Interferencija valova svjetlosti

$\vec E_1 = E_{10}\sin\omega\left(t-\frac{x_1}{v}\right)\vec j = \left|\frac{1}{v}=\frac{n}{c}\right| = E_{10}\sin\omega\left(t-\frac{nx_1}{c}\right)\vec j$

$\vec E_2 = E_{20}\sin\omega\left(t-\frac{nx_2}{c}\right)\vec j;\qquad \vec E=\vec E_1+\vec E_2;\qquad E_{10}=E_{20}=E_0$

$E_A = E_0\sin\omega\left(t-\frac{nx_1}{c}\right)+E_0\sin\omega\left(t-\frac{nx_2}{c}\right) = \left|\sin x+\sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right| =$

$=2E_0\cos\frac{\omega}{2c}(nx_1-nx_2)\sin\left[\omega t-\frac{\omega n}{2c}(x_1+x_2)\right]$

$\frac{\omega}{c}(nx_1-nx_2)=\varphi \qquad \frac{\omega n}{2c}(x_1+x_2)=a$

$E_A = 2E_0\cos\left(\frac{\varphi}{2}\right)\sin(\omega t-a);\qquad E_{012}=2E_0\cos\left(\frac{\varphi}{2}\right)$

$x_1-x_2=\Delta \rightarrow$ geometrijska razlika putova;

$n(x_1-x_2)=\delta \rightarrow$ optička razlika putova

$\cos\left(\frac{\varphi}{2}\right)=\pm1 \rightarrow \frac{\varphi}{2}=\frac{\omega}{2c}(nx_1-nx_2)=m\pi \quad ,m=0,1,2,3...$

$\cos\left(\frac{\varphi}{2}\right)=0 \rightarrow \frac{\omega}{2c}(nx_1-nx_2)=\frac{2m+1}{2}\pi=\left(m+\frac12\right)\pi$

$\frac{\omega}{2c}=\frac{2\pi f}{2\lambda f}=\frac{\pi}{\lambda};\quad \frac{\pi}{\lambda}(nx_1-nx_2)=m\pi;\quad \boxed{\delta=m\lambda}$ – konstruktivna interf.

$\frac{\omega}{2c}=\frac{\pi}{\lambda};\qquad \frac{\pi}{\lambda}\delta=\left(m+\frac12\right)\pi;\qquad \boxed{\delta=\left(m+\frac12\right)\lambda}$ – destruktivna interf.

$\boxed{\frac{\varphi}{2}=\frac{\pi}{\lambda}\delta}$

Interferencija valova svjetlosti – pomoću fazora

$\omega t+\varphi_1 \qquad \omega t=\frac{n\omega}{c}x_1\qquad \varphi_1=-\frac{n\omega}{c}x_1$

$\omega t+\varphi_2\qquad \omega t=\frac{n\omega}{c}x_2\qquad \varphi_2=-\frac{n\omega}{c}x_2$

$\varnothing=\varphi_2-\varphi_1=\frac{n\omega}{c}(x_1-x_2)=\varphi$

$E_{012}^2=E_{01}^2+E_{02}^2+2E_{01}E_{02}\cos\varphi=\left|E_{01}=E_{02}=E_0\right|=$

$=2E_0^2+2E_0^2\cos\rho=2E_0^2(1+\cos\varphi)=\boxed{4E_0^2\cos^2\frac{\varphi}{2}}$

Youngov pokus – nelokalizirane pruge interferencije

$\delta=\Delta=\sqrt{d^2+\left(y+\frac a2\right)^2}-\sqrt{d^2+\left(y-\frac a2\right)^2}$

$x=\frac{y\pm\frac a2}{d}<<1;\qquad \sqrt{1+x^2}=1+\frac12x^2+....$

Svjetle pruge $\quad n=1;\qquad \delta=\Delta=m\lambda\qquad m=0,1....$

$\delta=d\left(1+\frac12\frac{\left(y+\frac a2\right)^2}{d^2}-1-\frac12\frac{\left(y-\frac a2\right)^2}{d^2}\right)=$

$=\frac{1}{2d}\left[\left(y+\frac a2\right)^2-\left(y-\frac a2\right)^2\right]=\frac{1}{2d}a2y=\frac{ya}{d}\qquad \Delta=\boxed{\frac{ya}{d}=m\lambda}$

Tamne pruge $\quad \delta=\Delta=\left(m+\frac12\right)\lambda\qquad m=0,1,2....$

isto kao i gore .... $\Delta=\boxed{\frac{ya}{d}=\left(m+\frac12\right)\lambda}$

$I=\overline P=\frac12\sqrt{\frac{\varepsilon_0}{\mu_0}}E_{012}^2=\frac12\sqrt{\frac{\varepsilon_0}{\mu_0}}4E_0^2\cos^2\frac{\varphi}{2}=2\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\cos^2\frac{\varphi}{2}=$

