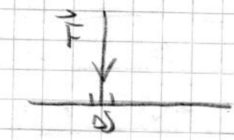
TEORIJA

1) NAPREZANJE

$\boxed{\sigma = \frac{F}{S}}$

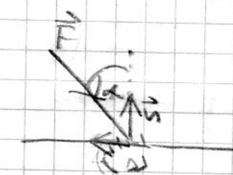
TENZOR

$\sigma = \lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} = \frac{dF}{dS}$



$\tau = \lim_{\Delta S \to 0} \frac{\vec{F} \cdot \vec{t}}{\Delta S} = \lim_{\Delta S \to 0} \frac{F \sin\alpha}{\Delta S}$

TANGENC. NAPREZANJE



$\sigma_n = \lim_{\Delta S \to 0} \frac{\vec{F} \cdot \vec{n}}{\Delta S} = \lim_{\Delta S \to 0} \frac{F \cos\alpha}{\Delta S}$

NORMALNO NAPREZANJE

$\boxed{\varepsilon = \frac{\text{promjena}}{\text{poč. vrijednost}}}$ RELATIVNA DEFORMACIJA (ili δ)

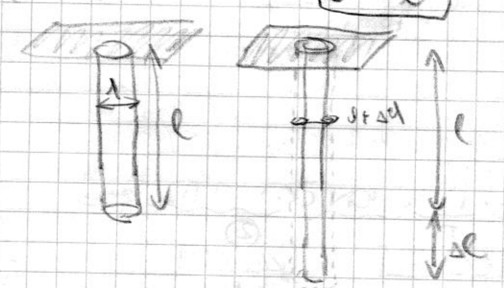
I) linearna promjena (vlak)

$\sigma = \frac{F}{S} = E\,\varepsilon = E\,\frac{\Delta l}{l}$

$\boxed{\sigma = E\varepsilon}$

↳ Youngov modul elastičnosti

$\boxed{\varepsilon = \frac{\Delta l}{l}}$



$\frac{\Delta d}{d} = -\mu \frac{\Delta l}{l} \Rightarrow \boxed{\mu = \frac{-\varepsilon_{\perp}}{\varepsilon_{\parallel}}}$

POISSONOV BROJ

II.) volumna promjena (tlak)

$\boxed{\varepsilon = \frac{\Delta V}{V}}$

$\boxed{\sigma = p}$

$p = -B\,\varepsilon = -B\,\frac{\Delta V}{V}$

$\boxed{p = -B\varepsilon}$

↳ MODUL KOMPRESIJE

$K = \frac{1}{B}$

$\frac{F}{S} = E\,\frac{\Delta l}{l} \Rightarrow F = \frac{ES}{l}\,\Delta l$

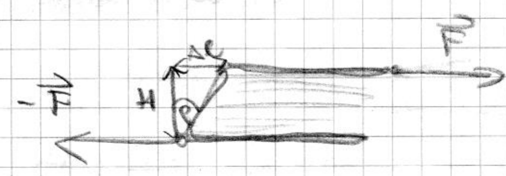
$\boxed{F = k\,\Delta l}$ HOOKEOV ZAKON

$\vec{F}_{el} = -\vec{F}$

$\boxed{B = \frac{E}{3(1-2\mu)}}$

III.) Smicanje (smik)

$$\epsilon = \frac{\Delta l}{H} = \tan\varphi \approx \varphi \quad (\varphi \to 0)$$



$$\sigma = G\epsilon = G\varphi$$

$$\boxed{G = \frac{B}{2(1+\mu)}}$$

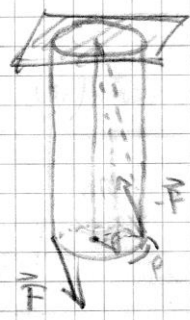
$$\boxed{\sigma = G\epsilon}$$

→ modul smicanja

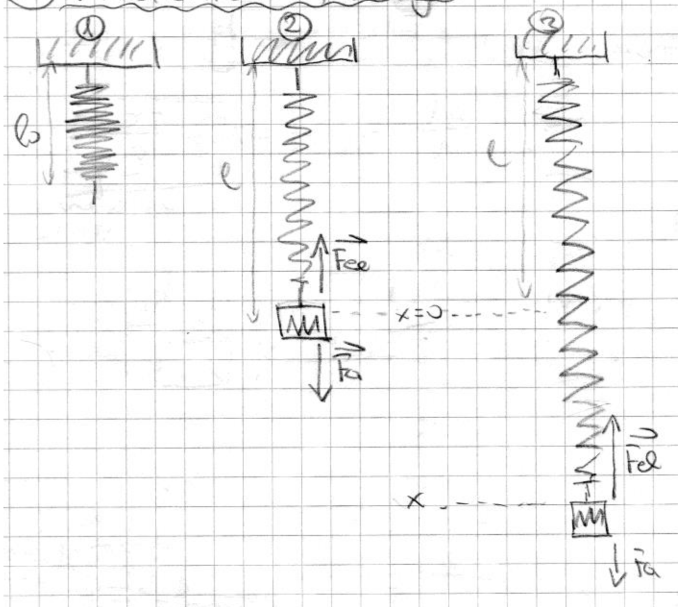
IV.) torzija

$$\boxed{D = \frac{dM}{d\varphi} = \frac{\pi r^4}{2l} G}$$

→ torziona konstanta



2) Harmoničko titranje



② ravnotežni položaj

$$\vec F_G = -\vec F_{el}$$

$$mg = -k\Delta l \quad \text{Hookeov zakon}$$

$$mg = -k(l - l_0)$$

③ pomicanje x

$$ma = F \qquad \text{2. NA}$$

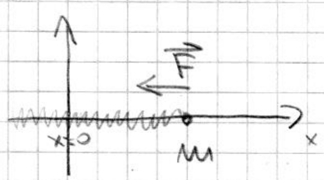
$$m\frac{d^2x}{dt^2} = F_G + F_{el}$$

$$m\frac{d^2x}{dt^2} = mg - k(l + x - l_0)$$

$$m\frac{d^2x}{dt^2} = \underbrace{mg - k(l - l_0)}_{=0} - kx$$

$$\boxed{m\frac{d^2x}{dt^2}+kx=0}$$ HARMONIČKI OSCILATOR U DIFERENCIJALNOM OBLIKU

$x(t)=?$



$$\vec{F}=-kx\vec{i}$$

P.U. $t=0$
$x=x_0$
$v=v_0$

$$m\frac{d^2x}{dt^2}+kx=0$$ (lin. dif. jedn. s konst. koeficijentima)

$$x=Ce^{\alpha t}$$

$$m\alpha^2Ce^{\alpha t}+kCe^{\alpha t}=0$$

$$\underbrace{Ce^{\alpha t}}_{\neq 0}(m\alpha^2+k)=0 \Rightarrow m\alpha^2+k=0$$

$$\alpha^2=-\frac{k}{m}=-\omega^2$$

$$\boxed{\alpha=\pm i\omega}$$

rješenje lin. dif. jednadžbe:

$$x=Ce^{i\omega t}+De^{-i\omega t}$$

$v(t)=?$

$$v(t)=\frac{dx}{dt}=i\omega Ce^{i\omega t}-i\omega De^{-i\omega t}$$

P.U. $t=0$
$x_0=C+D$
$v_0=i\omega(C-D)$

$$C+D=\mathrm{Re}C+i\,\mathrm{Im}C+\mathrm{Re}D+i\,\mathrm{Im}D$$

$x_0 \in R \Rightarrow C + D \in R \Rightarrow \boxed{\Im C = -\Im D}$ *

$i\omega(C-D) = i\omega(\Re C + i\Im C - \Re D - i\Im D)$

$v_0 \in R \Rightarrow i\omega(C-D) \in R \Rightarrow \boxed{\Re C = \Re D}$ *

→ Iz * i ** zaključujemo:

$\boxed{C = D^*}$ (konj. kompleksni broj)

$C = E e^{i\psi}$
$D = E e^{-i\psi}$ $\Longleftrightarrow$ $\left\{ \begin{array}{l} e^{\pm j\alpha} = \cos\alpha \pm j\sin\alpha \\ z = a \pm jb = K e^{\pm j\alpha} \end{array} \right\}$ eulerova jednadžba

$x = E e^{i\psi} e^{i\omega t} + E e^{-i\psi} e^{-i\omega t} = E\left[e^{i(\psi+\omega t)} + e^{-i(\psi+\omega t)}\right]$

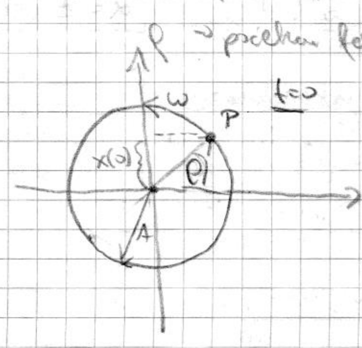
$= E\left(\cos(\psi+\omega t) + \cancel{i\sin(\psi+\omega t)} + \cos(\psi+\omega t) - \cancel{i\sin(\psi+\omega t)}\right)$

$= 2E\cos(\psi+\omega t) = \underbrace{2E}_{A}\sin(\underbrace{\psi + \omega t + \tfrac{\pi}{2}}_{\rho})$ $\qquad \cos(\rho) = \sin(\rho + \tfrac{\pi}{2})$

$\boxed{x(t) = A\sin(\omega t + \rho)}$ HARMONIČKI OSCILATOR $f(t) = \omega t + \rho$ → faza titranja

$v(t) = A\omega\cos(\omega t + \rho)$
$a(t) = -A\omega^2\sin(\omega t + \rho)$

ρ → početna faza $f(0)$



→ određujemo fazu pomoću FAZORA

$x = A\sin(\omega t + \rho)$
$v = A\omega\cos(\omega t + \rho)$

$x_0 = A\sin\rho$
$v_0 = A\omega\cos\rho$

$\left.\begin{array}{l} x_0 = A\sin\rho \\ \frac{v_0}{\omega} = A\cos\rho \end{array}\right\}$ *

$*|^2 +$ $\quad x_0^2 + \left(\frac{v_0}{\omega}\right)^2 = A^2(\underbrace{\sin^2\rho + \cos^2\rho}_{1}) \Rightarrow \boxed{A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}}$ AMPLITUDE

* (÷) $\frac{x_0 \omega}{v_0} = \frac{\sin\varphi}{\cos\varphi} \Rightarrow \boxed{tg\,\varphi = \frac{x_0\omega}{v_0}}$ POČETNA FAZA

$x(t+T) = x(t)$

$A\sin(\omega(t+T)+\varphi) = A\sin(\omega t+\varphi)$

$\cancel{\omega t} + \omega T + \cancel{\varphi} = \cancel{\omega t} + \cancel{\varphi} + 2\pi$

$T = \frac{2\pi}{\omega} = 2\pi f$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ PERIOD

3) ENERGIJA HARM. OSCILATORA

$E_k = \frac{mv^2}{2}$

$U = \frac{1}{2}kx^2$

$E = E_k + U = konst.$ *

$x = A\sin(\omega t+\varphi)$

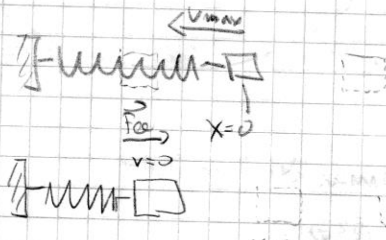
$v = A\omega\cos(\omega t+\varphi)$

$E_k = \frac{m A^2\omega^2\cos^2(\omega t+\varphi)}{2} = \frac{A^2 m \frac{k}{m}\cos^2(\omega t+\varphi)}{2} = \frac{1}{2}kA^2\cos^2(\omega t+\varphi)$

$E_k(t) = \frac{1}{2}kA^2\cos^2(\omega t+\varphi)$ f-ja od t

$E_k = \frac{1}{2}kA^2(1-\sin^2(\omega t+\varphi)) = \frac{1}{2}k(A^2 - \overbrace{A^2\sin^2(\omega t+\varphi)}^{x^2}) = \frac{1}{2}k(A^2-x^2)$

$E_k(x) = \frac{1}{2}k(A^2-x^2)$ f-ja od x



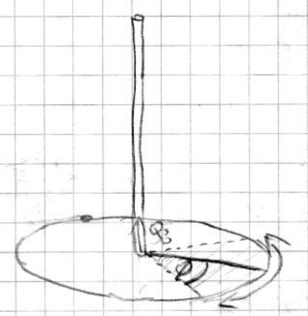
$\boxed{E_{k\,max} = \frac{1}{2}kA^2}$ (x=0 , v=max)

$E_{k\,min} = 0$ (x=A , v=0)

iz ZOE (*) $\Rightarrow \boxed{U_{max} = \frac{1}{2}kA^2}$ (x=A , v=0)

$U(t) = \frac{1}{2}kA^2\sin^2(\omega t+\varphi)$

(4) TORZIONO NJIHALO



$$\vec{\phi} = \phi \vec{\phi}_0$$

Iz torzionog naprezanja

$$\vec{M} = D\vec{Q}$$

$$\vec{M}_{el} = -\vec{M} = -D\vec{\phi}$$

$$M_{el} = -D\phi$$

harmonički oscilator (analogija na rotaciju)

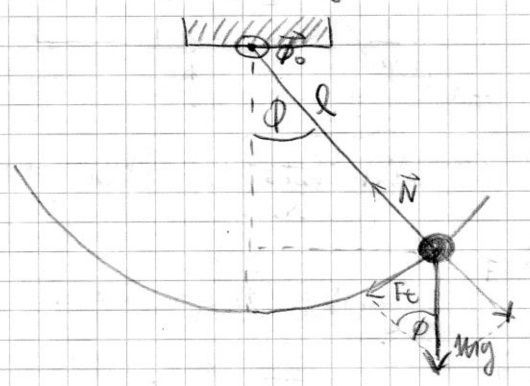
$$I\frac{d^2\phi}{dt^2} = M_{el} = -D\phi \qquad \boxed{\omega^2 = \frac{D}{I}}$$

$$I\frac{d^2\phi}{dt^2} + D\phi = 0 \qquad \boxed{T = 2\pi\sqrt{\frac{I}{D}}}$$

$$\frac{d^2\phi}{dt^2} + \frac{D}{I}\phi = 0$$

$$\boxed{\phi(t) = \phi_{max}\sin(\omega t + \varphi)}$$

(5) MATEMATIČKO NJIHALO



$$\vec{\phi} = \phi\vec{\phi}_0$$

$$\vec{M} = l F_t \vec{\phi}_0$$

$$\vec{M}_{el} = -\vec{M} = -lF_t\vec{\phi}_0$$

$$M_{el} = -lF_t = -lmg\sin\phi$$

harmonički oscilator:

$$I\frac{d^2\phi}{dt^2} = M_{el} = -mgl\sin\phi \qquad \phi \to 0$$

$$\left(\frac{2}{5}mR^2 + ml^2\right)\frac{d^2\phi}{dt^2} = -mgl\sin\phi \qquad \sin\phi \to \phi$$

zanemarimo $\quad \frac{d^2\phi}{dt^2} + \frac{g}{l}\sin\phi = 0 \qquad \frac{d^2\phi}{dt^2} + \frac{g}{l}\phi = 0$

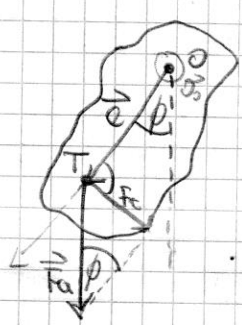
$$\boxed{\phi(t) = \phi_{max} \sin(\omega t + \varphi)}$$

$$\boxed{\omega^2 = \frac{g}{l} \qquad T_0 = 2\pi\sqrt{\frac{l}{g}}}$$

→ pend T_0 vrijedi samo za male amplitude

→ za velike amplitude: $\boxed{T = T_0\left(1 + \frac{1}{4}\sin^2\frac{\varphi_0}{2} + \frac{9}{64}\sin^4\frac{\varphi_0}{2} + \underbrace{\ldots}^{\text{zanemarivo}}\right)}$

⑥ Fizičko njihalo



$$\vec{\phi} = \phi\,\vec{\phi}_0$$

$$\vec{M} = l F_t\,\vec{\phi}_0$$

$$\vec{M}_{el} = -\vec{M}$$

$$M_{el} = -l F_t = -l m g \sin\phi$$

harmonijski oscilator:

$$I \frac{d^2\phi}{dt^2} = M_{el} = l m g \sin\phi$$

$$\frac{d^2\phi}{dt^2} + \frac{mgl}{I}\sin\phi = 0 \qquad \phi \to 0 \quad \sin\phi \to \phi$$

$$\frac{d^2\phi}{dt^2} + \frac{mgl}{I}\phi = 0$$

$$\boxed{\phi(t) = \phi_{max}\sin(\omega t + \varphi)}$$

$$\boxed{\omega^2 = \frac{mgl}{I} \qquad T = 2\pi\sqrt{\frac{I}{mgl}}}$$

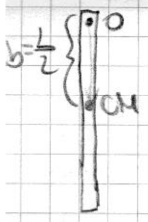
→ duljina matematičkog njihala kad ima isti T kao i fizičko njihalo:

$$\cancel{2\pi}\sqrt{\frac{l}{g}} = \cancel{2\pi}\sqrt{\frac{I}{mgb}}$$

$$\boxed{l_R = \frac{I}{mb}}$$

REDUCIRANA DULJINA

→ reducirana duljina homogenog štapa duljine L



$$I = I_{cm} + m d^2 = \frac{1}{12} m L^2 + m \frac{L^2}{4} = \frac{1}{3} m L^2$$

$$l_R = \frac{I}{m d} = \frac{\frac{1}{3} m L^2}{m \frac{L}{2}} = \frac{2}{3} L$$

$$\boxed{l_R = \frac{2}{3} L}$$

SREDIŠTE UDARCA (TITRANJA) ŠTAPA (za $\frac{2}{3}L$ udaljeno od osi vrtnje)

SREDIŠTE UDARCA → položaj za l_R udaljen od osi vrtnje

7 PRIGUŠENO TITRANJE

$$\vec{F}_d = -b\vec{v}$$ b - konstanta trenja

2.NZ

$$m\frac{d^2x}{dt^2} = F_{el} + F_d = -kx - b\vec{v}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \quad /:m$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = 0$$

