FOTOMETRIJA

Jakost svjetlosnog izvora = $[I]$ cd (kandela)

Svjetlosni tok : $d\phi = I \cdot d\Omega$

$d\Omega$ na udaljenosti r određuje površinu dS, koja je jednaka $dS = r^2 d\Omega$

$$d\phi = I \cdot \frac{dS}{r^2} \qquad [\phi]\ lm$$

Osvijetljenost površine

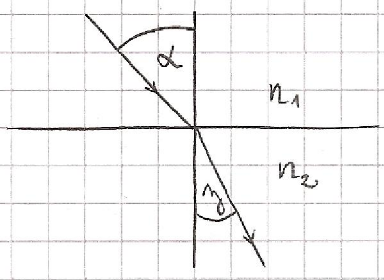
$$E = \frac{d\phi}{dS}$$

$$[E] = \frac{d\phi}{dS_0} \cdot \cos\theta \ \ lx$$

$$E = \frac{d\phi}{dS_0} \cdot \cos\phi$$

$$= \frac{I \cdot \frac{dS_0}{r^2}}{dS_0} \cos\phi = \frac{I}{r^2} \cos\theta$$

GEOMETRIJSKA OPTIKA



kada svjetlost prelazi iz sredstva u sredstvo frekvencija se ne mijenja

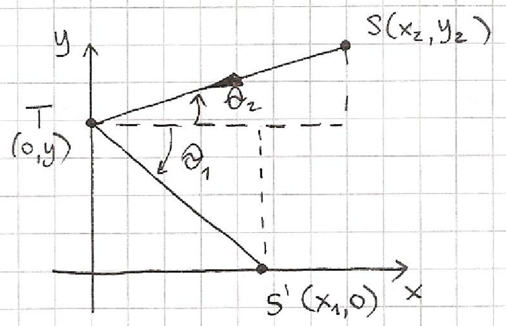
$$v_1 = \lambda_1 f \qquad v_2 = \lambda_2 f$$

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad \text{odnosno} \quad \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \quad \left(n = \frac{c}{v}\right)$$

Snellov zakon loma svjetlosti:

$$\frac{\sin u}{\sin l} = \frac{n_2}{n_1}$$

FERMATOV PRINCIP



$S(x_2, y_2)$, $T(0, y)$, $S'(x_1, 0)$

$tg\,\Theta_1 = \dfrac{y}{x_1}$ $\quad tg\,\Theta_2 = \dfrac{y_2 - y}{x_2}$

Put STS' iznosi:

$$s = \sqrt{x_2^2 + (y_2-y)^2} + \sqrt{x_1^2 + y^2}$$

Vrijeme potrebno za taj put:

$$t = \frac{s}{c} = \frac{1}{c}\left[\sqrt{x_2^2 + (y_2-y)^2} + \sqrt{x_1^2 + y^2}\right]$$

Najkraće vrijeme potrebno za taj put: $\dfrac{d(t)}{dy} = 0$ i to se zove FERMATOV PRINCIP.

$$\frac{d(t)}{dy} = \frac{1}{c}\left[\frac{-(y_2-y)}{\sqrt{x_2^2 + (y_2-y)^2}} + \frac{y}{\sqrt{x_1^2 + y^2}}\right] = 0$$

Sređivanjem dobijemo:

$$\frac{\frac{y^2}{x_1^2}}{1 + \frac{y^2}{x_1^2}} = \frac{\frac{(y_2-y)^2}{x_2^2}}{1 + \frac{(y_2-y)^2}{x_2^2}}$$

Trigonometrijom iz slike dobijemo:

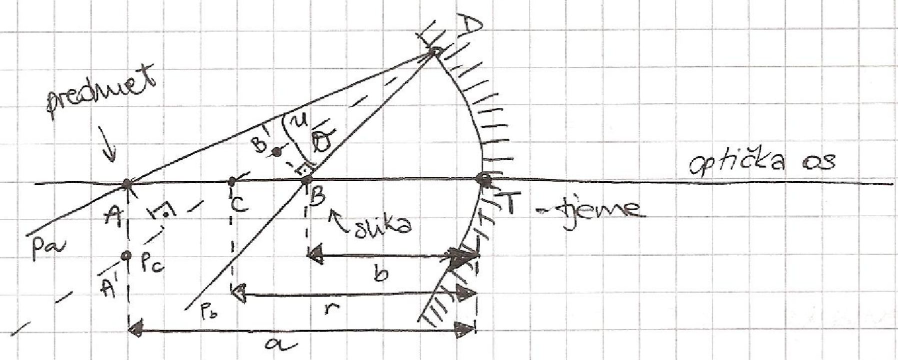
$$\frac{tg^2\Theta_1}{1 + tg^2\Theta_1} = \frac{tg^2(\Theta_2)}{1 + tg^2(\Theta_2)}$$