$=I_0\cos^2\frac{\varphi}{2}=I_0\cos^2\left(\frac{\pi}{\lambda}\delta\right)=I_0\cos^2\left(\frac{\pi}{\lambda}\frac{ay}{d}\right)$

$I=\frac12\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(E_0^2+E_0^2+2E_0^2\cos\varphi\right)=I_{01}+I_{02}+\frac12\sqrt{\frac{\varepsilon_0}{\mu_0}}\lambda E_0^2\cos\varphi=$

$I=2I_{0p}+2I_{0p}\cos\varphi\qquad I_{0p}\rightarrow$ pojedinačni izvor

$\cos\rho=1\rightarrow I_{max}=4I_0\qquad \cos\rho=-1\rightarrow I_{min}=0$

Interferencija na tankim listićima – lokalizirane pruge interferencije

$\delta=n\overline{AB}+n\overline{BC}-\overline{AD}-\frac{\lambda}{2};\qquad \frac{\sin u}{\sin l}=n$

$\Delta ABE\Rightarrow\cos l=\frac{d}{\overline{AB}};\qquad \overline{AB}=\frac{d}{\cos l}=\overline{BC};$

$\Delta ACB\Rightarrow\sin u=\frac{\overline{AD}}{\overline{AC}}\qquad \Delta A\frac{C}{2}E\Rightarrow\quad\tan l=\frac{\overline{AC}}{2d}$

$\delta=n2\frac{d}{\cos l}-\overline{AC}\sin u-\frac{\lambda}{2}=2n\frac{d}{\cos l}-2d\tan l\sin u-\frac{\lambda}{2}=$

$=2nd\frac{2}{\cos l}-2d\frac{\sin l}{\cos l}n\sin l-\frac{\lambda}{2}\qquad\rightarrow \sin u=n\sin l$

$\delta=2nd\left(\frac{1}{\cos l}-\frac{\sin^2 l}{\cos l}\right)-\frac{\lambda}{2}=\boxed{2nd\cos l-\frac{\lambda}{2}}$

$\cos l=\sqrt{\cos^2 l}=\sqrt{1-\sin^2 l}=\sqrt{1-\frac{\sin^2 u}{n^2}}$

$\delta=2nd\cos l-\frac{\lambda}{2}=2nd\sqrt{1-\frac{\sin^2 u}{n^2}}-\frac{\lambda}{2}=\boxed{2d\sqrt{n^2-\sin^2 n}-\frac{\lambda}{2}}$

Newtonovi kolobari

r-polumjer kolobara $\qquad \delta=2\overline{AB}+(\lambda/2)$

$R^2=r^2+\left(R-\overline{AB}\right)^2=r^2+R^2-2R\cdot\overline{AB}+\overline{AB}^2$

$2\overline{AB}\cdot R+\overline{AB}^2=r^2;\qquad 2\overline{AB}\cdot R\left(1-\frac{\overline{AB}}{2R}\right)=r^2$

$R\approx 1cm;\qquad \overline{AB}\ll R;\qquad \frac{\overline{AB}}{2R}\ll 1$

$2\overline{AB}=\frac{r^2}{R^2};\qquad \delta=2\overline{AB}+\frac{\lambda}{2}=\frac{r^2}{R}+\frac{\lambda}{2}$

Svijetli kolobari: $\quad \delta=m\lambda\qquad \frac{r_m^2}{R}+\frac{\lambda}{2}=m\lambda$

$r_m^2=R\left(m\lambda-\frac{\lambda}{2}\right)\qquad r_m=\sqrt{R(2m-1)\frac{\lambda}{2}}\qquad m=1,2...$

Tamni kolobari

$\delta=(2m+1)\lambda;\qquad \frac{r_m^2}{R}+\frac{\lambda}{2}=\left(m+\frac12\right)\lambda;\qquad r_m^2=R\left(\left(m+\frac12\right)\lambda-\frac{\lambda}{2}\right)$

$r_m=\sqrt{Rm\lambda};\qquad m=0,1,...$

Interferencija N koherentnih izvora

$\varphi=\frac{2\pi}{\lambda}\delta;\qquad \vec E=\vec E_1+\vec E_2+...+\vec E_n$

$\vec E_1=E_0\cos\omega t\,\vec j;\quad \vec E_2=E_0\cos(\omega t+\varphi)\vec j\quad \vec E_n=E_0\cos(\omega t+(n-1)\varphi)\vec j$

$E_1=E_0\cos\omega t\qquad E_2=E_0\cos(\omega t+\varphi)\qquad E_n=E_0\cos(\omega t+(n-1)\varphi)$

$E_1=E_0e^{i\omega t}\qquad E_2=E_0e^{i(\omega t+\varphi)}\qquad E_n=E_0e^{i(\omega t+(n-1)\varphi)}$

$E=E_1+E_2+......+E_n$

$E=E_0e^{i\omega t}+E_0e^{i(\omega t+\rho)}+...+E_0e^{i(\omega t+(n-1)\varphi)}=$

