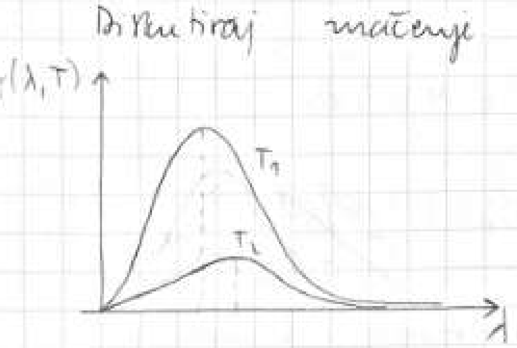
2. CIKLUS

1. Skicirajte planckovu ~~___~~ raspodjelu zračenja crnog tijela na dvije različite temperature. Diskutiraj položaje maksimuma. Diskutiraj značenje površine ispod krivulje.



$\lambda_{max} \cdot T = konst$

$T_1 > T_2$

- max. raspodjele računa se po Wienovom zakonu
- veća temp. $\rightarrow$ veća spektralna gustoća zračenja $f(\lambda, T)$

veća temp. $\Rightarrow$ max. je na manjoj λ

- površ. ispod krivulje predstavlja intenzitet zračenja $I = \sigma T^4$

2. Napiši Schrödingerovu jedn. za općeniti potencijal $V(x,t)$, a zatim riješi jedn. za slob. čest. $V(x,t) = 0$.

- općenito: $$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = E\Psi(x,t)$$
$$= i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

$\Psi(x,t)$ valna funkcija

- $V(x,t) = 0 \Rightarrow$ $$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

- pretp. rješenje oblika $\Psi(x,t) = \psi(x)\phi(t)$ (separac. var.)

$$\Rightarrow \left[-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}\right]\phi(t) = \left[i\hbar\frac{\partial}{\partial t}\phi(t)\right]\psi(x) \quad / : \phi(t)\psi(t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}\frac{1}{\psi(x)} = i\hbar\frac{\partial\phi(t)}{\partial t}\frac{1}{\phi(t)}$$

$\Rightarrow$ svaka strana mora bit jednaka konstanti da bi vrijedila $\forall t$ i $\forall x$ $\Rightarrow$ dimenz. en.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}\frac{1}{\psi(x)} = \underbrace{E = i\hbar\frac{\partial\phi(t)}{\partial t}\frac{1}{\phi(t)}}$$

$$\frac{E}{i\hbar} = \frac{d\phi(t)}{dt}\frac{1}{\phi(t)}$$

$$\Rightarrow \frac{d\phi(t)}{\phi(t)} = -i\frac{E}{\hbar}dt \quad /\int$$

$$\ln\frac{\phi}{\phi_0} = -i\frac{E}{\hbar}t \quad /e$$

$$\phi = \phi_0 e^{-iEt/\hbar}$$

uz $E = \hbar\omega$

$\underset{\sim\sim\sim}{\phi = Ce^{-i\omega t}}$

$\rightarrow \psi(x) = Ae^{ikx} + Be^{-ikx} \longrightarrow$ jer $\psi'' + \frac{2mE}{\hbar^2}\psi = 0 \rightarrow$ oblik jedn. harm. oscilatora

za $k = \sqrt{\frac{2mE}{\hbar^2}}$

$$\Rightarrow \underset{\sim\sim\sim}{\Psi(x,t) = C_1 e^{i(kx-\omega t)} + C_2 e^{-i(kx+\omega t)}}$$

općenito $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

(3.) Za danu fizikalnu veličinu f (npr. položaj čestice x) napiši izraz za njenu neodređenost Δf ako je zadana valna fja $\Psi(x,t)$ koja opisuje česticu.

- gustoća vjerojatnosti $P(x) = \Psi^* \Psi$
- srednja vrijednost $\langle f(x)\rangle = \int_{-\infty}^{+\infty} \Psi^*(x) f(x) \Psi(x)\, dx$
- srednja vrij. kvadrata $\langle (f(x))^2 \rangle = \int_{-\infty}^{+\infty} \Psi^*(x) (f(x))^2 \Psi(x)\, dx$
- standardna devijacija (srednje kvadr. odstup.) σ:

$$\sigma^2 = \langle (f(x) - \langle f(x)\rangle)^2 \rangle$$

$\langle\ \rangle$ može ući u zagradu (lin. op.)

$$= \langle ((f(x))^2 - 2f(x)\langle f(x)\rangle + \langle f(x)\rangle^2) \rangle$$

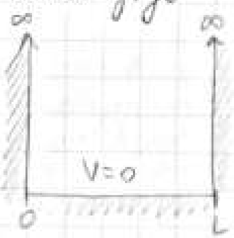
$$= \langle (f(x))^2 \rangle - 2\langle f(x) \langle f(x)\rangle\rangle + \langle f(x)\rangle^2$$

$$= \langle (f(x))^2 \rangle - \underbrace{2\langle f(x)\rangle\langle f(x)\rangle}_{-2\langle f(x)\rangle^2} + \langle f(x)\rangle^2$$

$$= \langle (f(x))^2 \rangle - \langle f(x)\rangle^2$$

$$\sigma = \Delta f = \sqrt{\langle (f(x))^2 \rangle - \langle f(x)\rangle^2} = \sqrt{\overline{f^2} - (\overline{f})^2} = \sqrt{\langle f^2\rangle - \langle f\rangle^2}$$

