7.5. $P = 10\ W$

$\lambda = 5 \cdot 10^{-7}\ m$

$\phi = 60\ s^{-1}$

$d = 0{,}5\ cm = 5 \cdot 10^{-3}\ m \Rightarrow r = 2{,}5 \cdot 10^{-3}\ m \Rightarrow S = r^2\pi = 19{,}6 \cdot 10^{-6}\ m^2$

$\phi = I \cdot \omega$

$E = \frac{P}{S} = 0{,}5$

Zadaća

1. $f = 600\ Hz$

$r = 1\ m$

$\omega = 100\ min^{-1} = \frac{5}{3}\ s^{-1}$

$v = \pm r\omega \cdot 2\pi$

$f_{max} = \frac{v_z}{v_z - v} f_0$

$= 618{,}89\ Hz$

$f_{min} = f_0 \frac{v_z}{v_z + v}$

$f_{min} = 582{,}22\ Hz$

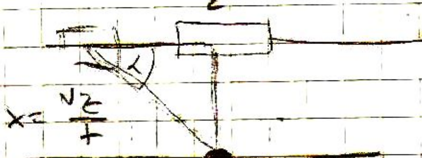
2. $v_i = \frac{v_z}{2}$

$f_1 = 100\ Hz$

$f_1 = f \cdot \frac{v_z}{v_z - v_a \cdot \cos\alpha} = f \cdot \frac{\cancel{v_z}}{\cancel{v_z}(1 - \frac{\cos\alpha}{2})} =$

$= f \cdot \frac{1}{\frac{3}{4}} = \frac{4}{3} f = 133\ Hz$

$s = \frac{v_z}{2} \cdot t$



$x = \frac{v_z}{t}$

$\cos\alpha = \frac{\frac{v_z}{2} \cancel{t}}{\cancel{v_z} \cancel{t}} = \frac{1}{2}$

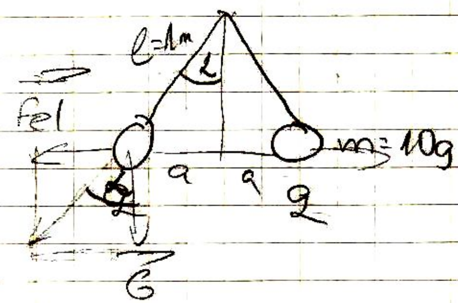
$$f' = f \cdot \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \qquad u'_x = \frac{u_x - v}{1 - \frac{u_x \cdot v}{c^2}}$$

1. brod

2. brod

3.

$$F_{el} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{d^2}$$



$$\sin\alpha = \frac{a}{l} = \frac{0,1}{1} = 0,1$$

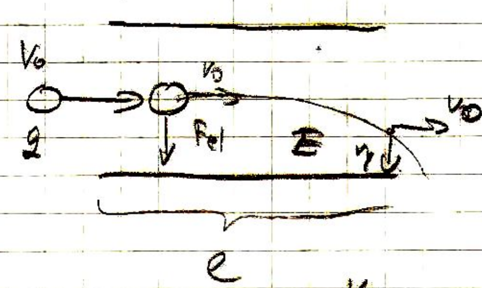
$$\alpha = 5,74^\circ$$

$$F_{el} = G \cdot tg\alpha = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(2a)^2} \Rightarrow q = \sqrt{G tg\alpha \cdot 4\pi\varepsilon_0 \cdot 4a^2} = A$$

$$Q = \frac{A}{q_e} = q_e = 4,135 \cdot 10^{11}\ e \qquad 1,3 \cdot 10^{12}$$

4.

