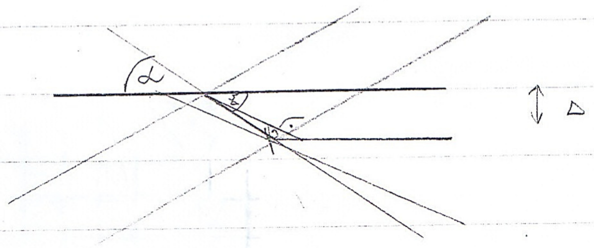
3. CIKLUS

4. PLANPARALELNA PLOČA

sloj optički gušćeg sredstva omeđenog 2. materijalima

d - debljina ploče

n - indeks loma



$$n = \frac{\sin\alpha}{\sin\beta}$$

$$d = d' \cos\beta$$

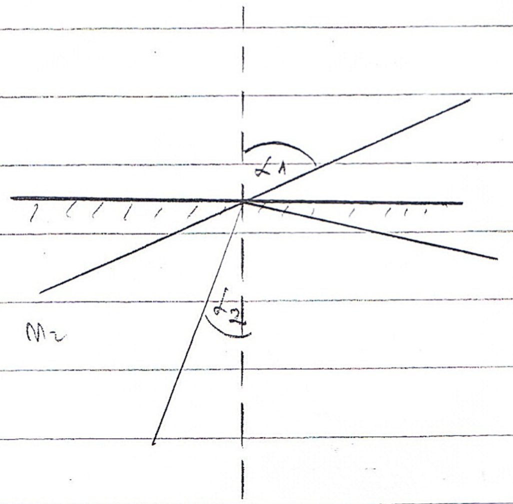
$$\Delta = d' \sin(\alpha - \beta)$$

$$\Delta = \frac{d}{\cos\beta} \cdot \sin(\alpha - \beta) = d\, \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\beta} =$$

$$= d\sin\alpha \left[1 - \frac{\cos\alpha \sin\beta}{\sin\alpha\cos\beta} \right] = d\sin\alpha \left[1 - \cos\alpha \frac{\frac{\sin\alpha}{n}}{\sin\alpha\sqrt{1-\sin^2\beta}} \right] =$$

$$= d\sin\alpha \left[1 - \frac{\cos\alpha}{\sqrt{n^2 - (n\sin\beta)^2}} \right] = d\sin\alpha \left[\frac{1-\cos\alpha}{\sqrt{n^2 - \sin^2\alpha}} \right]$$

5. TOTALNA REFLEKSIJA



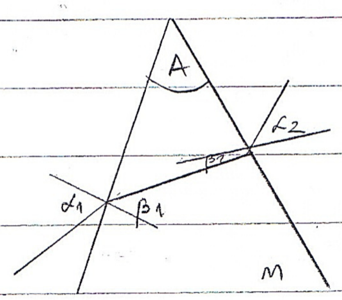
$n_2 > n_1 \rightarrow \dfrac{\sin\alpha_1}{\sin\alpha_2} = \dfrac{n_2}{n_1}$

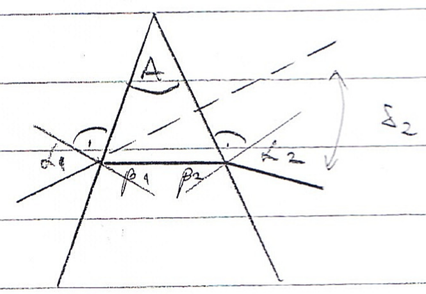
prepostavka : $\alpha_1 \rightarrow \frac{\pi}{2}$

$\sin\alpha_1 = \dfrac{n_1}{n_2}$

granični $\alpha_2 = \arcsin\dfrac{n_1}{n_2}$

6. OPTIČKA PRIZMA





α-upadni kut

δ-kut devijacije

kut devijacije $\delta = \delta_1 + \delta_2$

$\pi = A + \left(\frac{\pi}{2} - \beta_1\right) + \left(\frac{\pi}{2} - \beta_2\right)$

$\beta_2 = A - \beta_1$

$\delta_1 = \alpha_1 - \beta_1$, $\sin\beta_1 = \frac{1}{n}\sin\alpha_1$

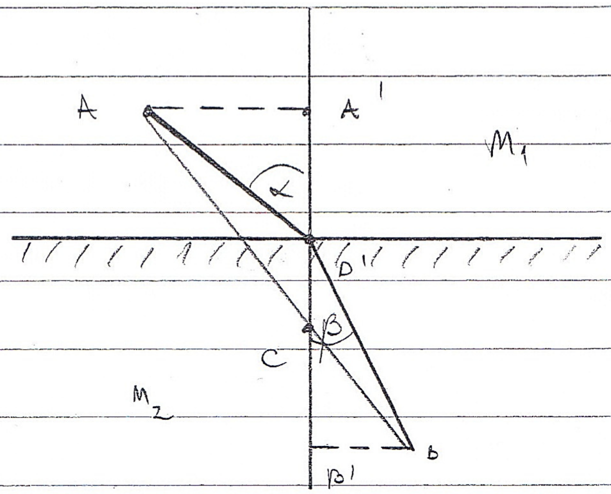
$\beta_1 = \arcsin\dfrac{\sin\alpha_1}{n}$

