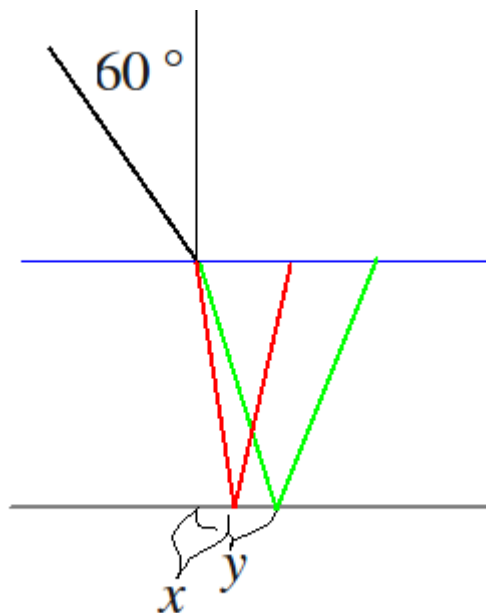


5.1



slika 5.1

- upadnu kud $u = 60^\circ$
- izlazni kutevi l_1 i l_2
- h udaljenost od površine do zrcala

$$n_{11} = n_{12} = 1$$

$$n_{21} = \frac{4}{3}$$

$$n_{22} = \frac{8}{3}$$

$$l = \frac{n_1}{n_2} \sin u$$

$$l_1 = 40.50^\circ$$

$$l_2 = 18.95^\circ$$

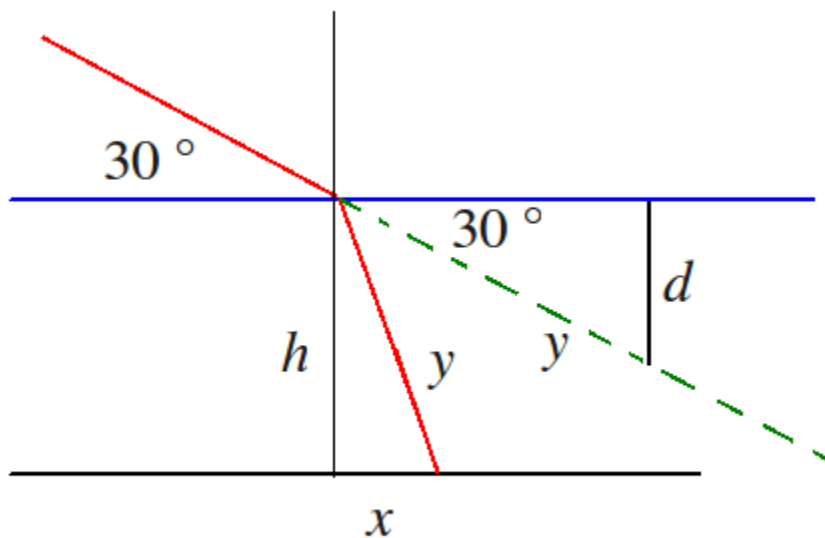
$$x = h \cdot \tan l_2$$

$$y = h \cdot \tan l_1$$

- udaljenosti od upadne do izlazne zrake iz vode $d_1 = 2y$ $d_2 = 2x$

$$k = \frac{d_1}{d_2} = \frac{\tan l_1}{\tan l_2} = 2.487$$

5.2



slika 5.2

- ovaj nisam siguran, pretpostavljam da put koji prođe zraka u vodi po crvenoj liniji jednak je puti koji prođe po zelenoj

- upadni kut $u = 60^\circ$

$$l = 40.63^\circ$$

$$y = \frac{h}{\cos l}$$

$$d = y \sin 30 = \frac{h}{\cos l} \sin 30^\circ = 1.976 \text{ m}$$

5.3

$$n_2 = 1.33$$

$$n_1 = 1.52$$

- idemo unatrag, minimalan $\sin u = \frac{n_2}{n_1}$

$$u = 61^\circ$$

- ako zraka ide unatrag tada je upadni kut na granicu presjeka valjka i vode $l = 90^\circ - u$, a izlazni kut je jednak α

$$\sin \alpha = \sin l \frac{n_1}{n_2}$$

$$\alpha = 33.59^\circ$$

- ako je α veci tada je l veci, pa je u manji i tad nemamo refleksiju pa mora vrijediti $\alpha < 33.59^\circ$

