valna jdžb el.mag. valova, u vakuumu

iz Maxwellovih jdžb :

1. $\vec{\nabla}\vec{E}=0$
2. $\vec{\nabla}\vec{B}=0$
3. $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$
4. $\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

<u>za $\vec{E}$</u> :

$$\frac{\partial\vec{E}}{\partial t}=\frac{1}{\mu_0\varepsilon_0}\vec{\nabla}\times\vec{B} \quad \Big/ \frac{\partial}{\partial t}$$

$$\frac{\partial^2\vec{E}}{\partial t^2}=\frac{1}{\mu_0\varepsilon_0}\vec{\nabla}\times\underbrace{\frac{\partial\vec{B}}{\partial t}}_{\text{iz }3}=\frac{-1}{\mu_0\varepsilon_0}\underbrace{\vec{\nabla}\times(\vec{\nabla}\times\vec{E})}=\frac{-1}{\mu_0\varepsilon_0}\Big[\vec{\nabla}(\underbrace{\vec{\nabla}\vec{E}}_{\text{iz }1})-\Delta\vec{E}\Big]$$

$$=\frac{1}{\mu_0\varepsilon_0}\Delta\vec{E} \rightarrow \quad \Delta\vec{E}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

<u>za $\vec{B}$</u> :

$$\frac{\partial\vec{B}}{\partial t}=-\vec{\nabla}\times\vec{E} \quad \Big/ \frac{\partial}{\partial t}$$

$$\frac{\partial^2\vec{B}}{\partial t^2}=-\vec{\nabla}\times\underbrace{\frac{\partial\vec{E}}{\partial t}}_{\text{iz }4}=-\vec{\nabla}\times\left(\frac{1}{\mu_0\varepsilon_0}\vec{\nabla}\times\vec{B}\right)=\frac{-1}{\mu_0\varepsilon_0}\Big[\underbrace{\vec{\nabla}\times(\vec{\nabla}\times\vec{B})}\Big]$$

$$=\frac{-1}{\mu_0\varepsilon_0}\Big[\vec{\nabla}(\underbrace{\vec{\nabla}\vec{B}}_{\text{iz }2})-\Delta\vec{B}\Big]=\frac{1}{\mu_0\varepsilon_0}\Delta\vec{B} \rightarrow$$

$$\Delta\vec{B}=\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

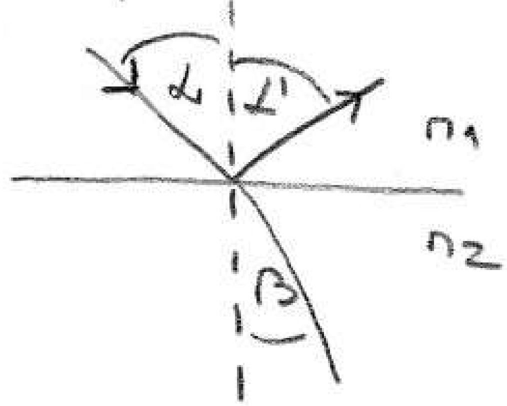
zakoni geometrijske optike :

1. zakon pravocrtnog širenja svj. - zr. su $\perp$ na valne fronte

2. zakon nezavisnosti svj. izvora - 2 snopa se prostiru bez međudjelovanja

3. zakon refleksije - na granici 2 sredstva, jedna zraka se ref., druga prolazi. Upadni kut (između upadne zr. i okomice) jednak je kutu ref. zrake

4. zakon loma (Snellov) $\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$



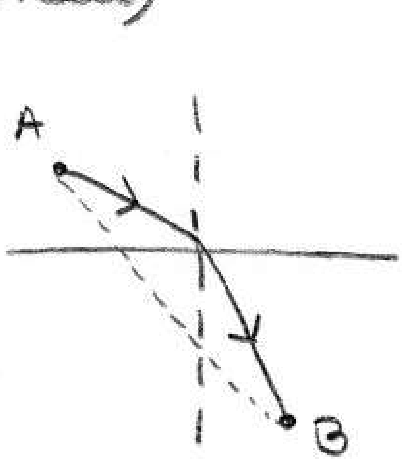
iz rjeđeg u gušće - lom + ref.
iz gušćeg u rjeđe - totalna ref.

$\beta = 90^\circ$ $\quad \sin\alpha_g = \frac{n_2}{n_1}$

Fermatov princip

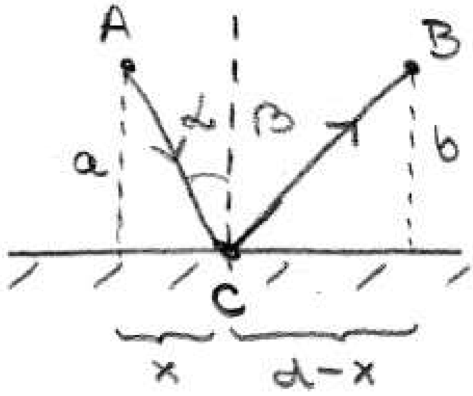
- svj. od jedne do druge točke prelazi put najkraćim vremenom (a ne najbližim putem)

$$t_{AB} = \int_A^B \frac{n}{c} dl$$



zakon refleksije iz Fermatovog principa

$$t_{AB} = \frac{\overline{AC} + \overline{CB}}{v} =$$



$$= \frac{1}{v}\left(\sqrt{x^2+a^2} + \sqrt{(d-x)^2+b^2}\right) \quad \Big/ \frac{d}{dx}$$

$x = ?$ za t_{min}

$$\frac{dt_{AB}}{dx} = \frac{1}{v}\left(\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}} + \frac{-\cancel{2}(d-x)}{\cancel{2}\sqrt{(d-x)^2+b^2}}\right) = 0$$

$$\frac{x}{\underbrace{\sqrt{x^2+a^2}}_{=\overline{AC}}} = \frac{d-x}{\underbrace{\sqrt{(d-x)^2+b^2}}_{=\overline{CB}}}$$