$$\Rightarrow tg^2\Theta_1 = tg^2\Theta_2$$

$$\boxed{\Theta_1 = \Theta_2}$$

Zaključak: zahtjev da svjetlost prolazi najkraći put u odbijanju, i zakon odbijanja su ekvivalentni.

JEDNADŽBA SFERNOG ZRCALA



Sličnost trokuta:

$$\frac{\overline{AC}}{\overline{AA'}} = \frac{\overline{BC}}{\overline{BB'}} \quad i \quad \frac{\overline{AD}}{\overline{AA'}} = \frac{\overline{BD}}{\overline{BB'}}$$

$$\frac{\overline{AC}}{\overline{BC}} = \frac{\overline{AD}}{\overline{BD}}$$

Gaussova aproksimacija:

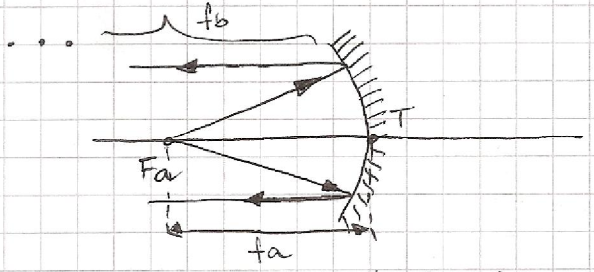
$\overline{AD} \simeq \overline{AT}$ i $\overline{BD} \simeq \overline{BT}$

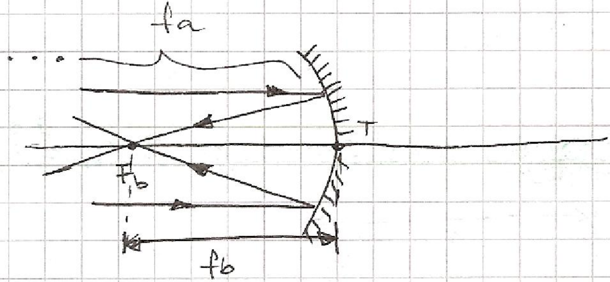
$$\frac{\overline{AC}}{\overline{BC}} = \frac{\overline{AT}}{\overline{BT}}$$

tj.

$$\frac{a-r}{r-b} = \frac{a}{b} \longrightarrow \boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{r}}$$

ŽARIŠTA SFERNOG ZRCALA





F_a = predmetno žarište (fokus)

f_a = žarišna duljina

F_b = slikovno žarište

f_b = slikovna žarišna duljina

$f_b = \infty \quad a = f_a$

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}$$

$$\frac{1}{a} + \frac{1}{\infty} = \frac{2}{r}$$

$$\frac{1}{a} = \frac{2}{r}$$

$$f_a = a = \frac{r}{2}$$

$$f_a = f_b = \frac{r}{2} = f$$

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}$$

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{f}}$$

$f_a = \infty \quad f_b = b$

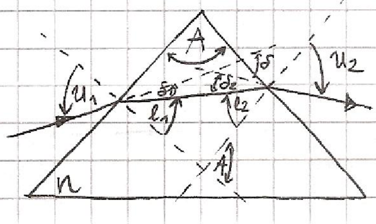
$$\frac{1}{a} + \frac{1}{b} = \frac{2}{r}$$

$$\frac{1}{\infty} + \frac{1}{b} = \frac{2}{r}$$

$$\frac{1}{b} = \frac{2}{r}$$

$$f_b = b = \frac{r}{2}$$

OPTIČKA PRIZMA



$$\delta_2 = u_2 - e_2$$

$$\delta_1 = u_1 - e_1$$

Ukupna devijacija $\delta = \delta_1 + \delta_2 = u_2 - e_2 + u_1 - e_1$

$$\delta = u_1 + u_2 - (e_1 + e_2)$$

$$A = e_1 + e_2 = konst$$

$$\delta = u_1 + u_2 - A$$

Određivanje n kod prizme:

