$v = -\delta e^{-\delta t}(A \operatorname{ch}\omega' t + B \operatorname{sh}\omega' t) + \omega' e^{-\delta t}(A \operatorname{sh}\omega' t + B \operatorname{ch}\omega' t)$

$0 = -\delta A + \omega' B \quad \rightarrow \quad B = \frac{\delta x_0}{\omega'}$

(D1) $t = 0$, $x = 0$, $v = v_0$

$$\boxed{x = x_0 e^{-\delta t}\left(\operatorname{ch}\omega' t + \frac{\delta}{\omega'} \operatorname{sh}\omega' t\right)}$$

Kritično prigušenje

$\omega_0^2 = \delta^2$

$\omega' = 0$

$\frac{\delta}{\omega'} \operatorname{sh}\omega' t = \delta t \frac{\operatorname{sh}\omega' t}{\omega' t} \qquad y = \omega' t$

$t = 0$, $x = x_0$, $v = 0$

(D2) $t = 0$, $x = 0$, $v = v_0$

$\lim_{y \to 0} \frac{\operatorname{sh} y}{y} = 1$

$$\boxed{x = x_0 e^{-\delta t}(1 + \delta t)}$$

Energija prigušenog titranja

$E_u(t) = E_k(v) + E_p(x)$

$E_k = \frac{mv^2}{2}$ $\quad v(t), x(t)$

$E_p = \frac{kx^2}{2}$

$$\frac{dE_u}{dt} = \frac{dE_k}{dv}\frac{dv}{dt} + \frac{dE_p}{dx}\cdot\frac{dx}{dt}$$

$$= mv\cdot\frac{dv}{dt} + kx\underbrace{\frac{dx}{dt}}_{v}$$

$$= \underbrace{v\left(m\frac{dv}{dt} + kx\right)}_{-b\frac{dx}{dt}} = v\cdot(-bv) = -bv^2$$

Prisilno titranje

$F = F_0 \sin\omega t$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0\sin\omega t$$

$\frac{F_0}{m} = A_0$

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = A_0\sin\omega t$$

- nehomogena jed.

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = A_0 \sin\omega t$$ - nehomogena jed.

- opće rješenje:

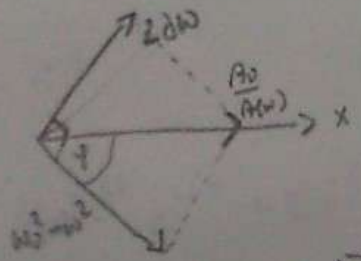
$$x_{OH} = x_{OH} + x_{PH}$$

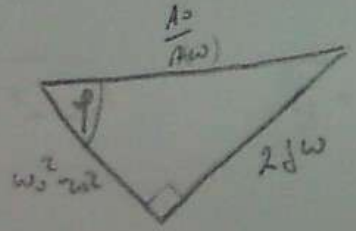
- prijelazno / tranzijentno vrijeme - vrijeme za koje zbog prigušenja x_{OH} utrne

- stacionarno rješenje $x_{OH} = x_{PH}$

$$x_{PH} = A(\omega)\sin(\omega t - \varphi)$$

$$\frac{A_0}{A(\omega)}\sin\omega t = (\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2\delta\omega\cos(\omega t - \varphi) = (\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2\delta\omega\sin(\omega t - \varphi + \frac{\pi}{2})$$





$$\frac{A_0^2}{A^2(\omega)} = 4\delta^2\omega^2 + (\omega_0^2 - \omega^2)^2$$

$$tg\,\varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2}$$

$$tg\,\varphi = \frac{2\frac{\delta\omega}{\omega_0^2}}{1 - \frac{\omega^2}{\omega_0^2}}$$

$$A(\omega) = \frac{A_0}{\sqrt{4\delta^2\omega^2 + (\omega_0^2 - \omega^2)^2}}$$

$$A(\omega) = \frac{A_0}{\omega_0^2\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\frac{\delta^2\omega^2}{\omega_0^4}}}$$