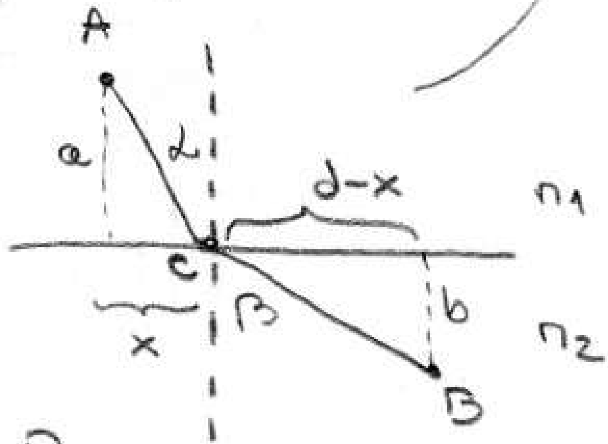
$$\sin\alpha = \frac{x}{\overline{AC}}$$

$$\sin\beta = \frac{d-x}{\overline{CB}}$$

$$\sin\alpha = \sin\beta \Rightarrow \alpha = \beta \Rightarrow x = \frac{d}{2}$$

zakon loma iz Fermatovog principa

$$t_{AB} = \frac{\overline{AC}}{v_1} + \frac{\overline{CB}}{v_2} =$$



$$= \frac{1}{v_1}\sqrt{x^2+a^2} + \frac{1}{v_2}\sqrt{(d-x)^2+b^2} \quad \Big/ \frac{d}{dx}$$

$$\frac{dt_{AB}}{dx} = \frac{\cancel{2}x}{\cancel{2}v_1\sqrt{x^2+a^2}} + \frac{-\cancel{2}(d-x)}{\cancel{2}v_2\sqrt{(d-x)^2+b^2}} = 0$$