$\delta_2 = \alpha_2 - \beta_2$, $\sin\beta_2 = \frac{1}{n}\sin\alpha_2$

$\alpha_2 = \arcsin(n\sin\beta_2)$

7. LOM SVJETLOSTI NA SFERNOJ GRANICI

Zakon loma u Möbiusovom obliku



$\Delta(AA'C) \cong \Delta(BB'C)$

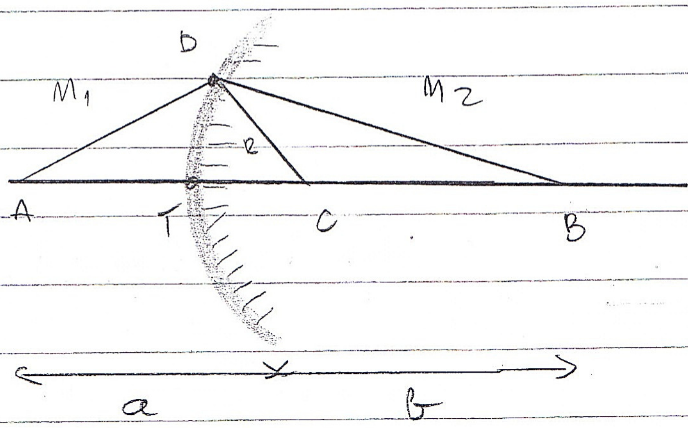
$$\sin\alpha = \frac{\overline{AA'}}{\overline{AD}}$$

$$\sin\beta = \frac{\overline{BB'}}{\overline{BD}}$$

$$\frac{M_2}{M_1} = \frac{\sin\alpha}{\sin\beta} = \frac{\dfrac{\overline{AA'}}{\overline{AD}}}{\dfrac{\overline{BB'}}{\overline{BD}}} = \frac{\overline{AA'}}{\overline{BB'}} \cdot \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{AC}}{\overline{BC}} \cdot \frac{\overline{BD}}{\overline{AD}}.$$

$$\boxed{M_1 \frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}} M_2} \Rightarrow \text{Zakon loma prema Möbiusu}$$

sferna granica



$$\frac{\overline{AC}}{\overline{AD}} \cdot n_1 = \frac{\overline{BC}}{\overline{BD}} n_2$$

$$\left.\begin{aligned} \overline{AD} &\simeq \overline{AT} = a \\ \overline{BD} &\simeq \overline{BT} = b \end{aligned}\right\} \text{Gaussove aproksimacije}$$

$$\overline{BC} = b - R$$
$$\overline{AC} = a + R$$

jednadžba sfernog dioptra

$$n_1 \frac{a+R}{a} = n_2 \frac{b-R}{b}$$

$$\left(1 + \frac{R}{a}\right) n_1 = \left(1 - \frac{R}{b}\right) n_2$$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$ jednadžba sfernog dioptra

slikovno žarište: $a \to \infty$

$$b \to \frac{n_2 R}{n_2 - n_1} \equiv f_b$$

predmetno žarište: $b \to \infty$

$$a \to \frac{n_1 R}{n_2 - n_1} \equiv f_a$$

$$\frac{f_a}{a} + \frac{f_b}{b} = 1$$

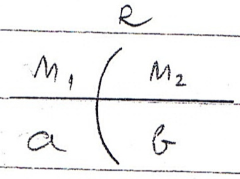
povećanje:

$$m = \frac{y'}{y} = \frac{-\overline{BC}}{\overline{AC}} = -\frac{b-R}{a+b} = -\frac{b}{a}\frac{n_1}{n_2}$$

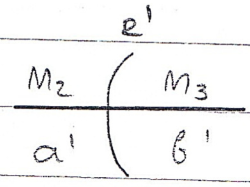
8. LEĆE

$\Rightarrow$ sloj omeđen dvjema sfernim granicama

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R} \quad \text{(prva granica)}$$



$$\frac{n_2}{a'} + \frac{n_3}{b'} = \frac{n_3 - n_2}{R'} \quad \text{(druga granica)}$$

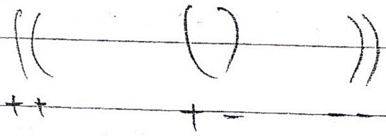


$$(1) \Rightarrow \frac{n_2}{b} = \frac{n_2 - n_1}{R} - \frac{n_1}{a} = \frac{-n_2}{a'} \Rightarrow (2)$$

$$\frac{n_3}{b'} = \frac{n_3 - n_2}{R'} - \frac{n_2}{a'} = \frac{n_3 - n_2}{R'} + \frac{n_2}{b} = \frac{n_3 - n_2}{R'} + \frac{n_2 - n_1}{R} - \frac{n_1}{a}$$

$$\frac{n_1}{a'} + \frac{n_3}{b'} = \frac{n_2 - n_1}{R} + \frac{n_3 - n_2}{R'}$$

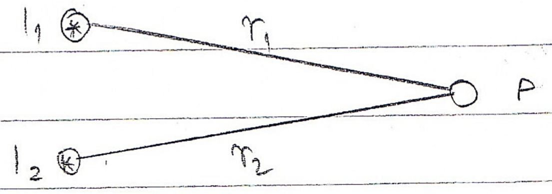
dogovor o predznacima:



FIZIKALNA OPTIKA

1. INTERFERENCIJA SVJETLOSTI

koherentni izvori – izvori elektromagnetskog titraja jednake frekvencije i stalnog faznog pomaka među njima



