Harmonijski Oscilator

- Idealno titranje pod utjecajem el. sile, da tijelo beskonačno titra

1. $m \cdot a = -kx$

$$\rightarrow \quad m \cdot \frac{d^2}{dt} x[t] + k\,x[t] = 0$$

$$\frac{d^2}{dt} x[t] + \omega_0^2 x[t] = 0 \qquad \omega_0^2 = \frac{k}{m} = 2\pi f$$

$$x[t] = A\cos(\omega t + \phi)$$

2.

$$E_k = \frac{mv^2}{2} \qquad E_p = \frac{1}{2} k x^2$$

$$E_{uk} = E_k + E_p$$

$$\frac{dE}{dt} = 0$$

dalje izvedi sam

to sve ponavljaj!

Prigušeno - prisilno titranje

$$m \cdot a = -kx - bv + F_p \cos[\omega t]$$

$$\frac{d^2}{dt^2} x[t] + 2\delta \frac{d}{dt} x[t] + \omega_0^2 X[t] = f_p \cos[\omega t]$$

$$2\delta = \frac{b}{m} \qquad \omega_0^2 = \frac{k}{m} \qquad f_p = \frac{F_p}{m}$$

Fizičko njihalo

$\phi \rightarrow x$ $\quad p = v \cdot m$ $\quad L = I \cdot \omega$

$\omega \rightarrow v$ $\quad F = m \cdot a$ $\quad M = I \cdot \ddot{\phi}$

$I \rightarrow m$

Jednadžba gibanja!

$$\frac{dL}{dt} = M = \frac{d}{dt} I \cdot \omega$$

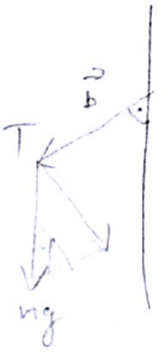
Vraća prema ravnoteži

$$-mgb \sin\phi = I \cdot \ddot{\phi}$$

$$\ddot{\phi} + \frac{mgb}{I} \cdot \phi = 0$$

$$\ddot{\phi} + \omega^2 \phi = 0$$

$$\omega^2 = \frac{mgb}{I}$$



Bilo koje tijelo koje njiše ali je obješeno u točki koja nije na osi težišta

Reducirana duljina fizičkog njihala

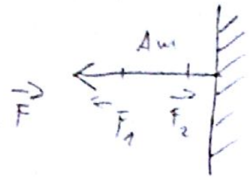
- Ona duljina niti matematičkog njihala koja ima isti period titraja kao i fizičko njihalo.

$$\omega_0^2 = \frac{mgb}{I} = \frac{g}{l_{red}}$$

$$l_{red} = \frac{I}{mb}$$

Transverzalni – Stojni val na žici

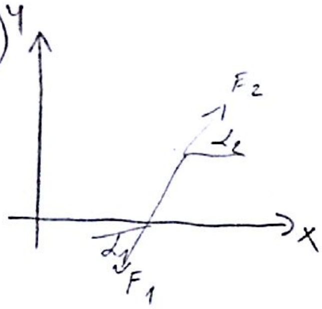
$$y(x,t) = a\cos(\omega t - kx) + a\cos(\omega t + kx)$$



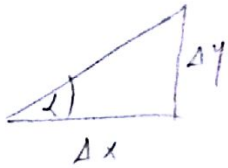
1. $F_1 = F_2$, $\mu = \frac{m}{l} = \frac{\Delta m}{\Delta l} = \frac{dm}{dl} = \frac{dm}{dx} \rightarrow dm = \mu \cdot dx$

$\Delta l = \Delta x$

2.



3.



4. $dF_y = F_{2y} - F_{1y} = F(\sin\alpha_2 - \sin\alpha_1)$

$\sin\alpha \approx tg\,\alpha$ $\qquad tg\,\alpha = \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$

$$dF_y = F(tg\,\alpha_2 - tg\,\alpha_1) = F\left(\left(\frac{\partial y}{\partial x}\right)_2 - \left(\frac{\partial y}{\partial x}\right)_1\right)$$

$$\left(\frac{\partial y}{\partial x}\right)_2 - \left(\frac{\partial y}{\partial x}\right)_1 = \Delta\left(\frac{\partial y}{\partial x}\right) = \frac{\Delta\left(\frac{\partial y}{\partial x}\right)}{\Delta x}\cdot\Delta x =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\cdot\Delta x = \frac{\partial^2 y}{\partial x^2}\Delta x$$