$\cancel{\frac{m}{b}} = 2\delta = \frac{b}{m}$

$\boxed{\delta = \frac{m}{2b}}$ FAKTOR PRIGUŠENJA

$\boxed{\frac{k}{m} = \omega_0^2}$ VLASTITA FREKV. TITRANJA

$x(t) = ?$

P.U. $t = 0$
$x = x_0$
$v = v_0$

$$x = Ce^{\alpha t}$$

$$\alpha^2 Ce^{\alpha t} + 2\delta\alpha Ce^{\alpha t} + \omega_0^2 Ce^{\alpha t} = 0$$

$$\underset{>0}{Ce^{\alpha t}}\underbrace{(\alpha^2 + 2\delta\alpha + \omega_0^2)}_{=0} = 0$$

$$\alpha^2 + 2\delta\alpha + \omega_0^2 = 0$$

$$\alpha_{1,2} = \frac{-\cancel{2}\delta \pm \sqrt{4\delta^2 - 4\omega_0^2}}{\cancel{2}}$$

$$\boxed{\alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}}$$

a.) SLABO PRIGUŠENJE: $\omega_0^2 > \delta^2$ $\quad (F_{HARM} > F_R)$

$$\boxed{\omega^2 = \omega_0^2 - \delta^2} \quad \text{FREKVENCIJE} \longrightarrow \delta^2 - \omega_0^2 = -\omega^2$$

$$\alpha_{1,2} = -\delta \pm \sqrt{-\omega^2} = -\delta \pm i\omega$$

$$x = Ce^{(-\delta + i\omega)t} + De^{(-\delta - i\omega)t} = Ce^{-\delta t}e^{i\omega t} + De^{-\delta t}e^{-i\omega t}$$

$v(t) = ?$

$$v(t) = \frac{dx}{dt} = (-\delta + i\omega)Ce^{(-\delta + i\omega)t} + (-\delta - i\omega)De^{(-\delta - i\omega)t}$$

P.U. $t = 0$

$$\left\{ \begin{aligned} x_0 &= C + D \\ v_0 &= (-\delta + i\omega)C + (-\delta - i\omega)D \end{aligned} \right\}$$

$$C + D = \mathrm{Re}\,C + i\,\mathrm{Im}\,C + \mathrm{Re}\,D + i\,\mathrm{Im}\,D$$

$$x_0 \in \mathbb{R} \Rightarrow C + D \in \mathbb{R} \rightarrow \boxed{\mathrm{Im}\,C = -\mathrm{Im}\,D} \; (*)$$

$$(-\delta + i\omega)C + (-\delta - i\omega)D = (-\delta + i\omega)(\mathrm{Re}\,C + i\,\mathrm{Im}\,C) + (-\delta - i\omega)(\mathrm{Re}\,D + i\,\mathrm{Im}\,D)$$

$$= -\delta(\mathrm{Re}\,C + i\,\mathrm{Im}\,C + \mathrm{Re}\,D + i\,\mathrm{Im}\,D) + i\omega(\mathrm{Re}\,C + i\,\mathrm{Im}\,C - \mathrm{Re}\,D - i\,\mathrm{Im}\,D)$$

$$x_0 \in \mathbb{R} \Rightarrow (-\delta + i\omega)C + (-\delta - i\omega)D \in \mathbb{R} \Rightarrow \boxed{\begin{aligned} \mathrm{Im}\,C &= -\mathrm{Im}\,D \\ \mathrm{Re}\,C &= \mathrm{Re}\,D \end{aligned}} \; (**)$$

$\rightarrow$ iz $(*)$ i $(**)$ zaključujemo:

$$\boxed{C = D^*}$$

$$\left\{ \begin{aligned} Re^{j\alpha} &= \cos\alpha + j\sin\alpha \\ z = a + jb &= Re^{j\alpha} \end{aligned} \right\} \text{ĆUVENA JEDNADŽBA}$$

$$C = E e^{i\psi}$$
$$D = E e^{-i\psi}$$

$$x = E e^{i\psi} e^{-\delta t} e^{i\omega t} + E e^{-i\psi} e^{-\delta t} e^{-i\omega t} = E e^{-\delta t}\left(e^{i(\psi+\omega t)} + e^{-i(\psi+\omega t)}\right)$$
$$= E e^{-\delta t}\left(\cos(\psi+\omega t) + \cancel{i\sin(\psi+\omega t)} + \cos(\psi+\omega t) - \cancel{i\sin(\psi+\omega t)}\right)$$
$$= \underbrace{\underbrace{2E}_{A_0} e^{-\delta t}}_{A} \cos(\omega t+\psi)$$

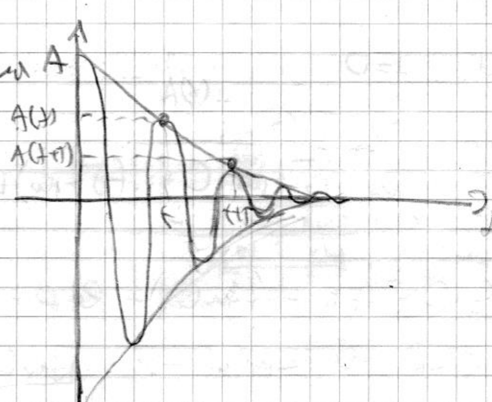
$$\boxed{x(t) = A_0 e^{-\delta t}\cos(\omega t+\psi)}$$

$$\omega = \sqrt{\omega_0^2 - \delta^2} = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$\varphi = \psi + \frac{\pi}{2}$$
$$\boxed{x(t) = A_0 e^{-\delta t}\sin(\omega t+\varphi)}$$

$$\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k^2}{m^2} - \frac{b^2}{4m^2}}}}$$

$A(t) = A_0 e^{-\delta t}$ – f-ja vremena



$$\lambda = \ln\frac{A(t)}{A(t+T)} = \ln\frac{\cancel{A_0} e^{-\delta t}}{\cancel{A_0} e^{-\delta(t+T)}}$$
$$= \ln\frac{e^{-\delta t}}{e^{-\delta t} e^{-\delta T}} = \ln e^{\delta T} = \delta T$$

$\boxed{\lambda = \delta T}$ LOGARITAMSKI DEKREMENT PRIGUŠENJA

$$\langle E \rangle = C e^{-2\delta t}$$

$\lambda \uparrow \Rightarrow \ln\frac{A(t)}{A(t+T)} \uparrow \Rightarrow A \downarrow \Rightarrow$ prigušenje $\uparrow$

$$\langle E \rangle = \frac{\int_t^{t+T} E(t)\,dt}{T}$$ prosječna energija po periodu

$\boxed{Q = 2\pi \frac{\langle E \rangle}{|\Delta E|}}$ FAKTOR DOBROTE

→ prosječna energija koju oscilator izgubi

$\boxed{Q = \frac{\pi}{\lambda}}$

$Q \nearrow \Rightarrow \lambda \searrow \Rightarrow$ prigušenje $\searrow$

tj.

$Q \gg$ velik broj perioda prije nego što se smanji amplituda

$Q \ll$ prigušenje jako

b.) JAKO PRIGUŠENJE (APERIODIČKO): $\boxed{\delta^2 > \omega_0^2}$

$\boxed{\omega^2 = \delta^2 - \omega_0^2}$ frekvencija

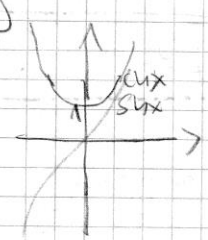
$\alpha_{1,2} = -\delta \pm \omega$

$$x = Ae^{(-\delta+\omega)t} + Be^{(-\delta-\omega)t}$$

$$\begin{cases} ch\,\omega = \frac{e^{\omega}+e^{-\omega}}{2} \\ sh\,\omega = \frac{e^{\omega}-e^{-\omega}}{2} \end{cases} \quad \begin{aligned} ch\,\omega + sh\,\omega &= e^{\omega} \\ ch\,\omega - sh\,\omega &= e^{-\omega} \end{aligned}$$

$$x = Ae^{-\delta t}e^{\omega t} + Be^{-\delta t}e^{-\omega t} = e^{-\delta t}\left[A(ch\,\omega t + sh\,\omega t) + B(ch\,\omega t - sh\,\omega t)\right]$$

$$= e^{-\delta t}\left[ch\,\omega t\,\underbrace{(A+B)}_{\mathcal{A}} + sh\,\omega t\,\underbrace{(A-B)}_{\mathcal{B}}\right]$$



$$x(t) = e^{-\delta t}\left[\mathcal{A}\,ch\,\omega t + \mathcal{B}\,sh\,\omega t\right]$$

$\underline{v(t) = ?}$

$$v(t) = \frac{dx}{dt} = -\delta e^{-\delta t}\left[\mathcal{A}\,ch\,\omega t + \mathcal{B}\,sh\,\omega t\right] + e^{-\delta t}\omega\left[\mathcal{A}\,sh\,\omega t + \mathcal{B}\,ch\,\omega t\right]$$

$$\begin{cases} \text{P.U.} \quad t = 0 \\ \boxed{x_0 = \mathcal{A}} \\ v_0 = -\delta \cdot 1\left[\mathcal{A}\cdot 1 + 0\right] + \omega\cdot\left[0 + \mathcal{B}\cdot 1\right] = -\mathcal{A}\delta + \mathcal{B}\omega \end{cases}$$

$$B = \frac{v_0 + A\delta}{\omega} = \frac{v_0 + x_0\delta}{\omega}$$

$$\boxed{B = \frac{v_0 + x_0\delta}{\omega}}$$

$$\boxed{x(t) = e^{-\delta t}\left[x_0 \operatorname{ch}\omega t + \frac{v_0 + x_0\delta}{\omega}\operatorname{sh}\omega t\right]}$$

→ APERIODIČKO TITRANJE – nema titranja već oscilator samo vrati u ravnotežni položaj

c.) KRITIČNO PRIGUŠENJE: $\boxed{\delta^2 = \omega_0^2}$

$\delta = \omega_0$

$\boxed{\omega = 0}$ FREKVENCIJA

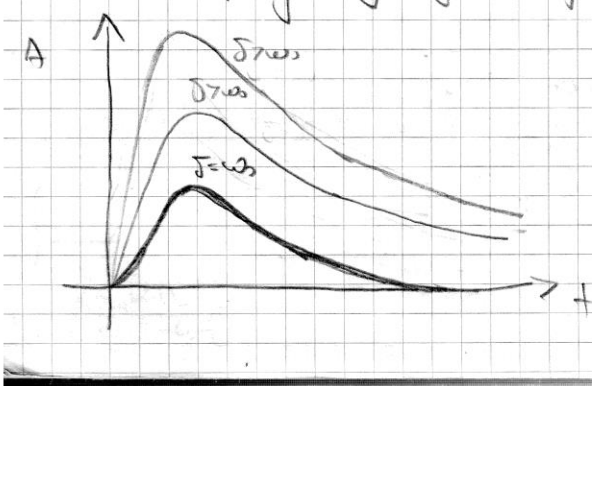
IZ APERIODIČKOG TITRANJA:

$$x(t) = e^{-\delta t}\left[x_0 \cdot 1 + (v_0 + x_0\delta)\frac{\operatorname{sh}\omega t}{\omega}\right] (*)$$

$$(*) \lim_{\omega\to 0}(v_0 + x_0\delta)\frac{\operatorname{sh}\omega t}{\omega} = t(v_0 + x_0\delta)\underbrace{\lim_{\omega\to 0}\frac{\operatorname{sh}\omega t}{\omega t}}_{\to 1} = t(v_0 + x_0\delta)$$

$$\boxed{x(t) = e^{-\delta t}\left[x_0 + t(v_0 + x_0\delta)\right]}$$ KRITIČKO PRIGUŠENJE

→ KRITIČNO prigušenje je najkraće APERIODIČKO prigušenje:



8) ENERGIJA prigušenog TITRANJA

$E_U = E_k + E_p \quad / \frac{d}{dt}$ $\qquad E_k = \frac{mv^2(t)}{2}$

$\frac{dE_U}{dt} = \frac{dE_k}{dt} + \frac{dE_p}{dt}$ $\qquad E_p = \frac{1}{2}kx^2(t)$

$= \frac{dE_k}{dv}\frac{dv}{dt} + \frac{dE_p}{dx}\frac{dx}{dt} = mv\cdot\frac{dv}{dt} + kxv$

! $= v\left(m\frac{dv}{dt} + kx\right)$

→ prigušeno titranje:

$$m\frac{dv}{dt} = -kx + \vec{F}_{ot}$$

$$\underline{m\frac{dv}{dt} + kx = -bv}$$

$\frac{dE_U}{dt} = -bv^2$

$$\boxed{P = \vec{F}\cdot\vec{v} = -bv^2}$$

10) PRISILNO TITRANJE

→ vanjska sila: $\underline{F_v = F_0 \sin \omega t}$

2. NZ

$m\frac{d^2x}{dt^2} = F_{el} + F_{ot} + F_v$

! $m\frac{d^2x}{dt^2} = -kx - bv + F_0 \sin \omega t$

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega t \quad / : m$

$$\boxed{\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = A_0 \sin \omega t} \qquad A_0 = \frac{F_0}{m}$$

→ nehomogena difer. jdb

$\underline{x = x_{ORJ} + x_{PRJ}}$

$x_{prij} = A(\omega)\sin(\omega t - \varphi)$

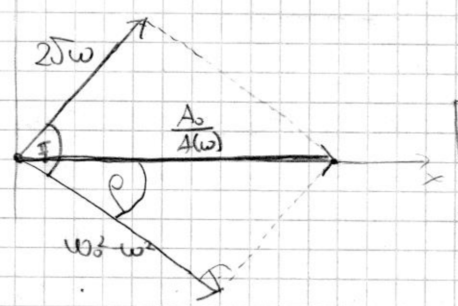
$\underline{A(\omega), \varphi = ?}$

$$-A(\omega)\omega^2 \sin(\omega t - \varphi) + 2\delta\omega A(\omega)\cos(\omega t - \varphi) + \omega_0^2 A(\omega)\sin(\omega t - \varphi) = A_0 \sin(\omega t)$$

$$A(\omega)(\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2\delta\omega A(\omega)\sin\left(\omega t - \varphi + \frac{\pi}{2}\right) = A_0 \sin(\omega t) \quad \Big/ : A(\omega)$$

$$(\omega_0^2 - \omega^2)\sin(\omega t - \varphi) + 2\delta\omega \sin\left(\omega t - \varphi + \frac{\pi}{2}\right) = \frac{A_0}{A(\omega)} \sin(\omega t)$$

→ fazorska metoda:

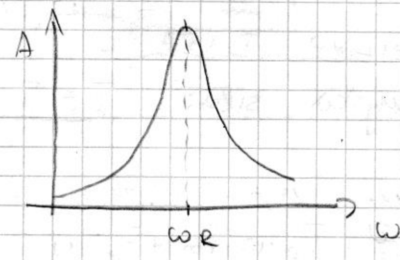


$$\frac{A_0}{A(\omega)} = \sqrt{(2\delta\omega)^2 + (\omega_0^2 - \omega^2)^2}$$

$$\boxed{A(\omega) = \frac{A_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}}$$

AMPLITUDA

$\omega \ll$ ili $\omega \gg \quad A \to 0$



$$\boxed{\operatorname{tg}\varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2}}$$ FAZA

$\underline{\omega_R, A_R = ?}$

$$\frac{dA(\omega)}{d\omega} = 0$$

$$A(\omega) = A_0 \left((\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2\right)^{-\frac{1}{2}} \quad \Big| \frac{d}{d\omega}$$

$$0 = A_0 \frac{-1}{2}\left((\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2\right)^{\frac{1}{2}} \cdot \left(2(\omega_0^2 - \omega^2)(-2\omega) + 8\delta^2\omega\right)$$

$$0 = \cancel{A_0}\cancel{(-2)}\cancel{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}\left(-\cancel{\omega}(\omega_0^2 - \omega^2) + 2\delta^2\cancel{\omega}\right)$$

$$\omega_0^2 - \omega^2 = 2\delta^2$$

$$\omega^2 = \omega_0^2 - 2\delta^2$$

$$\boxed{\omega_R = \sqrt{\omega_0^2 - 2\delta^2}}$$ REZONANTNA FREKVENCIJA

$$A(\omega_R) = \frac{A_0}{\sqrt{(\omega_0^2 - \omega_R^2)^2 + 4\delta^2\omega_R^2}} = \frac{A_0}{\sqrt{(\cancel{\omega_0^2} - \cancel{\omega_0^2} + 2\delta^2)^2 + 4\delta^2(\omega_0^2 - 2\delta^2)}}$$

$$A(\omega_R) = \frac{A_0}{2\delta\sqrt{\delta^2 + \omega_0^2 - 2\delta^2}}$$

$$\boxed{A(\omega_R) = \frac{A_0}{2\delta\sqrt{\omega_0^2 - \delta^2}}}$$ REZONANTNA (MAX) AMPLITUDA

$$x_{obj} = A' e^{-\delta t} \sin(\omega' t + \varphi_0)$$

$$x(t) = x_{obj} + x_{prej}$$

$$\boxed{x(t) = A' e^{-\delta t} \sin(\omega' t + \varphi_0) + \overbrace{A(\omega) \sin(\omega t - \varphi)}^{S_2}}$$ PRISILNO TITRANJE

↳ na početku titra sa frekvencijom ω' tj. s vlastitom frekv., ali nakon nekog vremena vlastito titranje utrne te dolazi STACIONARNO STANJE gdje ostaje samo S_2 titranje

