DOPLER 1

$$f' = f \frac{v + v_p}{v - v_i}$$

f promatrača

(+) → izvor jedan prema drugom

(−) → udaljavanje

$$f' = f \sqrt{\frac{1 + v/c}{1 - v/c}}$$

rel. brzina

$$u_x' = \frac{v_x - v}{1 - \frac{v_x v}{c^2}}$$

1. brod: v'; 2. brod: v

2.12 $\lambda = 650$ nm = crveno

$v = 0{,}15c$

$$f = \frac{c}{\lambda}$$

$$f' = \frac{c}{\lambda} \sqrt{\frac{1 + v/c}{1 - v/c}} = 5{,}39 \cdot 10^{14}\ Hz$$

$$f' = \frac{c}{\lambda'} \Rightarrow \lambda' = \frac{c}{f'} = 558\ nm$$

(zelena)

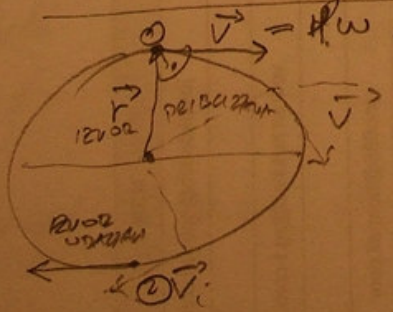
DZ 1

$f = 600$ Hz

$l = 1$ m

$\omega = 100$ okr/min $= \frac{100}{60} \cdot 2\pi\ s^{-1} = \frac{10\pi}{3}\ s^{-1}$

$\vec{v} = l\omega = \frac{10\pi}{3}\ ms^{-1}$



$v_p = 0$

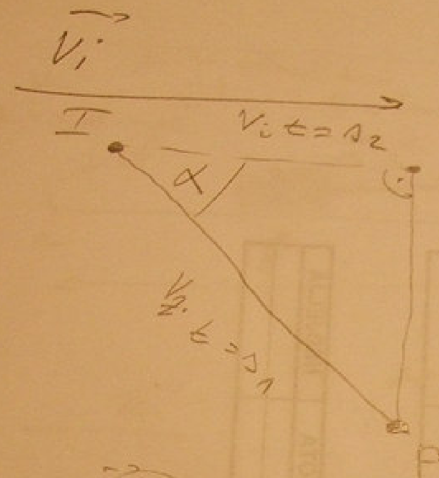
horizontalna komponenta UVIJEK → BRZINA IZVORA $\vec{v}_i$

$$f' = f \frac{v}{v \pm v_i} \Rightarrow$$

582 Hz

618,89 Hz

DZ [2]



$v_i = \frac{v_z}{2}$ $v_z = 343\ ms^{-1}$

$f_i = 100\ Hz$

$\cos\alpha = \frac{s_2}{s_1} = \frac{v_i t}{v_z \cdot t} = \frac{v_z}{2 v_z} = \frac{1}{2}$

$v_{i_1} = v_i \cos\alpha$

$f = f_i \frac{v_z}{v_z - v_i \cos\alpha} =$

$= f_i \frac{v_z}{v_z - \frac{v_z}{2} \cdot \frac{1}{2}} = 100 \cdot \frac{4}{3}$

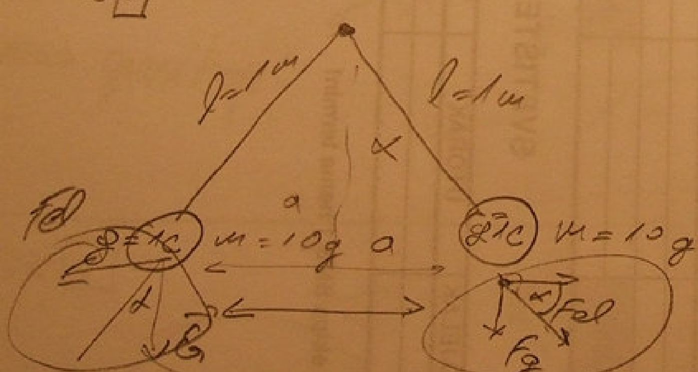
$= 133{,}33\ Hz$

moramo gledati ovu komponentu v_i koja je u smjeru zvuka

ELEKTROSTATIKA

Dz [3]

$a = 10\ cm$



$\sin\alpha = \frac{a}{l} = \frac{0{,}1}{1} = 0{,}1 \Rightarrow \alpha = 5{,}74°$

OPĆENITO

$F_{elektrostatska} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{d^2}$

$F_g = m \cdot g$

$\alpha = 5{,}74°$

$tg\alpha = \frac{F_{el}}{F_g}$

$F_{el} = G\ tg\alpha$

$(Q_1 \cdot Q_2) = G\ tg\alpha\ 4\pi\varepsilon_0 d^2$

$Q^2 = G\ tg\alpha\ 4\pi\varepsilon_0 d^2$

$Q = m \cdot g \cdot tg\alpha\ 4\pi\varepsilon_0 a^2$

$1{,}3 \cdot 10^{12}\ q_e$

izraziti A preko el. naboja

$q_e = 1{,}602 \cdot 10^{-19}\ C$

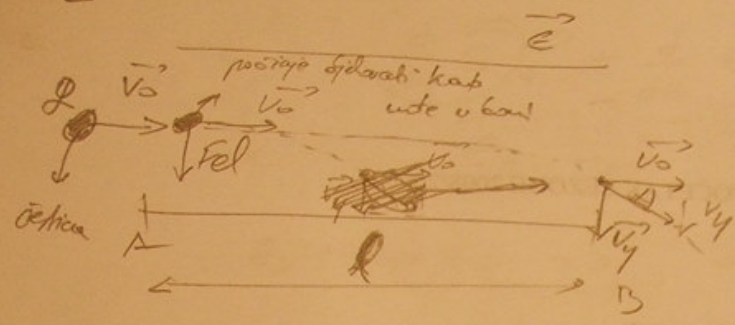
NODEKVATNO!!!