$=E_0e^{i\omega t}\left[1+e^{i\varphi}+e^{i2\varphi}+...+e^{i(n-1)\varphi}\right]=E_0e^{i\omega t}\frac{e^{in\varphi}-1}{e^{i\varphi}-1}$

$E=E_0e^{i\omega t}e^{\frac{in\varphi}{2}}e^{\frac{i\varphi}{2}}\frac{e^{\frac{in\varphi}{2}}-e^{-\frac{in\varphi}{2}}}{e^{\frac{i\varphi}{2}}-e^{-\frac{i\varphi}{2}}}=e^{i\left(\omega t+\frac{n-1}{2}\varphi\right)}E_0\frac{\sin\frac{n}{2}\varphi}{\sin\frac{\varphi}{2}}$

Intenzitet $\rightarrow\quad \boxed{I=I_0\frac{\sin^2\frac{n\rho}{2}}{\sin^2\frac{\rho}{2}}}\qquad\boxed{\rho=\frac{2\pi}{\lambda}\delta}$

Intenzitet glavnih maksimuma

$y=\frac{\varphi}{2};\qquad I=I_0\frac{\sin^2 ny}{\sin^2 y}\qquad \varphi=0;\qquad y\to0$

$\lim_{y\to0}\frac{\sin^2 ny}{\sin^2 y}=\lim_{y\to0}\frac{2n\sin ny\cos ny}{2\sin y\cos y}=$

$\lim_{y\to0}\frac{n^2\cos ny\cos ny-n^2\sin ny\sin ny}{\cos^2y-\sin^2y}=\lim_{y\to0}\frac{n^2\left(\overbrace{\cos^2 ny}^{1}-\overbrace{\sin^2 ny}^{0}\right)}{\underbrace{\cos^2y}_{1}-\underbrace{\sin^2 y}_{0}}$

$I=n^2I_0\qquad \sin^2 ny=0;\qquad ny=k\pi\qquad ,k=\pm1,\pm2,....$

$n=6\qquad 6y=6\frac{\varphi}{2}=k\pi;\qquad 3\varphi=k\pi;\qquad \varphi\frac{k}{3}\pi=\frac{\pi}{3},\frac{2\pi}{3},\pi,....$

Difrakcija ili ogib na pukotini

Fraunhoferova difrakcija:

$E=E_0\frac{\sin(n\cdot\varphi/2)}{\sin(\varphi/2)}\qquad \varphi=\frac{2\pi}{\lambda}\delta$

$\delta=n\Delta;\qquad n=1\to\delta=\Delta;\qquad n=m\to\Delta=\frac{d\sin\alpha}{m}$

$E(\alpha)=E_0\frac{\sin\frac{m\frac{2\pi}{\lambda}\frac{d\sin\alpha}{m}}{2}}{\sin\frac{\frac{2\pi}{\lambda}\frac{d\sin\alpha}{m}}{2}}=E_0\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\sin\frac{\pi d\sin\alpha}{\lambda m}}$

$\alpha=0: E(0)=mE_0\Rightarrow E_0=\frac{E(0)}{m}$

$\boxed{E(\alpha)=\frac{E(0)}{m}\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\sin\frac{\pi d\sin\alpha}{\lambda m}}}$ – aprox. $\qquad E(\alpha)=\lim_{m\to\infty}\frac{E(0)}{m}\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\sin\frac{\pi d\sin\alpha}{\lambda m}}$

$\lim_{m\to\infty}m\sin\frac{\pi d\sin\alpha}{\lambda m}=\frac{\pi d\sin\alpha}{\lambda}\lim_{m\to\infty}\frac{\sin\frac{\pi d\sin\alpha}{\lambda m}}{\frac{\pi d\sin\alpha}{\lambda m}}=$

$\left|x=\frac{\pi d\sin\alpha}{\lambda m};m\to\infty,x\to0\right|=\frac{\pi d\sin\alpha}{\lambda}\lim_{m\to\infty}\frac{\sin x}{x}=\boxed{\frac{\pi d\sin\alpha}{\lambda}}$

$E(\alpha)=E(0)\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\frac{\pi d\sin\alpha}{\lambda}}\qquad z=\frac{\pi d\sin\alpha}{\lambda}$

Električno polje vala koji se difraktira pod kutem $\alpha\to$

$\boxed{E(\alpha)=E_0\frac{\sin z}{z}}\qquad I_0\to$ intenzitet centralne (svjetle) $\quad\boxed{I=I_0\frac{\sin^2 z}{z^2}}$

Maksimum intenziteta svjetlosti (svjetle pruge): $\qquad\frac{dI}{dz}=0$

$\frac{d}{dz}I_0\frac{\sin^2 z}{z^2}=I_0\frac{2z^2\sin z\cos z-2z\sin^2 z}{z^4}=0$