(11) LISSAJOUSOVE KRIVULJE

$$x = A_1 \sin \omega t$$

$$y = A_2 \sin(\omega t + \Delta\varphi)$$

a) $\omega_1 = \omega_2 = \omega$

$\Delta\varphi = 0$

$$\omega t = \arcsin \frac{x}{A_1}$$

$$t = \frac{\arcsin \frac{x}{A_1}}{\omega}$$

$$y = A_2 \sin\left(\omega \frac{\arcsin \frac{x}{A_1}}{\omega}\right)$$

$$\boxed{y = \frac{A_2}{A_1} x}$$ (1)

b.) $\omega_1=\omega_2=\omega$

$\Delta\varphi=\pi$

$$y = A_2\sin(\omega t+\pi) = -A_2\sin\omega t$$

$$\frac{x}{y} = \frac{A_1\cancel{\sin\omega t}}{-A_2\cancel{\sin\omega t}}$$

$$\boxed{y = \frac{-A_2}{A_1}x} \quad (2)$$

c.) $\omega_1=\omega_2=\omega$

$\Delta\varphi=\frac{\pi}{2}$

$$y = A_2\sin\left(\omega t+\frac{\pi}{2}\right) = A_2\cos\omega t$$

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2} = \sin^2\omega t+\cos^2\omega t = 1$$

$$\boxed{\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2} = 1} \quad (3)$$

elipsa smjera kazaljke na satu

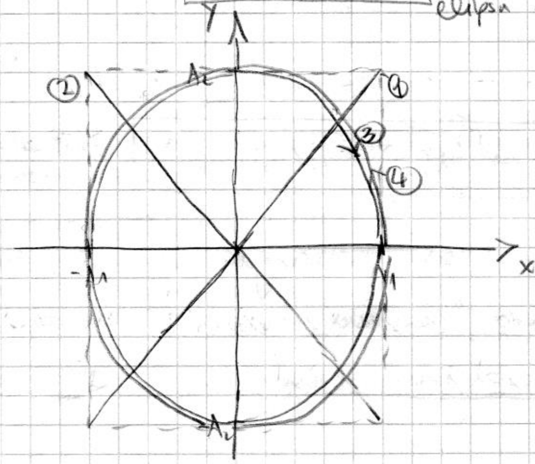
d.) $\omega_1=\omega_2=\omega$

$\Delta\varphi=\frac{3\pi}{2}$

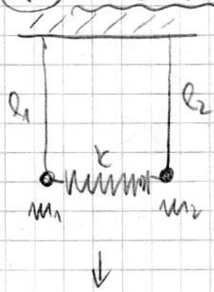
$$y = A_2\sin\left(\omega t+\frac{3\pi}{2}\right) = -A_2\cos\omega t$$

$$\boxed{\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2} = 1} \quad (4)$$

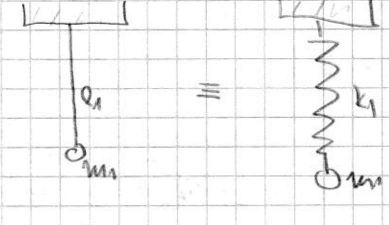
elipsa smjera obrnutog kaz. na satu



(12) OBERBÄCKOVA NJIHALA



analogija

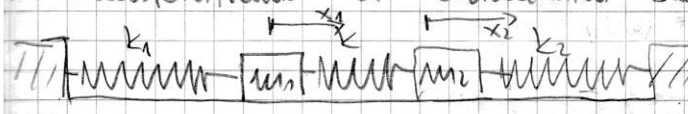


$$\omega_{M.Nj} = \omega_{H.O}$$

$$\frac{g}{l_1} = \frac{k_1}{m_1}$$

$l_1 = l_2$; $m_1 = m_2$ ↓

→ transformiramo u ekvivalentni sustav:



$\boxed{\frac{g}{l_1} = \frac{k_1}{m_1}}$ $\boxed{\frac{g}{l_2} = \frac{k_2}{m_2}}$

k ostaje isti

2NA:
$$m_1 \frac{d^2x_1}{dt^2} = -k_1x_1 + k(x_2 - x_1)$$
$$m_2 \frac{d^2x_2}{dt^2} = -k_2x_2 - k(x_2 - x_1)$$
jednadžbe gibanja

$l_1 = l_2$, $m_1 = m_2$ $\Rightarrow$ $\frac{k_1}{m_1} = \frac{k_2}{m_2} = \omega_0^2$

$$\frac{k}{m_1} = \frac{k}{m_2} = \Omega^2$$

$$\frac{d^2x_1}{dt^2} = -\frac{k_1}{m_1}x_1 + \frac{k}{m_1}x_2 - \frac{k}{m_1}x_1$$
$$\frac{d^2x_2}{dt^2} = -\frac{k_2}{m_2}x_2 - \frac{k}{m_2}x_2 + \frac{k}{m_2}x_1$$

(1) $\frac{d^2x_1}{dt^2} + x_1(\omega_0^2 + \Omega^2) = \Omega^2 x_2$

(2) $\frac{d^2x_2}{dt^2} + x_2(\omega_0^2 + \Omega^2) = \Omega^2 x_1$

$x_1 = A\sin(\omega t + \varphi)$
$x_2 = A'\sin(\omega' t + \varphi')$

(1) $-A\omega^2\sin(\omega t + \varphi) + (\omega_0^2 + \Omega^2)A\sin(\omega t + \varphi) = \Omega^2 A'\sin(\omega' t + \varphi')$

(2) $-A'\omega'^2\sin(\omega' t + \varphi') + (\omega_0^2 + \Omega^2)A'\sin(\omega' t + \varphi') = \Omega^2 A\sin(\omega t + \varphi)$

zaključujemo: $\boxed{\omega = \omega'}$

a) TITRANJE U FAZI: $\boxed{\varphi = \varphi'}$

$x_1 = A\sin(\omega t + \varphi)$
$x_2 = A'\sin(\omega t + \varphi)$

(1) $-A\omega^2\sin\cancel{(\omega t + \varphi)} + (\omega_0^2 + \Omega^2)A\sin\cancel{(\omega t + \varphi)} = \Omega^2 A'\sin\cancel{(\omega t + \varphi)}$

$-A\omega^2 + (\omega_0^2 + \Omega^2)A - \Omega^2 A' = 0 \quad /:A$

$$-\omega^2 + (\omega_0^2 - \Omega^2) - \frac{A'}{A}\Omega^2 = 0 \quad (*)$$

(2) $-A'\omega^2 \cancel{\sin(\omega t + \psi)} + (\omega_0^2 + \Omega^2) A' \cancel{\sin(\omega t + \psi)} = \Omega^2 A \cancel{\sin(\omega t + \psi)}$

$$-A'\omega^2 + A'(\omega_0^2 + \Omega^2) - \Omega^2 A = 0 \quad /:A'$$

$$-\omega^2 + (\omega_0^2 + \Omega^2) - \frac{A}{A'}\Omega^2 = 0 \quad (**)$$

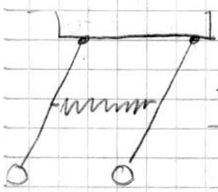
$(*) = (**)$

$$\cancel{-\omega^2} + (\omega_0^2 - \Omega^2) - \frac{A'}{A}\cancel{\Omega^2} = \cancel{-\omega^2} + (\omega_0^2 + \Omega^2) - \frac{A}{A'}\cancel{\Omega^2}$$

$$\frac{A'}{A} = \frac{A}{A'} \Rightarrow \boxed{A = A'} \Rightarrow -\omega^2 + (\omega_0^2 + \Omega^2) - \Omega^2 = 0$$

$$\omega^2 = \omega_0^2 + \cancel{\Omega^2} - \cancel{\Omega^2}$$

$$\boxed{\omega = \omega_0}$$



- isti period
- ista amplituda

b.) TITRANJE U PROTUFAZI $\boxed{\psi' = \psi + \pi}$

$x_1 = A\sin(\omega t + \psi)$

$x_2 = A'\sin(\omega t + \psi + \pi)$

1) $-A\omega^2 \sin(\omega t + \psi) + (\omega_0^2 + \Omega^2) A \sin(\omega t + \psi) = \underbrace{\Omega^2 A' \sin(\omega t + \psi + \pi)}_{-\Omega^2 A' \sin(\omega t + \psi)}$

$$-A\omega^2 + A(\omega_0^2 + \Omega^2) + \Omega^2 A' = 0 \quad /:A$$

$$-\omega^2 + (\omega_0^2 + \Omega^2) + \frac{A'}{A}\Omega^2 = 0 \quad (*)$$

(2) $-A'\omega^2 \sin(\omega t + \psi + \pi) + (\omega_0^2 + \Omega^2) A' \sin(\omega t + \psi + \pi) = \Omega^2 A \sin(\omega t$

$A'\omega^2 \sin(\omega t + \psi) - (\omega_0^2 + \Omega^2) A' \cancel{\sin(\omega t + \psi)} = \Omega^2 A \cancel{\sin(\omega t + \psi)}$

$$A'\omega^2 - (\omega_0^2 + \Omega^2) A' - \Omega^2 A = 0 \quad /:A$$

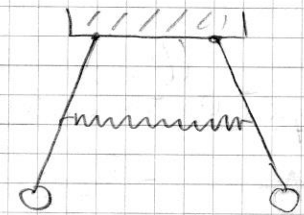
$$\frac{A'}{A}\omega^2 - \frac{A'}{A}(\omega_0^2 + \Omega^2) - \Omega^2 = 0 \quad (**)$$

$(*) = (**) \; /:-1$

$$-\omega^2 + (\omega_0^2+\Omega^2) + \frac{A'}{A}\Omega^2 = -\frac{A'}{A}\omega^2 + \frac{A'}{A}(\omega_0^2+\Omega^2) + \Omega^2$$

$$\frac{A'}{A}\left(\Omega^2+\omega^2 - \cancel{(\omega_0^2+\Omega^2)}\right) = \Omega^2+\omega^2 - (\omega_0^2+\Omega^2)$$

$$\frac{A'}{A} = 1$$



$\boxed{A = A'} \Rightarrow \omega^2 - (\omega_0^2+\Omega^2) - \Omega^2 = 0$

$$\omega^2 = 2\Omega^2 + \omega_0^2$$

$$\boxed{\omega = \sqrt{\omega_0^2+2\Omega^2}}$$

- period kraći
- amplitude iste

---

fazno titranje

$x_1 = A_1 \sin(\omega_1 t + \varphi_1)$

$x_2 = A_1 \sin(\omega_1 t + \varphi_1)$

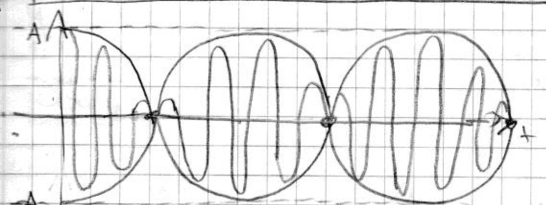
protufazno titranje

$x_1 = A_2 \sin(\omega_2 t + \varphi_2)$

$x_2 = -A_2 \sin(\omega_2 t + \varphi_2)$

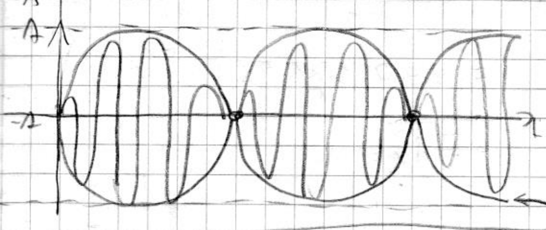
$$\boxed{\begin{aligned} x_1 &= A_1 \sin(\omega_1 t + \varphi_1) + A_2 \sin(\omega_2 t + \varphi_2) \\ x_2 &= A_1 \sin(\omega_1 t + \varphi_1) - A_2 \sin(\omega_2 t + \varphi_2) \end{aligned}}$$

OPĆE RJEŠENJE LIN. DIF. JEDNADŽBE



1. oscilator

UDARI



2. oscilator

amplituda

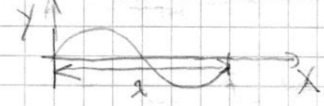
$$x_1 = \boxed{2A\cos\left(\frac{\omega_1-\omega_2}{2}t + \frac{\varphi_1-\varphi_2}{2}\right)} \sin\left(\frac{\omega_1+\omega_2}{2}t + \frac{\varphi_1+\varphi_2}{2}\right)$$

$$x_2 = \boxed{2A\sin\left(\frac{\omega_1-\omega_2}{2}t + \frac{\varphi_1-\varphi_2}{2}\right)} \cos\left(\frac{\omega_1+\omega_2}{2}t + \frac{\varphi_1+\varphi_2}{2}\right)$$

uz $A_1 = A_2 = A$

(13) VALNA FUNKCIJA

→ progresivni, harmonički, transverzalni val



$$\boxed{y(t,x) = A\sin(\omega t \mp kx + \varphi)}$$

$\boxed{v = \frac{\lambda}{T}}$ VALNA BRZINA

$y(T,0) = y(T,\lambda)$

$A\sin(\omega T) = A\sin(\omega T - k\lambda)$

$\omega T - 2\pi = \omega T - k\lambda$

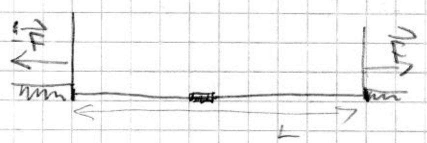
$\boxed{k = \frac{2\pi}{\lambda}}$ VALNI BROJ

$\lambda = \frac{2\pi}{k} \Rightarrow v = \frac{2\pi}{Tk}$

$\boxed{v = \frac{\omega}{k}}$

(14) TRANSVERZALNI VAL NA ŽICI

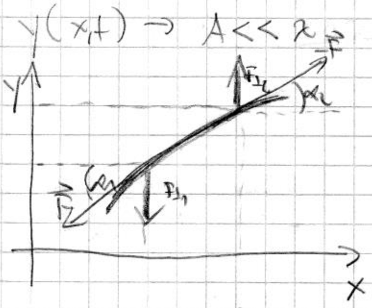
P.U. tanka žica $d \ll \lambda$
savršeno elastična žica
homogena žica



m, F, L

$\mu = \frac{m}{L}$

$y(x,t) \to A \ll \lambda$



$dF_y = F_{2y} - F_{1y} = F\sin\alpha_2 - F\sin\alpha_1$
$= |\alpha \to 0, \sin\alpha \to \alpha| = F(\alpha_2 - \alpha_1)$
$= F\,d\alpha =$

$\mathrm{tg}\,\alpha = \frac{\partial y}{\partial x} = \alpha \;/d$

$\underline{d\alpha = \frac{\partial^2 y}{\partial x^2}dx}$

$dF_y = F\frac{\partial^2 y}{\partial x^2}dx \quad (*)$

2.NA

$$dF_y = \frac{\partial^2 y}{\partial t^2}\, dm = \frac{\partial^2 y}{\partial t^2}\, \mu dx \quad (**)$$

$(*) = (**)$

$$F \frac{\partial^2 y}{\partial x^2} \cancel{dx} = \frac{\partial^2 y}{\partial t^2} \mu \cancel{dx}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} - \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2} = 0}$$ VALNA JEDNADŽBA

→ valna f-ja zadovoljava gornju jednadžbu:

$$\sin(\omega t \mp kx) = \sin(\omega\underbrace{(t \mp \tfrac{x}{v})}_{u})$$

$$f\left(t \mp \frac{x}{v}\right)$$

jer je $s(x,t) = f(x - vt) + g(x + vt)$ opće rješenje diferencijalne jednadžbe.

$f(x - vt)$ → opisuje val ulijevo

$g(x + vt)$ → opisuje val udesno

Dokaz rješenja:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{du}{dx} = \frac{\partial f}{\partial u}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial u}\right) = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\right) = \frac{\partial^2 f}{\partial u^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial u}\frac{du}{dt} = \frac{\partial f}{\partial u}(-v)$$

$$\frac{\partial f^2}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial u}(-v)\right) = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial t}\right)(-v) = \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}(-v)\right)(-v)$$

$$= \frac{\partial^2 f}{\partial u^2} v^2$$

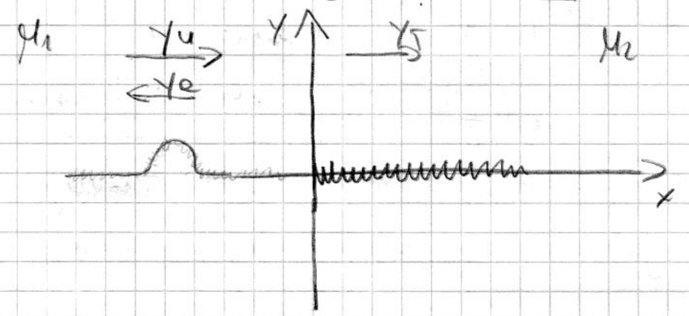
$$\frac{\partial f^2}{\partial u^2} - \frac{\mu}{F}\frac{\partial f^2}{\partial u^2} v^2 = 0 \Rightarrow v^2 = \frac{F}{\mu} \Rightarrow \boxed{v = \sqrt{\frac{F}{\mu}}}$$ FAZNA BRZINA

$$\boxed{\omega = k\sqrt{\frac{F}{\mu}}}$$ DISPERZIJSKA RELACIJA

brzina ne ovisi o frekvenciji => NEDISPERZIVNO SREDSTVO

(15) REFLEKSIJA VALA

P.U. uže nedisperzivno sredstvo } => ista fazna brzina
uže kontinuirano (1)
uže bez skoka (2)



$$v_1 = \sqrt{\frac{F_1}{\mu_1}}$$
$$v_2 = \sqrt{\frac{F_2}{\mu_2}}$$
$$\boxed{\frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}}$$

$$y_u = A_u \sin\left(\omega\left(t - \frac{x}{v_1}\right)\right)$$
$$y_R = A_R \sin\left(\omega\left(t + \frac{x}{v_1}\right)\right)$$
$$y_T = A_T \sin\left(\omega\left(t - \frac{x}{v_2}\right)\right)$$

