \frac{E_k^{[eV]}}{e} = 3{,}927\ V$

8.11 $\lambda = 450$ nm

$\lambda_1 = 600$ nm → $W_{i1} = 2{,}068$ eV

$W_{i2} = 2W_{i1} - 4{,}136$ eV

ostalo kao u 8.10

$E = h \cdot \frac{c}{\lambda} = 2{,}757$ eV

1.) $2{,}757$ eV $= 2{,}068$ eV $+ U$

$U = + \ldots -$

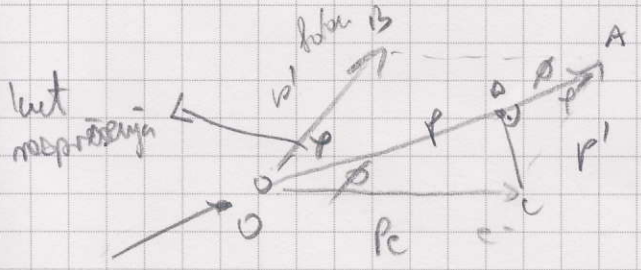
2) $2{,}757$ eV $= 4{,}136$ eV $+ U$

⇓ nema fotoelektričnog efekta

PRIMJER 8.10

$E = 0{,}75$ MeV

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\varphi)$$



$$p = m \cdot v \qquad E = p \cdot c \rightarrow E_{\text{fotona}}$$

$$\lambda' - \lambda = \frac{hc}{E'} - \frac{hc}{E} = \frac{h}{m_e c}(1 - \cos\varphi)$$

$$E' = 0{,}4328 \text{ MeV}$$

$$E_{\substack{\text{fotona} \\ \text{prije sudara}}} = E' + E_e$$

$$E_e = 0{,}32 \text{ MeV}$$

$$\operatorname{tg}\phi = \frac{\overline{CD}}{\overline{OD}} = \frac{\overline{CA}\cdot\sin\varphi \;/:\overline{CA}}{\overline{OA} - \overline{CA}\cos\varphi \;/:\overline{CA}}$$

$$\operatorname{tg}\phi = \frac{\sin\varphi}{\frac{\overline{OA}}{\overline{CA}} - \cos\varphi} = \frac{\sin\varphi}{\frac{r}{r'} - \cos\varphi}$$

$$\operatorname{tg}\phi = \frac{\sin\varphi}{\frac{E}{E'} - \cos\varphi} = \frac{\sin\varphi}{\frac{E}{E'} - \cos\varphi}$$

$$\boxed{\operatorname{tg}\phi = \frac{\sin\varphi}{\frac{E}{E'} - \cos\varphi}}$$

$$\phi = 35^\circ$$

PRIMJER 8.M

Opći slučaj

$$E_e = E - E'$$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\varphi)$$

$$\lambda' = \frac{hc}{E'} \qquad \lambda = \frac{hc}{E}$$

$$\frac{hc}{E'} - \frac{hc}{E} = \frac{h}{m_e c}(1 - \cos\varphi)$$

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\varphi)$$

$$\frac{1}{E'} = \frac{1}{m_e c^2}(1 - \cos\varphi) + \frac{1}{E}$$

$$E' = \frac{E}{1 + \frac{E}{m_e c}(1 - \cos\varphi)}$$

$$E_e = E - E'$$

$$\boxed{E_e = \frac{1 - \cos\varphi}{1 + \frac{E}{m_e c^2}(1 - \cos\varphi)} \cdot \frac{E^2}{m_e c^2}}$$

a) $\varphi = 0$

$E_e = 0$

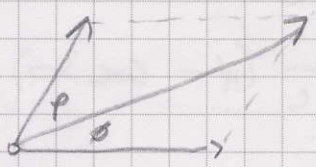
b) $\varphi = \pi$

$$E_e = \frac{1}{1 + \frac{E}{m_e c^2}}$$

PRIMJER 8.13

$E = 4 \cdot 10^{-14}\ J$

$\Delta\lambda = 1{,}5 \cdot 10^{-12}\ m$



$\alpha = \theta + \varphi$

$\frac{h}{m_e c}$

$\Delta\lambda = \lambda' - \lambda = \lambda_c (1 - \cos\varphi)$

$\left(\frac{\Delta\lambda}{\lambda_c} - 1 \right) = -\cos\varphi$

$$\varphi = \arccos\left(1 - \frac{\Delta\lambda}{\lambda_c}\right)$$

$\varphi = 67{,}56°$

$E = h \cdot \frac{c}{\lambda}$ → prije sudara

$\lambda = \frac{h \cdot c}{E}$

$\lambda' - \lambda = \Delta\lambda$

$\lambda' = \Delta\lambda + \lambda$

$$tg\,\theta = \frac{\sin\varphi}{\frac{E}{E'} - \cos\varphi}$$

8,17

$$\lambda'_{120} = 3\lambda'_{30}$$

$$\lambda'_{120} - \lambda = \frac{h}{m_e c}(1 - \cos 120)$$

$$\lambda'_{30} - \lambda = \frac{h}{m_e c}(1 - \cos 30)$$

$$\left.\begin{aligned} \frac{h}{m_e c} &= \frac{2}{3}(3\lambda'_{30} - \lambda) \\ \frac{h}{m_e c} &= 7{,}464(\lambda'_{30} - \lambda) \end{aligned}\right\} =$$