$$\frac{1}{v_1}\sin\alpha = \frac{1}{v_2}\sin\beta$$

$$v_1 = \frac{c}{n_1}$$

$$v_2 = \frac{c}{n_2}$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

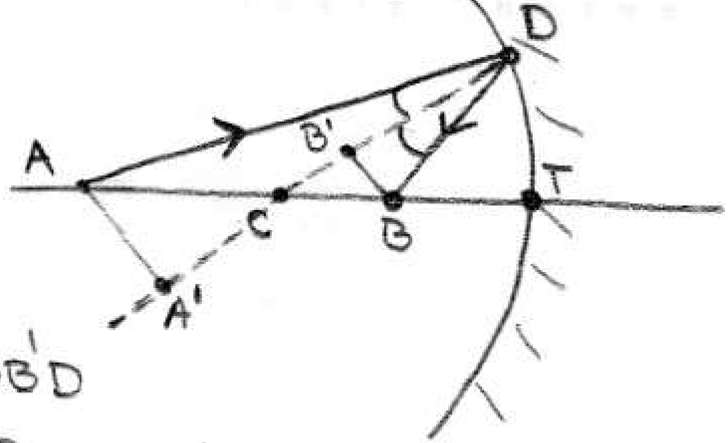
sferno zrcalo

$\overline{CT} = R$

$\overline{AT} = a$

$\overline{BT} = b$

$$m = -\frac{b}{a} = \frac{y'}{y}$$



$\Delta AA'C \approx \Delta BB'C$

$$\frac{AC}{AA'} = \frac{BC}{BB'}$$

$$\frac{AC}{BC} = \frac{AA'}{BB'}$$

$\Delta AA'D \approx \Delta BB'D$

$$\frac{AD}{AA'} = \frac{BD}{BB'}$$

$$\frac{AD}{BD} = \frac{AA'}{BB'}$$

$$\frac{AC}{BC} = \frac{AD}{BD}$$

$AD \approx AT$

$BD \approx BT$

Gaussove aprox.
D blizu T = paraksijalne zrake
oštra sl.
inače sf. aberacija

$$\frac{AC}{BC} = \frac{AT}{BT}$$

$$\frac{AC}{AT} = \frac{BC}{BT}$$

$$\frac{a-R}{a} = \frac{R-b}{b}$$

$ab - bR = aR - ab$

$2ab = aR + bR \quad / :ab$

$2 = \frac{R}{b} + \frac{R}{a} \quad / :R$

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{R} = \frac{1}{f}$$

žarišta

predmetno

$a \to f_a$

$b \to \infty$

$$\frac{1}{f_a} + \frac{1}{\infty} = \frac{2}{R}$$

$$f_a = \frac{R}{2}$$

slikovno

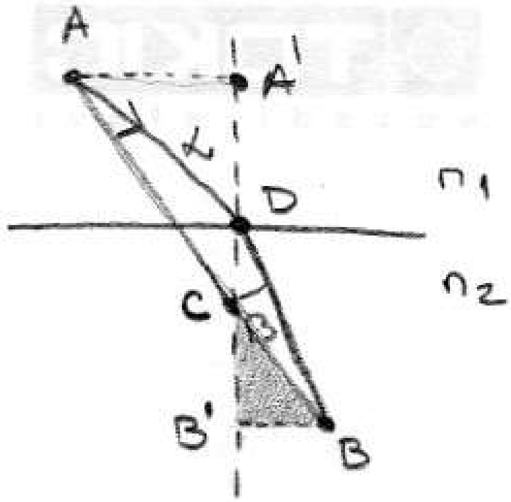
$b \to f_b$

$a \to \infty$

$$\frac{1}{\infty} + \frac{1}{f_b} = \frac{2}{R}$$

$$f_b = \frac{R}{2}$$

Möbiusov oblik zakona loma



$\Delta AA'C \simeq \Delta BB'C$

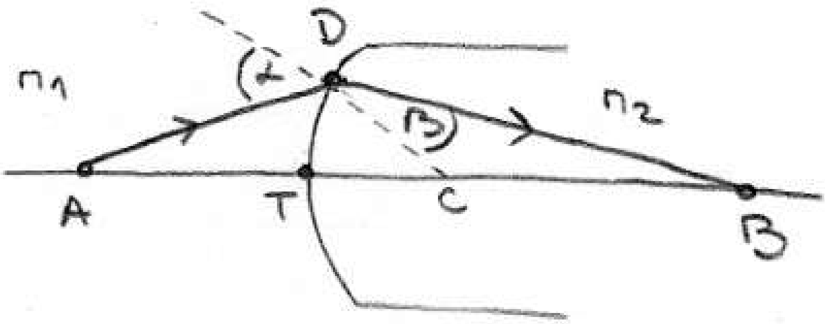
$$\frac{AC}{AA'} = \frac{BC}{BB'} \qquad \sin\alpha = \frac{AA'}{AD}$$

$$\sin\beta = \frac{BB'}{BD}$$

$$\frac{AC}{AD\sin\alpha} = \frac{BC}{BD\sin\alpha}$$

$$\frac{AC}{AD}n_1 = \frac{BC}{BD}n_2$$

lom na sfernom dioptru



Möbius $\frac{AC}{AD}n_1 = \frac{BC}{BD}n_2$

Gauss. ap. $AD \approx AT$, $BD \approx BT$

$BC = b - R$

$AC = a + R$

$$\frac{a+R}{a}n_1 = \frac{b-R}{b}n_2$$

$n_1(ab+bR) = (ab-aR)n_2$

$\frac{n_1}{R} + \frac{n_1}{a} = \frac{n_2}{R} - \frac{n_2}{b}$

$\overline{CT} = R$

$\overline{AT} = a$

$\overline{BT} = b$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

žarišta

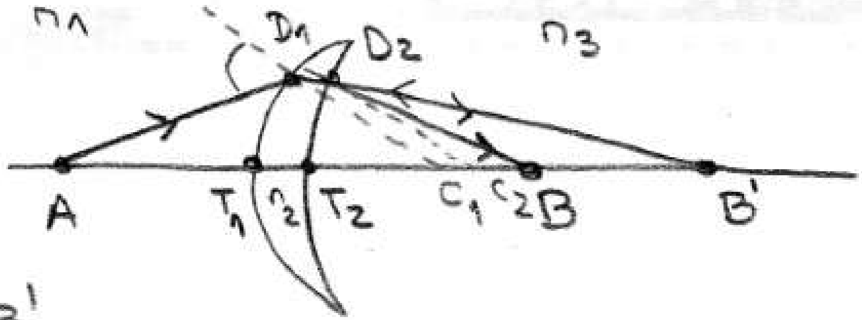
predmetno ← → slikovno

predmetno	slikovno
$a \to f_a$	$b \to f_b$
$b \to \infty$	$a \to \infty$
$f_a = \frac{n_1}{n_2 - n_1}R$	$f_b = \frac{n_2}{n_2 - n_1}R$

povećanje $\quad m = \frac{y'}{y} = -\frac{n_1}{n_2}\frac{b}{a}$

tanka leća

$\overline{T_1T_2} \to 0$



prvi lom $A \xrightarrow{n_1} D_1 \xrightarrow{n_2} B'$

$$\frac{n_1}{a}+\frac{n_2}{b'}=\frac{n_2-n_1}{R_1} \qquad (1)$$

$\overline{AT_1}=a$
$\overline{B'T_2}=b'$
$\overline{BT_2}=b$

drugi lom $B' \xrightarrow{n_2} D_2 \xrightarrow{n_3} B$

$$-\frac{n_2}{b'}+\frac{n_3}{b}=\frac{n_3-n_2}{R_2} \qquad (2)$$

$-\frac{n_2}{b'}$ jer je P desno od T

(1)+(2) $\quad \frac{n_1}{a}+\frac{n_3}{b}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$ ugl je $n_1=n_3=1$

i ako je $f_a=f_b=f$ $\to$ $\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$$

žarišta

predmetno ← → slikovno

$a \to f_a$
$b \to \infty$

$b \to f_b$
$a \to \infty$

$$\frac{n_1}{f_a}+\frac{n_2}{\infty}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2} \qquad \frac{n_1}{\infty}+\frac{n_2}{f_b}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$$

$$f_a=\frac{n_1R_1R_2}{R_2(n_2-n_1)+R_1(n_3-n_2)} \qquad f_b=\frac{n_3R_1R_2}{R_2(n_2-n_1)+R_1(n_3-n_2)}$$

$\frac{f_b}{f_a}=\frac{n_3}{n_1}$

$\frac{f_a}{a}+\frac{f_b}{b}=1$

F_a = mj. na opt. osi iz čijeg izlaze zr. čije su nakon loma || opt. os

F_b = mj. na opt. osi gdje se dobije slika predmeta koji je u ∞

interferencija koherentnih izvora svj.