(4) Krenuvši od vrem. nez. Schr. jdn., nađi rj. za česticu u 1D beskonačnoj potencijalnoj jami, tj. odredi valne fje i energije čestice.



$$V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & x < 0,\ x > L \end{cases}$$

$$\Psi(x) = 0 \quad \text{za} \quad x < 0,\ x > L$$

- $\Psi(x)$ mora biti kontinuirana fja $\Rightarrow \Psi(0) = \Psi(L) = 0$
- u jami $V(x) = 0 \Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x)}{\partial x^2} = E\Psi(x)$
- pretp. rješenje oblika $\Psi(x) = A_1 e^{ikx} + A_2 e^{-ikx}$

$$\Rightarrow \Psi(x) = A_1(\cos kx + i \sin kx) + A_2(\cos kx - i\sin kx)$$

$$= (A_1 + A_2)\cos kx + i(A_1 - A_2)\sin kx$$

- rubni uvj: $\Psi(0) = 0 = (A_1 + A_2)\cdot 1 + i(A_1 - A_2)\cdot 0$

$$\Rightarrow A_1 = -A_2$$

$$\Rightarrow \Psi(x) = \underbrace{i \cdot 2A_1}\sin kx = C \sin kx$$

$$x = L \Rightarrow \psi(L) = 0 = C \sin k \cdot L \Rightarrow kL = n\pi \Rightarrow k = \frac{n\pi}{L}, \quad n = 1, 2, 3, \ldots$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{n\pi}{L}} = \frac{2L}{n}$$

$$p_n = \frac{h}{\lambda_n} = \frac{h \cdot n}{2L} \Rightarrow E_n = \frac{p_n^2}{2m} = \frac{h^2 n^2}{4L^2 \cdot 2m} = \left\{ h = \hbar \cdot 2\pi \right\} = \underline{\underline{\frac{n^2 \hbar^2 \pi^2}{2m \cdot L^2}}}$$

- svaki en. nivo ima svoj kvantni broj n i svoju valnu funkciju
- normalizacija:

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \Rightarrow \int_0^L C^2 \sin^2 \frac{n\pi x}{L} dx = C^2 \left[\frac{x}{2} - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{4\frac{n\pi}{L}} \right] \Bigg|_0^L$$

$$= C^2 \left[\frac{L}{2} - 0 \right] = C^2 \frac{L}{2} = 1 \Rightarrow C = \sqrt{\frac{2}{L}}$$

$$\Rightarrow \text{konačno: } \underline{\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}, \quad n = 1, 2, 3, \ldots$$

$$\Rightarrow \underline{\Psi_n(x, t) = \left(\sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \right) e^{-i\frac{Et}{\hbar}}}$$

(5) Napiši izraze pomoću kojih bi izračunali srednju vrijednost položaja čestice opisane valnom fjom $\phi(x,t) = \psi(x)\varphi(t)$

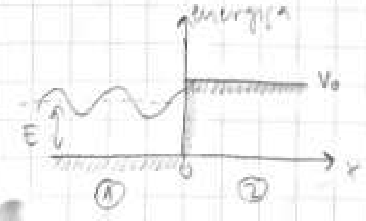
$$\phi(x,t) = \psi(x)\varphi(t) = \psi(x) \cdot e^{-i\frac{E}{\hbar}t}$$

$$\Rightarrow \langle x \rangle = \int_{-\infty}^{\infty} \phi^*(x,t) \cdot x \cdot \phi(x,t)\, dx$$

$$= \int_{-\infty}^{\infty} \psi^*(x) \overbrace{e^{i\frac{E}{\hbar}t} \cdot x \cdot \psi(x) e^{-i\frac{E}{\hbar}t}}^{=1} dx$$

$$= \underline{\int_{-\infty}^{\infty} \psi^*(x) \cdot x \cdot \psi(x)\, dx}$$

(6.) Napiši opća rj. (neodr. konst.) za valnu fju koja opisuje česticu koja nailazi na potenc. barijeru. Napiši rubne uvj. za odred. konst. Pomoću njih napiši def. transmisije i refleksije.



područje ①: $\psi_1(x) = \underbrace{A e^{ik_1 x}}_{\text{„upadni" val}} + \underbrace{B e^{-ik_1 x}}_{\text{reflektirani}}$, $k_1 = \frac{\sqrt{2mE}}{\hbar}$

područje ②: $\psi_2(x) = \underbrace{C e^{ik_2 x}}_{\text{transmit.}} + D e^{-ik_2 x}$, $k_2 = \frac{\sqrt{2m(E - V_0)}}{\hbar}$

$D = 0$ jer se nema odje reflektirati

$E > V_0$

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x > 0 \end{cases}$$

(beskonačna potenc. barijera)

- valna f-ja i njena prva deriv. moraju biti neprek.