Devijacija δ je najmanja kada je $u_1 = u_2$

$$\delta_{min} = 2u_1 - A \quad i \quad A = 2e_1$$

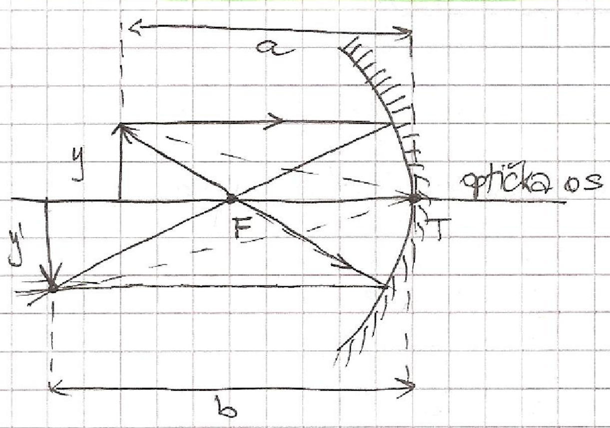
$$u_1 = \frac{1}{2}(\delta_{min} + A) \quad i \quad e_1 = \frac{A}{2}$$

Snellov zakon

$$\frac{\sin u_1}{\sin e_1} = \frac{n}{1}$$

$$n = \frac{\sin \frac{1}{2}(\delta_{min} + A)}{\sin \frac{A}{2}}$$

Povećanje kod sfernog zrcala



$$m = \frac{y'}{y}$$

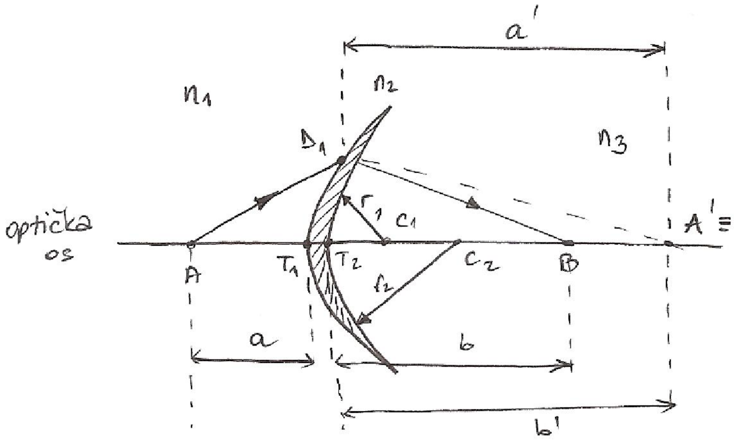
$y' : y = b : a$ ← iz sličnosti trokut

Po dogovoru povećanje je negativno ako je slika obrnuta $(m < 0)$

$$-\frac{y'}{y} = \frac{b}{a} = m$$

$$\Rightarrow \quad m = -\frac{b}{a}$$

TANKE LEĆE



$$\frac{n_1}{a} + \frac{n_2}{b'} = \frac{n_2 - n_1}{r_1}$$ ← lom na prvom sfernom dioptru.

$$\frac{n_2}{a'} + \frac{n_3}{b} = \frac{n_3 - n_2}{r_2}$$ ← lom na drugom sfernom dioptru.

\+

za tanku leću vrijedi: $-\frac{n_2}{b'} + \frac{n_3}{b} = \frac{n_3 - n_2}{r_2}$ $\left[|a'| = |b'| \quad a' < 0 \text{ i } b' > 0 \quad a' = -b' \right]$

$$\frac{n_1}{a} + \frac{n_3}{b} = \frac{n_2 - n_1}{r_1} + \frac{n_3 - n_2}{r_2}$$

