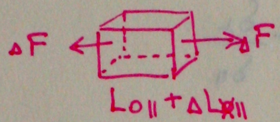
ELASTIČNOST

modul elastičnosti = $\frac{\text{naprezanje}}{\text{deformacija}}$

* VLAK



naprezanje $\sigma = \frac{\Delta F}{S_0}$

deformacija $\delta_{\parallel} = \frac{\Delta L_{\parallel}}{L_{0\parallel}}$

$\Rightarrow$ Youngov modul elastičnosti $E = \frac{\sigma}{\delta_{\parallel}}$

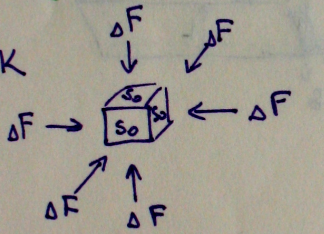
$$\Delta F = \sigma \cdot S_0 = E \cdot \delta_{\parallel} \cdot S_0 = \frac{E \cdot S_0}{L_{0\parallel}} \cdot \Delta L_{\parallel}$$

$$F = k \cdot x$$

Poissonov omjer: jednom uvijek negativna $\mu = \frac{-\delta_{\perp}}{\delta_{\parallel}} = \frac{-\frac{\Delta L_{\perp}}{L_{0\perp}}}{\frac{\Delta L_{\parallel}}{L_{0\parallel}}}$ $\frac{\text{okomita}}{\text{poprečna}}$ deformacija

$$E = \int F(x)dx$$

* TLAK



$V = V_0 + \Delta V$

naprezanje $p = \frac{\Delta F}{S_0}$

deformacija $\delta_V = \frac{\Delta V}{V_0}$ negativno kad je tlak pozitivan

$\Rightarrow$ Volumni modul stlačivosti $B = -\frac{p}{\delta_V}$

NAPOMENA Ako je tlak otišao u $p_0 + \Delta p$

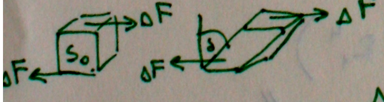
onda: $\cancel{\frac{\Delta V}{V_0} = -\frac{p}{B}}$ $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$ ✓

$$E = 3B(1-2\mu)$$

Youngov modul elastičnosti

$\Delta p \cdot \Delta V \approx 0$

* SMIK

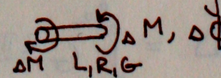


naprezanje $\sigma = \frac{\Delta F}{S_0}$

deformacija δ $(\text{tg}\,\delta \sim \delta)$

modul smicanja $G = \frac{\sigma}{\delta}$

konstanta torzije šipke



$$D = \frac{\Delta M}{\Delta \phi} = G \frac{R^4 \pi}{2L}$$

šipka s tankom stijenkom $D = \frac{\Delta M}{\Delta \phi} = G \frac{2\pi R^3 d}{L}$

cijev debele stijenke $D = \int_{R_1}^{R_2} dD = G \int_{R_1}^{R_2} \frac{2\pi R^3}{L} dR = G \frac{\pi}{2L}(R_2^4 - R_1^4)$

Izvod za $E = 3B(1-2\mu)$

$$V \to V + \Delta V = (\ell + \Delta \ell)(r+\Delta r)^2 \pi$$
$$= \ell r^2 \pi \left(1+\frac{\Delta \ell}{\ell}\right)\left(1+\frac{\Delta r}{r}\right)^2$$
$$= V(1+\delta_{\parallel})(1+\delta_{\perp})^2$$
$$= V(1+\delta_{\parallel})(1+2\delta_{\perp}+\underbrace{\delta_{\perp}^2}_{\approx 0})$$
$$= V(1+\delta_{\parallel})(1+2\delta_{\perp}) \qquad , \mu = -\frac{\delta_{\perp}}{\delta_{\parallel}}$$
$$= V(1+\delta_{\parallel})(1-2\mu\delta_{\parallel})$$
$$= V(1+\delta_{\parallel}-2\mu\delta_{\parallel}+\dots) \qquad E = \frac{\sigma}{\delta_{\parallel}}$$
$$\approx V(1+(1-2\mu)\delta_{\parallel}) = V\left(1+(1-2\mu)\frac{\sigma}{E}\right)$$

$$\Rightarrow \Delta V = (1-2\mu)\frac{\sigma}{E} = (1-2\mu)\frac{-p}{E} \qquad , \text{jer } \sigma = -p$$

