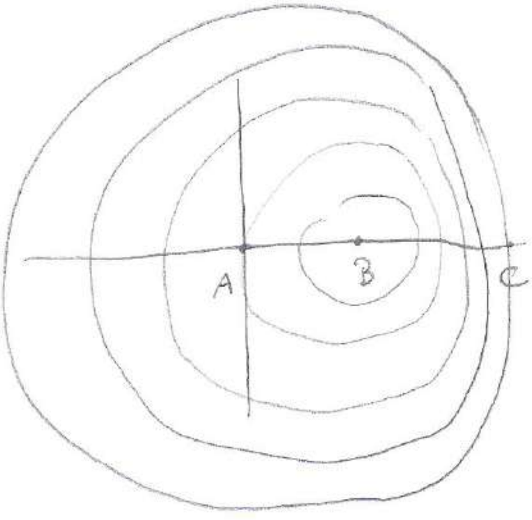
Dopplerov efekt

v_0, f_i - frekv. izvora $\quad T_i = \frac{1}{f_i}$



$n_i = \frac{t_i}{T_i} = t_i \cdot f_i$

$\overline{AB} = v_i \cdot t_i \qquad v_0 = \lambda_{ispred} \cdot f_{ispred}$

$\overline{AC} = v_0 \cdot t_i$

$\overline{BC} \Rightarrow$ ima n_i valova $= n_i \cdot \lambda_{ispred}$

$\overline{AC} = \overline{AB} + \overline{BC}$

$v_0 \cdot \cancel{t_i} = v_i \cdot \cancel{t_i} + \cancel{t_i} \cdot f_i \cdot \lambda_{ispred}$

$v_0 = v_i + f_i \lambda_{ispred}$

$$\lambda_{ispred} = \frac{v_0 - v_i}{f_i} = \frac{v_0}{f_{ispred}}$$

1. Slušaoc prema kojem ide izvor

$$f_{ispred} = f_i \cdot \frac{v_0}{v_0 - v_i} \qquad f_{ispred} > f_i$$

2. Slušaoc od kojeg ide izvor

$$f_{iza} = f_i \cdot \frac{v_0}{v_0 + v_i}$$

Integralni teoremi

(1.) Gaussov teorem ili teorem o divergenciji vektorskog polja $\underset{\sim}{A}$

$\underset{\sim}{A}(x, y, z)$

$$\oint_S \underset{\sim}{A}\, d\underset{\sim}{S} = \int_V (\nabla \underset{\sim}{A})\, dV \qquad \nabla \underset{\sim}{A} = \operatorname{div} \underset{\sim}{A}$$

Kartezijeve koordinate

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla \underset{\sim}{A} = \nabla \left(\hat{i} A_x(x,y,z) + \hat{j} A_y(x,y,z) + \hat{k} A_z(x,y,z)\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= \operatorname{div} A$$

(2.) Stokesov teorem ili teorem o rotaciji vekt. polja $\underset{\sim}{A}$

$$\oint_{\partial S} \underset{\sim}{A}\, d\underset{\sim}{l} = \int_S (\nabla \times \underset{\sim}{A})\, dS \qquad \nabla \times \underset{\sim}{A} = \operatorname{rot} \underset{\sim}{A}$$

(3.) Greenov teorem o skalarnom polju ϕ

$\underset{\sim}{A} = \nabla \phi$

$$\int_{\partial S} \underset{\sim}{A}\, d\underset{\sim}{l} = \int_{\partial S} \nabla\phi\, d\underset{\sim}{l} = \int_A^B d\phi = \phi(B) - \phi(A)$$

$\partial S \downarrow$ rub površine

1. Maxwellova jdba

1. Part I

$$\underset{\sim}{F} = k\,\frac{Q_1 Q_2}{r^2}\cdot\hat{r}$$

$$\rho = \frac{dQ}{dV} \text{ - gustoća naboja}$$

$$\sigma = \frac{dQ}{dS} \text{ - el. tok}$$

$$\underset{\sim}{F}_{12} = \underbrace{\left(k\,\frac{Q_1}{r^2}\,\hat{r}\right)}_{\underset{\sim}{E}_1}\cdot Q_2$$

$$\underset{\sim}{F}_{12} = \underset{\sim}{E}_1\cdot Q_2$$

$$\underset{\sim}{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{Q}{r^2}\,\hat{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho\,dV}{r^2}\,\hat{r}$$

$$\phi_E = \frac{dQ}{dS} \text{ - broj silnica kroz površinu (zatvorenu)}$$

$$\phi_E \propto \underset{\sim}{E}$$

$$\Delta\phi_E = \underset{\sim}{E}\cdot\Delta\underset{\sim}{S}$$

$$\phi_E = \oint\underset{\sim}{E}\,d\underset{\sim}{S} \propto Q_{\text{unutar } S}$$

$$\phi_E \propto Q_{\text{unutar } S}$$

$$\phi_E = \frac{Q_{\text{unutar } S}}{\varepsilon_0}$$

$$\boxed{\oint\underset{\sim}{E}\,d\underset{\sim}{S} = \frac{Q_{\text{unutar } S}}{\varepsilon_0}}$$