$b = \infty$

$$f_a = \frac{n_1 r_1 r_2}{r_2(n_2 - n_1) + r_1(n_3 - n_2)}$$

$a = \infty$

$$f_b = \frac{n_3 r_1 r_2}{r_2(n_2 - n_1) + r_1(n_3 - n_2)}$$

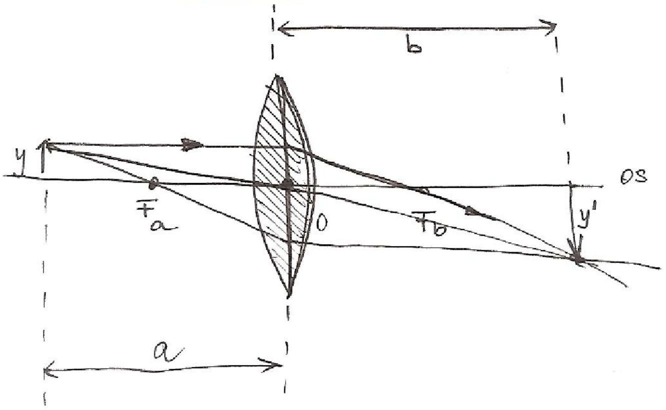
$$\frac{f_b}{f_a} = \frac{n_3}{n_1}$$

za $n_1 = n_3$

$$\frac{1}{a} + \frac{1}{b} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

ako je $n_1 = n_3$ onda $f_a = f_b$

Povećanje kod tanke leće



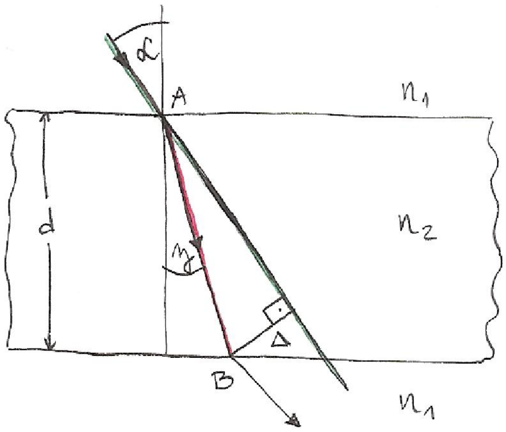
$y' : b = y : a$

$$\frac{y'}{y} = \frac{b}{a}$$

uvodimo definiciju $m = -\frac{y'}{y}$

$$m = -\frac{b}{a}$$

PLANPARALELNA PLOČA



Δ je pomak upadne zrake

prvo sredstvo je zrak $n_1 \simeq 1$

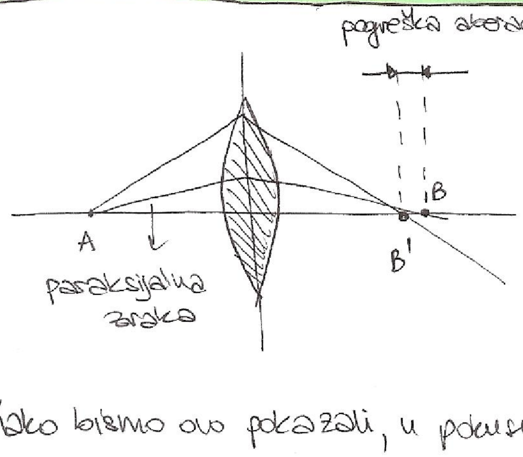
$$\sin(\alpha-\gamma) = \frac{\Delta}{\overline{AB}} \quad , \quad \cos\gamma = \frac{d}{\overline{AB}}$$

$$\Delta = d \cdot \frac{\sin(\alpha-\gamma)}{\cos\gamma}$$

Iskoristimo Snellov zakon i adicijske formule

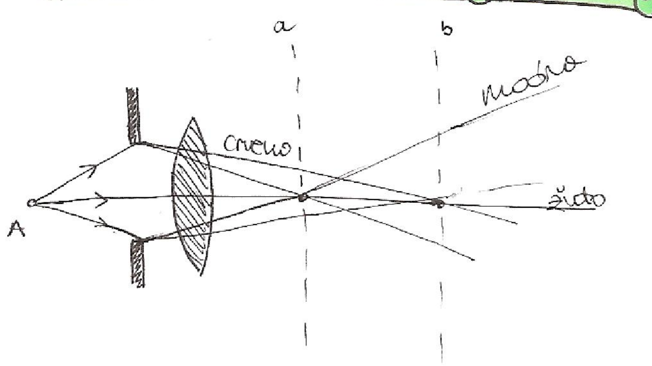
$$\Delta = d \cdot \sin\alpha \left(1 - \frac{\cos\alpha}{\sqrt{n^2 - \sin^2\alpha}}\right)$$

