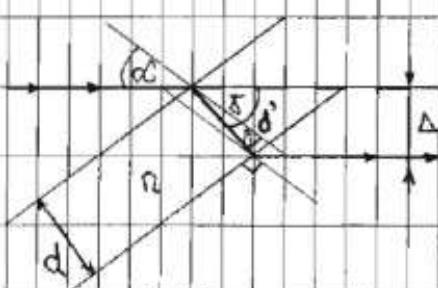


4) Planparallelná plocha



$$n = \frac{\sin \alpha}{\sin \beta}, \quad \delta = (\alpha - \beta)$$

$$\Delta = d \cos \beta$$

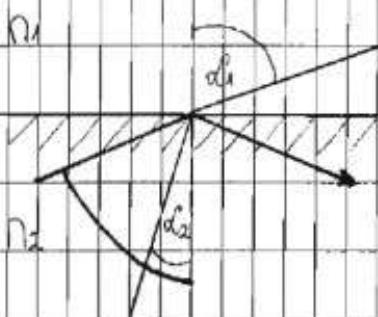
$$\Delta = d = \ln \frac{d}{\sin \beta}$$

$$N = \frac{d}{\cos \beta} \cdot \sin(\alpha - \beta) = d \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \beta} = d \sin \alpha \left[1 - \frac{\cos \alpha \sin \beta}{\sin \alpha \cos \beta} \right] = d \sin \alpha \left[1 - \frac{\cos \beta}{\sin \alpha} \right] = d \sin \alpha \frac{\sin \alpha}{1 - \cos \beta}$$

$$= d \sin \alpha \left[1 - \frac{\cos \alpha}{\sqrt{1 - (\sin \alpha)^2}} \right] = d \sin \alpha \frac{1 - \cos \alpha}{\sqrt{1 - (\sin \alpha)^2}}$$

5) Totalna refeleksia

$$n_2 > n_1 \rightarrow \frac{\sin \alpha_1}{\sin \alpha_2} = \frac{n_1}{n_2}$$

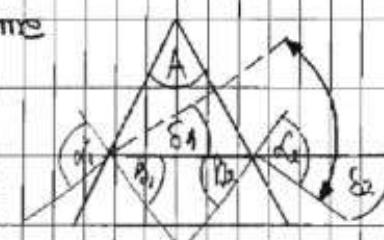
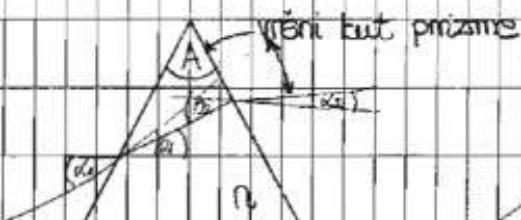


pripravka: $\alpha_1 \rightarrow T/2$

$$\sin \alpha_1 = n_1 / n_2$$

$$\text{granica: } \alpha_1 = \arcsin \frac{n_1}{n_2}$$

6) Optická prizma



α -úplní úhel
 S -úhel deviačie

$$\text{úhel deviačie: } S = S_1 + S_2$$

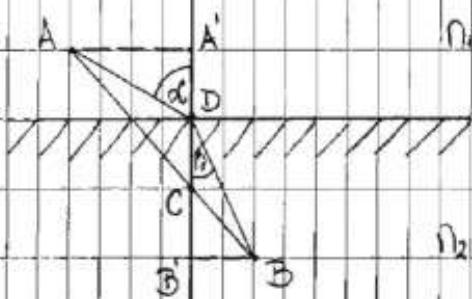
$$S_1 = \beta_1 - \alpha_1, \quad \sin \beta_1 = \frac{1}{n} \sin \alpha_1 \rightarrow \beta_1 = \arcsin \frac{\sin \alpha_1}{n}$$

$$S_2 = \beta_2 - \alpha_2, \quad \sin \beta_2 = \frac{1}{n} \sin \alpha_2 \rightarrow \beta_2 = \arcsin \frac{\sin \alpha_2}{n}$$

$$\beta_1 = \alpha_1 + \left(\frac{\pi}{2} - \beta_2 \right) + \left(\frac{\pi}{2} - \alpha_1 \right); \quad \beta_2 = \alpha_2 - \alpha_1$$

7) Izmjenjivost na sfernoj granici

Zabor loma u Möbiusovom obliku



n_1

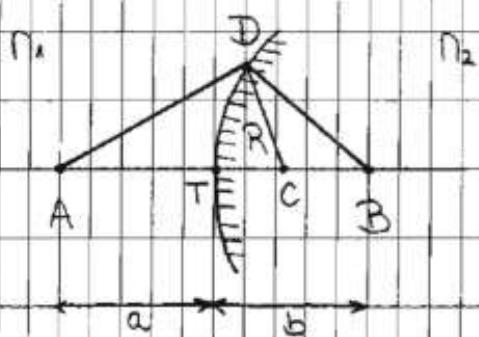
$$\Delta AAC \cong \Delta BB'C$$

$$\sin \alpha = \frac{|AA'|}{|AD|}$$

$$\sin \beta = \frac{|BB'|}{|BD|}$$

$$\frac{n_2}{n_1} = \frac{\sin \alpha}{\sin \beta} = \frac{|AB||BD|}{|BB'||AD|} = \frac{|AC|}{|BC|} \cdot \frac{|BD|}{|AD|} = n_1 \frac{|AC|}{|AD|} \cdot n_2 \frac{|BC|}{|BD|}$$

Sferna granica



$$\frac{|AC|}{|AD|} n_1 = \frac{|BC|}{|BD|} n_2$$

$$\frac{|AD|}{|AT|} = \frac{|AT|}{|AT|} = a$$

$$\frac{|BD|}{|BT|} = \frac{|BT|}{|BT|} = b$$

Gaussove aproksimacije

$$|BC| = b - R$$

$$|AC| = a + R$$

Sferni dioptrat

$$n_1 \frac{a+R}{a} = n_2 \frac{b-R}{b}$$

$$\left(1 + \frac{R}{a}\right) n_1 = \left(1 - \frac{R}{b}\right) n_2$$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

jednadžba sfernog dioptra

- silikono žarište: $a \rightarrow \infty$

$$b \rightarrow \frac{n_2 R}{n_2 - n_1} = f_b$$

- predmetno žarište: $b \rightarrow \infty$

$$a \rightarrow \frac{n_1 R}{n_2 - n_1} = f_a$$

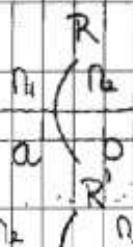
$$\frac{f_a}{a} + \frac{f_b}{b} = 1$$

- površanje: $m = \frac{y'}{y} = -\frac{\bar{BC}}{\bar{AC}} = -\frac{b-R}{a+R} = -\frac{b}{a} \frac{n_1}{n_2}$

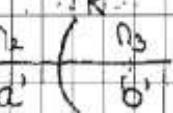
(8) Leće (tanke)

↳ = druge sferske granice

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R} \quad (\text{prva granica})$$



$$\frac{n_2}{a'} + \frac{n_3}{b'} = \frac{n_3 - n_2}{R'} \quad (\text{druga granica})$$



$$(1) \rightarrow \frac{n_2}{b} = \frac{n_2 - n_1}{R} - \frac{n_1}{a} = \frac{n_2}{a'} - \frac{n_2}{a} \rightarrow (2)$$

$$\frac{n_2}{b'} = \frac{n_3 - n_2}{R'} - \frac{n_2}{a'} + \frac{n_2 - n_1}{R} + \frac{n_1}{a} = \frac{n_2 - n_1}{R} + \frac{n_2 - n_1}{R'} - \frac{n_1}{a}$$

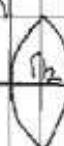
$$\rightarrow \boxed{\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R} + \frac{n_2 - n_1}{R'}}$$

Dogovor o predznacima:

(())
++ +- --

|R| |R'|

Promjer:



n_1

$n_3 = n_1$

$$\frac{n_1}{a} + \frac{n_1}{b} = \frac{n_2 - n_1}{R} + \frac{n_2 - n_1}{R'} \quad \frac{n_1}{a} + \frac{n_1}{b} = \frac{n_2 - n_1}{R + R'} \quad \frac{n_1}{a} + \frac{n_1}{b} = \frac{n_2 - n_1}{2R}$$

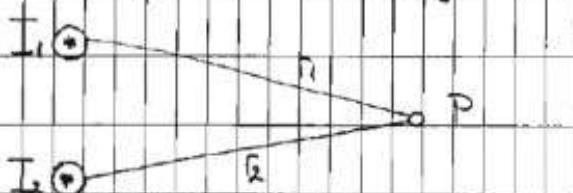
$$(n_2 - n_1) \left[\frac{1}{R} + \frac{1}{R'} \right]$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \cdot \left(\frac{n_2 - n_1}{n_1} \right) \left[\frac{1}{R} + \frac{1}{R'} \right]$$

OPTIKA

1. Interferencija svetlosti

- kohärenčni izvori - izvori elektromagn. vibracija jednake frekvencije i istalog faznog polazaka
medju njima



$$\text{-superpozicja: } E(t, p) = E_0 \cos \left[\omega \left(t - \frac{nmz}{p} \right) \right] \quad \text{S - nr. dalgowa odbijająca partia}$$

$$\omega t$$

$(t - \frac{n\pi c}{\omega})$

$\frac{\omega}{c} n\pi c = 2\pi n$

$$E_1 + E_2 = \dots \quad (\text{konštančnejm identitete}, \cos(a+b) + \cos(a-b) = 2\cos a \cos b)$$

$$= 2E_0 \cos \left[\omega \frac{\pi}{c} \frac{n_1 - n_2}{2} \right] \cos \left[\omega \left(t - \frac{1}{c} \frac{n_1 + n_2}{2} \right) \right]$$

ne trba v rámciu osoliniací faktor
 Ampplituda

$$\text{Amplitud de } u_P = 2E \cos\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta\phi = \frac{c}{\lambda} n(r_2 - r_1) = 2\pi \frac{n(r_2 - r_1)}{\lambda} = 2\pi \frac{\Delta s}{\lambda} \quad \text{rozlika vzdialostí optických povrchů}$$