- superpozicija :

$$E_{1,2}(t,p) = E_0 \cos\left[\omega\left(t + \frac{n r_{1,2}}{c}\right)\right]$$

jednaka freku.

$$\frac{\omega}{c} n r_{1,2} = 2\pi \frac{n r_{1,2}}{\lambda}$$

$\delta = n \cdot r \Rightarrow$ "duljina" optičkog puta

$E_1 + E_2 = \ldots$ | korištenjem identiteta $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$

$$= \underbrace{\underbrace{2E_0 \cos\left(\omega \frac{n}{c} \cdot \frac{r_2 - r_1}{2}\right)}_{\text{ne titra u vremenu}}}_{\text{amplituda}} \cdot \underbrace{\cos\left[\omega\left(t - \frac{1}{c}\frac{n(r_1+r_2)}{2}\right)\right]}_{\text{oscilirajući faktor}}$$

$$\cos x + \cos y = \cos\left[\underbrace{\frac{(x+y)}{2}}_{\alpha} - \underbrace{\frac{(x-y)}{2}}_{\beta}\right] + \cos\left[\underbrace{\frac{y+x}{2}}_{\alpha} + \underbrace{\frac{y-x}{2}}_{\beta}\right]$$

$= \cos(\alpha + \beta) + \cos(\alpha - \beta)$

$\Rightarrow \quad 2\cos\alpha \cos\beta$

$$\alpha = \omega\left(t - \frac{n(r_1 + r_2)}{2c}\right) \qquad \beta = -\frac{n(r_1 - r_2)}{2c}\omega$$

Amplituda u P

$$P = 2E_0 \cos\left(\frac{\Delta\Phi}{2}\right)$$

$$\Delta\Phi = \frac{\omega}{c} n(r_2 - r_1) = 2\pi \frac{n(r_2 - r_1)}{\lambda} = \frac{2\pi \Delta\delta}{\lambda}$$ → razlika u duljini optičkih puteva

↓ razlika u fazi između izvora

$$\cos\frac{\Delta\Phi}{2} = \begin{cases} \pm 1, & \text{max (konstruktivna interferencija)} \\ 0, & \text{min (destruktivna interferencija)} \end{cases}$$

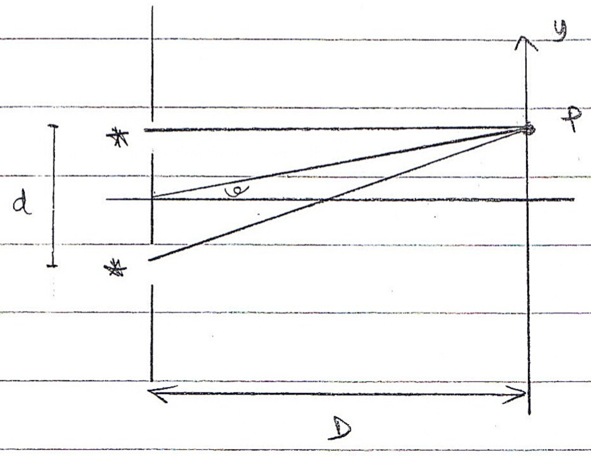
MAX: $\frac{\Delta\Phi}{2} = m\pi, \quad m = 0, \pm 1, \pm 2 \ldots$

$$\Delta\delta_{MAX} = (r_2 - r_1)n = m\lambda$$

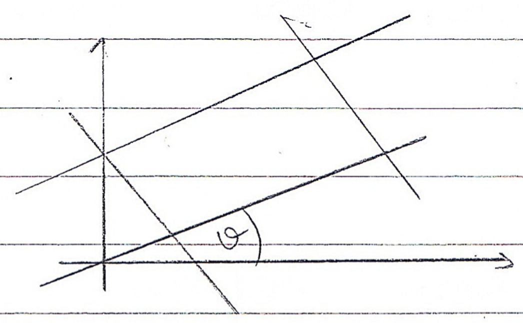
MIN: $\frac{\Delta\Phi}{2} = (2m+1)\frac{\pi}{2}, \quad m = 0, \pm 1, \pm 2 \ldots$

$$\delta_{MIN} = \left(m + \frac{1}{2}\right)\lambda$$

2. YOUNGOV POKUS NA DVIJE PUKOTINE



$D \gg d$



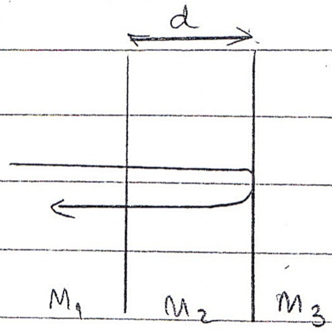
$$\left.\begin{aligned} \delta &= d \sin \vartheta \\ y &= d \,\mathrm{tg}\, \vartheta \end{aligned}\right\} \quad \frac{\delta}{d} = \frac{y}{D}$$

Maximum

$$\delta = m \lambda$$

$$y_{MAX} = \frac{D}{d} \delta_{MAX} = \frac{D}{d} m \lambda \qquad m = 0, \pm 1, \pm 2 \ldots$$