(1) uvjet:

$$y_u(t,0) + y_R(t,0) = y_T(t,0)$$

$$A_u \cancel{\sin(\omega t)} + A_R \sin(\omega t) = A_T \cancel{\sin(\omega t)}$$

$$\underline{A_u + A_R = A_T}$$

(2) uvjet:

$$\frac{\partial y_u}{\partial x} + \frac{\partial y_R}{\partial x} = \frac{\partial y_T}{\partial x} \qquad x=0$$

$$A_1 \cos\left(\omega\left(t-\frac{x}{v_1}\right)\right)\left(-\frac{1}{v_1}\right) + A_R \cos\left(\omega\left(t+\frac{x}{v_1}\right)\right)\frac{1}{v_1} = A_T\cos\left(\omega\left(t-\frac{x}{v_2}\right)\right)\left(\frac{-1}{v_2}\right)$$

$$\frac{A_1}{v_1} - \frac{A_R}{v_1} = \frac{A_T}{v_2}$$

$$\underline{\frac{A_1 - A_R}{v_1} = \frac{A_T}{v_2}}$$

① i ②

$$\frac{A_1 - A_R}{v_1} = \frac{A_1 + A_R}{v_2}$$

$$A_1 v_2 - A_R v_2 = A_1 v_1 + A_R v_1$$

$$A_1(v_2 - v_1) = A_R(v_1 + v_2)$$

$$\boxed{A_R = \frac{v_2 - v_1}{v_1 + v_2} A_1}$$

$$\frac{A_1 - (A_T - A_1)}{v_1} = \frac{A_T}{v_2}$$

$$A_1 v_2 - A_T v_2 + A_1 v_2 = A_T v_1$$

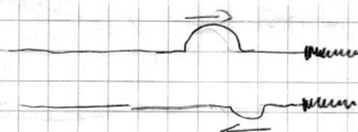
$$2A_1 v_2 = A_T(v_1 + v_2)$$

$$\boxed{A_T = \frac{2v_2}{v_1 + v_2} A_1}$$

Slučajevi:

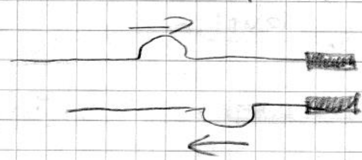
① $\boxed{\mu_1 < \mu_2} \Rightarrow v_1 > v_2 \Rightarrow A_R < 0$

$$y_R = -|A_R| \sin\left(\omega\left(t+\frac{x}{v_1}\right)\right) = |A_R| \sin\left(\omega\left(t+\frac{x}{v_1}\right)+\pi\right)$$



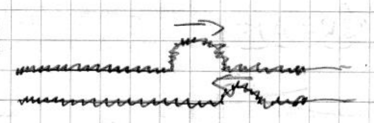
② $\boxed{\mu_2 = \infty}$ čvrsti kraj $\Rightarrow v_2 = 0 \Rightarrow A_R = -A_1$

$$y_R = -A_1 \sin\left(\omega\left(t+\frac{x}{v_1}\right)\right) = A_1 \sin\left(\omega\left(t+\frac{x}{v_1}\right)+\pi\right)$$



③ $\boxed{\mu_1 > \mu_2} \Rightarrow v_1 < v_2 \Rightarrow A_R > 0$

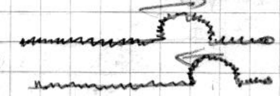
$$y_R = |A_R| \sin\left(\omega\left(t+\frac{x}{v_1}\right)\right)$$



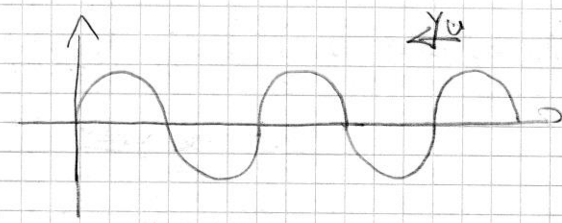
④ $\boxed{\mu_2 = 0}$ slobodni kraj

$$A_R = \frac{\sqrt{\frac{F}{\mu_2}} - \sqrt{\frac{F}{\mu_1}}}{\sqrt{\frac{F}{\mu_1}} + \sqrt{\frac{F}{\mu_2}}} A_1 \Big/ \frac{\sqrt{\mu_2}}{\sqrt{\mu_2}} = \frac{1 - \sqrt{\frac{\mu_2}{\mu_1}}}{1 + \sqrt{\frac{\mu_2}{\mu_1}}} A_1 = A_1$$

$A_R = A_1$



(16) STOJNI VAL



$y_u = A\sin(\omega t + kx)$

$y_R = A\sin(\omega t - kx + \pi)$

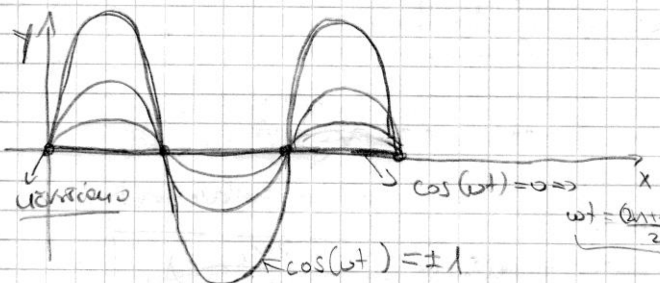
$= -A\sin(\omega t - kx)$

$Y = y_u + y_R = A\sin(\omega t + kx) - A\sin(\omega t - kx)$

$= A\,2\cos\left(\frac{\omega t + \omega t + \cancel{kx} - \cancel{kx}}{2}\right)\sin\left(\frac{\cancel{\omega t} + kx - \cancel{\omega t} + kx}{2}\right)$

$\boxed{Y = 2A\cos(\omega t)\sin(kx)}$ JEDNADŽBA STOJNOG VALA (lokalni val)

$\boxed{\Delta E = 0}$



$\cos(\omega t) = 0 \Rightarrow \underline{\omega t = \frac{(2n+1)}{2}\pi}$, $n = 0, 1, \dots$

$\cos(\omega t) = \pm 1$

$\Rightarrow \underline{\omega t = n\pi}$, $n = 0, 1, \dots$

ČVOR → $y = 0$ (po prostoru)

$\sin(kx_n) = 0$

$kx_n = n\pi \Rightarrow x_n = \frac{2n\pi}{2k}$

$\boxed{x_n = n\frac{\lambda}{2}}$, $n = 0, 1, 2, \dots$

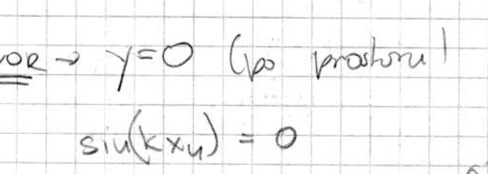
TRBUH $y = max$ (po prostoru)

$\sin(kx_n) = \pm 1$

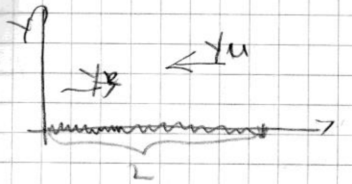
$kx_n = \frac{(2n+1)}{2}\pi \Rightarrow x_n = \frac{(2n+1)2\pi}{2 \cdot 2k}$

$\boxed{x_n = (2n+1)\frac{\lambda}{4}}$, $n = 0, 1, 2$

(17) TRANSVERZALNI STOJNI VAL NA NAPETOJ ŽICI



→ oba kraja učvršćena (1)



$$y_u(x,t) = A_1 \sin(\omega t + kx)$$
$$y_R(x,t) = A_2 \sin(\omega t - kx)$$

$$y = y_u + y_R = A_1 \sin(\omega t + kx) + A_2 \sin(\omega t - kx)$$

iz ① $\Rightarrow$ $y(t,0) = 0$

$$0 = A_1 \cancel{\sin}(\omega t) + A_2 \cancel{\sin}(\omega t)$$

$$\boxed{A_1 = -A_2} \Rightarrow y_R = -A_1 \sin(\omega t - kx) = \underline{A_1 \sin(\omega t - kx + \pi)}$$

$$y = A \sin(\omega t + kx) + A \sin(\omega t - kx + \pi) = 2A \cos(\omega t) \sin(kx)$$

$\hookrightarrow$ stojni val

iz ① $\Rightarrow$ $y(t,L) = 0$

$$2A \cos(\omega t) \sin(kL) = 0$$

$$\sin(k_n L) = 0 \Rightarrow k_n L = n\pi$$

$$L = \frac{2 n \pi}{2 k_n} \lambda_n$$

$$L = \frac{\lambda_n}{2} \cdot n$$

$$\boxed{\lambda_n = \frac{2L}{n}}$$ n - broj hamonika

$n = 1, 2, 3 \ldots$

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L} \sqrt{\frac{F}{\mu}}$$

$$\boxed{f_n = \frac{n}{2L} \sqrt{\frac{F}{\mu}}}$$ VLASTITA FREKVENCIJA

$$\boxed{f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}}}$$ OSNOVNA FREKVENCIJA

$$\boxed{f_n = n f_1}$$

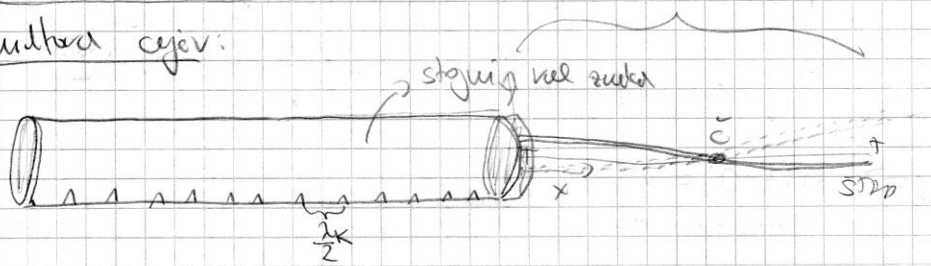
$$\boxed{y(x,t) = \sum_{n}^{\infty} (A_n \cos \omega_n t + B_n \sin \omega_n t) \sin k_n t}$$ VALNA FUNKCIJA STOJNOG VALA (Fourierov red)

P.U. $s(t=0,x) = \boxed{s_0(x) = \sum_0^{\infty} A_n \sin k_n t}$ $\boxed{v_0(x,t=0) = \frac{\partial s(x,t)}{\partial t} = v_0(x)}$

18 LONGITUDINALNI STOJNI VAL

a.) OTVOREN NA KRAJEVIMA

-> Kundtova cijev:



na otvorenom kraju nema pomaka u fazi:

$y_u = A \sin(\omega t - kx)$

$y_R = A \sin(\omega t + kx)$

$\boxed{y = 2A \cos kx \sin \omega t}$ LONGITUDINALNI STOJNI VAL

P.U. ① $y(t, l) = max$
② $y(t, \frac{l}{2}) = 0$

① $\cos k_n l = \pm 1$

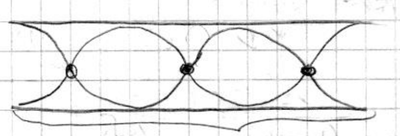
$k_n l = n\pi \quad , n = 0, 1, ...$

$l = \frac{(2\pi)}{2 k_n} \lambda_n$

$\boxed{\lambda_n = \frac{2l}{n}}, \; n = 1, 2, ...$

SLOBODNI KRAJEVI

* OTVORENA SVIRALA



$\lambda_n = \frac{2L}{n}$ ↳ broj harmonika

② $\cos k_n \frac{l}{2} = 0$

$k_n \frac{l}{2} = \frac{(2n+1)\pi}{2} \quad , n = 0, 1, ...$

$l = \frac{(2n+1)}{2} \frac{(2\pi)}{(k_n)} \lambda_n$

$\lambda_n = \frac{2l}{2n+1} \quad n = 0, 1, ...$

$\boxed{\lambda_n = \frac{2l}{2n-1}}, \; n = 1, 2, ...$

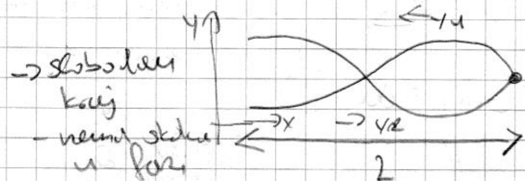
↳ zadovoljava i prethodni uvjet

SLOBODNI KRAJEVI, UČVRŠĆEN NA SREDINI

* ŠTAP KUNDTOVE CIJEVI

$\boxed{v_K = f_K \lambda_K}$

b.) SLOBODAN KRAJ - ČVOR



$$y_u = A\sin(\omega t - kx)$$

$$y_R = A\sin(\omega t + kx)$$

$$\boxed{y = 2A\cos kx \sin\omega t}$$

P.U. ① $y(t, 0) = max$ → uvjet vrijedi za slobod. kraj

② $y(t, L) = 0$

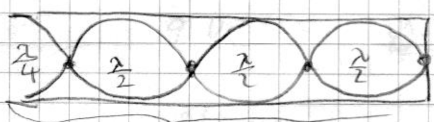
② $\cos k_n L = 0$

$$k_n L = \frac{(2n+1)\pi}{2} \quad n = 0, 1, \ldots$$

$$L = \frac{(2n+1)}{4} \frac{2\pi}{k_n} \lambda_n$$

$$\boxed{\lambda_n = \frac{4L}{(2n-1)}} \quad n = 1, 2, 3, \ldots$$

* ZATVORENA SVIRALA



$n = 3$

$$L = 3\frac{\lambda}{2} + \frac{\lambda}{4} = \frac{7\lambda}{4}$$

Provjera:

$$L = \frac{(2n+1)}{4}\lambda$$

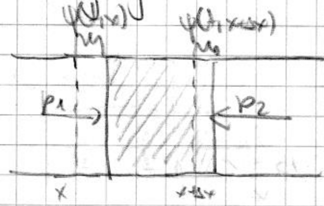
$$L = \frac{7}{4}\lambda$$

⑲ LONGITUDINALNI VALOVI ZVUKA. BRZINA ZVUKA

$$\Delta V_0 = S\Delta x$$

$$\Delta m = \rho\Delta V_0 = \rho S\Delta x$$

→ mijenja se tlak:



$$p_1 = p_0 + p$$

$$p_2 = p_0 + p + \Delta p$$

$$\Delta p = p_2 - p_1 \quad (\text{pretp. } p_2 > p_1)$$

$$\Delta\psi = \psi(t, x+\Delta x) - \psi(t, x)$$

HOOKEOV ZAKON	2 NZ
$\Delta\psi = \psi(t,x) + \frac{\partial\psi}{\partial x}\Delta x - \cancel{\psi(t,x)}$	$\Delta x \to dx$
$\Delta\psi = \frac{\partial\psi}{\partial x}\Delta x$	$dm = \rho S dx$

$p = -B\frac{\Delta V}{\Delta V_0} = -B\frac{\Delta\psi S}{S\Delta x} = -B\frac{\Delta\psi}{\Delta x}$

$\boxed{p = -B\frac{\partial\psi}{\partial x}}\ \frac{\Delta x}{\Delta x} \quad /d$

$dp = -B\frac{\partial^2\psi}{\partial x^2}dx$

$p_2 - p_1 = dp$

$dF = -S\,dp$ ($p_2 - p_1 > 0$, lada gura u smjeru $-x$)

$dF = -S\,dp = dm\frac{\partial^2\psi}{\partial t^2} = S\rho\,dx\frac{\partial^2\psi}{\partial t^2}$

$$S B\frac{\partial^2\psi}{\partial x^2}dx = S\rho\,dx\frac{\partial^2\psi}{\partial t^2}$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} - \frac{1}{\underbrace{\frac{B}{\rho}}_{v^2}}\frac{\partial^2\psi}{\partial t^2} = 0}$$ VALNA JEDNADŽBA LONG. VALA (ZVUKA)

$\boxed{v^2 = \frac{B}{\rho}}$ BRZINA ZVUKA (tekućina)

$\psi(t,x) = A\sin(\omega t - kx)$

$p(t,x) = ?$

$\omega = k\sqrt{\frac{B}{\rho}}$

$p = -B\frac{\partial\psi}{\partial x} = -BA\cos(\omega t - kx)\cdot(-k)$

$= AB\frac{\omega}{v}\cos(\omega t - kx) = Av^2\rho\frac{\omega}{v}\cos(\omega t - kx)$

$\boxed{p(t,x) = \Delta p_{max}\cos(\omega t - kx)}$ FUNKCIJA TLAKA $\quad \Delta p_{max} = Av\rho\omega$

$\boxed{v^2 = \frac{E}{\rho}}$ BRZINA ZVUKA (štap) $\qquad \boxed{v^2 = \kappa\frac{p}{\rho} = \kappa\frac{RT}{M}}$ BRZINA ZVUKA (plin)

(20) DOPPLEROV EFEKT

a) 

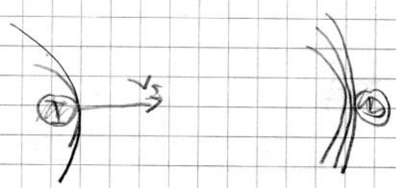
$f = \frac{v}{\lambda} \qquad f' = \frac{v_0 + v}{\lambda} = \frac{v_0 + v}{\frac{v}{f}} = \left(\frac{v_0 + v}{v}\right)f$

$\lambda = \frac{v}{f} \qquad \boxed{f' = f\left(1 + \frac{v_0}{v}\right)}$

b.) 

$$f' = \frac{v - v_O}{\lambda} = \frac{v - v_O}{\frac{v}{f}} = f\left(1 - \frac{v_O}{v}\right)$$

$$\boxed{f' = f\left(1 - \frac{v_O}{v}\right)}$$

c.) 

$$\lambda' = \lambda - v_I T$$

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - \frac{v_I}{f}} = \frac{v}{\frac{v}{f} - \frac{v_I}{f}} = f\left(\frac{v}{v - v_I}\right)$$

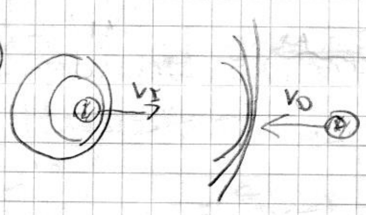
$$\boxed{f' = f\left(\frac{v}{v - v_I}\right)}$$

d.) 

$$\lambda' = \lambda + v_I T = \frac{v}{f} + \frac{v_I}{f}$$

$$f' = \frac{v}{\lambda'} = \frac{v}{\frac{v}{f} + \frac{v_I}{f}} = f\left(\frac{v}{v + v_I}\right)$$

$$\boxed{f' = f\left(\frac{v}{v + v_I}\right)}$$

e.) 

