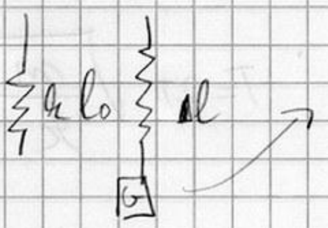
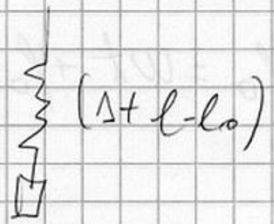
JEDNADŽBA GIBANJA H. O.



$mg = k(l - l_0)$



$F = mg - k(\Delta + l - l_0) \Rightarrow F = -k\Delta = m \cdot a$

$m \cdot a = -k\Delta \qquad m \cdot \frac{d^2\Delta}{dt^2} + k\Delta = 0 \quad /:m$

$\frac{d^2\Delta}{dt^2} + \frac{k}{m}\Delta = 0 \qquad \omega_0^2 = \frac{k}{m} \qquad \frac{d^2\Delta}{dt^2} + \omega^2\Delta = 0$

$$\boxed{x(t) = A e^{\alpha t}}$$

$\cancel{A}\alpha^2 \cancel{e^{\alpha t}} + \frac{k}{m}\cancel{A}\cancel{e^{\alpha t}} = 0$

$\alpha^2 = -\frac{k}{m}$

$\alpha_{1,2} = \pm i\sqrt{\frac{k}{m}} \Rightarrow x(t) = A_1 e^{-i\omega t} + A_2 e^{i\omega t}$

$t = 0 \quad x(0) = A \quad v(0) = 0$

$x(0) = A = A_1 + A_2$

$v(0) = -\cancel{i\omega} A_1 \cancel{e^{i\omega t}} + \cancel{i\omega} A_2 \cancel{e^{i\omega t}} = 0$

$A_1 = A_2$

$A_1 = A_2 = \frac{A}{2}$

$x(t) = \frac{A}{2}\left(e^{i\omega t} + e^{-i\omega t}\right) =$

$= \frac{A}{2}\left(\cos\omega t \pm \cancel{i\sin\omega t} + \cos\omega t \mp \cancel{i\sin\omega t}\right) = A\cos(\omega t)$

2. $x(0) = 0 \quad v(0) = v_0 \quad x(0) \Rightarrow A_1 = -A_2$

$v(0) = v_0 = -A_1 i\omega + A_2 i\omega \Rightarrow A_2 = \frac{v_0}{2i\omega}$

$x(t) = \frac{v_0}{\omega}\sin\omega t$

$x(t) = A\cos(\omega t + \varphi_0)$

$$\omega(t+T) + \varphi_0 = \omega t + \varphi_0 + 2\pi \Rightarrow \omega T = 2\pi \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = \frac{2\pi}{\omega}$$

ENERGIJA H.O. $E_k = \frac{1}{2} m v^2$ $\quad x(t) = A\sin(\omega t + \varphi_0)$

$$v(t) = A\omega\cos(\omega t + \varphi_0)$$

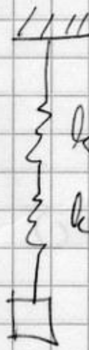
$$E_k = \frac{1}{2} k A^2 \omega^2 \cos^2(\omega t + \varphi_0) = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \varphi_0)$$

$$E_p = -\int_0^x (-k)x\,dx = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \sin^2(\omega t + \varphi_0)$$

$$E_M = E_k + E_p = \frac{1}{2} k A^2$$

SPOJ OPRUGA

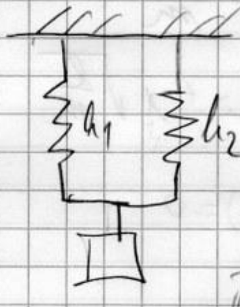
SERIJSKI SPOJ



$$\begin{cases} mg = k_1 x_1 \\ mg = k_2 x_2 \\ mg = k(x_1 + x_2) \end{cases}$$

$$\cancel{mg} = k\left[\frac{\cancel{mg}}{k_1} + \frac{\cancel{mg}}{k_2}\right]$$

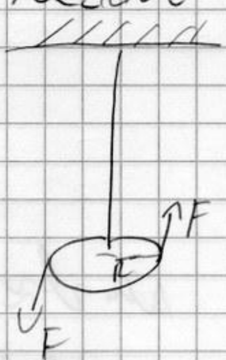
$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$



$$\begin{cases} mg = x(k_1 + k_2) \\ mg = kx \end{cases}$$

$$\boxed{k = k_1 + k_2}$$

TORZIONO NJIHALO



$$\vec{M} = -D \cdot \vec{\vartheta} \qquad \vec{M} = I \cdot \vec{\alpha} \qquad \alpha = \frac{d^2\vartheta}{dt^2}$$

$$I\vec{\alpha} = -D\vartheta$$

$$I \cdot \frac{d^2\vartheta}{dt^2} = -D\vartheta \qquad \frac{d^2\vartheta}{dt^2} + \frac{D}{I}\vartheta = 0$$

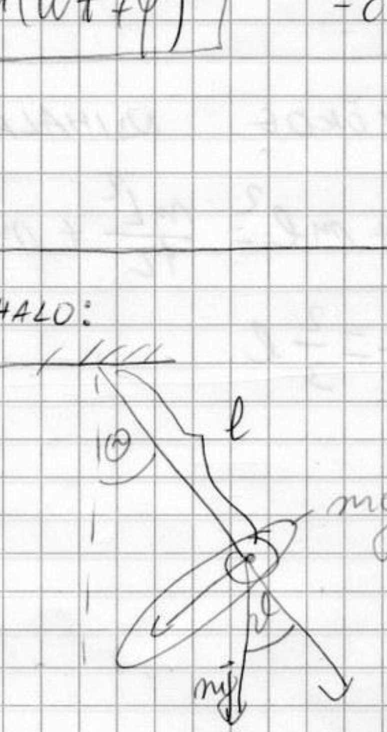
$$\omega^2 = \frac{D}{I} \Rightarrow \omega = \sqrt{\frac{D}{I}}$$

$$\ddot{\vartheta} + \omega^2\vartheta = 0 \qquad T = 2\pi\sqrt{\frac{I}{D}}$$

$$\boxed{\vartheta(t) = \vartheta_0 \sin(\omega t + \varphi)}$$

- ČISTO HARMONIČKO TITRANJE
- PERIOD NE OVISI O AMPLITUDI NI ZAKRETNOM KUTU

MATEMATIČKO NJIHALO:



$$m\vec{g} = \vec{F}_T + \vec{F}_N$$

$$\vec{M} = I \cdot \vec{\alpha} = r \times \vec{F}$$

$$r = l \qquad \vec{F} = \vec{F}_T$$

$$M = -lmg \sin\vartheta$$

$$\cancel{m} \cdot l^{\cancel{2}} \frac{d^2\vartheta}{dt^2} = -\cancel{m}g\cancel{l} \sin\vartheta$$

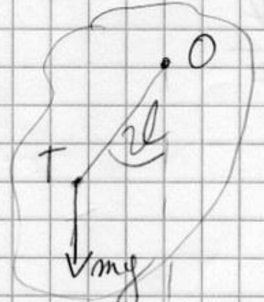
$$\frac{d^2\vartheta}{dt^2} + \frac{g}{l}\sin\vartheta = 0 \Rightarrow \sin\vartheta \approx \vartheta$$ ZA MALE KUTEVE

$$\frac{d^2\vartheta}{dt^2} + \frac{g}{l}\vartheta = 0 \qquad \boxed{\omega^2 = \frac{g}{l}} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

$$\vartheta(t) = \vartheta_0 \sin(\omega t + \varphi_0)$$

- HARMONIČKI TITRA ZA MALE KUTEVE

FIZIČKO NJIHALO



$OT = b \Rightarrow$ KRAK SILE

$$M = I \cdot \alpha = \vec{b} \times \vec{G}$$

$$I \frac{d^2\vartheta}{dt^2} + mg \cdot b \cdot \sin\vartheta = 0$$

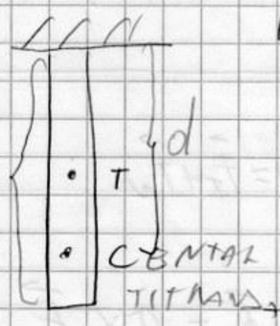
$\sin\vartheta \approx \vartheta$ ZA MALE KUTEVE

$$\frac{d^2\vartheta}{dt^2} + \frac{mgb}{I}\vartheta = 0 \qquad \omega = \sqrt{\frac{mgb}{I}}$$

$$\vartheta = \vartheta_0 \sin(\omega t + \varphi_0)$$

- NIJE H.O.