$z\sin z\cos z-\sin^2 z=0;\qquad \tan z=z$

$z_1\approx\pm\frac{3\pi}{2},\ z_2\approx\pm\frac{5\pi}{2},.......\qquad z_m\approx\pm\left(m+\frac12\right)\pi$

$\frac{\pi d\sin\alpha}{\lambda}=\pm\left(m+\frac12\right)\pi\qquad \boxed{d\sin\alpha=\pm\left(m+\frac12\right)\lambda}\qquad ,m=1,2,3....$

Minimum intenziteta svjetlosti (tamne pruge)

$\sin z=0,\qquad z\neq0\qquad z=m\pi,\qquad m=\pm1,\pm2,....$

$\frac{\pi d\sin\alpha}{\lambda}=m\pi\qquad \boxed{d\sin\alpha=m\lambda}$

(Fraunhoferova) difrakcija na dvije pukotine

kombinacija difrakcije na jednoj pukotini i interferencije na dvije pukotine

$E=E_0\frac{\sin(n\cdot\varphi/2)}{\sin(\varphi/2)};\qquad\varphi=\frac{2\pi}{\lambda}\delta;\qquad\delta=\Delta=D\sin\alpha$

$E_D(\alpha)=E(0)\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\frac{\pi d\sin\alpha}{\lambda}}\frac{\sin\frac{2\frac{2\pi}{\lambda}D\sin\alpha}{2}}{\sin\frac{\frac{2\pi}{\lambda}D\sin\alpha}{2}}=$

$=\boxed{E(0)\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\frac{\pi d\sin\alpha}{\lambda}}\frac{\sin\frac{2\pi D\sin\alpha}{\lambda}}{\sin\frac{\pi D\sin\alpha}{\lambda}}}$

$I_D(\alpha)=I_0\left[\frac{\sin\frac{\pi d\sin\alpha}{\lambda}}{\frac{\pi d\sin\alpha}{\lambda}}\right]^2\left[\frac{\sin\frac{2\pi D\sin\alpha}{\lambda}}{\sin\frac{\pi D\sin\alpha}{\lambda}}\right]^2$ $\quad$ d-širina pojedine pukotine

D-razmak između dviju pukotina

Optička rešetka (N pukotina)

$y=\frac{\pi d}{\lambda}\sin\alpha\qquad z=\frac{\pi d}{\lambda}\sin\alpha$

$I_D(\alpha)=I_0\left[\frac{\sin y}{y}\right]^2\left[\frac{\sin Nz}{z}\right]^2$

Difrakcija na jednoj pukotini:

$d\sin\alpha=m\lambda\qquad m=\pm1,\pm2,\pm3.....$

1. jako uske pukotine: $d<<\lambda\ \rightarrow\ \frac{\lambda}{d}>>1$
2. pukotine reda veličine $\lambda\ \rightarrow$ dolazi do ogiba na pukotini
3. jako široke pukotine: $d>>\lambda\ \rightarrow\ \frac{\lambda}{d}<<1\ \rightarrow$ 1. maks. je jako širok(?)

Glavni maksimumi (z=0 $\rightarrow$ centralni maks, isključimo taj slučaj)

$\sin z=0\qquad z=m\pi\qquad ,m=0,\pm1...$

$\frac{\pi D}{\lambda}\sin\alpha=m\pi\qquad D\sin\alpha=m\lambda$

m=1 $\rightarrow$ (spektar) maksimum prvog reda

Glavni minimumi - doprinose oba člana (difrakcija i interferencija)

$d\sin\alpha=m\lambda\qquad ,m=\pm1,\pm2,\pm3\qquad\rightarrow$ od difrakcije

$\sin Nz=0\qquad\rightarrow$ od onterferencije na N pukotina

$Nz=m\pi\qquad ,m\neq\pm N,\pm2N...\qquad z\neq0$ (centralni maks)

$\frac{N\pi D}{\lambda}\sin\alpha=m\pi$

$D\sin\alpha=\pm\left(k+\frac{l}{N}\right)\lambda\qquad k=0,1,2...\qquad l=1,\ 2,\ ...,\ N-1$

Moć razlučivanja

Moć razlučivanja-kolika je minimalna razlika dviju valnih duljina koje rešetka može razlučiti

maksimum svjetlosti $\lambda+\Delta\lambda$ se mora nalaziti na mjestu na kojem se nalazi minimum svjetlosti λ

$\begin{cases}D\sin\alpha=\left(m+\frac1N\right)\lambda & k=m\qquad l=1\\ D\sin\alpha=m(\lambda+\Delta\lambda)\end{cases}$

$m\lambda+m\Delta\lambda=m\lambda+\frac{\lambda}{N}\qquad\boxed{\frac{\lambda}{\Delta\lambda}=mN}$

