FIZ2

Riješeni zadaci (ali oni koji u zbirci zapravo nisu riješeni :D) iz valova i optike

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1. TITRANJE

ZADACI:

1.1 $E = \frac{F/S}{\Delta l/l} => \Delta l = \frac{F}{ES} l = \frac{mg}{Er^2\pi} l = 1.28\,mm$

1.2 $F = \frac{ES\Delta l}{l} = 738.9\,N$

1.3 $B = \frac{F/S}{\Delta V/V} => \frac{\Delta V}{V} = \frac{F/S}{B} = \frac{p}{B} = \frac{345\,MPa}{138\,GPa} = 2.5 \cdot 10^{-3} = 0.25\%$

1.4

1.5 $P = M\omega => M = \frac{P}{\omega} = D\varphi = \frac{\pi R^4 G}{2l}\varphi =>$

$\varphi = \frac{\frac{P}{\omega}}{\frac{\pi R^4 G}{2l}} = \frac{2Pl}{\pi R^4 G\omega} = 0.01187\,rad = 0.01187 \cdot \frac{180°}{\pi} = 0.68°$

1.6 $\frac{\Delta l}{l} = \frac{F}{ES} = 0.028$

1.7 $E_1 = \frac{F_1 l_0}{S\Delta l} = \frac{m_1 g l_0}{(r_v^2 - r_u^2)\pi\Delta l_1} = 3.425714 \cdot 10^6$

Tako se ponovi za ostala mjerenja i uzme se srednja vrijednost.

1.8 $\Delta l = \frac{1}{4}\frac{Fl}{ES} = \frac{1}{4}\frac{mgl}{ES} = 6.505 \cdot 10^{-4}\,m = 0.65\,mm$

1.9 $E = \frac{Fl}{S\Delta l} = 2 \cdot 10^{10} Pa$

1.10 A) $k = k_1 + k_2 = 200\frac{N}{kg} => \omega = \sqrt{\frac{k}{m}} = 6.3\frac{rad}{s}$

B) $k = \frac{k_1 k_2}{k_1 + k_2} = 49.875\,\frac{N}{kg} => \omega = \sqrt{\frac{k}{m}} = 3.2\frac{rad}{s}$

1.11 $A = \frac{"hod\,klipa"}{2} = 27.75mm,\ \omega = \frac{3000}{60} \cdot 2\pi = 100\pi\frac{rad}{s}$

$x(t) = A\cos(\omega t + \varphi) => v(t) = x'(t) = -A\omega\sin(\omega t + \varphi) =>\ v_{max} = A\omega = 8.71\frac{m}{s}$

$a(t) = v'(t) = -A\omega^2\cos(\omega t + \varphi) =>$

$a_{max} = A\omega^2 = 2.74\frac{km}{s} => F_{max} = ma_{max} = 985.97\,N$

1.12 $mg = k\Delta l => \frac{m}{k} = \frac{\Delta l}{g} => T = 2\pi\sqrt{\frac{\Delta l}{g}} = 0.4 => f = \frac{1}{T} = 2.5\,Hz => N = f \cdot 60 = 150$

1.13 $k = \frac{SE}{l}$ => Ako se l skrati 4 puta, k se poveća 4 puta: $k_1 = 4k \; i \; k_2 = \frac{4}{3}k \Rightarrow$

$$k_{par} = \frac{16}{3}k \Rightarrow T_{par} = 2\pi\sqrt{\frac{m}{k_{par}}}$$

$$T_{neprerezana} = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_{par}}{T_{neprerezana}} = \sqrt{\frac{k}{k_{par}}} = \sqrt{\frac{k}{\frac{16}{3}k}} = \sqrt{\frac{3}{16}} = 0.433$$

1.14 $mg = k_1\Delta x_1 = k_2\Delta x_2 \Rightarrow k_2 = 0.8k_1$

$k_{ser} = \frac{k_1k_2}{k_1+k_2} = 0.444k_1 \Rightarrow T = 2\pi\sqrt{\frac{k_1\Delta x_1}{g\cdot 0.444k_1}} = 0.602\ s$

1.16 $x(t) = Asin(\omega t + \varphi), v(t) = A\omega\cos(\omega t + \varphi)$

$6 = Asin\varphi \; i \; 5\pi = A\omega cos\varphi \Rightarrow tg\varphi = \frac{6}{5} \Rightarrow \varphi = 50.19°$

$$A = \frac{6}{sin\varphi} = 7.81\ cm$$

1.17 $T_1 = 2\pi\sqrt{\frac{M}{k}}, T_2 = 2\pi\sqrt{\frac{M+m}{k}}$

$Mg = k\Delta l_1$

$(M+m)g = k\Delta l_2$

$$\Delta l_2 - \Delta l_1 = \frac{(M+m)g}{k} - \frac{Mg}{k} = \frac{g}{4\pi^2}\left(T_2{}^2 - T_1{}^2\right) = 0.027\ m = 2.7\ cm$$