REDUCIRANA DULJINA FIZIČKOG NJIHALA



$$I_0 = \frac{mL^2}{12} \qquad I = I_0 + ml^2 = \frac{mL^2}{12} + m\frac{l^2}{4} = m\frac{L^2}{3}$$

$$l_r = \frac{\frac{1}{3}mL^2}{m \cdot \frac{l}{2}} = \frac{2}{3}l$$

$$T_F = T_M$$

$$\frac{l_r}{\cancel{g}} = \frac{I}{m\cancel{g}L} \qquad l_r = \frac{I}{mL}$$

PRISILNO TITRANJE

$\vec{F}_{el}$, $\vec{F}_{tr} = -b\vec{v}$ $\quad |\vec{F}_v| = F_0 \sin(\omega t)$ $\quad A_0 = \frac{F_0}{m}$

$$\frac{d^2 s}{dt^2} + 2\delta \frac{ds}{dt} + \omega_0^2 s = A_0 \sin(\omega t)$$

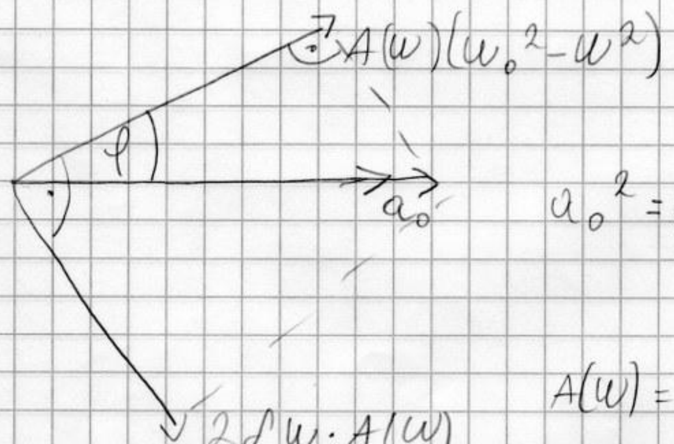
$$s_r(t) = A(\omega) \sin(\omega t + \varphi_0) \qquad s(t) = s_H(t) + s_r(t)$$

↳ KASNI U FAZI

$$\frac{ds_r}{dt} = \omega A(\omega) \cos(\omega t + \varphi) \qquad \frac{d^2 s_r}{dt^2} = -\omega^2 A(\omega) \sin(\omega t - \varphi)$$

$$-\omega^2 A(\omega) \sin(\omega t - \varphi) + 2\delta \omega A(\omega) \underbrace{\cos(\omega t + \varphi)}_{\sin(\omega t + \varphi + \frac{\pi}{2})} + \omega_0^2 A(\omega) \sin(\omega t + \varphi) = A_0 \sin(\omega t)$$

$$A(\omega) \left((\omega_0^2 - \omega^2) (\sin(\omega t + \varphi) + 2\delta\omega \sin(\omega t + \varphi - \frac{\pi}{2}) \right) = a_0 \sin(\omega t)$$



$$a_0^2 = (2\delta A(\omega))^2 + (A(\omega)(\omega_0^2 - \omega^2))^2$$

$$A(\omega) = \frac{a_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}}$$

$$\boxed{\operatorname{tg} \varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2}}$$

$$\frac{dA(w)}{dw} = \frac{-A_0 \frac{1}{2}\left(\quad // \quad \right)^{-\frac{1}{2}} \cdot \left(\overbrace{2(w_0^2-w^2)\cdot(-2w) + 8\delta^2 w}^{=0}\right)}{(w_0^2-w^2)^2 + (2\delta w)^2}$$

$$2(w_0^2-w^2)(-2w)+8\delta^2 w = 0$$

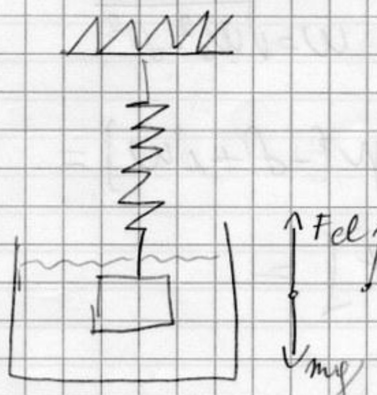
$$\cancel{(-4)}(w_0^2-w^2) = -2\delta^2 \cancel{4}$$

$$w_R^2 - w_0^2 = -2\delta^2$$

$$w_R^2 = w_0^2 - 2\delta^2$$

$$w_R = \sqrt{w_0^2 - 2\delta^2} \Rightarrow \text{REZONANTNA FREK}$$

/PRIGUŠENO TITRANJE/



$\vec{F} = -b\vec{v}$ → OBRNUTO OD SMJERA GIBANJA

$$m\cdot\vec{a} = \vec{F}_{el} + \vec{F}_{OT}$$

$$m\vec{a} = -k\Delta - bv$$

$$m\frac{d^2\Delta}{dt^2} = -k\Delta - b\frac{d\Delta}{dt}$$

$$\frac{d^2\Delta}{dt^2} + \frac{b}{m}\frac{d\Delta}{dt} + \frac{k}{m}\Delta = 0$$

FAKTOR ~~DEKREMENT~~ PRIGUŠENJA

$$2\delta = \frac{b}{m}$$

$$x(t) = C\cdot e^{at}$$

$$C\left(a^2 e^{at} + 2\delta a e^{at} + \omega_0^2 e^{at}\right) = 0$$

$$a^2 + 2\delta a + \omega_0^2 = 0$$

$$a_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

1. $\delta = 0$ NEMA PRIGUŠENJA
2. $\delta^2 < \omega_0^2$ SLABO PRIGUŠENJE
3. $\delta^2 > \omega_0^2$ JAKO PRIGUŠENJE
4. $\delta^2 = \omega_0^2$ KRITIČNO/GRANIČNO PRIGUŠENJE

2. SLABO PRIGUŠENJE

$$\delta^2 < \omega_0^2 \qquad a_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -\delta \pm i\omega \qquad \omega = \sqrt{\omega_0^2 - \delta^2}$$

$$x(t) = C \cdot e^{at} \Rightarrow x(t) = C_1 \exp\left[-\delta - i\omega\right] + C_2 \exp\left[-\delta + i\omega\right] =$$

$$= e^{-\delta t}\left[(C_1 + C_2)\cos\omega t + (C_1 - C_2)\sin\omega t\right] =$$

$$= A e^{-\delta t} \sin(\omega t + \varphi)$$

POČ. UVJET: $t = 0$ $\quad x(0) = A \sin\varphi = A_0 \quad v(0) = 0$

$$v(t) = \frac{dx}{dt} = A\left[-\delta e^{-\delta t}\sin(\omega t + \varphi) + e^{-\delta t}\cos(\omega t + \varphi)\right]$$

$$v(0) = 0 = -\delta \sin\varphi + \omega\cos\varphi \Rightarrow \boxed{\operatorname{tg}\varphi = \frac{\omega}{\delta}}$$

$$\frac{A(t)}{A(t+T)} = \frac{A e^{-\delta t}}{A e^{-\delta(t+T)}} = e^{\delta T} \quad / \ln$$