$$V''' = V_0\left(1-3(1-2\mu)\frac{p}{E}\right) \qquad , \text{jer } V_0 \to V''' = V_0\left(1+\frac{\Delta V'}{V_0}\right)^3 \approx V_0\left(1+3\frac{\Delta V'}{V_0}\right)$$

$$\frac{\Delta V}{V} = -\frac{p}{B} = -3\frac{p}{E}(1-2\mu)$$

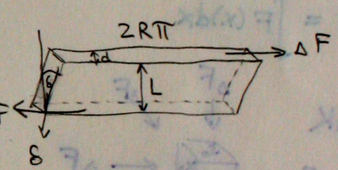
$$\Rightarrow \boxed{E = 3B(1-2\mu)}$$

Konstanta torzije cijevi

$\Delta\phi$  R, d, L, G

tanka cijev

$d << R$

razmotano 

$$S_0 = 2R\pi d$$
$$R \cdot \Delta\phi = L \cdot \delta$$
$$\Delta M = R \cdot \Delta F$$
$$G = \frac{\Delta F / S_0}{\delta} = \frac{\frac{\Delta M}{R}\cdot\frac{1}{2R\pi d}}{\frac{R\cdot\Delta\phi}{L}} = \frac{\Delta M}{\Delta\phi}\cdot\frac{L}{2R^3\pi d}$$
$$D = \frac{\Delta M}{\Delta\phi} = G\frac{2R^3\pi d}{L} \quad //$$

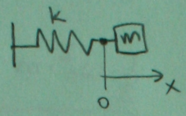
Cijev debele stijenke:
$$D = \frac{\Delta M}{\Delta\phi} = G\frac{2R^3\pi d}{L}$$
$$dD = G\frac{2\pi R^3}{L} dr$$
$$D = \int_{R_1}^{R_2} dD = \int_{R_1}^{R_2} G\frac{2\pi R^3}{L}dr = \frac{G\pi}{2L}(R_2^4 - R_1^4) \quad //$$

Puna cijev (šipka):
$$R_1 = 0 \Rightarrow D = G\frac{\pi R^4}{2L}$$

TITRANJE

Jednadžba gibanja:

1) osnovni model



$$ma = -kx$$

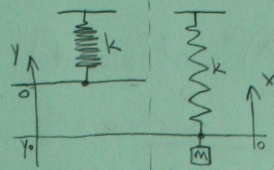
$$m\frac{d^2x}{dt^2} = -kx$$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \omega_0^2 x = 0$$

$$\hookrightarrow \omega_0^2 = \frac{k}{m} \quad , > 0$$

2) opruga + gravitacija



sile: $F_{opr} = -ky$

$F_{grav} = -mg$

$\sum F = 0$ (ravnoteža)

$$-ky - mg = 0$$

$$ky + mg = 0$$

$$y = \frac{-mg}{k} \equiv y_0$$

koordinate:

$$x = y - y_0$$

$$y = x + y_0$$

jednadžba gibanja:

$$ma = F$$

$$m\ddot{x} = F_{opr} + F_{grav}$$

$$= -k(x + y_0) - mg$$

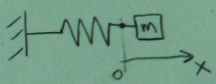
$$= -k\left(x - \frac{mg}{k}\right) - mg$$

$$= -kx + \cancel{mg} - \cancel{mg}$$

$$m\ddot{x} + \omega_0^2 x = 0$$

$$\hookrightarrow \omega_0^2 = \frac{k}{m}$$

3) iz E=konst.



$$E = E_{pot} + E_{kin}$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2$$

$$E = \text{konst.} \Rightarrow \frac{dE}{dt} = 0$$

$$kx\dot{x} + m\dot{x}\ddot{x} = 0$$

$$\dot{x}(kx + m\ddot{x}) = 0$$

$$m\dot{x}(\ddot{x} + \omega_0^2 x) = 0$$

$$\hookrightarrow \omega_0^2 = \frac{k}{m} > 0$$

Rješenje te dif. jedn.:

$$x[t] = A\cos(\omega_0 t + \phi)$$

Energija titranja:

$$E = E_k + E_p = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}m(\omega_0 \cdot A)^2 \sin^2[\ ] + \frac{1}{2}kA^2\cos^2[\ ] =$$

$$= \frac{1}{2}\cancel{m}\frac{k}{\cancel{m}}A^2\sin^2[\ ] + \frac{1}{2}kA^2\cos^2[\ ] = \boxed{\frac{1}{2}kA^2} \qquad \frac{1}{2}\omega_0^2 \cdot m \cdot A^2$$