$$v' = v + v_O$$

$$\lambda' = \lambda - v_I T = \lambda - \frac{v_I}{f}$$

$$f' = \frac{v'}{\lambda'} = \frac{v + v_O}{\lambda - \frac{v_I}{f}} = \frac{v + v_O}{\frac{v}{f} - \frac{v_I}{f}} = f\left(\frac{v + v_O}{v - v_I}\right)$$

$$\boxed{f' = f\left(\frac{v + v_O}{v - v_I}\right)}$$

apunte

$$f' = f\,\frac{v \pm v_O}{v} \qquad \left(\text{I} \quad \overset{+\;\;\;-}{\longleftrightarrow}\ \text{O}\right)$$

$$f' = f\left(\frac{v}{v \mp v_I}\right) \qquad \left(\overset{\pm\;\;}{\longleftrightarrow}\ \text{I} \quad \text{O}\right)$$

$$f' = f\left(\frac{v \pm v_O}{v \mp v_I}\right) \qquad \left(\overset{+\;\;\;-}{\longleftrightarrow}\ \text{I} \quad \overset{+\;\;\;-}{\longleftrightarrow}\ \text{O}\right)$$

$$\boxed{f' = f\left(\frac{v \mp v_O}{v \mp v_I}\right)} \left\{ \overset{\pm\;\;\;-}{\longleftrightarrow}\ \text{I} \quad \overset{\pm\;\;\;-}{\longleftrightarrow}\ \text{O} \right\}$$

TEORIJA

① VALNA JEDNADŽBA ELEKTROMAGNETSKOG VALA

vakuum
P.U $\rho = 0$
$\vec{J} = 0$
ε_0, μ_0

MAXWELLOVE JEDNADŽBE:

u vakuumu ① = 0
④ = nema J člana

① $\nabla \cdot \vec{E} = 0 = \frac{\rho}{\varepsilon_0}$ $\quad \int E\,dS = \frac{1}{\varepsilon_0}\int \rho\,dV$

② $\nabla \cdot \vec{B} = 0 \longrightarrow \Phi = \oint \vec{B}\,d\vec{S} = 0$ kroz zatv. plohu

③ $\nabla \times \vec{E} = \frac{-\partial B}{\partial t} \longrightarrow \mathcal{E} = \oint \vec{E}\,d\vec{l} = -\frac{\partial}{\partial t}\int B\,dS$

④ $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \frac{\partial E}{\partial t} = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$ $\quad \oint B\,d\vec{l} = \int \mu_0 \vec{J}\,d\vec{S} + \mu_0\varepsilon_0 \frac{\partial}{\partial t}\int E\,dS$

a.) $\nabla \times \vec{B} = \mu_0\varepsilon_0 \frac{\partial E}{\partial t} \;\Big/ \frac{\partial}{\partial t}$

$\frac{\partial}{\partial t}(\nabla \times \vec{B}) = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$

$\nabla \times \frac{\partial \vec{B}}{\partial t} = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$ ①

$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \;\Big/ \text{rot}$

$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$ ②

jednakost identiteta:
$\text{rot}(\text{rot}\vec{E}) = \text{grad}(\text{div}\vec{E}) - \Delta\vec{E}$
$\Delta\vec{E} = (\nabla \cdot \nabla)\vec{E}$

$\nabla \times (\nabla \times \vec{E}) = \text{grad}(\underbrace{\nabla \cdot \vec{E}}_{=0\,(1.M)}) - \Delta\vec{E}$

$\nabla \times (\nabla \times \vec{E}) = -\Delta\vec{E}$ ③

② i ③
$\Rightarrow -\Delta\vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t}$

①
$\Rightarrow \Delta\vec{E} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

$$\boxed{\Delta\vec{E} - \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

VALNA JEDNADŽBA elekt. polja

b.) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Big| \frac{\partial}{\partial t}$

$$\frac{\partial}{\partial t}(\nabla \times \vec{E}) = -\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla \times \frac{\partial \vec{E}}{\partial t} = -\frac{\partial^2 \vec{B}}{\partial t^2} \quad (1)$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \Big| \text{rot}$$

$$\nabla \times (\nabla \times \vec{B}) = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \quad (2)$$

jednakost se koristi

$$\Delta \vec{B} = \text{grad}(\text{div}\,\vec{B}) - \text{rot}(\text{rot}\,\vec{B})$$

$$\text{rot}(\text{rot}\,\vec{B}) = \text{grad}(\text{div}\,\vec{B}) - \Delta\vec{B} \quad \underbrace{}_{=0}\ \text{2.M.J.}$$

$$\nabla \times (\nabla \times \vec{B}) = \text{grad}(\nabla \cdot \vec{B}) - \Delta\vec{B}$$

$$\nabla \times (\nabla \times \vec{B}) = -\Delta\vec{B}$$

(2) i (3) $\Rightarrow$

$$-\Delta \vec{B} = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$-\Delta \vec{B} = \mu_0 \varepsilon_0 \left(\nabla \times \frac{\partial \vec{E}}{\partial t}\right)$$

(1) $\Rightarrow$ $\cancel{-}\Delta \vec{B} = \cancel{-}\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

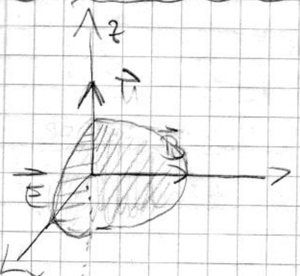
$$\boxed{\Delta \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0}$$ VALNA JEDNADŽBA MAGNETSKA POLJA

$\Rightarrow$ $\frac{1}{c^2} = \mu_0 \varepsilon_0$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$c = 2{,}998 \cdot 10^8\ \text{ms}^{-1}$

(2) HARMONIJSKI RAVNI ELEKTROMAGNETSKI VAL



P.U.

- $\rho, \vec{J} = 0$ (vakuum)
- monokromatski val
- $\vec{E}, \vec{B}, \vec{n}$ - desni sustav
- $c = \frac{\omega}{k}$
- μ_0, ε_0

monokromatski el. val (samo jedna frekvencija) čiji se položaj mijenja po z-osi

$$\vec{E}_x = E_{0x} \sin(\omega t - kz)\,\vec{i}$$

3.M.J. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}\vec{j}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ |\vec{E}_x| & 0 & 0 \end{vmatrix} = -\frac{\partial B}{\partial t}\vec{j}$$

$$\underbrace{\vec{i}\cdot 0}_{0} - \vec{j}\left(\frac{\partial}{\partial x}0 - \frac{\partial}{\partial z}|\vec{E}_x|\right) + \vec{k}\underbrace{\left(\frac{\partial}{\partial x}\cdot 0 - \frac{\partial}{\partial y}|\vec{E}|\right)}_{0} = -\frac{\partial B}{\partial t}\vec{j}$$

$$\vec{j}\,\frac{\partial}{\partial z}|\vec{E}_x| = -\frac{\partial B}{\partial t}\,\vec{j}$$

$$\frac{\partial|\vec{E}_x|}{\partial z} = -\frac{\partial B}{\partial t}$$

$$\cancel{-}k\,E_{0x}\cos(\omega t - kz) = \cancel{-}\frac{\partial B}{\partial t} \quad \Big| \int$$

$$kE_{0x}\int\cos(\omega t - kz)\,dt = B_y$$

$$kE_{0x}\,\frac{\sin(\omega t - kz)}{\omega} = B_y$$

$$\frac{E_{0x}\sin(\omega t - kz)}{\frac{\omega}{k}\;c} = B_y$$

$$B_y = \underbrace{\frac{E_{0x}}{c}}_{B_{0y}}\sin(\omega t - kz)$$

$$B_y = B_{0y}\sin(\omega t - kz)$$

$$\boxed{\vec{B}_y = B_{0y}\sin(\omega t - kz)\,\vec{j}}$$

$$B_{0y} = \frac{E_{0x}}{c}$$

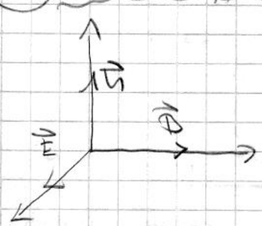
$$B_y = \frac{E_x}{c}.$$

$\vec{E},\vec{B},\vec{n} \to$ desni sustav

$$\boxed{\vec{B} = \frac{1}{c}(\vec{n}\times\vec{E}) \qquad \vec{E} = c\cdot(\vec{B}\times\vec{n})}$$

HARMONIJSKI RAVNI ELEKTROMAGNETSKI VAL

3 GUSTOĆA ENERGIJE EM VALA i POYNTINGOV VEKTOR



P.U.
vakuum ($\mu_0, \varepsilon_0, \vec{J}=0, \rho=0$)
$\vec{E},\vec{B},\vec{n} \to$ desni sustav
$B_y = \frac{E_{0x}}{c} = E_x\sqrt{\mu_0\varepsilon_0} \Rightarrow B_y^2 = E_x^2\mu_0\varepsilon_0$

$$w = w_E + w_B = \frac{1}{2}\varepsilon_0 E_x^2 + \frac{1}{2\mu_0}B_y^2$$

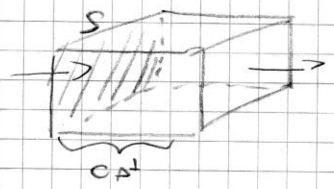
I.) $w = \frac{1}{2}\varepsilon_0 E_x^2 + \frac{1}{2\cancel{\mu_0}}E_x^2\cancel{\mu_0}\varepsilon_0 = \varepsilon_0 E_x^2$

II.) $w = \frac{1}{2}\varepsilon_0\frac{B_y^2}{\mu_0\varepsilon_0} + \frac{1}{2\mu_0}B_y^2 = \frac{B_y^2}{\mu_0}$

III.) $w = \varepsilon_0 E_x B_y c$

$w = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,E_x B_y$

$$\boxed{w = \varepsilon_0 E_x^2 = \frac{B_y^2}{\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_x B_y}$$



Δt

$E = ?$

$$E = w \cdot V = w S c \Delta t$$

$$|\vec{S}| = \frac{E}{S\Delta t} = \frac{w S c \Delta t}{S \Delta t} = wc$$

a.) $|\vec{S}| = \varepsilon_0 E_x^2 c = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_x^2 \rightsquigarrow |\vec{S}| = I = \frac{1}{T}\int_0^T |\vec{S}| = \frac{1}{T}\int_0^T \varepsilon_0 c E_x^2$

$= \frac{1}{T}\int_0^T \varepsilon_0 c E_{0x}^2 \sin^2(\omega t - kz)\,dt$

b.) $|\vec{S}| = \frac{B_y^2}{\mu_0} c = \frac{B_y^2}{\mu_0 \sqrt{\mu_0 \varepsilon_0}}$

$$\boxed{I = \frac{1}{2}\varepsilon_0 c E_{0x}^2}$$ INTENZITET EM VALA

c.) $|\vec{S}| = c \cdot \sqrt{\frac{\varepsilon_0}{\mu_0}} E_x B_y = \frac{1}{\mu_0} E_x B_y$

$$\boxed{|\vec{S}| = \frac{1}{\mu_0} E_x B_y}$$

$I = \frac{1}{2}\varepsilon_0 c^2 E_{0x} B_{0y}$

$I = \frac{1}{2}\frac{c}{\mu_0} B_{0y}^2$

$I = \frac{1}{2\mu_0\sqrt{\mu_0\varepsilon_0}} B_{0y}^2$

$\vec{S} = x\vec{n} \quad (\vec{S} \parallel \vec{n})$

$$\boxed{\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})}$$ POYNTINGOV VEKTOR

4' FOTOMETRIJA

$\boxed{\Phi = I\Omega}$ SVJETLOSNI TOK IZOTROPNOG TOČKASTOG IZVORA $[lm]$

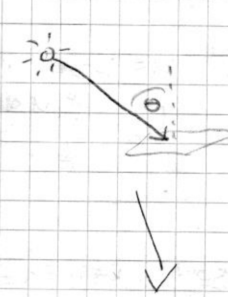
$\boxed{\Omega = \frac{S}{R^2}}$

$\boxed{d\Phi = I\,d\Omega}$ SVJETLOSNI TOK TOČKASTOG IZVORA KOJI NIJE IZOTROPAN

$\boxed{d\Omega = \sin\theta\, d\theta\, d\varphi}$

$\underline{1W = 683\ lm}$

$\boxed{\eta = \frac{\Phi_{uk}}{P}}$ KORISNOST IZVORA



$\boxed{E = \frac{d\Phi}{dS}}\ [lx]$ OSVIJETLJENOST POVRŠINE

$\Phi = I\Omega = I\frac{S}{R^2} \rightsquigarrow \frac{\Phi}{S} = E = \frac{I}{R^2} \rightsquigarrow \boxed{E = \frac{I}{R^2}\cos\theta}$ LAMBERTOV ZAKON

(5) ZAKONI GEOMETRIJSKE OPTIKE

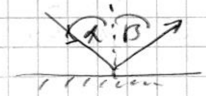
$\boxed{\lambda \ll D}$ → dimenzija predmeta

UVJET APROKSIMACIJE SVJETLOSTI ZRAKAMA

1.) zakon → svjetlost se pravocrtno širi u homogenom izotropnom sredstvu.

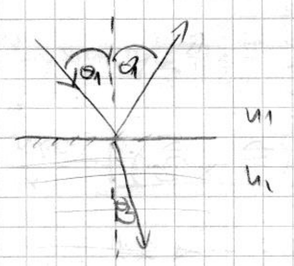
2.) zakon → za nekoherentne snopove vrijedi da jedan na drugog ne utječe ni kad se presjecaju.

3.) zakon → $\boxed{\alpha = \beta}$ ZAKON REFLEKSIJE



4.) zakon → $\boxed{\frac{\sin \Theta_1}{\sin \Theta_2} = \frac{n_2}{n_1}}$ ZAKON LOMA

$$\boxed{n = \frac{c}{v} \qquad n^2 = \varepsilon_r \mu_r}$$



5.) zakon → svjetlost se između 2-ju zadanih točaka giba onom stazom za koju joj treba najmanje vremena: FERMATOV PRINCIP

$$t_{AB} = \int_{A(\gamma)}^{B} \frac{dl}{v} = \int_{A(\gamma)}^{B} \frac{n\,dl}{c} \iff \frac{d}{ds}t = 0 \quad \text{(min, vrijedi)}$$

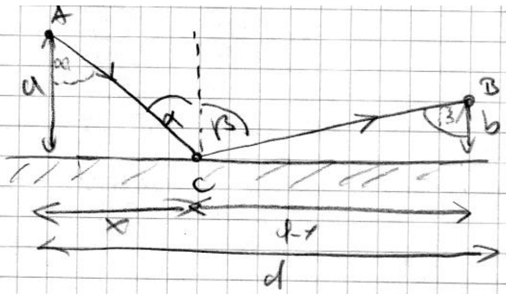
Npr. $\Rightarrow$ $n = \text{konst}$ (homogeno tijelo)

$$t_{AB} = \int_A^B \frac{n\,dl}{c} = \frac{n}{c}\int_A^B dl$$ putanja je pravac $\Rightarrow$ $\boxed{t_{AB} = \frac{n}{c}\int_A^B dl = \frac{n}{c}|AB|}$

1. ZAKON GEOM. OPTIKE

(6) ZAKON REFLEKSIJE (Fermatov princip)

$\left\{ \begin{array}{l} \text{P.U.} \\ n = \text{konst} \quad 1.\text{Z.G.O.}: \ t_{AB} = \frac{n}{c}|AB| \end{array} \right\}$



$t_{AB} = t_{AC} + t_{CB}$

$t_{AB} = \frac{n}{c}|AC| + \frac{n}{c}|CB|$

$$t_{AB} = \frac{n}{c}\left(\sqrt{a^2+x^2} + \sqrt{(d-x)^2+b^2}\right) = \frac{n}{c}\left((a^2+x^2)^{\frac{1}{2}} + ((d-x)^2+b^2)^{\frac{1}{2}}\right)$$

$\frac{d}{dx} t_{AB} = 0$ (minimalno vrijeme)

$$\frac{n}{c}\left(\frac{1}{2}(a^2+x^2)^{-\frac{1}{2}} 2x + \frac{1}{2}\left((d-x)^2+b^2\right)^{-\frac{1}{2}} 2(d-x)(-1)\right) = 0$$

$$\frac{n}{c}\left[-\left((d-x)^2+b^2\right)^{-\frac{1}{2}}(d-x) + (a^2+x^2)^{-\frac{1}{2}} x\right] = 0$$

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{(d-x)^2+b^2}}$$

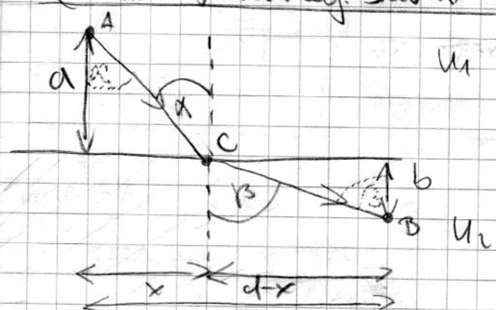
$\sin\alpha = \sin\beta$

$\boxed{\alpha = \beta}$ ZAKON REFLEKSIJE

7 ZAKON LOMA (Fermatov princip)

P.U. 1. ZGO: $t_{AB} = \frac{n}{c}|AB|$

$n_{1,2}$ = konst (homog. sredstva)



$t_{AB} = t_{AC} + t_{CB} = \frac{n_1}{c}|AC| + \frac{n_2}{c}|CB|$

$t_{AB} = \frac{n_1}{c}\sqrt{a^2+x^2} + \frac{n_2}{c}\sqrt{(d-x)^2+b^2}$

$t_{AB} = \frac{1}{c}\left[n_1(a^2+x^2)^{\frac{1}{2}} + n_2\left((d-x)^2+b^2\right)^{\frac{1}{2}}\right]$