Polarizacija svjetlosti

u_B – Brewsterov kut $\qquad u_B+l=\frac{\pi}{2}\qquad\frac{\sin u_B}{\sin l}=n$

$\frac{\sin u_B}{\sin\left(\frac{\pi}{2}-u_B\right)}=\frac{\sin u_B}{\cos u_B}=n\qquad\boxed{\tan u_B=n}\qquad\boxed{\tan u_B=\frac{n_2}{n_1}}$

Kvantna priroda svjetlosti

Kvantna priroda svjetlosti: $\qquad$ Kirchhoffov zakon: $\frac{\varepsilon(\lambda,T)}{\alpha(\lambda,T)}=f(\lambda,T)$

Stefan-Boltzmannov zakon:

$I=\sigma T^4\qquad p=e\sigma ST^4\qquad e$ – koef. emisije (za relano tijelo)

Hertzov oscilator:

$f(\lambda,T)=\frac{2\pi e}{\lambda^4}\overline{E}\qquad \overline{E}=\frac12kT+\frac12\text{kT=kT}\qquad E_n=nh\upsilon$

$\overline{E}=\frac{\text{ukupna en.}}{\text{broj oscil.}}=\frac{N_1h\upsilon+N_2h\upsilon+...=\sum_{n=1}^{\infty}N_nnh\upsilon}{N_0+N_1+...=\sum_{n=0}^{\infty}N_n}=\frac{\sum_{n=1}^{\infty}N_nnh\upsilon}{\sum_{n=0}^{\infty}N_n}$

Maxwell-Boltzmannova raspodjela

$P(E)=Ae^{-\frac{E}{kT}}\qquad E_n=nh\upsilon\qquad P(E_n)=\frac{N_n}{N}\qquad N_n=P(E_n)N=ANe^{-\frac{nh\upsilon}{kT}}$

$N_0=AN\qquad N_n=N_0e^{-\frac{nh\upsilon}{kT}}$

$\overline{E}=\frac{\sum_{n=1}^{\infty}N_0e^{-\frac{nh\upsilon}{kT}}nh\upsilon}{\sum_{n=0}^{\infty}N_0e^{-\frac{nh\upsilon}{kT}}}=\frac{N_0h\upsilon\sum_{n=1}^{\infty}ne^{-\frac{nh\upsilon}{kT}}}{N_0\sum_{n=0}^{\infty}e^{-\frac{nh\upsilon}{kT}}}\qquad x=e^{-\frac{nh\upsilon}{kT}}$

$\overline{E}=\frac{h\upsilon\left(x+2x^2+3x^3+...\right)}{1+x+x^2+...}=\frac{h\upsilon x\left(1+2x+3x^2+...\right)}{1+x+x^2+...}=h\upsilon x\frac{\frac{1}{(1-x)^2}}{\frac{1}{1-x}}$

$=h\upsilon\frac{x}{1-x}=h\upsilon\frac{e^{-\frac{h\upsilon}{kT}}}{1-e^{-\frac{h\upsilon}{kT}}}=\frac{h\upsilon}{e^{\frac{h\upsilon}{kT}}-1}$

Plankova jednadžba (za zračenje crnog tijela):

$f(\lambda,T)=\frac{2\pi c}{\lambda^4}\frac{h\frac{c}{\lambda}}{e^{\frac{hc}{\lambda kT}}-1}=\boxed{\frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}}$

$f(\upsilon,T)=\frac{2\pi h\upsilon^5}{c^3}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\qquad h\upsilon>>kt\ \rightarrow\ \overline{E}=h\upsilon e^{-\frac{h\upsilon}{kT}}$

$\qquad h\upsilon<<kt\ \rightarrow\ \overline{E}=kT$

$I=\int_0^{\infty}f(\lambda,T)d\lambda=\sigma T^4\qquad\frac{df}{d\lambda}=0\Rightarrow\quad\lambda_mT=\text{konst.}\ \rightarrow$ Wienov zakon

Fotoelektrični efekt

$EU_Z=\frac{mv_{\max}^2}{2}\qquad \text{U}_\text{Z}\rightarrow$ zaustavni napon

$\boxed{h\upsilon=W_i+\frac{mv_{\max}^2}{2}}\qquad\rightarrow\qquad h\upsilon>>W_i+\frac{mv_{\max}^2}{2}$

$\upsilon_g\rightarrow$ granična frek. $\qquad W_i\rightarrow$ izlazni rad

$\upsilon=\upsilon_g\ \rightarrow\ h\upsilon=W_i$

$\upsilon>\upsilon_g\ \rightarrow\ E_K>0$

Comptonov efekt

$E=h\upsilon=\frac{hc}{\lambda}\qquad P=\frac{E}{c}=\frac{h\upsilon}{c}=\frac{n}{\lambda}\quad\rightarrow$ Količina gibanja

$\lambda\rightarrow\lambda'\qquad\lambda'-\lambda=\Delta\lambda\qquad P=\frac{h}{\lambda}\rightarrow P'=\frac{h}{\lambda'}$