Part II

$\Delta \phi_E = \underset{\sim}{E} \cdot \Delta \underset{\sim}{S}$

ϕ_E = br. izlaznih – br. ulaznih silnica

$\phi_E = \oint_S \underset{\sim}{E}\, d\underset{\sim}{S} \sim Q_{unutar\ S}$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}\,\hat{r}$$

$$d\underset{\sim}{S} = r^2\, d\Omega = r^2 \underbrace{(\sin\theta \cdot d\theta\, d\varphi)}_{4\pi}$$

ϕ_E (točkasti naboj) $= \int_S \underset{\sim}{E}_{tn}\, d\underset{\sim}{S}$

$$= \frac{1}{4\pi\varepsilon_0} \int \left(\frac{Q}{r^2}\,\hat{r}\right) r^2\, d\Omega$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot Q \cdot 4\pi = \frac{Q_{unutar\ S}}{\varepsilon_0}$$

$$\boxed{\phi_E = \oint_S \underset{\sim}{E}\, d\underset{\sim}{S} = \frac{Q_{unutar\ S}}{\varepsilon_0}}$$

$$\oint_S \underset{\sim}{E}\, d\underset{\sim}{S} = \int (\nabla E)\, dV = \frac{1}{\varepsilon_0} \int \rho\, dV = \frac{Q_{unutar\ S}}{\varepsilon_0}$$

$\rho = \frac{dQ}{dV} \qquad Q_{unutar\ S} = \int_V dQ = \int_V \rho\, dV$

$\int_V (\nabla E)\, dV = \frac{1}{\varepsilon_0} \int_V \rho\, dV$

$\boxed{\Delta \underset{\sim}{E} = \frac{\rho}{\varepsilon_0}}$ – integralni oblik

Rad u el. polju

$$W = \int \underset{\sim}{F}\, d\underset{\sim}{r} \qquad d\underset{\sim}{r} = \hat{r}\, dr$$

$$\underset{\sim}{F_C} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}\, \hat{r}$$

$$W_{P,K} = \int_P^K \underbrace{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}\, \hat{r}}_{F_m} \cdot \hat{r}\, dr$$

$$= \frac{Q_1 Q_2}{4\pi\varepsilon_0} \int \frac{dr}{r^2} = -\frac{Q_1 Q_2}{4\pi\varepsilon_0} \cdot \frac{1}{r} \Bigg|_P^K = \frac{-Q_1 Q_2}{4\pi\varepsilon_0} \left(\frac{1}{r_K} - \frac{1}{r_P} \right)$$

$$= -\left(\frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{1}{r_K} - \frac{Q_1 Q_2}{4\pi\varepsilon_0} \cdot \frac{1}{r_P} \right) = -\left(E_K^{pot} - E_P^{pot} \right) = \underline{\underline{-\Delta E^{pot}}}$$

$$E_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r}$$

$$W_{P,K} = \int \left(\frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r^2} \hat{r} \right) Q_2\, \hat{r}\, dr \quad \Big/ : Q_2$$

$$\frac{W_{P,K}}{Q_2} = \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1}{r^2}\, dr = -\Bigg(\underbrace{\frac{Q_1}{4\pi\varepsilon_0} \cdot \frac{1}{r_K} - \frac{Q_1}{4\pi\varepsilon_0} \cdot \frac{1}{r_P}}_{\text{el. potencijal}} \Bigg) = -\Delta V = -(V_K - V_P)$$

$\Downarrow$

el. pot. $\Rightarrow$ rad po naboju

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} \qquad [V] = \text{volt} = \text{V}$$

$$W_{p,k} = \int \underset{\sim}{F}\, d\underset{\sim}{r} = -\Delta E^{pot} = -\int_p^k d E^{pot}$$

$$\underset{\sim}{F} \cdot d\underset{\sim}{r} = -d E_{pot}$$

$$\left.\begin{array}{l} \underset{\sim}{F} = -\nabla E_{pot} \\ \oint \underset{\sim}{F_c}\, d\underset{\sim}{r} = 0 \end{array}\right\} \text{KONZERVATIVNOST}$$

$$\cancel{Q_2}\, E_1 = -\nabla \frac{Q_1 \cancel{Q_2}}{4\pi\varepsilon_0} \cdot \frac{1}{r} \cdot \hat{r}$$