$q = \frac{A}{1{,}602 \cdot 10^{-19}} q_e = 4{,}135 \cdot 10^{11}\ q_e$

$q = A\ e^{upr.} =$

$A = 209\ nC$

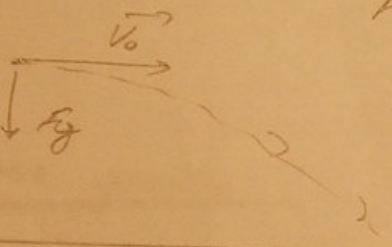
DZ 4



v_0 = konst. u horiz. smjeru

$$\operatorname{tg}\alpha = \frac{v_y}{v_0}$$

ANALOGNO HORIZ. HITCU



$$m \cdot a = F_{el} = q \cdot E$$

$$a = \frac{q \cdot E}{m}$$

$t = ?$ = vrijeme potrebno za prijeći l (A → B)

$$l = v_0 t \Rightarrow t = \frac{l}{v_0}$$

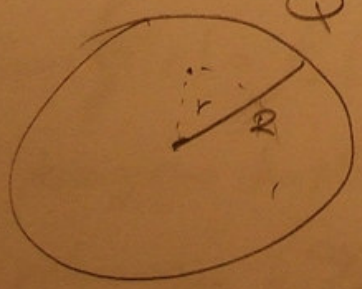
$$v_y = a \cdot t = \frac{qE}{m} \cdot t = \frac{qE}{m} \frac{l}{v_0}$$

$$\operatorname{tg}\alpha = \frac{qEl}{mv_0^2}$$

nema gravitacije!

$$\Rightarrow \alpha = \operatorname{arctg} \frac{qEl}{mv_0^2}$$

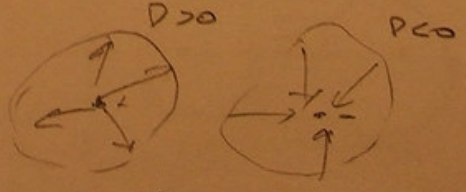
DZ 5



$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$$

$E(r) = ?$

$$\vec{D} = \frac{1}{\varepsilon_0} \rho$$



$\vec{\nabla} \epsilon = \frac{\partial}{\partial x}\vec{\epsilon}\,\vec{i} + \frac{\partial}{\partial y}\vec{\epsilon}\,\vec{j} + \frac{\partial}{\partial z}\vec{\epsilon}\,\vec{k}$

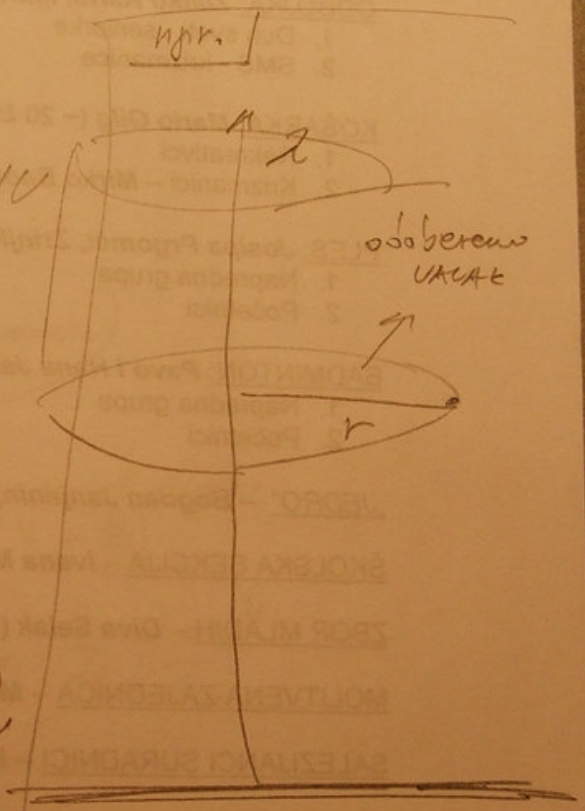
$\epsilon_x \quad \epsilon_y \quad \epsilon_z$

$\epsilon_x + \epsilon_y + \epsilon_z > 0 \Rightarrow \epsilon$ - izvor

$= 0 \Rightarrow$ nema izvor

$< 0 \Rightarrow$ silnice ulaze u izvor

Gaussova ploha



$$\iiint_V \vec{\nabla}\vec{\epsilon}\, dV = \frac{1}{\epsilon_0}\iiint_V \rho\, dV$$

$$\oint \vec{\epsilon}\,(\vec{n}\cdot d\sigma) = \epsilon \oint_S d\sigma$$

$\vec{n}$ - okomito

$d\sigma$ - skalarni

$$\epsilon \oint_S d\sigma = \frac{1}{\epsilon_0}\iiint_V \rho\, dV$$

$4r^2\pi \qquad Q'$

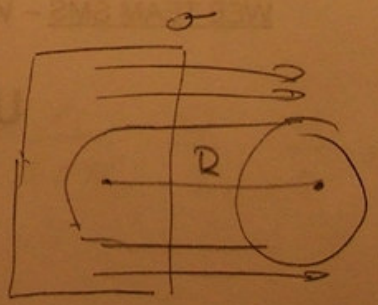
$\Rightarrow \rho(r) = \rho_0\left(1 - \frac{r}{R}\right) \Leftarrow$

homogena kugla $Q = \left(\frac{r}{R}\right)^3$

nehomogena kugla

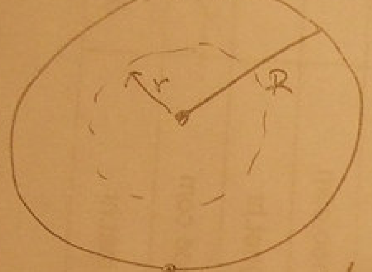
$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right)$