5. $dF_y = dm\,a = dm\frac{\partial^2 y}{\partial t^2}$

$$F\frac{\partial^2 y}{\partial x}\cancel{\Delta x} = \mu\,\cancel{dx}\,\frac{\partial^2 y}{\partial t^2} \qquad dF_y = F\left(\frac{\partial^2 y}{\partial x^2}\Delta x\right)$$

$$\boxed{\frac{\partial y}{\partial x} - \frac{\mu}{F}\frac{\partial y}{\partial t} = 0} \rightarrow v = \sqrt{\frac{F}{\mu}}$$

Interferencija valova!

$$\vec{E}_1(t,x) + \vec{E}_2(t,x_2) = \vec{E}(t,x)$$

$$E_0 \cos(\omega t - kx_1) + E_0 \cos(\omega t - kx_2) = \vec{E}(t,x)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2} \cdot \cos\frac{\alpha+\beta}{2}$$

$$\vec{E}(t,x) = \underbrace{2E_0 \cos\left[\frac{k(x_1-x_2)}{2}\right]}_{\text{Amplituda}} \cdot \cos\left[\omega t - \frac{k}{2}(x_2-x_1)\right]$$

optička razlika u hodu

Konstruktivna interferencija $\delta = m \cdot \lambda$

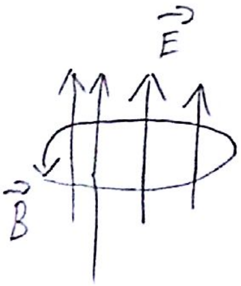
Destruktivna interferencija $\delta = (2m+1)\frac{\lambda}{2}$

$\nabla \cdot D = \rho_s$ Gausov zakon Električni $\oiint_S \vec{D}\vec{n}\,dS = \iiint_V \rho_s\,dV$

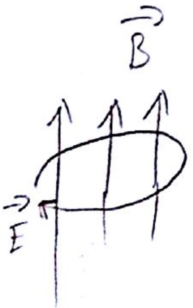
$\nabla \times E = -\frac{\partial B}{\partial t}$ Faradijev zakon indukcije $\oint_C \vec{E}\,dl = -\frac{\partial}{\partial t}\oiint_S \vec{B}\vec{n}\,dS$

$\nabla \cdot B = 0$ Gausov zakon magnetski $\Phi = \oiint_S B\vec{n}\,dS = 0$

$\nabla \times H = J + \frac{\partial D}{\partial t}$ Amperov zakon $\oint_C \vec{H}\,dl = \iint_S \left(J + \frac{\partial D}{\partial t}\right)\vec{n}\,dS$



$\frac{\partial \vec{E}}{\partial t} > 0$ Nastaje magnetsko polje



$\frac{\partial \vec{B}}{\partial t} > 0$ Nastaje električno polje

→ po pravilu desne ruke dodaje se (−) u faradijevom zakonu!

Jednadžba elektromagnetskog vala!

$\nabla \cdot D = 0$

$\nabla \cdot B = 0$

$\nabla \times E = -\frac{\partial B}{\partial t}$

$\nabla \times H = \frac{\partial D}{\partial t}$

$$\frac{\partial \vec{D}}{\partial t} = \nabla \times H \,/\,'$$

$$\frac{\partial^2 \vec{D}}{\partial t^2} = \nabla \times \left(\frac{\partial H}{\partial t}\right) = \nabla \times \left(\frac{\partial \left(\frac{B}{\mu}\right)}{\partial t}\right) = \frac{1}{\mu} \nabla \times \left(\frac{\partial B}{\partial t}\right)$$

$$\frac{\partial^2 \vec{D}}{\partial t^2} = \frac{-1}{\mu} \nabla \times (\nabla \times E)$$

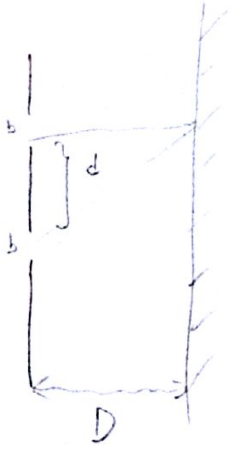
$$\nabla \times (\nabla \times \vec{E}) = -\nabla(\nabla \cdot \vec{E}) + \vec{E}(\nabla \cdot \nabla) = -\nabla(\underbrace{\nabla \cdot \vec{E}}_{=0}) + \Delta \vec{E} = 0$$

$$\frac{\partial^2 \vec{D}}{\partial t^2} = \frac{1}{\mu} \Delta \vec{E}$$

$$\boxed{\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{\varepsilon \mu} \Delta \vec{E} = 0}$$

Difrakcija kroz pukotine !