$$\underset{\sim}{E} = -\nabla V(r)$$

$$V(r) = -\int \underset{\sim}{E} \cdot d\underset{\sim}{r}$$

Polarizacija dielektrika

$$\underset{\sim}{E} = \underset{\sim}{E}_0 + \underset{\sim}{E}_{ind} \rightarrow \text{polarizacija}$$

$$\underset{\sim}{P} \propto \underset{\sim}{E}$$

$\underset{\sim}{P}$ - jakost pojedinog dipola, tj. dipolni moment

$$|P| = Q \cdot d \qquad \underset{\sim}{P} = \alpha \underset{\sim}{E}$$

α → polarizibilnost (svojstvo materijala da se polarizira)

$\underset{\sim}{D}$ - vektor dielektričnog pomaka

$$\left.\begin{aligned} \underset{\sim}{D} &= \varepsilon_0 \underset{\sim}{E} + \underset{\sim}{P} \\ \underset{\sim}{D} &= \varepsilon \underset{\sim}{E} = \varepsilon_0 \varepsilon_r \underset{\sim}{E} \end{aligned}\right\} \quad \varepsilon_0 \varepsilon_r \underset{\sim}{E} = \varepsilon_0 \underset{\sim}{E} + \underset{\sim}{P} = \varepsilon_0 \underset{\sim}{E} + \alpha \underset{\sim}{E}$$

$$\varepsilon_0 \underset{\sim}{E} (\varepsilon_r - 1) = \underset{\sim}{P} = \alpha \underset{\sim}{E}$$

χ_e - električna susceptibilnost

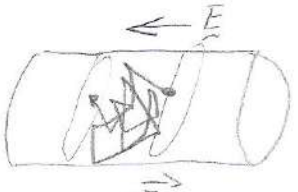
$$\varepsilon_0 \chi_e = \alpha \qquad \nabla \underset{\sim}{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla (\varepsilon_0 \underset{\sim}{E}) = \rho$$

$$\boxed{\nabla \underset{\sim}{D} = \rho} \text{ 1. Maxwell jdba.}$$

Električna struja

$n_0 \sim 10^{23}/m^3$



$\vec{v}_d$ → driftna brzina

$$E = \frac{\Delta U}{l}$$

$$\Delta Q = e \cdot n_0 \cdot \Delta V = e \cdot n_0 \cdot S \cdot \Delta \vec{l} = e \cdot n_0 \cdot S \cdot \vec{v}_d \cdot \Delta t$$

$$\vec{I} = \frac{\Delta Q}{\Delta t} = \underbrace{e}_{1.6 \cdot 10^{-19} C} n_0 S \vec{v}_d$$

$$\boxed{\vec{I} = e n_0 S v_d}$$

Gustoća el. struje

$$\vec{J} = \frac{I}{S} = e n_0 \vec{v}_d \qquad \frac{\Delta Q}{\Delta V} = \rho_e$$

$$\boxed{\vec{J} = \rho_e \cdot \vec{v}_d}$$

$\vec{J} \propto \vec{E}$ (vanjsko el. polje)

Ohmov zakon

$$\vec{J} = \sigma \cdot \vec{E}$$

σ → el. vodljivost

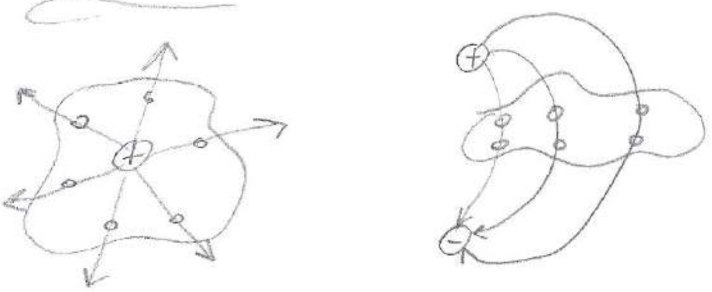
$$\frac{I}{S} = \sigma E = \sigma \cdot \frac{\Delta U}{l}$$

$$I = \underbrace{\frac{S\sigma}{l}}_{\frac{1}{R}} \cdot \Delta U$$

$$\boxed{I = \frac{\Delta U}{R}} \qquad \frac{l}{S} \cdot \frac{1}{\sigma} = \rho \cdot \frac{l}{S} = R$$

