PRIGUŠENO TITRANJE:

$F_{TR} = -b\vec{v}$

poč. uvjeti $x = x_0$, $\dot{x} = v_0$

$m\frac{d^2x}{dt^2} = -kx - bv$ ← jedn. gibanja

$m\frac{d^2x}{dt^2} + kx + bv = 0$

$$\boxed{\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0}$$

$\frac{b}{m} = 2\delta$ → faktor prigušenja

$\frac{k}{m} = \omega_0^2$ → vlastita frekvencija

pretpostavimo rješenje $x = Be^{\alpha t}$ $\quad x' = B\alpha e^{\alpha t}$ $\quad x'' = B\alpha^2 e^{\alpha t}$ i uvrstimo:

$$B\alpha^2 e^{\alpha t} + 2\delta B\alpha e^{\alpha t} + \omega_0^2 B e^{\alpha t} = 0$$

$$\alpha^2 + 2\delta\alpha + \omega_0^2 = 0$$

$$\alpha_{1,2} = \frac{-2\delta \pm \sqrt{4\delta^2 - 4\omega_0^2}}{2}$$

$$\alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

Postoje 3 slučaja:

- $\omega_0^2 > \delta^2$ → slabo prigušenje
- $\delta^2 = \omega_0^2$ → kritično prigušenje
- $\delta^2 > \omega_0^2$ → aperiodičko prigušenje

I SLABO PRIGUŠENJE: $(\alpha_{1,2} = -\delta \pm i\omega)$

$x = Ce^{-\delta t}e^{i\omega t} + De^{-\delta t}e^{-i\omega t}$

$v = Ce^{-\delta t}e^{i\omega t}(-\delta + i\omega) + De^{-\delta t}e^{-i\omega t}(-\delta - i\omega)$

poč. uvjeti

$\Rightarrow x_0 = C e^0 e^0 + D e^0 e^0 = C + D$

$\Rightarrow v_0 = C(i\omega - \delta) + D(-\delta - i\omega) = -\delta(C + D) + i\omega(C - D)$

$\text{Im}\,C = -\text{Im}\,D$, $\text{Re}\,C = \text{Re}\,D$ ⇒ C i D su kompleksno konjugirani $D = C^*$

$$\Rightarrow \boxed{x = Ee^{-\delta t}e^{i(\omega t + \varphi_0)} + Ee^{-\delta t}e^{-i(\omega t + \varphi_0)} = 2Ee^{-\delta t}\cos(\omega t + \varphi_0) = A_0 e^{-\delta t}\cos(\omega t + \varphi_0) = \underbrace{A_0 e^{-\delta t}}_{\text{amplituda}}\sin(\omega t + \varphi_0)}$$

II APERIODIČKO PRIGUŠENJE $\left(\alpha_{1,2} = -\delta \pm \underbrace{\sqrt{\delta^2 - \omega_0^2}}_{\omega'}\right)$

$x = Ae^{-\delta t}e^{\omega' t} + Be^{-\delta t}e^{-\omega' t}$

$x = A e^{-\delta t}(\text{ch}\,\omega' t + \text{sh}\,\omega' t) + B e^{-\delta t}(\text{ch}\,\omega' t - \text{sh}\,\omega' t)$ zbog:

$\text{ch}\,\omega' t = \frac{e^{\omega' t} + e^{-\omega' t}}{2}$

$\text{sh}\,\omega' t = \frac{e^{\omega' t} - e^{-\omega' t}}{2}$

$e^{\omega' t} = \text{ch}\,\omega' t + \text{sh}\,\omega' t$

$e^{-\omega' t} = \text{ch}\,\omega' t - \text{sh}\,\omega' t$

$x = e^{-\delta t}(\text{ch}\,\omega' t(A + B) + \text{sh}\,\omega' t(A - B))$

$x = e^{-\delta t}(C\,\text{ch}\,\omega' t + D\,\text{sh}\,\omega' t)$

$v = -\delta e^{-\delta t}(C\,\text{ch}\,\omega' t + D\,\text{sh}\,\omega' t) + e^{-\delta t}(C\omega'\,\text{sh}\,\omega' t + D\omega'\,\text{ch}\,\omega' t)$

poč. uvjeti $t = 0$:

$v = 0 \Rightarrow v_0 = -\delta \cdot C + 1 \cdot D\omega' = 0 \Rightarrow D = \frac{\delta}{\omega'}x_0$

$x = 0 \Rightarrow x_0 = e^0(C\,\text{ch}\,0 + D\,\text{sh}\,0) = C$

$$\boxed{x = x_0 e^{-\delta t}\left(\text{ch}\,\omega' t + \frac{\delta}{\omega'}\text{sh}\,\omega' t\right)}$$

• PRISILNO TITRANJE:

periodična vanjska sila $F = F_0 \sin\omega t$, $\omega_0^2 = \frac{k}{m}$ - vlastita frekv.

ω - frekv. vanjskog osc.

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \sin\omega t \quad /m$$

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = \underbrace{\frac{F_0}{m}}_{A_0}\sin\omega t \qquad \delta = \frac{b}{2m} \quad \omega_0^2 = \frac{k}{m}$$

rješenje jednadžbe: $x = x_H + x_p$ (homogeno + partikularno)

uzimamo da je $x_p = A(\omega)\sin(\omega t - \varphi)$

$\dot{x}_p = -A\omega\cos(\omega t - \varphi) \qquad \ddot{x} = -A\omega^2\sin(\omega t - \varphi)$

uvrstimo: $-A\omega^2\sin(\omega t - \varphi) - 2\delta A\omega\cos(\omega t - \varphi) + \omega_0^2 A\sin(\omega t - \varphi) = A_0\sin(\omega t)$

$$A(\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2A\delta\omega\sin(\omega t - \varphi + \tfrac{\pi}{2}) = A_0\sin(\omega t)$$

$$\Longrightarrow \quad A_0^2 = 4A^2\delta^2\omega^2 + A^2(\omega_0^2 - \omega)^2 \qquad tg\varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2}$$

$$\boxed{A = \frac{A_0}{\omega_0^2\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\frac{\delta^2\omega^2}{\omega_0^4}}}} \qquad \boxed{tg\varphi = \frac{2\delta\frac{\omega}{\omega_0^2}}{1 - \frac{\omega^2}{\omega_0^2}}}$$

PRIJELAZNO ili TRANZIJENTNO VRIJEME - vrijeme potrebno da utrne x_H

Stacionarno rješenje $x = x_p = A(\omega)\sin(\omega t - \varphi)$

Rezonantna frekvencija: $\frac{dA}{d\omega} = 0 \quad \ldots \quad \boxed{\omega_r = \sqrt{\omega_0^2 - 2\delta^2}}$