3. TANKI LISTIĆI



$$\delta = 2dn_2 + \frac{\lambda}{2} + \frac{\lambda}{2}$$

Max: $\delta_{MAX} = m\lambda$

$$\forall\ n_1 < n_2 < n_3$$

$$2dn_2 + \lambda = m\lambda$$

$$d = \frac{\lambda}{2n}$$

Min:

$$\delta_{MIN} = \left(m + \frac{1}{2}\right)\lambda$$

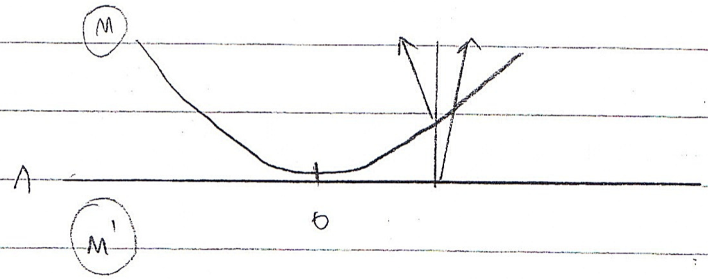
$$\forall\ n_1 < n_2 < n_3$$

$$2dn_2 + \lambda = \left(m + \frac{1}{2}\right)\lambda$$

najtanji listić

$$r = \frac{\lambda}{4n_2}$$

4. NEWTONOVI KOLOBARI



Refleksija

$$d(r) = R - \sqrt{R^2 - r^2} \simeq \frac{r^2}{2R}$$

$$r << R$$

$$\delta = 2d(r) + \frac{\lambda}{2} = \frac{r^2}{R} + \frac{\lambda}{2}$$

Max: $\delta = m\lambda$.

$r_{MAX} = \sqrt{R(m - \frac{1}{2})\lambda}$ $\quad m = 1, 2 \dots$

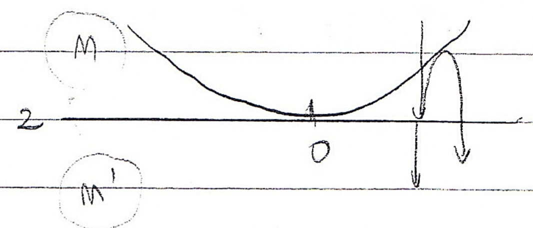
Min: $\delta = (m + \frac{1}{2})\lambda$

$r_{MIN} = \sqrt{Rm\lambda}$ $\quad m = 0, 1, 2 \dots$

⊙

tamna sredina, broji se od 0

Transmisija



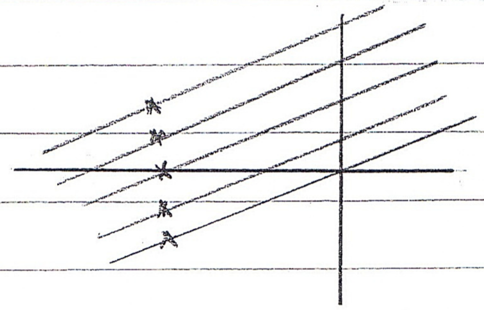
$$\delta = 2d(r) + 2\frac{\lambda}{2}$$

$r_{MAX} = \sqrt{Rm\lambda}$ $\quad m = 0, 1, 2 \dots$

$r_{MIN} = \sqrt{R(m - \frac{1}{2})\lambda}$ $\quad m = 1, 2, \dots$

5. OPTIČKA REŠETKA

N - koherentnih izvora

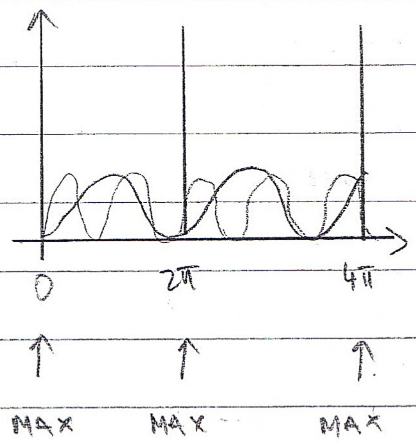


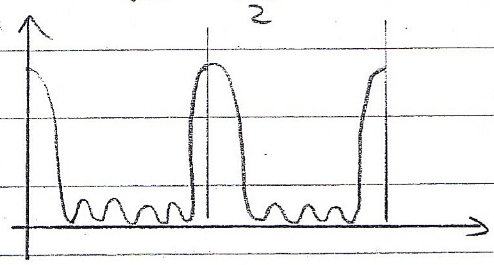
Rezultantno polje

$$E(\alpha) = E_0 \cdot \frac{\sin\left(N \frac{\Delta\phi}{2}\right)}{\sin \frac{\Delta\phi}{2}}$$

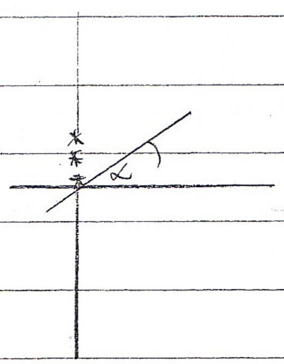
$$\Delta\phi = 2\pi \frac{d \sin\alpha}{\lambda}$$

$$I = I_0 \frac{\sin^2\left(N \frac{\Delta\phi}{2}\right)}{\sin^2 \frac{\Delta\phi}{2}}$$





6. DIFRAKCIJA NA JEDNOJ PUKOTINI



Model: N koherentnih izvora na razmal

$$d = \frac{a}{N}$$

Formula za rešetku

$$E(\alpha) = E_0 \frac{\sin\left(N \frac{\Delta\phi}{2}\right)}{\sin \frac{\Delta\phi}{2}}$$

$$\Delta\phi = 2\pi \frac{\delta}{\lambda} = 2\pi \frac{d \sin\alpha}{\lambda} = 2\pi \frac{a \sin\alpha}{N\lambda}$$

$$E(\alpha) = E_0 \frac{\sin\left(\pi \frac{a \sin\alpha}{\lambda}\right)}{\sin\left(\pi \frac{a \sin\alpha}{N\lambda}\right)}$$