5.4

$$\delta_{minl} = 12^\circ$$

- ako je δ minimalan vrijedi da je $u_1 = u_2$ $l_1 = l_2$

$$2u_{vode} = \delta + A$$

$$l_{vode} = \frac{A}{2}$$

$$\sin l_{vode} = \frac{n_{vode}}{n_{prizme}} \sin u_{vode}$$

$$n_{prizme} = 1.621$$

– ako je u ulju

$$l_{ulja} = \frac{A}{2}$$

$$\sin u_{ulja} = \sin l_{ulja} \frac{n_{prizme}}{n_{ulja}}$$

$$u_{ulja} = 27.57$$

$$u_{ulja1} = u_{ulja2} = u_{ulja}$$

$$\delta_{min2} = 2u_{ulja} - A = 5.14^\circ$$

5.5

$$r = 16m$$

$$a = 1.49 \cdot 10^{11}$$

$$r_s = 6.95 \cdot 10^8 m$$

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}$$

$$\frac{1}{b} = \frac{2}{r} - \frac{1}{a}$$

$$\frac{1}{b} = 0.125$$

$$b = 8m$$

$$y = \frac{-b}{a} = 8.389 \cdot 10^{-13}$$

$$r_{zrcalni} = r_s * y = 3.73 cm$$

$$visina = 2r_{zrcalni} = 7.46 cm$$

5.6

$$1^\circ \quad y = \frac{-b}{a} \quad \frac{1}{4} = \frac{b}{a} \quad b = \frac{a}{4}$$

$$2^\circ \quad y = \frac{-b}{a} \quad \frac{1}{2} = \frac{b}{a+5} \quad b = \frac{1}{2}(a+5)$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$\frac{1}{a} + \frac{1}{\frac{a}{4}} = \frac{1}{(a-5)} + \frac{1}{\frac{1}{2}(a-5)} \quad \frac{1}{f} = \frac{5}{a} \quad f = \frac{a}{5}$$

$$\frac{5}{a} = \frac{3}{a-5}$$

$$5a - 25 = 3a$$

$$a = 12.5$$

$$f = 2.5 \text{ cm}$$

5.7

- imamo sustav
- uzmemo kao da je predmet u beskonacnosti $a_1 = \infty$

$$\frac{1}{f_{lece}} = n_1 - 1 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$f_{lece} = 0.4 \text{ m}$$

$$f_{zrcala} = \frac{r}{2} = 0.2$$

$$\frac{1}{a_1} + \frac{1}{b_1} = \frac{1}{f_{lece}}$$

$b_1 = f_{lece}$ - to predstavlja sliku za zrcalo (virtualni sliku)

$$a_2 = -b_1 = -f_{lece}$$

$$\frac{-1}{f_{lece}} + \frac{1}{b_2} = \frac{1}{f_{zrcala}}$$

$$\frac{1}{b_2} = \frac{f_{lece} + f_{zrcala}}{f_{lece} f_{zrcala}}$$

b_2 - je opet virtualna slika za lecu

$$a_3 = -b_2$$

$$\frac{1}{a_3} + \frac{1}{b_3} = \frac{1}{f_{lece}} \quad - \quad b_3 \quad \text{je} \quad \text{zarisna duljina}$$

$$-\frac{f_{lece} + f_{zrcala}}{f_{lece} f_{zrcala}} + \frac{1}{b_3} = \frac{1}{f_{lece}}$$

$$\frac{1}{f} = \frac{1}{b_3} = \frac{f_{zrcala} + f_{lece} + f_{zrcala}}{f_{lece} f_{zrcala}}$$

$$f = 0.1 \text{ m}$$

5.8

$f = 30 \quad a = 40 \quad \frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ - treba postaviti zrcalu na mjestu gdje nastaje slika

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \quad \frac{1}{b} = \frac{1}{30} - \frac{1}{40} = \frac{4-3}{120} = \frac{1}{120}$$

$$d = b - a = 80 \text{ cm}$$

5.9

$$n = 1.33$$

– od vode imamo plan-konkavno zrcalo

$$\frac{1}{f_{lece}} = (n-1) \left(\frac{-1}{r_2} \right) = \frac{1}{180} \quad n_1 = 1 \quad r_1 = \infty$$

- imamo predmet $a_1 = \infty$

$$\frac{1}{b_1} = \frac{1}{f_{lece}}$$

$b_1 = 180 = -a_2$ - a_2 je udaljenost realna slike lece (virtualna slika za zrcalo)

$$\frac{1}{a_2} + \frac{1}{b_2} = \frac{1}{f_{zrcal}}$$

$$\frac{1}{b_2} = \frac{1}{30} + \frac{1}{180} = \frac{6+1}{180} = \frac{7}{180}$$

$$b_2 = \frac{180}{7} = -b_3 \quad \text{- virtualna slika za lecu}$$

$$\frac{1}{b_3} + \frac{1}{f} = \frac{1}{f_{lece}} \quad \frac{-7}{180} + \frac{1}{f} = \frac{1}{180}$$