2. Maxwellova jednadžba

el. monopol



$\phi_E = \#$ izl. $- \#$ ulaz. $= 0$

$$\phi_E = \oint \underset{\sim}{E} \cdot d\underset{\sim}{S} = \frac{Q_{unutars}}{\varepsilon_0}$$

$$\boxed{\phi_M = \oint_S \underset{\sim}{B} \cdot d\underset{\sim}{S} = 0}$$

integralni oblik

$$\boxed{\nabla \underset{\sim}{B} = 0}$$ dif. oblik

- nema mag. monopola

Lorentzova sila

$\underset{\sim}{E}, \underset{\sim}{B}, q, \underset{\sim}{v}$

$\underset{\sim}{F}_{el} = q\,\underset{\sim}{E}$ $\quad\quad$ $\underset{\sim}{F}_{mL} = q\,\underset{\sim}{v} \times \underset{\sim}{B}$

$\underset{\sim}{F}_{mL} \propto q$ $\quad\quad$ $\underset{\sim}{F}_L = F_{el} + F_{m/L} = q\,\underset{\sim}{E} + q\,\underset{\sim}{v} \times \underset{\sim}{B}$ //

$\underset{\sim}{F}_{mL} \perp \underset{\sim}{v}, \underset{\sim}{B}$

Sila na vodič u $\underset{\sim}{B}$

$\underset{\sim}{F} = q\,\underset{\sim}{v_d} \times \underset{\sim}{B}$

$\Delta Q = q\,\underset{\sim}{v_d} \cdot S \cdot \Delta t \cdot n_0$

$dQ = q\,n_0\,\underset{\sim}{v_d}\,S\,dt$

$d\underset{\sim}{F} = (q\,\underset{\sim}{v_d} \times \underset{\sim}{B})\,n_0\,S\,dt$ $\quad\quad$ $\underset{\sim}{v_d}\,dt = dl$

$d\underset{\sim}{F} = q\,d\underset{\sim}{l} \times \underset{\sim}{B} \cdot S$

$\underset{\sim}{F} = I \int_A^B d\underset{\sim}{l} \times \underset{\sim}{B}$ //

Amper-Maxwellov zakon

$$\oint_{\partial S} \underset{\sim}{B} \cdot d\underset{\sim}{l} \propto I$$

$$\oint \underset{\sim}{B}\, d\underset{\sim}{l} = B \oint d\underset{\sim}{l} = 2Br\pi = A \cdot I$$

kružnica

$$B = \frac{A\, I}{2r\pi} \qquad \underline{\underline{\mu_0 I}}$$

$$\oint \underset{\sim}{B}\, d\underset{\sim}{l} = \mu_0 I = \mu_0 \oint_{\partial S} \underset{\sim}{J}\, d\underset{\sim}{S} = \oint (\nabla \times B)\, d\underset{\sim}{S} = \mu_0 \int \underset{\sim}{J}\, d\underset{\sim}{S}$$

$$= \mu_0 (I_1 \pm I_2)$$

$$= \mu_0 \sum_i^m I_i$$

$$I = \oint_S \underset{\sim}{J}\, dS$$

$$\boxed{\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J}} \quad \text{A.t.}$$

$$\nabla \underset{\sim}{E} = \frac{1}{\varepsilon_0} \rho \quad \Big/ \frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial t} (\nabla \underset{\sim}{E}) = \frac{1}{\varepsilon_0} \frac{\partial}{\partial t} \rho = -\frac{1}{\varepsilon_0} \nabla \underset{\sim}{J}$$

$$\nabla \left(\frac{\partial}{\partial t} \underset{\sim}{E} \right) = \nabla \left(-\frac{1}{\varepsilon_0} \underset{\sim}{J} \right)$$

- struja nastaje zbog vrem. promjenjivosti el. polja

$$\tilde{\underset{\sim}{J}} \rightarrow \underset{\sim}{J} + \underset{\sim}{J}_{\text{pomaka}} \quad \text{- Maxwell}$$

$$\underset{\sim}{J} \sim \frac{\partial}{\partial t} (\underset{\sim}{E})$$

$$\oint \underset{\sim}{B}\, d\underset{\sim}{l} = \mu_0 \int \underset{\sim}{\tilde{J}}\, d\underset{\sim}{S} = \mu_0 \int \underset{\sim}{J}\, d\underset{\sim}{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \underset{\sim}{E}\, d\underset{\sim}{S}$$

$$\Downarrow$$

$$\underset{\sim}{J} + \underset{\sim}{J}_{pomaka}$$

$$\oint_{\partial S} \underset{\sim}{B}\, d\underset{\sim}{l} = \mu_0 \oint_S \underset{\sim}{J}\, d\underset{\sim}{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \oint_S \underset{\sim}{E}\, d\underset{\sim}{S}$$ // - integralni oblik