• HARMONIČKI OSCILATOR:

$F_{opr} = -kx$

poč. položaj $\vec{i}mg - \vec{i}k\Delta x = 0$

povlačimo dolje (rukom) $\vec{i}mg + \vec{i}F_{ruka} - \vec{i}k(\Delta x + A) = 0$

mičemo ruku $\vec{i}mg - \vec{i}k(\Delta x + x) = \vec{i}m\ddot{x}$

poč. uvjeti: $x(0) = x_0$, $\dot{x}(0) = \emptyset$

$$\underbrace{mg - k\Delta x}_{=0} - k\cdot x = m\ddot{x}$$

$$m\cdot\ddot{x} = -k\cdot x$$

$$\ddot{x} + \underbrace{\frac{k}{m}}_{\omega_0}x = 0$$

supstitucija: $x = Ae^{\alpha t}$, $\dot{x} = A\alpha e^{\alpha t}$, $\ddot{x} = A\alpha^2 e^{\alpha t}$

$$A\alpha^2 e^{\alpha t} + \omega_0^2 A e^{\alpha t} = 0$$

$$\alpha^2 + \omega_0^2 = 0 \qquad \alpha_{1,2} = \pm i\omega_0$$

dobivamo rješenja $x_1(t) = A_1 e^{i\omega_0 t}$, $x_2(t) = A_2 e^{-i\omega_0 t}$

BERBECKOVO NJIHALO

- u fazi $\quad x_1 = A_1 \sin(\omega_1 t + \phi_1)$

$$x_2 = A_1' \sin(\omega_1 t + \phi_1)$$

I jedn. $\quad -A_1\omega_1^2 \sin(\omega_1 t + \phi_1) + A_1(\omega_0^2 + \Omega^2)\sin(\omega_1 t + \phi_1) = \Omega^2 A_1' \sin(\omega_1 t + \phi_1)$

$$\omega_1^2 - (\omega_0^2 + \Omega^2) + \Omega^2 \frac{A_1'}{A_1} = 0 \qquad (*)$$

II jedn. $\quad -A_1'\omega_1^2 \sin(\omega_1 t + \phi_1) + A_1' \sin(\omega_1 t + \phi_1)(\omega_0^2 + \Omega^2) = \Omega^2 A_1 \sin(\omega_1 t + \phi_1)$

$$\omega_1^2 - (\omega_0^2 + \Omega^2) + \Omega^2 \frac{A_1}{A_1'} = 0 \qquad (**)$$

II

iz $(*)$ i $(**)$ $\Rightarrow \frac{A_1'}{A_1} = \frac{A_1}{A_1'} \Rightarrow \boxed{A_1 = A_1'}$

$(*) \Rightarrow \omega_1^2 - \omega_0^2 - \Omega^2 + \Omega^2 = 0$

$$\boxed{\omega_1 = \omega_0}$$

- u protufazi $\quad x_1 = A_2 \sin(\omega_2 t + \phi_2)$

$$x_2 = A_2' \sin(\omega_2 t + \phi_2 + \pi) = -A_2' \sin(\omega_2 t + \phi_2)$$

I jedn. $\quad -A_2\omega_2^2 \sin(\omega_2 t + \phi_2) + A_2(\omega_0^2 + \Omega^2)\sin(\omega_2 t + \phi_2) = -\Omega^2 A_2' \sin(\omega_2 t + \phi_2)$

$$\omega_2^2 - (\omega_0^2 + \Omega^2) - \Omega^2 \frac{A_2'}{A_2} = 0 \qquad (*)$$

II jedn. $\quad A_2'\omega_2^2 \sin(\omega_2 t + \phi_2) - A_2' \sin(\omega_2 t + \phi_2)(\omega_0^2 + \Omega^2) = \Omega^2 A_2 \sin(\omega_2 t + \phi_2)$

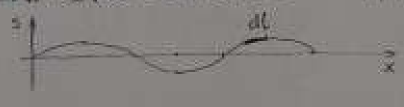
$$\omega_2^2 - (\omega_0^2 + \Omega^2) - \frac{A_2}{A_2'}\Omega^2 = 0 \qquad (**)$$

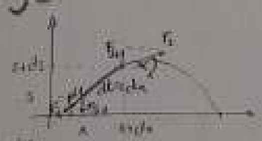
$(*)$ i $(**)$ $\Rightarrow \boxed{A_2' = A_2}$

$(*) \Rightarrow \omega_2^2 - \omega_0^2 - \Omega^2 - \Omega^2 = 0$

$$\omega_2 = \sqrt{\omega_0^2 + 2\Omega^2}$$

• TRANSVERZALNO TITRANJE:





$$|\vec{F}_1| = |\vec{F}_2|$$

$dF_s = F_{s2} - F_{s1} = F(\sin\alpha_2 - \sin\alpha_1)$ $\quad$ mali kutovi pa vrijedi $\sin\alpha \approx \alpha$ i $dx \approx tg\alpha \cdot dx$

$dF_s = F(tg\alpha_2 - tg\alpha_1)$

$tg\alpha_2 = \left(\frac{\partial \psi}{\partial x}\right)_{x+dx} \quad tg\alpha_1 = \left(\frac{\partial \psi}{\partial x}\right)_x$

$$dF_s = F\left[\left(\frac{\partial \psi}{\partial x}\right)_{x+dx} - \left(\frac{\partial \psi}{\partial x}\right)_x\right]$$

$$= F\left[\left(\frac{\partial \psi}{\partial x}\right)_x + \left(\frac{\partial^2 \psi}{\partial x^2}\right)_x dx - \left(\frac{\partial \psi}{\partial x}\right)_x\right] = F\frac{\partial^2 \psi}{\partial x^2}dx$$

$dF_S = F\dfrac{\partial^2\psi}{\partial x^2}dx$, $dm = \mu dx$ μ - linearna gustoća mase

$dF_S = \mu \dfrac{\partial^2\psi}{\partial t^2}dx$ ← II Newtonov aksiom

$\Rightarrow F\dfrac{\partial^2\psi}{\partial x^2}dx = \mu\dfrac{\partial^2\psi}{\partial t^2}dx$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} - \frac{\mu}{F}\frac{\partial^2\psi}{\partial t^2} = 0}$$

rješenja jednadžbe:

$\psi = f(vt - x) + g(vt + x)$

HUYGENSOV PRINCIP:

-svaka točka valne fronte izvor je novog kuglastog elementarnog vala; envelopa (ovojnica) svih elementarnih valova je nova valna fronta.