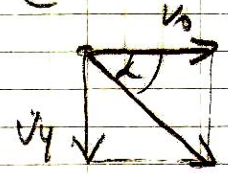
$$tg\alpha = \frac{v_y}{v_0}$$



$$F_{el} = m \cdot a$$

$$q \cdot E = m \cdot a$$

$$a = \frac{q \cdot E}{m}$$



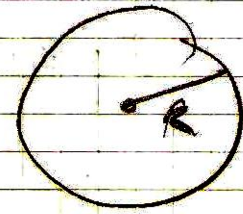
$$l = v_0 \cdot t$$

$$t = \frac{l}{v_0}$$

$$v_y = a\,t = \frac{q \cdot E}{m} \cdot \frac{l}{v_0}$$

$$tg\alpha = \frac{qEl}{m v_0^2}$$

5. $\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right)$



$E(r) = ?$

$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \cdot \rho$

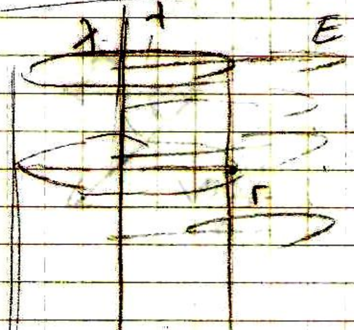
$\underbrace{\frac{\partial}{\partial x} \vec{E} \cdot \vec{i}}_{E_x} + \underbrace{\frac{\partial}{\partial y} \vec{E} \cdot \vec{j}}_{E_y} + \underbrace{\frac{\partial}{\partial z} \vec{E} \cdot \vec{k}}_{E_z}$

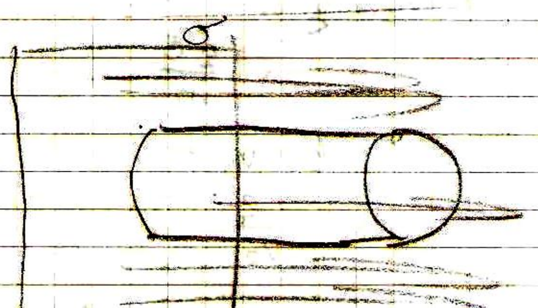
Gaussova površina je zatvorena površina, u svakoj njezinoj točki je E okomit na nju

$\iiint_V \vec{\nabla} \vec{E} \, dV = \frac{1}{\varepsilon_0} \iiint \rho \, dV$

$\oint_V \vec{E} \cdot d\vec{a} = E \oint_S da$, $\quad d\vec{a} \to \vec{n} \, da$

$E \overbrace{\oint_S da}^{4r^2\pi} = \frac{1}{\varepsilon_0} \iiint_V \rho \, dV$





$E \oint da = \frac{1}{\varepsilon_0} \iiint_V \rho \, dV$

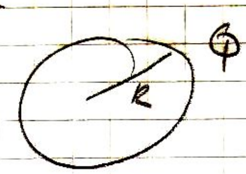
$E \oint_S da = \frac{1}{\varepsilon_0} \iiint_V \rho \, dV$ $\qquad$ da je homogena $\left(\frac{r}{R}\right)^3$

Za $\vec{r}$ gustoća opada linearno, kubni rast volumena

$Q' = Q \left(\frac{r}{R}\right)^2$

$E \cdot 4r^2\pi = \frac{1}{\varepsilon_0} \cdot Q \left(\frac{r}{R}\right)^2 \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2}$

6. $E(r) = E_{max}\left(\frac{r}{R}\right)^n$ $\qquad n \geq 2$



$E(r) = E_{max}\left(\frac{r}{R}\right)^n$

$\rho(r) = f(Q, R)$

Gausova ploha koja obuhvaća ~~...~~ kugli

$\vec{\nabla}\vec{E} = \frac{1}{\varepsilon_0}\rho$

$\rho = \varepsilon_0 \cdot \vec{\nabla} \vec{E}$

ako gledamo samo u jednom smjeru:

$\rho = \varepsilon_0 \cdot \vec{\nabla}\vec{E} = \varepsilon_0 \cdot \frac{\partial E}{\partial x} = \varepsilon_0 \cdot E_{max} \cdot \frac{\partial}{\partial x}\left(\frac{x}{R}\right)^n$