Amper - Maxwellov teorem

ili 3. Maxwellova jednadžba

$$\oint_{\partial S} \underset{\sim}{B}\, d\underset{\sim}{l} = \mu_0 \oint_{\partial S} \underset{\sim}{J}\, d\underset{\sim}{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int_{\partial S} \underset{\sim}{E}\, d\underset{\sim}{S}$$

$$\oint_S (\nabla \times B)\, d\underset{\sim}{S}$$

$$\boxed{\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \underset{\sim}{E}}$$ | - dif. oblik

- Vremenskom promjenom E dobijemo $\underset{\sim}{B}$

Faraday-ev zakon EM indukcije (4. Maxwell-ova jdba)

$F_{mL} = q\, \underset{\sim}{v} \times \underset{\sim}{B}$

$E = \underset{\sim}{v} \times \underset{\sim}{B}$

$W = \oint_{\partial S} \underset{\sim}{F}\, d\underset{\sim}{l} = \oint_{\partial S} q\underset{\sim}{E}\, d\underset{\sim}{l} = q \oint_{\partial S} \underset{\sim}{v} \times \underset{\sim}{B}\, d\underset{\sim}{l}$

$\frac{W}{q} = \oint \underset{\sim}{E}\, d\underset{\sim}{l}$ - ind. napon (rad po naboju)

- ind. EMS

$EMS_i = \oint \underset{\sim}{v} \times \underset{\sim}{B}\, dl$

$= \oint \nabla \times \underset{\sim}{E}\, d\underset{\sim}{S}$ → Stokes $\oint_{\partial S} \underset{\sim}{A}\, d\underset{\sim}{l} = \int_{\partial S} \nabla \times \underset{\sim}{A}\, d\underset{\sim}{S}$

$= E \cdot L$

$= v \cdot B \cdot L$

$= \frac{dx}{dt} B L$

$= \frac{d}{dt}(XLB) = \frac{d}{dt}\overset{S}{(XL)}B = \frac{d}{dt}(BS)$ $\quad BS = -\underset{\sim}{B} \cdot \underset{\sim}{S}$

$= -\frac{d}{dt}(\underset{\sim}{B} \cdot \underset{\sim}{S}) = \frac{-d\phi_M}{dt}$

$\boxed{\nabla \times \underset{\sim}{E} = \frac{-\partial}{\partial t} \underset{\sim}{B}}$

⇓

$\oint_S \nabla \times \underset{\sim}{E}\, d\underset{\sim}{S} = -\frac{d}{dt}\phi_M = \frac{-\partial}{\partial t} \oint_S \underset{\sim}{B}\, d\underset{\sim}{S}$

$\boxed{\oint_S \nabla \times \underset{\sim}{E}\, d\underset{\sim}{S} = -\frac{\partial}{\partial t} \oint_S \underset{\sim}{B}\, d\underset{\sim}{S}}$ - 4. Maxwellova jdba

EM valovi

$\rho = 0$, $\underset{\sim}{J} = 0$ $\qquad \nabla \times \underset{\sim}{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \underset{\sim}{E}$

$$\nabla \times \Big| \ \nabla \times \underset{\sim}{E} = -\frac{\partial}{\partial t} \underset{\sim}{B}$$

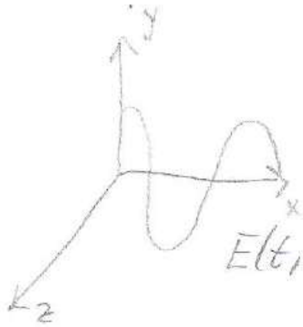
$$\nabla \times (\nabla \times \underset{\sim}{E}) = \nabla(\nabla \cdot \underset{\sim}{E}) - \Delta \underset{\sim}{E} = -\frac{\partial}{\partial t}(\nabla \times \underset{\sim}{B})$$

$$\boxed{\Delta \underset{\sim}{E} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \underset{\sim}{E} = 0} \rightarrow \text{prije} \quad \frac{\partial^2}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} d = 0$$

$$\frac{1}{c^2} = \mu_0 \varepsilon_0$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

EM val se širi brz. svj.



$E(t,x) = \hat{j}\, E_y(t,x)$

$$\Delta \underset{\sim}{B} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \underset{\sim}{B} = 0$$

$$\nabla \times \underset{\sim}{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(t,x) & 0 \end{vmatrix}$$

$$= \vec{i}\left(\partial_y \cdot 0 - \partial_z \cdot E_y\right) - \vec{j}\left(\partial_z \cdot 0 - \partial_x \cdot 0\right) + \vec{k}\left(\partial_x E_y - \partial_y 0\right)$$

$$= \vec{k} \frac{\partial}{\partial x}\left(E_{0y} \cos(\omega t - kx)\right)$$

$$= \vec{k} \cdot k \cdot E_{0y} \sin(\omega t - kx) = -\frac{\partial}{\partial t} \underset{\sim}{B}$$

$$\int d\underset{\sim}{B} = -\vec{k} k\, E_{0y} \int \sin(\omega t - kx)\, dt$$

$$\underset{\sim}{B}(t,x) = \vec{k} \frac{k}{\omega} E_{0y} \cos(\omega t - kx)$$