$\boxed{\lambda = \delta \cdot T} \Rightarrow$ LOGARITAMSKI DEKREMENT PRIGUŠENJA

3. JAKO PRIGUŠENJE

$$\delta^2 > \omega_0^2 \qquad a_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -\delta \pm \omega$$

$$x(t) = Ce^{at} \Rightarrow x(t) = C_1 e^{(-\delta+\omega)t} + C_2 e^{(-\delta-\omega)t}$$

$$sh(\omega t) = \frac{1}{2}\left(e^{\omega t} - e^{-\omega t}\right) \qquad e^{\omega t} = (sh(\omega t) + ch(\omega t))$$

$$ch(\omega t) = \frac{1}{2}\left(e^{\omega t} + e^{-\omega t}\right) \qquad e^{-\omega t} = ch\omega t - sh\,\omega t$$

$$x(t) = e^{-\delta t}\left[A\, sh(\omega t) + B\, ch(\omega t)\right]$$

ENERGIJA PRIGUŠENOG TITRANJA

$$E = E_K + E_P = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \quad \Big/ \frac{dE}{dt}$$

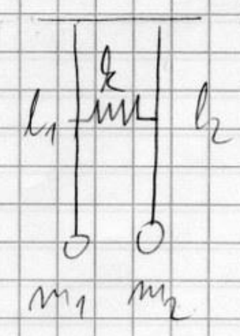
$$\frac{dE}{dt} = \frac{d}{dt}\left(E_K + E_P\right) = \frac{dE_K}{dv} \cdot \frac{dv}{dt} + \frac{dE_P}{dx} \cdot \frac{dx}{dt} =$$

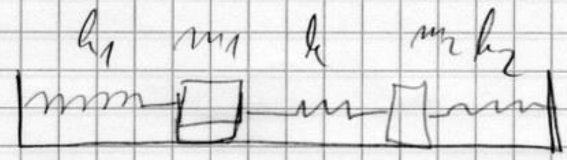
$$= m v \frac{dv}{dt} + kx \cdot v = v \underbrace{\left[m \cdot \frac{dv}{dt} + kx \right]}_{-bv} = -bv^2$$

Q => FAKTOR DOBROTE

$$Q = 2\pi \frac{[E]}{|\Delta E|}$$

OBERBECKOVA NJIHALA - VEZANI OSCILATORI





$$\begin{cases} m_1 \ddot{\Delta}_1 = -k_1 \Delta_1 + k(\Delta_2 - \Delta_1) \\ m_2 \ddot{\Delta}_2 = -k_2 \Delta_2 - k(\Delta_2 - \Delta_1) \end{cases} \quad \begin{cases} \Delta_1 = A_1 \sin(\omega t + \varphi_1) \\ \Delta_2 = A_2 \sin(\omega t + \varphi_2) \end{cases}$$

1. TITRANJE U FAZI

$A_1 = A_2 = A$

$\hookrightarrow$

$\Delta_2 = \pm A \sin(\omega t + \varphi_2)$

$$k_1 = k_2 \quad m_1 = m_2 \Rightarrow \omega_0^2 = \frac{k_1}{m}$$

$$\mathcal{L}(t) = \Delta_1(t) + \Delta_2(t)$$

$$+\begin{cases} \ddot{\Delta}_1 + \omega_0^2 \Delta_1 - \frac{k}{m}(\Delta_2 - \Delta_1) = 0 \\ \ddot{\Delta}_2 + \omega_0^2 \Delta_2 + \frac{k}{m}(\Delta_2 - \Delta_1) = 0 \end{cases}$$

$$\ddot{\Delta}_1 + \ddot{\Delta}_2 + \omega_0^2(\Delta_1 + \Delta_2) = 0$$

$$\ddot{\mathcal{L}}_1 + \omega_0^2 \mathcal{L}_1 = 0$$

2. PROTUFAZNO TITRANJE $\quad \mathcal{L}_2 = \Delta_1(t) - \Delta_2(t)$

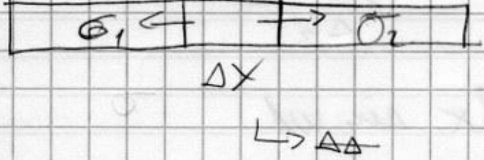
$$\ddot{\mathcal{L}}_2 + \left(\omega_0^2 + \frac{2k}{m}\right)\mathcal{L}_2 = 0 \qquad \omega_2^2 = \omega_0^2 + \frac{2k}{m}$$

$\mathcal{L}_1 = \Delta_1 + \Delta_2 \qquad \mathcal{L}_2 = \Delta_1 - \Delta_2 \qquad 2\Delta_1 = \mathcal{L}_1 + \mathcal{L}_2 \qquad 2\Delta_2 = \mathcal{L}_1 - \mathcal{L}_2$

$$2\Delta_1(t) = A \sin(\omega_0 t + \varphi_{01}) + A \sin(\omega_0 t + \varphi_{01}) \Rightarrow$$

$$\Rightarrow \cancel{\Delta_1} = 2A \cos\left(\frac{\omega_0 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right) \sin\left(\frac{\omega_0 + \omega_2}{2} + \frac{\varphi_{01} + \varphi_{02}}{2}\right)$$

LONGITUDINALNI VAL NA ŠTAPU



$$\varepsilon = \frac{\Delta s}{\Delta x} \approx \frac{\partial s}{\partial x} \qquad \sigma = \frac{F}{A}$$

$$\sigma = E \cdot \varepsilon \approx E \frac{\partial s}{\partial x}$$

$$\sigma_2 - \sigma_1 = 0$$

$$F_2 - F_1 = \Delta m a$$

$$A \cdot \sigma_2 - \sigma_1 \cdot A = (\sigma_2 - \sigma_1) \cdot A = \Delta\sigma \cdot A = \Delta m \cdot a$$

$$\Delta m = \rho \cdot \Delta x \cdot A$$

$$\frac{\Delta\sigma}{\Delta x} \approx \frac{\partial \sigma}{\partial x}$$

$$\Delta\sigma = \frac{\partial s}{\partial x} \Delta x$$

$$\frac{\partial \sigma}{\partial x} \cancel{\Delta x} \cdot \cancel{A} = \rho \cdot \cancel{\Delta x} \cdot \cancel{A} \frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial \sigma}{\partial x} = \frac{\partial}{\partial x}\left(E \cdot \frac{\partial s}{\partial x}\right) = E \frac{\partial^2 s}{\partial x^2}$$

$$\frac{\partial^2 s}{\partial x^2} - \boxed{\frac{\rho}{E}} \frac{\partial^2 s}{\partial t^2} = 0 \qquad \boxed{\frac{\rho}{E}} \rightarrow \frac{1}{v^2}$$

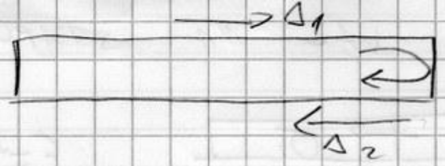
$$\boxed{v = \sqrt{\frac{E}{\rho}}}$$

$$s(x,t) = A \sin(\omega t - kx)$$

$$pV = nRT$$

$$p = \frac{m}{M \cdot V} RT = \frac{\rho RT}{M} \qquad v = \sqrt{\kappa \frac{RT}{M}}$$

STOJNI VAL NA ŠTAPU



$\Delta_1 = A\sin(\omega t - kx)$

$\Delta_2 = A\sin(\omega t + kx)$ $\qquad \Delta_1 + \Delta_2 = 2A\cos kx \sin \omega t$

$\cos kx = \pm 1$ TRBUH $\qquad \cos kx = 0$ ČVOR

$x = L \qquad \cos kL = \pm 1$

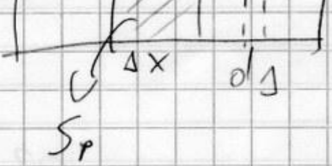
$$\left.\begin{aligned} k\cdot L &= n\pi \\ k &= \frac{2\pi}{\lambda} \end{aligned}\right\} \quad \lambda_n = \frac{2L}{n}$$

LONG. VALOVI U FLUIDU

$$E \to B \qquad \frac{\partial^2 \Delta}{\partial x^2} - \frac{\rho}{B}\frac{\partial^2 \Delta}{\partial t^2} = 0 \qquad v = \sqrt{\frac{B}{\rho}}$$