$\rho = \varepsilon_0 \cdot E_{MAX} \cdot \frac{n}{R^n} \cdot x^{n-1}$

$E_{max} \cdot 4R^2 \cdot \pi = \frac{1}{\varepsilon_0} \cdot Q$

$E_{max} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2}$

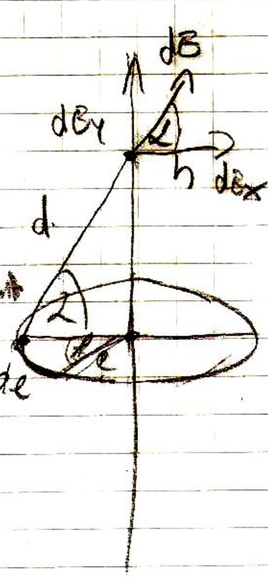
7. Q

$l = 2R\pi$

$\lambda = \frac{Q}{l} \Rightarrow Q = \lambda \cdot l$

linijski naboj

Horizontalne komponent(e) se zbog simetričnosti kružnice poništavaju



$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{d^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda l}{d^2}$

$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{d^2} \cdot dl$

$\sin\lambda = \frac{h}{d} = \frac{E_y}{E} \Rightarrow E_y = E \cdot \frac{h}{d}$

$dE_y = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{d^2} \cdot dl \cdot \frac{h}{d}$

$dE_y = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda h}{d^3} \cdot R\,d\varphi$

$E_y = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda h}{d^3} R \int_0^{2\pi} d\varphi$

2π

$dl = R \cdot d\varphi$

$E_y = \frac{\lambda h R}{2\varepsilon_0 d^3} = \frac{Q h R}{2 \cdot 2R\pi\varepsilon_0 d^3} = \frac{Qh}{4\pi\varepsilon_0 d^3}$

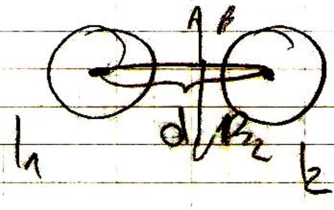
$E_y = \frac{Qh}{4\pi\varepsilon_0 d^3} = \frac{Qh}{4\pi\varepsilon_0 (R^2+h^2)}$

$$\frac{dE_y}{dh} = 0 \Rightarrow (R^2+h^2)^{\frac{3}{2}} = 3h^2 \cancel{(R^2+h^2)^{\frac{1}{2}}}$$

$$R^2 + h^2 = 3h^2$$

$$h = \pm \frac{R}{\sqrt{2}}$$

8. $\frac{I_1}{I_2} = \frac{1}{2} \Rightarrow I_2 = 2I_1$



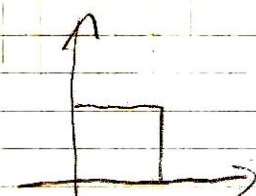
$$B_1 = B_2$$

$$B = \frac{\mu_0 \cdot I}{2\pi d}$$

$$\frac{\cancel{\mu_0} \cdot \cancel{2I_1}}{\cancel{2\pi} x} = \frac{\cancel{\mu_0} \cdot \cancel{2I_1}}{\cancel{2\pi}(d-x)} \rightarrow 2x = d - x \Rightarrow 3x = d$$

$$x = \frac{d}{3}$$

9. $B(x, y, z=0) = B_0 \left(3\vec{i} + \left(\frac{x}{a}\right)^2 \vec{j} \right)$



$$\oint \vec{B} \cdot d\vec{l} = \iint \vec{\nabla} \times \vec{B} \cdot d\vec{a}$$