$\langle E_k \rangle$

$$\langle E_k \rangle = \frac{1}{T}\int_0^T E_k(t)\,dt$$

$$E_k = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m(\omega_0 A)^2 \sin^2[\omega_0 t + \phi]$$

površina $= \frac{1}{2}$

$$\boxed{\langle E_k \rangle = \frac{1}{4}m(\omega_0 A)^2 = \langle E_p \rangle}$$

Torziono njihalo - pravi harmonički oscilator (formula ne ovisi o kutu)

konst. torzije $\quad D \equiv \frac{dM}{d\phi} = G\frac{\pi R^4}{2L}$

moment tromosti I, $\omega = \dot{\phi}$

Jedn. gibanja $\quad \frac{dL}{dt} = M$ kutna količina gibanja

$$L = I \cdot \omega = I \cdot \dot{\phi}$$

$$M = -D\phi$$ ako $\phi > 0$, $M < 0$ jer se vraća u ravnotežni položaj

$$\Rightarrow I\ddot{\phi} = -D\phi$$

$$I\ddot{\phi} + D\phi = 0$$

$$\ddot{\phi} + \omega_0^2\phi = 0$$

$$\omega_0^2 = \frac{D}{I}$$

Rješenje jedn.

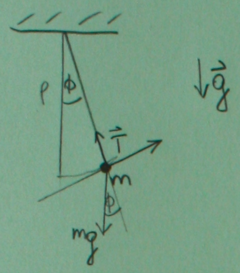
$$\phi(t) = \Phi \cdot \cos(\omega_0 t + \vartheta)$$

$\sqrt{\frac{D}{I}}$; faza

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{D}}$$

egzaktna (točna) formula

MATEMATIČKO NJIHALO - težina tijela uzrokuje njihanje



$\omega = \dot{\phi}$

Vrtnja oko objesišta!

$L = I\omega \quad ; \quad I = m\rho^2 \quad ; \quad \omega = \dot{\phi}$

$M = -mg \cdot \rho \sin\phi$ (projekcija na krak $\perp \rho'$)

jedn. gib.

$$\frac{dL}{dt} = M$$

$$I\dot{\omega} = m\rho^2 \ddot{\phi} = -mg\rho\sin\phi \quad / : m\rho^2$$

$$\ddot{\phi} + \frac{g}{\rho}\sin\phi = 0$$

za male ϕ → omnomnom sin ⇒ NIJE PRAVI H.O. ali je lijepa aproksimacija za male ϕ

$$\Rightarrow \ddot{\phi} + \omega_0^2 \phi = 0$$

$$\omega_0^2 = \frac{g}{\rho} \quad , \quad T = 2\pi\sqrt{\frac{\rho}{g}}$$

→ aproksimativna formula

Alternativni izvod iz E

$$E = E_k + E_p$$

$$= \frac{1}{2}I\omega^2 + mgh$$

$$= \frac{1}{2}m\rho^2\dot{\phi}^2 + mg\rho(1-\cos\phi)$$

$$\frac{dE}{dt} = 0$$

$$0 = m\rho^2\dot{\phi}\ddot{\phi} + mg\rho\sin\phi \cdot \dot{\phi}$$

$$= \dot{\phi}(m\rho^2\ddot{\phi} + mg\rho\sin\phi)$$

$$m\rho^2\ddot{\phi} + mg\rho\sin\phi = 0$$

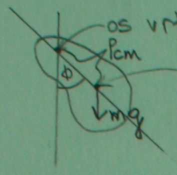
$$\ddot{\phi} + \frac{g}{\rho}\sin\phi = 0$$

za male ϕ, $\sin\phi \approx \phi$

$$\ddot{\phi} + \omega_0^2\phi = 0$$

$$\omega_0^2 = \frac{g}{\rho}$$

Fizičko njihalo - također nije pravi H.O.



središte mase = težište

$\downarrow \vec{g}$

vrtnja oko osi: $L = I\omega$, $\omega = \dot{\phi}$

$$M = \underbrace{-mg}_{\text{sila}} \underbrace{l_{cm}\sin\phi}_{\text{krak}}$$

<u>jedn. gib.</u> $\frac{dL}{dt} = M$

$$I\ddot{\phi} = -mgl_{cm}\sin\phi$$

$$\ddot{\phi} + \frac{mgl_{cm}}{I}\sin\phi = 0$$

$\Rightarrow$ NIJE H.O. jer opet imamo $\sin\phi$!