$\cancel{\dfrac{2\pi \cdot f}{\lambda \cdot 2\pi \cdot f}}$ $\lambda \cdot f = c$

$\underset{\sim}{B}(t, x) = \vec{k} \, \dfrac{E_{0y}}{c} \cdot \cos(\omega t - kx)$

$\boxed{B_{0z} = \dfrac{E_{0y}}{c}}$

$$\boxed{\begin{aligned} &B(t, x) = \vec{k} \, B_{0z} \cos(\omega t - kx) \\ &\underset{\sim}{E}(t, x) = \vec{j} \, E_{0y} \cos(\omega t - kx) \end{aligned}}$$

Energija EM vala, Poynting

$dW = \underset{\sim}{F}\, d\underset{\sim}{l}$

L.S.

$d\underset{\sim}{F} = dq\underset{\sim}{E} + dq\underset{\sim}{v}\times\underset{\sim}{B}$

$d\underset{\sim}{l} = \underset{\sim}{v}\,dt \qquad dq = \rho\, dV \qquad \underset{\sim}{J} = \rho\cdot\underset{\sim}{v}$

$dW = dq\,\underset{\sim}{E}\,\underset{\sim}{v}\,dt + dq\,(\underset{\sim}{v}\times\underset{\sim}{B})\,\underset{\sim}{v}\,dt$

$\perp \underset{\sim}{v}\cdot\underset{\sim}{v} = 0$

$$\frac{dW}{dt} = \underset{\sim}{E}\cdot\underset{\sim}{v}\,dq = \underset{\sim}{E}\cdot\underset{\sim}{v}\cdot\rho\, dV = \underset{\sim}{E}\cdot\underset{\sim}{J}\,dV$$

$$\frac{dW}{dt} = \int(\underset{\sim}{E}\cdot\underset{\sim}{J})\,dV \qquad \underset{\sim}{J} \Rightarrow \qquad \nabla\times\underset{\sim}{B} = \mu_0\underset{\sim}{J} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\underset{\sim}{E}$$

$$\underset{\sim}{J} = \frac{1}{\mu_0}\nabla\times B - \varepsilon_0\frac{\partial}{\partial t}\underset{\sim}{E}$$

$$\underset{\sim}{E}\cdot\underset{\sim}{J} = \underset{\sim}{E}\left(\frac{1}{\mu_0}\nabla\times B - \varepsilon_0\frac{\partial}{\partial t}\underset{\sim}{E}\right)$$

$$\nabla(\underset{\sim}{A}\times\underset{\sim}{B}) = \underset{\sim}{B}(\nabla\times\underset{\sim}{A}) + \underset{\sim}{A}(\nabla\times\underset{\sim}{B})$$

$$\underset{\sim}{E}(\nabla\times\underset{\sim}{B}) = -\nabla(\underset{\sim}{E}\times\underset{\sim}{B}) + \underset{\sim}{B}(\nabla\times\underset{\sim}{E}) \xrightarrow{\nabla\times E} -\frac{\partial}{\partial t}\underset{\sim}{B}$$

$$\underset{\sim}{E}\cdot\underset{\sim}{J} = \frac{1}{\mu_0}\cdot\left(-\nabla(\underset{\sim}{E}\times\underset{\sim}{B})\right) - \frac{1}{\mu_0}\underset{\sim}{B}\cdot\frac{\partial}{\partial t}\underset{\sim}{B} - \varepsilon_0\underset{\sim}{E}\frac{\partial}{\partial t}\underset{\sim}{E} \rightarrow \frac{1}{2}\frac{\partial}{\partial t}E^2$$

$$\underset{\sim}{E}\cdot\underset{\sim}{J} = \frac{1}{\mu_0}\nabla(\underset{\sim}{E}\times\underset{\sim}{B}) - \frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0\underset{\sim}{E}^2 + \frac{1}{\mu_0}\underset{\sim}{B}^2\right)$$

$$\frac{dW}{dt} = \int(\underset{\sim}{E}\cdot\underset{\sim}{J})\,dV = \underbrace{\frac{1}{\mu_0}\int\nabla(\underset{\sim}{E}\times\underset{\sim}{B})\,dV}_{\text{Gaussov zakon}} - \frac{\partial}{\partial t}\underbrace{\int\left(\frac{1}{2}\varepsilon_0E^2 + \frac{1}{2}\mu_0B^2\right)dV}_{\text{en. EM polja } w_{EM}}$$