$$\psi_1(x=0) = \psi_2(x=0), \quad \psi_1'(x=0) = \psi_2'(x=0)$$ 6-nastavak

$$\psi_1(0) = A + B \qquad \psi_1'(0) = ik_1 A - ik_1 B$$

$$\psi_2(0) = C \qquad \psi_2'(0) = ik_2 C$$

$$A + B = C \qquad ik_1 A - ik_1 B = ik_2 C$$

1° $ik_1 A - ik_1 B = ik_2 (A+B)$

$A(ik_1 - ik_2) = B(ik_1 + ik_2) /:i$

$$\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}$$

2° $ik_1 A - ik_1 (C - A) = ik_2 C /:i$

$A(k_1 + k_1) = C(k_2 + k_1)$

$$\frac{C}{A} = \frac{2k_1}{k_1 + k_2}$$

A → valna f-ja za upadni val ⇒ vjerojatnost $|A|^2$
B → valna f-ja za reflekt. ⇒ -||- $|B|^2$
C → valna f-ja za transmit. ⇒ -||- $|C|^2$

- koef. refleksije R ⇒ omjer vjerojatnosti refleksije i upada

$$R = \frac{|B|^2}{|A|^2} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$$

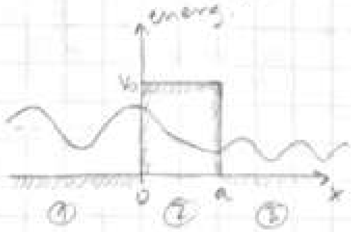
- uvjet: $R + T = 1$ (ne čest. se ili refl. ili transmit.)

koef. transmisije

$$T = 1 - R = 1 - \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{\cancel{k_1^2} + 2k_1k_2 + \cancel{k_2^2} - \cancel{k_1^2} + 2k_1k_2 - \cancel{k_2^2}}{(k_1 + k_2)^2}$$

$$T = \frac{4k_1k_2}{(k_1 + k_2)^2}$$

⑦ Komentiraj izraz za transmis. koef. za čest. en. E koja nalijeće na potenc. barijeru širine a i visine V_0, $T \propto e^{-2k_2 a}$, $k_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$. (nacrtaj valnu f-ju u ova podr., kako je def. koef. transmis., odnos E i V_0?)



- potencijalna barijera je dovoljno uska da valna f-ja ne nestane te postoji mogućnost da neke čestice prođu i na drugu stranu

① $\psi_1 = Ae^{ik_1 x} + Be^{-ik_1 x}$

② $\psi_2 = Ce^{k_2 x} + De^{-k_2 x}$

③ $\psi_3 = E \cdot e^{k_1 x}$

- odnos E i V_0

$$\alpha = \frac{V_0}{E} > 1$$

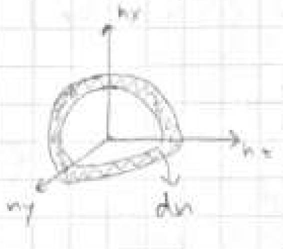
- prema $T \propto e^{-2k_2 a}$, vidimo da se vjerojatnost transmisije eksponencijalno smanjuje sa širinom barijere, a povećava se sa smanjenjem razlike $(V_0 - E)$

8) Iz modela 3D besk. potencijalne jame, odredi broj stanja ΔN u intervalu energija $(E, E+\Delta E)$ i volumenu ΔV.

$$E_n = \frac{p_n^2}{2m} = \underbrace{\frac{n^2\pi^2\hbar^2}{2mL^2}}_{\text{zadano}} = (n_x^2 + n_y^2 + n_z^2)\cdot\frac{h^2\pi^2}{4\pi^2\, 2mL^2} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$$p^2 = \frac{n^2h^2}{4L^2} \quad /\sqrt{\ }$$

$$\Rightarrow n = \frac{2pL}{h} \;/d \Rightarrow dn = \frac{2L}{h}dp$$



$n_x, n_y, n_z > 0$ (1. oktant)

„površ. kugle" jer samo 1. okt. 2 spina elektrona

$$\Delta N = 4\pi^2 n^2 dn \cdot \frac{1}{8}\cdot 2$$

$$\Rightarrow \Delta N = \frac{2}{8}4\pi n^2 dn = \frac{1}{\cancel{4}}\cdot 4\pi\cdot\left(\frac{2pL}{h}\right)^2\cdot\frac{2L}{h}dp$$

$$= 8\pi\frac{\overbrace{L^3}^{=\Delta V}}{h^3}p^2dp = \frac{8\pi\Delta V}{h^3}\cdot p\,dp\cdot\frac{m}{m}\cdot p$$

$E = p^2/2m$

$dE = \frac{2p\,dp}{2m}$

$p = \sqrt{2mE}$

$$= \frac{8\pi\Delta V}{h^3}\,dE\cdot\sqrt{2mE}\cdot m$$

$$= \frac{8\pi\Delta V}{h^3}\sqrt{2m^3E}\,dE$$

9) Iz izraza za ukupni broj stanja (na 0K) izvedite izraz za Fermijevu energiju pri T=0K. Ako je d međuatomski razmak, izrazite Fermijevu energiju pri apsolutnoj nuli preko parametra d.