$$\frac{1}{f} = \frac{8}{180}$$

$$f = 22.5 \text{ cm}$$

5.10

– isti ko i 5.9

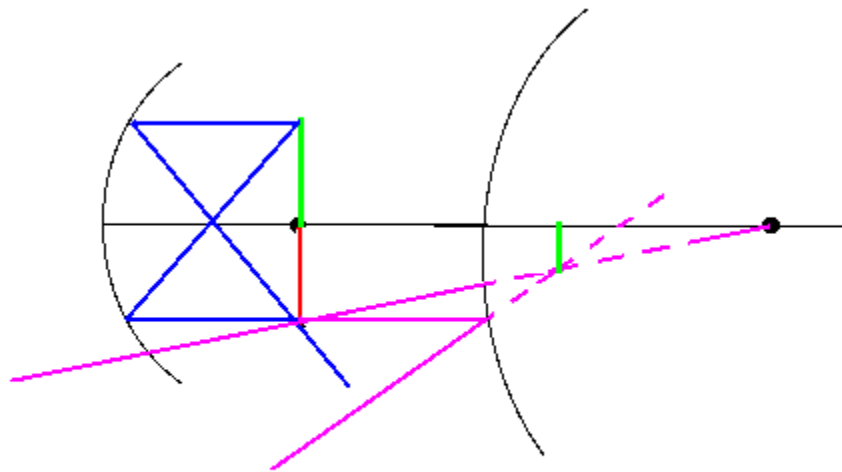
$$f_{lece} = 2.22 \text{ m} \quad f_{zrcala} = 0.4 \text{ m}$$

$$b_1 = 2.22 \text{ m}$$

$$b_2 = 0.33 \text{ m}$$

$$f = 0.294 \text{ m}$$

5.11



slika 5.11

- na konkavnom zrcalu

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}$$

$$\frac{1}{b} = \frac{1}{12.5} - \frac{1}{25}$$

$$b = 25 \text{ cm}$$

- na konveksnom

$$\frac{1}{a} + \frac{1}{b} = \frac{-2}{r}$$

$$\frac{1}{25} + \frac{1}{b} = \frac{-1}{25}$$

$$b = -12.5$$

- posto je b negativan sa desne strane konveksnog zrcala predmet je virtualan u 12.5 cm iza zrcala
- a ako je zrcalo ravno onda je predmet sa suprotne strane ravnog zrcala na istoj udaljenosti (25 cm)

5.12

- imamo 2 dioptra
- za prvi vrijedi

$$d = 10 \text{ cm}$$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{r}$$

$$n_1 = 1$$

$$a_1 = \text{jako veliki broj}$$

$$n_2 = 1.5$$

$$\frac{n_2}{b_1} = \frac{n_2}{r}$$

$$b_1 = 5$$

- za drugi dioptrar

$$a_2 = d - b_1 = 5$$

$$\frac{n_1}{a_2} + \frac{n_2}{b} = \frac{n_2 - n_1}{r} \quad r = \infty \quad n_2 = 1 \quad n_1 = 1.5$$

$$\frac{1.5}{a_2} + \frac{1}{b} = 0$$

$$b = -3.33$$

5.13

$$y = 5$$

$$\frac{y'}{y} = \frac{b}{a}$$

$$1^\circ \quad y' = 15 \quad \frac{b_1}{a} = \frac{15}{5} \quad b = 3a$$

$$2^\circ \quad y' = 10 \quad \frac{b_2}{(a+1.5)} = \frac{10}{5} \quad b_2 = 2(a+1.5)$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$\frac{1}{a} + \frac{1}{3a} = \frac{1}{(a+1.5)} + \frac{1}{2(a+1.5)}$$

$$\frac{4}{3a} = \frac{3}{2(a+1.5)}$$

$$8a + 12 = 9a$$

$$a = 12 \text{ cm}$$

$$\frac{1}{f} = \frac{4}{3a} = \frac{1}{9}$$

$$f = 9 \text{ cm}$$

5.14

$$r_1 = 10 \text{ cm} \quad r_2 = 12 \text{ cm}$$

$$a = 48 \text{ cm}$$

$$b = 96 \text{ cm}$$

$$n_v = \frac{4}{3}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$\frac{1}{48} + \frac{1}{96} = \frac{3}{96} = \frac{1}{32} = \frac{1}{f}$$

$$\frac{n_2 - n_1}{n_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{1}{f}$$

$$n_2 = \frac{n_1 \left(\frac{1}{f} + \frac{1}{r_1} - \frac{1}{r_2} \right)}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$n_2 = 1.56$$