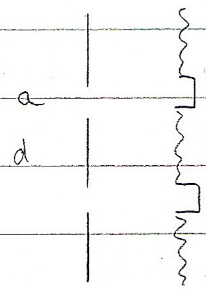
Gledamo limes $N \to \infty$

$$z = \frac{\pi a \sin\alpha}{\lambda} \Rightarrow N E_0 \frac{\sin z}{z}$$

E_0 1

$$I(\alpha) = I_0 \frac{\sin^2 z}{z^2} \Rightarrow \text{ogib}$$

7. SPEKTOGRAF S REŠETKOM



N = broj pukotina

d = konst. rešetke

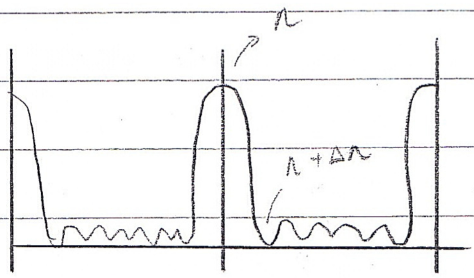
a = širina rešetke

$$I(\alpha) = I_0 \cdot \frac{\sin^2\left(\pi \frac{a \sin\alpha}{\lambda}\right)}{\left(\pi \frac{a \sin\alpha}{\lambda}\right)^2} \cdot \frac{\sin^2\left(N\pi \frac{d \sin\alpha}{\lambda}\right)}{\sin^2\left(\pi \frac{d \sin\alpha}{\lambda}\right)}$$

MAXIMUM:

$$d \sin\alpha = m\lambda$$

$$d \sin\alpha = \frac{m}{N}\lambda$$



Minimum mora biti prvi nakon maximum.

MAX: $d \sin\alpha = m \cdot \lambda \; (\lambda + \Delta\lambda)$

MIN: $d \sin\alpha = \frac{m(N+1)}{N} \lambda$

Moć razlučivanja rešetke

$$m(\lambda + \Delta\lambda) = \frac{mN+1}{N}\lambda$$

$$R = \frac{\lambda}{\Delta\lambda} = m \cdot N$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{mN}$$

8. POLARIZACIJA SVJETLOSTI

linearno polarizirani val

$$\vec{E}(t,\vec{r}) = \vec{j}\,\underbrace{E_0}_{\text{amp. vektor}}\cos(\omega t - \underbrace{\vec{k}}_{\text{val. vektor}}\vec{r})$$

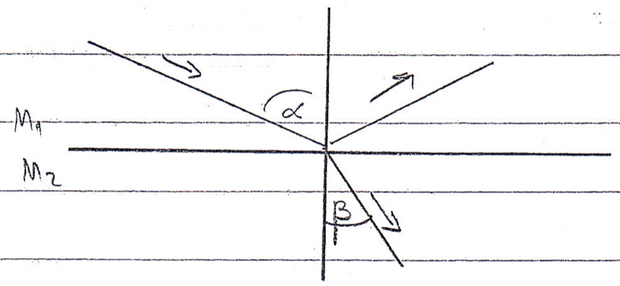
t – vrijeme, $\vec{r}$ – položaj

kružno polarizirani val

$$\vec{E}(t,\vec{r}) = \vec{j}\,E_0\cos(\omega t - \vec{k}\vec{r}) + \vec{k}\,E_0\cos\left(\omega t - \vec{k}\vec{r} - \frac{\pi}{2}\right)$$

Brewsterov kut

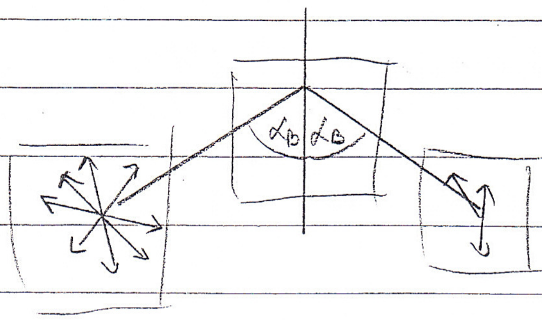
- ako je kut između lomljene i reflektirane zrake $= \frac{\pi}{2}$,
zraka je polarizirana



Snelov zakon

$$\frac{\sin\alpha_B}{\sin\beta} = \frac{n_2}{n_1} \qquad \frac{\sin\beta}{\sin\left(\frac{\pi}{2} - \alpha_B\right)} = \operatorname{tg}\alpha_B \qquad \alpha_B = \operatorname{arctg}\frac{n_2}{n_1}$$

Malusov zakon



$$I' = I\cos^2\Theta$$

$$I \longrightarrow \phi \longrightarrow I'$$

lin. polar. svjetlost

MODERNA FIZIKA

1. ZRAČENJE CRNOG TIJELA

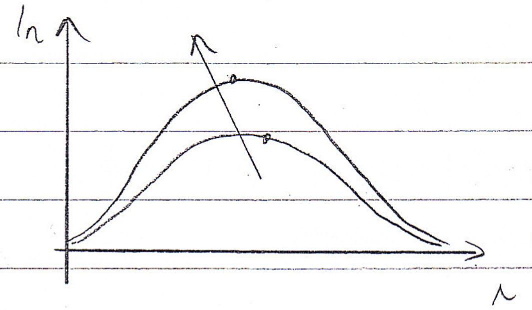
Štefan - Boltzmanov zakon (experimentalno)