LONG. VALOVI U PLINU

$$B = \frac{1}{\kappa} \qquad \frac{\partial^2 \Delta}{\partial x^2} - \frac{\rho}{B}\frac{\partial^2 \Delta}{\partial t^2} = 0 \qquad \kappa = \frac{1}{B} = -\frac{1}{V}\cdot\frac{\Delta V}{\Delta p}$$



$v = \sqrt{\frac{B}{\rho}} \qquad \boxed{B = \rho v^2} \qquad \boxed{k = \frac{\omega}{v}}$

$$\frac{\Delta V}{V} = \frac{\Delta s \cdot S}{\Delta x \cdot S} \approx \frac{\partial \Delta}{\partial x}$$

$$\Delta p = -B\frac{\partial \Delta}{\partial x} = +BkA\cos(\omega t - kx)$$

$$= \rho v^2 A\frac{\omega}{v}\cos(\omega t - kx) = \underbrace{\boxed{\rho\cdot v\cdot A\cdot\omega}}_{\Delta p_{max}}\cos(\omega t - kx)$$

v = zv. u plinu $\Rightarrow$ ADIABATSKI

$p\cdot V^{\kappa} = konst \qquad \kappa_a = -\frac{1}{V}\frac{dV}{dp} = \frac{1}{B}$

$V^{\kappa} = \frac{konst}{p}$

$$\kappa V^{\kappa-1} dV = -\frac{konst}{p^2}dp \qquad \kappa = \frac{i+2}{i}$$

$$\Rightarrow \frac{dV}{dp} = \frac{-konst}{\kappa V^{\kappa-1} p^2} = \frac{-1}{\kappa p V^{-1}}$$

$$\kappa_a = \frac{1}{\kappa p} = \frac{1}{B} \qquad v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\kappa p}{\rho}}$$

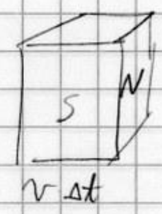
1-AT $= \frac{5}{3}$

2-AT $= \frac{7}{5}$

$\longrightarrow$

ENG. MEH. VALOVA:

$$E=\frac{1}{2}kA^2=\frac{1}{2}mv^2_{max}=\frac{1}{2}mA^2\omega^2 \qquad v(t)=\underbrace{A\omega}_{v_{max}}\cos(\omega t)$$



$$\Delta E=E\cdot N=n\Delta V\frac{1}{2}mA^2\omega^2=\frac{1}{2}\rho A^2\omega^2\Delta V$$

$$\rho=\frac{m\cdot N}{\Delta V}=m\cdot n$$

$$\Delta V=S\cdot v\cdot\Delta t$$

$n=\frac{N}{\Delta V}$ → BROJ ČESTICA

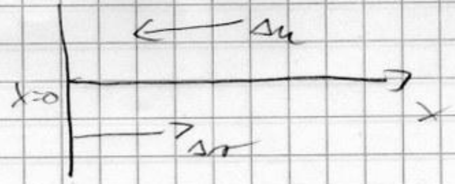
GUSTOĆA ENG. $W=\frac{\Delta E}{\Delta V}=\frac{1}{2}\rho A^2\omega^2$

$$\Delta E=W\cdot\Delta V=WSv\cdot\Delta t$$

$$P=\frac{\Delta E}{\Delta t}=WSv$$

$$I=\frac{P}{S}=W\cdot v=\frac{1}{2}\rho vA^2\omega^2$$

STOJNI VAL NA UŽETU



$s_u = A \sin(\omega t - kx)$

$s_r = -A \sin(\omega t + kx)$

$$\underline{s_u + s_r = 2A \sin kx \cos \omega t}$$

STOJNI VAL

ČVOR: $\sin kx_n = 0$

$$kx_n = n\pi \quad , \; n = 0, 1, 2 \ldots) \qquad k = \frac{2\pi}{\lambda}$$

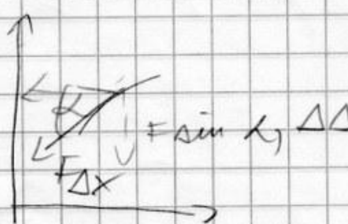
$$x_n = \frac{n\pi}{k} = n\frac{\lambda}{2}$$

TRBUH: $\sin kx_n = \pm 1$

$$kx_n = (2n+1)\frac{\pi}{2}$$

$$x_n = (2n+1)\frac{\lambda}{4}$$

PRIJENOS ENERGIJE U TRAN. VALU



$$s(x,t) = A \sin(\omega t - kx + \varphi)$$

$$tg\,\varphi = \frac{\Delta s}{\Delta x}$$

$W = \vec{F} \cdot \vec{s} =$

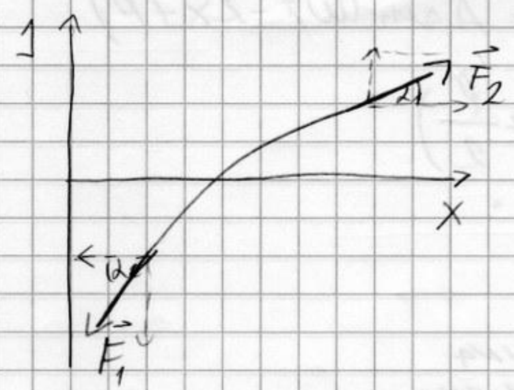
$W = -F \sin\alpha \, ds \qquad \sin\alpha \cong tg\,\alpha = \frac{\partial s}{\partial x}$

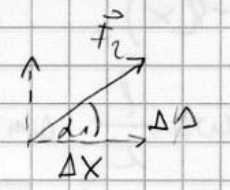
$$P = \frac{dW}{dt} = -\frac{d}{dt}\left(F \sin\alpha \, ds\right) = -F\frac{\partial s}{\partial x}\frac{\partial s}{\partial t} =$$

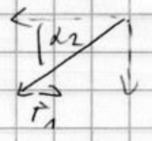
$$= -F(-kA)A\omega\cos^2(\omega t - kx + \varphi) = FA^2k\omega\cos^2(\omega t - kx + \varphi)$$

$$\overline{P} = \frac{1}{T}\int_0^T P\,dt = \frac{1}{2}FA^2k\omega$$

VALNA JEDNADŽBA TRANS. VALA NA ŽICI







$$\left.\begin{aligned} F_{1s} &= F_1 \sin\alpha_1 \\ F_{2s} &= F_2 \sin\alpha_2 \end{aligned}\right\} F_1 = F_2$$

$$\operatorname{tg}\alpha = \frac{\Delta s}{\Delta x} = \frac{\partial s}{\partial x} \approx \sin\alpha$$

$$dF_s = F_{2s} - F_{1s} = F(\sin\alpha_2 - \sin\alpha_1)$$

$$= F(\operatorname{tg}\alpha_2 - \operatorname{tg}\alpha_1) =$$

$$= F\left(\left(\frac{\partial s}{\partial x}\right)_2 - \left(\frac{\partial s}{\partial x}\right)_1\right)$$

$$\left(\frac{\partial s}{\partial x}\right)_2 - \left(\frac{\partial s}{\partial x}\right)_1 = \Delta\left(\frac{\partial s}{\partial x}\right) =$$

$$= \frac{\Delta\left(\frac{\partial s}{\partial x}\right)}{\Delta x}\Delta x = \frac{\partial^2 s}{\partial x^2}\cdot\Delta x$$

$$\Rightarrow dF_s = F\cdot\frac{\partial^2 s}{\partial x^2}\Delta x$$

$$dm = \mu\Delta x$$

$$dF_s = \Delta m\cdot a = dm\frac{\partial^2 s}{\partial t^2} = \mu\Delta x\frac{\partial^2 s}{\partial t^2}$$

$$F\frac{\partial^2 s}{\partial x^2}\cancel{\Delta x} = \mu\cancel{\Delta x}\frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial x^2} - \frac{\mu}{F}\frac{\partial^2 s}{\partial t^2} = 0$$