→ razlike u frekvenci veljaju pravog i sljedećeg izvora

$$\cos \frac{\Delta \phi}{2} = \begin{cases} \pm 1, \text{ max. (constructive interference)} \\ 0, \text{ min. (destructive interference)} \end{cases}$$

$$\text{Tax} = \frac{\Delta P}{k} = m\Delta T$$

$$\Delta S_{\text{max}} = n(C_p - C_v) = \text{const}$$

$$\text{Mis } \frac{\Delta \phi}{j} = (\zeta_{m+1})^j$$

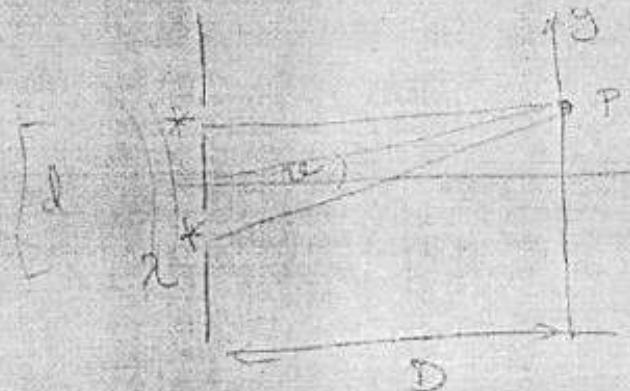
$$S_{\min} = \left(m + \frac{1}{j}\right)n$$

Dio koji fali u FIZIKALNOJ OPTICI

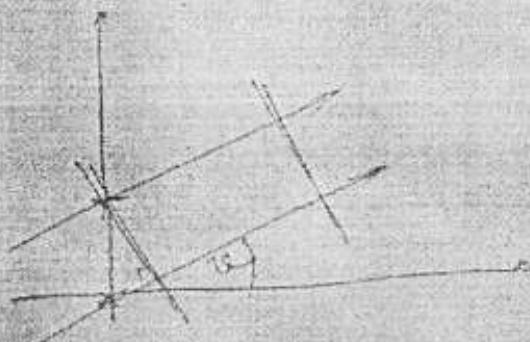
ACO

(u bilježnici je samo točka ①, ovo je ostatak)

② Youngov potis u 2 putinice



$$D \gg d$$



$$\delta = d \sin \theta$$

$$y = d \tan \theta$$

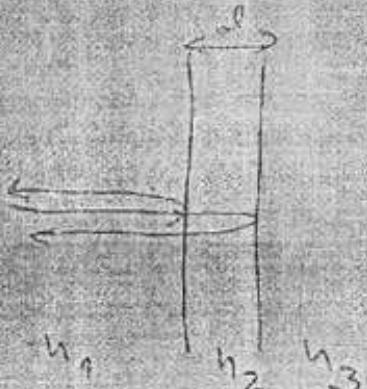
$$\frac{\delta}{d} = \frac{y}{D}$$

Maximum

$$\delta = m\lambda$$

$$y_{max} = \frac{D}{d} \delta_{max} = \frac{D}{d} m\lambda \quad m=0, 1, \dots$$

③ TANKI LISTİCİ



$$\delta = 2dh_2 + \frac{\lambda}{2} + \frac{\lambda}{2}$$

Max

$$\delta_{MAX} = m\lambda$$

$$\forall h_1 < h_2 < h_3$$

$$2dh_2 + \lambda = m\lambda$$

$$d = \frac{\lambda}{2m}$$

Min

$$\delta_{MIN} = \left(m + \frac{1}{2}\right)\lambda$$

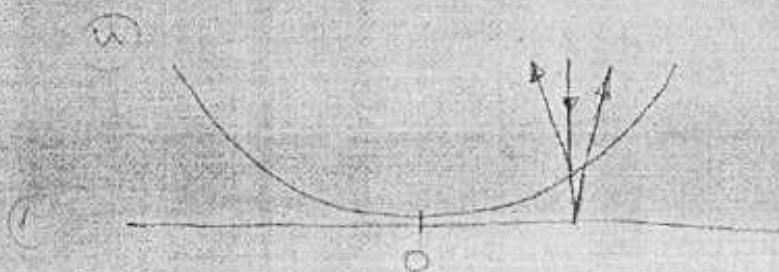
$$h_1 < h_2 < h_3$$

$$2dh_2 + \lambda = \left(m + \frac{1}{2}\right)\lambda$$

merkezî listedi

$$r = \frac{\lambda}{4\pi^2 f_{min}}$$

④ Newtonovи koločni



(w)

REFLEKSIJA

$$u(r) = R - \sqrt{r^2 - c^2} \approx \frac{c^2}{2R}$$

vel R

$$\delta = 2u(r) + \frac{\lambda}{c} = \frac{r^2}{R} + \frac{\lambda}{c}$$

Max

$$\delta = m\lambda$$

$$r_{\max} = \sqrt{R(m - \frac{1}{2})\lambda} \quad m = 1, 2, \dots$$

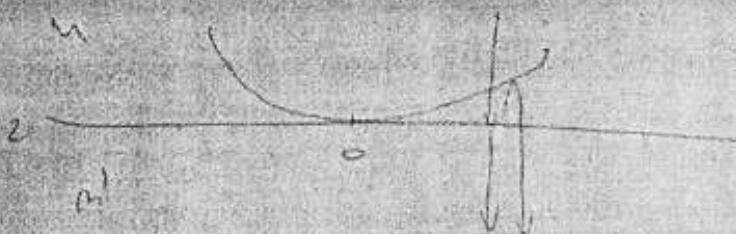
Min

$$\delta = \left(m + \frac{1}{2}\right)\lambda$$

$$r_{\min} = \sqrt{R(m + \frac{1}{2})\lambda} \quad m = 0, 1, 2, \dots$$



Transmisiya



$$\delta = 2n(\tau) + 2 \frac{\pi}{2}$$

$$r_{\max} = \sqrt{R_m R} \quad m=0, 1, \dots$$

$$r_{\min} = \sqrt{R \left(m + \frac{1}{2} \right) R} \quad m=1, 2, \dots$$



⑤ OPTIČKA REŠETKA

N količinatnih izora

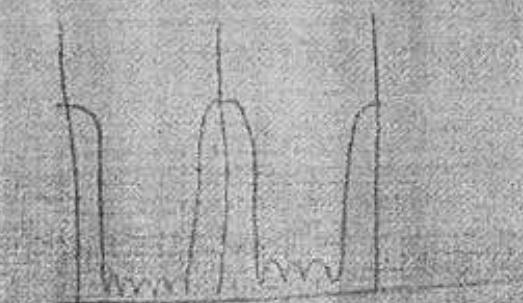
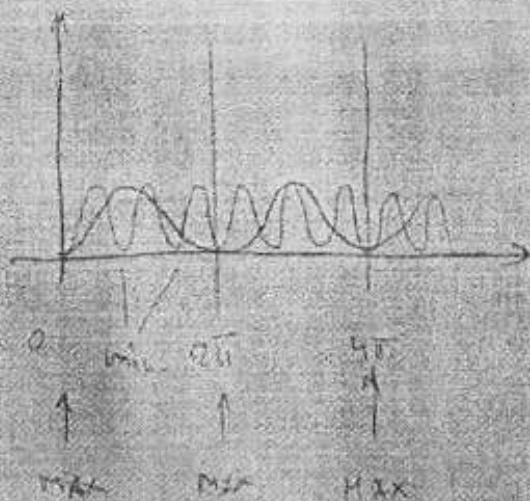


Rezultantno polje

$$E(x) = E_0 \cdot \frac{\sin(N \frac{\Delta\phi}{2})}{\sin \frac{\Delta\phi}{2}}$$

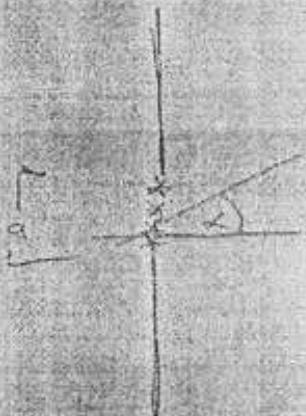
$$\Delta\phi = z \frac{d \sin\theta}{\lambda}$$

$$I = I_0 \cdot \frac{\sin^2(N \frac{\Delta\phi}{2})}{\sin^2 \frac{\Delta\phi}{2}}$$



(6)

Difrakcja w jednej półolini



Model: K kier. izwora

na ośrodku

$$d = \frac{a}{N}$$

1. Formuła do rozkładu

$$E(\lambda) = E_0 \frac{\sin(N \frac{\Delta\phi}{z})}{\sin \frac{\Delta\phi}{z}}$$

$$\Delta\phi = 2\pi \frac{d}{\lambda} = 2\pi \frac{a \sin \alpha}{\lambda} - 2\pi \frac{a \sin \alpha}{N\lambda}$$

$$E(\lambda) = E_0 \frac{\sin \left(\pi \frac{a \sin \alpha}{\lambda} \right)}{\sin \left(\pi \frac{a \sin \alpha}{N\lambda} \right)}$$

2. Gledam linię $0 \rightarrow \infty$

$$z = \frac{\pi a \sin \alpha}{\lambda}$$

$$= \underbrace{N E_0}_{E_0'} \frac{\sin^2 \alpha}{z}$$

$$I(\lambda) = I_0 \cdot \frac{\sin^2 \alpha}{z} \rightarrow \text{OGIB}$$

7

SPECTROGRAF S EMISSION

$I = I_0 \cdot \sin^2\left(\pi \frac{a \sin \alpha}{\lambda}\right) \cdot \sin^2\left(\pi \frac{d \sin \alpha}{\lambda}\right)$

I_0 - konst. reference
 a - sines pulsine

$$I(\lambda) = I_0 \cdot \frac{\sin^2\left(\pi \frac{a \sin \alpha}{\lambda}\right)}{\left(\pi \frac{a \sin \alpha}{\lambda}\right)^2} \cdot \frac{\sin^2\left(\pi \frac{d \sin \alpha}{\lambda}\right)}{\sin^2\left(\pi \frac{d \sin \alpha}{\lambda}\right)}$$

Haben wir

$$d \sin \alpha = m \lambda$$

$$d \sin \alpha = \frac{m}{n} \lambda$$



Haben wir
 max. bei
 aufnahmen
 max.

$$\text{max. Distanz} = m \lambda / (\lambda - \delta \lambda)$$

$$\text{min. Distanz} = \frac{m(n+1)}{n} \lambda$$

$$m(\lambda + \delta \lambda) = \frac{n(n+1)}{n} \lambda$$

$$\frac{\Delta \lambda}{\lambda} = \frac{1}{m}$$

Wir erhalten auf
 welche

$$R = \frac{\lambda}{\Delta \lambda} = m \cdot N$$

8) Polarizacija svjetlosti

Unutarnje polarizacije val.