$Q' = Q\left(\frac{r}{R}\right)^2$



$$E \, 4 r^2 \pi = \frac{1}{\varepsilon_0} Q \left(\frac{r}{R}\right)^2 \Rightarrow E = \frac{1 \, r^2}{4 r^2 \pi \varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0 R^2}$$

DZ 6



Q

$$E(r) = E_{max} \cdot \left(\frac{r}{R}\right)^n$$

$$\rho(r) = f(Q, R)$$

E_{max} = jakost na površini

$$\vec{\nabla} \vec{E} = ~~\ldots~~ = \frac{1}{\varepsilon_0} \rho$$

~~⇒ uzimamo gaussovu kuglu kroz površinu~~

$$\rho = \varepsilon_0 (\vec{\nabla} \vec{E}) \rightarrow$$ gledamo u 1 smjeru $(x = r)$

$$\rho = \varepsilon_0 \cdot \frac{dE}{dx} = \varepsilon_0 E_{max} \frac{d}{dx} \left(\frac{x}{R}\right)^n$$

$$= \varepsilon_0 E_{max} \frac{n}{R^n} \cdot x^{n-1}$$

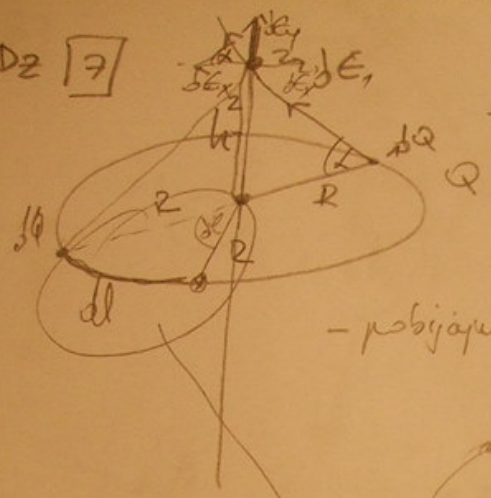
izrazit preko Q

$$E_{max} \cdot 4 R^2 \pi = \frac{1}{\varepsilon_0} Q$$

$$E_{max} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}$$

$$\rho = \varepsilon_0 \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \frac{n}{R^n} x^{n-1}$$

Dz [7]

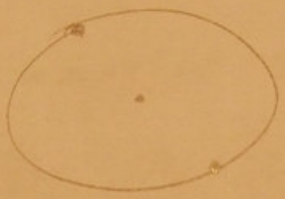


tanki obroč = $l_{obroč} = 2R\pi$

λ (linijski naboj) = $\frac{Q}{l} = \frac{Q}{2R\pi}$

$Q = 2l$

- pobijajo se dE_1 i dE_2



pogled odozgo

$= 0$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{2l}{d^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{d^2}dl$$

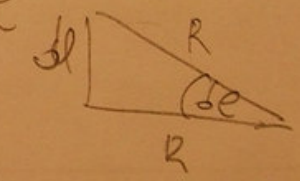
$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{d^2}dl\,\frac{h}{d}$$

$$\sin\alpha = \frac{h}{d}$$

$$\sin\alpha = \frac{E_y}{E}$$

$$E_y = E\frac{h}{d}$$

$dl = R\,d\varphi$



$$tg\,d\varphi = \frac{dl}{R}$$

$$dl = R\,tg\,d\varphi$$

$$\boxed{dl = R\,d\varphi}$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2h}{d^3}R\,d\varphi \Big/ \int$$

$\varphi = [0, 2\pi]$

$$E_y = \frac{2h}{4\pi\varepsilon_0 d^3}R\int_0^{2\pi} d\varphi \neq 2\pi$$

$$E_y = \frac{2hR}{4\pi\varepsilon_0 d^3}\,2\pi$$

$$E_y = \frac{2hR}{2\varepsilon_0 d^3} = \frac{Qh\cancel{R}}{2\cancel{2\pi}\varepsilon_0 d^3} = \frac{Qh}{4\pi\varepsilon_0 d^3}$$