$$E_1 = E_2 = E_0 \cos(\omega t - k_1 x) = E_0 \cos\left[\omega\left(t_0 - \frac{k_1}{\omega} x\right)\right] =$$

$$= E_0 \cos\left[\omega\left(t_0 - \frac{n_1 x_1}{c}\right)\right] \qquad \frac{\cancel{2\pi}}{\lambda \cancel{2\pi} f} = \frac{1}{\lambda_1 f} = \frac{1}{v_1} = \frac{n_1}{c}$$

$$E(t_0, x) = \vec{E}_1(t_0, x_1) + \vec{E}_2(t_0, x_2) = E_0 \cos\left(\omega\left(t_0 - \frac{n_1 x_1}{c}\right)\right) +$$

$$+ E_0 \cos\left[\omega\left(t_0 - \frac{n_2 x_2}{c}\right)\right] = \underbrace{2 E_0 \cos\left(\frac{\omega}{2c}(n_1 x_1 - n_2 x_2)\right)}_{\text{amp rezult. vala}} \cdot$$

$$\cdot \cos\left(\omega t_0 - \frac{\omega}{2c}(n_1 x_1 - n_2 x_2)\right)$$

$$\Delta \varphi = \frac{\omega}{c} \underbrace{(n_1 x_1 - n_2 x_2)}_{\delta} = \frac{2\pi}{\lambda} \delta \qquad \delta \text{ - opt. razlika hoda}$$

$$E_{or} = 2E_0 \cos \frac{\Delta \varphi}{2} \qquad \text{intenzitet} \sim (\text{amp.})^2$$

$$\cos \frac{\Delta \varphi}{2} = \pm 1 \quad \rightarrow \text{max}$$

$$\frac{\Delta \varphi}{2} = n\pi$$

$$\frac{1}{\cancel{2}} \frac{\cancel{2\pi}}{\lambda} \delta = n \cancel{\pi}$$

$$\delta_{max} = n\lambda \qquad \text{konstruktivna int.}$$

$$\cos \frac{\Delta \varphi}{2} = 0 \quad \rightarrow \text{min}$$

$$\frac{\Delta \varphi}{\cancel{2}} = (2n+1) \frac{\pi}{\cancel{2}}$$

$$\frac{2\cancel{\pi}}{\lambda} \delta = (2n+1) \cancel{\pi}$$

$$\delta_{min} = (2n+1) \frac{\lambda}{2} \qquad \text{destruktivna int.}$$

Youngov pokus

= otkrivena interferencija svj. (1800.g.)

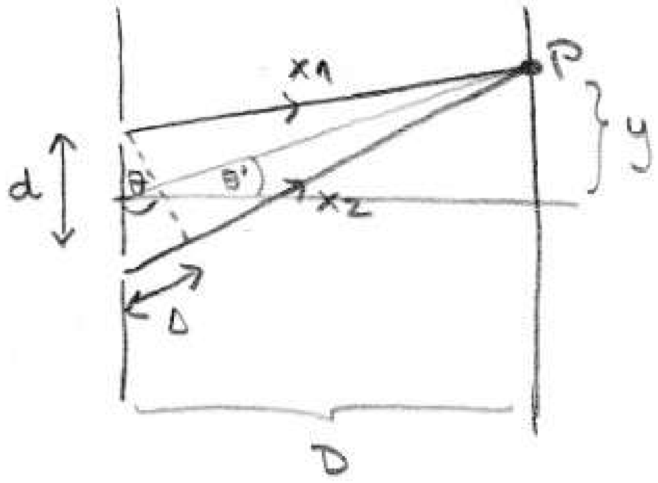
točkasti izvor monokromatske svj. + 2 pukotine

po Huygensovom principu, svaka pukotina je postala sekundarni izvor valova (koherentnih)

ud. zastora od p. je 1/2 m, a p. međusobno 10^{-4} m

$\delta = n(x_1 - x_2)$

$\Delta = d\sin\theta$ iz slike



max $\rightarrow$ $d\sin\theta = n\lambda$

min $\rightarrow$ $d\sin\theta = (2n+1)\dfrac{\lambda}{2}$

ovisi o položaju točke P

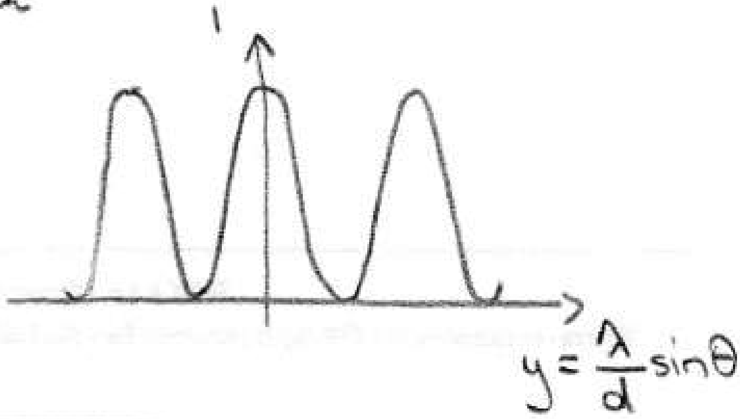
$\mathrm{tg}\,\theta' = \dfrac{y}{D}$ za $\theta' \ll$ $\mathrm{tg}\,\theta' \approx \sin\theta' \approx \theta'$

$\sin\theta' = \dfrac{y}{D}$ + $\sin\theta = \dfrac{n\lambda}{d}$

max $\rightarrow$ $y = n\dfrac{\lambda D}{d}$

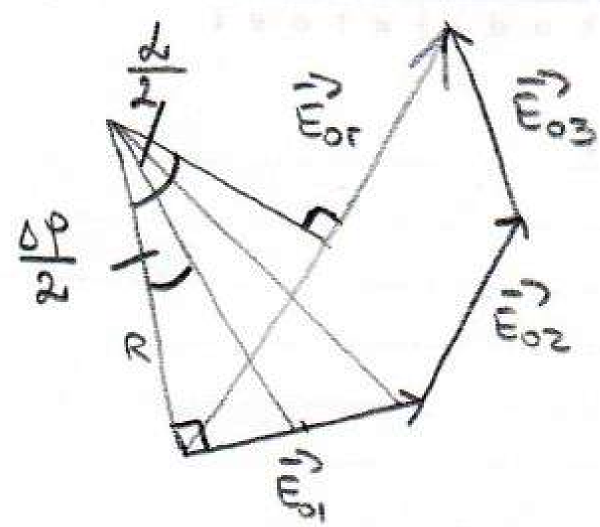
min $\rightarrow$ $y = (2n+1)\dfrac{\lambda D}{2d}$

razmak maksimuma $\Delta y = \dfrac{\lambda D}{d}$



Interferencija N izvora (optička rešetka)

