(14) $\vec{E}(\vec{r}) = \frac{1}{r}\left(A\hat{r} + B\sin\theta\cos\phi\,\hat{\phi}\right)$

$\rho = ?$

$$\nabla\cdot\vec{E}(\vec{r}) = \frac{1}{r^2}\frac{\partial(r^2E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta E_\theta)}{\partial\theta}$$

$$\nabla\cdot\vec{E}(\vec{r}) = \frac{1}{r^2}\cdot A + \frac{1}{r\cancel{\sin\theta}}\cdot\frac{1}{r}\cdot B\cancel{\sin\theta}(-\sin\phi)$$

$$\nabla\cdot\vec{E}(\vec{r}) = \frac{1}{r^2}\left(A - B\sin\phi\right)$$

$$\nabla E = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0\cdot\nabla\cdot E = \frac{\varepsilon_0}{r^2}(A - B\sin\phi)$$

(15.) $\vec{E} = -\nabla\Phi$

$$\vec{E} = -\nabla\Phi = -\frac{\partial\Phi}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\theta} - \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\hat{\phi}$$

$\Rightarrow \vec{\nabla}\times\vec{E} = 0$

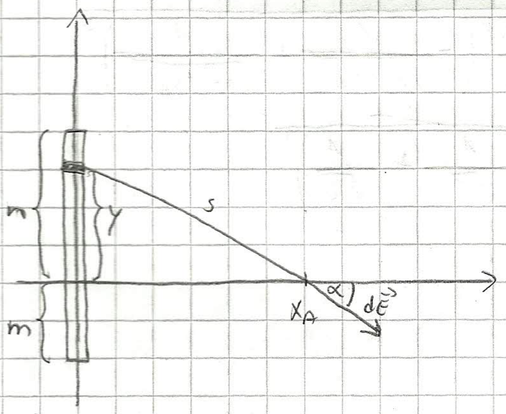
$$\vec{\nabla}\times\vec{E} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta E_\phi)}{\partial\theta} - \frac{\partial E_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial(rE_\phi)}{\partial r}\right]\hat{\theta} +$$

$$+\frac{1}{r}\left[\frac{\partial(rE_\theta)}{\partial r} - \frac{\partial E_r}{\partial\theta}\right]\hat{\phi}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(-\frac{1}{r}\frac{\partial\Phi}{\partial\phi}\right) - \frac{\partial}{\partial\phi}\left(-\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right)\right] + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(-\frac{\partial\Phi}{\partial r}\right) - \frac{\partial}{\partial r}\left(-\frac{1}{\sin\theta}\frac{\partial\Phi}{\partial\phi}\right)\right]\hat{\theta} +$$

$$+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(-\frac{\partial\Phi}{\partial\theta}\right) - \frac{\partial}{\partial\theta}\left(-\frac{\partial\Phi}{\partial r}\right)\right]\hat{\phi} = \underline{\underline{0}}$$

(16.)



$Q = 10^{-10}\ C$ $\quad m : n = 1 : 2$

$L = 0{,}8\ m$

$x_A = 1\ m$

$E = ?$

$\alpha = ?$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{s^2}\hat{s}$$

$$d\vec{E} = \vec{i}\,dE_x + \vec{j}\,dE_y$$

$$dE_x = dE \cdot \cos\alpha$$

$$dE_x = k \cdot \frac{dQ}{s^2}\cos\alpha \qquad \cos\alpha = \frac{x_A}{s}$$

$$E_x = k \cdot \int_m^n \frac{dQ}{s^2} \cdot \frac{x_A}{s} \qquad \frac{Q}{L} = \frac{dQ}{dy} \Rightarrow dQ = \frac{Q}{L} \cdot dy$$

$$E_x = k \cdot \frac{Q}{L} \cdot \int_m^n \frac{x_A}{s^3}\,dy \qquad s^2 = y^2 + x_A^2 \Rightarrow s = \sqrt{y^2 + x_A^2}$$

$$E_x = k \cdot \frac{Q}{L} \int_m^n \frac{x_A}{(\sqrt{y^2 + x_A^2})^3}\,dy \qquad u = (x^2 + a^2)^{\frac{1}{2}} \qquad \int \frac{dx}{u^3} = \frac{1}{a^2} \cdot \frac{x}{u}$$

$$E_x = k \cdot \frac{Q}{L} \cdot \cancel{x_A} \cdot \frac{1}{x_A^{\cancel{2}}} \cdot \frac{y}{\sqrt{y^2 + x_A^2}}\Bigg|_m^n$$

$$E_x = k \cdot \frac{Q}{L} \cdot \frac{1}{x_A} \cdot \left(\frac{n}{\sqrt{n^2 + x_A^2}} - \frac{m}{\sqrt{m^2 + x_A^2}}\right)$$

$$dE_y = dE \cdot \sin\alpha$$

$$dE_y = k \cdot \frac{dQ}{s^2}\sin\alpha \qquad \sin\alpha = \frac{y}{s}$$

$$dE_y = k \cdot \frac{Q}{L} \cdot \frac{y}{s^3}\,dy$$

$$E_y = k \cdot \frac{Q}{L} \int_m^n \frac{y\,dy}{(\sqrt{y^2 + x_A^2})^3}$$

$$E_y = k \cdot \frac{Q}{L}\left(-\frac{1}{\sqrt{y^2 + x_A^2}}\right)\Bigg|_m^n$$

$$E_y = k \cdot \frac{Q}{L}\left(-\frac{1}{\sqrt{n^2 + x_A^2}} + \frac{1}{\sqrt{m^2 + x_A^2}}\right)$$

$$|\vec{E}| = \sqrt{\left(k \cdot \frac{Q}{L} \cdot \frac{1}{x_A}\right)^2 \cdot \left(\frac{n}{\sqrt{n^2+x_A^2}} - \frac{m}{\sqrt{m^2+x_A^2}}\right)^2 + \left(k \cdot \frac{Q}{L}\right)^2 \left(-\frac{1}{\sqrt{n^2+x_A^2}} + \frac{1}{\sqrt{m^2+x_A^2}}\right)^2}$$

$$|\vec{E}| = 9 \cdot 10^9 \cdot \frac{10^{-10}}{0{,}8} \cdot \sqrt{\frac{1}{1^2} \cdot \left(\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}}\right)^2}$$

$$|\vec{E}| = 0{,}3604 \ N/C$$

$$tg\,\alpha = \frac{E_y}{E_x}$$

$$\alpha = arctg\left(\frac{\cancel{k} \cdot \frac{\cancel{Q}}{\cancel{L}} \cdot \left(-\frac{1}{\sqrt{n^2+x_A^2}} + \frac{1}{\sqrt{m^2+x_A^2}}\right)}{\cancel{k} \cdot \frac{\cancel{Q}}{\cancel{L}} \cdot \frac{1}{x_A}\left(\frac{n}{\sqrt{n^2+x_A^2}} - \frac{m}{\sqrt{m^2+x_A^2}}\right)}\right) = 54{,}22^\circ$$

17.

$$V(r) = A \cdot \frac{e^{-\lambda r}}{r}$$

$\vec{E} = ?$

$$\vec{E} = -\nabla V(r) = -\frac{\partial V(r)}{\partial r}\hat{r}$$

$$\vec{E} = -A\left(\frac{-\lambda e^{-\lambda r} \cdot r - e^{-\lambda r}}{r^2}\right)\hat{r}$$

$$\vec{E} = A \cdot \frac{\lambda r + 1}{r^2} \cdot e^{-\lambda r}\hat{r}$$