$P\rightarrow$ foton prije sudara $\qquad P'\rightarrow$ foton poslije sudara

$E_e=mc^2\qquad\rightarrow$ elektron prije sudara

$E_e'^2=m^2c^4+Pe^2c^2\qquad\rightarrow$ elektron poslije sudara

1) ZOE $\rightarrow\ h\frac{c}{\lambda}+mc^2=h\frac{c}{\lambda'}+E_e\qquad$ ZOKG $\rightarrow\vec P=\vec P'+\vec P_e$

$1)/c=2)\ \frac{h}{\lambda}+mc=\frac{h}{\lambda'}+\frac{E_e}{c}\qquad \vec P-\vec P_r=\vec P_e\,/.^2$

$P_e^2=P^2+P'^2-2\vec P\vec P'=\left(\frac{h}{\lambda}\right)^2+\left(\frac{h}{\lambda'}\right)^2-2\frac{h^2}{\lambda\lambda'}\cos\theta$

$2)^2\ \left(\frac{h}{\lambda}-\frac{h}{\lambda'}\right)^2+m^2c^2+2mc\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\right)=\frac{E_e^2}{c^2}=m^2c^2+P_e^2$

$\left(\frac{h}{\lambda}\right)^2+\left(\frac{h}{\lambda'}\right)^2-2\frac{h^2}{\lambda\lambda'}+2mch\frac{\lambda'-\lambda}{\lambda\lambda'}=\left(\frac{h}{\lambda}\right)^2+\left(\frac{h}{\lambda'}\right)^2-2\frac{h^2}{\lambda\lambda'}\cos\theta$

$mc\Delta\lambda=h-h\cos\theta=h(1-\cos\theta)$

$\Delta\lambda=\frac{h}{mc}(1-\cos\theta)=\frac{2h}{mc}\sin^2\frac{\theta}{2}\qquad\rightarrow$ Comptonova relacija

$\frac{h}{mc}\ \rightarrow$ Comptonova valna duljina elektrona

Struktura atoma

Struktura atoma $\qquad$ generalizirana Balmerova formula $\rightarrow$

$\frac{1}{\lambda}=R\left(\frac{1}{n^2}-\frac{1}{m^2}\right)\qquad m>n,\ m=n+1,n+2...$

$R=1,09\cdot10^7m^{-1}\qquad\rightarrow$ Rydbergova konst.

Bohrov model atoma

1. postulat - elektron se može gibati oko jezgre samo određenim, dozvoljenim kružnim stazama. Elektron pri tom gibanju ne zrači

2. postulat - dozvoljena stanja su ona za koja je iznos kutne veličine gibanja jednak višelratniku Planckove konst. podijeljene sa 2π

$L_n=n\frac{h}{2\pi}=n\hbar\qquad n=1,2,3...\ \rightarrow$ (glavni) kvantni broj

$\hbar\rightarrow$ Reducirana planckova konstanta $\qquad 1)\ L_n=m_er_nv_n=n\hbar$

3. postulat - kada elektron skoči s više staze energije (E_m) na nižu stazu energije (E_n) onda elektron (atom) emitira, zrači foton čija je energija $h\upsilon$ jendaka razlici energija višeg i nižeg stacionarnog stanja ($E_m-E_n=h\upsilon$)

$2)\frac{m_ev_n^2}{r_n}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}\qquad\rightarrow1)=3):v_n=\frac{n\hbar}{m_er_n}$

$3\rightarrow2:\frac{m_e}{r_n}\frac{n^2\hbar^2}{m_e^2r_n^2}=\frac{e^2}{4\pi\varepsilon_0r_n^2}\qquad r_n=\frac{4\pi\varepsilon_0n^2}{m_ee^2}\hbar^2=\left|\hbar=\frac{4}{2\pi}\right|=\frac{4\pi\varepsilon_0n^2}{m_ee^2}\frac{4^2}{4\pi^2}$

$4)\ \boxed{r_n=n^2\frac{\varepsilon_0h^2}{\pi m_ee^2}}\qquad\rightarrow$ polumjer dozvoljenih staza

$4\rightarrow3:5)\ \boxed{v_n}=\frac{nh}{2\pi m_e}\frac{\pi m_ee^2}{n^2\varepsilon_0h^2}=\boxed{\frac1n\frac{e^2}{2\varepsilon_0h}}$

$6)\ E_0=\frac{m_ev_n^2}{2}-\frac{e^2}{4\pi\varepsilon_0r_n}\qquad$ Energije atacionarnih stanja $\rightarrow$

$4,5\rightarrow6)\ \boxed{E_n}=\frac{m_e}{2}\frac{e^4}{n^24\varepsilon_0^2h^2}-\frac{e^2}{4\pi\varepsilon_0}\frac{\pi m_ee^2}{n^2\varepsilon_0h^2}=\boxed{-\frac{1}{n^2}\frac{m_ee^4}{8\varepsilon_0^2h^2}}\ \ 7)$