• SUPERPOZICIJA VALOVA:

2 vala iste amplitude ; ω_1, ω_2 , s pomakom u fazi :

$\psi_1 = A\sin(\omega_1 t - k_1 x + \phi_1)$

$\psi_2 = A\sin(\omega_2 t - k_2 x + \phi_2)$

$\psi = \psi_1 + \psi_2 = A\left(\sin(\omega_1 t - k_1 x + \phi_1) + \sin(\omega_2 t - k_2 x + \phi_2)\right) =$

$= 2A\left(\sin\left(\dfrac{\omega_1 t - k_1 x + \phi_1 + \omega_2 t - k_2 x + \phi_2}{2}\right)\cdot\cos\left(\dfrac{\omega_1 t - k_1 x + \phi_1 - \omega_2 t + k_2 x - \phi_2}{2}\right)\right)$

$= 2A\left[\sin\left(\dfrac{\omega_1+\omega_2}{2}t - \dfrac{k_1+k_2}{2}x + \dfrac{\phi_1+\phi_2}{2}\right)\cos\left(\dfrac{\omega_1-\omega_2}{2}t - \dfrac{k_1-k_2}{2}x + \dfrac{\phi_1-\phi_2}{2}\right)\right]$

za $\omega_1 = \omega_2 = \omega$, $k_1 = k_2 = k$

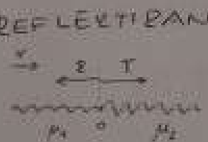
1. $\phi_1 = \phi_2 = \phi$ $\boxed{\psi = 2A\sin(\omega t - kx + \phi)}$ KONSTRUKTIVNA INTERFERENCIJA

$\phi_1 = \phi_2 + 2m\pi$ $m = 0, 1, \dots$

2. $\phi_1 = \phi_2 + \pi$ $\boxed{\psi = 2A\sin(\omega t - kx + \phi_2 + \frac{\pi}{2})\cos(\omega t - kx + \frac{\pi}{2}) = 0}$ DESTRUKTIVNA INTERFERENCIJA

$\phi_1 = \phi_2 + (2m+1)\pi$ $m = 0, 1, \dots$

• REFLEKTIRANI I TRANSLATIRANI:



$v_1 = \sqrt{\dfrac{F}{\mu_1}}$ $v_2 = \sqrt{\dfrac{F}{\mu_2}}$ $\dfrac{v_1}{v_2} = \sqrt{\dfrac{\mu_2}{\mu_1}}$ $\psi_u = A_u\sin\left[\omega\left(t - \dfrac{x}{v_1}\right)\right]$

REFLEKTIRANI : $\psi_R = A_R\sin\left[\omega\left(t + \dfrac{x}{v_1}\right)\right]$

TRANSLATIRANI : $\psi_T = A_T\sin\left[\omega\left(t - \dfrac{x}{v_2}\right)\right] = A_T\sin\left[\omega\left(t - \sqrt{\dfrac{\mu_2}{\mu_1}}\dfrac{x}{v_1}\right)\right]$

I $\psi_u + \psi_R = \psi_T$ na mjestu $x = 0$

II $\dfrac{\partial\psi_u}{\partial x} + \dfrac{\partial\psi_R}{\partial x} = \dfrac{\partial\psi_T}{\partial x}$

I. $\rightarrow$ $A_u \sin\omega t + A_R \sin\omega t = A_T \sin\omega t$ $\rightarrow$ $\boxed{A_u + A_R = A_T}$

II. $\rightarrow$ $\left.\frac{\partial \psi_u}{\partial x}\right|_{x=0} = -\frac{A_u}{v_1}\cos\omega t$ $\quad \left.\frac{\partial \psi_R}{\partial x}\right|_{x=0} = \frac{A_R}{v_1}\cos\omega t$ $\quad \left.\frac{\partial \psi_T}{\partial x}\right|_{x=0} = -\frac{A_T}{v_2}\cos\omega t$

$$\Rightarrow \boxed{\frac{A_u}{v_1} - \frac{A_R}{v_1} = \frac{A_T}{v_2}}$$

I i II ... $A_R = \frac{v_2 - v_1}{v_1 + v_2} A_u$ $\qquad A_T = \frac{2v_2}{v_1 + v_2} A_u$

GUŠĆE - RIJEĐE SREDSTVO

$\mu_1 < \mu_2 \Rightarrow v_1 > v_2$ $\quad A_R < 0$ $\quad \psi_R = -|A_R| \sin\left[\omega\left(t - \frac{x}{v_1}\right)\right]$ $\quad$ u i R razlika u fazi π

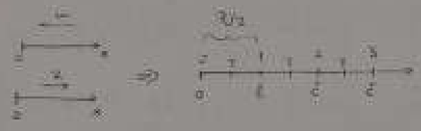
$A_T > 0$ $\quad = |A_R| \sin\left[\omega\left(t - \frac{x}{v_1}\right) + \pi\right]$

$\mu_2 = \infty$ (čvrsti kraj) $\quad \frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} \Rightarrow v_2 = 0$ $\quad A_R = -A_u$

$\mu_2 = 0$ (slobodni kraj) $\quad v_1 = 0 \Rightarrow A_R = A_u$ $\quad \psi_R = A_u \sin\left[\omega\left(t + \frac{x}{v_1}\right)\right]$

$\mu_1 > \mu_2$ $\quad v_1 < v_2$ $\quad A_R, A_T > 0$, u fazi s upadnim

STOJNI VALOVI:



$\psi_u = A\sin(\omega t + kx)$

$\psi_R = A\sin(\omega t - kx + \pi) = -A\sin(\omega t - kx)$

$$\psi = \psi_u + \psi_R = A\left(\sin(\omega t + kx) - \sin(\omega t - kx)\right) = 2A\cos\left(\frac{\omega t + kx + \omega t - kx}{2}\right)\sin\left(\frac{\omega t + kx - \omega t + kx}{2}\right)$$

$\boxed{\psi = 2A\sin kx \cos\omega t}$ $\rightarrow$ amplituda $\rightarrow$ puls titranja

$\rightarrow$ slobodni kraj:

čvorovi: $\psi(t, x=0) = 0$

$\sin kx_n = 0$

$kx_n = n\pi \quad n = 0, 1, 2, \ldots$

$\frac{2\pi}{\lambda} x_n = n\pi$

$x_n = \frac{n\lambda}{2}$

trbusi:

$\sin kx_n = \pm 1$

$kx_n = \frac{(2n-1)}{2}\pi \quad n = 0, 1, 2$

$\frac{2\pi}{\lambda} x_n = (2n-1)\frac{\pi}{2}$

$x_n = (2n-1)\frac{\lambda}{4}$

$\rightarrow$ učvršćen na oba kraja:

$\psi(t, x=L) = 0$ $\quad \sin kL = 0 \quad n = 1, 2$

$k_n L = n\pi$

$\frac{2\pi}{\lambda} L = n\pi$

$\lambda_n = \frac{2}{n} L$

STOJNI LONGITUDINALNI VALOVI:

$$\Psi_1 = A\sin(\omega t - kx)$$
$$\Psi_2 = A\sin(\omega t + kx)$$
$$\Psi = \Psi_1 + \Psi_2 = \dots = 2A\cos kx \cdot \sin\omega t$$

amplituda na mjestu x; ovisnost o vremenu

ČVOROVI: $x = \frac{L}{2}$ $\quad \Psi(t, \frac{L}{2}) = 0 \quad \cos(k_n \frac{L}{2}) = 0$

$$\frac{2\pi}{\lambda_n}\frac{L}{2} = \frac{(2n+1)\pi}{2} \qquad n = 0, 1, 2 \dots$$

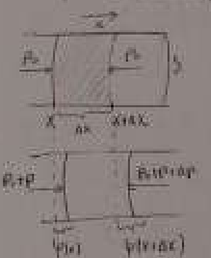
$$\lambda_n = \frac{2L}{2n+1}$$

TRBUSI: $x = L \quad \Psi(t, x=L) = \pm A \quad \cos(k_n L) = \pm 1$

$$\frac{2\pi}{\lambda_n} L = \frac{2n\pi}{2} \qquad n = 0, 1, 2 \dots$$

$$\lambda_n = \frac{2L}{n}$$

LONGITUDINALNI VAL U PLINU:



$\Delta m = \rho S \Delta x$

$P(x)$ - promjena deformacija tlaka od ravnotežnog tlaka na mjestu x

DEFORMACIJA

$$P = -B\frac{\Delta V}{V} = -B\frac{\Delta\Psi \cdot S}{\Delta x \cdot S} = -B\frac{\Delta\Psi}{\Delta x}$$

$\Delta\Psi = \Psi(x+\Delta x) - \Psi(x) \quad \Delta x \to 0$

$$\Rightarrow \boxed{P = -B\lim_{\Delta x\to 0}\frac{\Delta\Psi}{\Delta x} = -B\frac{\partial\Psi}{\partial x}}$$

SILA: $\quad P_2 - P_1 = dP \quad \boxed{dF = P_1 S - P_2 S = -dP\,S = \frac{\partial^2\Psi}{\partial t^2}dm} \to$ II newtonov aksiom

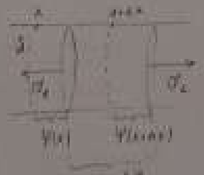
$$dP = -B\frac{\partial^2\Psi}{\partial x^2}dx$$

jednadžba long. vala u plinu:

$$B\frac{\partial^2\Psi}{\partial x^2} S\, dx = \frac{\partial^2\Psi}{\partial t^2}\rho S\, dx$$

$$\boxed{\frac{\partial^2\Psi}{\partial x^2} - \frac{\rho}{B}\frac{\partial^2\Psi}{\partial t^2} = 0} \Rightarrow v = \sqrt{\frac{B}{\rho}}$$

... U ŠTAPU:



$\sigma = E\varepsilon = E\frac{\partial s}{\partial x}$ $\quad \sigma$ - naprezanje $\quad \varepsilon$ - rel. deformacija $\quad E$ - Youngov modul el.

$$F = F_1 - F_2 = S(\sigma_1 - \sigma_2) = S\Delta\sigma$$

$$F = SE\frac{\partial^2 s}{\partial x^2}\Delta x$$

2. Newtonov aksiom $\to SE\frac{\partial^2 s}{\partial x^2}dx = \rho S\Delta x\frac{\partial^2 s}{\partial t^2}$

$$\boxed{\frac{\partial^2 s}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2 s}{\partial t^2} = 0} \Rightarrow v = \sqrt{\frac{E}{\rho}}$$

DOPLEROV EFEKT:

I Izvor miruje, detektor se giba od ili prema izvoru:

a)  $v+v_D$ $\quad f' = \frac{v+v_D}{\lambda} = \frac{v+v_D}{v/f} = f\,\frac{v+v_D}{v}$

b)  $v-v_D$ $\quad f' = \frac{v-v_D}{v/f} = f\,\frac{v-v_D}{v}$

II Izvor se giba od ili prema mirujućem detektoru

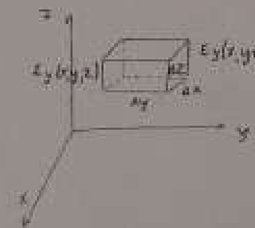
a) 
$$\lambda' = \lambda - v_I T$$
$$f' = \frac{v}{\lambda'} = \frac{v}{v/f - v_I/f} = f\,\frac{v}{v-v_I}$$

b) 
$$\lambda' = \lambda + v_I T$$
$$f' = \frac{v}{\lambda'} = \frac{v}{v/f + v_I/f} = f\,\frac{v}{v+v_I}$$

I Maxwelova jednadžba u diferencijalnom obliku:

$$\operatorname{div}\vec{E} = \lim_{\Delta V \to 0} \frac{\oint \vec{E}\, d\vec{s}}{\Delta V} = \frac{\rho}{\varepsilon_0}$$

$$\Delta V = \Delta x \cdot \Delta y \cdot \Delta z$$



kroz plohu xz:

$$\left[E_y(x,y+\Delta y,z) - E_y(x,y,z)\right]\Delta x \cdot \Delta z = \underbrace{\frac{E_y(x,y+\Delta y,z) - E_y(x,y,z)}{\Delta y}}_{\frac{\partial E_y}{\partial y}} \Delta x \cdot \Delta y \cdot \Delta z$$

kroz plohu zy:

$$\left[E_x(x+\Delta x,y,z) - E_x(x,y,z)\right]\Delta y \cdot \Delta z = \underbrace{\frac{E_x(x+\Delta x,y,z) - E_x(x,y,z)}{\Delta x}}_{\frac{\partial E_x}{\partial x}} \cdot \Delta x \cdot \Delta y \cdot \Delta z$$

kroz plohu xy:

$$\left[E_z(x,y,z+\Delta z) - E_z(x,y,z)\right]\Delta x \cdot \Delta y = \underbrace{\frac{E_z(x,y,z+\Delta z) - E_z(x,y,z)}{\Delta z}}_{\frac{\partial E_z}{\partial z}} \cdot \Delta x \cdot \Delta y \cdot \Delta z$$

$$\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) dV = \frac{\rho}{\varepsilon_0}\, dV$$

$$\Rightarrow \operatorname{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \nabla \vec{E}$$

1 AMPER - je jakost one stalne struje koja prolazeći kroz dva ravna usporedna i neizmjerno duga vodiča zanemarivo malog kružnog presjeka u vakuumu međusobno udaljena 1 m uzrokuje između njih silu od $2 \cdot 10^{-7}$ N/m.