$$N = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E_F^{3/2} \quad /^{2/3}$$

$$\frac{N}{V} = \frac{\text{br. at.}}{\text{volum.}} = \frac{1}{\text{vol. jedne ćelije}} = \frac{1}{d^3}$$

$$N^{2/3} = \left(\frac{V}{3\pi^2}\right)^{2/3}\cdot\frac{2m}{\hbar^2}E_F$$

$$E_F = \frac{\hbar^2}{2m}N^{2/3}\cdot\left(\frac{3\pi^2}{V}\right)^{2/3} = \frac{\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3}(3\pi^2)^{2/3}$$

$$= \frac{\hbar^2}{2m}\left(\frac{1}{d^3}\right)^{2/3}(3\pi^2)^{2/3}$$

$$= \frac{\hbar^2}{2md^2}(3\pi^2)^{2/3}$$

(10.) Počevši od izraza za funkciju raspodjele $W = dN_{E, E+\Delta E} / N$ odredite srednju en. elektrona pri 0 K.

$$N = \int_0^{p_{max}} dN$$

$$E = \frac{p^2}{2m} \Rightarrow \quad p^2 = 2mE \,/^{3/2}$$

$$p^3 = (2mE)^{3/2}$$

$$\Rightarrow \quad N = \frac{V}{3\pi^2} \frac{p_{max}^3}{\hbar^2} \quad // \quad dN = \frac{V}{\pi^2} \frac{p^2}{\hbar^2} dp$$

$$W = \frac{\cancel{\frac{V}{\pi^2}} \frac{p^2}{\cancel{\hbar^2}} dp}{\cancel{\frac{V}{3\pi^2}} \cdot \frac{p_{max}^3}{\cancel{\hbar^2}}} = \frac{3p^2}{p_{max}^3} dp$$

$$\bar{E} = \int_0^\infty E\, dW = \int_0^\infty \frac{p^2}{2m} \cdot \frac{3p^2}{p_{max}^3} dp = \frac{3}{2m p_{max}^3} \int_0^\infty p^4 dp$$

$$= \frac{3}{2m p_{max}^3} \cdot \frac{p_{max}^5}{5} = \frac{3}{5 \cdot 2m} \cdot p_{max}^2 = \underline{\underline{\frac{3}{5} E_F}}$$

(11) → prekompliciran!

(12) Objasni kako se određuju koef. a i b koji se javljaju prilikom izvoda Fermi-Diracove raspodjele u izrazu

$$n_i = \frac{N_i}{e^{a+bE_i}+1}$$

- za slučaj malih gustoća, tj. $N_i \gg n_i$, Paulijev princip isključenja ne dolazi do izražaja pa raspodjela prelazi u Maxwell-Boltzmannovu, za koju $\underbrace{b = \frac{1}{kT}}$

- a određujemo iz očekivanog ponašanja pri $T = 0\,K$

$$\lim_{T\to 0} \frac{1}{e^{a+E/kT}+1} = \begin{cases} 0, & a+E/kT > 0 \\ 1, & a+E/kT < 0 \end{cases}$$

max. en.

$$E < E_F$$

$$a + \frac{E}{kT} < 0 \qquad \underbrace{\frac{E}{kT} < \frac{E_F}{kT}}$$

$$\frac{E}{kT} < -a \quad \Rightarrow \quad \underbrace{a = -\frac{E_F}{kT}}$$

(13) Fermi-Diracova f-ja raspodjele dana je preko izraza

$$f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

. Pokaži da je simetrična s obzirom na Fermijevu energiju.

$$f(E_F - E) = 1 - f(E_F + E)$$

$$1 + f(E_F + E) = 1 - \frac{1}{e^{(E_F+E-E_F)/kT}+1} = 1 - \frac{1}{e^{E/kT}+1}$$

$$f(E_F - E) = \frac{1}{e^{-E/kT}+1}$$

$$\frac{1}{e^{-E/kT}+1} = 1 - \frac{1}{e^{E/kT}+1}$$

$$e^{E/kT} + 1 = (e^{-E/kT}+1)(e^{E/kT} + \cancel{1} - 1)$$

$$\underbrace{e^{E/kT} + 1 = 1 + e^{E/kT}}$$ ✓

(14) Dokazati Blochov tm. i tj. pokazati da u periodičnom potencijalu iz pretp. periodičnosti gustoće vjerojatnosti slijede valne fje Blochovog oblika.

(?)

- razmak = N · a

$$\psi(x+Na) = \lambda^N \psi(x)$$

$$Ae^{ikx} = Ae^{ik(x+Na)}$$

$$e^{ikNa} = 1 \Rightarrow \quad kNa = 2\pi M, \quad M \in \mathbb{Z}$$

$$k = \frac{2\pi M}{Na}$$

$$\psi(x) = Ae^{i\, 2\pi M x/Na} \Rightarrow \quad \lambda^N = (e^{ika})^N$$

$$\lambda = e^{ika} = e^{i\, 2\pi M/N}$$

$$\psi(x) = e^{ikx} \underbrace{u(x)}_{\text{fja koja ima periodičnost rešetke}} \qquad \psi^* \psi = u^2(x)$$

$$u(x) = u(x+a)$$

(15) Napiši uvj. koje mora zadovoljiti valna fja u Kronig-Penneyevu modelu.

- valna funkcija mora biti glatka i neprekinuta (rubni uvj.)
- model podrazumijeva periodičke potencijalne barijere
- periodički potencijal
- Blochova WF (ampl. modulirane periodično s periodom kristalne rešetke)

(16) Izvesti pojednostavljenu uvjetnu jedn. polazeći od

$$\sin(\alpha a)\,sh(\beta b)\,\frac{\beta^2-\alpha^2}{2\alpha\beta} + ch(\beta b)\cos(\alpha a) = \cos k(a+b)$$

a = širina jame, b = širina barijere, $\alpha^2 = 2mE/\hbar^2$, $\beta^2 = 2m(V_0-E)/\hbar^2$

pretp. $V_0 \cdot b$ = konst., $V_0 \to \infty$, $b_0 \to 0$. Pomoću pojednostavljenje uvjetne jedn. pokazati uzrok pojave energijskih vrpci u kristalu.