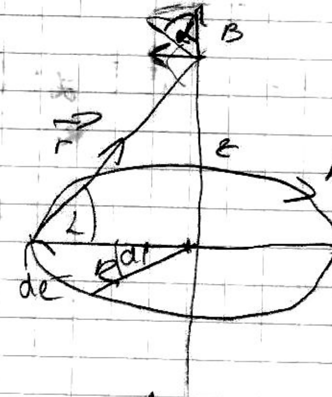
Linijski int po liniji C jednak je $\iint$ polja po površini da kojeg zatvara ta krivulja

$$\vec{\nabla} \times \vec{B} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 & \frac{x}{a} & 0 \end{vmatrix} \cdot B_0 = B_0 \left[i \left(\cancel{-\frac{\partial}{\partial z}} \left(\frac{x}{a}\right)^2 \right) - j \left(\cancel{\frac{\partial}{\partial z} 3} \right) + k \left(\frac{\partial}{\partial x} \left(\frac{x}{a}\right)^2 - \cancel{\frac{\partial}{\partial y} 3} \right) \right] = B_0 \frac{2x}{a^2} \cdot \vec{k}$$

$$\iint \vec{\nabla} \times \vec{B} \cdot da = B_0 \cdot \frac{2}{a^2} \iint_S x \cdot \vec{k} \cdot n \cdot dS = B_0 \cdot \frac{2}{a^2} \iint x^2 dS =$$

$$= B_0 \frac{2}{a^2} \int^a x\,dx \int dy = B_0 \frac{2}{a^2} \cdot \frac{1}{2} \cancel{a^2} \cdot a = B_0 \cdot a$$

10. $B_y = B \cdot \cos\alpha = B \cdot \frac{R}{d}$



$B = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot d\vec{l} \times \cdot \vec{r}}{d^2}$

$dl = R d\varphi$

$dB = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{d^2} = \frac{\mu_0}{4\pi} \cdot \frac{I dl}{d^2}$

Zbog simetrije se poništavaju horizontalne komponente

$d\vec{l} \times \vec{r} = |d\vec{l}| \, |\hat{r}| \sin\theta = dl$

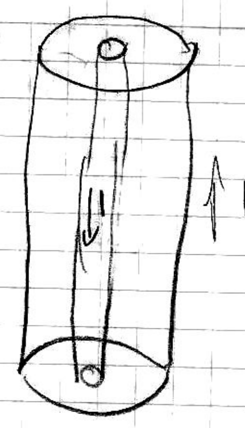
$dB_y = dB \cdot \frac{R}{d}$

$dB = \frac{\mu_0}{4\pi d^2} \cdot I \cdot R d\varphi$

$B_y = \int_0^{2\pi} \frac{\mu_0}{4\pi d^2} \cdot I R d\varphi \cdot \frac{R}{d} = \frac{\mu_0 I R^2}{{}_2\cancel{4\pi} d^3} \cancel{2\pi} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}}$

11.

$w_{EM} = \underbrace{\frac{1}{2} \varepsilon_0 E^2}_{\text{strujni}} + \underbrace{\frac{1}{2} \frac{1}{\mu_0} B^2}_{\text{magnetski dio}}$



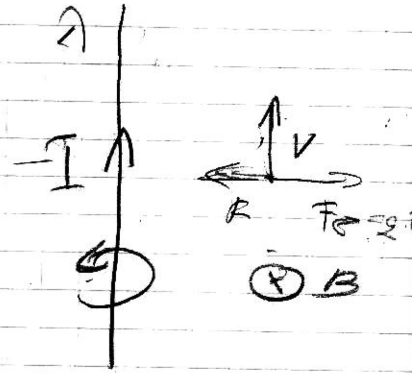
mag polje izvan vodiča $B = \frac{\mu_0 I}{2r\pi} + \frac{\mu_0 (-I)}{2r\pi}$

od središta

$W_B = \int_0^{2\pi} d\varphi \int_a^b \left(\frac{\mu_0 I}{2r\pi}\right)^2 dr = \frac{1}{2\mu_0} \cdot 2\pi \int_a^b \frac{\mu_0^2 I^2}{4r^2\pi^2} dr$