1.19 $k_{ser} = \frac{k^2}{2k} = \frac{k}{2} \Rightarrow k_{par} = 2k$

$$T_{ser} = 2\pi\sqrt{\frac{m}{k_{ser}}}$$

$$T_{par} = 2\pi\sqrt{\frac{m}{k_{par}}}$$

$$\frac{T_{ser}}{T_{par}} = \sqrt{\frac{2k}{\frac{k}{2}}} = \sqrt{4} = \frac{2}{1}$$

1.21 $T = 2\pi\sqrt{\frac{m}{k}} = 0.31$

$x(t) = Asin(\omega t)$

$\frac{A}{2} = Asin(\omega t) => t = \frac{\pi}{6}\sqrt{\frac{m}{k}} = 0.026$

$T_{UK} = \frac{T}{2} + 2t = 0.207\ s$

1.22 $F(x) = -G\frac{mM(x)}{x^2} = \left|\frac{M(x)}{V(x)} = \rho = 5515.3\frac{kg}{m^3}; V(x) = \frac{4}{3}x^3\pi\right| = -\frac{4}{3}Gm\rho\pi x = -kx$

$m\ddot{x} = -kx$

$m\ddot{x} + kx = 0$

$\omega^2 = \frac{k}{m} = \frac{\frac{4}{3}Gm\rho\pi}{m} = \frac{4}{3}G\rho\pi => \omega = 1.24 \cdot 10^{-3} => T = \frac{2\pi}{\omega} = 5060.465\ s = 84.34\ min$

1.23 $F_t = F_{el} => mg\mu = kA => mg\mu = k, A = 1$

$\frac{m}{k} = \frac{1}{g\mu} => T = 2\pi\sqrt{\frac{1}{g\mu}} => f = \frac{1}{T} = 0.35\ Hz$

1.24 $T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\left(\sqrt{\frac{l_1}{g}} + \sqrt{\frac{l_2}{g}}\right) = 2.28\ s$

1.25

$l_1 = 22 + l_2$

$30 \cdot 2\pi\sqrt{\frac{l_1}{g}} = 36 \cdot 2\pi\sqrt{\frac{l_2}{g}}$

$30^2 l_1 = 36^2 l_2 => l_2 = \frac{30^2}{36^2} l_1$

$l_1 = 22 + \frac{30^2}{36^2} l_1 => l1 = \frac{22}{1 - \frac{30^2}{36^2}} = 72\ cm => l_2 = 50\ cm$

1.27 $mgl(1 - cos\vartheta) = \frac{1}{2}mv^2$

$v = \sqrt{2g(1 - cos\vartheta)} = 0.3278\frac{m}{s}$

$F_{cp} = \frac{mv^2}{l} = 1.0745\ mN$

$F_N = F_{cp} + mg = 0.099\ N = 99\ mN$

1.28 $T = 2\pi\sqrt{\frac{I}{mgL}}$

$I = m_1 r_1^2 + m_2 r_2^2$

$$x_T = \frac{0 \cdot m_1 + m_2(r_1 + r_2)}{m_1 + m_2} = \frac{m_2(r_1 + r_2)}{m_1 + m_2}$$

$$L = r_1 - x_T = \frac{m_1 r_1 - m_2 r_2}{m_1 + m_2}$$

$$T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{m_1 r_1^2 + m_2 r_2^2}{(m_1 + m_2)g\frac{m_1 r_1 - m_2 r_2}{m_1 + m_2}}} = 2\pi\sqrt{\frac{m_1 r_1^2 + m_2 r_2^2}{g(m_1 r_1 - m_2 r_2)}}$$

1.29 $T = 2\pi\sqrt{\frac{I}{mgL}}$

$$I = \frac{ml^2}{12} + md^2$$

$$L = d$$

$$T(d) = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{\frac{ml^2}{12} + md^2}{mgd}}$$

$$T'(d) = \frac{2\pi}{2\sqrt{\frac{\frac{ml^2}{12} + md^2}{mgd}}} \cdot \frac{2md \cdot mgd - \left(\frac{ml^2}{12} + md^2\right)mg}{(mgd)^2} = 0$$

$$2m^2gd^2 = \left(\frac{ml^2}{12} + md^2\right)mg$$

$$2d^2 = \frac{l^2}{12} + d^2 \Rightarrow d_{min} = \sqrt{\frac{l^2}{12}} = 0.58$$

Da bi se izračunali periodi u a) i b) samo uvrstiti u T(d) odgovarajući d.