$b \to 0 \Rightarrow c \approx a$ $\qquad$ $\beta b \to 0 \Rightarrow e^{\beta b} \approx 1$

$V_0 \to \infty \Rightarrow \beta^2 \gg \alpha^2$ $\qquad$ $ch\,\beta b \to 1$

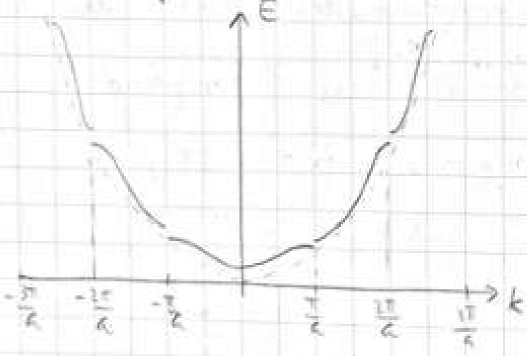
$$\frac{sh\,\beta b}{\beta b} \approx \frac{\beta b}{\beta b} \to 1$$

$$\Rightarrow \sin(\alpha a)\,sh(\beta b)\cdot\frac{\beta b}{\beta b}\,\frac{\beta^2}{2\alpha\beta} + \cos(\alpha a) = \cos ka$$

$$\sin(\alpha a)\cdot\frac{\alpha a}{\alpha a}\cdot b\cdot\frac{\beta^2}{2\alpha} + \cos(\alpha a) = \cos ka$$

$$\left(\frac{\sin(\alpha a)}{\alpha a}\right)\left(ab\,\frac{\beta^2}{2}\right) + \cos(\alpha a) = \cos(ka)$$

(17) Skic. $E(k)$ u proširenom prikazu. Pokaži da je gornja en. vrpce jednaka onoj za el. u kvt. dubokoj pot. jami.



- energije gornjih granica za: (rubovi BZ)

$\boxed{\alpha a = n\pi}$ $\qquad$ $\alpha = \sqrt{\frac{2mE}{\hbar^2}}$

$$E = \frac{\alpha^2\hbar^2}{2m} = \left(\frac{n\pi}{a}\right)^2\frac{\hbar^2}{2m}$$

$$= \frac{n^2\pi^2}{a^2}\cdot\frac{h^2}{(2\pi)^2}\,\frac{1}{2m} = \underbrace{\frac{n^2h^2}{8ma^2}}_{\text{en. za slob. el.}}$$

(18) Polazeći od uvj. jedn. $P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka$ def. Brillouinove zone preko iznosa valnog vektora.

$|\cos ka| \le 1 \Rightarrow$ rubovi: $ka = \pm n\pi$ $\qquad$ $n = 1, 2, \ldots$

$$k = \pm\frac{n\pi}{a}$$

- BZ su dozvoljene vrijednosti E, $E(k)$

1\. BZ $\Rightarrow$ $-\frac{\pi}{a} < k < \frac{\pi}{a}$

2\. BZ $\Rightarrow$ $-\frac{2\pi}{a} < k < -\frac{\pi}{a}$ $\cup$ $\frac{\pi}{a} < k < \frac{2\pi}{a}$

itd.

(19) Napiši QM varijantu II. Nz. za elektrone u kristalu koristeći se izjednačavanjem grupne brzine valnog paketa i brzine elektrona. Pomoću te veze def. efektivnu masu nosilaca, m^*.

$$F_L = e\mathcal{E} = \frac{dp}{dt} = \frac{d}{dt}(\hbar k) \qquad p = \hbar k \qquad E = \frac{p^2}{2m}$$

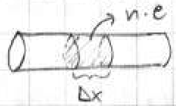
II. Nt. $e\mathcal{E} = m^* a_g \qquad v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$

$$= m^* \frac{dv_g}{dt} = m^* \frac{d}{dt}\left(\frac{dE}{dp}\right) = m^* \frac{d}{dt}\left(\frac{dE}{\hbar dk}\right)$$

$$= \frac{m^*}{\hbar} \frac{d}{dt}\left(\frac{dE}{dk}\right) = \frac{m^*}{\hbar} \frac{d^2E}{dk^2} \cdot \frac{dk}{dt}$$

$$\frac{d}{dt}(\hbar k) = \frac{m^*}{\hbar} \frac{d^2E}{dk^2} \frac{dk}{dt} \Rightarrow \hbar = \frac{m^*}{\hbar} \frac{d^2E}{dk^2} \Rightarrow \boxed{m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}}$$

(20) Izvedi izraz za vodljivost elektrona u kristalu.