3 izvora:



$$\sin\frac{\Delta\varphi}{2} = \frac{E_{on}/2}{R}$$

$$R = \frac{E_0}{2\sin\frac{\Delta\varphi}{2}}$$

$$\sin\frac{\alpha}{2} = \frac{E_{or}/2}{R}$$

$$E_{or} = 2R\sin\frac{\alpha}{2} = \cancel{2}\frac{E_0}{\cancel{2}\sin\frac{\Delta\varphi}{2}}\sin\frac{\alpha}{2} = \qquad \frac{\alpha}{2} = 3\frac{\Delta\varphi}{2}$$

$$= \frac{E_0}{\sin\frac{\Delta\varphi}{2}}\cdot\sin\frac{3\Delta\varphi}{2}$$

$$I \sim E_{or}^2 \qquad I = E_0^2\,\frac{\sin^2\frac{3\Delta\varphi}{2}}{\sin^2\frac{\Delta\varphi}{2}}$$

N izvora: $$I = E_0^2\,\frac{\sin^2\frac{N\Delta\varphi}{2}}{\sin^2\frac{\Delta\varphi}{2}}$$

$$\frac{\Delta\varphi}{2} = \frac{\pi d\sin\varphi}{\lambda}$$

$$E_{or} = 2E_0\cos\frac{\Delta\varphi}{2}$$

max $\rightarrow$ $\frac{\Delta\varphi}{2} = n\pi \qquad \sin^2\frac{\Delta\varphi}{2} = 0 \qquad d\sin\theta = m\lambda$

min $\rightarrow$ $\frac{\Delta\varphi}{2} = (2n+1)\pi \qquad \sin^2\frac{\Delta\varphi}{2} = \pm 1 \qquad d\sin\theta = \frac{m\lambda}{N}$

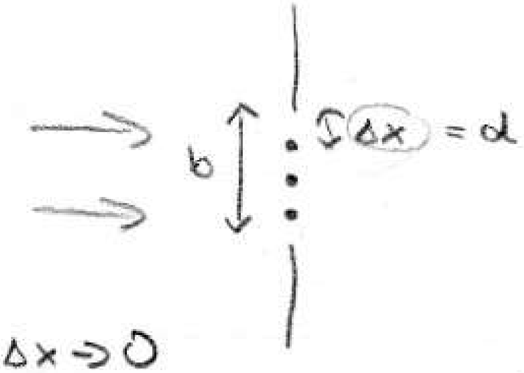
Fraunhofferov ogib

ogib (difrakcija) = kad valna fronta naiđe na prepreku deformira se i nastaje svjetlo u geom. sjeni
dimenzije prepreke $\sim \lambda$

a) Fresnelov - ud. izvor - zastor je konačna
b) Fraunhofferov - -||- jako velika - zrake su paralelne

$$b = N \cdot \Delta x$$

$$\Delta \varphi = \frac{2\pi}{\lambda} \Delta x \sin\alpha$$



$$I = E_0^2 \frac{\sin^2 \overbrace{\left(\frac{n}{\cancel{2}} \frac{\cancel{2}\pi}{\lambda} \Delta x \sin\alpha\right)}^{=\frac{n\Delta\varphi}{2}}}{\sin^2\left(\frac{1}{2}\frac{2\pi}{\lambda}\Delta x \sin\alpha\right)}$$

$\Delta x \to 0$
$N >>$

$$\sin\left(\frac{\pi}{\lambda}\Delta x \sin\alpha\right) \approx \frac{\pi}{\lambda}\Delta x \sin\alpha = \frac{\pi}{\lambda}\frac{b}{N}\sin\alpha$$

$$I = (E_0 N)^2 \frac{\sin^2\left(\frac{\pi}{\lambda} b \sin\alpha\right)}{\left(\frac{\pi}{\lambda} b \sin\alpha\right)^2}$$

$$I = I_0 \frac{\sin^2 y}{y^2} \quad / \frac{d}{dy}$$

$$\frac{\cancel{2y^2 \sin y} \cos y - \cancel{2y} \sin^2 y}{y^4} = 0$$

$$y \cos y = \sin y$$

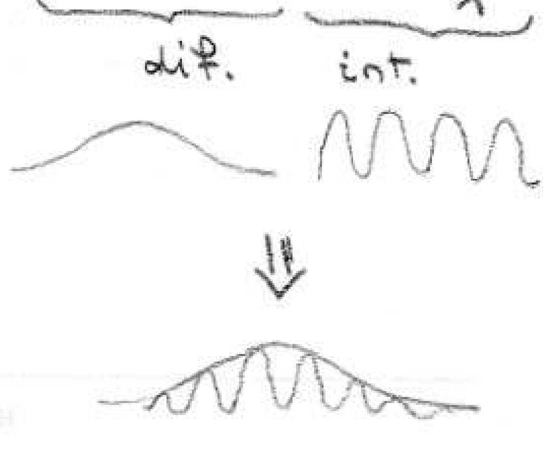
$$y = \operatorname{tg} y$$

max $\to$ $b \sin\alpha = (2m+1)\frac{\lambda}{2}$

min $\to$ $b \sin\alpha = m\lambda$

intenzitet optičke rešetke

$$I = I_0 \underbrace{\frac{\sin^2\left(\frac{\pi}{\lambda} b \sin\alpha\right)}{\left(\frac{\pi}{\lambda} b \sin\alpha\right)^2}}_{\text{dif.}} \cdot \underbrace{\frac{\sin^2\left(\frac{N\pi d}{\lambda}\sin\right.}{\sin^2\left(\frac{\pi d}{\lambda}\sin\right.}}_{\text{int.}}$$