FARADAYEV ZAKON INDUKCIJE - Elektromagnetska indu je pojava u kojoj se u prisutnosti magnetskog polja mehanička e pretvara u električnu. Inducirana elektromotorna sila razmjer je brzini promjene magnetskog toka kroz petlju.

$$\mathcal{E} = -\frac{d\phi}{dt}$$

LENZOVO PRAVILO - inducirana struja ima takav smjer da proizvodi tok magnetskog polja kroz petlju koji se protivi promjen magnetskog toka zbog kojeg je nastala. Da nije tako imali bi perpetuum mobile. Proizlazi iz zakona o očuvanju energije.

BOHROVI POSTULATI:

1. elektron se može gibati oko jezgre samo određenim dozvoljenim kružnim stazama. Elektron pri tom gibanju ne zrači.
2. Dozvoljena stanja su ona za koje je kutna količina gibanja jednaka višekratniku reducirane Planckove konstante.
3. Kada elektron skoči s više staze energije E_k na nižu stazu energije E_l onda izrazi foton čija je energija jednaka $h\nu = E_k - E_l$

$2. \rightarrow \quad L_n = n\hbar \quad \hbar = \frac{h}{2\pi} \quad n = 1, 2 \ldots$ GLAVNI KVANTNI BROJ

$L_n = m r_n v_n$

$$\boxed{r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2} \quad n = 1, 2, \ldots \quad r_1 = 0{,}53\,nm$$

$$\begin{cases} m_e v r_n = n\frac{h}{2\pi} \\ \frac{m_e v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \end{cases}$$

$F_{cp} = F_c$

$$\boxed{v_n = \frac{1}{n}\frac{e^2}{2\varepsilon_0 h}} \quad v_1 = \frac{c}{137}$$

$$\boxed{E_n = \frac{1}{n^2}\frac{m e^4}{8\varepsilon_0^2 h^2}}$$

ZAKON RADIOAKTIVNOG RASPADA

$t = 0$ N_0 - broj jezgara $\quad A$ - (aktivnost) - brzina kojom se jezgre raspadaju [Bq]

$$A = -\frac{dN}{dt} \quad \frac{dN}{dt} = -\lambda N \quad \lambda \text{ - konstanta raspada}$$

$$\Rightarrow \frac{dN}{N} = -\lambda dt \quad // \int \quad \Rightarrow \int \frac{1}{N} dN = \int -\lambda dt$$

poč. uvjeti $t = 0 \quad N = N_0$

$N_0 = C$

$$\ln N = -\lambda t + C$$

$$N = Ce^{-\lambda t} \qquad \boxed{N = N_0 e^{-\lambda t}}$$

$$A = -\frac{dN}{dt} = -N_0(-\lambda)e^{-\lambda t} = N_0 \lambda e^{-\lambda t} \quad t = 0 \quad A_0 = \lambda N_0$$

$$\boxed{A = A_0 e^{-\lambda t}}$$

VRIJEME POLURASPADA (poluživot $T_{1/2}$)

vremenski interval u kojem se raspadne 1/2 jezgara radioaktivne tvari

$$N = \frac{N_0}{2} \quad \frac{1}{2} = e^{-\lambda t} \quad \ln\frac{1}{2} = -\lambda t \qquad T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0{,}693}{\lambda}$$

$$\frac{\ln 2}{\lambda} = t = T_{1/2}$$

SREDNJE VRIJEME ŽIVOTA τ

ukupno vrijeme života svih jezgara / početni broj jezgara

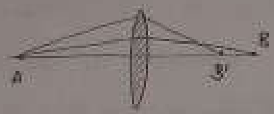
$$\tau = \frac{\int_{N_0}^{0} t\,dN}{N_0} \underset{N \to t}{=} \frac{\int_0^\infty \lambda N_0 t e^{-\lambda t} dt}{N_0} = \overset{\text{parcijalno}}{\ldots} = \frac{1}{\lambda} \qquad \boxed{\tau = \frac{1}{\lambda}}$$

$dN = -\lambda N dt$
$= -\lambda N_0 e^{-\lambda t} dt$

POGREŠKE LEĆE

Sferna aberacija: zrake koje ne zadovoljavaju Gaussove aproksimacije.

Promatramo široki snop upadnih zraka svjetlosti, zrake padaju na veliki dio površine leće. Upadni kutovi zraka svjetlosti su različiti i slike dobivene lećom nisu oštre.



Pokus sa različitim zaslonima:

→ propušta samo periferne zrake, dobivamo sliku bliže leći

→ stupa zrake iz centralnog snopa, dobivamo sliku dalje od leće

Kromatska aberacija: konvergencija ili jakost leće ovisi o indeksu loma leće pa se mijenja s bojom svjetlosti koja prolazi kroz leću

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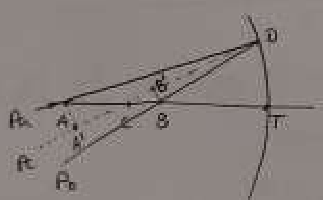
JEDNADŽBA SFERNOG ZRCALA



p_c je simetrala kuta što ga zatvaraju p_a i p_b

$\Delta AA'C$ je sličan $\Delta BB'C$

$$\frac{\overline{AC}}{\overline{AA'}} = \frac{\overline{BC}}{\overline{BB'}}$$

$\Delta AA'D$ je sličan $\Delta BB'D$

$$\frac{\overline{AD}}{\overline{AA'}} = \frac{\overline{BD}}{\overline{BB'}}$$

$$\Rightarrow \frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}}$$

Gaussove aproksimacije $\overline{AD} \doteq \overline{AT}$, $\overline{BD} \doteq \overline{BT}$

$$\Rightarrow \overline{AC} \cdot \overline{BT} = \overline{BC} \cdot \overline{AT}$$

$$(a-r) : a = (r-b) : b$$

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{r}}$$

$\overline{AT} = a$ predmetna daljina
$\overline{BT} = b$ slikovna daljina
$\overline{CT} = r$ polumjer zakrivljenosti