5.15

– ako je slika realna onda se nalazi s desne strane konvergentne

$$b = 0.3$$

$$a_d = -b$$

– jer je kod divergentne leće predmet s desne strane (slika od prve leće)

$$\frac{1}{a_d} + \frac{1}{b_d} = \frac{1}{f_d}$$

$$\frac{1}{f_d} = \frac{-1}{0.3} + \frac{1}{0.4}$$

$$f_d = -1.2 \text{ m}$$

5.16

$$d = 10 \text{ cm}$$

$$f_1 = f = -f_2 = 25$$

$$a_1 = \infty$$

$$\frac{1}{b_1} = \frac{1}{f}$$

$$b_1 = 25 \text{ cm}$$

$$a_2 = d - b_1 = -15 \text{ cm}$$

$$\frac{1}{a_2} + \frac{1}{b_2} = \frac{1}{f_2}$$

$$\frac{1}{b_2} = \frac{-1}{25} + \frac{1}{15} = \frac{-3+5}{75} = \frac{2}{75}$$

$$b_2 = 37.5 \text{ cm}$$

5.17

– slika leće

$$j = 8$$

$$a_1 = \infty$$

$$\frac{1}{b_1} = \frac{1}{f} = j$$

$$b = \frac{1}{8} m = 12.5 \text{ cm}$$

– predmet za ploču

$$a_2 = d - b = -10 \text{ cm}$$

– ploča je zapravo 2 dioptra

$$\frac{n_1}{a_2} + \frac{n_2}{b_2} = \frac{n_2 - n_1}{r} \quad n_1 = 1 \quad n_2 = \frac{3}{2} \quad r = \infty$$

$$\frac{-1}{10} + \frac{3}{2b_2} = 0$$

$$b_2 = 15 \text{ cm}$$

– debljina plex $d_p = 20 \text{ cm}$

– slika za drugu stranu ploče

$$a_3 = d_p - b_2 = 5 \text{ cm}$$

$$\frac{n_1}{a_3} + \frac{n_2}{b_3} = \frac{n_2 - n_1}{r} \quad n_1 = \frac{3}{2} \quad n_2 = 1 \quad r = \infty$$

$$\frac{3}{15} + \frac{1}{b_3} = 0$$

$b_3 = -5 \text{ cm}$ - znaci da je slika unutar plexiglasa, udaljena 5 cm o dna plex

5.18

$$a = 2 \text{ cm}$$

$$f = 6 \text{ cm}$$

$$d = 10 \text{ cm}$$

$$- \text{ lijeva strana pravokutnika } a_2 = d + \frac{l}{2} = 11 \text{ cm}$$

$$- \text{ desna strana pravokutnika } a_1 = d + \frac{l}{2} = 9 \text{ cm}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$- \text{ povećanje stranice } m = \frac{b_1 - b_2}{a_2 - a_1}$$

$$b = \frac{1}{\frac{1}{f} - \frac{1}{a}} = \frac{f a}{a - f}$$

$$b_2 = 13.2 \quad b_1 = 18$$

5.19

$$u = a + b$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$b = \frac{f a}{a - f}$$

$$u = a + \frac{f a}{a - f} \quad - \text{ trazimo minimumu funkcije}$$

$$\frac{du}{da} = 1 + \frac{f \cdot (a - f) - f a}{(a - f)^2} = 0$$

$$a f - f^2 - a f = -a^2 + 2 a f - f^2$$

$$a^2 - 2 a f = 0$$

$$a(a - 2 f) = 0$$

$$a = 2 f$$

$$a = 40$$

$$d = a + b = 80 \text{ cm}$$

5.20

$$a = ?$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$b = \frac{p}{100} f$$

$$\frac{1}{a} + \frac{100}{pf} = \frac{1}{f}$$

$$a = \pm \frac{p}{p-100} f$$

5.21

$$f_1 \quad f_2$$

$$a_1 = \infty$$

$$\frac{1}{b_1} = \frac{1}{f_1}$$

$$b_1 = f_1$$

$$a_2 = d - b_1 = d - f_1$$

$$\frac{1}{d - f_1} + \frac{1}{a_2} = \frac{1}{f_2}$$

$$\frac{1}{a_2} = \frac{1}{f_2} - \frac{1}{d - f_1} = \frac{d - f_1 - f_2}{f_2(d - f_1)}$$

$$a_2 = f_{\text{sustava}} = \frac{f_2(f_1 - d)}{f_1 + f_2 - d}$$

5.22

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

a)

$$r_1 = \infty$$

$$r_2 = 4.5 \text{ cm}$$

$$n_1 = 1$$

$$n_2 = 1.62$$

$$f_1 = -7.26 \text{ cm}$$

b)

$$r_1 = -4.5$$

$$r_2 = -6.3 \text{ cm}$$

$$n_1 = 1$$

$$n_2 = 1.52$$

$$f = -30.28$$

c)

$$a_1 = \infty$$

$$\frac{1}{b_1} = \frac{1}{f_1} \quad a_2 = d - b_1 - b_1$$

...

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} \quad j_{\text{sustava}} = j_1 + j_2 = 0.17$$

$$f = 5.9 \text{ cm}$$