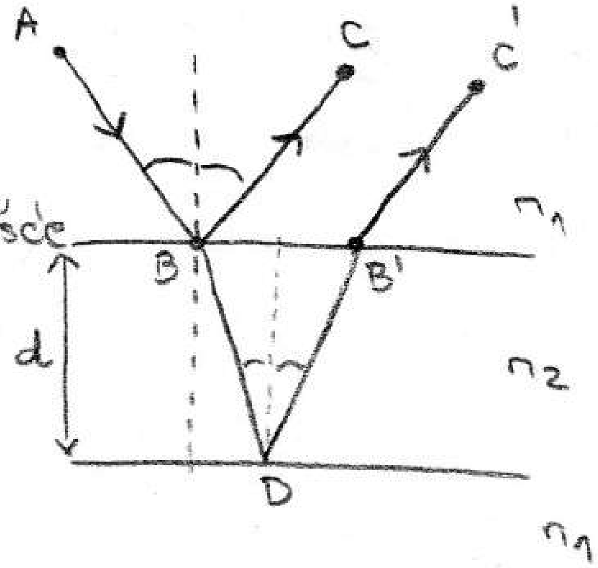
interferencija na tankim listićima

svj pada skoro $\perp$ na tanki slj mat.

optički put koji prijeđe val 1 $L_1 = n_1\overline{AB} + n_1\overline{BC}$

-||-   val 2 $L_2 = n_1\overline{AB} + n_2\overline{BD} + n_2\overline{DB'} + n_1\overline{B'C'}$

$n_2 > n_1$



$\overline{BC} \approx \overline{B'C'}$

$\overline{BD} \approx \overline{DB'} = d$

$\varphi = \omega t - kx$ iz rjeđeg u gušće

$\varphi_1 = \cancel{\omega t} - \frac{2\pi}{\lambda}(\cancel{n_1 AB} + \cancel{n_1 BC}) + \pi$

$\varphi_2 = \cancel{\omega t} - \frac{2\pi}{\lambda}(\cancel{n_1 AB} + n_2 2d + \cancel{n_1 B'C'})$

$\Delta\varphi = \varphi_2 - \varphi_1 = -\frac{2\pi}{\lambda}(2dn_2 - \pi)$

$E_{or} = 2E_0 \cos\left(\frac{\Delta\varphi}{2}\right)$

max $\rightarrow$ $\frac{2\pi d n_2}{\lambda} - \pi = m\pi$ $d = \frac{(m+1/2)\lambda}{2n_2}$

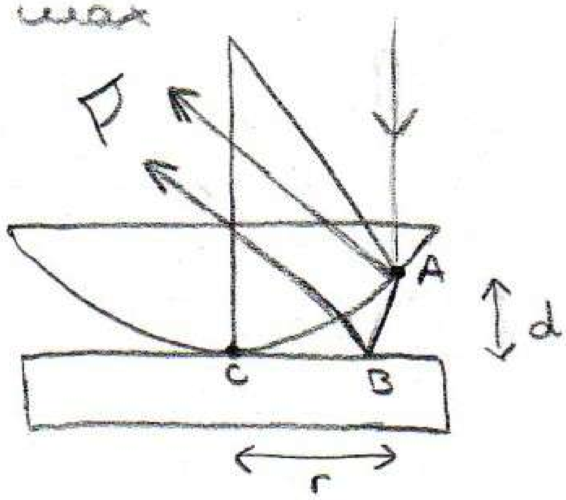
min $\rightarrow$ $\frac{2\pi d n_2}{\lambda} - \pi = (2m+1)\frac{\pi}{2}$ $d = \frac{m\lambda}{2n_2}$

u ovisnosti o debljini sloja, pojedine λ će se poništavati a pojedine pojačavati, pa se od bijele svj. javljaju razl. boje

Newtonovi kolobari

zbog djelomične ref. stvara se geom. razlika hoda između 2 vala koji mogu int. i dati min i max

- N.K. u reflektiranoj svj.



svj. val se u A podijeli na 2 zr. - jedna se ref. na leći, jedna na pl.par. ploči - prijeđe $2\times\overline{AB} = 2d$ + ref. na gušćem sred.

fazna razlika vala 1 i 2:

$$|\Delta\varphi| = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda}\cdot 2d + \pi$$

$$E_{or} = 2E_0\cos\left(\frac{\Delta\varphi}{2}\right)$$

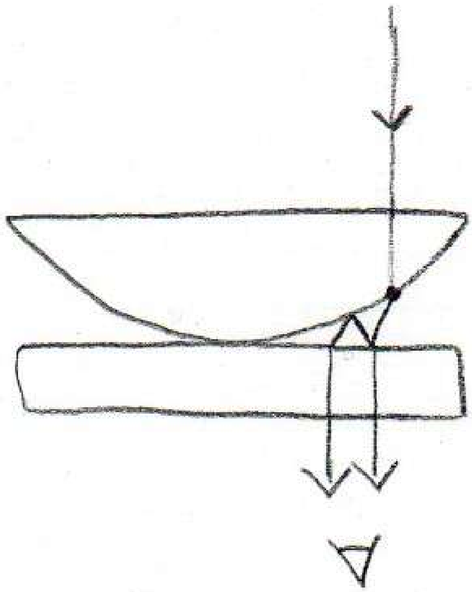
max $\rightarrow$ $\frac{\Delta\varphi}{2} = n\pi$ $\quad d = \frac{1}{2}(2m-1)\frac{\lambda}{2}$

min $\rightarrow$ $\frac{\Delta\varphi}{2} = (2n+1)\frac{\pi}{2}$ $\quad d = \frac{1}{2}m\lambda$

- N.K. u transmitiranoj svj.