$$\vec{E}(t, \vec{r}) = \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r})$$

Amplitude vector
 Phase vector

Vibračno polarizacije val

$$\vec{E}(t, \vec{r}) = \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r}) + \vec{k} \cdot \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r} - \frac{\pi}{2})$$



Brewsterov kut

Najveći kut incidentne kugle u kojem se refleksija i reflektirana kugla sastaju na mediju & planarom

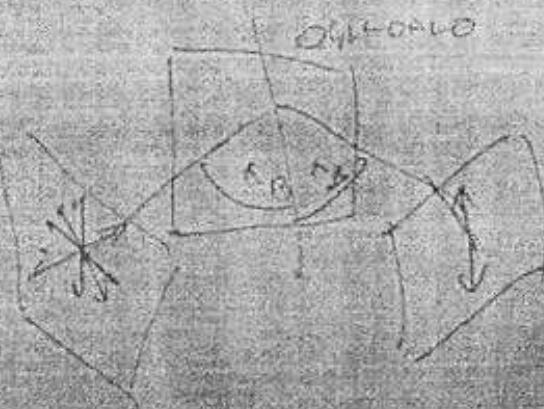


Snelov faktor

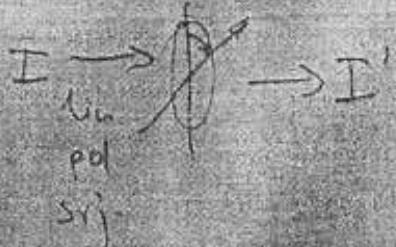
$$\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1}$$

$$\frac{\sin \beta}{\sin(\frac{\pi}{2} - \alpha_B)} = \operatorname{ctg} \alpha_B \quad \alpha_B = \operatorname{ctg} \frac{n_2}{n_1}$$

Molešov faktor



$$I' = I \cos^2 \Theta$$



"MODERNA" FIZIKA

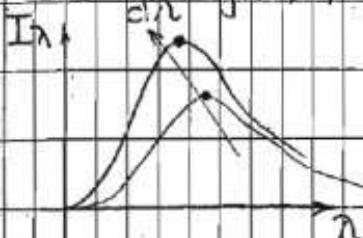
① Zračenje crnog tijela.

Stefan-Boltzmannov zakon (experimentalno)

Tijelo ugnjajšo na T zrači: $I = \sigma T^4$, $\sigma = 5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$

Raspodjelja intenziteta po valnim duljinama

$$I_\lambda = \frac{dI}{d\lambda} = f(\lambda, T)$$



- maximum se postiže sve manjim valnim duljinama

Planckov zakon

$$I_\lambda = f(\lambda, T) = \left(\frac{c}{4} \frac{8\pi}{\lambda^5} \frac{kT}{e^{h\lambda/c} - 1} \right) \frac{1}{\lambda^5}$$

R-J Planck

$$I_\lambda = \frac{c}{4} \frac{8\pi^5 k T}{\lambda^5} \frac{h\lambda/kT}{e^{h\lambda/kT} - 1}$$

Ug: spektrofna gaterajuća energija

→ Rayleigh-Jeansova formula, KLASIČNA FIZIKA

"Ultraljubičasta katastrofa"

$$\text{Planck } \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{h\lambda/kT} - 1}$$

k → Boltzmannova konst. $k = 1.381 \cdot 10^{-23} \text{ J/K}$

h → Planckova konst. $h = 6.626 \cdot 10^{-34} \text{ Js}$

$$I_\lambda = \frac{1}{dI} = \sqrt{I_\lambda d\lambda} = \sqrt{\int f(\lambda, T) d\lambda} = \frac{2\pi^5 k^4}{15 c^5 h^3}$$

Pokazujemo da je R-J zakon limes Planckova zakona pri $\lambda \rightarrow \infty$

$$\text{Planckov faktor } \frac{hc}{kT} \frac{1}{e^{h\lambda/kT} - 1} = \left[\frac{hc}{kT} - x \right] \frac{x}{e^x - 1}$$

$\lim_{\lambda \rightarrow \infty} \lambda \rightarrow 0 \rightarrow \text{odg limes } \lambda \rightarrow 0$

$$\lim_{x \rightarrow 0} \frac{x}{e^x - 1} = \lim_{x \rightarrow 0} \frac{x}{x} = \lim_{x \rightarrow 0} \frac{1}{1} = 1$$

$$\text{Opisac } \text{I} = \frac{c}{\nu}$$

$$dI = f(\nu, T) d\nu$$

$$= \frac{2\pi h c^2}{\nu^5} \frac{1}{e^{\frac{hc}{kT}} - 1} = \frac{2\pi h c^2}{(c/e)^5} \frac{1}{e^{\frac{hc}{kT}} - 1} = \frac{2\pi h c^2}{c^5} \frac{1}{e^{\frac{hc}{kT}} - 1}$$

$$I_\nu = \frac{2\pi h c^2}{c^5} \cdot \frac{1}{e^{\frac{hc}{kT}} - 1}$$

Wienov zakon

$$\nu_{\max} \cdot T = K_W \rightarrow \text{Wienova konst. } 0.0029 \text{ nmK}$$

$$I_\nu = f(G, T) \propto \frac{x^5}{e^x - 1} \quad x = \frac{hc}{kT}$$

$$\frac{dI_\nu}{dx} + \frac{df}{dx} = 0 \quad -f \text{ otkad nije } 0$$

$$\left(\frac{5}{x} - \frac{x}{e^x - 1} \right) f$$

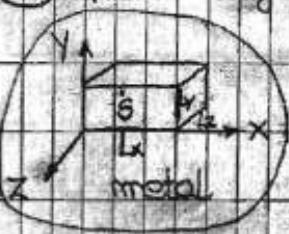
$$= 0 \quad x = 4.965$$

$$x = \frac{hc}{\text{Wienova konst.}}$$

do d. 2009

Izvod zakona zračenja, crnog tijela

I. Spektralna gustoća elektromagn. mimočina



EM zračenje zadonjjava mimočne uvjete: $\vec{E} = 0$

$$\rightarrow \vec{E}(\vec{r}, t) = \vec{E}_0 e^{i\omega t} (e^{i\vec{k}\vec{r}} + e^{-i\vec{k}\vec{r}}) = \vec{E}_0 e^{i\omega t} 2 \cos(\vec{k}\vec{r}) \text{ gdje je } \vec{k} \text{ valni vektor}$$

$$\vec{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$$

$$\text{Zag mimočnih uvjeta: } k_x = \frac{2\pi}{\lambda_x}, \quad k_y = \pi \frac{\lambda_x}{2} \Rightarrow k_x = \frac{\pi}{\lambda_x} \pi, \quad n_x = 1, 2, 3, \dots$$

$$\text{analogno } k_y, k_z$$

Razmatrasmo el. val u n-prostomu $ds_x ds_y ds_z = d^3 r$

$$k\text{-prostomu } dk_x dk_y dk_z = d^3 k$$

$$\left. \begin{array}{l} dk_x = \frac{\pi}{L_x} dx \\ dk_y = \frac{\pi}{L_y} dy \\ dk_z = \frac{\pi}{L_z} dz \end{array} \right\} * \rightarrow dk_x dk_y dk_z = \frac{\pi^3}{L_x L_y L_z} dx dy dz$$

$$d^3k = \frac{\pi^3}{V} d^3n$$

gustota raspodjeljivih valnih vektorova u k-prost

Broj elektromagn. modova u V takvini da $k \leq k$

$$N(k) = 2 \cdot \frac{1}{8} \cdot \frac{4}{3} k^3 \pi \cdot \frac{V}{\pi^3} \rightarrow \text{gustota} = \frac{1}{3} k^3 \frac{V}{\pi^2}$$

oktantis \downarrow luge poluvijenja k
2 moguća smjera polarizacije

Broj elektromagn. modova tako da $k \leq k < k + dk$

$$dN = \frac{dN(k)}{dk} dk = k^2 \frac{V}{\pi^2} dk$$

Broj elektromagn. modova tako da $\nu \in (\nu, \nu + d\nu)$: $(k \cdot \frac{2\pi}{\lambda} = \frac{2\pi}{c} \nu)$

$$dN = \left(\frac{2\pi}{c}\nu\right)^2 \frac{V}{\pi^2} \frac{2\pi}{c} d\nu = g(\nu) d\nu$$

$$g(\nu) = \frac{8\pi^3}{c^3} \nu^2 \quad \text{spektralna gustoća elektromagn. modova}$$

II) Rayleigh-Jeans

Pripremke: - termodynamička ravnatelja

- Boltzmannova raspodjela

$$f(\epsilon) = ce^{-\epsilon/kT} = \frac{1}{kT} e^{-\epsilon/kT}$$

$$\text{Normalizacija vjerojatnosti: } \int f(\epsilon) d\epsilon = 1 \rightarrow C = \frac{1}{kT}$$

$$\text{Ukupna energija: } \langle \epsilon \rangle = \int \epsilon f(\epsilon) d\epsilon = \int \epsilon \frac{1}{kT} e^{-\epsilon/kT} d\epsilon = kT$$

$$\text{Spektralna gustoća energije: } U(\nu) = \langle \epsilon \rangle \cdot g(\nu) = kT \frac{8\pi^3}{c^3} \nu^2$$

III) Planck

Pripremke: - termodynamička ravnatelja

- Boltzmannova raspodjela

- Hipoteza o kvantizaciji

$$\langle E \rangle = \sum_n E_n f_n \quad f_n = e^{-\frac{E_n}{kT}}$$

Normalizacija verovatnoće: $\sum_n f_n = 1 = C \sum_n e^{-\frac{E_n}{kT}}$

$$= C [e^{-\frac{E_1}{kT}} + (e^{-\frac{E_2}{kT}})^2 + \dots]$$

$$= C \frac{1}{1 - e^{-\frac{E_1}{kT}}}$$

$$\Rightarrow C \cdot 1 - e^{-\frac{E_1}{kT}} = 1 - e^{-\frac{h\nu}{kT}}$$

← vršimo

$$\langle E \rangle = \sum_n h\nu n f_n (1 - e^{-\frac{h\nu}{kT}}) \in \frac{h\nu}{kT}$$

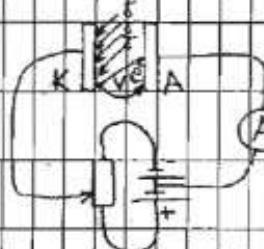
$$= \nu (1 - e^{-\frac{h\nu}{kT}}) \sum_n n e^{-\frac{h\nu}{kT}} = \dots$$

$$= \nu \frac{kT}{e^{\frac{h\nu}{kT}} - 1} = \frac{h\nu}{kT} \frac{kT}{e^{\frac{h\nu}{kT}} - 1} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Spektralna gustoća energije:

$$U(\nu) = \langle E(\nu) \rangle = g(\nu) = \frac{8\pi h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

② Fotolektrični efekt



- i) Sružja razinjerna intenziteti zračenja [OK]
- ii) Ako ν ide ispod neke ν_c stvarnostaje [PROBLEM]
- iii) Max kin. energija elektrona je ovisi o intenzitetu [PROBLEM]

$$E_{kin, max} = \frac{mv_{max}^2}{2} = eV_z \quad \text{zaustavni napon}$$

19.01.2009.

