AUDITORNE

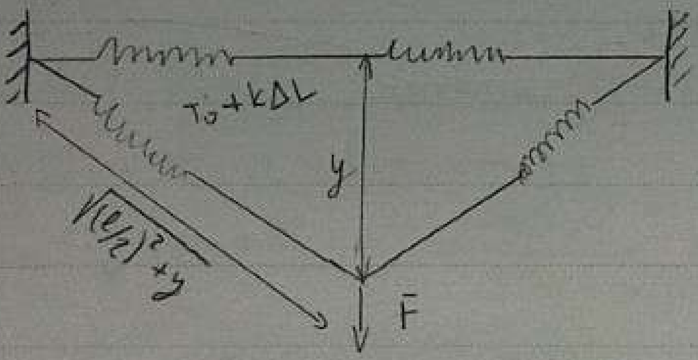
① $d = 0{,}5\ mm$

$T_0 = 100N$

$l = 2m$

$y = 10\ cm = 0{,}1\ m$

$E = 200 \cdot 10^9\ Pa$



$$E = \frac{\frac{F}{S}}{\frac{\Delta L}{L}} \quad , F = k\Delta L \quad \Rightarrow k = \frac{SE}{l/2}$$

$$T[y] = T_0 + k\,\Delta L = T_0 + k\left[\sqrt{(l/2)^2 + y^2} - l/2\right]$$

$$\Delta L = \sqrt{(l/2)^2 + y^2} - l/2$$

$$F[y] = 2 \cdot T[y]\,\frac{y}{\sqrt{(l/2)^2 + y^2}}$$

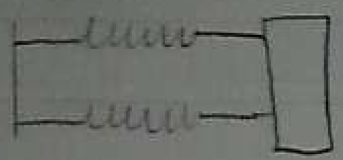
(2) $\Delta x_1 = 4\,cm$, $\Delta x_2 = 6\,cm$

$T_P, T_S = ?$

* - malo o seriji i paraleli opruga

$F = konst$

$\Delta x_1 \neq \Delta x_2$



$\Delta x_1 = \Delta x_2$

$F_1 \neq F_2$

$$mg = k_1 \Delta x_1 = k_2 \Delta x_2$$

a) Paralela

$$k_P = k_1 + k_2$$

$$\omega_P^2 = \frac{k_P}{m} = \frac{k_1}{m} + \frac{k_2}{m} \leftarrow \underbrace{k_1 = \frac{mg}{\Delta x_1}, \; k_2 = \frac{mg}{\Delta x_2}}$$

$$\omega_P^2 = \frac{g}{\Delta x_1} + \frac{g}{\Delta x_2} = g\,\frac{\Delta x_1 + \Delta x_2}{\Delta x_1 \Delta x_2}$$

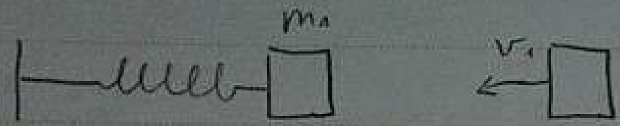
b) Serija

$$\omega_S^2 = \frac{k_S}{m} = \frac{1}{m} \cdot \frac{k_1 k_2}{k_1 + k_2} = \frac{1}{m}\,\frac{\frac{mg}{\Delta x_1} \cdot \frac{mg}{\Delta x_2}}{\frac{mg}{\Delta x_1} + \frac{mg}{\Delta x_2}} = \frac{g}{\Delta x_1 + \Delta x_2}$$

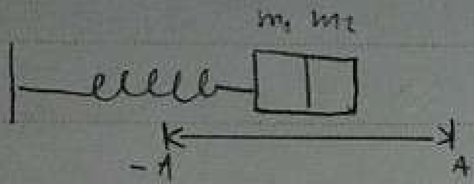
$$\underline{T = \frac{2\pi}{\omega}} \leftarrow \text{samo uvrstimo } \omega_P \text{ i } \omega_S$$

③ m_1, v_1, k

$A = ?$

a) 

prije sudara

b) 

poslije sudara

ZOKG (zakon očuvanja količine gibanja)

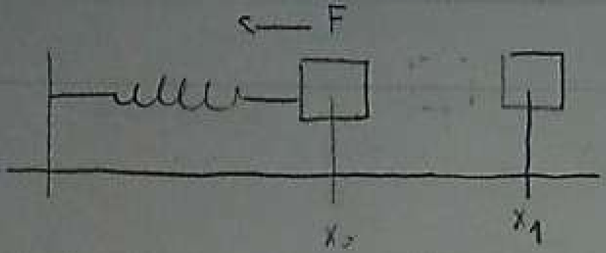
$$m_1 v_1 + m_2 \cdot 0 = (m_1 + m_2) v'$$

$$E_{kin} = \frac{1}{2}(m_1 + m_2) v'^2 = \frac{m_1^2 v_1^2}{2(m_1 + m_2)} = E_{pot} = \frac{1}{2} k A^2$$

$$A = \frac{m_1 v_1}{\sqrt{k(m_1 + m_2)}}$$

4. $m = 1 kg$, $\mu = 0{,}1$, $k = 100 N m^{-1}$, $F_0 = 10 N$

$s = ?$



$x_0 = -\frac{F_0}{k}$, $E_0 = \frac{1}{2} k x_0^2$

$E = \frac{1}{2} k x_1^2 + \mu m g (x - x_2)$

ZOE

$\frac{1}{2} k x_0^2 = \mu m g (x_1 - x_0) + \frac{1}{2} k x_1^2$

$x_1 = -x_0 - \frac{2 \mu m g}{k}$

$\cancel{x_1 < x_0}$

$s = x_1 - x_0 = \frac{2}{k}(F - \mu m g)$

(8) $x[t] = A e^{-\delta t} \cos[\omega t + \phi]$, $t=0$ $\quad x = x_0 > 0$

$A, \phi = ?$

$$x'(t) = -A\delta e^{-\delta t}\cos[\omega t + \phi] - A e^{-\delta t}\omega \sin(\omega t + \phi)$$
$$= -(\delta + \omega \operatorname{tg}(\omega t + \phi))\, x(t)$$