$$s(x,t) = f(x - vt) + g(x + vt) \qquad u = x - vt \quad f(u)$$

$$\frac{\partial s}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = f' \qquad \frac{\partial^2 s}{\partial t^2} = v^2 f''$$

$$\frac{\partial^2 s}{\partial x^2} = f'' \qquad v = \sqrt{\frac{F}{\mu}}$$

$$f'' - \frac{\mu}{F}v^2 f'' = 0 \implies s(x,t) = A\sin(\omega t - kx)$$

SUPERPOZICIJA VALOVA

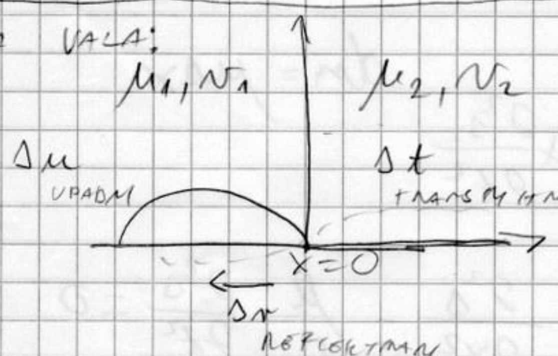
$$s_1(x,t) = A \sin(\omega t - kx) \qquad s_2(x,t) = A \sin(\omega t - kx + \varphi)$$

$$s = s_1 + s_2 = \underbrace{2A \cos\frac{\varphi}{2}}_{\text{AMPLITUDA}} \sin\left(\omega t - kx + \frac{\varphi}{2}\right)$$

$$\left.\begin{aligned} \cos\frac{\varphi}{2} &= \pm 1 \\ \frac{\varphi}{2} &= 2n\pi \end{aligned}\right\} \text{KONSTRUKTIVNA INTERFERENCIJA}$$

$$\left.\begin{aligned} \cos\frac{\varphi}{2} &= 0 \\ \frac{\varphi}{2} &= (2n+1)\frac{\pi}{2} \end{aligned}\right\} \text{DESTRUKTIVNA INTE.}$$

REFLEKSIJA VALA:



$$s_u = A_u \sin\omega\left(t - \frac{x}{v_1}\right)$$

$$s_r = A_r \sin\omega\left(t + \frac{x}{v_1}\right)$$

$$s_t = A_t \sin\omega\left(t - \frac{x}{v_2}\right)$$

UVJETI:

$$\left.\begin{aligned} &(1)\ s_u + s_r = s_t \\ &(2)\ \frac{\partial}{\partial x}(s_u + s_r) = \frac{\partial}{\partial x} s_t \end{aligned}\right\} x = 0$$

$$A_u \cancel{\sin\omega t} + A_r \cancel{\sin\omega t} = A_t \cancel{\sin\omega t} \Rightarrow \boxed{A_u + A_r = A_t}$$

$$-\frac{A_u}{v_1}\cancel{\cos\omega t} + \frac{A_r}{v_1}\cancel{\cos\omega t} = -\frac{A_t}{v_2}\cancel{\cos\omega t}$$

$$\frac{A_u}{v_1} - \frac{A_r}{v_1} = \frac{A_t}{v_2} = \frac{1}{v_2}(A_u + A_r)$$

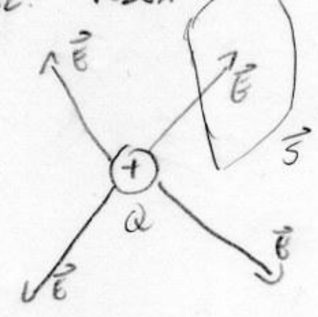
$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_u$$

$$A_t = \frac{2v_2}{v_1 + v_2} A_u$$

1. TOK EL. POLJA

a)

(1)



$$\Phi_E = \int_S \vec{E}\, d\vec{S} \Rightarrow \Phi_E = \oint_S \vec{E}\, d\vec{S}$$

$$\Phi_E = \frac{Q}{\varepsilon_0} \qquad \boxed{\oint \vec{E}\, d\vec{S} = \frac{Q}{\varepsilon_0}} \Rightarrow$$ GAUSSOV ZAKON ZA EL POLJE. 1. MAXWELLOVA JEDNADŽBA

$$d\vec{S} = dS \cdot \vec{n} \qquad \Phi = \vec{D}\cdot\vec{S} = DS\cos\vartheta$$

EL. POLJE TOČKASTOG NABOJA

$$\oint_S \vec{E}\, d\vec{S} = \frac{Q}{\varepsilon_0} \Rightarrow E \underbrace{\oint_S d\vec{S}}_{4r^2\pi} = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$ COULOMBOV ZAKON

$$= k\frac{Q}{r^2} \qquad I = \frac{dQ}{dt} = S \cdot v \cdot e \cdot n$$

b) $\rho = \frac{dQ}{dV} \quad dQ = \rho\, dV \quad Q = \int_V \rho\, dV \quad \oint_S \vec{E}\, d\vec{S} = \int_V \vec{\nabla}\vec{E}\, dV$

$$\frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho\, dV = \int_V \vec{\nabla}\vec{E}\, dV \Rightarrow \boxed{\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}}$$ => DIFERENCIJALNI OBLIK

2. ELEKTRIČNI POTENCIJAL: $d\vec{l} = d\vec{r}$

$$\int_A^B \vec{E}\, d\vec{l} = \int_A^B \vec{E}\, d\vec{r} = \cancel{\int_A^B E\, dr} = k\int_A^B \frac{Q}{r^2}\, dr = kQ\left(-\frac{1}{r}\right)\Big|_A^B = -kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right) =$$

$$= V(r_B) - V(r_A) \Rightarrow V(r) = k\frac{Q}{r} \qquad F = Q \cdot \vec{E}$$

b) RAD $\quad W_V = -\int_A^B Q\vec{E}\, d\vec{l} = Q(V_B - V_A) = Q\,\Delta V$

$$\Delta V = -W_E / Q \qquad \Rightarrow W_E = \int_A^B Q\vec{E}\, d\vec{l} \Rightarrow W = -Q\int_\infty^A \vec{E}\, d\vec{l} = kQ^2 \int_\infty^A \frac{dr}{r^2} =$$

$$= \frac{kQ^2}{a}$$

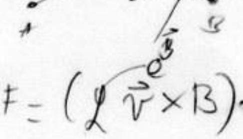
ELEKTROSTATSKA POTENCIJALNA ENERGIJA

3. MAXWELLOVA JEDN

$\oint_A \vec{E}\, d\vec{l} = 0 \Rightarrow$ KONZERVATIVNO EL. POLJE.

STOKESOV TM. $\quad \oint_K \vec{E}\, d\vec{l} = \int_S (\vec{\nabla} \times \vec{E})\, d\vec{S} \Rightarrow \vec{\nabla} \times \vec{E} = 0$

4. SILA NA VODIČ U $\vec{B}$



$q, \vec{v} \qquad F = q \cdot \vec{v} \times B$

(S) $dV = S \cdot dl$

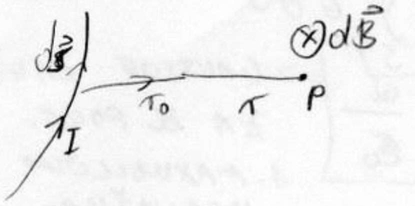
$n \cdot dV$

$$dF = (q\vec{v} \times B)\, n\, dV \qquad I = S v e n \quad dV = S\, d\vec{l}$$

$$dF = I \cdot d\vec{l} \times \vec{B}$$

$$F = I \cdot \int_A^B d\vec{l} \times \vec{B}$$

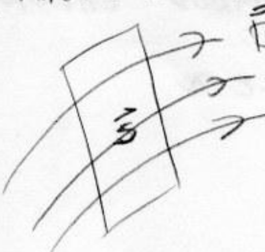
5. BIOT – SAVARTOV ZAKON



$$d\vec{B} = \frac{\mu}{4\pi} I \frac{d\vec{s}\times \vec{r_0}}{r}$$

$$\vec{B} = \int_{\text{PO VODIČU}} d\vec{B} = \mu \frac{I}{2\pi r}$$

6. MAGNETSKI TOK – 2 MAXW.



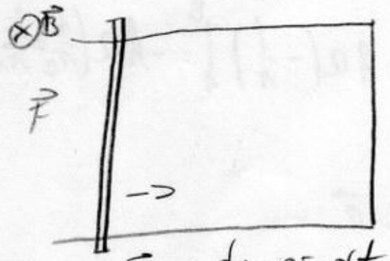
$$\Phi_M = \int_S \vec{B} d\vec{S} \quad [\Phi_M] = Wb \quad \oint_S \vec{B} d\vec{S} = 0 \Rightarrow \text{DIF}$$