$d^2 = R^2 + h^2$

$$E_y = \frac{Qh}{4\pi\varepsilon_0 (R^2+h^2)^{3/2}}$$

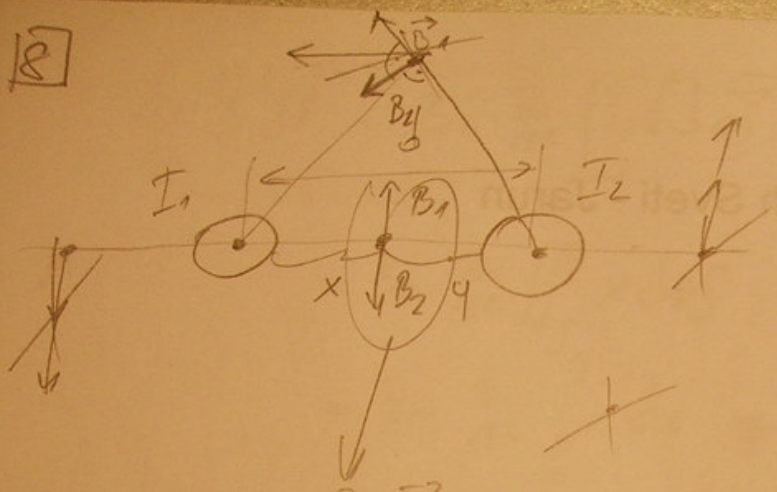
$$\frac{dE_y}{dh} = \ldots = 0$$

$$(R^2+h^2)^{3/2} = 3h^2(R^2+h^2)^{1/2}$$

$$R^2 + h^2 = 3h^2$$

$$\boxed{h = \frac{1}{\sqrt{2}}R}$$

8



$$I_2 = 2I_1$$

– oba vodiča

$\vec{B}_1 + \vec{B}_2$ enaka kot 0

a) $\vec{B}_1 + \vec{B}_2$

$$B = \frac{\mu_0 I}{2\pi d}$$

$B_1 = B_2$ TO TRAČIMO $x + y = d$

$$y = d - x$$

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 2I_1}{2\pi (d-x)}$$

$$\frac{1}{x} = \frac{2}{d-x}$$

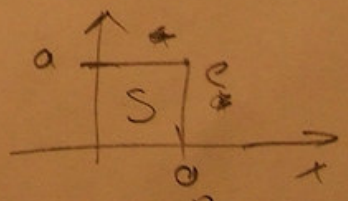
$$d - x = 2x$$

$$3x = d$$

$$x = \frac{d}{3}$$

---

9 $\vec{B}(x, y, z=0) = B_0 \left(3\hat{i} + \left(\frac{x}{a}\right)^2 \hat{j}\right)$ homogeno



$$\oint_C \vec{B}\, d\vec{l} = \iint_S (\vec{\nabla} \times \vec{B})\, d\vec{a}$$

$$\vec{\nabla} \times \vec{B} = B_0 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3 & \left(\frac{x}{a}\right)^2 & 0 \end{vmatrix} = B_0 \left[\hat{i}\left(-\frac{\partial}{\partial z}\left(\frac{x}{a}\right)^2\right) - \hat{j}\left(\left(\frac{-\partial}{\partial z}3\right)\right) + \hat{k}\left(\frac{\partial}{\partial x}\left(\frac{x}{a}\right)^2 - \frac{\partial}{\partial y}3\right) \right]$$

$$= B_0 \frac{2x}{a^2} = \frac{2B_0 x}{a^2}\hat{k}$$

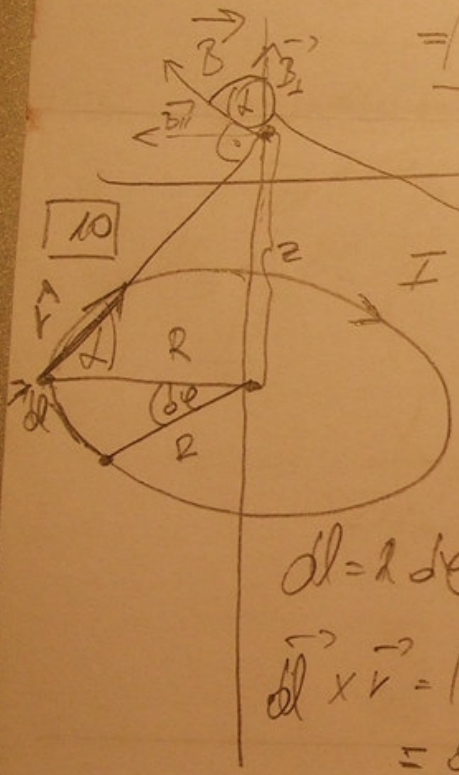
$$\oint_C \vec{B}\,d\vec{l} = B_0 \frac{2}{a^2} \iint_S x\,\vec{k}\cdot\vec{n}\,dS = \vec{n}\cdot d\vec{S}$$

$$\vec{n}\cdot\vec{k} = 1$$

$$= B_0 \frac{2}{a^2} \iint_S x\,dS$$

$$= B_0 \frac{2}{a^2} \int_0^a x\,dx \int_0^a dy = B_0 \frac{2}{a^2} \frac{1}{2} a^2 \cdot a$$

$$= B_0 a$$



kuteví s okomitim kracima

$$B_y = B\cos\alpha$$

$$= B\frac{R}{d}$$

$$\frac{R}{d} = \cos\alpha$$

BIOT-SAVARTOV ZAKON

$$dl = R\,d\varphi$$

$$d\vec{l}\times\vec{r} = |d\vec{l}|\,|\vec{r}|\sin\measuredangle(d\vec{l},\vec{r})$$

$$= dl$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{d^2}$$

$$dB = \frac{\mu_0 I\,dl}{4\pi d^2}$$

$$dB = \frac{\mu_0 I R^2 d\varphi}{4\pi d^3}$$

$$dB_y = dB\frac{R}{d}$$

$$dB = \frac{dB_y\,d}{R}$$

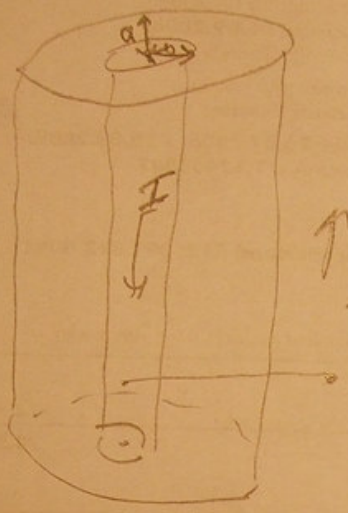
$$B_{uk} = \int_0^{2\pi} \frac{\mu_0 I R^2}{4\pi d^3}\,d\varphi$$