- tijelo ugrijano na T zrači : $I = \sigma T^4$

$\sigma = 5,67 \cdot 10^{-8}\ W/m^2K^4$

Raspodjela intenziteta po valnim duljinama

$$I_\lambda = \frac{dI}{d\lambda} = f(\lambda, T)$$



→ maximumi se pomiču sve manjim valnim duljinama

Planckov zakon

$$I_\lambda = f(\lambda, T) = \underbrace{\left(\frac{c}{4} \cdot \frac{8\pi}{\lambda^4} kT\right)}_{\text{Rayleigh - Jeansova formula}} \left(\frac{hc}{\lambda kT} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)$$

"ultraljubičasta katastrofa"

planck → $\frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$

k → boltzmanova konst. $k = 1,381 \cdot 10^{-23}\ J/K$

h → Planckova konst. $h = 6,626 \cdot 10^{-34}\ Js$

$$I_\lambda = \int dI = \int_0^\infty I_\lambda \, d\lambda = \int_0^\infty f(\lambda,T)\, d\lambda = \frac{2\pi^5 k^4}{15 c^2 h^3} T^4$$

* pokazujemo da je R-J zakon, limes Planckova zakona pri $\lambda \to \infty$

Planckov faktor : $\frac{hc}{\lambda kT} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = \left\{ \frac{hc}{\lambda kT} = x \right\} = \frac{x}{e^x - 1}$

$\lim \lambda \to \infty \to$ odg. limes $\lambda \to 0$

$$\lim_{x\to 0} \frac{x}{e^x - 1} = \lim_{x \to 0} \frac{x}{(1+x)-1} = \lim \frac{x}{x} = 1$$

* oprez $\quad \lambda = \frac{c}{\nu}$

$dI = f(\lambda, T) d\lambda$

$$= \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{\frac{hc}{k T\lambda}} - 1} = \frac{2\pi h c^2}{\left(\frac{c}{\nu}\right)^5} \cdot \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}$$

$$\boxed{I_\nu = \frac{2\pi h \nu^3}{c^2} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}}$$

WIENOV ZAKON

$\lambda_{MAX} \cdot T = k_W \to$ Wienova konstanta 0.0029 mK

$I_\lambda = f(\lambda, T) \propto \frac{x^5}{e^x - 1} \qquad x = \frac{hc}{k\lambda T}$

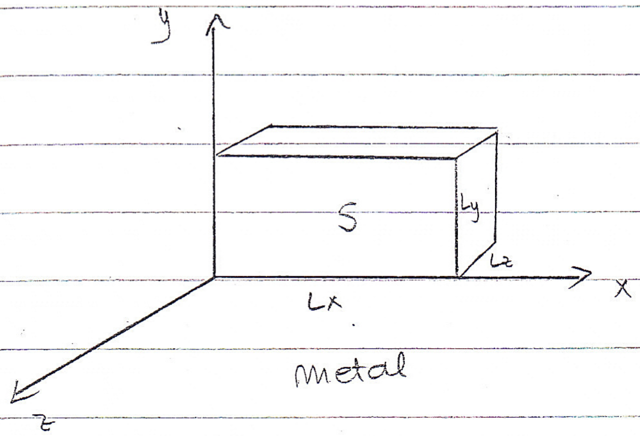
$\frac{dI_\lambda}{dx} = \frac{df}{dx} = 0 \qquad$ jer f nikad nije 0

$\underbrace{\left(\frac{5}{x} - \frac{x}{e^x - 1} \right)}_{=0} f \qquad x = 4{,}965$

$$x = \frac{hc}{\lambda_{MAX} k T}$$

Izvod zakona zračenja crnog tijela

I. Spektralna gustoća elektrom. modova



EM zračenje zadovoljava rubne uvjete : $\vec{E} = 0$

$$\Rightarrow \vec{E}(\vec{r}, t) = \vec{E}_0 e^{i\omega t} \left(e^{i\vec{k}\vec{r}} + e^{-i\vec{k}\vec{r}} \right) = \vec{E}_0 e^{i\omega t} 2\cos(\vec{k}\vec{r}) \text{ gdje je}$$

$\vec{k}$ valni vektor

$$\vec{k} = k_x \vec{i} + k_y \vec{j} + k_z \vec{k}$$

Zbog rubnih uvjeta: $k_x = \frac{2\pi}{\lambda_x}$, $L_x = m_x \frac{\lambda_x}{2} \Rightarrow k_x = \frac{m_x}{L_x}\pi$, $m_x = 1, 2, 3.$

analogno k_y, k_z

Razmatramo EM. val u m-prostoru $dm_x dm_y dm_z = d^3 m$

k-prostoru $dk_x dk_y dk_z = d^3 k$

$$\left. \begin{aligned} dk_x &= \frac{\pi}{L_x} \\ dk_y &= \frac{\pi}{L_y} \\ dk_z &= \frac{\pi}{L_z} \end{aligned} \right\} \star \rightarrow dk_x dk_y dk_z = \frac{\pi^3}{L_x L_y L_z} dm_x dm_y dm_z$$

$$d^3 k = \frac{\pi^3}{V} \cdot d^3 m$$

$\frac{\pi^3}{V}$ → gustoća dopuštenih valnih vektora u k-prostoru

Broj elektromagnetskih modova u V takvih da $|\vec{k}| < k$

$$N(k) = \underbrace{2}_{\text{2 moguća smjera polarizacije}} \cdot \underbrace{\frac{1}{8}}_{\text{oktant}} \cdot \underbrace{\frac{4}{3} k^3 \pi}_{\text{kugla polumjera } k} \cdot \frac{V}{\pi^3} = \frac{1}{3} k^3 \frac{V}{\pi^2}$$