$\Rightarrow$ INT

GAUSSOV TM. $\oint \vec{B} d\vec{S} = \int_V \vec{\nabla} \vec{B} dV \Rightarrow \vec{\nabla} \vec{B} = 0$

GAUSSOV ZAKON ZA MAGNETIZAM ILI DRUGA MAXWELLOVA JEDNADŽBA

7. ELM. INDUKCIJA = FARADAYEV ZAKON INDUKCIJE – 3 MAXWELL.



$\vec{v} \to ds = v \cdot dt$

$$\vec{F} = -e\vec{v}\times\vec{B} \qquad E_{ind} = -\frac{\vec{F}}{Q}$$

$$\mathcal{E} = (EMS) = -\frac{d\Phi}{dt} \qquad \Phi = \vec{B}\cdot\vec{S} \qquad \mathcal{E} = \frac{W}{Q}$$

$$W = F\cdot l = Q(\vec{v}\times\vec{B})l = QvBl$$

$$\boxed{\mathcal{E} = B\cdot l\cdot v}$$

$$dS = l\cdot ds = l\cdot v\cdot dt$$

$$d\Phi = B\, d\vec{S} = B\cdot l\cdot v\cdot dt$$

$$\frac{d\Phi}{dt} = (-)B\cdot l\cdot v \to \text{SMANJENJE MAG TOKA}$$

$$\mathcal{E} = \frac{F d\vec{l}}{Q} = \frac{\pm Q E_{ind}\, dl}{Q} = \oint_k \vec{E} d\vec{l} = \oint_k (\vec{v}\times\vec{B}) d\vec{l}$$

PO KRIVULJI

ST. TM $\oint_k \vec{E} d\vec{l} = \int_S (\vec{\nabla}\times\vec{E}) d\vec{S}$ $\Rightarrow$ INT

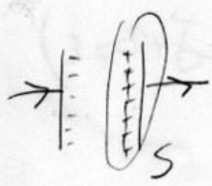
$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{S}) = -\frac{\partial}{\partial t}\int_S \vec{B} d\vec{S} = \oint_k \vec{E} d\vec{l} = \int_S \vec{\nabla}\times\vec{E}\, d\vec{S} \Rightarrow \text{DIF}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

② 1.

$$\oint_k \vec{B}\cdot d\vec{s} = \mu_0 \sum I_1 = \mu_0 \int_S \vec{J} d\vec{S}$$

$$I = \int_S \vec{J} d\vec{S} \quad \text{U TOČKI} \Rightarrow \oint_k \vec{B}\cdot d\vec{s} = 0 \rightarrow \int_S \vec{J} d\vec{S} = 0$$



$$\frac{dQ}{dt} \qquad \oint_S \vec{J} d\vec{S} = -\frac{dQ}{dt} \qquad \oint_S \vec{J} d\vec{S} = \int_V \vec{\nabla}\vec{J} dV \qquad Q = \int_V \rho dV$$

$$\vec{\nabla}\vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \qquad \vec{\nabla}(\vec{\nabla}\times\vec{B}) = 0 \Rightarrow \mu_0 \vec{\nabla}\vec{J} = 0$$

$$\oint_S \vec{J} d\vec{S} = -\frac{dQ}{dt}$$

G. ZAKON $\oint_S \vec{E} d\vec{S} = \frac{Q_{UNUTRA}}{\varepsilon_0} \Rightarrow -\frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = -\frac{d}{dt}\oint_S \vec{E} d\vec{S}$

$$-\frac{\partial}{\partial t}(Q) = \varepsilon_0 \frac{d}{dt}\oint_S \vec{E}\cdot d\vec{S} \Rightarrow \oint_S \vec{B} d\vec{S} = \mu_0 \int_S \vec{J} d\vec{S} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\int_S \vec{E} d\vec{S}$$

$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow$ MAXWELOV DOPRINOS

2. VALNA JEDNADŽBA EM VALA U VAKUUMU:

$$\overset{1.}{\vec{\nabla}\vec{E} = 0} \qquad \overset{2.}{\vec{\nabla}\vec{B} = 0} \qquad \overset{3.}{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \qquad \overset{4.}{\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t}}$$

a) $\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0\mu_0}\vec{\nabla}\times\vec{B} \;)' \qquad \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\nabla\times\frac{\partial B}{\partial t} = -\frac{1}{\varepsilon_0\mu_0}\nabla\times(\vec{\nabla}\times\vec{E}) = \frac{1}{\varepsilon_0\mu_0}\Delta\vec{E}$

$$\frac{1}{v^2} = \frac{1}{c^2} = \varepsilon_0\mu_0$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} - \frac{1}{\varepsilon_0\mu_0}\Delta\vec{E} = 0} \qquad \Delta\vec{E} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad c = \sqrt{\frac{1}{\varepsilon_0\mu_0}}$$

b) $\frac{\partial B}{\partial t} = -\vec{\nabla}\times\vec{E} \;\Big|\; \frac{\partial}{\partial t} \qquad \frac{\partial^2 \vec{B}}{\partial t^2} = -\vec{\nabla}\times\left(\frac{\partial \vec{E}}{\partial t}\right) = -\vec{\nabla}\times\left(\frac{1}{\varepsilon_0\mu_0}\vec{\nabla}\times\vec{B}\right)$

$$\frac{\partial^2 B}{\partial t^2} = -\frac{1}{\varepsilon_0\mu_0}\left(\vec{\nabla}\times(\vec{\nabla}\times\vec{B})\right) = -\frac{1}{\varepsilon_0\mu_0}\left(\vec{\nabla}(\cancelto{0}{\vec{\nabla}\vec{B}}) - \Delta\vec{B}\right) = \frac{1}{\varepsilon_0\mu_0}\Delta\vec{B}$$

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\Delta\vec{B} = 0 \quad \Rightarrow \Delta\vec{B} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

RJEŠENJA VALNE JEDNADŽBE

$\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}$ $k = \frac{2\pi}{\lambda}$

$B = B_0 e^{i(\vec{k}\vec{r} - \omega t)}$

Rj $E_x = E_0 \sin(kz - \omega t) \Rightarrow E_y = E_z = 0$

$B = ? \Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\vec{\nabla} \times \vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{\partial}{\partial t}(B_x i + B_y \vec{j} + B_z k)$

$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$

$\vec{E} = E_x \vec{i} + E_y \vec{j} + E_z \vec{k}$

$$(-1)^3 \vec{j}\left(-\frac{\partial E_x}{\partial z}\right) = -\frac{\partial}{\partial t} B_y \vec{j}$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$B_y = -\int \frac{\partial E_x}{\partial z} \cdot dt = -\int k E_0 \cos(kz - \omega t)\, dt = \frac{k E_0}{\omega} \sin(kz - \omega t) =$$

$$\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{2\pi f} = \frac{1}{\lambda f} = \frac{1}{c}$$

$B_y = \frac{E_0}{c} \sin(kz - \omega t) = B_0 \sin(kz - \omega t)$

$\boxed{B_0 = \frac{E_0}{c}}$ $B_x = B_z = \emptyset$

2. GUSTOĆA TOKA - INTENZITET EL MAG. VAL

$L: E_m = LI^2 \frac{1}{2}$ $L = \mu \frac{N^2 S}{l}$ $V = S \cdot l$

$B = \mu \frac{NI}{l}$

$C: E_{el} = \frac{1}{2} \varepsilon \frac{S}{d}(Ed)^2 / : V(S \cdot d)$ $E_{el} = \frac{1}{2} CU^2$