SFERNA ABERACIJA



Kako bismo ovo pokazali, u pokusu koristimo različite zaslone. Prvo propuštamo samo paraksijalne zrake, a onda samo periferne zrake. Vidimo da se slika pomiče, što dokazuje sfernu aberaciju

KROMATSKA ABERACIJA



na zastoru (a) dobivamo modru sliku, dok je na zastoru (b) crvena slika

Fizikalna optika

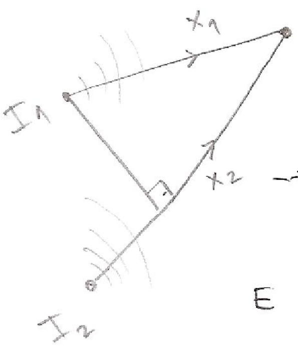
Interferencija svjetlosti

- moramo imati koherentne izvore (razlika u fazi dva ista vala mora biti vremenska konstanta)

val E_1 : $E_1(t_0, x_1) = E_0 \cos(\omega t_0 - k_1 x_1) = E_0 \cos\left(\omega\left(t_0 - \frac{k_1}{\omega} x_1\right)\right) =$

$$= E_0 \cos\left[\omega\left(t_0 - \frac{1}{\lambda_1 f} x_1\right)\right] = E_0 \cos\left[\omega\left(t_0 - \frac{x_1}{v_1}\right)\right]$$

$$= E_0 \cos\left[\omega\left(t_0 - \frac{n_1 x_1}{c}\right)\right]$$



- to vrijedi i za val E_2.

$$E = E_1 + E_2$$

$$= E_0 \cos\left[\omega\left(t_0 - \frac{n_1 x_1}{c}\right)\right] + E_0 \cos\left[\omega\left(t_0 - \frac{n_2 x_2}{c}\right)\right]$$

$$= \left\{2E_0 \cos\left[\frac{\omega}{2c}(n_1 x_1 - n_2 x_2)\right]\right\} \cdot \cos\left[\omega t_0 - \frac{\omega}{2c}(n_1 x_1 + n_2 x_2)\right]$$

razlika u fazi dva vala $\rightarrow$ $\Delta\phi = \frac{\omega}{c}(n_1 x_1 - n_2 x_2) = \frac{2\pi}{\lambda}(n_1 x_1 - n_2 x_2) = \frac{2\pi}{\lambda}\delta$

optička razlika hoda

$$\delta = n_1 x_1 - n_2 x_2 = L_1 - L_2$$

Amplituda rezultantnog vala:

$$E = 2E_0 \cos\left[\frac{\omega}{2c}(n_1 x_1 - n_2 x_2)\right] = 2E_0 \cos\left[\frac{\Delta\Phi}{2}\right]$$

$$\cos\left[\frac{\Delta\Phi}{2}\right] = \begin{cases} \pm 1 & \text{– max intenzitet} \\ 0 & \text{– min intenzitet} \end{cases}$$

max intenzitet: $\cos\left(\frac{\Delta\Phi}{2}\right) = \pm 1 \iff \frac{\Delta\Phi}{2} = n\pi \quad , n = 0, \pm 1, \pm 2, \ldots$

$$\rightarrow \frac{1}{\cancel{2}} \cdot \frac{\cancel{2\pi}}{\lambda}\delta_{max} = n\cancel{\pi}$$

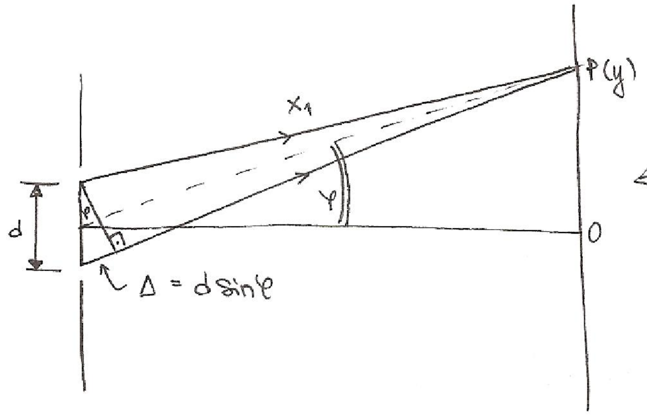
$$\delta_{max} = n\lambda$$

min intenzitet : $\cos\left(\frac{\Delta\Phi}{2}\right) = 0 \iff \frac{\Delta\Phi}{2} = (2n+1)\frac{\pi}{2}, \quad n = 0, \pm 1, \pm 2, \ldots$