$= \frac{\cancel{1}}{\cancel{\mu_0}} \cdot \frac{\mu_0 I^2}{4\pi} \left(-\frac{1}{r}\right)\Big|_a^b = \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

12. $F = q \cdot (\vec{v} \times \vec{B})$



$$|E| = \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$|B| = \frac{\mu I}{2\varepsilon_0 \pi d}$$

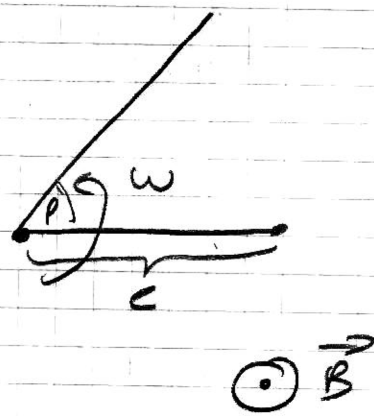
$$\cancel{q} \cdot E = \cancel{q} \cdot (\vec{v} \times \vec{B})$$ sin α

$$\frac{\lambda}{\cancel{2\pi}\varepsilon_0 \cancel{d}} = v \cdot \frac{\mu I}{\cancel{2\pi d}}$$

$$I = \frac{\lambda}{\mu_0 \varepsilon_0 v}$$

13.

1) način



$$U_i = \frac{d\Phi}{dt} = \frac{d(B \cdot S)}{dt} = B \cdot \frac{dS}{dt}$$

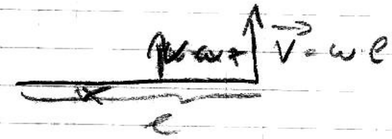
$$d\varphi = w \cdot dt$$

$$dS = \frac{d\varphi}{2\cancel{\pi}} \cdot l^2 \cancel{\pi} = \frac{w \cdot dt}{2\cancel{\pi}} \cdot l^2 \cancel{\pi} = \frac{wl^2}{2} \cdot dt \quad /: dt$$

$$\frac{dS}{dt} = \frac{wl^2}{2}$$

$$U_i = B \frac{dS}{dt} = B \frac{wl^2}{2}$$

2) način



$$U_i = B \cdot l \cdot v \qquad v = wl$$

$$dU_i = B \cdot w\, l\, dl$$

$$U_i = B \cdot w \cdot \int_0^L l\, dl = Bw \cdot \frac{l^2}{2}\Big|_0^L = Bw \frac{L^2}{2}$$

14. $\varepsilon_r = 1{,}15$

$\mu_r = 1{,}05$

$$n = \frac{c}{v} = \sqrt{\varepsilon_r \cdot \mu_r}$$

$$\frac{\lambda_2}{\lambda_1} = ?$$

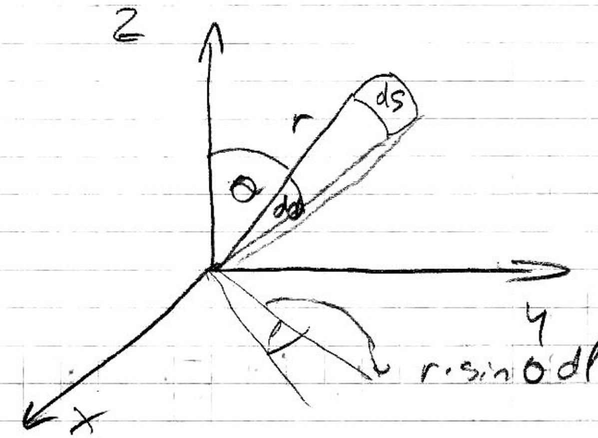
$$v = f \cdot \lambda$$

$$\frac{\cancel{f}\cdot \lambda_1}{\cancel{f}\cdot \lambda_2} = \sqrt{\varepsilon_r \mu_r}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{\varepsilon_r \mu_r}} = 0{,}91 = 91\%$$