$w_{el} = \frac{E_{el}}{V} = \frac{1}{2}\varepsilon E^2$ $S = \frac{P}{A} = ?$ $w_m = \frac{E_m}{V} = \frac{1}{2\mu} B^2$

$w = \frac{1}{2}\varepsilon E_x^2 + \frac{1}{2\mu} B_y^2$ $P \cdot dt = w dV = w A dz = w \cdot A \cdot v \cdot dt \Rightarrow P = w A \cdot v$

$\boxed{v^2 = \frac{1}{\varepsilon \mu} \quad E_x = v B_y}$

$S = \frac{P}{A} = w \cdot v = \left[\frac{1}{2}\varepsilon E_x^2 + \frac{1}{2\mu} B_y^2\right] \cdot v$

$S = \frac{1}{2}\varepsilon v^2 E_x B_y + \frac{1}{2\mu} E_x B_y = \frac{1}{\mu} E_x B_y$ $\overline{\sin} = \frac{1}{2}$

$\vec{S} = \frac{1}{\mu_0}(E \times B)$ $w = \varepsilon E_0^2 \sin^2(\omega t - kz)$ $\overline{w} = \frac{1}{2}\varepsilon E_0^2$

FERMATOV PRINCIP

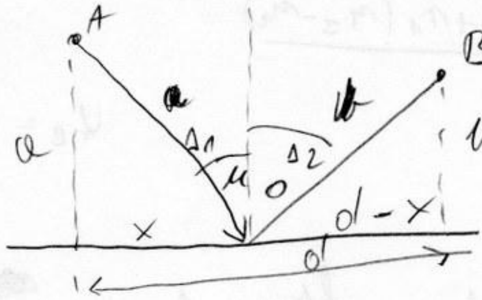
3)

a) ZAKON REFLEKSIJE

$t_{AB} = \frac{\Delta}{v}$

$t_{AB} = \int_A^B \frac{n\,dl}{c}$

$\delta_{AB} t = 0$



$t = \frac{\Delta s}{v} = \frac{1}{v}\Delta s$

$t_{min} \Rightarrow \frac{dt}{dx} = 0$

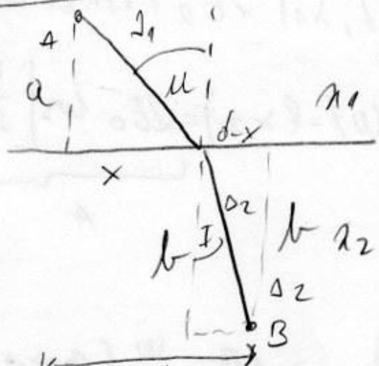
$0 = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{(d-x)^2+b^2}} =$

$\Delta_1 = \sqrt{a^2+x^2}$, $\Delta_2 = \sqrt{b^2+(d-x)^2}$

$\Delta s = \Delta_1 + \Delta_2$

$= \sin u - \sin o = 0$

$\sin u = \sin o \Rightarrow \boxed{u = o}$



b) ZAKON LOMA

$\Delta_1 = \sqrt{a^2+x^2}$

$\Delta_2 = \sqrt{(d-x)^2+b^2}$

$t_{min} \Rightarrow \frac{dt}{dx} = 0$

$t_u = \frac{\Delta_1}{v_1} + \frac{\Delta_2}{v_2}$

$\frac{dt}{dx} = \frac{x}{v_1\sqrt{a^2+x^2}} - \frac{d-x}{v_2\sqrt{(d-x)^2+b^2}}$

$= \frac{\sin u}{v_1} - \frac{\sin i}{v_2} = 0$

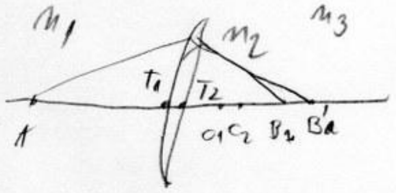
$\frac{\sin u}{\sin i} = \frac{v_1}{v_2} = \frac{\frac{c}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1}$

c) JEDNADŽBA TANKE LEĆE

$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{r} \Rightarrow$ J. S. D

$a > 0$ PREDMET LIJEVO OD T

$b > 0$ SLIKA DESNO



1. DIOPTAR

$\frac{n_1}{a} + \frac{n_2}{b'} = \frac{n_2 - n_1}{r_1}$ (1)

$b' = -a'$

2. SF DIOPTAR

$\frac{n_2}{-b'} + \frac{n_3}{b} = \frac{n_3 - n_2}{r_2}$ (2)

(1)+(2)

$\frac{n_1}{a} + \frac{n_3}{b} = \frac{n_2 - n_1}{r_1} + \frac{n_3 - n_2}{r_2}$

$a = f_a \quad b = \infty$ $\qquad\qquad$ $b = f_B \quad a = \infty$

$$\frac{n_1}{f_a} = \frac{r_2(n_2-n_1)+r_1(n_3-n_2)}{r_1 r_2} \qquad f_B = \frac{n_3 r_1 r_2}{—\,''\,—}$$

$$f_a = \frac{n_1 r_1 r_2}{—\,''\,—}$$

$$\frac{f_B}{f_a} = \frac{n_3}{n_1} \Rightarrow \frac{f_a}{a} + \frac{f_b}{b} = 1$$

$$n_1 = n_3 \Rightarrow f_a = f_b = f \qquad \boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{f} = \frac{n_2-n_1}{n_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

2. INTERFERENCIJA 2 IZVORA $\quad E_1(t, x_1) + E_2(t, x_2) = E(t, x) =$

$$= E_0 \cos(\omega t - k x_1) + E_0 \cos(\omega t - k x_2) = \underbrace{2E_0 \cos\left[\frac{k}{2}(x_2 - x_1)\right]}_{A} \cos\left[\omega t - \frac{k}{2}(x_2 - x_1)\right]$$

$\Delta = x_2 - x_1 \quad k = \frac{2\pi}{\lambda}$

$\delta = n \cdot \Delta$

$$2E_0 \cos\left[\frac{\omega}{2c}(n_1 x_1 - n_2 x_2)\right] \Rightarrow \Delta\Phi = \frac{\omega}{c}(n_1 x_1 - n_2 x_2) = \frac{2\pi}{\lambda}\delta$$

$$\hookrightarrow 2E_0 \cos\left(\frac{\Delta\phi}{2}\right) \Rightarrow \cos$$

$$I \sim E^2 \quad \text{MAKS } I \Rightarrow \cos\left(\frac{\Delta\Phi}{2}\right) = \pm 1 \Rightarrow \frac{\Delta\Phi}{2} = m\pi \qquad m = 0, \pm 1, \pm 2 \ldots$$

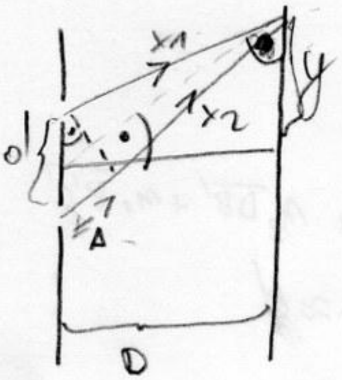
$$\frac{1}{2}\frac{2\pi}{\lambda}\delta_{max} = m\pi \Rightarrow \delta_{maks} = m \cdot \lambda$$

2. MIN. $\quad \cos\left(\frac{\phi}{2}\right) = 0 \quad \frac{\Delta\phi}{2} = (2m+1)\frac{\pi}{2}$

$$\frac{1}{2}\frac{2\pi}{\lambda}\delta_{min} = (2m+1)\frac{\pi}{2}$$

$$\delta_m = (2m+1)\frac{\lambda}{2}$$

(4) 1. YOUNGOV POKUS



$o - \vartheta$ $\quad \sin\vartheta = \frac{\Delta}{d}$ $\quad \Delta = d\sin\vartheta$ $\quad \sin\vartheta \approx \operatorname{tg}\vartheta$