FERMATOV PRINCIP

Svjetlost se između dviju zadanih točaka širi onom stazom za koju joj treba najmanje vremena

1. zakon odbijanja

$$t_s = \frac{s_1}{v} + \frac{s_2}{v} = \frac{1}{v}\left(\sqrt{a^2+x^2} + \sqrt{(d-x)^2+b^2}\right)$$

$$\frac{dt_s}{dx} = 0 \qquad \frac{dt_s}{dx} = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{(d-x)^2+b^2}} = \sin u - \sin \sigma = \emptyset$$

$$\sin u = \sin \sigma$$
$$u = \sigma$$

2. zakon loma

$$t_s = \frac{s_1}{v_1} + \frac{s_2}{v_2} = \frac{\sqrt{a^2+x^2}}{v_1} + \frac{\sqrt{(d-x)^2+b^2}}{v_2}$$

$$\frac{dt_s}{dx} = \frac{x}{v_1\sqrt{a^2+x^2}} - \frac{d-x}{v_2\sqrt{(d-x)^2+b^2}} = \frac{\sin u}{v_1} - \frac{\sin \ell}{v_2} = \emptyset$$

$$\frac{\sin u}{\sin \ell} = \frac{v_1}{v_2}$$

$$\frac{\sin u}{\sin \ell} = \frac{n_2}{n_1}$$

PLANCKOV ZAKON ZRAČENJA CRNOG TIJELA:

Rayleigh-Jeansov zakon $f(\nu,T) = \frac{2\pi\nu^2}{c^2}\overline{E}$ $\qquad E_n = nh\nu$ $\qquad$ h-kvantna konstanta

$$\overline{E} = \frac{\sum_{n=0}^{\infty} N_n E_n}{\sum_{n=0}^{\infty} N_n} \qquad N_n = N_0 e^{-\frac{E_n}{kT}}$$

$$\overline{E} = \frac{\sum_{n=0}^{\infty} nh\nu N_0 e^{-\frac{nh\nu}{kT}}}{\sum_{n=0}^{\infty} N_0 e^{-\frac{nh\nu}{kT}}} = \frac{h\nu \sum_{n=0}^{\infty} n x^n}{\sum_{n=0}^{\infty} x^n} \qquad x = e^{-\frac{h\nu}{kT}}$$

$$1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$

$$= h\nu \frac{x + 2x^2 + 3x^3 + \dots nx^n}{1 + x + x^2 + x^3 + \dots x^n} = h\nu \frac{\frac{x}{(1-x)^2}}{\frac{1}{1-x}}$$

$$\frac{d}{dx}\frac{1}{1-x} = 1 + 2x + 3x^2 + 4x^3$$

$$\frac{1}{(1-x)^2} \cdot x = x + 2x^2 + 3x^3 + \dots$$

$$\overline{E} = h\nu \frac{e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}} \Rightarrow f(\nu,T) = \frac{2\pi\nu^3}{c^2} h \frac{e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}}$$

COMPTONOV EFEKT

prije sudara $\qquad$ poslije sudara

$E = h\frac{c}{\lambda}$ $\qquad E' = h\frac{c}{\lambda'}$

$p_f = \frac{h}{\lambda}$ $\qquad p_f' = \frac{h}{\lambda'}$

$E_0 = m_e c^2 = 0{,}511\,MeV$ $\qquad \vec{p}_e, E_e \qquad E_e^2 = \vec{p}_e^{\,2} c^2 + m_e^2 c^4$

$$\Rightarrow \frac{E_e^2}{c^2} = p_e^2 + m_e^2 c^2 \quad (**)$$

zakon očuvanja energije: $\quad E + E_0 = E' + E_e$

$$\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + E_e \,/ : c \Rightarrow \frac{h}{\lambda} + m_e c = \frac{h}{\lambda'} + \frac{E_e}{c} \Rightarrow \frac{h}{\lambda} - \frac{h}{\lambda'} + m_e c = \frac{E_e}{c} \,/^2 \quad (*)$$

zakon očuvanja količine gibanja $\quad \vec{p}_f = \vec{p}_f' + \vec{p}_e \,/^2$

$$(\vec{p}_f - \vec{p}_f')^2 = \vec{p}_e^{\,2}$$

$$\left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h^2}{\lambda\lambda'}\cos\theta = p_e^2$$

$$(*) \to (**) \Rightarrow \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 + 2m_e c\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right) + m_e^2 c^2 = \frac{E_e^2}{c^2} = p_e^2 + m_e^2 c^2$$

$$\left(\frac{h}{\lambda}\right)^2 - 2\frac{h^2}{\lambda\lambda'} + \left(\frac{h}{\lambda'}\right)^2 + 2m_e c h\left(\frac{\lambda' - \lambda}{\lambda'\lambda}\right) = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\theta$$

$$m_e c \,\Delta\lambda = h(1 - \cos\theta) = h\, 2\sin^2\frac{\theta}{2}$$

$$\Delta\lambda = \frac{2h}{m_e c}\sin^2\frac{\theta}{2}$$

$\frac{h}{m_e c} \to$ Comptonova valna duljina čestice

$\lambda_c = 2{,}42 \cdot 10^{-12}\,m$

ZAKONI GEOMETRIJSKE OPTIKE:

(- primjenjuju se kad su dimenzije objekata puno veće od valne duljine svjetlosti
Grana fizike koja proučava valove valnih duljina 380 - 780 nm.)

4. zakona:

1. ZAKON PRAVOCRTNOG ŠIRENJA SVJETLOSTI:

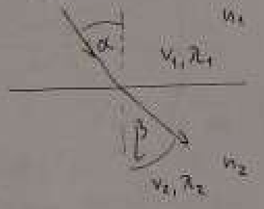
Svjetlost se u homogenom izotropnom sredstvu širi pravocrtno.

2. ZAKON NEOVISNOSTI SNOPOVA SVJETLOSTI:

Ako se dva svjetlosna snopa presjecaju jedan na drugi ne utječe i svaki se širi kao da onaj drugi ne postoji
(vrijedi ako nisu koherentni, ako su koherentni, onda interferiraju)

3. ZAKON LOMA SVJETLOSTI

Kada se svjetlost reflektira na granici dva sredstva upadna zraka, reflektirana zraka i okomica na granicu dva sredstva leže u istoj ravnini, a upadni kut zrake (kut između upadne zrake i okomice na granicu sredstava) jednak je kutu reflektirane zrake.