za male kutove ϕ
$\sin\phi \simeq \phi$

$$\ddot{\phi} + \omega_0^2\phi = 0$$

$$\hookrightarrow \omega_0^2 = \frac{mgl_{cm}}{I}$$

$$T = 2\pi\sqrt{\frac{I}{mgl_{cm}}}$$

$\hookrightarrow$ aproksimativna formula

<u>rješenje dif. jedn.</u> $\phi(t) = \Phi\cos(\omega_0 t + \vartheta)$

$\hookrightarrow$ amplituda

$\hookrightarrow$ fazni pomak , sve isto...

Steinerov tm (FIZ 1)

$$I = I_{cm} + ml_{cm}^2$$

$$\Rightarrow \omega_0^2 = \frac{mgl_{cm}}{I} = \frac{mgl_{cm}}{I_{cm} + ml_{cm}^2}$$

Izvod iz E

$$E = E_k + E_{pot}$$

$$E = \frac{1}{2}I\omega^2 + mgh$$

$$= \frac{1}{2}I\dot{\phi}^2 + mgl_{cm}(1-\cos\phi)$$

za $\phi \ll 1$, $\cos\phi \simeq 1 - \frac{\phi^2}{2}$

$$= \frac{1}{2}I\dot{\phi}^2 + mgl_{cm}\frac{1}{2}\phi^2$$

očuvanje E

$$\frac{dE}{dt} = 0$$

$$0 = I\dot{\phi}\ddot{\phi} + mgl_{cm}\phi\dot{\phi}$$

$$= \dot{\phi}(I\ddot{\phi} + mgl_{cm}\phi)$$

$$\ddot{\phi} + \omega_0^2\phi = 0$$

$$\hookrightarrow \omega_0^2 = \frac{mgl_{cm}}{I}$$

Reducirana duljina

↳ duljina niti matem. njihala koje ima isti T kao i dano fiz njihalo

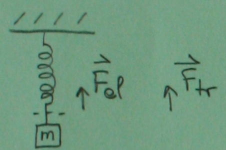
mat njih $\omega_0^2 = \frac{g}{l_{red}}$ fiz. njih. $\omega_0^2 = \frac{mgl_{cm}}{I}$

$$\Rightarrow l_{red} = \frac{g}{\omega_0^2} = \frac{gI}{mgl_{cm}} = \frac{I}{m \cdot l_{cm}}$$

$$l_{red} = \frac{I + ml_{cm}^2}{ml_{cm}} = \frac{I}{ml_{cm}} + l_{cm}$$

reverzibilno njihalo - ako su oba njihališta udaljena za određenu duljinu, oba će se njihati istim periodom

PRIGUŠENO TITRANJE



$\vec{F}_{tr} = -\underbrace{b}_{\text{ovisi o sredstvu}}\dot{\vec{r}}$ → viskozno trenje

$$\underbrace{m\vec{a}}_{\text{4.0.}} = \vec{F}_{el} + \vec{F}_{tr}$$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \quad /:m$$

$$\frac{d^2x}{dt^2} + \underbrace{\frac{k}{m}}_{\omega_0^2}x + \underbrace{\frac{b}{m}}_{2\delta}\frac{dx}{dt} = 0$$

$$\boxed{\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0}$$

Pretpostavimo rj. $x(t) = C \cdot e^{at}$; $c, a =$ konst.

$$\cancel{C}a^2\cancel{e^{at}} + 2\delta\cancel{C}a\cancel{e^{at}} + \omega_0^2 \cdot \cancel{C}\cancel{e^{at}} = 0$$

$$a^2 + 2\delta a + \omega_0^2 = 0$$

$$a_{1/2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

1. $\delta^2 < \omega_0^2$ SLABO PRIGUŠENJE
2. $\delta^2 > \omega_0^2$ JAKO PRIGUŠENJE
3. $\delta^2 = \omega_0^2$ KRITIČNO PRIGUŠENJE

1. SLABO PRIGUŠENJE

$\delta^2 < \omega_0^2$

$a_{1/2} = -\delta \pm i\omega \qquad ; \omega = \sqrt{\omega_0^2 - \delta^2}$

$x(t) = C_1 x_1(t) + C_2 x_2(t)$ (linearna kombinacija)

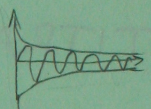
$x(t) = C_1 e^{-\delta t + i\omega} + C_2 e^{-\delta t - i\omega}$