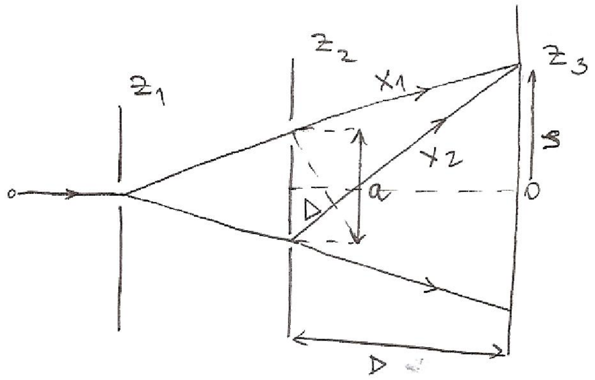
onda je $\frac{1}{\cancel{2}} \frac{\cancel{2\pi}}{\lambda}\delta_{min} = (2n+1)\frac{\cancel{\pi}}{2}$

$$\delta_{min} = (2n+1)\frac{\lambda}{2}$$

Youngov pokus



← skica iz Horvata



$D \gg s + \frac{a}{2}$

$$\Delta = x_2 - x_1 = \sqrt{\left(s+\frac{a}{2}\right)^2 + D^2} - \sqrt{\left(s-\frac{a}{2}\right)^2 + D^2}$$

Razvoj u red: $\sqrt{1 \pm x} \approx 1 \pm \frac{1}{2}x - \frac{1}{8}x^2 \pm \dots$ za $|x| \leq 1$

$$\Delta = D\left[1 + \frac{\left(s+\frac{a}{2}\right)^2}{2D^2} - 1 - \frac{\left(s-\frac{a}{2}\right)^2}{2D^2}\right] = \frac{1}{2} \cdot 2as = \frac{as}{D}$$

$\frac{as}{D} = \Delta = k\lambda$ ← za svijetlu prugu

$s = k \cdot \frac{D\lambda}{a}$ ← udaljenost svijetle pruge od središta zasto
u Youngovom pokusu.

Intenzitet u Youngovom pokusu

$I_0 \propto E_0^2$ nas zanima srednja vrijednost kvadrata elongacije

$I_0 \propto \frac{1}{2}E_0^2$

$$I \propto (E_{0,rez})^2 = (2E_0)^2\left[\cos\left(\frac{\Delta\Phi}{2}\right)\right]^2$$

max. intenzitet → I_0

$$I_{koh} = I_0\cos^2\left(\frac{\Delta\Phi}{2}\right) = I_0\cos^2\left(\frac{1}{2}\frac{2\pi}{\lambda}\delta\right)$$

$$= I_0\cos^2\left[\frac{\pi}{\lambda}(n_1x_1 - n_2x_2)\right] = I_0\cos^2\left(\frac{\pi}{\lambda}\Delta\right)$$

$$= I_0\cos^2\left(\frac{\pi}{\lambda}a\sin\varphi\right)$$

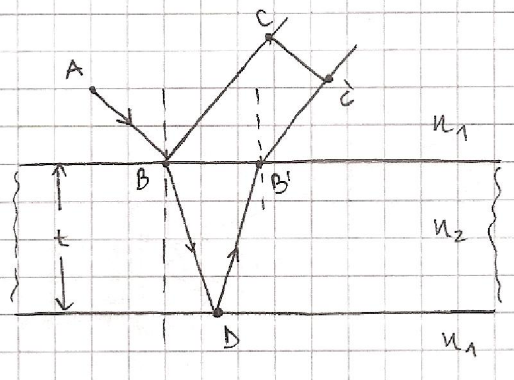
Δ ← geom. razlika ho

a ← razmak među puk

$\Delta\Phi$ ← razlika faza

δ ← optička razlika

Tanki listići



tanki listić debljine t.