$$\frac{n}{\Delta V} = N \qquad I = SveN$$
$$J = Nev$$

$$J = \frac{I}{S} = \frac{\Delta Q}{\Delta t}\frac{1}{S} = \frac{ne}{\Delta t}\frac{1}{S} = Ne\frac{\Delta V}{S\Delta t} = Ne\frac{S\cdot\Delta x}{S\cdot\Delta t}$$

v

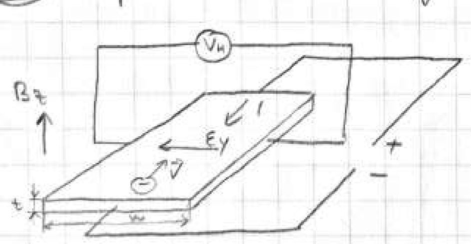
$$\vec{F_L} = e\vec{\mathcal{E}} = m\vec{a} = m\frac{v}{\tau} \Rightarrow v = \frac{e\mathcal{E}\tau}{m}$$

vrijeme izm. sud.

$$\Rightarrow J = Ne\cdot\frac{e\mathcal{E}\tau}{m} = \frac{Ne^2\tau}{m}\vec{\mathcal{E}} \Rightarrow \underline{\sigma = \frac{Ne^2\tau}{m}} \text{ vodljivost}$$

→ QM učinci uzimaju se u obzir korištenjem m^*

(21) Opišite Hallov efekt. Izvedi izraz za gustoću nosilaca.



- stalno polje okomito na plohu pločastog vodiča → nosioci se zakreću → na jednoj strani se nakupljaju nosioci dok se ne izjednače polje (el.) i F_L, stvara se napon ovisno o kojem se određuje gustoća i predzn. nab.)

$e\mathcal{E}_y = evB_z \Rightarrow$ uvjet

$$v = \frac{\mathcal{E}_y}{B_z} = \frac{\vec{J}}{Ne} = \frac{I}{NeS} = \frac{I}{Newt} \quad \Big\} \ \mathcal{E}_y = \frac{V_H}{w}$$

$$\vec{F_L} = e\vec{\mathcal{E}} + e\vec{v}\times\vec{B}$$

$$\mathcal{E}_y \cancel{e} = \cancel{e}\vec{v}\times\vec{B_z}$$

$$\underline{eN = \frac{I}{wt}\frac{B_z}{\mathcal{E}_y}}$$

$$\vec{J} = Nev \Rightarrow v = \frac{\vec{J}}{Ne}$$

(22) Izrazi ukupnu vodljivost u poluvodiču koristeći izraza za pokretljivost nosioca naboja.

a) intrinzični

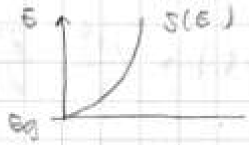
$$\vec{J} = \vec{J}_e + \vec{J}_h \; / : \vec{\mathcal{E}} \qquad J = \overbrace{\frac{Ne^2\tau}{m}}^{\sigma} \vec{\mathcal{E}} \qquad \mu_{e,h} = \frac{e\tau_{e,h}}{m^*_{e,h}}$$

$$\Rightarrow \sigma = \frac{N_e e^2 \tau_e}{m^*_e} + \frac{N_h e^2 \tau_h}{m^*_h} \qquad \sigma_{e,h} = \frac{e^2 N_{e,h}}{m^*_{e,h}} \tau_{e,h}$$

$$= e\left(\frac{e\tau_e}{m^*_e} N_e + \frac{e\tau_h}{m^*_h} N_h \right) = \underline{e(\mu_e N_e + \mu_h N_h)}$$

(23) Izvedi izraz za broj elektrona u intrinzičnom poluvodiču na temp. T.

$$S(E) \cdot f(E) = \frac{dn}{dE}$$



$$S(E) = \frac{8\pi \sqrt{2m_e^3}}{h^3} \cdot \sqrt{E-E_g}$$

$$f(E) = \frac{1}{\underbrace{e^{(E-E_F)/kT}}_{\gg 1 \text{ za velike } E} + 1} \approx e^{-(E-E_F)/kT} = e^{-(E-E_g)/kT} \cdot \underbrace{e^{(E_F-E_g)/kT}}_{\text{konst}}$$

$$N_e = \underbrace{\frac{8\pi\sqrt{2m_e^3}}{h^3} e^{(E_F-E_g)/kT}}_{= \text{konst. } C} \int_{E_g}^{\infty} \sqrt{E-E_g}\, e^{-(E-E_g)/kT} dE = \left\{ \begin{array}{l} \text{sup. } u = (E-E_g)/kT \\ du = dE/kT \\ \sqrt{E-E_g} = \sqrt{ukT} \\ E_g \to 0,\ \infty \to \infty \end{array} \right\}$$

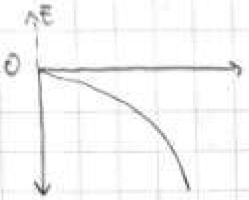
$$= C \cdot \int_0^{\infty} \sqrt{ukT}\, e^{-u}\, kT\, du = C \cdot \int_0^{\infty} u^{1/2} \cdot e^{-u} (kT)^{3/2} du$$

$$= C \cdot (kT)^{3/2} \int_0^{\infty} u^{1/2} e^{-u} du = \left| \text{TABL. INT.} = \frac{\sqrt{\pi}}{2} \right|$$

$$= \frac{8\pi\sqrt{2m_e^3}}{h^3} e^{(E_F-E_g)/kT} (kT)^{3/2} \frac{\sqrt{\pi}}{2}$$

$$= \frac{4\sqrt{2\pi^2 m_e^3}}{h^3} (kT)^{3/2} e^{(E_F-E_g)/kT}$$

(24) Izvedi izraz za broj šupljina u intrinsičnom poluvodiču na temp. T.