1.31 $I = \frac{mR^2}{2} + mr^2$

$$T(r) = 2\pi\sqrt{\frac{I}{mgr}} = 2\pi\sqrt{\frac{\frac{mR^2}{2} + mr^2}{mgr}} = 2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}}$$

$$T'(r) = \frac{2\pi \cdot \left(2m^2r^2g - \left(\frac{R^2}{2} + r^2\right)m^2g\right)}{2\sqrt{\frac{4r^2 + R^2}{gr}}} = 0 \Rightarrow 2m^2r^2g = \left(\frac{R^2}{2} + r^2\right)m^2g$$

$$2r^2 = \frac{R^2}{2} + r^2 \Rightarrow r^2 = \frac{R^2}{2} \Rightarrow r = \frac{R}{\sqrt{2}}$$

1.32 $I_k = \frac{2}{5}mr^2 \Rightarrow I_{uk} = \frac{2}{5}mr^2 + m(l+r)^2$

$T_f = 2\pi\sqrt{\frac{I_{uk}}{mg(l+r)}} = 2\pi\sqrt{\frac{\frac{2}{5}mr^2+m(l+r)^2}{mg(l+r)}} = 2\pi\sqrt{\frac{\frac{2}{5}r^2+(l+r)^2}{g(l+r)}} = 3.492007707\text{s}$

$T_m = 2\pi\sqrt{\frac{l+r}{g}} = 3.491939245s$

$\Delta T = 6.85 \cdot 10^{-5}$

1.33 $I = \frac{2}{5}mr^2 + md^2$

$T = 2\pi\sqrt{\frac{\frac{2}{5}mr^2 + md^2}{mgd}}$

$\frac{T_1}{T_2} = \sqrt{\frac{\frac{\frac{2}{5}mr^2 + md_1{}^2}{mgd_1}}{\frac{\frac{2}{5}mr^2 + md_2{}^2}{mgd_2}}} = \sqrt{\frac{\frac{\frac{2}{5}r^2 + d_1{}^2}{d_1}}{\frac{\frac{2}{5}r^2 + d_2{}^2}{d_2}}} = \sqrt{\frac{\left(\frac{2}{5}r^2 + d_1{}^2\right)d_2}{\left(\frac{2}{5}r^2 + d_2{}^2\right)d_1}} = 0.5$

$\frac{\left(\frac{2}{5}r^2 + d_1{}^2\right)d_2}{\left(\frac{2}{5}r^2 + d_2{}^2\right)d_1} = \frac{1}{4}$

$\Rightarrow d_2{}^2 - 0.52d_2 + 4 \cdot 10^{-3} = 0$

$\Rightarrow d_2 = 0.78\,cm$

1.34 m1=2m2

$x_T = \frac{\frac{m_1 l}{2} + \frac{3m_2 l}{2}}{m_1 + m_2} = \cdots = \frac{5}{6}l$ /*položaj težišta, mjereno od slobodnog kraja štapa 1*/

$I_1 = \frac{m_1 l^2}{3} + \frac{m_2 l^2}{12} + \frac{m_2 9}{4}l^2 = 3m_2 l^2$

$I_2 = \frac{m_2 l^2}{3} + \frac{m_1 l^2}{12} + \frac{m_1 9}{4}l^2 = 5m_2 l^2$

$\frac{T_1}{T_2} = \sqrt{\frac{\frac{3m_2 l^2}{\frac{3m_2 g 5}{6}l}}{\frac{5m_2 l^2}{\frac{3m_2 g l \cdot 7}{6}}}} = \sqrt{\frac{6 \cdot 21}{5 \cdot 30}} = 0.92$

1.35 $T_l = 2\pi\sqrt{\frac{\frac{ml^2}{3}}{\frac{mgl}{2}}}; \left(jer\ je\ I = \frac{ml^2}{3}\ te\ L = \frac{l}{2}\right)$

$$T_l = 2\pi\sqrt{\frac{\frac{l}{3}}{\frac{g}{2}}} = 2\pi\sqrt{\frac{2l}{3g}}$$

$$T_{2l} = 2\pi\sqrt{\frac{4l}{3g}}$$

$$\frac{T_l}{T_{2l}} = \frac{\sqrt{\frac{2l}{3g}}}{\sqrt{\frac{4l}{3g}}} = \sqrt{\frac{1}{2}} = 0.707$$