$N = 2$



$$I = E_0^2 \, \frac{\sin^2\left(\frac{N \Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$$

$$I = I_0 \, \frac{\sin^2\left(\frac{\pi}{\lambda} b \sin\alpha\right)}{\left(\frac{\pi}{\lambda} b \sin\alpha\right)^2} \cdot \frac{\sin^2\left(\frac{2\pi d}{\lambda} \sin\alpha\right)}{\sin^2\left(\frac{\pi d}{\lambda} \sin\alpha\right)}$$

$\Downarrow$ ogib na pukotini

$\Downarrow$ interferencija

Ogib		Interferencije
$b \sin\alpha = m\lambda$	min	$d \sin\alpha = (2m+1)\frac{\lambda}{2}$
$b \sin\alpha = (2m+1)\frac{\lambda}{2}$	maks	$d \sin\alpha = m\lambda$

Planck

$$E = \hbar \cdot \omega = h \cdot f = h \frac{c}{\lambda}$$

Fotoefekt

1.) Struja ako je jača je razmjerna intenzitetu

2.) Ako je frekvencija neke granice ili λ veći od neke granice, struja nestaje

3.) Postoji gornja granična energija elektrona!

$$E_{\gamma} = h \cdot f$$

$$\boxed{E_{\gamma} \geq W_{iz} + E_{kin}}$$

Borov model atoma

P1 Postoje dozvoljene staze za koje je $p = m \cdot v = 0$

P2 Kutna količina gibanja je $L = r \cdot p = r \cdot m \cdot v = \hbar \cdot n \quad n \in 1, 2, \ldots$

P3 Pri prelasku iz jednog u drugo stanje atom zrači foton energije koja je jednaka razlici energija tih dvaju stanja

$$F_{cp} = F_{culumb}$$

1.) $$\frac{m \cdot v^2}{r} = \frac{e^2}{4 \pi \varepsilon r^2}$$

$E_{uk} = E_k + E_p$

2.) $$L = r \cdot m \cdot v = \hbar \cdot n \rightarrow v = \frac{\hbar \cdot n}{r \cdot m}$$

$2 \rightarrow 1$

$v = ?$ $r = ?$

Radioaktivni raspad

$N \rightarrow$ broj jezgara

$$\boxed{dN = -\lambda N \, dt}$$

$$\frac{dN}{N} = -\lambda \, dt$$

$$\ln N = -\lambda t + C$$

$$N[t] = e^{C} \cdot e^{-\lambda t} = N_0 \cdot e^{-\lambda t}$$

Aktivnost uzorka

$$\cancel{A = \left|\frac{dN}{dt}\right| = \lambda N}$$

$$\boxed{A[t] = A_0 \cdot e^{-\lambda t}}$$

$$A_0 = \lambda \cdot N_0$$

Vrijeme poluraspada

$$N[T_{1/2}] = \frac{1}{2} N[0]$$

$$N_0 \cdot e^{-\lambda T_{1/2}} = \frac{1}{2} N_0$$

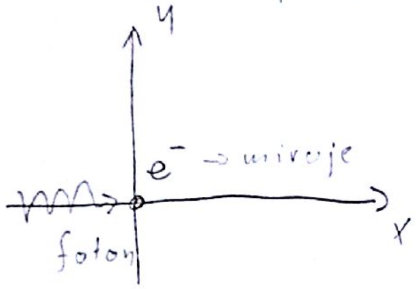
$$e^{-\lambda T_{1/2}} = \frac{1}{2}$$

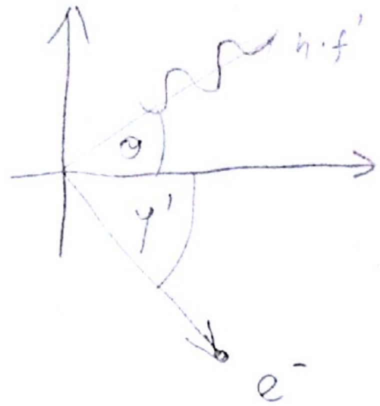
$$\ln 2 = \lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Camponovo raspršenje

prije sudar nakon





Energija

$$h \cdot f + m_e c^2 = h \cdot f' + \gamma' m_e c^2$$

$$\gamma' = \frac{1}{\sqrt{1 - \left(\frac{v_e'}{c}\right)^2}}$$

Količina gibanja

$$\frac{hf}{c} = \frac{hf'}{c} \cos\theta + \gamma' m_e v_e' \cos\varphi$$

$$\frac{hf'}{c} \sin\theta = \gamma' m_e v_e' \sin\varphi$$

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$