Optički put od A do C:

$$L_1 = n_1 \overline{AB} + n_1 \overline{BC}$$

Optički put druge zrake:

$$L_2 = n_1 \overline{AB} + n_2 \overline{BD} + n_2 \overline{DB'} + n_1 \overline{B'C'}$$

vidimo da $\overline{BD}$ i $\overline{DB'} = t$ (lomljena zraka je inače puno strmija pa se lakše vidi)

za skoro okomito gledanje $\overline{BC} \simeq \overline{B'C'}$

Faza prvog vala:

$$\Phi_1 = \omega t - \frac{2\pi}{\lambda}\left(n_1 \overline{AB} + n_1 \overline{BC}\right) + \pi$$

(π dolazi zbog refleksije 1. zrake u točki B)

Faza drugog vala:

$$\Phi_2 = \omega t - \frac{2\pi}{\lambda}\left(n_1 \overline{AB} + n_2 2t + n_1 \cdot \overline{B'C'}\right)$$

$$\Delta\Phi = \Phi_2 - \Phi_1 = -\left(\frac{2\pi}{\lambda} n_2 \cdot 2t - \pi\right)$$

$$E_{0,rez} = 2E_0 \cos\left(\frac{\Delta\Phi}{2}\right)$$

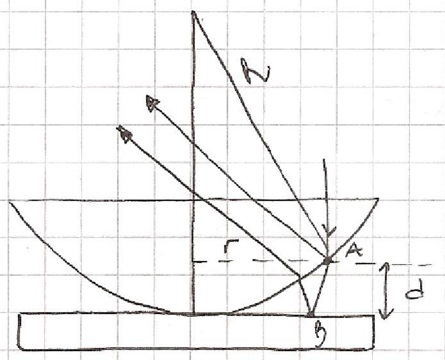
Određivanje max i min:

Max $\rightarrow$ argument $\frac{\Delta\Phi}{2} = k\pi$ $\rightarrow$ $\frac{1}{2}\left(\frac{2\pi}{\lambda} n_2 \cdot 2t - \pi\right) = k\pi$, $(k = 1, 2, 3, \dots)$

$$t_{max} = \frac{2k-1}{2n_2} \cdot \frac{\lambda}{2}$$

Minimum $\rightarrow$ argument $\frac{\Delta\Phi}{2} = 0$ $\rightarrow$ $t_{min} = \frac{k}{2n_2} \cdot \lambda$

Newtonovi kolobari



$$|\Delta\Phi| = \Phi_2 - \Phi_1 = \frac{2\pi}{\lambda} \cdot 2d + \pi$$

naravno, opet imamo:

Maximum $\rightarrow$ $\frac{\Delta\Phi}{2} = k\pi \qquad (k = 1, 2, 3, \ldots)$

$$d_{max} = \frac{1}{2}(2k-1)\frac{\lambda}{2} \qquad (k = 1, 2, 3, \ldots)$$

Minimum $\rightarrow$ $\frac{\Delta\Phi}{2} = 0$

$$d_{min} = \frac{1}{2}k\lambda \quad (k = 0, 1, 2, 3, \ldots)$$

Prema slici se može izraziti veličina d pomoću polumjera kolobara r i polumjera zakrivljenosti R.

$$r^2 + (R-d)^2 = R^2 \quad \rightarrow \quad d = \frac{r^2}{2R}$$

(d^2 smo zanemarili jer $d^2 \ll R$)

Svijetli kolobar: (max)

ako je $\Delta = \lambda, 2\lambda, \ldots k\lambda \qquad k = 1, 2, 3, \ldots$

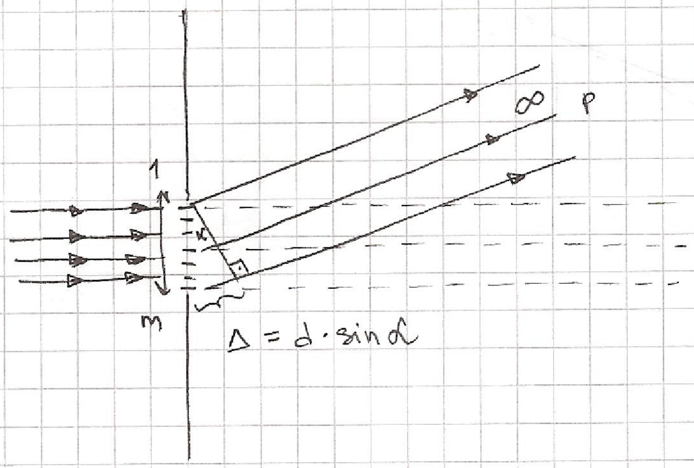
$$k\lambda = \frac{r^2}{R} + \frac{\lambda}{2} \qquad r_k^2 = R(2k-1)\frac{\lambda}{2}$$