$$\frac{2}{3}(3\lambda'_{30} - \lambda) = 7{,}464(\lambda'_{30} - \lambda)$$

$$2\lambda'_{30} - 11{,}2\lambda'_{30} = -6{,}8\lambda$$

$$8{,}2\lambda'_{30} = 6{,}8\lambda$$

$$\boxed{\lambda'_{30} = 0{,}83\lambda}$$

$$\lambda'_{30} - \lambda = \frac{h}{m_e c}(1 - \cos 30)$$

$$0{,}83\lambda - \lambda = \lambda_c(1 - \cos 30)$$

$$\lambda = \text{---}$$

STRUKTURA ATOMA

$\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$ → za vodik

Ljuske	k
K	$k=1$
L	$k=2$
M	$k=3$
⋮	

stanje	n
stac.	$n=1$
1. pobuđeno	$n=2$
2. "	$n=3$

PRIMJER 10.3

$3Li^{++}$ — H

$$E_n = \frac{E_0}{n^2}$$

$$E_n = -\frac{Z^2 m e^4}{8\hbar^2 \varepsilon_0^2} \cdot \frac{1}{n^2}$$

E_0 ... $n=1$

$Z(Li) = 3$

$E_1 = -122{,}4\ eV$ → stac. stanje litija

$E_2 = \frac{E_1}{4}$

$E_3 = \frac{E_1}{9}$

$E_4 = \frac{E_1}{16}$

K_α	$k=1$	$n=2$
K_β	$k=1$	$n=3$
K_γ	$k=1$	$n=4$

$$\frac{1}{\lambda_2} = Z_{Li}^2 R\left(\frac{1}{1} - \frac{1}{4}\right)$$

$$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$$

$R = R_\infty$ → u žutim formulama

↓ opća skroz

PRIMJER 10.9

$X = 2{,}5 \cdot 10^{-10}\ m$

a) FOTONI

$\lambda = X = 2{,}5 \cdot 10^{-10}\ m$

$E = h \cdot \frac{c}{\lambda}\ [J]$

$E = 4{,}9\ keV$

b) ELEKTRONI

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$2m \cdot \frac{mv^2}{2} = m^2v^2 = p^2$

$E = 24{,}1\ eV$

10.1 $E_\gamma = 2{,}55\ eV = 4{,}0851 \cdot 10^{-19}\ J$

$n = 2$

$E_0 = -13{,}68\ eV$

$E_n = -13{,}68 \cdot \frac{Z^2}{n^2}\ [eV]$

$E_1 = \frac{E_0}{4} = -3{,}42\ eV$

$E_2 = \frac{E_0}{9} = -1{,}52\ eV$

$E_3 = \frac{E_0}{16} = -0{,}855\ eV$ $\quad n = 4$

$E = E_\gamma + E_1$

$E = -0{,}87\ eV$

$n = 4$

10.3 $E = 12{,}09\ eV$

$E_0 = -13{,}68\ eV$

$E = -1{,}59\ eV$ $\qquad E_2 = -1{,}52\ eV$

$n = 3$

$$r_n = \frac{\varepsilon_0 h^2}{\pi m_e Z e^2} \cdot n^2$$

$=$

$r_2 = 9 r_0$

$$r_n = 0{,}528 \cdot 10^{-10}\ \frac{n^2}{Z}$$

→ opći izraz

10.7 $\lambda = 489\ nm$ $\qquad E_0 = -13{,}6\ eV$

$E_\gamma = h \cdot \frac{c}{\lambda} = 2{,}537\ eV$ $\qquad E_x = -3{,}4\ eV$

$E_x = \frac{E_0}{n^2} \Rightarrow n = 2$

$E_{vezanje} = E_\gamma + E_x = -0{,}863\ eV$

Borovi POSTULATI → TO SAMI PROUČITE

1. elektron se može gibati samo po određenim kvantiziranim putanjama
2. kutna količina gibanja ovisi ...
3. kada elektron prelazi s više razine energije na nižu, on emitira foton