15. $\Phi = 5 \cdot 10^4 \ \mathrm{lm}$

$I = 2 \cdot 10^5 \ \mathrm{cd}$



$$d\Phi = I \cdot d\Omega$$

$$d\Omega = \frac{ds}{r^2} = \frac{\cancel{r^2}\sin\theta \, d\theta \, d\varphi}{\cancel{r^2}}$$

$$= \sin\theta \, d\theta \, d\varphi$$

$$\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta = 2\pi\,(1-\cos\theta)$$

$$\Phi = I \cdot \Omega = 1 \cdot 2\pi\,(1-\cos\theta)$$

$$\cos\theta = 1 - \frac{\Phi}{2\pi \cdot I}$$

$$\theta = 16{,}22^\circ$$

15. $\Delta\Phi = 5 \cdot 10^4 \ lm$

$I = 2 \cdot 10^5 \ cd$

$\alpha = ?$

$$I = \frac{\Delta\Phi}{\Delta\Omega} \Rightarrow \Delta\Omega = \frac{1}{\Delta\Phi} = \frac{5 \cdot 10^4}{2 \cdot 10^5} = 0{,}25$$

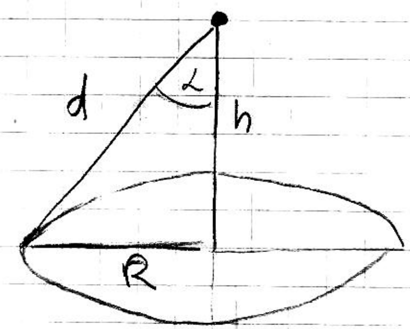
$$\Delta\Omega = \int_0^{2\pi} d\varphi \int_0^{\alpha} \sin\alpha \cdot d\alpha$$

$$\Delta\Omega = 2\pi \ (1 - \cos\alpha)$$

$$1 - \cos\alpha = \frac{0{,}25}{2\pi}$$

$$\alpha = 16{,}21^\circ$$

16.



$$E = \frac{1}{d^2} \cdot \cos\alpha$$

$$E(h) = \frac{1 \cdot h}{(R^2+h^2)^{\frac{3}{2}}}$$

$$\frac{dE}{dh} = 0$$

$$\frac{dE}{dh} = \frac{1(R^2+h^2)^{\frac{3}{2}} - 1h \cdot \frac{3}{2} \, 2h\sqrt{R^2+h^2}}{(R^2+h^2)^3}$$

$$\cancel{1} \cdot (R^2+h^2)^{\frac{3}{2}} = \cancel{1} \, 3h^2 \sqrt{R^2+h^2} \qquad | : \sqrt{R^2+h^2}$$

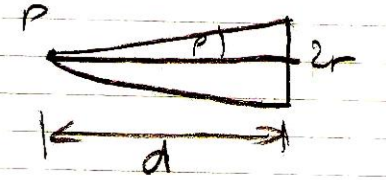
$$R^2 + h^2 = 3h^2$$

$$h^2 = \frac{1}{2} R^2$$

$$h = \frac{\sqrt{2}}{2} R$$

17. $P = 10\ W$ $\quad 2r = 6\ mm$

$\lambda = 500\ nm$



$E = \frac{hc}{\lambda}$

$P_{min} = 60 \cdot E = 60 \frac{hc}{\lambda}$

$\frac{P}{4\pi} \cdot \Omega = P_{min}$

veza između prostornog kuta i onog u radijanima

$\Omega = \frac{P_{min}}{P} \cdot 4\pi = \int_0^{2\pi} d\varphi \int_0^{\theta} \sin\theta \, d\theta = 2\pi(1 - \cos\theta)$