$$= \frac{\mu_0 I R^2}{4\pi d^3} 2\pi$$

$$B = \frac{\mu_0 I R^2}{2 d^3}$$

$$B = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

11



=> isto smjer jakosti

Suprotni: => $\vec{I}$ teče po površini

Gustoća el. polja → ne zanima nas

$$W_{EM} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2$$

Homogeno

$$B = \frac{\mu_0 I}{2r\pi_b} \quad \text{(između kabela)}$$

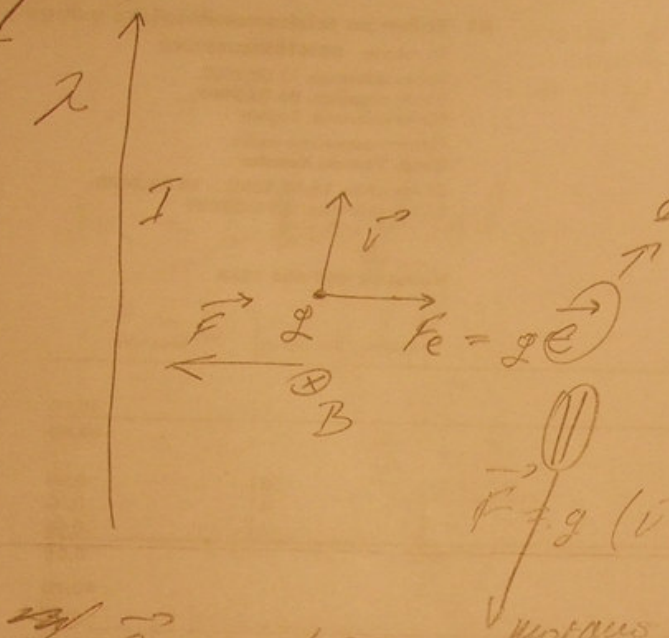
$$W_B = \int_0^{2\pi} d\varphi \int_a^b \left(\frac{\mu_0 I}{2r\pi}\right)^2 dr \left[\frac{1}{2\mu_0}\right]$$

$$= \frac{1}{2\mu_0} 2\pi \int_a^b \frac{\mu_0^2 I^2}{4r^2\pi^2} dr$$

$$= \frac{\pi}{\mu_0} \frac{\mu_0^2 I^2}{4\pi^2} \left(\frac{-1}{r}\right)\Big|_a^b$$

$$= \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

12]



$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$B = \frac{\mu_0 I}{2\pi d}$$

$\vec{F} = e(\vec{v} \times \vec{B})$ postavimo izjačurosti da Li

$e \cdot \vec{E} = e(\vec{v} \times \vec{B})$ vkupna sila = 0

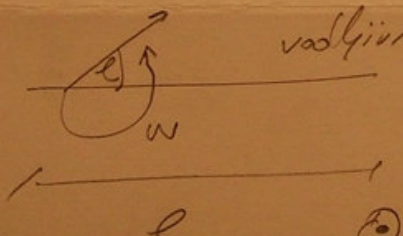
$|\vec{v} \times \vec{B}| = |\vec{v}||\vec{B}| \sin \sphericalangle(\vec{v},\vec{B})$

$$\frac{\lambda}{2\pi\varepsilon_0 d} = v \cdot \frac{\mu_0 I}{2\pi d}$$

$$\boxed{I = \frac{\lambda}{\mu_0 \varepsilon_0 v}}$$

13]

1. način



$$U_i = \frac{d\Phi}{dt} = \frac{d(B \cdot S)}{dt} = B \cdot \frac{dS}{dt}$$

$d\Phi = d(B \cdot S)$

$\Rightarrow dl = w\,dt$

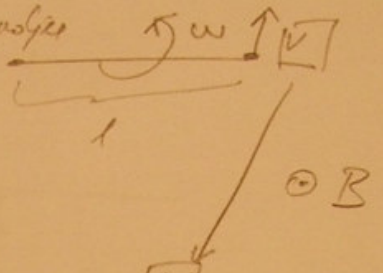
$dS = \frac{dl}{2\pi} \cdot l 2\pi$

$= \frac{w\,dt}{2} l^2$

$$U_i = B \frac{w\,dt\,l^2}{2\,dt} = \boxed{\frac{B w l^2}{2}}$$

2. NAČIN

$U_i = B \cdot l \cdot v$ — brzina gibanja vodiča u magn. polju



$v = \omega \cdot l$

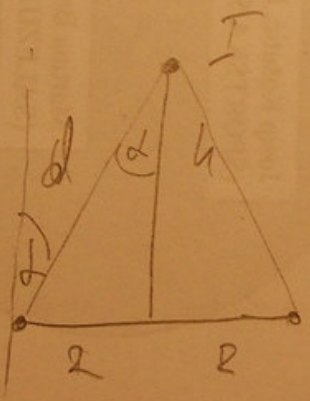
$dU_i = B \omega l \, dl$

$U_i = B\omega \int_0^L l \, dl$

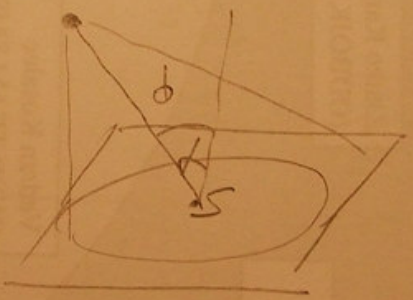
$= B\omega \frac{L^2}{2} \Big|_0^L$

$= \boxed{B\omega \frac{L^2}{2}}$

V6



Laplaceov zakon



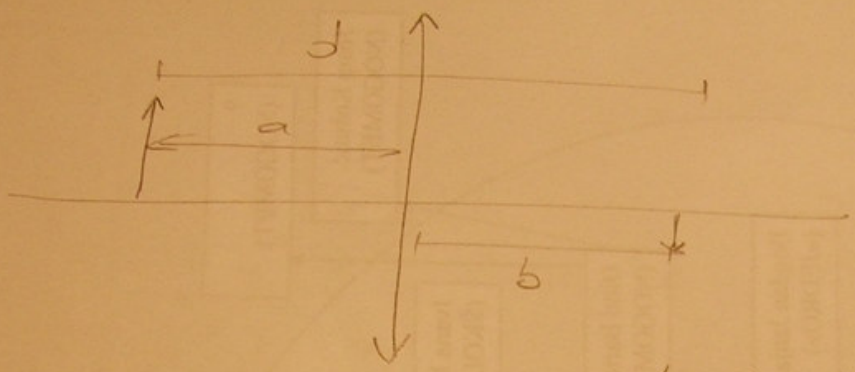
$\cos\alpha = \frac{h}{d}$

$\epsilon = \frac{I}{d^2} \cos\alpha$

$\epsilon = \frac{I}{d^2} \frac{h}{d} = \frac{Ih}{d^3} = \frac{Ih}{(R^2+h^2)^{3/2}}$