$\frac{d}{dx} t_{AB} = 0$ (minimalno vrijeme)

$$\frac{1}{c}\left[n_1 \frac{1}{2}(a^2+x^2)^{-\frac{1}{2}} 2x + n_2 \frac{1}{2}\left((d-x)^2+b^2\right)^{-\frac{1}{2}} 2(d-x)(-1)\right] = 0$$

$$n_1 x (a^2+x^2)^{-\frac{1}{2}} = n_2 (d-x)\left((d-x)^2+b^2\right)^{-\frac{1}{2}}$$

$$n_1 \frac{x}{\sqrt{a^2+x^2}} = n_2 \frac{d-x}{\sqrt{(d-x)^2+b^2}}$$

$$n_1 \sin\alpha = n_2 \sin\beta$$

$$\boxed{\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}}$$ ZAKON LOMA

⑧ OPTIČKI PUT

$n \neq$ konst

$$t_{AB} = \int_A^B \frac{n\,dl}{c} = \frac{1}{c}\int_A^B n\,dl$$

$$\underbrace{t_{AB}\, c}_{s} = \int_A^B n\,dl$$

$$\boxed{s = \int_A^B n\,dl}$$ OPTIČKI PUT

$n =$ konst

$$s = n\int_A^B dl$$

$$\boxed{s = n\,|AB|}$$ OPTIČKI PUT U VAKUUMU

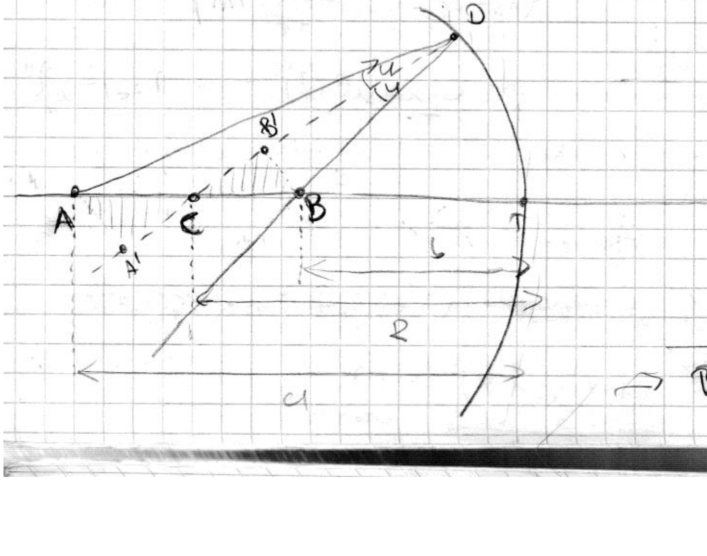
⑨ RAVNO I SFERNO ZRCALO

$\boxed{a=-b}$ ravno zrcalo

→	
$a>0$	$a<0$
$b>0$	$b<0$

P.U. Gaussove aproksimacije
- snopovi blizu osi
- mali kutovi refleksije

$\overline{AD}=\overline{AT}$
$\overline{BD}=\overline{BT}$



$AA'C \sim CB'B$; $AA'D \sim BB'D$

$$\frac{\overline{AC}}{\overline{AA'}} = \frac{\overline{BC}}{\overline{BB'}}$$

$$\frac{\overline{AD}}{\overline{AA'}} = \frac{\overline{BD}}{\overline{BB'}}$$

$$\rightarrow \overline{BB'}\,\overline{AC} = \overline{AA'}\,\overline{BC}$$

$$\frac{\overline{BB'}}{\overline{AA'}} = \frac{\overline{BC}}{\overline{AC}} \quad (1)$$

$$\rightarrow \overline{AD}\,\overline{BB'} = \overline{AA'}\,\overline{BD}$$

$$\frac{\overline{BB'}}{\overline{AA'}} = \frac{\overline{BD}}{\overline{AD}} \quad (2)$$

$$(1) = (2) \Rightarrow \frac{\overline{BC}}{\overline{AC}} = \frac{\overline{BD}}{\overline{AD}}$$

$$\overline{AD}\,\overline{BC} = \overline{AC}\,\overline{BD}$$

$$\frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}}$$

→ iz G. A.:

$$\frac{\overline{AC}}{\overline{AT}} = \frac{\overline{BC}}{\overline{BT}} \Rightarrow \frac{a-R}{a} = \frac{R-b}{b}$$

$$ab - bR = aR - ab$$

$$2ab = R(a+b)$$

$$\frac{2ab}{a+b} = R$$

$$\frac{ab}{a+b} = \frac{R}{2}$$

$$\frac{a+b}{ab} = \frac{2}{R}$$

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{2}{R} = \frac{1}{f}}$$ JEDNADŽBA SFERNOG ZRCALA

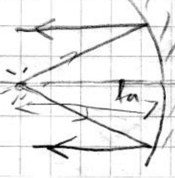
$$\underline{f = \frac{R}{2}}$$

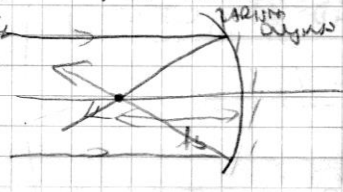
$R \Rightarrow \infty$: ravno zrcalo

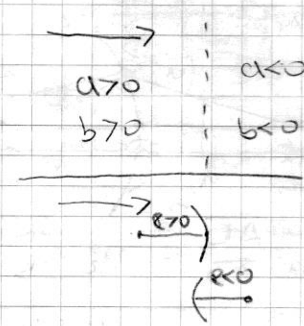
$a = f_a$, $b = \infty$ } $\underline{f_a = \frac{R}{2}}$ PREDMETNA ŽARIŠNA DULJINA

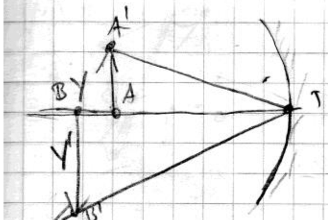
$a = \infty$, $b = f_b$ } $\underline{f_b = \frac{R}{2}}$ SLIKOVNA ŽARIŠNA DULJINA

$$\Rightarrow \underline{f_a = f_b = f = \frac{R}{2}}$$









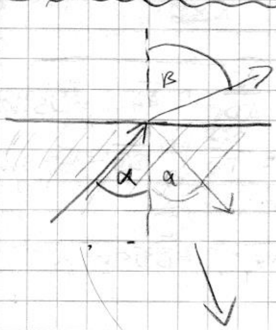
$$|m| = \frac{y'}{y} = \frac{\overline{BB'}}{\overline{AA'}} = \frac{\overline{BT}}{\overline{AT}} = \frac{b}{a}$$

$$\boxed{m = -\frac{b}{a}}$$ POVEĆANJE SLIKE

$m < 0$ (obrnuta slika)

$m > 0$ (uspravna slika)

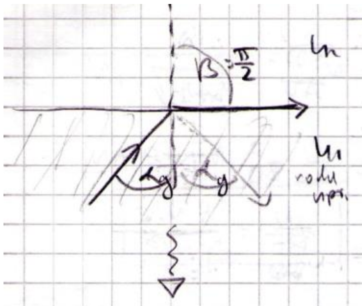
(10) TOTALNA REFLEKSIJA

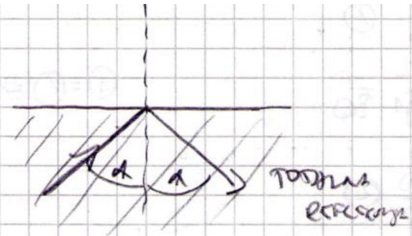


$\beta > \alpha$ za $n_1 > n_2$ (zraka iz gušćeg u rjeđe sredstvo)

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1} < 1 \text{ za } n_1 > n_2$$

$\Rightarrow \beta > \alpha$ (jedino tada moguća tot. refleksija)





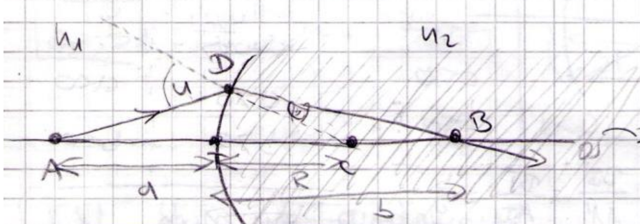
$$\frac{\sin\alpha_g}{\sin\beta} = \frac{n_2}{n_1}$$

$\beta = \frac{\pi}{2} \Rightarrow \sin\beta = 1$

$\Rightarrow \boxed{\sin\alpha_g = \frac{n_2}{n_1}}$ GRANIČNI KUT (uvjet $n_1 > n_2$)

(11) SFERNI DIOPTAR

{ P.U. Gaussove aproksimacije $\overline{BD}=\overline{BT}$; $\overline{AD}=\overline{AT}$; $\overline{CD}=\overline{CT}$ }



$$n_1 \frac{\overline{AC}}{\overline{AD}} = n_2 \frac{\overline{BC}}{\overline{BD}}$$

→ primjenjujemo Möbiusov zapis [illegible]

(P.U)

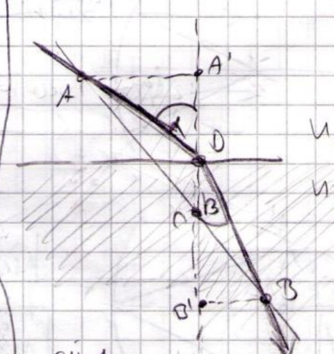
$$n_1 \frac{\overline{AC}}{\overline{AT}} = n_2 \frac{\overline{BC}}{\overline{BT}}$$

A → izvor svjetla

$$\underbrace{n_1 \frac{\overline{AC}}{\overline{AT}} = konst \Rightarrow n_2 \frac{\overline{BC}}{\overline{BT}} = konst}$$

Sve konjugirane točke sfernog dioptra sijeku optičku os u točki B

Möbiusov zapis:



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

$AA'C \sim BB'C$

$$\frac{\overline{AC}}{\overline{AA'}} = \frac{\overline{BC}}{\overline{BB'}} \quad (1)$$

$\sin\alpha = \frac{\overline{AA'}}{\overline{AD}} \rightarrow \overline{AA'} = \overline{AD}\sin\alpha$

$\sin\beta = \frac{\overline{BB'}}{\overline{BD}} \rightarrow \overline{BB'} = \overline{BD}\sin\beta$

$$\frac{\overline{AC}}{\overline{AD}\sin\alpha} = \frac{\overline{BC}}{\overline{BD}\sin\beta}$$

$$n_1 \frac{a+R}{a} = n_2 \frac{b-R}{b}$$

$$n_1 ab + n_1 bR = n_2 ab - n_2 aR$$

$$ab(n_1 - n_2) = R(-n_1 b - n_2 a)$$

$$\frac{ab}{n_1 b + n_2 a} = \frac{R}{n_2 - n_1} \quad \Big|^{-1}$$

$$\frac{n_2 a + n_1 b}{ab} = \frac{n_2 - n_1}{R}$$

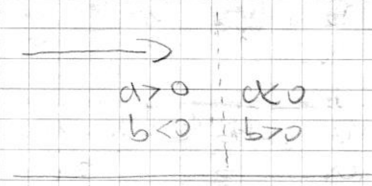
$$\boxed{\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}}$$ JEDNADŽBA SFERNOG DIOPTRA

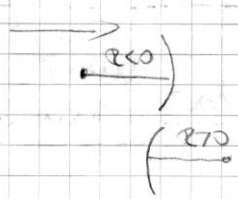
$$\frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}} \frac{\sin\alpha}{\sin\beta}$$

$$\frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}} \frac{n_2}{n_1}$$

$$\boxed{n_1 \frac{\overline{AC}}{\overline{AD}} = n_2 \frac{\overline{BC}}{\overline{BD}}}$$

Möbiusov zapis

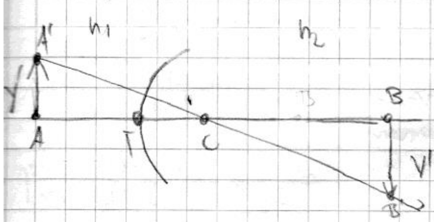




$R \to \infty$ RAVNI DIOPTAR $\quad \frac{n_2}{b} = -\frac{n_1}{a}$

$$\boxed{b = -\frac{n_2}{n_1} a}$$ TR $\quad -b n_1 = n_2 a$

$\left.\begin{matrix} a = f_a \\ b = \infty \end{matrix}\right\}$ $f_a = \frac{n_1}{n_2 - n_1} R$ $\qquad$ $\left.\begin{matrix} a = \infty \\ b = f_b \end{matrix}\right\}$ $f_b = \frac{n_2}{n_2 - n_1} R$ $\qquad$ $f_b - f_a = R$ $\quad$ $\boxed{\frac{f_a}{a} + \frac{f_b}{b} = 1}$



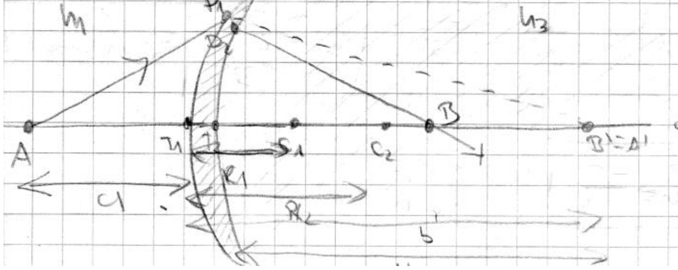
$$|m| = \frac{y'}{y} = \frac{\overline{BB'}}{\overline{AA'}} = \frac{\overline{BC}}{\overline{AC}} = \Big| \text{Möbiusov zapis} \Big|$$

$$= \Big| \frac{n_1}{\overline{AD}} \overline{AC} = \frac{n_2}{\overline{BD}} \overline{BC} \Rightarrow \frac{\overline{BC}}{\overline{AC}} = \frac{n_1}{n_2} \frac{\overline{BD}}{\overline{AD}} \Big|$$

$$|m| = \frac{n_1}{n_2} \frac{\overline{BT}}{\overline{AT}} = \frac{n_1}{n_2} \frac{b}{a} \Rightarrow \boxed{m = -\frac{n_1}{n_2} \frac{b}{a}}$$ POVEĆANJE SLIKE

(12) TANKA LEĆA

{P.U. Gaussove aproksimacije; tanka leća: $T_1 T_2 \to 0$}



Raščlamba:

1. sf. dioptar:

$$\frac{n_1}{a} + \frac{n_2}{b'} = \frac{n_2 - n_1}{R_1}$$

2. sferni dioptar:

$$\frac{n_2}{a'} + \frac{n_3}{b} = \frac{n_3 - n_2}{R_2}$$

P.U) Tanka leća:

$|a'| = |b'|$

$a' = -b'$

$\longrightarrow$

$a>0 \quad b<0 \; ; \; a<0 \quad b>0$ $\Big\}$ $a'<0 \quad b'>0$

$$\left.\begin{aligned} \frac{n_1}{a} + \frac{n_2}{b'} &= \frac{n_2 - n_1}{R_1} \\ -\frac{n_2}{a'} + \frac{n_3}{b} &= \frac{n_3 - n_2}{R_2} \end{aligned}\right\} +$$

$$\boxed{\frac{n_1}{a} + \frac{n_3}{b} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}}$$ JEDNADŽBA TANKE LEĆE

$$\left.\begin{aligned} a &= f_a \\ b &= \infty \end{aligned}\right\} f_a = \frac{n_1 R_1 R_2}{R_2(n_2 - n_1) + R_1(n_3 - n_2)} \quad (*)$$

$$\left.\begin{aligned} a &= \infty \\ b &= f_b \end{aligned}\right\} f_b = \frac{n_3 R_1 R_2}{R_2(n_2 - n_1) + R_1(n_3 - n_2)} \quad (**)$$

$$\frac{f_a}{a} + \frac{f_b}{b} = 1 \quad (***)$$

Slučaj: $n_1 = n_3 = n \; ; \; n_2 = n'$

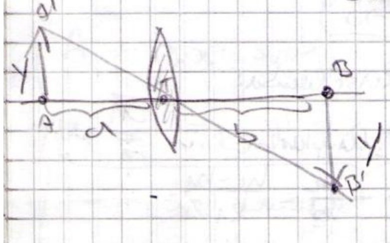
$$n\left(\frac{1}{a} + \frac{1}{b}\right) = (n' - n)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{n' - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$ JEDNADŽBA TANKE LEĆE U SREDSTVU INDEKSA LOMA n

$(*)$ i $(**)$ $\Longrightarrow$ $f_a = f_b = f$

$(***)$ $\Longrightarrow$ $\boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{f}}$ JEDNADŽBA TANKE LEĆE

$\boxed{j = \frac{1}{f} = \frac{n' - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$ JAKOST LEĆE, DIOPTRIJA



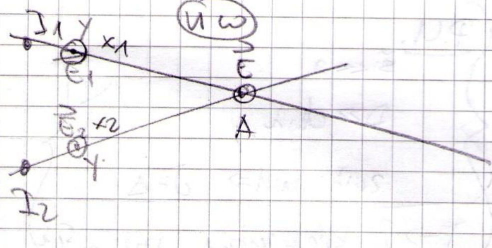
$$|m| = \frac{y'}{y} = \frac{\overline{BB'}}{\overline{AA'}} = \left| \triangle AA'S \sim \triangle BB'S \right| = \frac{b}{a}$$

$$\boxed{m = \frac{-b}{a}}$$ POVEĆANJE SLIKE

(13) INTERFERENCIJA 2 KOHERENTNA IZVORA

$\hookrightarrow$ izvori iste frekvencije ili stalne razlike u fazi (monokromatski izvori)

{ P.U.
amplitude jednake $E_{01} = E_{02} = E_0$
elek. i mag. polje polarizirano }



$$\left\{ v = \frac{c}{n},\quad \frac{1}{v} = \frac{n}{c} \right\}$$

$$\vec{E}_1 = E_{01} \sin \omega\left(t - \frac{1}{v} x_1\right) \vec{j}$$
$$\vec{E}_2 = E_{02} \sin \omega\left(t - \frac{1}{v} x_2\right) \vec{j}$$

$$\vec{E}_1 = E_{01} \sin \omega\left(t - \frac{n}{c} x_1\right) \vec{j}$$
$$\vec{E}_2 = E_{02} \sin \omega\left(t - \frac{n}{c} x_2\right) \vec{j}$$

$$\vec{E}_1 = E_0 \sin (\omega t \underbrace{- \omega \tfrac{n}{c} x_1}_{\varphi_1}) \vec{j}$$
$$\vec{E}_2 = E_0 \sin (\omega t \underbrace{- \omega \tfrac{n}{c} x_2}_{\varphi_2}) \vec{j}$$

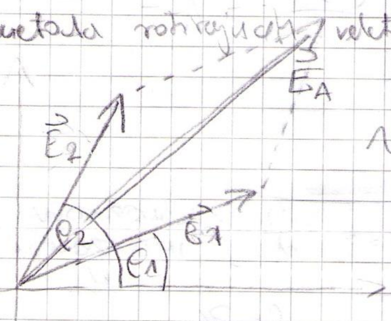
$\phi = \varphi_2 - \varphi_1$

$\phi = \omega \frac{n}{c}(x_1 - x_2)$

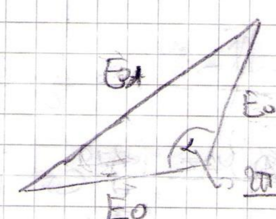
točka A:

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2$$

$\rightarrow$ metoda rotirajućih vektora



$\Rightarrow$



$$\frac{2\pi - 2\phi}{2} = \pi - \phi$$

$$E_{0A}^2 = E_0^2 + E_0^2 - 2E_0^2 \cos(\pi - \phi)$$

$$E_{0A} = \sqrt{2E_0^2 + 2E_0^2 \cos \phi} = E_0 \sqrt{2(1 + \cos \phi)} =$$

$$\left\{ \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} \Rightarrow 1 + \cos x = 2\cos^2 \frac{x}{2} \right\}$$

$$= E_0 \sqrt{2\left(2\cos^2 \frac{\phi}{2}\right)}$$

$$\boxed{E_{0A} = 2 E_0 \cos \frac{\phi}{2}}$$

(1) MAKSIMUMI:

$$\cos \frac{\phi}{2} = \pm 1$$

$$\frac{\phi}{2} = l\pi$$

$$\frac{1}{2} \frac{\omega}{c} n(x_1 - x_2) = l\pi$$

$$\frac{1}{2} \frac{2\pi\cancel{c}}{\lambda\cancel{c}} n(x_1 - x_2) = l\pi$$

$\boxed{\delta = n(x_1 - x_2)}$ RAZLIKA OPTIČKIH PUTOVA $\boxed{\delta = n\Delta}$

$\boxed{\delta = l\lambda}$ UVJET MAX.

