Kartezijeve koordinate

Gradijent: $$\nabla\Phi = \frac{\partial\Phi}{\partial x}\hat{\imath} + \frac{\partial\Phi}{\partial y}\hat{\jmath}\frac{\partial\Phi}{\partial z}\hat{k}$$

Divergencija: $$\nabla\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Rotacija: $$\nabla\times\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k} \qquad \text{(DP6.5-2)}$$

Laplasijan: $$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Element duljine: $d\vec{\ell} = \hat{\imath}dx + \hat{\jmath}dy + \hat{k}dz$

Element površine: $dS = dx\,dy$ ili $dS = dx\,dz$ ili $dS = dy\,dz$ (DP6.5-3)

Element volumena: $dV = dx\,dy\,dz.$

Cilindrične koordinate

Gradijent: $$\nabla\Phi = \frac{\partial\Phi}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}\hat{\phi} + \frac{\partial\Phi}{\partial z}\hat{k}$$

Divergencija: $$\nabla\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

Rotacija: $$\nabla\times\vec{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{\rho} + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right]\hat{k} \qquad \text{(DP6.5-4)}$$

Laplasijan: $$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}$$

Element duljine: $d\vec{\ell} = \hat{\rho}d\rho + \hat{\phi}\rho d\phi + \hat{k}dz$

Element površine: $dS = \rho d\rho\,d\phi$ ili$dS = \rho d\phi\,dz$ ili $dS = d\rho\,dz$ (DP6.5-5)

Element volumena: $dV = \rho d\rho\,d\phi\,dz$

Sferne koordinate

Gradijent: $$\nabla\Phi = \frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\hat{\phi}$$

Divergencija: $$\nabla\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, A_\theta)}{\partial\theta}$$

Rotacija: $$\begin{aligned}\nabla\times\vec{A} &= \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\, A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\hat{r}\\ &+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(r\, A_\phi)}{\partial r}\right]\hat{\theta}\\ &+ \frac{1}{r}\left[\frac{\partial(r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}\end{aligned} \qquad \text{(DP6.5-6)}$$

Laplasijan: $$\begin{aligned}\Delta &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\\ &+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\end{aligned}$$

$$\begin{aligned}&\text{Element duljine:} && d\vec{\ell} = \hat{r}\, dr + \hat{\phi} r\sin\theta d\phi + \hat{\theta} r d\theta\\ &\text{Element površine:} && dS = r^2\sin\theta\, d\theta\, d\phi\\ &\text{Element volumena:} && dV = r^2\sin\theta\, dr\, d\theta\, d\phi.\end{aligned} \qquad \text{(DP6.5-7)}$$

Operator *nabla* može djelovati na kombinacije vektorskih i skalarnih funkcija odnosno može višestruko djelovati na neku kombinaciju skalarnih i vektorskih funkcija. Slijedi niz korisnih teorema koji opisuju takvo djelovanje operatora *nabla*.

$$\begin{aligned}\nabla(f_1(r) + f_2(r)) &= \nabla f_1(r) + \nabla f_2(r) & \nabla\cdot(f\vec{A}) &= f(\nabla\vec{A}) + (\nabla f)\cdot\vec{A};\\ \nabla(\vec{A}_1 + \vec{A}_2) &= \nabla\vec{A}_1(r) + \vec{A}_2(r) & \nabla\times(f\vec{A}) &= f(\nabla\times\vec{A}) + (\nabla f)\times\vec{A};\\ \nabla\times(\vec{A}_1 + \vec{A}_2) &= \nabla\times\vec{A}_1 + \nabla\vec{A}_2 & \vec{A}\times(\nabla\times\vec{A}) &= \frac{1}{2}\nabla(A^2) - (\vec{A}\cdot\nabla)\vec{A}\end{aligned} \qquad \text{(DP6.5-8)}$$

$$\begin{aligned}\nabla\cdot(\vec{A}_1\times\vec{A}_2) &= \vec{A}_2\cdot(\nabla\times\vec{A}_1) - \vec{A}_1\cdot(\nabla\times\vec{A}_2)\\ \nabla(\vec{A}\times\vec{B}) &= \vec{B}(\nabla\times\vec{A}) - \vec{A}(\nabla\times\vec{B})\\ \nabla\times(\vec{A}\times\vec{B}) &= \vec{A}(\nabla\vec{B}) - \vec{B}(\nabla\vec{A}) + (\vec{B}\nabla)\vec{A} - (\vec{A}\nabla)\vec{B}\\ \nabla(\vec{A}\cdot\vec{B}) &= (\vec{B}\cdot\nabla)\vec{A} + (\vec{A}\cdot\nabla)\vec{B} + \vec{B}\times(\nabla\times\vec{A}) + \vec{A}\times(\nabla\times\vec{B})\\ &= \vec{B}(\nabla\cdot\vec{A}) + \vec{A}(\nabla\cdot\vec{B}) + (\vec{B}\times\nabla)\times\vec{A} + (\vec{A}\times\nabla)\times\vec{B}\end{aligned} \qquad \text{(DP6.5-9)}$$