Gauss

$$-\frac{1}{\mu_0}\int(\underset{\sim}{E}\times\underset{\sim}{B})\,d\underset{\sim}{S} \rightarrow \frac{\frac{dW}{dt}}{dS} = \frac{1}{\mu_0}(\underset{\sim}{E}\times\underset{\sim}{B})$$

$\frac{\text{Snaga}}{\text{Površina}}$ = Intenzitet

slovo S

$$\frac{1}{\mu_0}(\underset{\sim}{E}\times\underset{\sim}{B}) = \mathcal{S}$$ → Poyntingov vektor

$$\frac{dW}{dt} = -\int \mathcal{S}\,d\underset{\sim}{S} - \frac{\partial}{\partial t}\int w_{EM}\,dV$$

$$= -\underbrace{\int \nabla\underset{\sim}{\mathcal{S}}\,dV}_{\text{tok en. EM polja}} - \frac{\partial}{\partial t}\underbrace{\int w_{EM}\,dV}_{\text{promjena gustoće en. EM polja}}$$

$$\frac{\partial}{\partial t}\left(W + \int w_{EM}\,dV\right) = -\int \nabla\underset{\sim}{\mathcal{S}}\,dV$$

$$W \downarrow \int w_{MEH}\,dV$$

$$\frac{\partial}{\partial t}\left[\int w_{MEH}\,dV + \int w_{EM}\,dV\right] = -\int \nabla\underset{\sim}{\mathcal{S}}\,dV$$

$$\underbrace{\frac{\partial}{\partial t}(w_{MEH} + w_{EM})}_{\text{En. uk.}} = -\nabla\underbrace{\underset{\sim}{\mathcal{S}}}_{\text{tok EM en.}}$$ / Poyntingov teorem

Poynting - nastavak

$\underset{\sim}{E}(t,x) = \hat{j}\, E_{oy} \cos(\omega t - kx)$

$\underset{\sim}{B}(t,x) = \hat{k}\, B_{oz} \cos(\omega t - kx)$

$$\underset{\sim}{S} = \frac{1}{\mu_0} \underset{\sim}{E} \times \underset{\sim}{B} = \frac{1}{\mu_0} \vec{i}\, \underset{\sim}{E_{oy}}\, \underset{\sim}{B_{oz}} \cos^2(\omega t - kx)$$

$$\underset{\sim}{S} = \vec{i}\, \frac{1}{\mu_0} \frac{E_{oy}^2}{c} \cos^2(\omega t - kx)$$

$$\underset{\sim}{S} = \vec{i} \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_{oy}^2\, \underbrace{\cos^2(\omega t - kx)}$$

$$\langle \cos^2(\omega t - kx) \rangle = \frac{1}{2}$$

$$\langle \underset{\sim}{S} \rangle = \vec{i}\, \frac{\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_{oy}^2}{2}$$

$$w_{EM} = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$= \frac{1}{2} \varepsilon_0 E_{oy}^2 \cos^2(\omega t - kx) + \frac{1}{2\mu_0} B_{oz}^2 \cos^2(\omega t - kx)$$

$$B_{oz}^2 \to \frac{E_{oy}^2}{c^2} = \varepsilon_0 \mu_0 E_{oy}^2$$

$$w_{EM} = \varepsilon_0 E_{oy}^2 \cos^2(\omega t - kx)$$

$$\langle w_{EM} \rangle = \varepsilon_0 E_{oy}^2 \langle \cos^2(\omega t - kx) \rangle$$

$$\langle w_{EM} \rangle = \frac{1}{2} \varepsilon_0 E_{oy}^2$$

$$\langle \underset{\sim}{S} \rangle = \vec{i} \sqrt{\frac{\varepsilon_0}{\mu_0}}\, \frac{E_{oy}^2}{2}$$

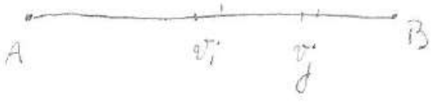
$$= \vec{i}\, \varepsilon_0 \sqrt{\frac{1}{\varepsilon_0 \mu_0}}\, \frac{E_{oy}^2}{2}$$

$$= \vec{i} \cdot \varepsilon_0 \cdot c \cdot \frac{1}{2} \cdot E_{oy}^2$$

$$= \vec{i} \cdot c \cdot \langle w_{EM} \rangle$$

$$\langle \underset{\sim}{S} \rangle = \vec{i} \cdot c \cdot \langle w_{EM} \rangle$$