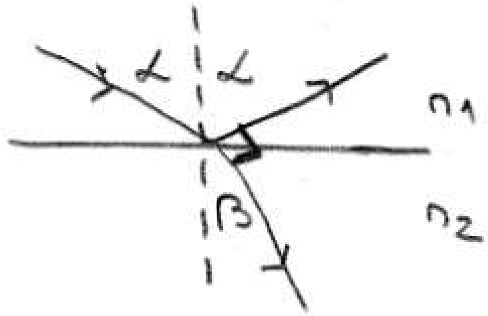
val se $2\times$ ref. na gušćem sred., ukupna promjena u fazi $= 2\pi$

tamni su postali svijetli i obrnuto



Načini polarizacije :

1. refleksijom = svj. pada na prozirno sred., ako lomljena i ref. zr. tvore pravi kut, ref. zr. je polarizirana ⊥ na ravninu refleksije



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

$$\frac{\sin\alpha}{\sin(90^\circ-\alpha)} = \frac{n_2}{n_1}$$

$$tg\,\alpha = \frac{n_2}{n_1}$$ uvjet da ref. zr. bude pol.

↳ Brewsterov zakon

2. raspršenjem = na mol. zraka, vod. pare, prašine
jače se raspršuje svj. malih λ (plava)

$$I \sim \frac{1}{\lambda^4}$$

3. materijalima (dvolom) = posljedica neizotropnosti krist.
svj. se lomi na gr. kr. tako da nastaju 2 zrake – jedna redovna (po Snell-u) jedna izvanredna; obe zr. su pol., rav. pol. su im okomite
npr islandski dvolomac

4. dikroizam (selektivna aps.) = jednu zr. propusti, drugu aps. – smanji joj se int., ona koja prođe je pol.
npr turmalin

$I(\varphi) = I_0 \cos^2\varphi$ Malusov zakon

ako je I_0 nepol. $I = \frac{1}{2} I_0$

Planckov zakon

crno tijelo = apsorbira upadno zračenje

$$I = \int_0^\infty f(\lambda,T)\,d\lambda = \sigma T^4 \quad \text{Stefan-Boltzmannov z.}$$

$f(\lambda,T)$ - spektralna gustoća zračenja = ?

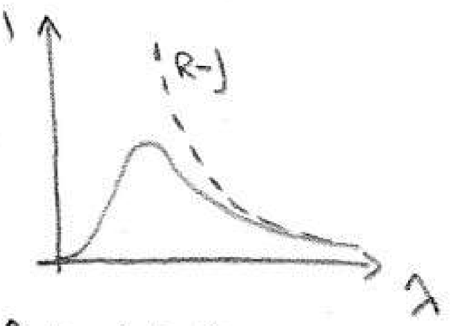
Rayleigh-Jeansova formula = $f(\lambda,T) = \frac{2\pi c}{\lambda^4}\bar{E} = \frac{2\pi c}{\lambda^4}kT$

Planck:

$E_0 = h\nu$, $h = 6.626\cdot 10^{-34}$ Js

$= \frac{hc}{\lambda}$ kvant energije (nije kont., nego diskretna)

$E = nh\nu$, $\nu = f$



λ<< I→∞
UV katastrofa

$$f(\lambda,T) = \underbrace{\frac{2c\pi}{\lambda^4}}_{R-J} \underbrace{\frac{hc}{\lambda}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}}_{=\bar{E}}$$

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-\frac{E_n}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{E_n}{kT}}} = \frac{hf\sum_{n=0}^{\infty} n e^{-\frac{nhf}{kT}}}{\sum e^{-\frac{nhf}{kT}}} = \frac{hf\sum n x^n}{\sum x^n} = hf\frac{x+2x^2+\dots nx^n}{1+x+\dots x^n} =$$

$e^{-\frac{hf}{kT}} = x$

$$= hfx\frac{1+2x+\dots nx^{n-1}}{1+x+\dots x^n} = \frac{hfx}{1-x} =$$

$1+2x+\dots nx^{n-1} = (\text{naz.})' = \frac{1}{(1-x)^2}$

$1+x+\dots x^n = \sum x^n = \frac{1}{1-x}$

$$= hf\frac{e^{-\frac{hf}{kT}}}{1-e^{-\frac{hf}{kT}}} \begin{matrix} : e^{-} \\ : e^{-} \end{matrix} = hf\frac{1}{\frac{1}{e^{-}}-1} = hf\frac{1}{e^{\frac{hf}{kT}}-1}$$

srednja en. kvantnog oscilatora

fotoefekt + Einsteinova jdžb

= neki metali ispuštaju neg. naboje kad ih se rasvijetli

svj. ulazi kroz kvarcni prozor i pada na foto-katodu iz koje izlaze e^-, priključena na izvor struje

1. ako je za upadnu svj. f = konst i U_{izvor} = konst → $I \sim$ int. svj.
2. za neke met. e^- se izbacuju samo za $f > \nu_g$ bez obzira na int. svj.
3. zaustavni napon = min. razlika pot. K i A koja spriječi e^-
4. int. svj. ↓ br. foto e^- ↓ ali su izbačeni istone E_K

$$E_\gamma = h\nu = h\frac{c}{\lambda} = W + E_{KIN}$$

e^- u metalu aps. kvant en. svj.; ako je en. dovoljno velika dio se troši na izlazni rad, a dio preda e^-

$h\nu_g = W$ granična frekv., e^- nema en. da napusti m.

max $E_{KIN} = eV_z$ → $h(\nu - \nu_g) = E_{KIN}$, $f\uparrow E\uparrow$

kol. gib. fotona $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

$\nu > \nu_g$ nastat će fotoefekt bez obzira na int. svj.

$$\nu_g = \frac{W_i}{h} \quad W_i \sim 2\text{-}6\ eV$$

Comptonov efekt + formula

= pri raspršenju X ili γ zraka na ugljiku u raspršenim valovima se osim originalne frekv. (λ) javlja i komp. $\neq$ frekv. upadne svj. i pomak u frekv. ne ovisi o valnoj duljini upadne svj. ni vrsti mat.