Objašnjenje: Albert Einstein (1905)

$$E_x = h\nu = h\frac{c}{\lambda}$$

\downarrow
 \downarrow
 \hbar/kT

Planckova koef.

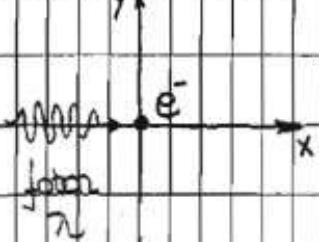
$$\text{Energija: } E_x \geq W + E_{kin}$$

→ izložni rad

③ "Comptonovo rasprećenje"

"foton noliće se na čestici kogn. smijeći, rasprećenje fotona na e^- "

Prije:



Poslije:



$$\Delta p = p - p_0 = \frac{h}{mc} (1 - \cos \varphi)$$

$$\frac{1}{m_0} - \frac{1}{m} = \frac{1}{mc^2} (1 - \cos \varphi)$$

Energija $E = \sqrt{(mc^2)^2 + (pc)^2} = \frac{mc^2}{\sqrt{1 - (\frac{v}{c})^2}}$

$$\underbrace{E = hv}_{\text{prije}} + \underbrace{mc^2}_{\text{postoji}} = \underbrace{hv'}_{\text{prije}} + \underbrace{\left(m_0 c^2 + \frac{mv_0}{\sqrt{1 - (\frac{v_0}{c})^2}}\right)}_{\text{postoji}}$$
 (1)

Impuls $\vec{p} = \frac{mv}{\sqrt{1 - (\frac{v}{c})^2}}$ za $m > 0$, $p = \frac{E}{c} = \frac{hv}{c} + \frac{h}{\lambda}$ za $m = 0$

$$p_x = \frac{hv}{c} = \frac{hv}{c} \cos \varphi + \frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \cos \varphi$$
 (2)

$$p_y = 0 = \frac{hv}{c} \sin \varphi - \frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \sin \varphi$$
 (3)

Iz (2), (3) eliminiramo φ :

$$\left(\frac{hv}{c}\right)^2 + \left(\frac{hv}{c} \cos \varphi + \frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \cos \varphi\right)^2 = \left(\frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}}\right)^2$$
 (4)

Prepravljamo (1):

$$\left(\frac{hv}{c} - \frac{hv}{c} + m_0 c\right)^2 - (m_0 c)^2 = \left(\frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}}\right)^2$$

$$\left(\frac{hv}{c}\right)^2 + \left(\frac{hv}{c}\right)^2 + 2\left(\frac{hv}{c} - \frac{hv}{c}\right)m_0 c - 2 \frac{hv}{c} \frac{hv}{c} = \left(\frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}}\right)^2$$
 (5)

(5)-(4)

$$2\left(\frac{hv}{c} - \frac{hv}{c}\right)m_0 c - 2 \frac{hv}{c} \frac{hv}{c} + 2 \frac{hv}{c} \frac{hv}{c} \cos \varphi = 0$$

$$\left(\frac{1}{m/e} \right) \left(\frac{1}{m/e} \right) m/e = 1 - \cos \vartheta$$

$$UZ \quad \gamma D = c$$

$$\lambda - \gamma = \frac{n}{m/e} (1 - \cos \vartheta)$$

4. Bohrov model atoma (1913)

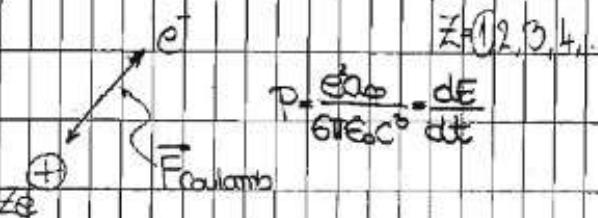
Prihodnik: *Thomson

*Rutherford

*Bohmova formula, $\lambda = (364.56 \text{ nm}) \frac{n^2}{n^2 - 4} \quad n=3,4,5...$

$$\text{dil } \frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \quad n=3,4,5, \dots \quad R_H = 1.087373 \times 10^{-7} \text{ m}^{-1}$$

Radijus gama vlastnosti



$$P = \frac{e \phi}{4\pi\epsilon_0 c} = \frac{dE}{dt}$$

$$z=1,2,3,4...$$

P1: Postoji dozvoljeno stanje na kojima će ne zadržati.

P2: Kružna kol.gibaja je klasička veličina kod Bohra. $L = mv = n \frac{h}{2\pi} = m\omega r_n, n=1,2,3...$

P3: Elektron može iz jednog dozvoljenog stanja preći u drugo.

Pri prelasku: $n_i \rightarrow n_f$, zrači se foton, $E_{\text{foton}} = E_i - E_f = h\nu$

$$\text{Jedinkost sila: } F_{\text{Coul}} = \frac{mv^2}{r_n} = \frac{1}{4\pi\epsilon_0} \frac{Ze \cdot e}{r_n^2}$$

$$\text{iz (P2)} \quad L = mv = m\omega r_n \rightarrow \omega_n = \frac{mv}{mr_n}$$

$$\rightarrow \frac{mv}{mr_n} \left(\frac{mv}{mr_n} \right)^2 = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n^4}$$

$$r_n = \frac{4\pi\epsilon_0 \cdot n^2}{Ze^2 \cdot m}$$

$$\omega_n = \frac{Ze^2}{4\pi\epsilon_0 m} \cdot \frac{1}{n^2}$$

Ukupna energija e⁻

$$E_n = -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} + \frac{m}{2} \omega_n^2 = -\frac{mZe^2}{2\pi^2\epsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

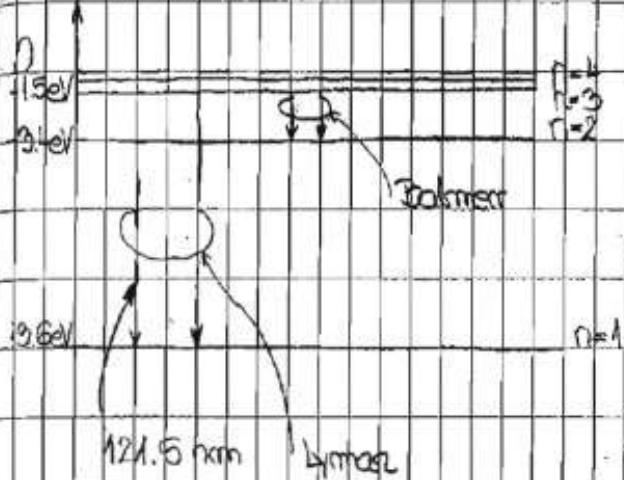
$$E_1 = -13.6 \text{ eV}$$

$$E_n = E_1 \cdot \frac{1}{n^2}$$

> energija konzerve se dozvoljog stanja

7 (P3) $h\nu = E_i - E_f = E_i \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$

Balmer $n_f=2$



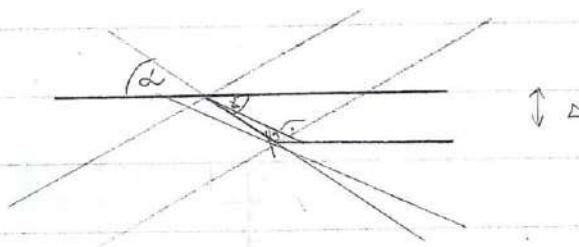
3. CIKLUS

4. PLANPARALELNA PLOČA

sloj optički quščeg sredstva omeđenog z. materijalima

d - debljina ploče

m - indeks lomine



$$m = \frac{\sin \alpha}{\sin \beta}$$

$$d = d' \cos \beta$$

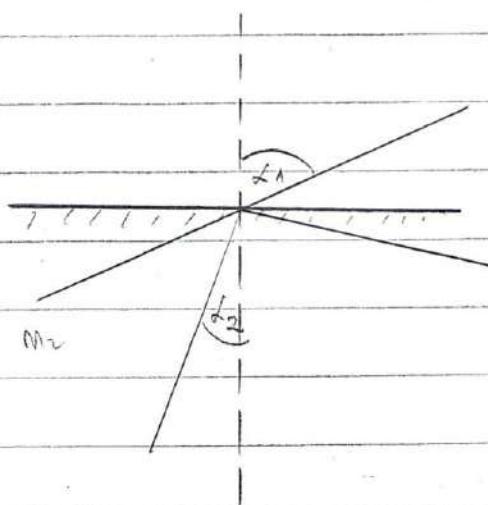
$$\Delta = d' \sin (\alpha - \beta)$$

$$\Delta = \frac{d}{\cos \beta} \cdot \sin (\alpha - \beta) = d \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \beta} =$$

$$= d \sin \alpha \left[1 - \frac{\cos \alpha \sin \beta}{\sin \alpha \cos \beta} \right] = d \sin \alpha \left[1 - \cos \alpha \frac{\frac{\sin \beta}{\sin \alpha}}{\sqrt{1 - \sin^2 \beta}} \right] =$$

$$= d \sin \alpha \left[1 - \frac{\cos \alpha}{\sqrt{m^2 - (\sin \alpha)^2}} \right] = d \sin \alpha \left[\frac{1 - \cos \alpha}{\sqrt{m^2 - \sin^2 \alpha}} \right]$$