10.13 $\lambda = 0{,}0788\ nm$

$$\frac{1}{\lambda} = z^2 R \left(\frac{1}{k^2} - \frac{1}{m^2} \right)$$

L ... $m = 2$

K ... $k = 1$

$z = 40$

↓

Zirkonium

NUKLEARNA FIZIKA

$^{A}_{Z}X$

A - maseni broj

Z - redni broj

$N = A - Z$

m_u

α raspad

$$^{A}_{Z}X \rightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$$

β raspad

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu}e$$

e^- ↓ pozitron

$\bar{\nu}e$ → anti-neutrino

$$^{Z}_{A}X + e^{+} \rightarrow {}^{A}_{Z+1}Y + \nu e$$

$$A = -\frac{dN}{dt} \quad [Bq = s^{-1}]$$

A ↓ aktivnost

$N_{t=0}$

$$\boxed{N = N_0 e^{-\lambda t}}$$

broj nerazpadnutih jezgara

$$\boxed{A = A_0 e^{-\lambda t}}$$

aktivnost u $t=0$

$$A = N \cdot \lambda$$

$$\frac{1}{2}N_0 = N_0 \cdot e^{-\lambda t}$$

$$\ln\frac{1}{2} = -\lambda t$$

$$\ln 2 = \lambda t$$

$$\lambda = \frac{\ln 2}{t}$$

$t = T_{1/2} \Rightarrow$ $\boxed{T_{1/2} = \frac{\ln 2}{\lambda}}$

$T_{1/2}$ – vrijeme poluraspada

$\boxed{\tau = \frac{1}{\lambda}}$

τ – srednje vrijeme života

PRIMJER M.1

$A = 10^5\ s^{-1}$

$^{14}_{6}C$

$T_{1/2} = 5568$ god

$A = \lambda \cdot N$

$\boxed{A = \frac{\ln 2}{T_{1/2}} \cdot m \cdot \frac{N_A}{M}}$

N_A – Avogadrova konstanta

M – molna masa

$n = \frac{m}{M} = \frac{N}{N_A}$

$N = \frac{m \cdot N_A}{M}$

PRIMJER 11.3

$N_0 = 10^6$

$v = 2200\ m\,s^{-1}$

$d = 11\ m$

$T_{1/2} = 12\ min$

$t = \frac{d}{v} = 5\ ms$

$N = N_0 e^{-\lambda t}$

$\Delta N = N_0 - N_0 e^{-\lambda t}$

$\Delta N = N_0 (1 - e^{-\lambda t})$

$\Delta N = \lambda t \cdot N_0$

PRIMJER 11.5

$^{235}_{92}U$ ... 0,72 % $T_{1/2} = 7{,}038 \cdot 10^8$ god

$^{238}_{92}U$ ... 99,2745 % $T_{1/2} = 4{,}468 \cdot 10^9$ god.

$N_0^{(235)} = N_0^{(238)}$

$N = N_0 e^{-\lambda t}$

$\frac{N^{(238)}}{N^{(235)}} = 137{,}88$

$N_0 = \frac{N}{e^{-\lambda t}}$

$\frac{N^{(235)}}{e^{-\lambda(235)t}} = \frac{N^{(238)}}{e^{-\lambda(238)t}} \cdot 137{,}88$

$e^{t(\lambda(235) - \lambda(238))} = 137{,}88$

$t = \frac{\ln 137{,}88}{\lambda^{(235)} - \lambda^{(238)}} = \dots$

$\frac{MeV}{c^2} = 1{,}07 \cdot 10^{-3}\, u$ $\quad u = 1{,}66 \cdot 10^{-27}\, kg$

PRIMJER 11.13

$^{32}_{15}P$ β raspad

$E = 1{,}71\ MeV$

$r = 0{,}1\ m$

$E_0 = m_e \cdot e^2 = 0{,}511\ MeV$

↓

kada računamo brzinu, ako je energija $E > E_0$ onda uzimamo relativističke formule

$B \cdot e \cdot v = m \cdot \frac{v^2}{r}$

$$B = \frac{1}{e \cdot r} m \cdot v$$

$$E_k = E - E_0 = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

11.7

$m = 1\ g$

$A = 1{,}48 \cdot 10^5\ Bq$

$\frac{^{14}C}{^{12}C} = 10^{-6}$ $\quad T_{1/2}(^{14}C) = 5568\ god$

$$A_0 = \frac{\ln 2}{T_{1/2}} \cdot N_A \cdot \frac{m}{M \rightarrow {}^{12}C}$$

$A = A_0 \cdot e^{-\lambda t}$

$\ln \frac{A}{A_0} = -\lambda t$

$$t = \frac{1}{\lambda} \cdot \ln \frac{A_0}{A}$$

11.6

^{14}C

$A_0 = 200\ Bq$

$A = 190\ Bq$

$T_{1/2} = 5570\ god$

$A = A_0 \cdot e^{-\lambda t}$

$$t = \frac{T_{1/2}}{\ln 2} \cdot \ln \frac{A_0}{A}$$

11.4

$E_k = 10\ keV$

$r = 0{,}1\ m$

$B = ?$

$E_k = \frac{mv^2}{2} \Rightarrow v = \dots$

$B = \frac{1}{e \cdot r} \cdot m \cdot v$

$B = \dots$

11.1

$^{3}_{1}H$ $\quad m(^{3}_{1}H) = 3{,}016\,05\ u$

$\Delta m = 0{,}000\,02\ u$

$m(He) = 3{,}016\,03\ u$

$^{3}_{1}H \rightarrow {}^{3}_{2}He + e^- + \overline{\nu_e}$

$\frac{MeV}{c^2} = 1{,}07 \cdot 10^{-3}\ u$

$E = \Delta m \cdot c^2 = \dots$