$8)\ E_m-E_n=h\upsilon\qquad$ Iz Bohrovog modela Š

$7\rightarrow8)\frac{-1}{m_e^2}\frac{m_ee^4}{8\varepsilon_0^2h^2}+\frac{1}{m_e^2}\frac{m_ee^4}{8\varepsilon_0^2h^2}=\boxed{h\upsilon=\frac{m_ee^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n^2}-\frac{1}{m^2}\right)}\qquad m\neq m_e$

Iz eksperimenta $\rightarrow\qquad\upsilon=cR\left(\frac{1}{2^2}-\frac{1}{m^2}\right);$

Rydbergova konst. $\rightarrow R=\frac{me^4}{8\varepsilon_0^2h^3c}=1,03*10^{-7}m^{-1}$

Zakon radioaktivnog raspada

aktivnost: $\left(\frac{-dN}{dt}\right)\qquad\frac{-dN}{dt}=\lambda N\qquad\frac{dN}{dt}=-\lambda N.\cdot\int$

$\lambda\rightarrow$ konstanta raspada karakteristična za pojedinu jezgru

$\int_{N_0}^{N}\frac{dN}{dt}=-\int_0^t\lambda dt\qquad\ln N\Big|_{N_0}^{N}=-\lambda t\qquad\ln\frac{N}{N_0}=-\lambda t$

$\boxed{N=N_0e^{-\lambda t}}\qquad\rightarrow$ zakon radioaktivnog raspada

$\left(\frac{-dN}{dt}\right)=\lambda N=\lambda N_0e^{-\lambda t}\qquad\left(\frac{-dN}{dt}\right)_{t=0}=\lambda N_0=\left(\frac{-dN}{dt}\right)_0$ -u poč. tren.

$\boxed{\left(\frac{-dN}{dt}\right)=\left(\frac{-dN}{dt}\right)_0e^{-\lambda t}}$ - aktivna promjena u vremenu

Vrijeme poluraspada/poluživota - vrem. interval u kojem se raspadne polovica radioaktivne jezgre:

$t=T_{1/2}\qquad N=\frac{N_0}{2}\qquad\frac{N_0}{2}=N_0e^{-\lambda T_{1/2}}\ \ ....\ \ T_{1/2}=\frac{\ln2}{\lambda}$

Srednje vrijeme života:

$\tau=\frac{\int_{N_0}^{0}tdN}{\int_{N_0}^{0}dN}=\frac{-\int_0^{\infty}tN_0\lambda e^{-\lambda t}dt}{-N_0}=\left|\begin{matrix}N=N_0e^{-\lambda t}\\dN=-N_0\lambda e^{-\lambda t}dt\end{matrix}\right|=-\lambda\frac{d}{d\lambda}\int_0^{\infty}e^{-\lambda t}dt=$

$=-\lambda\frac{d}{d\lambda}\left(-\frac{1}{\lambda}e^{-\lambda t}\Big|_0^{\infty}\right)=-\lambda\frac{d}{d\lambda}\frac{1}{\lambda}=-\lambda\left(\frac{-1}{\lambda^2}\right)=\boxed{\frac{1}{\lambda}=\tau}$

$N=\frac{N_0}{e}\qquad e\rightarrow$ baza prirodnog algoritma

Geometrijska optika

1. Zakon pravocrtnog širenja svjetlosti
2. Zakon neovisnosti svijetlosnih svojstava –ako se dva sv. snopa presjecaju, jedan na drugi ne utječu i svaki se širi tako kao da onaj drugi ne postoji
3. Zakon refleksije svijetlosti – kut upada jednak je kutu refleksije
4. Zakon loma svijetlosti

$n=\frac{c}{v}=>\quad\text{indeks.loma}=\frac{\text{brzina svijetlosti u vakuumu}}{\text{brzina u određenom sredstvu}}$

$n_1\sin\Theta_1=n_2\sin\Theta_2\qquad\frac{\sin\Theta_1}{\sin\Theta_2}=\frac{n_2}{n_1}$

Fermatov princip

$t_{AB}=\int_A^B\frac{n}{c}dl\qquad\delta t_{AB}=0=>1.$ varijacija vremena $\qquad v=\frac{c}{n}$

$\int_A^B ndl=>$ optički put $\qquad\rightarrow u$ homogenom sredstvu $\quad n=const.$

Zakon refleksije:

$t_{AB}=t_{AC}+t_{BC}=\frac{\overline{AC}}{v}+\frac{\overline{CB}}{v}=\frac1v\left(\sqrt{a^2+x^2}+\sqrt{b^2+(d-x)^2}\right)$

$\frac{dt_{AB}}{dx}=0\qquad..........\qquad\frac{x}{\sqrt{a^2+x^2}}=\frac{d-x}{\sqrt{b^2+(d-x)^2}}\qquad...\qquad\frac{\sin\alpha}{\sin\beta}=\frac{n_2}{n_1}$