$\frac{d\epsilon}{dh} = \ldots = \phi \Rightarrow \boxed{h = \frac{R}{\sqrt{2}}}$

(21)



$M = \frac{-1}{2}$

$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$

$\frac{1}{a} + \frac{2}{a} = \frac{1}{f}$

$\frac{3}{a} = \frac{1}{f}$

$a+b=d$

$M = \frac{-b'}{2} = \frac{-b}{a}$ $\Big\}$ $b = \frac{a}{2}$

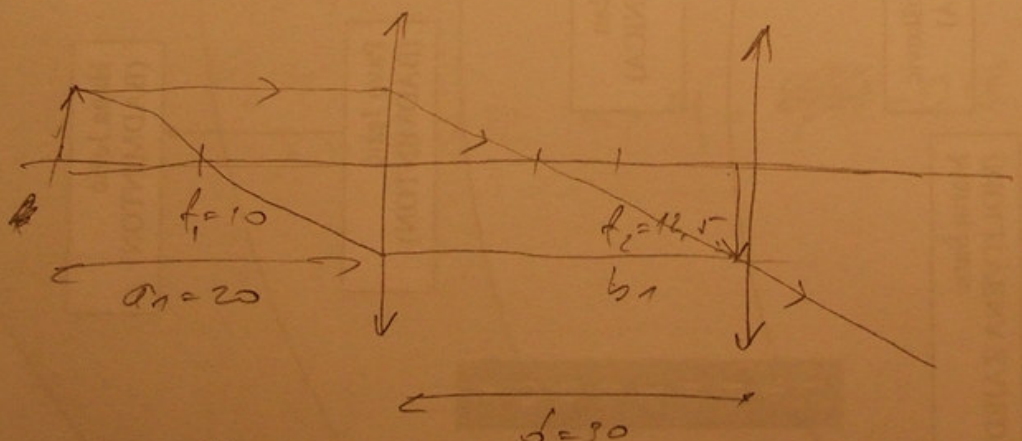
$\boxed{f = \frac{a}{3}}$

$\frac{d}{2} + a = d \Rightarrow a = \frac{2}{3}d \rightarrow f = \frac{a}{3} = \frac{2}{3}d \cdot \frac{1}{3}$

$\boxed{f = \frac{2}{9}d}$

---

(22)



$\frac{1}{a_1} + \frac{1}{b_1} = \frac{1}{f_1}$

$\boxed{b_1 = 20}$

$b_1 + a_2 = d$

$\boxed{a_2 = 10}$

$M = M_1 \cdot M_2$

$= \frac{-b_1}{a_1} \cdot \frac{-b_2}{a_2}$

$= \frac{-20}{20} \cdot \frac{50}{10}$

$= -1 \cdot 5$

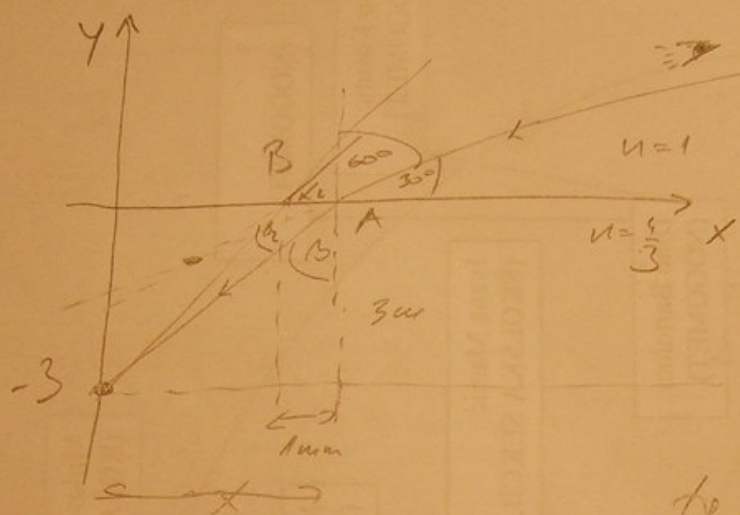
$\boxed{M = -5}$ obrnuta sa vecá

$\frac{1}{a_2} + \frac{1}{b_2} = \frac{1}{f_2}$

$\boxed{b_2 = -50}$ cm slika virtualna

$d_{objekat} = 50 - 10 = \boxed{40 \text{ cm}}$

ZADATAK 5.2



$\frac{\sin 60°}{\sin\beta} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{3}$

$\beta = 40,5°$

$tg\beta = \frac{x}{3} \Rightarrow x = 2,\sqrt{6}$

$A(2,\sqrt{6}, 0)$

$B(2.\sqrt{3}, 0)$

$tg\beta_2 = \frac{2,\sqrt{3}}{3}$

$B_2 = 40,469°$

$y_1 - 0 = tg\alpha_1 30°(x_1 - 2,\sqrt{6})$

$y_2 - 0 = tg(90-\alpha_2)(x_2 - 2,\sqrt{3})$

$\frac{\sin\alpha_2}{\sin\beta_2} = \frac{4}{3} \Rightarrow \alpha_2 = 59,91°$

$y_1 = y_2$

$x = 2,2482 \Rightarrow y = -0,18$ (?!?)