5. TOTALNA REFLEKCIJA



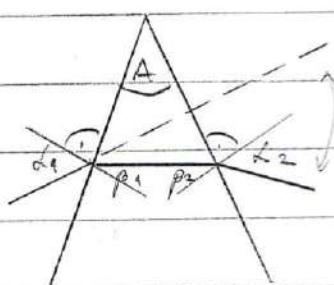
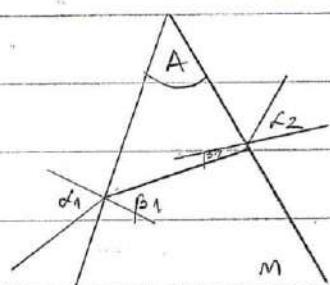
$$M_2 > M_1 \rightarrow \frac{\sin \alpha}{\sin \beta} = \frac{M_2}{M_1}$$

pretpostavka: $\alpha \rightarrow \frac{\pi}{2}$

$$\sin \alpha = \frac{M_1}{M_2}$$

$$\text{pričini } \beta = \arcsin \frac{M_1}{M_2}$$

6. OPTIČKA PRIZMA



" α -upadni" kut

δ -kut devijacije

$$\text{kut devijacije } \delta = \beta_1 + \beta_2$$

$$\pi = A + \left(\frac{\pi}{2} - \beta_1\right) + \left(\frac{\pi}{2} - \beta_2\right)$$

$$\beta_1 = \alpha_1 - \beta_1, \quad \sin \beta_1 = \frac{1}{m} \sin \alpha_1$$

$$\beta_2 = A - \beta_1$$

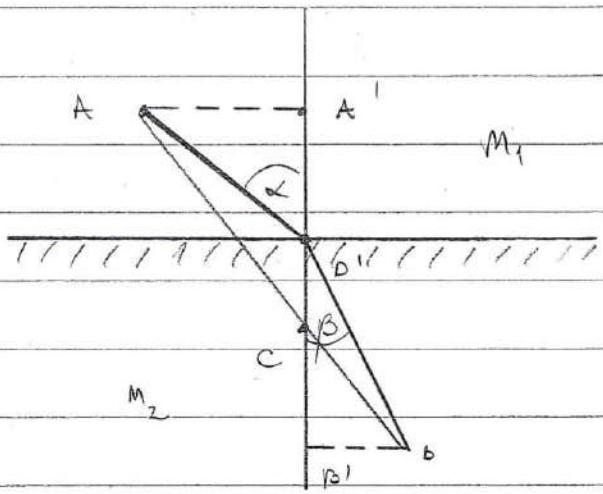
$$\beta_1 = \arcsin \frac{\sin \alpha_1}{m}$$

$$\beta_2 = \alpha_2 - \beta_2, \quad \sin \beta_2 = \frac{1}{m} \sin \alpha_2$$

$$\alpha_2 = \arcsin (m \sin \beta_2)$$

7. LOM SVJETLOSTI NA SFERNOG GRANICA

Zakon loma u Möbiusovom obliku



$$\Delta(AA'C) \cong \Delta(BB'C)$$

$$\sin \alpha = \frac{\overline{AA'}}{\overline{AO}}$$

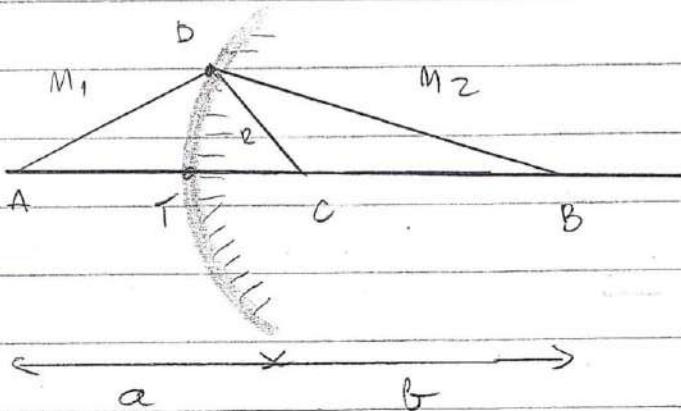
$$\sin \beta = \frac{\overline{BB'}}{\overline{BO}}$$

$$\frac{M_2}{M_1} = \frac{\sin \alpha}{\sin \beta} = \frac{\frac{\overline{AA'}}{\overline{AD}}}{\frac{\overline{BB'}}{\overline{BD}}} = \frac{\overline{AA'}}{\overline{BB'}} \cdot \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{AC}}{\overline{BC}} \cdot \frac{\overline{BD}}{\overline{AD}}$$

$$M_1 \frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}} M_2 \Rightarrow \text{Zakon loma prema}$$

Möbiusu

Sferna geometria



$$\frac{\bar{AC}}{\bar{AD}} \cdot M_1 = \frac{\bar{BC}}{\bar{BD}} M_2$$

$$\left. \begin{array}{l} \bar{AD} \approx \bar{AT} = a \\ \bar{BD} \approx \bar{BT} = b \end{array} \right\} \begin{array}{l} \text{Gaussove} \\ \text{aproximace} \end{array}$$

$$\bar{BC} = b - R$$

$$\bar{AC} = a + R$$

Jednačina sfernog dioptra

$$M_1 \frac{a+R}{a} = M_2 \frac{b-R}{b}$$

$$(1 + \frac{R}{a}) M_1 = (1 - \frac{R}{b}) M_2$$

$$\frac{M_1}{a} + \frac{M_2}{b} = \frac{M_2 - M_1}{R}$$

jednačina sfernog
dioptra

slitovno žarište: $a \rightarrow \infty$

$$b \rightarrow \frac{M_2 R}{M_2 - M_1} \equiv b_F$$

povećanje:

$$m = \frac{y'}{y} = \frac{-\bar{BC}}{\bar{AC}} =$$

predmetno žarište: $b \rightarrow \infty$

$$a \rightarrow \frac{M_1 R}{M_2 - M_1} \equiv f_a$$

$$= -\frac{b - R}{a + b} = -\frac{b}{a} \frac{M_1}{M_2}$$

$$\frac{f_a}{a} + \frac{f_b}{b} = 1$$

8. LEĆE

⇒ sloj omeđen dujem sfernim granicama

$$-\frac{M_1}{a} + \frac{M_2}{b} = \frac{M_2 - M_1}{R} \quad (\text{prva granica}) \quad \frac{M_1}{a} \left(\frac{M_2}{b} \right)$$

$$\frac{M_2}{a'} + \frac{M_3}{b'} = \frac{M_3 - M_2}{R'} \quad (\text{druga granica}) \quad \frac{M_2}{a'} \left(\frac{M_3}{b'} \right)$$

$$(1) \Rightarrow \frac{M_2}{b} = \frac{M_2 - M_1}{R} - \frac{M_1}{a} = \frac{-M_2}{a'} \Rightarrow (2)$$

$$\frac{M_3}{b'} = \frac{M_3 - M_2}{R'} - \frac{M_2}{a'} = \frac{M_3 - M_2}{R'} + \frac{M_2}{b} = \frac{M_3 - M_2}{R'} + \frac{M_2 - M_1}{R} - \frac{M_1}{a}$$

$$\frac{M_1}{a} + \frac{M_3}{b'} = \frac{M_2 - M_1}{R} + \frac{M_3 - M_2}{R'}$$

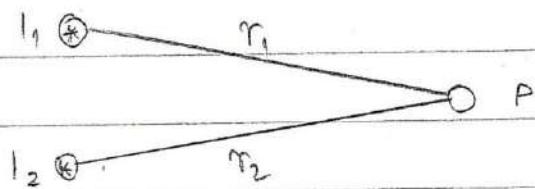
dogovor o predznacima:

$$\begin{array}{c} ((\quad () \quad)) \\ ++ \quad + - \end{array}$$

FIZIKALNA OPTIKA

I. INTERFERENCIJA SVJETLOSTI

bokserentni izvori - izvori elektromagnetskog kola s jednake frekvencije i stalnog faznog ponata među njima



- superpozicija:

$$E_{1,2}(t, p) = E_0 \cos \left[\omega(t + \frac{m r_{1,2}}{c}) \right]$$

jednaka
frekv.

$$\frac{\omega}{c} m r_{1,2} = 2\pi \frac{m r_{1,2}}{\lambda}$$

$\xi = m \cdot r \Rightarrow$ "dužina" optičkog puta

$$E_1 + E_2 = \dots \text{koristenjem identiteta } \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$$

$$= 2 E_0 \underbrace{\cos \left(\frac{\omega m \cdot \frac{r_2 - r_1}{2}}{c} \right)}_{\text{merjena u vremenu}} \cdot \underbrace{\cos \left[\omega \left(t - \frac{1}{c} \frac{m(r_1 + r_2)}{2} \right) \right]}_{\text{oskulirajući faktor}}$$

amplituda

$$\cos x + \cos y = \cos \left[\frac{(x+y)}{2} - \frac{(x-y)}{2} \right] + \cos \left[\frac{(y+x)}{2} + \frac{(y-x)}{2} \right]$$

x β x β

$$= \cos(\omega + \beta) + \cos(\omega - \beta)$$

$$\Rightarrow 2\cos\omega \cos\beta$$

$$\omega = w \left(t - \frac{m(r_1+r_2)}{2c} \right)$$

$$\beta = -\frac{m(r_1-r_2)}{2c} w$$

Amplituda u p

$$P = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right)$$

$$\Delta\phi = \frac{w}{c} m(r_2-r_1) = 2\pi \frac{m(r_2-r_1)}{\lambda} = \frac{2\pi \Delta s}{\lambda} \rightarrow$$

razlika u duljini
optičkih kuteva

\downarrow

razlika u fazi
izmedju izvora

$$\cos \frac{\Delta\phi}{2} = \begin{cases} \pm 1, & \text{max (konstruktivna interferencija)} \\ 0, & \text{min (destruktivna interferencija)} \end{cases}$$