Tamni kolobar: (min)

ako je $\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots \frac{(2k+1)\lambda}{2} \qquad k = 0, 1, 2, 3$

$$r_k^2 = Rk\lambda$$

OPTIČKA REŠETKA



$\Delta = d \cdot \sin\alpha$

razlika u hodu zraka 1 i m $\Phi = k\Delta$

$\Phi = \frac{2\pi}{\lambda} d \sin\alpha$

Ako zbrajamo valove iz N izvora dobijemo:

$$E_{o/rez} = E_o \cdot \frac{\sin \frac{N\Delta\Phi}{2}}{\sin \frac{\Delta\Phi}{2}}$$

$m = \frac{1}{N}$

gdje je .. broj zareza u jednom cm ili mm.

$$I \propto E_o^2$$

$$I = I_o \cdot \frac{\sin^2 \frac{N\Delta\Phi}{2}}{\sin^2 \frac{\Delta\Phi}{2}}$$

Max → nazivnik = 0 → $\sin^2 \frac{\Delta\Phi}{2} = 0$

$$\frac{\Delta\Phi}{2} = k\pi \quad , \quad k = 0, \pm 1, \pm 2, \ldots$$

$$d \sin\alpha_{max} = k \cdot \lambda \quad (m = 0, \pm 1, \pm 2)$$

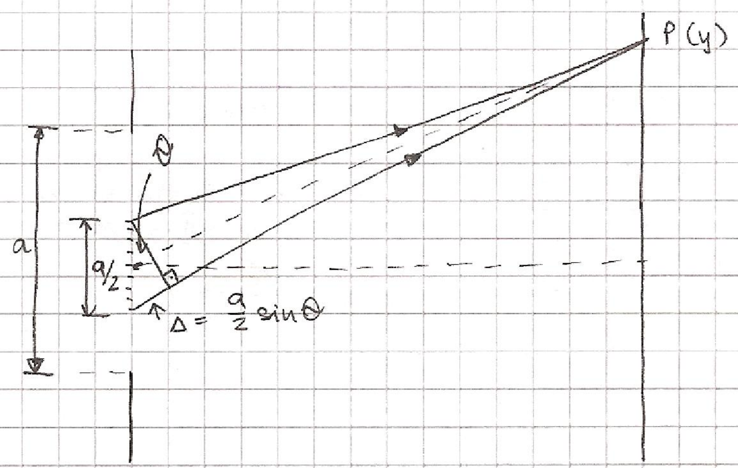
Minimum → $d \sin\alpha_{min} = k \cdot \frac{\lambda}{N} \quad (m = \pm 1, \pm 2, \ldots \text{ i } m \neq \pm N, \pm 2N, \ldots)$

$$I = I_o \left(\frac{\sin\left(\frac{\pi}{\lambda} a \sin\alpha\right)}{\frac{\pi}{\lambda} a \sin\alpha} \right)^2 \left(\frac{\sin\left(N \frac{\pi}{\lambda} d \sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda} d \sin\alpha\right)} \right)^2$$

a = širina jedne pukotine

d = razmak između pukotina

DIFRAKCIJA NA JEDNOJ PUKOTINI



$$\Delta = \frac{a}{2} \sin\theta = \delta$$

$$\frac{\Delta\Phi}{2} = \frac{\pi}{\lambda} \cdot \frac{a}{2} \sin\theta$$

u točki P(y) je minimum kada je $\frac{\Delta\Phi}{2} = \pm \frac{\pi}{2}$ tj.

$$a \sin\theta_{min/1} = \pm 1 \cdot \lambda$$

općenito minimumi:

$$a \sin\theta_{min/k} = k\lambda \quad (k = \pm 1, \pm 2, \dots)$$

Polarizacija refleksijom. Brewsterov zakon.

$$u + l = 90^\circ$$

$$\frac{\sin u}{\sin l} = \frac{n_2}{n_1} = \frac{\sin u}{\sin(90^\circ - u)}$$

$$\mathrm{tg}\, u = \frac{n_2}{n_1}$$