Broj elektromagnetskih modova tako da $|\vec{k}| \in (k, k+dk)$

$$dN = \frac{dN(k)}{dk} \cdot dk = k^2 \frac{V}{\pi^2} dk$$

Broj elektromag. modova tako da $\nu \in (\nu, \nu + d\nu)$: $\left(k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu\right)$

$$dN = \left(\frac{2\pi}{c}\nu\right)^2 \frac{V}{\pi^2} \cdot \frac{2\pi}{c} d\nu = g(\nu)\, d\nu$$

$$g(\nu) = \frac{8\pi^3}{c^3} \nu^2$$ $\Rightarrow$ spektralna gustoća elektrom. modova

II. Rayleigh - Jeans

Pretpostavke: - termodinamička ravnoteža
- Boltzmanova raspodjela

$$f(\epsilon) = Ce^{\frac{-\epsilon}{kT}} = \frac{1}{kT} e^{\frac{-\epsilon}{kT}}$$

Normalizacija vrijednosti: $\int_0^\infty f(\epsilon)\, d\epsilon = 1 \Rightarrow C = \frac{1}{kT}$

Srednja energija: $\langle \epsilon \rangle = \int_0^\infty \epsilon f(\epsilon)\, d\epsilon = \int_0^\infty \epsilon \cdot \frac{1}{kT} e^{\frac{-\epsilon}{kT}} d\epsilon = kT$

Spektralna gustoća energije: $U(\nu) = \langle \epsilon \rangle g(\nu) = kT \cdot \frac{8\pi^3}{c^3} \nu^2$

III. Planck

Pretpostavke: - termodinamička ravnoteža
- Boltzmanova raspodjela
- hipoteza o kvantizaciji

$$\langle \epsilon \rangle = \sum_n E_n f_n \quad \longrightarrow \quad f_n = c\, e^{-\frac{E_n}{kT}}$$

$$\Downarrow$$

$$E_n = nh\nu$$

Normalizacija vjerojatnosti

$$\sum_n f_n = 1 = C \sum_n e^{-nx} \qquad x = \frac{h\nu}{kT}$$

$$= C\left[e^{-x} + (e^{-x})^2 + \dots\right]$$

$$= C \frac{1}{1-e^{-x}}$$

$$\Rightarrow \cancel{c = 1 - e^{-x} = 1 - e^{-\frac{h\nu}{kT}}}$$

$$\langle \epsilon \rangle = \sum_n nh\nu \left(1 - e^{-\frac{h\nu}{kT}}\right) e^{-\frac{nh\nu}{kT}}$$

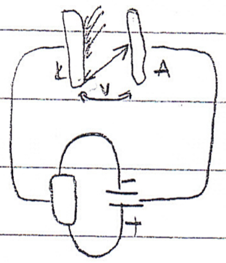
$$= x\,(1-e^{-x})\, kT \sum_n n e^{-nx} = \dots$$

$$= x \frac{kT}{e^x - 1} = \frac{h\nu}{\cancel{kT}} \cdot \frac{\cancel{kT}}{e^{\frac{h\nu}{kT}} - 1} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Spektralna gustoća energije

$$u(\nu) = \langle \epsilon(\nu) \rangle g(\nu) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

2. FOTOELEKTRIČNI EFEKT



a) struja razmjerna intenzitetu zračenja [OK]

b) ako V ide ispod neke V_c struja nestaje [PROBLEM]

c) max. kin. en. elektrona ne ovisi o intenzitetu [PROBLEM]

$$E_{k.MAX} = \frac{m v_{MAX}^2}{2} = e V_z \rightarrow \text{zaustavni napon}$$

objašnjenje: A. Einstein (1905)

$$E_x = \hbar \omega = h\nu = h \cdot \frac{c}{\lambda}$$

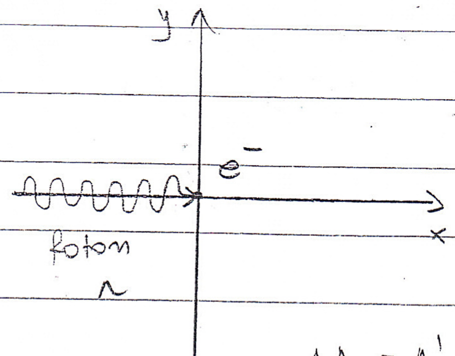
$\hbar = \frac{h}{2\pi}$; h → Planckova konst.