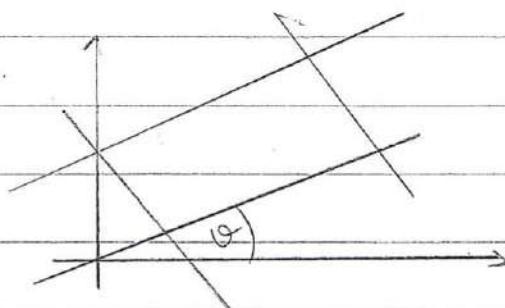
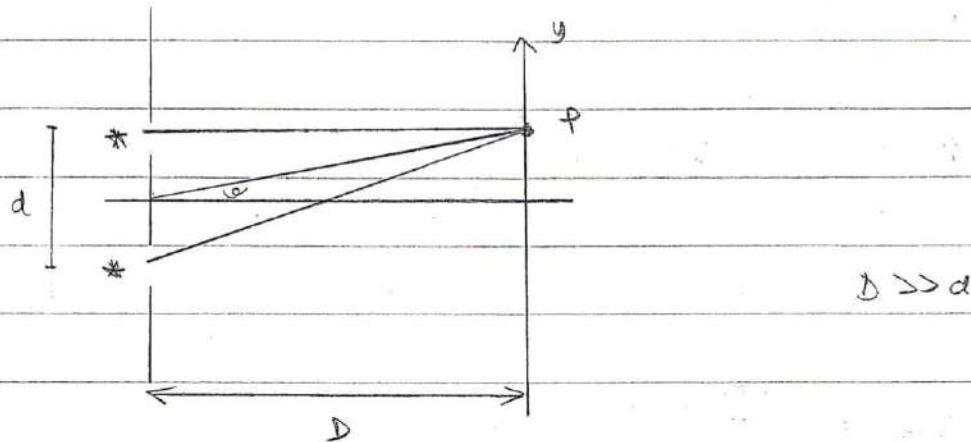
$$\text{MAX: } \frac{\Delta\phi}{2} = m\pi, \quad m = 0, \pm 1, \pm 2, \dots$$

$$\Delta s_{\text{MAX}} = (r_2-r_1) m = m \lambda$$

$$\text{MIN: } \frac{\Delta\phi}{2} = (2m+1) \frac{\pi}{2}, \quad m = 0, \pm 1, \pm 2, \dots$$

$$\Delta s_{\text{MIN}} = \left(m + \frac{1}{2}\right) \lambda$$

2. YOUNGOU POKUS NA DVJE PUKOTINE



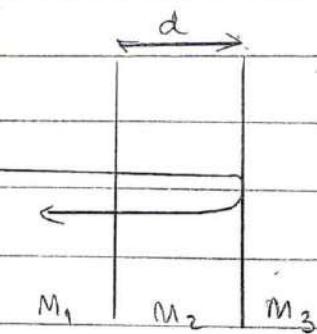
$$\left. \begin{array}{l} s = d \sin \alpha \\ y = d + g \alpha \end{array} \right\} \frac{s}{d} = \frac{y}{D}$$

Maximum

$$s = m \lambda$$

$$y_{\max} = \frac{D}{d} s_{\max} = \frac{D}{d} m \lambda \quad m = 0, \pm 1, \pm 2, \dots$$

3. TANK LISTCI



$$\delta = 2dm_2 + \frac{n}{2} + \frac{n}{2}$$

$$\text{Max: } \delta_{\text{MAX}} = mn$$

$$+ M_1 < M_2 < M_3$$

$$2dm_2 + n = mn$$

$$d = \frac{n}{2m}$$

M_{MIN} :

$$\delta_{\text{MIN}} = \left(m + \frac{1}{2}\right) n$$

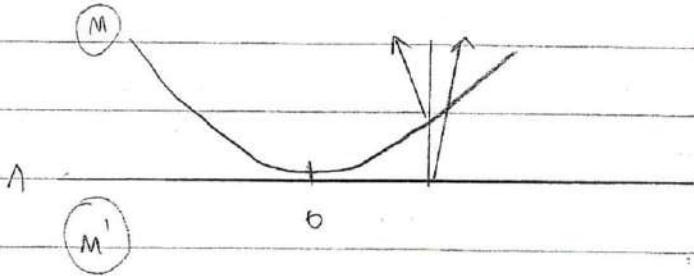
$$+ M_1 < M_2 < M_3$$

$$2dm_2 + n = \left(m + \frac{1}{2}\right) n$$

Maytanji Listci

$$r = \frac{n}{4M_2}$$

4. NEWTONOVI KOLOBARI



Refleksija

$$m(r) = \frac{r}{R} - \sqrt{R^2 - r^2} \approx \frac{r^2}{2R}$$

$$r \ll R$$

$$\delta = 2m(r) + \frac{\lambda}{2} = \frac{r^2}{R} + \frac{\lambda}{2}$$

$$\text{Max: } \delta = m\lambda.$$

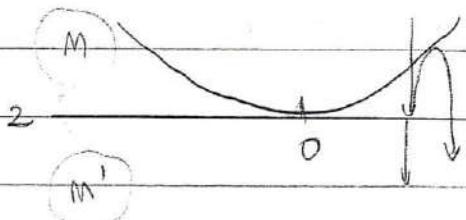
$$r_{\max} = \sqrt{R(m - \frac{1}{2})\lambda} \quad m = 1, 2, \dots$$

$$\text{Min: } \delta = (m + \frac{1}{2})\lambda$$

$$r_{\min} = \sqrt{Rm\lambda} \quad m = 0, 1, 2, \dots$$

↑
tanka sredina,
broj je od 0

Transmisijska



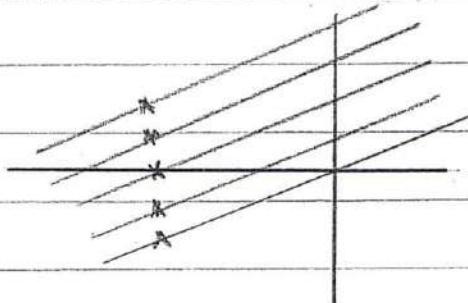
$$\delta = 2m(r) + 2\frac{\lambda}{2}$$

$$r_{\max} = \sqrt{Rm\lambda} \quad m = 0, 1, 2, \dots$$

$$r_{\min} = \sqrt{R(m - \frac{1}{2})\lambda} \quad m = 1, 2, \dots$$

5. OPTIČKA REŠETKA

N - koherenčnih izvora

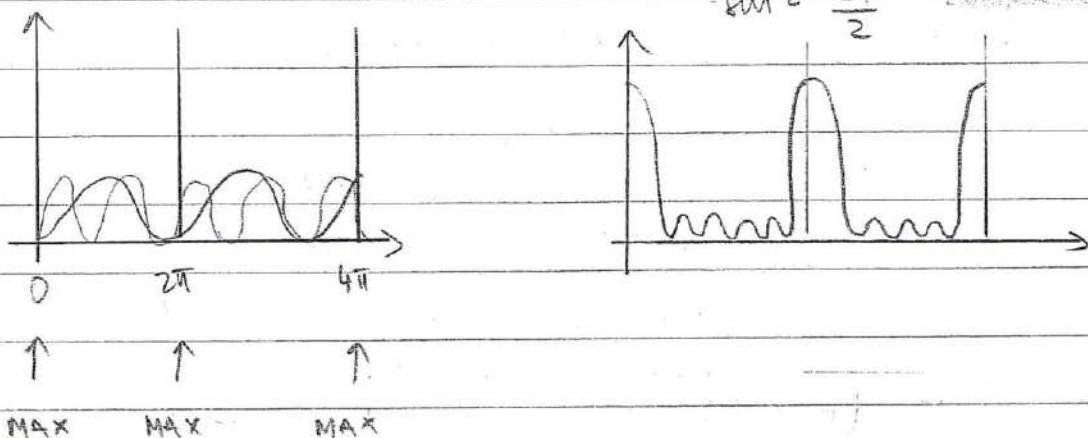


Rezultantno polje

$$E(\lambda) = E_0 \cdot \frac{\sin(N - \frac{\Delta\phi}{2})}{\sin \frac{\Delta\phi}{2}}$$

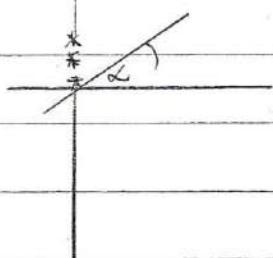
$$\Delta\phi = 2\pi \frac{d \sin \lambda}{\lambda}$$

$$I = I_0 \frac{\sin^2(N - \frac{\Delta\phi}{2})}{\sin^2 \frac{\Delta\phi}{2}}$$



6. DIFRAKCIJA NA JEDNOJ PUKOTINI

Model: N koherenčnih izvora ma razmota



$$d = \frac{a}{N}$$

Formula za rešetku

$$E(\lambda) = E_0 \frac{\sin(N - \frac{\Delta\phi}{2})}{\sin \frac{\Delta\phi}{2}}$$

$$\Delta\phi = 2\pi \frac{s}{\lambda} = 2\pi \frac{d \sin \lambda}{\lambda} = 2\pi \frac{a \sin \lambda}{N \lambda}$$

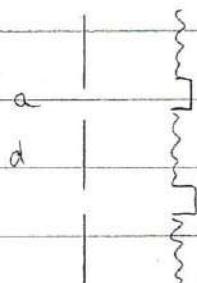
$$E(\lambda) = E_0 \frac{\sin(\pi \frac{a \sin \lambda}{N \lambda})}{\sin(\pi \frac{a \sin \lambda}{N \lambda})}$$

gleđamo tines $N \rightarrow \infty$

$$z = \frac{\pi a \sin \alpha}{n} = \frac{N e_0 \sin \alpha}{z}$$

$$I(\lambda) = I_0 \frac{\sin^2 z}{z^2} \Rightarrow \text{ognj}$$

7. SPETOGRAF S REŠETKOM



N = broj pakotina

d = konst. rešetke

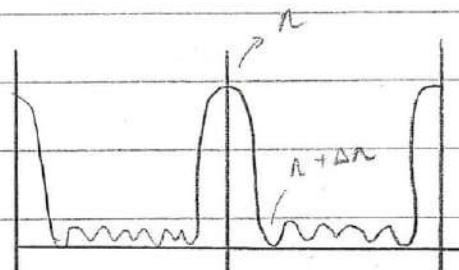
a = širina rešetke

$$I(\lambda) = I_0 \cdot \frac{\sin^2 \left(\frac{\pi a \sin \alpha}{n} \right)}{\left(\frac{\pi a \sin \alpha}{n} \right)^2} \cdot \frac{\sin^2 \left(N \frac{\pi d \sin \alpha}{n} \right)}{\sin^2 \left(\frac{\pi d \sin \alpha}{n} \right)}$$