$\cos\theta = 1 - 2\frac{P_{min}}{P}$

$\cos\theta = 1 - \alpha$

$\sqrt{1 - \sin^2\alpha} = 1 - \alpha \quad /^2$

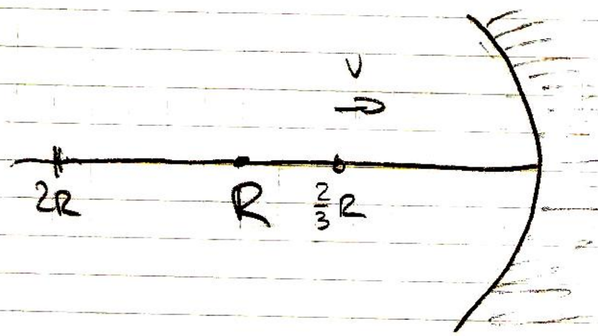
$\cancel{1} - \theta^2 = \cancel{1} - 2\alpha \cancel{+\alpha^2}$

$\Theta = \sqrt{2\alpha}$

$tg\,\theta = \frac{r}{d} \approx$

$\theta = \frac{r}{d} \Rightarrow d = \frac{r}{\sqrt{2\alpha}} = 809\,623\ m$

18. $\frac{1}{a} + \frac{1}{b} = \frac{2}{R}$



$b = \frac{\frac{R}{2} \cdot a}{a - \frac{R}{2}} = 2R$

$\frac{1}{a+da} + \frac{1}{b+db} = \frac{2}{R}$

$-\frac{2}{3}R$

$\underbrace{b}_{2R} + db = \frac{\frac{R}{2}(a+da)}{da - \frac{R}{2}}$

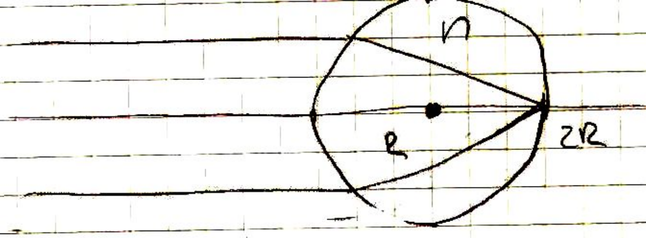
$db = -\frac{9R\,da}{R + 6da} \quad | : dt$

$\frac{db}{dt} = v_b = -\frac{9R\,v_a}{R + 6da} = -9 v_a$

zanemarimo

19. $a=\infty$ Sferni diopter

$b=2R$



$$\frac{\overset{1}{n_1}}{a}+\frac{\overset{n}{n_2}}{b}=\frac{\overset{n}{n_2}-\overset{1}{n_1}}{R}$$

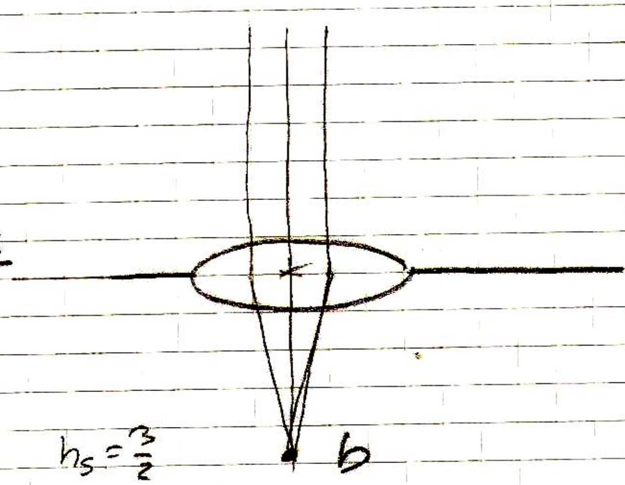
$$\frac{n}{2R}=\frac{n-1}{R}$$

$n=2$

b) $n=\infty$

20. Formula tanke leće:

$$\frac{\cancel{\overset{1}{n_1}}}{\cancel{\underset{\infty}{a}}}+\frac{\overset{n_v}{n_2}}{b}=\frac{\overset{n_s}{n_2}-n_1}{R_1}+\frac{n_3-n_2}{R_2}$$



$R_1=-R_2$ po konvenciji

$n_s=\frac{3}{2}$

$n_v=\frac{4}{3}$

$$\frac{n_v}{b}=\frac{n_s-1}{R}+\frac{n_s-n_v}{R}$$

$b=2R$

Sferni diopter i tanka leća

$$\frac{\overset{n_s}{n_2}-\overset{1}{n_1}}{\underset{1}{n_1}}\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$$

$$(n_s-1)\cdot\frac{2}{R}=\frac{1}{f}$$

$$\frac{1}{2}\cdot\frac{2}{R}=\frac{1}{f}$$

$f=R$

$b=2$

21.