$x(0) = X_0 = A\cos\phi$

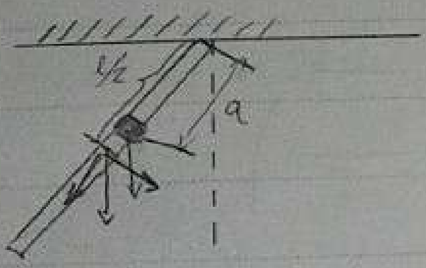
$x' = 0 = -(\delta + \omega \operatorname{tg}\phi)\cdot x_0$ $\quad * \cos = \frac{1}{\sqrt{1+\operatorname{tg}\phi^2}}$

$\operatorname{tg}\phi = -\frac{\delta}{\omega}$

$\cos\phi = \frac{1}{\sqrt{1+\left(\frac{\delta}{\omega}\right)^2}}$, $A = X_0\sqrt{1+\left(\frac{\delta}{\omega}\right)^2}$

(6) l, M

$a = ?$



$\ddot{\phi}$

I_{sys} , $\mathcal{L} = \sum M$

$I_{su} = I_{\check{s}cm} + M\left(\frac{l}{2}\right)^2 + ma^2 = \frac{Ml^2}{3} + ma^2$

$\frac{Ml^2}{12}$

$M = \vec{r} \times \vec{F}$

$M = Mg\sin\phi \frac{l}{2} + mg\sin\phi\, a$

$$\left(\frac{1}{3}Ml^2 + ma^2\right)\ddot{\phi} = -Mg\frac{l}{2}\sin\phi - mag\sin\phi \approx -\left(M\frac{l}{2} + ma\right)g\,\phi$$

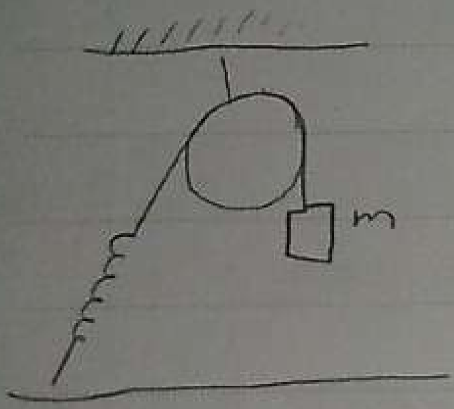
$\ddot{\phi} + \omega_0^2\phi = 0 \qquad \omega_0^2 = \frac{\left(M\frac{l}{2} + ma\right)g}{\frac{Ml^2}{3} + ma^2}$

$$\frac{d\omega_0^2}{da} = 0 = \frac{m\left(\frac{Ml^2}{3} - Mla - ma^2\right)}{\left(m\frac{l^2}{3} + ma^2\right)^2}$$

$$a_{1,2} = \frac{Ml}{2m}\left(1 + \sqrt{1 + \frac{4m}{3M}}\right)$$

7 M, R

$T = ?$



$x = R\phi$

$\dot{x} = R\dot{\phi}$

$$E_{kin} = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I \dot{\phi}^2 \quad , \; I = \frac{1}{2} MR^2$$

$$= \underbrace{\frac{1}{2}\left(m + \frac{M}{2}\right)\dot{x}^2}$$

$$\frac{dE}{dt} = 0 \qquad E_{pot} = \cancel{F_0 x} + \frac{1}{2} kx^2 - \cancel{mg}\, x$$

$$\frac{dE}{dt} = 0 = \frac{d}{dt}\left(E_{kin} + E_{pot}\right) = \left(m + \frac{M}{2}\right)\dot{x} \cdot \left(\ddot{x} + \overbrace{\frac{k}{m + \frac{M}{2}}}^{\omega_0^2} x\right)$$

$$\omega_0 = \sqrt{\frac{k}{m + \frac{M}{2}}}$$

(9) $M = 120t$, $m_p = 1t$, $v_p = 800\,ms^{-1}$, $x_{max} = 1{,}5m$

$$x(t) = e^{-\delta t}\left[x_0 + (v_0 + x_0\delta)t\right]$$

$t = 0$, $x_0 = 0$

ZOKG

$$M \cdot v_0 = m_p \cdot v_p$$

$$v_0 = \frac{m_p v_p}{M}$$

$$x[t] = v_0 t e^{-\delta t} = \frac{m_p v_p}{M} t e^{-\delta t}$$

$$\frac{dx(t)}{dt} = 0 \qquad t = \frac{1}{\delta}$$

$$x_{max} = \frac{m_p v_p}{M}\frac{1}{e\delta}$$

$$\delta = \frac{m_p v_p}{e M \cdot x_{max}} \rightarrow \underbrace{t = \frac{1}{\delta} = \frac{e M x_{max}}{m_p v_p}}$$

$$F = M \cdot a = M\ddot{x}[t] = m_p v_p \delta \underbrace{e^{-\delta t}(-2+\delta t)}_{t=0}$$

$$F_{max} = F[0] = \frac{2 m_p v_p)^2}{e M x_{max}}$$