Fermatov princip



l_{AB} - optički put

$$l_{ij} = v_j t_j = \frac{c}{n_j} \cdot t_j$$

$$t_{AB} = \int_A^B \frac{n}{c} \, dl$$

$$\boxed{\delta t_{AB} = 0}$$

Za slučaj loma

$$\frac{\partial t}{\partial x} = \frac{\partial t}{\partial y} = 0$$

ODP - optička dužina puta → gore l_{AB}

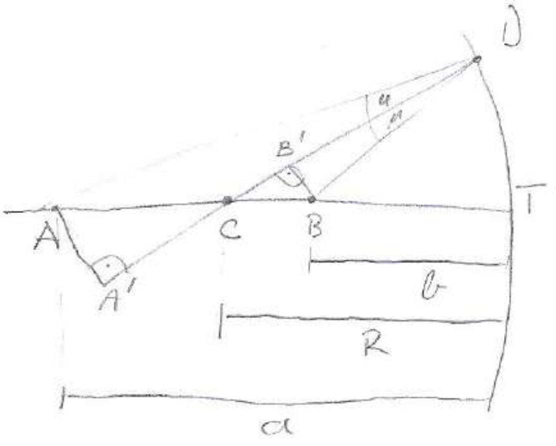
$$d(ODP) = c \cdot dt = \frac{c}{v} \cdot v \cdot dt = n \cdot ds$$

$$ODP = \int n \, ds$$

$$\delta t = 0 \quad \text{ili} \quad \delta (ODP) = 0$$

↓
varijacija

Refleksija svj. na sfernom zrcalu



$\Delta(AA'C) \cong \Delta(BB'C)$

$\Delta(AA'D) \cong \Delta(BB'D)$

$$\frac{\overline{AA'}}{\overline{AC}} = \frac{\overline{BB'}}{\overline{BC}} \qquad \frac{\overline{AA'}}{\overline{AD}} = \frac{\overline{BB'}}{\overline{BD}}$$

$$\frac{\overline{AD}}{\overline{AC}} = \frac{\overline{BD}}{\overline{BC}}$$

Gaussove aproksimacije

$\overline{AD} \simeq \overline{AT} = a$

$\overline{BD} \simeq \overline{BT} = b$

- paraksijalne zrake (kao paralelne za D jako blizu T)

$\Downarrow$

$\overline{AC} = a - R$

$\overline{BC} = R - b$

$$\frac{\overline{AD}}{\overline{AC}} = \frac{\overline{BD}}{\overline{BC}} \Rightarrow \boxed{\frac{a}{a-R} = \frac{b}{R-b}}$$

ili

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{R}}$$

žarište

(1.) Predmetno

$a \to f_a \quad b \to \infty$

$\frac{1}{f_a} + \frac{1}{\infty} = \frac{2}{R} \Rightarrow f_a = \frac{R}{2}$

(2.) Slikovno

$a \to \infty \quad b \to f_b$

$\frac{1}{\infty} + \frac{1}{f_b} = \frac{2}{R} \Rightarrow f_b = \frac{R}{2} = f_a = f$

f - žarišna udaljenost

$f = \overline{FT}$

F - žarište

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{R} \Rightarrow$$

$$\boxed{\frac{f}{a} + \frac{f}{b} = 1} \quad \text{ili} \quad \boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{f}}$$

Intenzitet zvuka u plinovima

$E = E_k + E_p$

$\overline{E_k} = \overline{E_p}$

$\xi(t,x) = \xi_0 \cos(\omega t - kx)$

$\Delta P_m = v \omega \rho \xi_0$

$\Delta E_k = \frac{1}{2} \Delta m \left(v(t) \right)^2$

$= \frac{1}{2} \Delta m \cdot \dot{\xi}(t,x)$

$= \frac{1}{2} \Delta m \cdot \left(-\xi_0 \omega \sin(\omega t - kx) \right)^2$

$\Delta V = \Delta S \cdot \Delta x$

$\Delta m = \rho \cdot \Delta V$

$= \rho \Delta S \Delta x$

$= \frac{1}{2} \xi_0^2 \omega^2 \sin^2(\omega t - kx) \cdot \Delta m$

$= \frac{1}{2} \rho \Delta S \Delta x \, \xi_0^2 \omega^2 \sin^2(\omega t - kx) = \Delta E_p$

$E_{uk} = \rho \Delta S \Delta x \, \xi_0^2 \omega^2 \sin^2(\omega t - kx)$

$\Delta P = \frac{\Delta E}{\Delta t}$

$I = \frac{\Delta P}{\Delta S} = \frac{1}{2} \rho v \xi_0^2 \omega^2 \Big/ \cdot \frac{v \cdot \rho}{v \cdot \rho}$

$I_0 = 10^{-12} \, W/m^2$

$$\underline{I = \frac{\Delta P_m^2}{2 v \rho}}$$

$D = 10 \log \frac{I}{I_0}$