$$S(E) = \text{konst} \cdot \sqrt{-E}$$

$$S(E) \cdot f_h(E) = \frac{dn}{dE}$$

$$f_h(E) = 1 - f(E)$$

$$f_h(E) = 1 - \frac{1}{e^{(E-E_F)/kT}+1} = \frac{e^{(E-E_F)/kT}}{e^{(E-E_F)/kT}+1} = \frac{1}{1+\underbrace{e^{(E_F-E)/kT}}_{\gg 1,\ |E|>E_F}} \approx e^{-(E_F-E)/kT} = e^{-E_F/kT} \cdot e^{E/kT}$$

$$N_h = \frac{8\pi\sqrt{2m_h^3}}{h^3} e^{-E_F/kT} \int_{-\infty}^{0} e^{E/kT}\sqrt{-E}\, dE$$

$$= \left\{ \begin{array}{lll} \text{sup.} \ u = -E/kT & \sqrt{-E} = \sqrt{ukT} & 0 \to 0 \\ du = -dE/kT & & -\infty \to +\infty \end{array} \right\}$$

$$= \frac{8\pi\sqrt{2m_h^3}}{h^3} e^{-E_F/kT} \int_{+\infty}^{0} e^{-u}\sqrt{ukT}\,(-du\, kT)$$

$$= \frac{8\pi\sqrt{2m_h^3}}{h^3} e^{-E_F/kT} \int_{0}^{\infty} u^{1/2} e^{-u} \underbrace{(kT)^{3/2}}_{\text{konst}} du = \left\{ \text{tabl. int.} = \frac{\sqrt{\pi}}{2} \right\}$$

$$= \frac{8\pi\sqrt{2m_h^3}}{h^3} e^{-E_F/kT} \cdot (kT)^{3/2} \frac{\sqrt{\pi}}{2}$$

$$= \frac{4\sqrt{2\pi^3 m_h^3}}{h^3} (kT)^{3/2} e^{-E_F/kT}$$

(25) Izvedi izraz za Fermijev nivo u intrinz. poluvodiču ako

$$N_e = \frac{4\sqrt{2\pi^3 m_e^3}}{h^3} (kT)^{3/2} e^{(E_F-E_g)/kT} \quad \text{i} \quad N_h = \frac{4\sqrt{2\pi^3 m_h^3}}{h^3} (kT)^{3/2} e^{-E_F/kT}$$

− intrinz. poluvodič. $\Rightarrow$ $N_e = N_h$!

$$\frac{\cancel{4}\sqrt{\cancel{2\pi^3} m_e^3}}{\cancel{h^3}} \cancel{(kT)^{3/2}} e^{(E_F-E_g)/kT} = \frac{\cancel{4}\sqrt{\cancel{2\pi^3} m_h^3}}{\cancel{h^3}} \cancel{(kT)^{3/2}} e^{-E_F/kT}$$

$$m_e^{3/2} e^{E_F/kT} e^{-E_g/kT} = m_h^{3/2} e^{-E_F/kT}$$

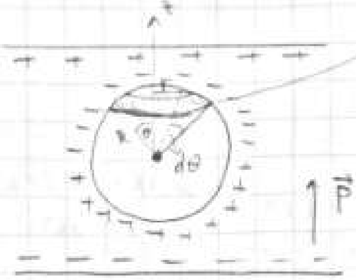
$$e^{2E_F/kT} e^{-E_g/kT} = \left(\frac{m_h}{m_e}\right)^{3/2} \quad / \ln$$

$$\frac{2E_F - E_g}{kT} = \frac{3}{2} \ln \frac{m_h}{m_e}$$

$$2E_F - E_g = kT \frac{3}{2} \ln \frac{m_h}{m_e}$$

$$E_F = \frac{E_g}{2} + \frac{3}{4} kT \ln\left(\frac{m_h}{m_e}\right)$$

(26) Izvedi izraz za Lorentzovo polje (polje na dipol u središtu kugle, zbog naboja na površini kugle, ~~kug~~ koji je nastao uslijed polarizacije $\vec{P}$).



$$dS = 2r\pi R\, d\theta \qquad r = R\sin\theta$$

$$\Rightarrow dS = 2R^2\pi \sin\theta\, d\theta$$

$$dQ = P\cos\theta\, dS$$

$$\Rightarrow dQ = (P\cos\theta\, dS)\cdot 2R^2\pi \sin\theta\, d\theta$$

u mjeri polarizacije $\vec{P}$

$$E_L = \int_0^{\pi} \frac{P\cos\theta \cdot 2R^2\pi \sin\theta\, d\theta}{4\pi\epsilon_0 R^2} \cdot \cos\theta$$

$$= \frac{P}{2\epsilon_0}\int_0^{\pi} \cos^2\theta \cdot d(-\cos\theta) = \left\{ \text{Sup.}\ z = \cos\theta \quad \begin{matrix} 0 \to 1 \\ \pi \to -1 \end{matrix} \right\}$$

$$= \frac{P}{2\epsilon_0}\int_{-1}^{1} z^2\, dz = \frac{P}{2\epsilon_0}\left.\frac{z^3}{3}\right|_{-1}^{1} = \frac{P}{2\epsilon_0}\frac{2}{3} = \frac{P}{3\epsilon_0}$$

$$\Rightarrow \vec{E_L} = \frac{\vec{P}}{3\epsilon_0}$$

(27) Izvedi Clausius-Mossottijevu jedn.