$l \in \mathbb{Z}$

③ MINIMUM:

$\cos\frac{\phi}{2} = 0$

$\frac{\phi}{2} = \frac{2l+1}{2}\pi \quad l \in \mathbb{Z}$

$\phi = (2l+1)\pi$

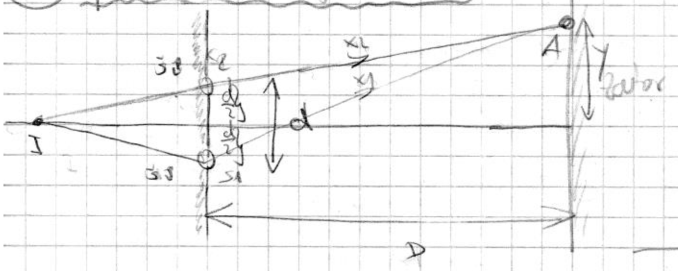
$\frac{\omega}{c} \underbrace{n(x_1 - x_2)}_{\underbrace{\Delta}_{\delta}} = (2l+1)\pi$

$\frac{2\pi}{\lambda}\delta = (2l+1)\pi$

$\boxed{\delta = \frac{2l+1}{2}\lambda}$ UVJET MIN.

$l \in \mathbb{Z}$

(14) YOUNGOVE POKUSI



P.U.
$s_1 \ll \lambda$
$D \gg d, \lambda, y$
zrak: $n=1 \Rightarrow \delta = \Delta$
$I \to 2$ koherentna izvora S_1, S_2

$\delta = \Delta = x_1 - x_2 = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2} = D\left(\sqrt{\left(\frac{y+\frac{d}{2}}{D}\right)^2 + 1} - \sqrt{\left(\frac{y-\frac{d}{2}}{D}\right)^2 + 1}\right)$

$=$ razvoj u red: $\sqrt{1 \pm x^2} = \boxed{1 + \frac{x^2}{2}} + \underbrace{\dots}$ zanemarimo (zbog uvjeta $D \gg d, y$) $= D\left(\cancel{1} + \frac{\left(\frac{y+\frac{d}{2}}{D}\right)^2}{2} - \cancel{1} - \frac{\left(\frac{y-\frac{d}{2}}{D}\right)^2}{2}\right)$

$\delta = \frac{D}{2}\left(\frac{y^2 + yd + \cancel{\frac{d^2}{4}} - y^2 + yd - \cancel{\frac{d^2}{4}}}{D^2}\right) = \frac{1}{2}\,\frac{2yd}{D}$

$\boxed{\delta = \frac{dy}{D}}$

→ ① MAX.

$\delta = l\lambda$
$\frac{dy}{D} = l\lambda$

$\boxed{y = l\frac{\lambda D}{d}}$ KONSTRUKTIVNA INTERFERENCIJA

$l \in \mathbb{Z}$

→ ② MIN.

$\delta = \frac{2l+1}{2}\lambda$
$\frac{dy}{D} = \frac{2l+1}{2}\lambda$

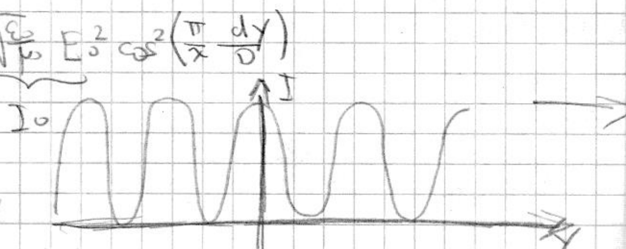
$\boxed{y = \frac{2l+1}{2}\,\frac{\lambda D}{d}}$ DESTRUKTIVNA INTERFERENCIJA

$I = \frac{1}{2}\varepsilon_0 c E_{0A}^2$
$E_{0A} = 2E_0\cos\frac{\phi}{2}$
$\phi = \frac{\omega}{c}\delta \rightsquigarrow \phi = k\delta \rightsquigarrow \phi = \frac{2\pi}{\lambda}\delta$
$\frac{\phi}{2} = \frac{\pi}{\lambda}\delta$

$\delta = \frac{dy}{D}$

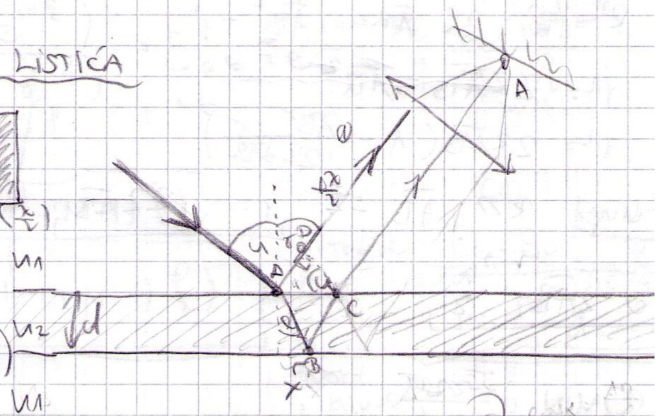
$\bar{I} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, 4E_0^2\cos^2\left(\frac{\pi}{\lambda}\delta\right) = \underbrace{2\sqrt{\frac{\varepsilon_0}{\mu_0}}\,E_0^2}\cos^2\left(\frac{\pi}{\lambda}\frac{dy}{D}\right)$

$\boxed{I = I_0\cos^2\left(\frac{\pi}{\lambda}\frac{d}{D}y\right)}$ INTENZITET PRUGE NA UDALJENOSTI y NA ZASTORU



TEORIJA

(15) INTERFERENCIJA TANKIH LISTIĆA



$$\delta = n(\overline{AB} + \overline{BC}) - 1\cdot(\overline{AD} + \tfrac{\lambda}{2})$$

$$\delta = n(\overline{AB} + \overline{BC}) - \overline{AD} - \tfrac{\lambda}{2}$$

$$\delta = 2n\overline{AB} - \overline{AD} - \tfrac{\lambda}{2}$$

P.U. $n_1 = 1$, $n = n_2 > n_1$

dakle $\dfrac{\sin u}{\sin l} = \dfrac{n_2}{n_1} = n_2 = n$

$$\cos l = \frac{d}{\overline{AB}}$$

$$\boxed{\overline{AB} = \frac{d}{\cos l}}$$

$$\sin u = \frac{\overline{AD}}{\overline{AC}}$$

$$\overline{AD} = \overline{AC}\sin u$$

$$\overline{AC} = 2x = 2\overline{AB}\sin l = 2d\frac{\sin l}{\cos l} = 2d\,\mathrm{tg}\,l$$

$$\overline{AD} = 2d\,\mathrm{tg}\,l \sin u \overset{\text{snell zakon}}{=} 2d\,\mathrm{tg}\,l\, n \sin l$$

$$\boxed{\overline{AD} = 2nd\frac{\sin^2 l}{\cos l}}$$

$$\delta = 2nd\frac{1}{\cos l} - 2nd\frac{\sin^2 l}{\cos l} - \frac{\lambda}{2} = 2nd\frac{1}{\cos l}(1 - \sin^2 l) - \frac{\lambda}{2}$$

$$\delta = 2nd\frac{1}{\cos l}\cos^2 l - \frac{\lambda}{2}$$

$$\boxed{\delta = 2nd\cos\phi_2 - \frac{\lambda}{2}}$$

→ (1) MAX $\delta = l\lambda$

$$l\lambda = 2nd\cos\phi_2 - \frac{\lambda}{2}$$

$$\boxed{2nd\cos\phi_2 = (l + \tfrac{1}{2})\lambda}$$ KONSTRUKTIVNA INTERFERENCIJA

$l \in \mathbb{Z}$

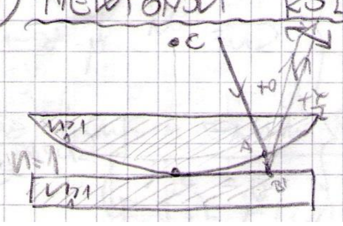
(2) MIN $\delta = \frac{2l+1}{2}\lambda$

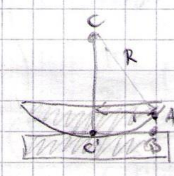
$$\frac{2l+1}{2}\lambda = 2nd\cos\phi_2 - \frac{\lambda}{2}$$

$$2nd\cos\phi_2 = \left(\frac{2l+1}{2} + \frac{1}{2}\right)\lambda$$

$$\boxed{2nd\cos\phi_2 = (l+1)\lambda}$$ DESTRUKTIVNA INTERFERENCIJA

(16) NEWTONOVI KOLOBARI





P.U. $R \gg r, \overline{AB} \Rightarrow \overline{AB'} = \overline{AB}$

$n = 1$ (zrak)

$$\delta = 2\overline{AB} + \frac{\lambda}{2}$$

$$R^2 = (R - \overline{AB})^2 + r^2$$
$$R^2 = R^2 - 2R\,\overline{AB} + \overline{AB}^2 + r^2$$
$$r^2 = 2R\overline{AB} - \overline{AB}^2$$
$$r^2 = 2R\overline{AB}\left(1 - \frac{\overline{AB}^2}{2R\overline{AB}}\right)$$

uvjet $R >> \overline{AB} \Rightarrow \boxed{r = \sqrt{R2\overline{AB}}}$

$$\boxed{\delta = 2\overline{AB} + \frac{\lambda}{2}}$$

① MAX $\delta = l\lambda$

$$l\lambda = 2\overline{AB} + \frac{\lambda}{2}$$
$$\left(l - \frac{1}{2}\right)\lambda = 2\overline{AB}$$
$$(2l-1)\frac{\lambda}{2} = 2\overline{AB}$$
$$\boxed{r_{S.K} = \sqrt{R(2l-1)\frac{\lambda}{2}}}$$ SVIJETLI KOLOBARI

općenito

$$\boxed{r_{S.K} = \sqrt{\frac{R(2l-1)\frac{\lambda}{2}}{n}}}$$

② MIN $\delta = \frac{2l+1}{2}\lambda$

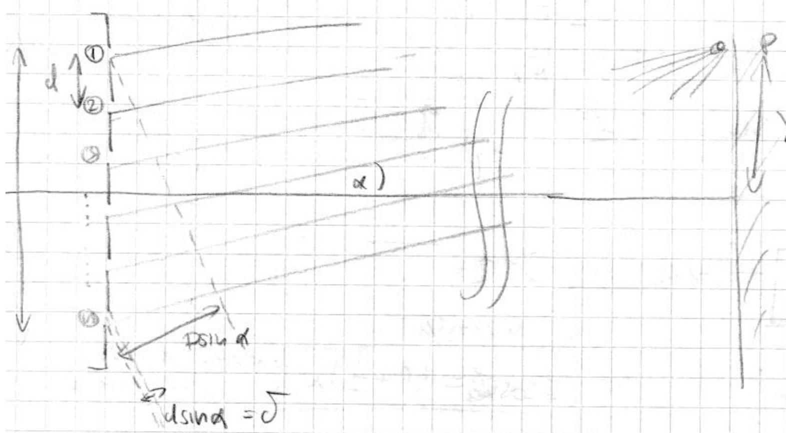
$$\frac{2l+1}{2}\lambda = 2\overline{AB} + \frac{\lambda}{2}$$
$$l\lambda = 2\overline{AB}$$
$$\Rightarrow \boxed{r_{T.K} = \sqrt{Rl\lambda}}$$ TAMNI KOLOBARI

$l \in \mathbb{Z}$

općenito

$$\boxed{r_{T.K} = \sqrt{\frac{Rl\lambda}{n}}}$$

17) INTERFERENCIJA NA N KOHERENTNIH IZVORA



$$\left\{ \begin{aligned} \varphi &= \frac{2\pi}{\lambda}\delta \text{ ; hod} \\ \frac{\varphi}{2} &= \frac{\pi}{\lambda}\delta \\ \delta &= n\Delta \quad \Delta = d\sin\alpha \end{aligned} \right.$$

λ, E_0

$$E_1 = E_0 \cos\omega t = E_0 e^{i\omega t}$$
$$E_2 = E_0 \cos(\omega t + \varphi) = E_0 e^{i(\omega t + \varphi)}$$
$$E_3 = \quad = E_0 e^{i(\omega t + 2\varphi)}$$
$$\dots$$
$$E_N = \quad = E_0 e^{i(\omega t + (N-1)\varphi)}$$

$$E_P = E_1 + E_2 + \dots + E_N$$

g. red

$$E_P = E_0 e^{i\omega t}\left(1 + e^{i\varphi} + e^{i2\varphi} + \dots e^{i(N-1)\varphi}\right)$$

$$S_N = a_1 \frac{q^n - 1}{q - 1}$$

$$\left.\begin{aligned} a_1 &= 1 \\ q &= e^{i\varphi} \end{aligned}\right\} S_N = \frac{e^{iN\varphi} - 1}{e^{i\varphi} - 1}$$

$$E_p = E_0 e^{i\omega t} \cdot \frac{e^{in\varphi}-1}{e^{i\varphi}-1}$$

$$= E_0 e^{i\omega t} \frac{e^{in\frac{\varphi}{2}}\left(e^{in\frac{\varphi}{2}} - e^{-in\frac{\varphi}{2}}\right)}{e^{i\frac{\varphi}{2}}\left(e^{i\frac{\varphi}{2}} - e^{-i\frac{\varphi}{2}}\right)} = E_0 e^{i\omega t} e^{i(N-1)\frac{\varphi}{2}} \frac{\underbrace{\mathrm{sh}\, in\frac{\varphi}{2}}_{i \sin n\frac{\varphi}{2}}}{\underbrace{\mathrm{sh}\, i\frac{\varphi}{2}}_{i\sin\frac{\varphi}{2}}}$$

$$= E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}} \underbrace{e^{i(\omega t + (N-1)\frac{\varphi}{2})}}_{\cos(\omega t + (N-1)\frac{\varphi}{2})}$$

$$\boxed{E_p(\alpha) = E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}} \cos\left(\omega t + (N-1)\frac{\varphi}{2}\right)}$$

ELEKTRIČNO POLJE OVISNO O BROJU IZVORA I KUTU α

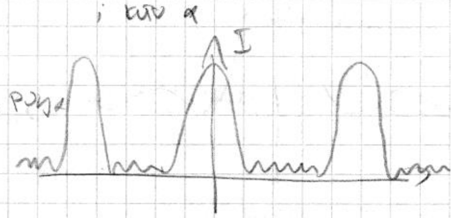
(A) $E_{op} = E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}$

$\frac{\varphi}{2} = \frac{\pi}{\lambda} d \sin\alpha$

$$I = \frac{1}{2}\varepsilon_0 c E_{op}^2$$

$$\boxed{I(\alpha) = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 \frac{\sin^2 N\frac{\varphi}{2}}{\sin^2\frac{\varphi}{2}}}$$

INTENZITET ELEK. POLJA U KUTU α



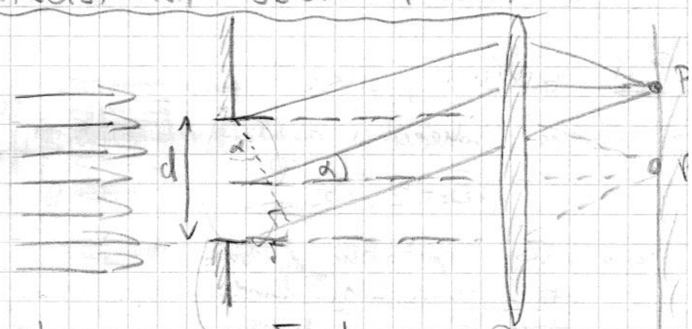
(18) FRAUNHOFEROVA DIFRAKCIJA NA JEDNOJ PUKOTINI

{P.U. $d \ll \lambda$; točk}

$\varphi = \frac{2\pi}{\lambda}\delta$

$\delta = n s = s = d\sin\alpha$

$E_{op} = E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}$



Izvor dijelimo na m dijelova

d — razmak $\frac{d}{m} = d'$

t.d. $m = N$

Za zrake → $\delta = d\sin\alpha$ koje se sijeku u jednoj (R) pokoj? vrijedi

$E(0) = m E_0 \Rightarrow E_0 = \frac{E(0)}{m}$

$$E_{op} = E_0 \frac{\sin(N\frac{\pi}{\lambda}d'\sin\alpha)}{\sin(\frac{\pi}{\lambda}d'\sin\alpha)} = E_0 \frac{\sin(N\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha)}{\sin(\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha)}$$

$$E_{op}(\alpha) = \frac{E(0)}{m} \frac{\sin(\frac{\pi}{\lambda} d \sin\alpha)}{\sin(\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha)} \qquad \{N = m\}$$

$m \to \infty$

$$\lim_{m\to\infty}\left(m \sin\left(\frac{\pi}{\lambda}\frac{d}{m}\sin\alpha\right)\right) = \frac{\pi}{\lambda} d\sin\alpha$$

$$\boxed{E_{op}(\alpha) = E(0) \frac{\sin(\frac{\pi}{\lambda}d\sin\alpha)}{\frac{\pi}{\lambda}d\sin\alpha}}$$