MAXIMUM:

$$d \sin \alpha = m n$$

$$d \sin \alpha = \frac{m}{N} n$$



minimum mora biti prije maksimuma

$$\text{MAX: } d \sin \alpha = m \cdot n (n + \Delta n)$$

$$\text{MIN: } d \sin \alpha = \frac{m(n+1)}{N} n$$

moc razlučivanja rešetke

$$m(n + \Delta n) = \frac{mN + 1}{N} n$$

$$R = \frac{n}{\Delta n} = M \cdot N$$

$$\frac{\Delta n}{n} = \frac{1}{MN}$$

2. POLARIZACIJA SUJETOSTI

linearno polarizirani val

$$\vec{E}(t, \vec{r}) = \vec{j} E_0 \cos(wt - k\vec{r})$$

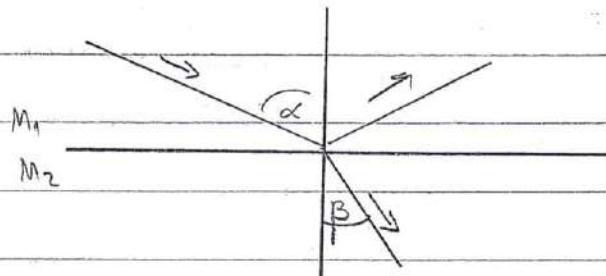
vrijeme položaj amplitudni vektor val. vektor

kružno polarizirani val

$$\vec{E}(t, \vec{r}) = \vec{j} E_0 \cos(wt - k\vec{r}) + \vec{i} E_0 \cos(wt - k\vec{r} - \frac{\pi}{2})$$

Brewsterov kut

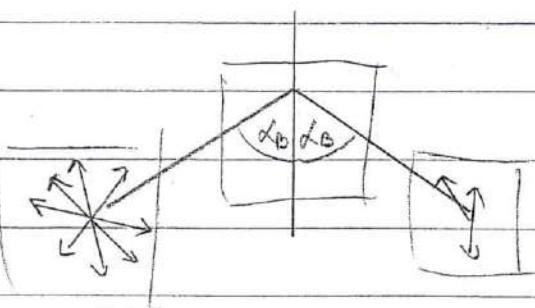
- ako je kut između lomljene i reflektirane zrake $= \frac{\pi}{2}$,
zraka je polarizirana



Snelou zákon

$$\frac{\sin \alpha}{\sin \beta} = \frac{m_2}{m_1} \quad \frac{\sin \beta}{\sin(\frac{\pi}{2} - \gamma)} = \tan \alpha \quad \alpha = \arctan \frac{m_2}{m_1}$$

Malleson zákon



$$I' = I \cos^2 \theta$$

$$I \rightarrow I' \rightarrow I''$$

lin. polar. sujetlost

MODERNA

FIZIKA

1. ZRAĆENJE CRNOG TIJELA

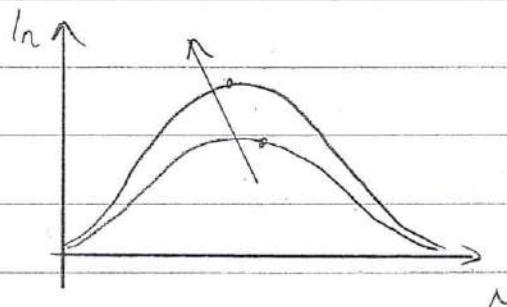
Štefan-Boltzmannov zakon (experimentalno)

- tijelo ugrljivo na T zrači: $I = \sigma T^4$

$$\sigma = 5,67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$$

Raspodjela intenziteta po valnim duljinama

$$I_n = \frac{dI}{dn} = f(n, T)$$



Maximum se pomiču sve
manjim valnim duljinama

Planckov zakon

$$I_n = f(n, T) = \underbrace{\left(\frac{C}{4} \cdot \frac{8\pi}{n^4} \cdot T \right)}_{\text{Rayleigh-Jeansova formula}} \left(\frac{AC}{n k T} \cdot \frac{1}{e^{\frac{AC}{n k T}} - 1} \right)$$

Rayleigh-Jeansova
formula

"ultraljubičasta katastrofa"

$$\text{planck} \rightarrow \frac{2\pi h c^2}{n^5} \cdot \frac{1}{e^{\frac{AC}{n k T}} - 1}$$

$$k \rightarrow \text{Boltzmannova konst. } k = 1,381 \cdot 10^{-23} \text{ J/K}$$

$$h \rightarrow \text{Planckova konst. } h = 6,626 \cdot 10^{-34} \text{ Js}$$

$$I_n = \int dl = \int_0^\infty I_n dn = \int_0^\infty f(n, T) dn = \frac{2\pi^5 c^4}{15 \epsilon^2 h^3} T^4$$

* potazujemo da je 2-j zakon, limes Planckova zakona pri $n \rightarrow \infty$

$$\text{Planckov faktor: } \frac{hc}{\lambda kT} \cdot \frac{1}{e^{\frac{hc}{kT}} - 1} = \left\{ \frac{hc}{\lambda kT} = x \right\} = \frac{x}{e^x - 1}$$

$\lim n \rightarrow \infty \rightarrow$ odg. limes $\lambda \rightarrow 0$

$$\lim_{x \rightarrow 0} \frac{x}{e^x - 1} = \lim_{x \rightarrow 0} \frac{x}{(1+x)-1} = \lim \frac{x}{x} = 1$$

* oprez

$$n = \frac{c}{f}$$

$$dl = f(n, T) dn$$

$$= \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{kT}} - 1} = \frac{2\pi hc^2}{(\frac{c}{f})^5} \cdot \frac{1}{e^{\frac{hc}{kT}} - 1} = \frac{2\pi h v^3}{c^2} \cdot \frac{1}{e^{\frac{hc}{kT}} - 1}$$

$$I_v = \frac{2\pi h v^3}{c^2} \cdot \frac{1}{e^{\frac{hc}{kT}} - 1}$$

WILNOV ZAKON

$$n_{\max} \cdot T = k_w \Rightarrow \text{Wilnova konstanta } 0,0029 \text{ mK}$$

$$I_n = f(n, T) \propto \frac{x^5}{e^x - 1} \quad x = \frac{hc}{kT}$$

$$\frac{dn}{dx} = \frac{df}{dx} = 0$$

graf mitad nije 0

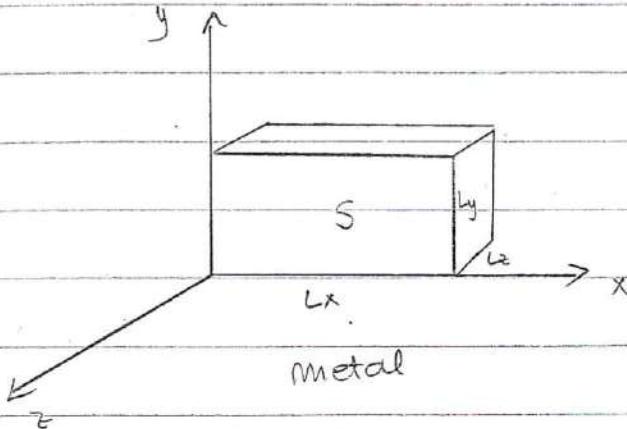
$$\underbrace{\left(\frac{x}{x} - \frac{x}{e^x - 1} \right)}_{} f$$

$$x = 4,965$$

$$x = \frac{hc}{n_{\max} kT}$$

Izvod zakona zračenja crnog tijela

I. Spektralna gustoća elektrom. madaova



EM zračenje zadovoljava rubne uvjete: $\vec{E} = 0$

$$\Rightarrow \vec{E}(\vec{r}, t) = \vec{E}_0 e^{i\omega t} (e^{i\vec{k}\vec{r}} + e^{-i\vec{k}\vec{r}}) = \vec{E}_0 e^{i\omega t} 2\cos(\vec{k}\vec{r}) \text{ gdje je } \vec{k}$$

\vec{k} valni vektor

$$\vec{k} = k_x \vec{i} + k_y \vec{j} + k_z \vec{k}$$

$$\text{Za pog rubnih uvjeta: } k_x = \frac{2\pi}{L_x}, \quad L_x = m_x \frac{\lambda_x}{2} \Rightarrow k_x = \frac{m_x}{L_x} \pi, \quad m_x = 1, 2, 3.$$

analogno k_y, k_z

Razmatrano EM. val u M-prostoru $dm_x dm_y dm_z \equiv d^3 m$

k-prostoru $dk_x dk_y dk_z = d^3 k$

$$\left. \begin{aligned} dk_x &= \frac{\pi}{L_x} \\ dk_y &= \frac{\pi}{L_y} \\ dk_z &= \frac{\pi}{L_z} \end{aligned} \right\} \Rightarrow dk_x dk_y dk_z = \frac{\pi^3}{L_x L_y L_z} dm_x dm_y dm_z$$

$$d^3 k = \frac{\pi^3}{V} \cdot d^3 m \quad \begin{matrix} \text{gustoća dopuštenih} \\ \text{valnih vektora u k prostoru} \end{matrix}$$

Broj elektromagnetskih modova u V takvih da $|E| < k$

$$N(k) = 2 \cdot \frac{1}{8} \cdot \left(\frac{4}{3} k^3 \pi \right) \cdot \frac{V}{\pi^2} = \frac{1}{3} k^3 \frac{V}{\pi^2}$$

2 moguća ↓ otant ↗ kugla polimjerat
sugrađa polarizacije

Broj elektromagnetskih modova tako da $|E| \in [k, k+dk]$

$$dN = \frac{dN(k)}{dk} \cdot dk = k^2 \frac{V}{\pi^2} dk$$

Broj elektromag. modova tako da $\lambda \in (\lambda, \lambda+d\lambda) : (k - \frac{2\pi}{\lambda} = \frac{2\pi}{c} \lambda)$

$$dN = \left(\frac{2\pi}{c}\lambda\right)^2 \frac{V}{\pi^2} \cdot \frac{2\pi}{c} d\lambda = g(\lambda) d\lambda$$

$$g(\lambda) = \frac{8\pi^3}{c^3} \lambda^2 \Rightarrow \text{spektralna gustoća elektrom. modova}$$