$= e^{-\delta t}\left(C_1 e^{i\omega} + C_2 e^{-i\omega}\right) \qquad e^{\pm i\omega} = \cos\omega t \pm i \sin\omega t$

$= e^{-\delta t}\left(\underbrace{(C_1 + C_2)}_{A}\cos\omega t + i\underbrace{(C_1 - C_2)}_{B}\sin\omega t\right)$

$x(t) = A\underbrace{e^{-\delta t}}_{\text{novo!}} \cdot \begin{array}{l} \sin(\omega t + \varphi_0) \\ \cos(\omega t + \varphi_0) \end{array} \rightarrow \omega = \sqrt{\omega_0^2 - \delta^2}$

amplituda - smanjuje se



$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \delta^2}}$

poč. uvj. $t = 0 = t_0$

$x(t=0) = A_0 = A\sin\varphi_0$

$v(t=0) = 0$

$\frac{dx}{dt} = -\delta A e^{-\delta t}\sin(\omega t + \varphi_0) + A e^{-\delta t}\omega\cos(\omega t + \varphi_0)$

$0 = -\delta A \sin\varphi_0 + \omega A \cos\varphi_0$

$\delta \sin\varphi_0 = \omega\cos\varphi_0 \Rightarrow tg\varphi_0 = \frac{\omega}{\delta}$

$A = \frac{A_0}{\sin\varphi_0}$

nema u žutim formulama

logaritamski dekrement prigušenja λ

$\frac{A(t)}{A(t+T)} = \frac{A e^{-\delta t}}{A e^{-\delta(t+T)}} = e^{\delta T} \quad ; \quad \lambda = \ln\left(\frac{A(t)}{A(t+T)}\right) = \delta \cdot T$

$\frac{dE}{dt} = \frac{d}{dt}(E_k + E_p) = \frac{dE_k}{dv}\cdot\frac{dv}{dt} + \frac{dE_p}{dx}\cdot\frac{dx}{dt} = mv\cdot\frac{dv}{dt} + kx\frac{dx}{dt} = v\underbrace{\left(m\frac{dv}{dt} + kx\right)}_{-bv} = -bv^2$

Q faktor - faktor dobrote (govori koliki je gubitak E u radijanu)

$Q = 2\pi\frac{\langle E\rangle}{|\Delta E|}$ gubitak E u radijanu

$\langle E\rangle = \frac{1}{T}\int_t^{t+T} E(t)\,dt$

$\langle E\rangle = const \cdot e^{-2\delta t}$

$\frac{|\Delta E|}{T} \sim \frac{d\langle E\rangle}{dt} \sim const\cdot(-2\delta)\cdot e^{-2\delta t}$

$Q = 2\pi\frac{const\cdot e^{-2\delta t}}{T const\, 2\delta e^{-2\delta t}}$

$Q = \frac{\pi}{\delta\cdot T}$ (mali δ, velika dobrota) :)

2. JAKO PRIGUŠENJE

$\delta^2 > \omega_0^2$, $\omega = \sqrt{\omega_0^2 - \delta^2}$

$\omega' = \sqrt{\delta^2 - \omega_0^2}$

$a_{1/2} = -\delta \pm \omega'$

$x(t) = C_1 e^{(-\delta + \omega')t} + C_2 e^{(-\delta - \omega')t}$

$sh(\omega' t) = \frac{1}{2}(e^{\omega' t} - e^{-\omega' t})$

$ch(\omega' t) = \frac{1}{2}(e^{\omega' t} + e^{-\omega' t})$

$x(t) = e^{-\delta t}\left[A\, sh\omega' t + B\, ch\omega' t\right]$

$v(t) = \frac{dx}{dt} = -\delta e^{-\delta t}(A sh\omega' t + B ch\omega' t) + \omega' e^{-\delta t}(A ch\omega' t + B sh\omega' t)$

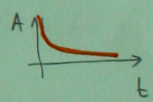
P.U. $x(t=0) = x_0$
$v(t=0) = 0$

$x_0 = 0(0 + B)$, $B = x_0$

$0 = -\delta(0 + B) + \omega'(A + 0)$

$-\delta B + \omega' A = 0$

$A = \frac{\delta \cdot x_0}{\omega'}$



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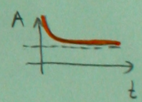


$\Rightarrow$ $x(t) = e^{-\delta t}\left[\frac{\delta x_0}{\omega'} sh\omega' t + x_0 ch\omega' t\right]$