sudar e^- i fotona:

en. upadnog fotona $E = h\nu = \frac{hc}{\lambda}$

kol. gib. -||- $p = \frac{E}{c} = \frac{h}{\lambda}$

kol. gib. izlaznog f. - hor. i vert. komp. $p'_h = \frac{h}{\lambda'}\cos\theta$ $\quad p'_v = \frac{h}{\lambda'}\sin\theta$

en. mirovanja e^- $\quad E = mc^2$

en. e^- nakon sudora $\quad E_k + mc^2 = \gamma mc^2$ $\quad p'_{eh} = \gamma mv\cos\alpha$ $\quad p'_{ev} = \gamma mv\cos\alpha$

ZOE $= \quad \frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \gamma mc^2$

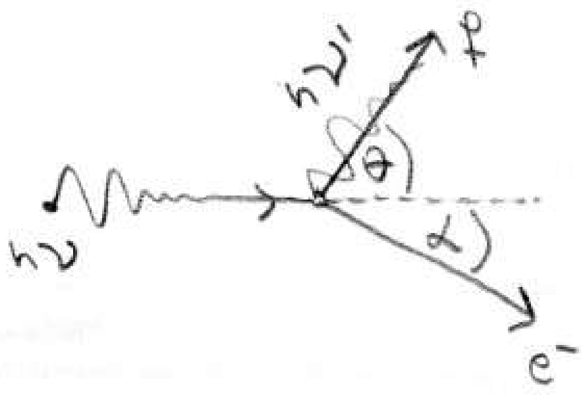
ZOKG: $\quad \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma mv\cos\alpha$ $\quad$ hor.

$0 = \frac{h}{\lambda'}\sin\theta - \gamma mv\sin\alpha$ $\quad$ ver.

$$\left.\begin{aligned} \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta &= \gamma mv\cos\alpha \quad /^2 \\ \frac{h}{\lambda'}\sin\theta &= \gamma mv\sin\alpha \quad /^2 \end{aligned}\right\} + \quad = *$$

ZOE^2 :
$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'} + \frac{2mch}{\lambda\lambda'}(\lambda - \lambda') = \gamma^2 m^2 v^2$$

ZOE^2 $-$ * :
$$\lambda - \lambda' = \frac{h}{mc}(1 - \cos\theta)$$



λ - valna d. upadnog f.

λ' - valna d. raspršenog f.

θ - smjer između up. i rasp. f.

m - masa e^-

Bohrovi postulati :

1. e^- se može gibati oko jezgre određenim kružnim stazama i pri tome ne zrači (stac. st.)

2. Bohrov kvantni uvjet = dozvoljena stanja :
$$L = rmv = n\frac{h}{2\pi} = n\hbar$$

3. kad e^- skoči s više staze na nižu zrači foton čija je en. $h\nu = E_V - E_N$ -> Balmerova formula :

$$\frac{c}{\lambda} = \nu = \frac{E_V}{h} - \frac{E_N}{h} = \frac{-E_1}{hn_V^2} - \frac{-E_1}{hn_N^2} = \frac{E_1}{h}\left(\frac{1}{n_N^2} - \frac{1}{n_V^2}\right)$$
$$\frac{1}{\lambda} = \frac{E_1}{hc}\left(\frac{1}{n_N^2} - \frac{1}{n_V^2}\right) = R\left(\frac{1}{n_N^2} - \frac{1}{n_V^2}\right)$$

E_1 - en. ionizacije (en. najnižeg st.) $= -13.6\,eV$

Kvantizacija energije

sile na e^- u stac. st. : $F_{coulomb} = F_{cp}$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} = \frac{mv_n^2}{r_n}$$

iz P2 $v_n = \frac{n\frac{h}{2\pi}}{mr_n}$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\cancel{r_n^2}} = \frac{\cancel{m}\frac{n^2h^2}{4\pi^2}}{\cancel{r_n^2}m^{\cancel{2}}\cdot r_n}$$

$$\frac{r_n}{4\pi\varepsilon_0}e^2 = \frac{n^2}{m}\frac{h^2}{4\pi^2} = \hbar^2$$

$$r_n = n^2\frac{4\pi\varepsilon_0}{me^2}\hbar^2$$ polumjer n-te staze

$$r_n = n^2 r_1$$

$$v_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0\hbar} = \frac{v_1}{n}$$

$$E_n = E_K + E_P = \frac{1}{2}mv_n^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{1}{n^2}\frac{me^4}{32\pi^2\varepsilon_0\hbar^2} = \frac{-E_1}{n^2}$$

zakon radioaktivnog raspada

$$\text{aktivnost} = -\frac{dN}{dt}$$ brzina raspada radioakt. mat. [Bq]

$$-\frac{dN}{dt} = \lambda N$$

N - br. nestabilnih jezgri u uzorku
λ - konst. raspada

$$\frac{dN}{N} = -\lambda dt \quad / \int$$

$$\int_{N_0}^{N(t)} \frac{dN}{N} = \int_0^t -\lambda dt$$

$$\ln N \Big|_{N_0}^{N(t)} = -\lambda t$$

$$\ln \frac{N(t)}{N_0} = -\lambda t \quad / e$$

$$N(t) = N_0 e^{-\lambda t}$$

N_0 - poč. br. jezgri
N - br. neraspadnutih j. nakon t

$$A(t) = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = \lambda N(t)$$

vrijeme poluraspada = vr. potrebno da se pola jezgre raspadne ($t \to T_{1/2}$, $N \to N_0/2$)

$$\frac{\cancel{N_0}}{2} = \cancel{N_0} e^{-\lambda T_{1/2}} \quad / \ln$$

$$\ln 2 = \lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$