II. Rayleigh - Jeans

- termodynamička ravnoteža
- Boltzmannova raspodjela

$$f(E) = ce^{-\frac{E}{kT}} = \frac{1}{kT} e^{-\frac{E}{kT}}$$

$$\text{Normalizacija uvijednosti: } \int_0^\infty f(E)dE = 1 \Rightarrow c = \frac{1}{kT}$$

$$\text{Srednja energija: } \langle E \rangle = \int_0^\infty E f(E)dE = \int_0^\infty E \cdot \frac{1}{kT} e^{-\frac{E}{kT}} dE = kT$$

$$\text{Spektralna gustoća energije: } U(\lambda) = \langle E \rangle g(\lambda) = kT \cdot \frac{8\pi^3}{c^3} \lambda^2$$

III. Planck

Pretpostavke:

- termodynamička ravnoteža

- Boltzmannova raspodjela
- hipoteza o kvantizaciji

$$\langle E \rangle = \sum_m E_m f_m \rightarrow f_m = c e^{-\frac{E_m}{kT}}$$

\downarrow
 $E_m = \hbar \nu$)

Normalizacija vjerovatnosti

$$\sum_m f_m = 1 = C \sum_m e^{-m x} \quad x = \frac{\hbar \nu}{kT}$$

$$= C [e^{-x} + (e^{-x})^2 + \dots]$$

$$= C \frac{1}{1 - e^{-x}}$$

$$\Rightarrow C = 1 - e^{-x} = 1 - e^{-\frac{\hbar \nu}{kT}}$$

$$\langle E \rangle = \sum_m m \hbar \nu (1 - e^{-\frac{\hbar \nu}{kT}}) e^{-\frac{m \hbar \nu}{kT}}$$

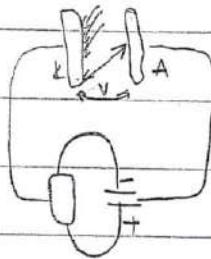
$$= x (1 - e^{-x}) kT \sum_m m e^{-m x} = \dots$$

$$= x \frac{kT}{e^x - 1} = \frac{\hbar \nu}{kT} \cdot \frac{kT}{e^{\frac{\hbar \nu}{kT}} - 1} = \frac{\hbar \nu}{e^{\frac{\hbar \nu}{kT}} - 1}$$

Spektralna gustoća energije

$$U(\nu) = \langle E(\nu) \rangle g(\nu) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{\hbar \nu}{kT}} - 1}$$

2. FOTOELEKTRIČNI EFEKT



- a) struja razmjeru intenzitetu zračenja [OK]
- b) ako V_0 ide ispod mreže V_0 struja nestaje [PROBLEM]
- c) Max. kin. en. elektrona ne ovisi o intenzitetu [PROBLEM]

$$E_{k,\max} = \frac{mv_{\max}^2}{2} = eV_0 \rightarrow \text{zastaviti mapom}$$

objašnjenje: A. Einstein (1905)

$$Ex = h\nu = h\nu = h \cdot \frac{c}{\lambda}$$

\downarrow
Planckova konst.

$$\lambda = \frac{h}{2\pi}$$

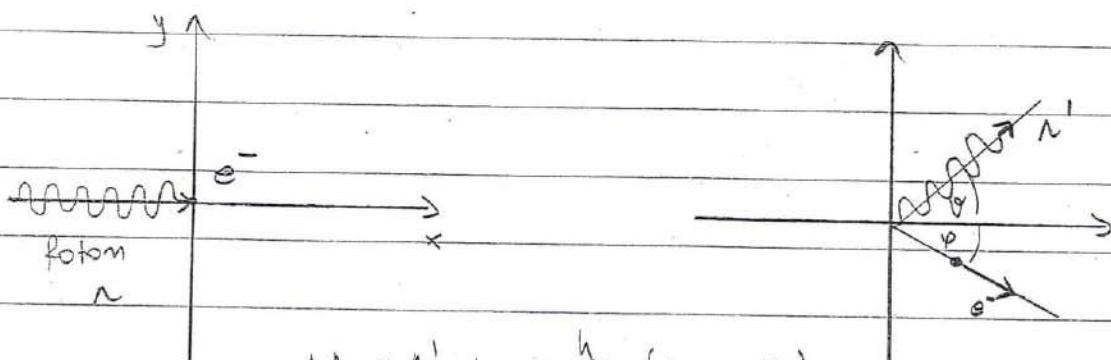
energija: $E_k \geq W + \epsilon_{kin}$ izlazni rad

3. "COMPTONOV" RASPRŠENJE

- foton interaguje sa česticu koja miruje, "raspršenje fotona na e^- "

prije:

poslije:



$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$$

$$\frac{1}{\lambda'} - \frac{1}{\lambda} = \frac{1}{m_e c^2} (1 - \cos\theta)$$

$$\text{Energija: } \left[E = \sqrt{(m_0 c^2)^2 + (pc)^2} = \frac{m_0 c^2}{\sqrt{1 - (\frac{v}{c})^2}} \right]$$

$$E = h\nu + m_0 c^2 = h\nu' + \sqrt{(m_0 c^2)^2 + \left(\frac{mv \cdot c}{\sqrt{1 - (\frac{v}{c})^2}} \right)^2} \quad (1)$$

Impuls:

$$\vec{P} = \frac{m \cdot \vec{v}}{\sqrt{1 - (\frac{v}{c})^2}} \quad \text{za } m > 0, \quad P = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \text{za } m_0$$

$$p_x = \frac{h\nu}{c} = \frac{h\nu'}{c} \cos \vartheta + \frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \cos \varphi \quad (2)$$

$$p_y = 0 = \frac{h\nu'}{c} \sin \vartheta - \frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \sin \varphi \quad (3)$$

Iz (2) i (3) eliminiramo φ :

$$\left(\frac{h\nu}{c} \right)^2 - 2 \frac{h\nu}{c} \cdot \frac{h\nu'}{c} \cos \vartheta + \left(\frac{h\nu'}{c} \right)^2 = \left(\frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \right)^2 \quad (4)$$

prepisujemo (1)

$$\left(\frac{h\nu}{c} - \frac{h\nu'}{c} + m_0 c \right)^2 - (m_0 c)^2 = \left(\frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \right)^2$$

$$\left(\frac{h\nu}{c} \right)^2 + \left(\frac{h\nu'}{c} \right)^2 + 2 \left(\frac{h\nu}{c} - \frac{h\nu'}{c} \right) m_0 c - 2 \frac{h\nu}{c} \cdot \frac{h\nu'}{c} = \left(\frac{m_0 v}{\sqrt{1 - (\frac{v}{c})^2}} \right)^2$$

(5). (4)

$$2 \cdot \left(\frac{h\nu}{c} - \frac{h\nu'}{c} \right) m_0 c - 2 \frac{h\nu}{c} \cdot \frac{h\nu'}{c} + 2 \frac{h\nu}{c} \cdot \frac{h\nu'}{c} \cos \vartheta = 0$$

$$\left(\frac{1}{\frac{h\nu'}{c}} - \frac{1}{\frac{h\nu}{c}} \right) m_0 c = 1 - \cos \vartheta$$

uz $\lambda\nu = c$

$$\lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos \vartheta)$$

4. BOHROV MODEL ATOMA (1913)

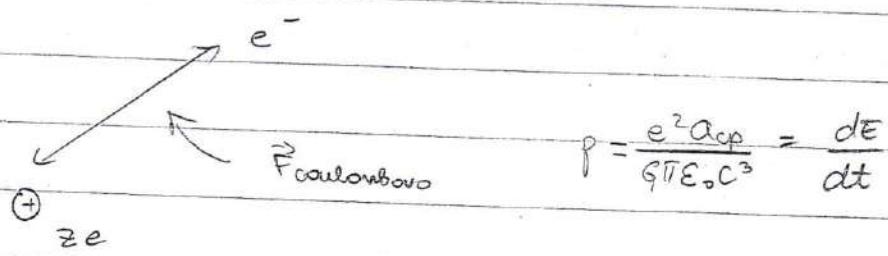
Predložnik: * Thomson

* Rutherford

$$* \text{ Balmerova formula } \lambda = (364.55 \text{ nm}) \frac{m^2}{m^2 - 4} \quad m=3,4,5$$

$$\text{ti: } \frac{1}{\lambda} = R_A \left(\frac{1}{2^m} - \frac{1}{m^2} \right) \quad m=3,4,5 \quad R_A = 1.097373 \cdot 10^9 \text{ m}^{-1}$$

Rydbergova konst.



$$P = \frac{e^2 a_0}{4\pi \epsilon_0 c^3} = \frac{dE}{dt}$$

$P_1 \Rightarrow I.$ B postulat: postoje dozvoljene staze na kojima e^- može zračiti

$P_2 \Rightarrow II.$ B postulat: kutna količina gibanja je klasična veličina

$$\text{kod Bohra. } L = m\vec{r} = m \frac{\vec{r}}{2\pi} = m \cdot v_m \cdot r_m ; \quad m=1,2,3\dots$$

$P_3 \Rightarrow III.$ B. postulat: elektron može iz jednog dozvoljenog stanja preći u drugo. Pri prelasku $m_i \rightarrow m_f$, zrači se foton; $E_{\text{foton}} = E_i - E_f = h\nu$

$$\text{jednakost sliči: } F_{\text{cp}} = \frac{mv^2}{r_m} = \frac{1}{4\pi \epsilon_0} \cdot \frac{ze \cdot e}{r_m^2}$$

$$12 \quad (P_2) \quad L_m = m\vec{r} = m v_m r_m \Rightarrow v_m = \frac{m\vec{r}}{mr_m}$$

$$\frac{m}{r_m} \left(\frac{m\vec{r}}{mr_m} \right)^2 = \frac{1}{4\pi \epsilon_0} \cdot \frac{ze^2}{r_m^2}$$

$$r_m = \frac{4\pi \epsilon_0 \hbar}{ze^2 \cdot m} \cdot M^2$$

$$v_m = \frac{ze^2}{4\pi \epsilon_0 \hbar} \cdot \frac{1}{M}$$

Ukupna energija

$$E_m = -\frac{1}{4\pi\epsilon_0} \cdot \frac{ze^2}{r_m} + \frac{m}{2} v_m^2 = \dots = \frac{-m z^2 e^4}{32\pi^2 \epsilon_0^2 h^2} \cdot \frac{1}{M^2}$$

$$E_1 = -13.58 \text{ eV}$$

$$E_M = E_1 \cdot \frac{1}{M^2}$$

energija ionizacije osnovnog stanja

12

(P₃)

$$\Delta E = E_i - E_f = E_1 \left(\frac{1}{m_f} - \frac{1}{m_i} \right)$$

Balmer $m_f = 2$