- rezonantna frekvencija amplitude

ω_R , $A(\omega_R) = MAX$

$\frac{dA(\omega)}{d\omega} = 0$ ... (D²)

$$\omega_R = \sqrt{\omega_0^2 - 2\delta^2}$$

$$A(\omega_e) = \frac{A_0}{2\delta\sqrt{\omega_0^2 - \delta^2}}$$

$P = \vec{F} \cdot \vec{v}$

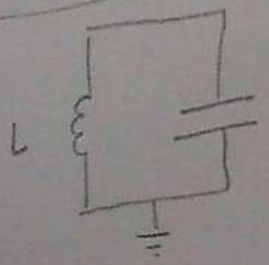
$v = A\omega \cos(\omega t - \varphi) = A\omega \sin(\omega t - \varphi + \frac{\pi}{2}) = A\omega \sin \omega t$

$F = F_0 \sin(\omega t)$

$\varphi = \frac{\pi}{2}$ razlika u fazi

$\omega = \omega_0$ - maksimalna absorbcija energije?

Električni titrajni krug



$q(t) = ?$

naboji na kondenzatoru

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{CL}q = 0 \rightarrow \omega_0^2 = \frac{1}{LC}$$

$$T = 2\pi\sqrt{LC}$$

Thomsonova formula

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \underbrace{\frac{k}{m}}_{\omega_0^2} x = 0$$

M.O.	T.k.

kondenzatora

$$m\frac{d^2x}{dt^2} + kx = 0 \qquad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{k}{m} = \omega_0^2$$

$$T = 2\pi\sqrt{LC}$$

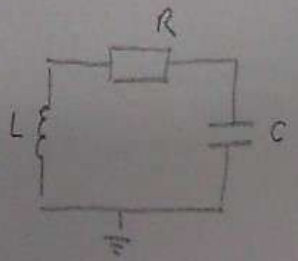
Thomsonova formula

M.O.	T.k.
x	q
v	$I = \frac{dq}{dt}$
m	L
k	$\frac{1}{C}$
$\frac{1}{2}kx^2$	$\frac{q^2}{2C}$
$\frac{1}{2}mv^2$	$\frac{1}{2}LI^2$

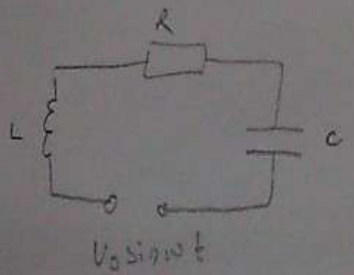
energija kondenzatora

magnetska energija zavojnice

$\frac{\pi}{2}$)



$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$



$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V_0 \sin\omega t$$

Zbrajanje dvaju harmonijskih titranja na istom pravcu jednake frekvencije

$x_1 = A_1 \sin(\omega t + \varphi_{01})$

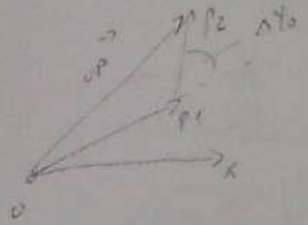
$x_2 = A_2 \sin(\omega t + \varphi_{02})$

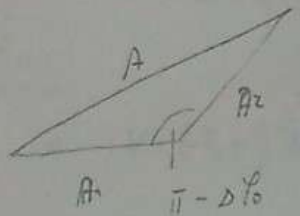
$x = x_1 + x_2$

$\Delta\varphi_0 = \varphi_{02} - \varphi_{01} = \text{konst.}$

↓

koherentno titranje





$A^2 = A_1^2 + A_2^2 - 2A_1A_2\cos(\pi - \Delta\varphi_0)$

$= A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\varphi_0$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\varphi_0}$$

$\Delta\varphi = \varphi_2 - \varphi_1$

$$\operatorname{tg}\varphi = \frac{A_y}{A_x} = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$$

MAX AMPLITUDA

$\Delta\varphi = 0, 2\pi, 4\pi \dots$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2} = A_1 + A_2$

-konstruktivna interferencija

MIN AMPLITUDA

$\Delta\varphi = \pi, 3\pi, 5\pi \dots$

$A = \sqrt{A_1^2 + A_2^2 - 2A_1A_2} = |A_1 - A_2|$

-destruktivna interferencija