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

$$v_1 = \lambda_1 f$$

$$v_2 = \lambda_2 f$$

$$\frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$$

$$\boxed{\frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}}$$

$$\frac{n_2}{n_1} = \frac{\frac{c}{v_2}}{\frac{c}{v_1}} = \frac{v_1}{v_2}$$

POLARIZACIJA

Dobivanje polarizirane svjetlosti: refleksijom
prolaskom kroz kristale

Brewster:

→ linearno polarizirano zračenje

$$u_B + l = \frac{\pi}{2}$$

Brewsterov kut

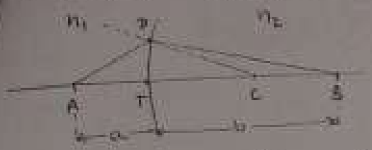
prema zakonu loma:

$$\frac{\sin u_B}{\sin l} = \frac{n_2}{n_1}$$

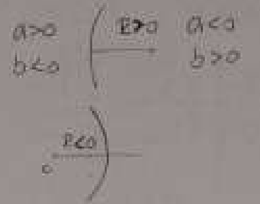
$$\frac{\sin u_B}{\cos u_B} = \frac{n_2}{n_1}$$

$$\boxed{tg\, u_B = \frac{n_2}{n_1}}$$ uvjet za Brewsterov kut

SFERNI DIOPTAR



$$\boxed{\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}}$$



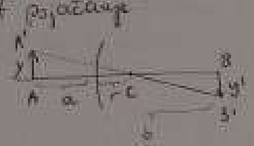
žarišta kod sfernog dioptra:

$a = f_a$, $b = \infty$ $\quad \frac{n_1}{f_a} = \frac{n_2 - n_1}{R} \quad f_a = R\frac{n_1}{n_2 - n_1} \quad \frac{f_a}{f_b} = \frac{n_1}{n_2}$

$b = f_b$, $a = \infty$ $\quad \frac{n_2}{f_b} = \frac{n_2 - n_1}{R} \quad f_b = \frac{n_2}{n_2 - n_1}R \quad f_b - f_a = R$

$$\frac{R}{n_2 - n_1}\frac{n_1}{a} + \frac{R}{n_2 - n_1}\frac{n_2}{b} = 1 \Rightarrow \boxed{\frac{f_a}{a} + \frac{f_b}{b} = 1}$$

* pojačanje

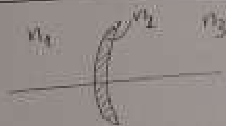


$\frac{y'}{BC} = \frac{y}{AC} \quad AC = a + R \quad BC = b - R \quad \frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$

$|m| = \frac{y'}{y} = \frac{BC}{AC} = \frac{b - R}{a + R} \quad \boxed{m = -\frac{b - R}{a + R}} \quad n_1\left(\frac{a + R}{aR}\right) = n_2\left(\frac{b - R}{bR}\right) \Rightarrow \boxed{m = -\frac{n_1}{n_2}\frac{b}{a}}$

TANKA LEĆA

-dva sustava



$n_1 \rightarrow n_2$, R_1 $\quad \frac{n_1}{a} + \frac{n_2}{b'} = \frac{n_2 - n_1}{R_1} \quad (*)$

$n_2 \rightarrow n_3$, R_2 $\quad \frac{n_2}{a'} + \frac{n_3}{b} = \frac{n_3 - n_2}{R_2}$

$a' = -b' \longrightarrow -\frac{n_2}{b'} + \frac{n_3}{b} = \frac{n_3 - n_2}{R_2} \quad (**)$

$(*) + (**)$ $\quad \boxed{\frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} = \frac{n_1}{a} + \frac{n_3}{b}}$

žarišta

$a = f_a$, $b = \infty$ $\quad \frac{n_1}{f_a} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \Rightarrow f_a = n_1\frac{R_1 R_2}{R_2(n_2 - n_1) + R_1(n_3 - n_2)} \quad \frac{f_b}{f_a} = \frac{n_3}{n_1}$

$a = \infty$, $b = f_b$ $\quad \frac{n_3}{f_b} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \Rightarrow f_b = n_3\frac{R_1 R_2}{R_2(n_2 - n_1) + R_1(n_3 - n_2)}$

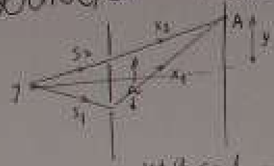
$$\Rightarrow \frac{f_a}{a} + \frac{f_b}{b} = 1$$

za $n_1 = n_3$ $\quad \frac{f_b}{f_a} = 1 \quad f_b = f_a = f$

$\frac{1}{f} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \frac{1}{f} = J$ - dioptrija

pojačanje $\quad m = \frac{-b}{a}$

YOUNGOV POKUS:



$n=1 \Rightarrow \delta = \Delta = x_1 - x_2 = d\sqrt{1+\left(\frac{y+\frac{a}{2}}{d}\right)^2} - d\sqrt{1+\left(\frac{y-\frac{a}{2}}{d}\right)^2}$

$x_1 = \sqrt{d^2 + \left(y+\frac{a}{2}\right)^2}$

$x_2 = \sqrt{d^2 + \left(y-\frac{a}{2}\right)^2}$

pretp. $y \pm \frac{a}{2} \ll d$, $\sqrt{1+x^2} \approx 1 + \frac{x^2}{2}$

$$\Rightarrow \delta = \Delta = d\left(1 + \frac{\left(y+\frac{a}{2}\right)^2}{2d^2} - 1 - \frac{\left(y-\frac{a}{2}\right)^2}{2d^2}\right) = \frac{1}{2d} 2ya = \frac{ay}{d}$$

svijetla pruga $\frac{ay}{d} = \ell\lambda$

INTENZITET: $E_A = 2E_0 \cos\frac{\varphi}{2}$, $\frac{\varphi}{2} = \frac{\pi}{\lambda}\delta$, $\delta = \frac{ay}{d}$

$$\Rightarrow E_A = 2E_0 \cos\left(\frac{\pi}{\lambda}\frac{ay}{d}\right)$$

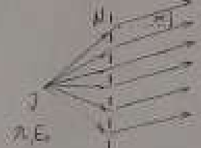
$J_1 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 = J_2$ intenzitet 1. izvora

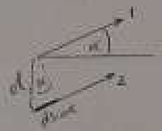
$$J = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_A^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_A^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, 4E_0^2 \cos^2\left(\frac{\pi}{\lambda}\frac{ay}{d}\right) = 2J_1 \cos^2\frac{\varphi}{2}$$

max intenzitet $\cos = 1$: $J_0 = 2\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 = 4J_1$

$$J = 2\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 \frac{1+\cos\varphi}{2} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(E_0^2 + E_0^2 + 2E_0^2\cos\varphi\right) = J_1 + J_2 + \underbrace{\overset{\sqrt{J_1 J_2}}{2J_1}\cos\varphi}_{\text{interferencijski član}}$$