$m = -\frac{1}{2}$

$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$

$a + b = d$

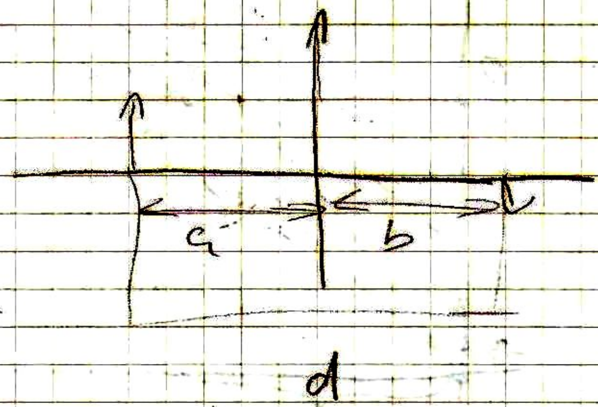
$m = -\frac{b}{a}$

$b = \frac{a}{2}$

$\frac{1}{a} + \frac{2}{a} = \frac{1}{f} \Rightarrow \frac{3}{a} = \frac{1}{f} \Rightarrow f = \frac{a}{3}$

$a + \frac{a}{2} = d \Rightarrow a = \frac{2}{3} d$

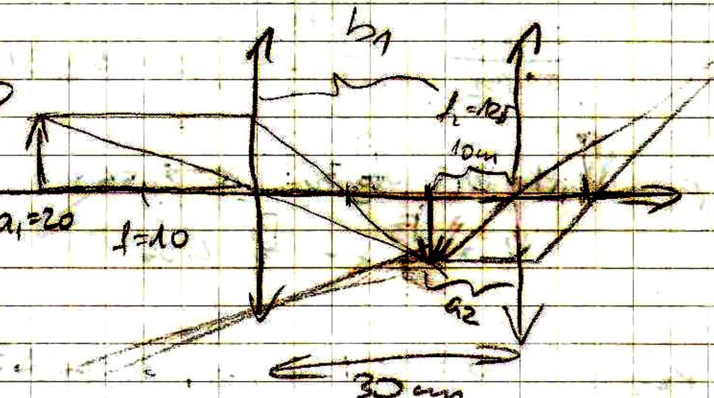
$f = \frac{2}{3} d \cdot \frac{1}{3} = \frac{2}{9} d$



22.

$\frac{1}{a_1} + \frac{1}{b_1} + \frac{1}{f_1} \Rightarrow \frac{1}{b_1} = \frac{1}{f_1} - \frac{1}{a_1} \Rightarrow$

$\Rightarrow b_1 = 20$ cm



$a_2 = 10$ cm

$\frac{1}{a_2} + \frac{1}{b_2} = \frac{1}{f_2} \Rightarrow b_2 = -50$ cm

$m_1 = -\frac{b_1}{a_1}$ $\quad m_2 = -\frac{b_2}{a_2}$ $\quad m = m_1 m_2 = \frac{-20}{20} \cdot \frac{+50}{-10} = 5$

Konkavno (udubljeno)

$r > 0$

Ako je $a > r \Rightarrow$ slika realna, obrnuta, smanjena

$a = r \Rightarrow b = a, \; m < 0$, realna, obrnuta

$a = f \Rightarrow$ slika u beskonačnosti

Konveksno (izbočeno)

slika uvijek imaginarna $b < 0$

uspravna $m > 0$