3. GRANIČNO/KRITIČNO PRIGUŠENJE

$\delta^2 = \omega_0^2$

$\omega' = 0$

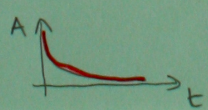
$ch(0) = 1$

$x(t) = e^{-\delta t}\left(\frac{\delta x_0}{\omega'} sh\omega' t + x_0 ch\omega' t\right)$

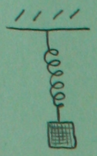
$\lim_{\omega' \to 0} \frac{sh\omega' t}{\omega' t} \cdot t = t$

$\Rightarrow$ $x(t) = e^{-\delta t} \cdot x_0(\delta t + 1)$

nema titranja $\rightarrow$ giba se do $A = 0$
$\Rightarrow$ vraća se u ravnotežni položaj



PRISILNO - NAMETNUTO TITRANJE



$$\vec{F}_{el} = -k\vec{x}$$

$$\vec{F} = -b\vec{v}$$

$$\vec{F}_v = \vec{F}_0 \sin \omega t$$

↳ frekv. vanjske sile

$$m\vec{a} = \sum \vec{F}$$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \sin\omega t \quad /:m$$

$$\frac{d^2x}{dt^2} + \underbrace{\frac{k}{m}}_{\omega_0^2}x + \underbrace{\frac{b}{m}}_{2\delta}\frac{dx}{dt} = \underbrace{\frac{F_0}{m}}_{A_0}\sin\omega t$$

$$\ddot{x} + \omega_0^2 x + 2\delta\dot{x} = A_0 \sin\omega t$$

$$x(t) = x_h(t) + x_p(t)$$

↳ isto ka prigušeno!

$$x_p(t) = A(\omega)\cdot\sin(\omega t - \varphi)$$

↳ A je funkcija od ω

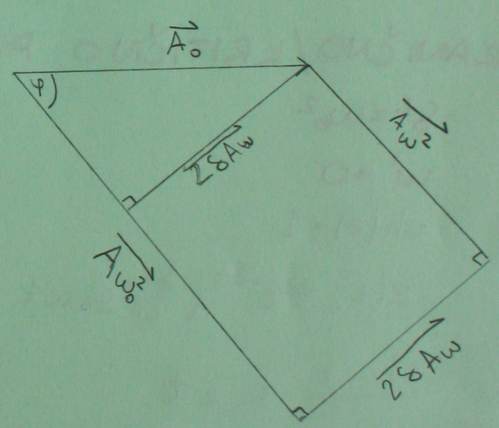
$$\frac{dx_p}{dt} = \omega A(\omega)\cos(\omega t - \varphi)$$

$$\frac{d^2x_p}{dt^2} = -\omega^2 A(\omega)\sin(\omega t - \varphi) \quad \Rightarrow \text{u jednadžbu}$$

$$-\omega^2 A(\omega)\sin(\omega t - \varphi) + 2\delta\omega A(\omega)\cos(\omega t - \varphi) + \omega_0^2 A(\omega)\cdot\sin(\omega t - \varphi) = A_0 \sin\omega t$$

$$A(\omega)\cdot\omega^2\sin(\omega t - \varphi + \pi) + 2\delta\omega A(\omega)\sin(\omega t - \varphi + \tfrac{\pi}{2}) + \omega_0^2 A(\omega)\sin(\omega t - \varphi) = A_0 \sin\omega t$$

$$\overrightarrow{A\omega^2} + \overrightarrow{2\delta\omega A} + \overrightarrow{\omega_0^2 A} = \overrightarrow{A_0}$$



$$A_0^2 = (2\delta A\omega)^2 + (A\omega_0^2 - A\omega^2)^2$$

$$A_0^2 = A^2\left(4\delta\omega^2 + (\omega_0^2 - \omega^2)^2\right)$$

$$A = A(\omega) = \frac{A_0}{\sqrt{4\delta^2\omega^2 + (\omega_0^2 - \omega^2)^2}}$$

$$\operatorname{tg}\varphi = \frac{2\delta A\omega}{A\omega_0^2 - A\omega^2} = \frac{2\delta\omega}{\omega_0^2 - \omega^2}$$

$$x(t) = x_h(t) + x_p(t)$$

ω prigušenja, ω vanjske sile

$$x(t) = \underbrace{Ae^{-\delta t}\sin(\omega_p t + \varphi_0)}_{\text{prigušeno}} + \underbrace{A(\omega)\sin(\omega t - \varphi)}_{\text{partikularno}}$$