14

$\varepsilon_r = 1.15$

$\mu_r = 1.05$

$\frac{Z_2}{Z_1}$ $\quad \mathbb{C}, V$

n = indeks loma $= \frac{c}{v} =$

$n = \sqrt{\varepsilon_r \mu_r}$

$V = \lambda \cdot f$

λ konst

$\frac{\lambda \cdot Z_1}{\lambda \cdot Z_2} = \sqrt{\varepsilon_r \mu_r}$

Razlika za 9%

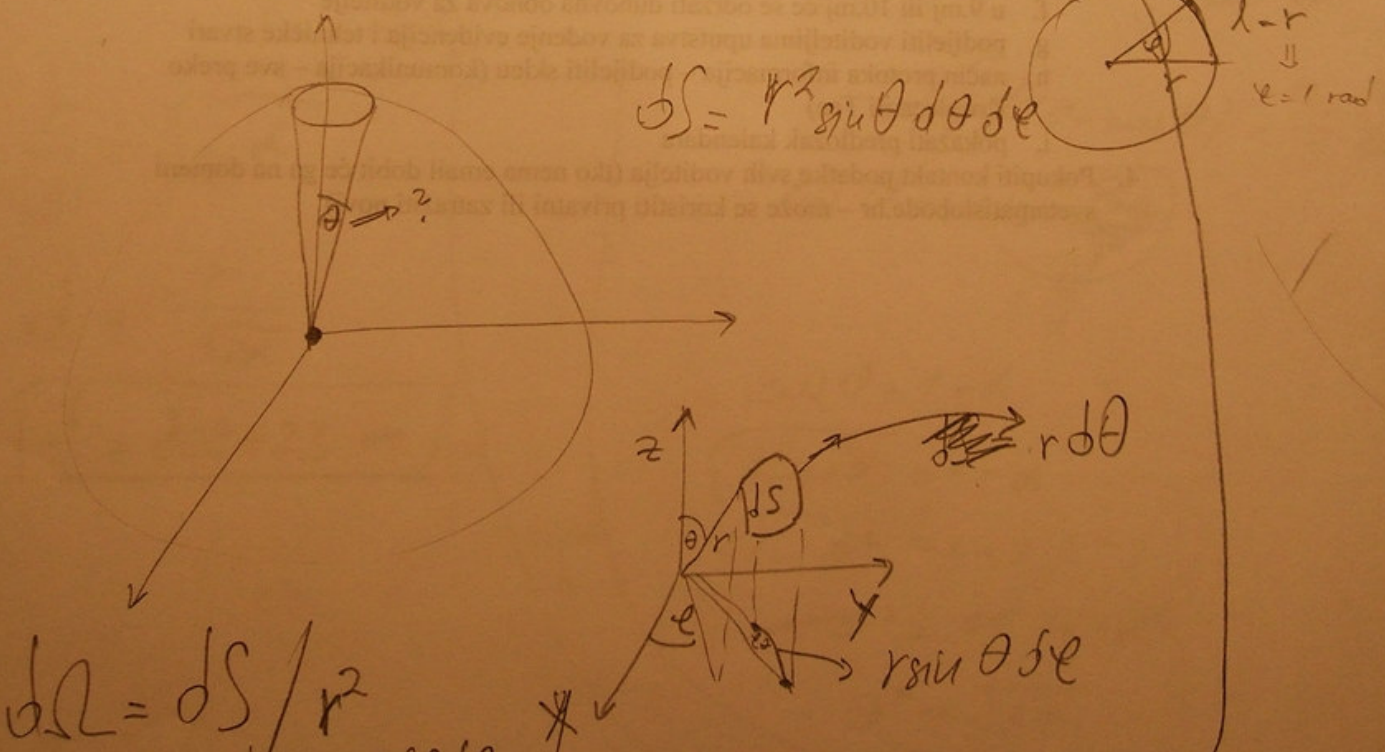
$\frac{Z_2}{Z_1} = \frac{1}{\sqrt{\varepsilon_r \mu_r}} = 0{,}91$

15

$\Phi = 5 \cdot 10^5$ lm

$I = 2 \cdot 10^5$ cd

$d\Phi = I \, d\Omega$ prostorni kut



$dS = r^2 \sin\theta \, d\theta \, d\varepsilon$

$d\Omega = dS / r^2$

$= \frac{r^2 \sin\theta \, d\theta \, d\varepsilon}{r^2} = \sin\theta \, d\theta \, d\varepsilon$

$\Omega = \int_0^{2\pi} d\varepsilon \int_0^{\pi} \sin\theta \, d\theta = 2\pi (1 - \cos\theta)$

$\Phi = I\,\Omega$

$= I\,2\pi(1-\cos\Theta)$

$\cos\Theta = 1 - \frac{\Phi}{2\pi I} \rightarrow \boxed{\Theta = 16{,}22^\circ}$

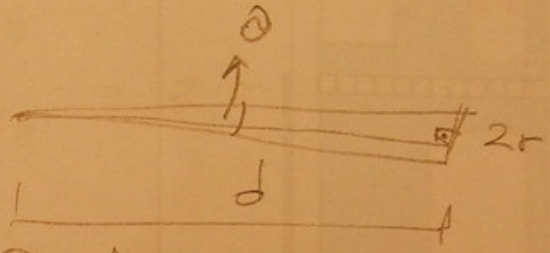
---

[17] $P = 10\ W$ $\quad 2r = 5\ mm$

$\lambda = 500\ nm$

$\epsilon = \frac{hc}{\lambda}$ [energia fotonu]

$P_{min} = 60 \cdot \frac{4\epsilon}{\lambda}$



$\frac{P}{4\pi}\Omega = P_{min}$

$\Omega = \frac{P_{min}}{P} 4\pi$

$\int_0^{2\pi} d\varphi \int_0^{\Theta} \sin\theta\, d\theta = \frac{P_{min}}{P} 4\pi$