Zakon loma:

$n_1=konst.\qquad n_2=konst.\qquad v_1=\frac{c}{n_1}\qquad v_2=\frac{c}{n_2}$

$t_{AB}=t_{AC}+t_{BC}=\frac{\overline{AC}}{v_1}+\frac{\overline{CB}}{v_2}=\frac{1}{v_1}\sqrt{a^2+x^2}+\frac{1}{v_2}\sqrt{b^2+(d-x)^2}$

$\frac{dt_{AB}}{dx}=0\qquad..............\qquad\frac{1}{v_1}\underbrace{\frac{x}{\sqrt{a^2+x^2}}}_{\sin\alpha}=\frac{1}{v_2}\underbrace{\frac{dx}{\sqrt{b^2+(d-x)^2}}}_{\sin\beta}\qquad....\qquad\frac{\sin\alpha}{\sin\beta}=\frac{n_2}{n_1}$

Sferno zrcalo

$\frac{\overline{AC}}{\overline{AA'}}=\frac{\overline{BC}}{\overline{BB'}}\qquad\frac{\overline{AD}}{\overline{AA'}}=\frac{\overline{BD}}{\overline{BB'}}\qquad\Rightarrow\qquad\frac{\overline{AC}}{\overline{AD}}=\frac{\overline{BC}}{\overline{BD'}}$

gauss. aprox. $D->T.pa.\overline{AD}\cong\overline{AT}$

$\frac{\overline{AC}}{\overline{AT}}=\frac{\overline{BC}}{\overline{BT}}$

$\frac{a-R}{a}=\frac{R-b}{b}\qquad\frac1a+\frac1b=\frac2R=\frac1f\qquad\rightarrow$ jedn. sfernog zrcala

Ravno zrcalo- $R=\infty\qquad\frac1a+\frac1b=0\qquad=>\qquad a=-b$

Povećanje sfernog zrcala $\qquad|m|=\frac{y'}{y}$

$\frac{y}{a}=\frac{y'}{b}\qquad\frac{y'}{y}=\frac{a}{b}=|m|\qquad m=-\frac{b}{a}$

$\uparrow-predmet,\downarrow-slika\qquad m<0$

$\uparrow-predmetm,\uparrow-slika\qquad m>0$

Sferni dioptar

$u=\varphi+\Theta$

$\Theta=\varphi'+l=>l=\Theta-\varphi'$

$\frac{u}{l}=\frac{\varphi+\Theta}{\Theta-\varphi'}$

$\frac{\sin u}{\sin l}=\frac{n_2}{n_1}\qquad\sin u=u\qquad\sin l=l$

$\frac{u}{l}=\frac{\varphi+\Theta}{\Theta-\varphi'}=\frac{n_2}{n_1}$

$tg\varphi=\frac{h}{a}\qquad tg\varphi'=\frac{h}{b}\qquad tg\Theta=\frac{h}{r}$

$\varphi=\frac{h}{a}\qquad\varphi'=\frac{h}{b}\qquad\Theta=\frac{h}{r}$

$\frac{n_1}{a}+\frac{n_2}{b}=\frac{n_2-n_1}{R}\qquad\rightarrow$ jedn. sfernog dioptra

Ravni dioptar: $\quad\text{R}\rightarrow\infty$

$\frac{n_1}{a}+\frac{n_2}{b}=0$

Tanka leća

$|a'|=|b'|\qquad a'=-b'$

Prvi sferni dioptar:

$\frac{n_1}{a}+\frac{n_2}{b'}=\frac{n_2-n_1}{R_1}\qquad(1)$

Drugi sferni dioptar:

$n_2->n_3\qquad a'=-b'$

$\frac{n_2}{a'}+\frac{n_3}{b}=\frac{n_3-n_2}{R_2}\qquad-\frac{n_2}{b'}+\frac{n_3}{b}=\frac{n_3-n_2}{R_2}\qquad(2)$

$(1)+(2)=\frac{n_1}{a}+\frac{n_3}{b}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$

$F_a->$ predmetno žariste $\qquad\overline{F_aT}=f_a$

$a=f_a\qquad=>b=\infty$

$f_a=\frac{n_1R_1R_2}{(n_2-n_1)R_2+(n_3-n_1)R_1}$

$F_b->$ slikovno žariste $\qquad\overline{F_bT}=f_b$

$a=\infty\qquad=>b=f_b$

$\frac1f=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

jakost leće $\rightarrow J=\frac1f[J]=$ dioptrija

$\frac1a+\frac1b=\frac1f$

$|m|=\frac{y'}{y}$

$\frac{y}{a}=\frac{y'}{b}\qquad\frac{y'}{y}=\frac{a}{b}=|m|\qquad m=-\frac{b}{a}$

$\uparrow-predmet,\downarrow-slika\qquad m<0$

$\uparrow-predmetm,\uparrow-slika\qquad m>0$