REZONANCIJA u prisilnom titranju

→ ima maksimalnu A na nekoj ω

$$\frac{dA(\omega)}{d\omega} = 0 = \frac{-A_0}{2\left(4\delta^2\omega^2+(\omega_0^2-\omega^2)^2\right)^{3/2}}\left(8\delta^2\omega-2(\omega_0^2-\omega^2)\cdot 2\omega\right)$$

$$0 = \cancel{-A_0}\left(8\delta^2\cancel{\omega}-\cancel{4\omega}(\omega_0^2-\omega^2)\right)$$

$$2\delta^2-\omega_0^2+\omega^2=0$$

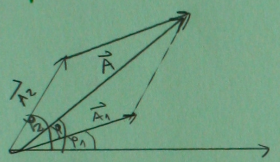
$$\omega^2=\omega_0^2-2\delta^2$$

$$\omega_R=\sqrt{\omega_0^2-2\delta^2} \quad \Rightarrow \text{ za } \delta=0 \quad \omega_R=\omega_0$$

- u rezonanciji je prijenos E maksimalan

Zbrajanje harmoničkih titraja

1. 2 titraja → ω jednake



$$x(t) = x_1(t) + x_2(t)$$
$$= A_1\sin(\omega t+\varphi_1)+A_2\sin(\omega t+\varphi_2)$$
$$= A\sin(\omega t+\varphi)$$

$$\vec{A}=\vec{A}_1+\vec{A}_2 \quad /^2$$

$$A^2=A_1^2+A_2^2+\underbrace{2\vec{A}_1\vec{A}_2}_{2A_1A_2\underbrace{\cos(\varphi_2-\varphi_1)}_{\substack{+1 \Rightarrow \text{konstr. int.}\\ -1 \Rightarrow \text{destr. int.}}}}$$

$\varphi_2-\varphi_1=0, 2\pi, \ldots \quad A=A_1+A_2$ konstr.
$\varphi_2-\varphi_1=\pi, 3\pi, \ldots \quad A=A_1-A_2$ destr. } interferencija

$$\mathrm{tg}\,\varphi=\frac{A_1\sin\varphi_1+A_2\sin\varphi_2}{A_1\cos\varphi_1+A_2\cos\varphi_2}$$

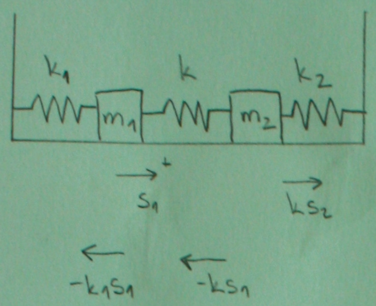
2. 2 titraja → ω_1, ω_2, amplitude jednake

$$x(t)=x_1(t)+x_2(t)=A\sin(\omega_1t+\varphi_1)+A\sin(\omega_2t+\varphi_2)$$

$$x(t)=\underbrace{2A\cos\left(\frac{\omega_1-\omega_2}{2}t+\frac{\varphi_1-\varphi_2}{2}\right)}_{\text{amplituda} \to \text{mijenja se!}}\cdot\sin\left(\frac{\omega_1+\omega_2}{2}t+\frac{\varphi_1+\varphi_2}{2}\right)$$

$\frac{\omega_1+\omega_2}{2}$ → frekvencija

VEZANI HO



Jedn. gib. za m_1 & m_2

$$m_1 \frac{d^2 s_1}{dt^2} = -k_1 s_1 - k s_1 + k s_2$$
$$= -k_1 s_1 + k(s_2 - s_1) \qquad ①$$

$$m_2 \frac{d^2 s_2}{dt^2} = -k_2 s_2 - k(s_2 - s_1)$$

$$\left.\begin{aligned} s_1(t) &= A_1 \sin(\omega t + \varphi_1) \\ s_2(t) &= A_2 \sin(\omega t + \varphi_2) \end{aligned}\right\} \Rightarrow A_1 = A_2$$

$$s_1(t) = A \sin(\omega t + \varphi_1)$$
$$s_2(t) = \pm A \sin(\omega t + \varphi_2)$$

1. Titranje u fazi

$$q_1(t) = s_1(t) + s_2(t)$$

$k_1 = k_2$ $\qquad \omega_0^2 = \frac{k_1}{m} = \frac{k_2}{m}$

$m_1 = m_2 = m$

①/masa $\quad \ddot{s}_1 + \omega_0^2 s_1 - \frac{k}{m}(s_2 - s_1) = 0 \qquad \mathrm{I}'$