$\Rightarrow 2\pi(1-\cos\Theta) = \frac{P_{min}}{P} 4\pi$

$\cos\Theta = 1 - 2\frac{P_{min}}{P}$

α

$\cos\Theta = 1 - \alpha$

$\sqrt{1-\sin^2\Theta} = 1 - \alpha$

$\sqrt{1-\Theta^2} = 1 - \alpha\ /^2$

$1 - \Theta^2 = 1 - 2\alpha + \alpha^2$

$\Theta^2 = 2\alpha$

$\Theta = \sqrt{2\alpha}$

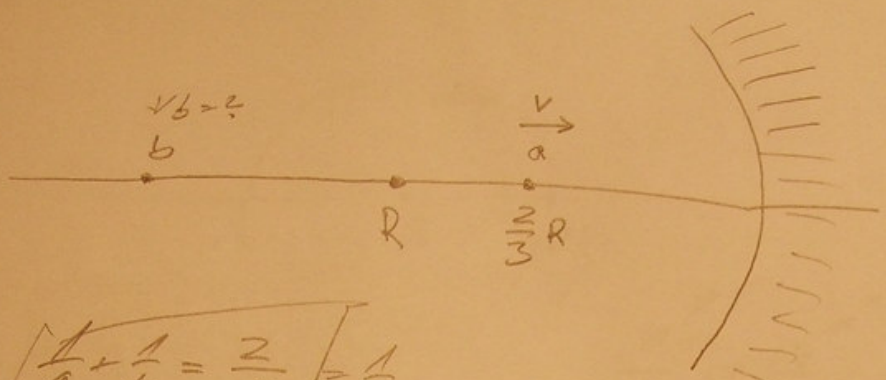
$tg\,\Theta = \frac{r}{d} = \Theta$

$\frac{r}{d} = \sqrt{2\alpha}$

$d = \frac{r}{\sqrt{2\alpha}}$

$\boxed{d = 809623\ m}$

18



$$\frac{1}{a} + \frac{1}{b} = \frac{2}{R} \Big| = \frac{1}{f}$$

$$b = \frac{\frac{R}{a}\,a}{a - \frac{R}{2}} = \frac{\frac{1}{2}R - \frac{2}{3}R}{\frac{2}{3}R - \frac{1}{2}R} = 2R$$

brzina i smjer slike?

$$\frac{1}{a+\Delta a} + \frac{1}{b+\Delta b} = \frac{2}{R}$$

$\frac{2}{3}R$

$$b + \Delta B = \frac{\frac{R}{2}(a+\Delta a)}{(a+\Delta a) - f}$$

$2R$

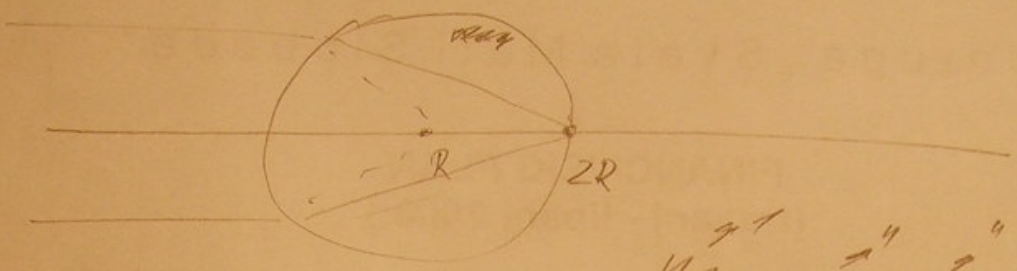
$$dB = -\frac{2R\,da}{R + 6\,da} \Big| : dt$$

$$\frac{db}{dt} = v_b = -\frac{2R\,v_a}{R + 6\,da} = \boxed{-2v_a}$$

slika se giba suprotnom smjeru od a i

2x brže

19



$a = \infty$
$b = 2R$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

$$\frac{n}{2R} = \frac{n-1}{R}$$

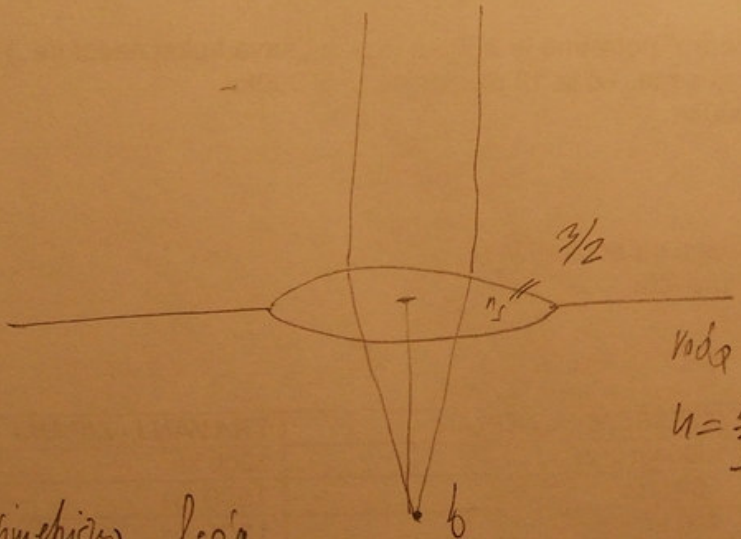
$$n = n - 1$$

$n = \infty$ NE POSTOJI

$$\frac{n}{2R} = \frac{n-1}{R}$$

$n = 2$

20



ravna $n = \frac{4}{3}$

simetrična leća

$R_1 = -R_2$

formula tanke leće

$$\frac{n_1}{a} + \frac{n_3}{b} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

$$\frac{4}{3b} = \frac{3/2 - 1}{R} + \frac{4/3 - 3/2}{R}$$

$b = 2R$

$b = 2f$

$$\frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

$$\frac{n_3 - 1}{1}\left(\frac{2}{R}\right) = \frac{1}{f}$$

$$\frac{1}{1} \cdot \frac{2}{R} = \frac{1}{f}$$

$f = R$