energija : $E_\gamma \geq W + E_{kin}$ → izlazni rad

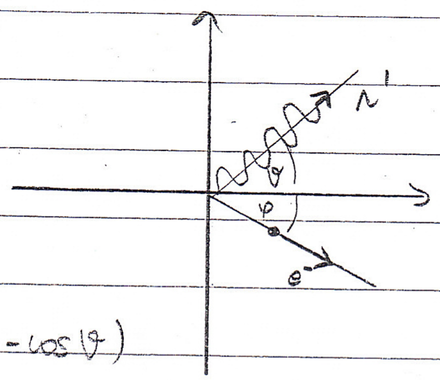
3. "COMPTONOVO" RASPRŠENJE

- foton nalijeće na česticu koja miruje, "raspršenje fotona na e^-"

prije :



poslije:



$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\vartheta)$$

$$\frac{1}{h\nu'} - \frac{1}{h\nu} = \frac{1}{m_0 c^2}(1 - \cos\vartheta)$$

Energija: $\left[E = \sqrt{(m_0 c^2)^2 + (pc)^2} = \dfrac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \right]$

$$E = h\nu + m_0 c^2 = h\nu' + \sqrt{(m_0 c^2)^2 + \left(\frac{m v \cdot c}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2} \qquad (1)$$

Impuls: $\left[\vec{p} = \dfrac{m \cdot \vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \quad \text{za } m > 0 \, , \quad p = \dfrac{E}{c} = \dfrac{h\nu}{c} = \dfrac{h}{\lambda} \quad \text{za } m = 0 \right]$

$$\left.\begin{array}{l} p_x = \dfrac{h\nu}{c} = \dfrac{h\nu'}{c}\cos\vartheta + \dfrac{m_0 \cdot v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \cos\varphi \qquad (2) \\[2em] p_y = 0 = \dfrac{h\nu'}{c}\sin\vartheta - \dfrac{m_0 \cdot v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \sin\varphi \qquad (3) \end{array}\right\} +$$

Iz (2) i (3) eliminiramo φ:

$$\left(\frac{h\nu}{c}\right)^2 - 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c}\cos\vartheta + \left(\frac{h\nu'}{c}\right)^2 = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2 \qquad (4)$$

prepisujemo (1)

$$\left(\frac{h\nu}{c} - \frac{h\nu'}{c} + m_0 c\right)^2 - (m_0 c)^2 = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2$$

$$\left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + 2\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right) m_0 c - 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c} = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2$$

(5) - (4)

$$2 \cdot \left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right) m_0 \cdot c - 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c} + 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c}\cos\vartheta = 0$$

$$\left(\frac{1}{\frac{h\nu'}{c}} - \frac{1}{\frac{h\nu}{c}}\right) m_0 c = 1 - \cos\vartheta$$

uz $\lambda\nu = c$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\vartheta)$$

4. BOHROV MODEL ATOMA (1913)

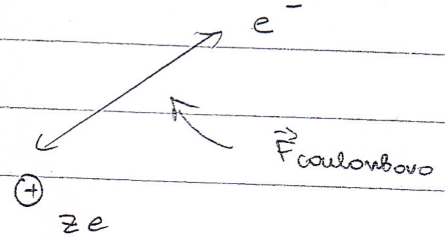
Prethodnik: * Thomson

* Rutherford

* Balmerova formula $\lambda = (364.56\,nm)\dfrac{n^2}{n^2-4}$ $n=3,4,5$

ili $\dfrac{1}{\lambda} = R_H\left(\dfrac{1}{2^n} - \dfrac{1}{n^2}\right)$ $n=3,4,5\ldots$ $R_H = 1.097373 \cdot 10^7\,m^{-1}$

$\downarrow$ Rydbergova konst.



$P = \dfrac{e^2 a_{cp}}{6\pi\varepsilon_0 c^3} = \dfrac{dE}{dt}$

$P_1 \Rightarrow$ I. B postulat: postoje dozvoljene staze na kojima e^- ne zrači

$P_2 \Rightarrow$ II. B postulat: kutna količina gibanja je klasična veličina kod Bohra. $L = n\hbar = n\dfrac{h}{2\pi} = m \cdot v_n \cdot r_n$; $n = 1,2,3\ldots$

$P_3 \Rightarrow$ III B. postulat: elektron može iz jednog dozvoljenog stanja prijeći u drugo. Pri prelasku $n_i \to n_f$; zrači se foton; $E_{phot} = E_i - E_f = h\nu$

Jednakost sila: $F_{cp} = \dfrac{mv^2}{r_n} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{ze \cdot e}{r_n^2}$

Iz P_2 $L_n = n\hbar = m v_n r_n \rightarrow v_n = \dfrac{n\hbar}{m r_n}$

$$\frac{m}{r_n}\left(\frac{n\hbar}{m r_n}\right)^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{ze^2}{r_n^2}$$

$$\boxed{r_n = \frac{4\pi\varepsilon_0 \hbar}{ze^2 \cdot m} \cdot n^2}$$

$$v_n = \frac{ze^2}{4\pi\varepsilon_0 \hbar} \cdot \frac{1}{n}$$

Ukupna energija

$$E_n = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r_n} + \frac{m}{2} v_n^2 = \dots = \frac{-m Z^2 e^4}{32\pi^2 \varepsilon_0^2 \hbar^2} \cdot \frac{1}{n^2}$$

energija ionizacije osnovnog stanja

$$E_1 = -13.58\,eV$$

$$E_n = E_1 \cdot \frac{1}{n^2}$$

Iz P3 $\quad h\nu = E_i - E_f = E_1 \left(\frac{1}{n_f} - \frac{1}{n_i} \right)$

Balmer $\quad n_f = 2$