②/masa $\quad \ddot{s}_2 + \omega_0^2 s_2 + \frac{k}{m}(s_2 - 1) = 0 \qquad \mathrm{II}'$

$\mathrm{I}' + \mathrm{II}'$

$$\underbrace{\ddot{s}_1 + \ddot{s}_2}_{\ddot{q}_1} + \omega_0^2 (s_1 + s_2) = 0$$

$$\boxed{\ddot{q}_1 + \omega_0^2 q_1 = 0}, \quad \omega_0 = \sqrt{\frac{k_1}{m}}$$

2. Protufazno titranje

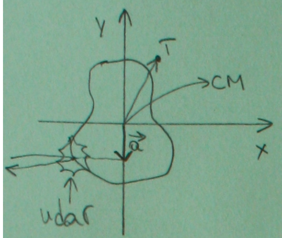
$$q_2(t) = s_1(t) - \underbrace{s_2(t)}_{\to -A}$$

$$\mathrm{I}' - \mathrm{II}' \Rightarrow \ddot{q}_2 + \omega_0^2 q_2 + 2\frac{k}{m}(s_1 - s_2) = 0$$

$$\ddot{q}_2 + \underbrace{\left(\omega_0^2 + \frac{2k}{m}\right)}_{\omega = \sqrt{\omega_0^2 + \frac{2k}{m}}} q_2 = 0$$

$$q_1 + q_2 = 2 s_1(t)$$
$$q_1 - q_2 = 2 s_2(t)$$

CENTAR UDARA (trebalo je biti kod fizičkog njihala)



pretp. → kruto tijelo se giba u x-y ravnini

→ na tijelo se prenosi linearna kol. gib. $\Delta\vec{p} \to \Delta p\,\vec{i}$

Tome odgovara kutna kol. gib. $\Delta\vec{L} = \vec{a} \times \Delta\vec{p}$

Tvrdnja: gibanje tijela <u>neposredno</u> nakon udara ekvivalentno je vrtnji oko točke T → centra udara

<u>Dokaz</u> - pronalazimo točku T

$$\Delta\vec{p} = m\,\Delta\vec{v}_{cm}$$

$$\Rightarrow \Delta\vec{v}_{cm} = \frac{\Delta\vec{p}}{m} = \frac{\Delta p}{m}\vec{i}$$

$$\Delta\vec{L} = I_{cm}\cdot\Delta\vec{\omega}$$

$$\Rightarrow \Delta\vec{\omega} = \frac{\Delta\vec{L}}{I_{cm}} = \frac{\vec{a}\times\Delta\vec{p}}{I_{cm}} = \frac{a_y\vec{j}\times\Delta p\,\vec{i}}{I_{cm}} = \frac{a_y\cdot\Delta p}{I_{cm}}(-\vec{k})$$

<u>brzina točke T</u>

$$\vec{b} = b_x\vec{i} + b_y\vec{j}$$

$$\Delta\vec{v}_T = \Delta\vec{v}_{cm} + \Delta\vec{\omega}\times\vec{b} = 0$$

$$\frac{\Delta p}{m}\vec{i} - \frac{a_y\cdot\Delta p}{I_{cm}}\vec{k}\times(b_x\vec{i} + b_y\vec{j}) = 0$$

$$\frac{\Delta p}{m}\vec{i} - \frac{a_y\Delta p}{I_{cm}}\cdot b_x\vec{j} + \frac{a_y\cdot\Delta p}{I_{cm}}\cdot b_y\cdot\vec{i} = 0$$

$$b_x = 0$$

$$\frac{\Delta p}{m} + \frac{a_y\cdot\Delta p}{I_{cm}}\cdot b_y = 0$$

$$b_y = \frac{-\cancel{\Delta p}}{m}\cdot\frac{I_{cm}}{a_y\cancel{\Delta p}}$$

$$b_y = -\frac{I_{cm}}{m\cdot a_y}$$

<u>ZANIMLJIVO</u> $|\vec{a}| + |\vec{b}| = l_{red}$

$$l_{red} = \frac{I_{cm}}{m\,\underbrace{l_{cm}}_{b}} + \overbrace{l_{cm}}^{b} = \frac{I_{cm}}{m b} + b$$

$$b = \frac{I}{ma},\ a = \frac{I}{mb}$$