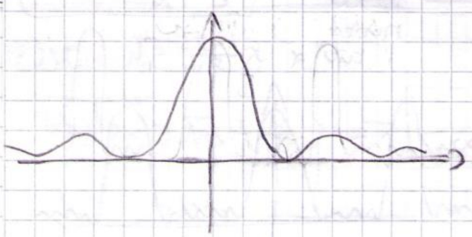
ELEKTRIČNO POLJE OVISNO O KUTU (A)

$$\underline{I = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_{op}^2}$$

$$I(\alpha) = \underbrace{\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E(0)^2}_{I_0}\frac{\sin^2\left(\frac{\pi d}{\lambda}\sin\alpha\right)}{\left(\frac{\pi}{\lambda}d\sin\alpha\right)^2}$$

$$\boxed{I(\alpha) = I_0\frac{\sin^2\left(\frac{\pi d}{\lambda}\sin\alpha\right)}{\left(\frac{\pi}{\lambda}d\sin\alpha\right)^2}}$$

intenzitet E-polja o kutu



→ centralni maksimum I_0

→ minimumi: $\underline{\sin^2\left(\frac{\pi d}{\lambda}\sin\alpha\right) = 0}$

→ maksimumi: $y = \frac{\pi}{\lambda}d\sin\alpha$

$$\frac{dI}{dy} = 0$$

(19) Fraunhoferova difrakcija na 2 pukotine

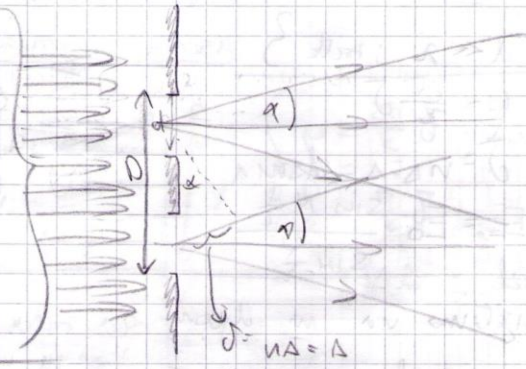
P.U. zrak ($n=1$); $d \ll \lambda$

- pukotine: 2 koherentna izvora:

$$\underline{E_{0A} = 2E_0\cos\frac{\varphi}{2}}$$

- na svakoj pukotini je ogib:

$$\underline{E_{op} = E(0)\frac{\sin\left(\frac{\pi}{\lambda}d\sin\alpha\right)}{\frac{\pi}{\lambda}d\sin\alpha}}$$



$\delta = n\Delta = \Delta$

$\delta = D\sin\alpha$

interferencija 2 koh. izvora:

$$\frac{\varphi}{2} = \frac{\pi}{\lambda}\delta = \frac{\pi}{\lambda}D\sin\alpha$$

$$E_{0A} = 2E_0\cos\frac{\varphi}{2} = 2E(0)\frac{\sin\left(\frac{\pi}{\lambda}d\sin\alpha\right)}{\frac{\pi}{\lambda}d\sin\alpha}\cos\frac{\varphi}{2}$$

$$\sin 2\frac{\varphi}{2} = 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}$$

$$\cos\frac{\varphi}{2} = \frac{\sin\varphi}{2\sin\frac{\varphi}{2}}$$

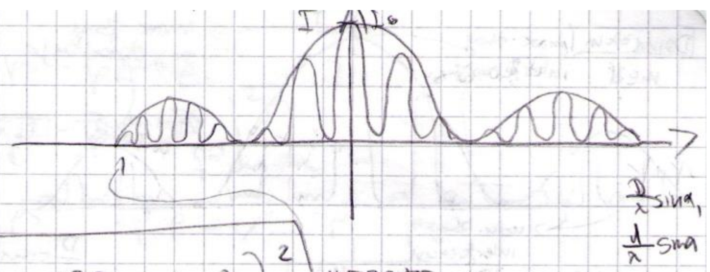
$$E_{0A} = 2E(0)\frac{\sin\left(\frac{\pi}{\lambda}d\sin\alpha\right)}{\frac{\pi}{\lambda}d\sin\alpha}\frac{\sin\left(\frac{2\pi}{\lambda}D\sin\alpha\right)}{\cancel{2}\sin\left(\frac{\pi}{\lambda}D\sin\alpha\right)}$$

$$\boxed{E_{0A}(\alpha) = \underbrace{E(0)\frac{\sin\left(\frac{\pi}{\lambda}d\sin\alpha\right)}{\frac{\pi}{\lambda}d\sin\alpha}}_{\text{difrakcija na jednoj pukotini}}\underbrace{\frac{\sin\left(\frac{2\pi}{\lambda}D\sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda}D\sin\alpha\right)}}_{\text{interferencija na 2 pukotine}}}$$

amplituda električnog polja u točki P koja je pod kutom od α

$$I = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,E_0^2$$

$$I = \underbrace{\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,E(0)^2}_{I_0} \;-\!\!/\!/\!\!-^{2} \;-\!\!/\!/\!\!-^{2}$$



$$\boxed{I(\alpha) = I_0\left[\frac{\sin\left(\frac{\pi}{\lambda} d\sin\alpha\right)}{\frac{\pi}{\lambda} d\sin\alpha}\right]^2 \left[\frac{\sin\left(\frac{2\pi}{\lambda} D\sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda} D\sin\alpha\right)}\right]^2}$$

INTENZITET U TOČKI P POD KUTOM α

I_0 - intenzitet centralnog maksimuma

20) FRAUNHOFEROVA DIFRAKCIJA NA N PUKOTINA

P.U. $(n=1)$, nakj $d \ll \lambda$

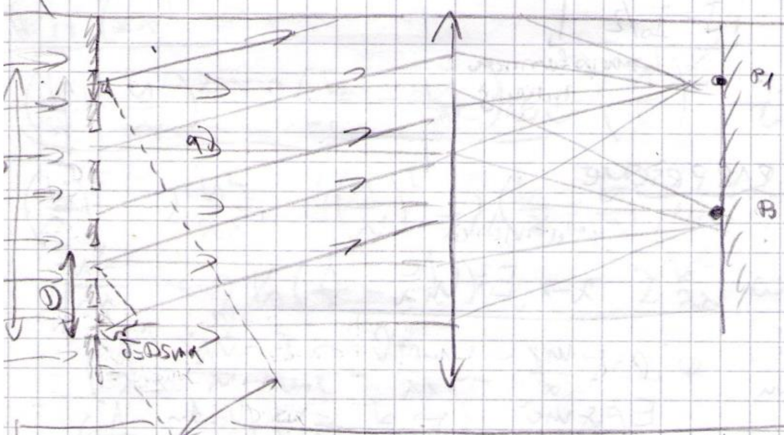
- pukotine: interferencija na n koherentnih izvora

$$E_{0A} = E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}$$

- na svakoj pukotini je ogib:

$$E_{0P} = E(0)\frac{\sin\left(\frac{\pi}{\lambda} d\sin\alpha\right)}{\frac{\pi}{\lambda} d\sin\alpha}$$

D = konstanta rešetke



$\delta = D\sin\alpha$

$$\frac{\varphi}{2} = \frac{\pi}{\lambda} D\sin\alpha$$

$$E_{0A} = E_0 \frac{\sin N\frac{\varphi}{2}}{\sin\frac{\varphi}{2}}$$

E_{0P}

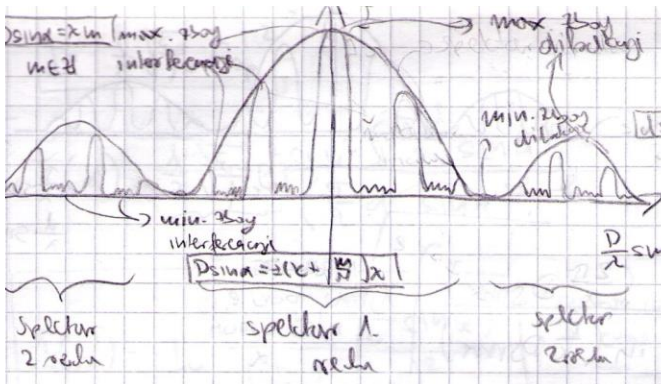
$$\boxed{E_{0A}(\alpha) = E(0)\frac{\sin\left(\frac{\pi}{\lambda} d\sin\alpha\right)}{\frac{\pi}{\lambda} d\sin\alpha}\;\frac{\sin\left(\frac{N\pi}{\lambda} D\sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda} D\sin\alpha\right)}}$$

AMPLITUDA ELEK. POLJA U TOČKI P POD KUTEM OD α

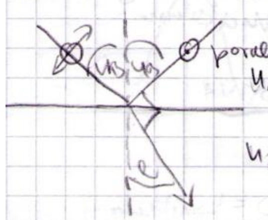
$$I = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,E_0^2$$

$$I = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,E(0)^2 \;-\!\!/\!/\!\!-^{2} \;-\!\!/\!/\!\!-^{2} \Rightarrow \boxed{I(\alpha) = I_0\left[\frac{\sin\left(\frac{\pi}{\lambda} d\sin\alpha\right)}{\frac{\pi}{\lambda} d\sin\alpha}\right]^2\left[\frac{\sin\left(\frac{N\pi}{\lambda} D\sin\alpha\right)}{\sin\left(\frac{\pi}{\lambda} D\sin\alpha\right)}\right]^2}$$

INTENZITET U TOČKI P POD KUTOM α



21) POLARIZACIJA



$u_B + e = \frac{\pi}{2}$

$$\frac{\sin u_B}{\sin e} = \frac{n_2}{n_1}$$

$$\frac{\sin(u_B)}{\underbrace{\sin(\frac{\pi}{2} - e)}_{\cos e}} = \frac{n_2}{n_1}$$

$$\boxed{\operatorname{tg} u_B = \frac{n_2}{n_1}}$$ Brewsterov kut

- dikroični materijali:

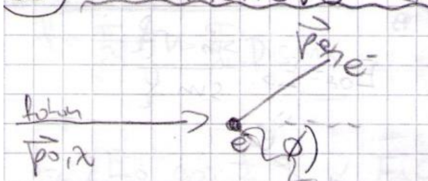
$$\boxed{I = I_0 \cos^2 \alpha}$$ Malusov zakon

→ polarizirane trake

$$\boxed{I = I_0 / 2}$$

→ nepolarizirana trake

22) COMPTONOVO RASPRŠENJE



relativistički

$\lambda \to E = h\frac{c}{\lambda}$

$p = \gamma m v \Rightarrow \gamma m = \frac{p}{v} \Rightarrow E = \frac{p}{v} c^2$

$E = \gamma m c^2$

$\vec{p} = \frac{E}{c^2} \vec{v}$ foton

$p = \frac{E}{c^2} c$

$p = \frac{E}{c} = \frac{h \frac{c}{\lambda}}{c}$

$$\boxed{p = \frac{h}{\lambda}}$$ De Broglieva relacija

$$\boxed{\Delta\lambda = \lambda' - \lambda}$$

ZOKG:

$$\vec{p_0} = \vec{p_e} + \vec{p}$$

$$\vec{p_0} - \vec{p} = \vec{p_e} \quad \Big|^2$$

$$p_0^2 - 2p_0 p\cos\phi + p^2 = p_e^2$$

$$\left(\frac{h}{\lambda}\right)^2 - 2\frac{h}{\lambda}\frac{h}{\lambda'}\cos\phi + \left(\frac{h}{\lambda'}\right)^2 = p_e^2 \quad (*)$$

ZOE

fotona elektrona

$$h\nu_0 + mc^2 = h\nu' + E_e$$

relativistika:

$$E_e^2 = p_e^2c^2 + m^2c^4$$

$$h\frac{c}{\lambda} + mc^2 = h\frac{c}{\lambda'} + E_e \quad \Big| :c$$

$$\frac{h}{\lambda} + mc - \frac{h}{\lambda'} = \frac{E_e}{c}$$

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right) + mc = \frac{E_e}{c} \quad \Big|^2$$

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 + 2\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)mc + \cancel{(mc)^2} = p_e^2 + \cancel{m^2c^2}$$

$$p_e^2 = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 + 2\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)mc \quad (**)$$

(*) i (**) $\rightarrow$

$$\underbrace{\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 + 2\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)mc}_{} = \cancel{\left(\frac{h}{\lambda}\right)^2} - 2\frac{h^2}{\lambda\lambda'}\cos\phi + \cancel{\left(\frac{h}{\lambda'}\right)^2}$$

$$\cancel{\left(\frac{h}{\lambda}\right)^2} - 2\frac{h^2}{\lambda\lambda'} + \cancel{\left(\frac{h}{\lambda'}\right)^2} + 2\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)mc$$

$$-\cancel{2}\frac{h^2}{\lambda\lambda'} + \cancel{2}\cancel{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)mc = -\cancel{2}\frac{h^2}{\lambda\lambda'}\cos\phi$$

$$\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)mc = \frac{h}{\lambda\lambda'} - \frac{h}{\lambda\lambda'}\cos\phi$$

$$\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)mc = \frac{h}{\lambda\lambda'}(1 - \cos\phi)$$

$$\frac{\overbrace{\lambda' - \lambda}^{\Delta\lambda}}{\lambda\lambda'}mc = \frac{h}{\lambda\lambda'}(1 - \cos\phi)$$

$$\boxed{\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)} = 2\frac{h}{mc}\left(\frac{1-\cos\phi}{2}\right) = 2\frac{h}{mc}(\sin^2\phi$$

$$\boxed{\Delta\lambda = \frac{2h}{mc}\sin^2\phi}$$

RAZLIKA VALNIH DULJINA COMPTONOVA RASPRŠENJA

(23) RADIOAKTIVNI RASPAD

$$\begin{cases} A = -\frac{dN}{dt} \; [Bq] \\ A = \lambda N \end{cases}$$

(aktivnost je linearna fja broja jezgara)

$$\lambda N = \frac{-dN}{dt}$$

$$\left\{ \begin{matrix} P.U & t=0 \\ & N=N_0 \end{matrix} \right\}$$

$$-\lambda dt = \frac{1}{N} dN \quad \Big| \int$$

$$-\lambda t = \ln N + C$$

$$\ln N = -\lambda t + C$$

$$N = e^{-\lambda t + C}$$

$$N = e^{-\lambda t} e^{C}$$

$$t=0 \Rightarrow \quad e^{C} = N_0$$

$$\boxed{N = N_0 e^{-\lambda t}}$$ ZAKON RADIOAKTIVNOG RASPADA

$$A = -\frac{dN}{dt} = -N_0 e^{-\lambda t} \cdot (-\lambda) = \lambda N_0 e^{-\lambda t}$$

$$A_0 = A(t=0) = \lambda N_0$$

$$\boxed{A = A_0 e^{-\lambda t}}$$ AKTIVNOST

$$N = \frac{N_0}{2}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t} \quad | \ln$$

$$\ln 2^{-1} = -\lambda t \Rightarrow T_{1/2} = \frac{-\ln 2}{-\lambda}$$

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda}}$$ VRIJEME POLURASPADA

$$\boxed{\tau = \frac{\int_{N_0}^{0} t \, dN}{\int_{N_0}^{0} dN} = \frac{1}{\lambda}}$$

SREDNJE VRIJEME ŽIVOTA JEZGRE

(24) ZAKON ZRAČENJA CRNOG TIJELA

Rayleigh - Jeansov zakon

$$f(\lambda,T) = \frac{2\pi c}{\lambda^4}\,\bar{E}$$

Max Planck:

$$E = h\nu$$
$$E_i = i h\nu$$

$$\bar{E} = \frac{\sum_{i=0}^{\infty} N_i E_i}{\sum_{i=0}^{\infty} N_i} = \left|\begin{array}{c}\text{Boltzmannova raspodjela} \\ \text{čestica po energijama} \\ N = N_0 e^{-\frac{E}{kT}}\end{array}\right| = \frac{\sum_{i=0}^{\infty} N_0 e^{-\frac{E_i}{kT}} E_i}{\sum_{i=0}^{\infty} N_0 e^{-\frac{E_i}{kT}}}$$

$$\left\{ e^{-\frac{\varepsilon}{kT}} = x \right\}$$

$$= \frac{\sum_{i=0}^{\infty} x^i \, i h\nu}{\sum_{i=0}^{\infty} x^i} = h\nu \frac{\sum_{i=0}^{\infty} i x^i}{\sum_{i=0}^{\infty} x^i} = h\nu \frac{x + 2x^2 + 3x^3 + \ldots}{\underbrace{1 + x + x^2 + x^3 + \ldots}}$$

$$\left\{ \text{g. red } S = \frac{a_1}{1-q} = \frac{1}{1-x} \right\}$$

$$\bar{E} = h\nu x \frac{1 + 2x + 3x^2 + \ldots}{1 + x + x^2 + x^3 + \ldots}$$

$$= h\nu x \frac{\frac{d}{dx}\left(\frac{1}{1-x}\right)}{\frac{1}{1-x}} = h\nu x \frac{\frac{d}{dx}\left(\frac{1}{x-1}\right)}{\frac{1}{x-1}} = h\nu x \frac{-(x-1)^{-2}}{\frac{1}{x-1}} = h\nu x \frac{x-1}{-(x-1)^2}$$

$$\bar{E} = \frac{-h\nu x}{x-1} = \frac{-h\nu e^{-\frac{h\nu}{kT}}}{e^{-\frac{h\nu}{kT}} - 1} = \frac{e^{-\frac{h\nu}{kT}}(-h\nu)}{e^{-\frac{h\nu}{kT}}\left(1 - \frac{1}{e^{-\frac{h\nu}{kT}}}\right)} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$f(\lambda,T) = \frac{2\pi c}{\lambda^4} \frac{h\frac{c}{\lambda}}{e^{\frac{h\nu}{kT}} - 1}$$

$$\boxed{f(\lambda,T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}}$$

PLANCKOVA GUSTOĆA ZRAČENJA CRNOG TIJELA

f-ja od λ

Kirchhoffov zakon

$$\boxed{\frac{e}{a} = f(\lambda,T)}$$

$$\boxed{f(\nu,T) = \frac{2\pi h \nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}}$$

f-ja od ν