$$\vec{E}_{lok} = \vec{E_v} + \vec{E_L} \qquad (\vec{E_v} = \text{vanjsko el. polje; } \vec{E_L} \text{ Lorentzovo})$$

$$\vec{p} = \alpha \vec{E}_{lok} \qquad \vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E_v} = \epsilon_0 \chi_e \vec{E_v}$$

$$\vec{P} = N\vec{p} = N\alpha \vec{E}_{lok} \to \vec{E}_{lok}$$

$$\frac{\vec{P}}{N\alpha} = \frac{1}{N\alpha}\epsilon_0(\epsilon_r - 1)\vec{E_v} = \overbrace{\vec{E_v} + \vec{E_L}}^{\vec{E}_{lok}} = \vec{E_v} + \frac{\vec{P}}{3\epsilon_0} = \vec{E_v} + \frac{1}{3\epsilon_0}\left[\epsilon_0(\epsilon_r - 1)\vec{E_v}\right]$$

$$\frac{1}{N\alpha}\epsilon_0(\epsilon_r - 1)\vec{E_v} = \vec{E_v}\left[1 + \frac{1}{3\epsilon_0}\epsilon_0(\epsilon_r - 1)\right]$$

$$\frac{1}{N\alpha}\epsilon_0(\epsilon_r - 1) = 1 + \frac{1}{3}\epsilon_r - \frac{1}{3} = \frac{1}{3}(\epsilon_r + 2)$$

$$\Rightarrow \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

(28) Izvedi izraz za ~~magn.~~ dijamagnetsku susceptibilnost.

$$\vec{\mu} = -\frac{e\vec{L}}{2m}, \quad L = r_0 m v, \quad v = r_0 \omega$$

$$\vec{\mu} = -e\frac{v r_0}{2}\hat{n} = -e\frac{r_0^2 \omega}{2}\vec{n}$$

$$= -e^2 \frac{r_0^2}{2}\frac{\vec{B}}{2m}$$

$$\Rightarrow \vec{\mu}_{inducirano} = -e^2 \frac{r_0^2}{2m}\vec{B}$$

$\omega = \frac{eB}{2m}$ iz Farad. zakona

$$\oint \vec{E} d\vec{l} = -\frac{\partial \Phi_m}{\partial t}, \quad \Phi_m = \vec{B} d\vec{S}$$

$$\vec{E} \cdot 2\pi r_0 = -r_0^2 \pi \frac{d\vec{B}}{dt}$$

$$\vec{E} = -\frac{1}{2} r_0 \frac{d\vec{B}}{dt}$$

$$\vec{F} = -e\vec{E} = \frac{e}{2} r_0 \frac{d\vec{B}}{dt} = m\frac{d\vec{v}}{dt}$$

$$dv = r_0 d\omega$$

$$d\omega = \frac{e\,dB}{2m} \quad / \int$$

$$\omega - \omega_0 = \frac{eB}{2m} = \omega_L$$

Iznos magnetizacije $M = N \cdot \mu_i$

$$\vec{M} = -Ne^2 \frac{r_0^2}{4m}\vec{B}$$

$$\vec{M} = \chi_m \vec{H} = \chi_m \cdot \frac{\vec{B}}{\mu_0} = -Ne^2 \frac{r_0^2}{4m}\vec{B}$$

$$\Rightarrow \chi_m = -\mu_0 \frac{Ne^2 r_0^2}{4m}$$

→ za dijamagn. $\Rightarrow \quad r_0^2 = x^2 + y^2$

$$\left.\begin{aligned} &\overline{x^2+y^2} = \int \Psi^* (x^2+y^2) \Psi \, dV \\ &\overline{x^2} = \overline{y^2} = \overline{z^2} \Rightarrow \overline{x^2+y^2} = \frac{2}{3}\overline{r^2} \end{aligned}\right\} =$$

$$\Rightarrow \boxed{\chi_m^{dia} = -\mu_0 \frac{Ne^2 \overline{r^2}}{6m}}$$

(29) Od kojih se komponenti sastoji uk. magn. mom. el?

Izvedi izraz za magn. mom. el. μ_L uslijed orbitalnog gibanja el.

– uk. magn. moment je vekt. zbroj vlastitog i orbitalnog momenta

$$\vec{\mu} = I \cdot \vec{S}$$

$$I = -e\frac{\omega}{2\pi}, \quad \omega = \text{kr. frekv.}$$

$$\vec{S} = r^2 \pi \hat{n}, \quad \hat{n} \text{ okom. na ravn. el. putanje}$$

$$\vec{\mu} = -e\frac{\omega}{2\pi} \cdot r^2 \pi \hat{n} = -\frac{e r^2 \omega}{2}\hat{n}$$

$$\vec{L} = \vec{r} \times m\vec{v} \quad (\perp \text{ na ravn. rotacije}) = r^2 m \omega \hat{n}$$

$$\Rightarrow \boxed{\vec{\mu}_L = -\frac{e\vec{L}}{2m}}$$