INTERFERENCIJA IZ N EKVIDISTANTNIH PUKOTINA





$\delta = d\sin\alpha$

$\phi = \frac{2\pi}{\lambda}\delta$

1. $E_1 = E_0\cos\omega t$
2. $E_2 = E_0\cos(\omega t - \phi)$
3. $E_3 = E_0\cos(\omega t - 2\phi)$

$E = E_1 + E_2 + \ldots E_N$

$E_N = E_0\cos(\omega t - (N-1)\phi)$

Ovo preko kompleksnih brojeva:

$E_1 = E_0\cos\omega t \rightarrow E_0 e^{i\omega t} = \cos\omega t + i\sin\omega t$

Fizikalni smisao ima realni dio rješenja

$$E = E_1 + E_2 + \ldots + E_N = E_0 e^{i\omega t} + E_0 e^{i(\omega t - \phi)} + \ldots + E_0 e^{i[\omega t - (N-1)\phi]} = E_0 e^{i\omega t}\Big[\underbrace{1 + e^{-i\phi} + \ldots + e^{-i(N-1)\phi}}_{\text{suma geom. reda}}\Big]$$

$$E = E_0 e^{i\omega t}\left[\frac{e^{-iN\phi} - 1}{e^{-i\phi} - 1}\right] = E_0 e^{i\omega t}\left[\frac{e^{-i\frac{N}{2}\phi}\left(e^{-i\frac{N}{2}\phi} - e^{i\frac{N}{2}\phi}\right)}{e^{-i\frac{\phi}{2}}\left(e^{-i\frac{\phi}{2}} - e^{i\frac{\phi}{2}}\right)}\right]$$

$$E = E_0 e^{i\omega t} e^{-i\frac{N-1}{2}\phi} \frac{\sin\frac{N}{2}\phi}{\sin\frac{\phi}{2}} = \underbrace{E_0\frac{\sin\frac{N}{2}\phi}{\sin\frac{\phi}{2}}}_{\text{ampl}} e^{i\left[\omega t - \frac{N-1}{2}\phi\right]}$$

$$\Rightarrow J(\alpha) = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 \frac{\sin^2\frac{N}{2}\phi}{\sin^2\frac{\phi}{2}} \qquad \phi = \frac{2\pi}{\lambda} d\sin\alpha$$

OGIB ILI DIFRAKCIJA

$$E_A = E_0 \frac{\sin\left(\frac{m}{2}\frac{2\pi}{\lambda}\frac{d}{m}\sin\alpha\right)}{\sin\left(\frac{1}{2}\frac{2\pi}{\lambda}\frac{d}{m}\sin\alpha\right)} = E_0 \frac{\sin\left(\frac{d\pi}{\lambda}\sin\alpha\right)}{\sin\left(\frac{d\pi}{\lambda m}\sin\alpha\right)} = \left[\begin{array}{l} \alpha = 0 \\ E_A(\alpha=0) = mE_0 = E(0) \\ E_0 = \frac{E(0)}{m} \end{array}\right.$$

$$E_A = \frac{E(0)}{m} \cdot \frac{\sin\left(\frac{\pi}{\lambda} d \sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha\right)} \qquad m \to \infty$$

$$\lim_{m\to\infty} m \sin\left(\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha\right) = \lim_{m\to\infty}\frac{\pi d}{\lambda}\sin\alpha \underbrace{\frac{\sin\left(\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha\right)}{\frac{1}{m}\frac{\pi}{\lambda}d\sin\alpha}}_{1} = \underbrace{\frac{\pi}{\lambda}d\sin\alpha}_{y = \frac{\pi}{\lambda}d\sin\alpha}$$

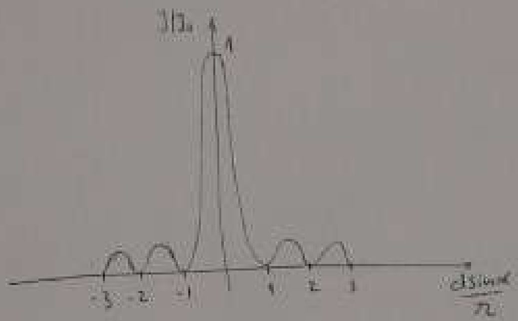
$$\boxed{E(\alpha) = E_0\frac{\sin y}{y}} \Rightarrow \boxed{J(\alpha) = J(0)\frac{\sin^2 y}{y}}$$

minimumi $\sin y = 0$

$y = l\pi \qquad l = \pm 1, \pm 2, \pm 3 \ldots$

$\frac{\pi}{\lambda} d\sin\alpha = l\pi$

$\boxed{d\sin\alpha = l\lambda}$



maksimumi $\frac{dJ}{dy} = 0$

$$J(0)\,\frac{2\sin y\cos y\cdot y^2 - \sin^2 y\cdot 2y}{y^4} = 0$$

$$2\sin y\cos y\, y^2 - \sin^2 y\, 2y = 0$$

$$y\cos y - \sin y = 0$$

$$y = \operatorname{tg} y$$

$y_1 = \pm 1{,}43\pi$

$y_2 = \pm 2{,}46\pi$

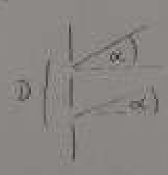
$y_3 = \pm 3{,}47\pi$

$y_4 = \pm 4{,}48\pi$

$y_l = \pm\left(l + \frac{1}{2}\right)\pi$

$l = 1, 2, 3, \ldots$

NA DVIJE PUKOTINE



$$E(\alpha) = E(0)\frac{\sin y}{y}\,\frac{\sin\left(\frac{2\pi}{\lambda}D\sin\alpha\right)}{\sin\left(\frac{1}{2}\frac{2\pi}{\lambda}D\sin\alpha\right)}$$

$\delta = \frac{2\pi}{\lambda}\delta$

$\delta = D\sin\alpha$

$$E(\alpha) = E(0)\underbrace{\frac{\sin y}{y}}_{\text{difrakcija na pukotini}}\underbrace{\frac{\sin\left(\frac{2\pi}{\lambda}D\sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda}D\sin\alpha\right)}}_{\text{interferencija iz 2 pukotine}}$$

$$J(\alpha) = J(0)\frac{\sin^2 y}{y^2}\,\frac{\sin^2\left(\frac{2\pi}{\lambda}D\sin\alpha\right)}{\sin^2\left(\frac{\pi}{\lambda}D\sin\alpha\right)}$$