$\operatorname{tg}\vartheta = \frac{y}{D}$

$\Delta_{maks} \to d\sin\vartheta = m\lambda$

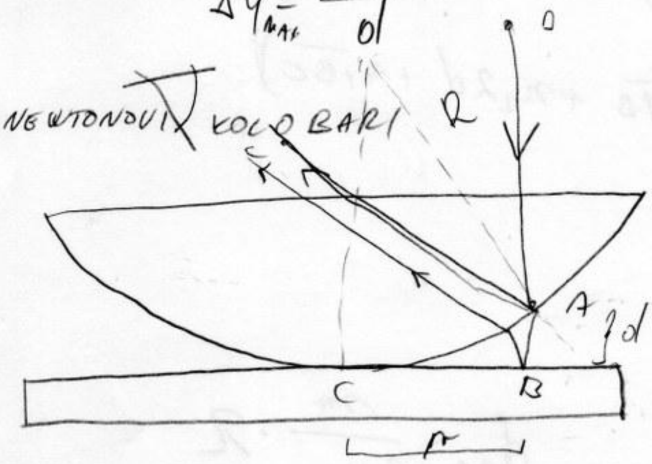
$$\frac{\Delta}{d} = \frac{y}{D} \Rightarrow y = \frac{\Delta}{d} D$$

$\Delta_{min} \to d\sin\vartheta = \frac{(2m+1)\lambda}{2}$

$$y_{maks} = \frac{m\lambda D}{d} \qquad y_{min} = (2m+1)\frac{\lambda D}{2d}$$

$$\Delta y_{max} = \frac{\lambda D}{d}$$

2. NEWTONOVI KOLOBARI



$$|\Delta\Phi| = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda} 2d + \pi = \frac{2\pi}{\lambda}\delta$$

$$\delta = 2dn + \frac{\lambda}{2}$$

$$r^2 + (R-d)^2 = R^2$$

$$\hookrightarrow d = \frac{r^2}{2R}$$

r_{max} i r_{min}

min $\frac{\Delta\Phi}{2} = (2m+1)\frac{\pi}{2}$

MAKS $\frac{\Delta\Phi}{2} = m\pi$

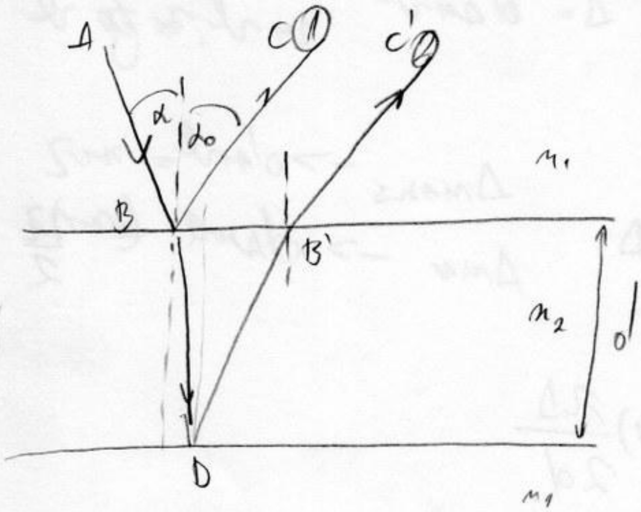
$$\frac{4\pi}{\lambda} d + \pi = 2\pi m$$

$$d_{max} = \frac{1}{2}(2m-1)\frac{\lambda}{2}$$

$$d_m = \frac{1}{2} m\lambda$$

PROLAZNA I OBRNUTO

INTERFERENCIJA NA TANKOM SLOJU



$\alpha \ll 1$

① $L_1 = n_1\overline{AB} + n_1\overline{BC}$

② $L_2 = n_1\overline{AB} + n_2\overline{BD} + n_2\overline{DB'} + n_1\overline{B'C'}$

$\overline{BC} \approx \overline{B'C'}$ i $\overline{BD} = \overline{DB} \approx d$

$\varphi = \omega t - kx$

$$\varphi_1 = \omega t - \frac{2\pi}{\lambda}(n_1\overline{AB} + n_1\overline{BC}) + \pi$$

π e na gušćem

$$\varphi_2 = \omega t - \frac{2\pi}{\lambda}(L_2) = \omega t - \frac{2\pi}{\lambda}(n_1\overline{AB} + n_2 2d + n_1\overline{BC})$$

$$\Delta\varphi = \varphi_2 - \varphi_1 = -\left(\frac{2\pi}{\lambda}n_2 2d + \pi\right)$$

$$E_{0R} = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right)$$

$$MAX \Rightarrow \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\pi}{\lambda}n_2 2d + \cancel{\pi}\right) = m\cancel{\pi}$$

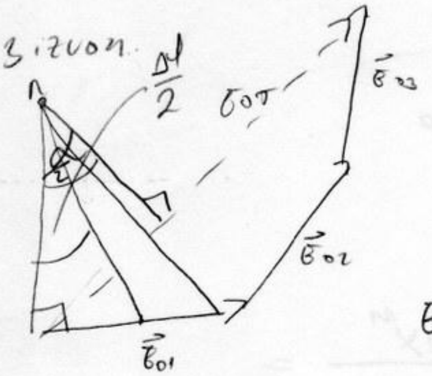
$$\frac{n_2 2d}{\lambda} = m - \frac{1}{2}$$

$$d_{MAX} = \frac{2m-1}{2n_2}\cdot\frac{\lambda}{2}$$

$$d_{min} = \frac{m}{2n_2}\cdot\lambda$$

1. INTERFERENCIJA N IZVORA

3 izvora



$$\sin\frac{\Delta\Phi}{2} = \frac{E_{01}}{2R} \qquad \sin\frac{\alpha}{2} = \frac{E_{0T}}{2R}$$

$$R = \frac{E_0}{2\sin\frac{\Delta\Phi}{2}} \qquad \frac{\alpha}{2} = \frac{3\Delta\Phi}{2}$$

$$E_{0R} = 2R\sin\frac{\alpha}{2} = 2\cdot\frac{E_0}{2\sin\frac{\Delta\phi}{2}}\cdot\sin\frac{\alpha}{2} = E_0\frac{\sin\frac{3\Delta\phi}{2}}{\sin\frac{\Delta\phi}{2}}$$

$I \sim E_{0R}^2$

3: $\Rightarrow I = \frac{\sin^2\frac{3\Delta\phi}{2}}{\sin^2\frac{\Delta\phi}{2}} E_0^2$ | N izvora $I = E_0^2 \frac{\sin^2\frac{N\Delta\Phi}{2}}{\sin^2\frac{\Delta\Phi}{2}}$ $\qquad \Delta\Phi = \frac{2\pi}{\lambda} d\sin\alpha$

2. OGIB NA PUKOTINI

$b = N\cdot\Delta x$

N : $\Delta x = d$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x\sin\alpha$$

$$I = E_0^2\frac{\sin^2\left(\frac{N}{2}\frac{2\pi}{\lambda}\sin(\alpha\Delta x)\right)}{\sin^2\left(\frac{1}{2}\frac{2\pi}{\lambda}\Delta x\sin(\alpha)\right)}$$

$\Delta x \to 0$

$N \to \infty$

$$\sin\left(\frac{\pi}{\lambda}\Delta x\sin\alpha\right) \approx \frac{\pi}{\lambda}\Delta x\sin\alpha = \frac{\pi}{\lambda}\frac{b}{N}\sin\alpha$$

$$I = (E_0 N)^2\frac{\sin^2\left(\frac{\pi}{\lambda}b\sin\alpha\right)}{\left(\frac{\pi}{\lambda}b\sin\alpha\right)^2}$$

$\Downarrow$

I_0

$$I = I_0\frac{\sin^2 y}{y^2} \Big/ \frac{d}{dy} = 0$$

$y\cos y = \sin y$

$y = \operatorname{tg} y$

min $\to b\sin\alpha = m\lambda$

max $\to b\sin\alpha = (2m+1)\frac{\lambda}{2}$

PLANCKOV ZAKON ZRAČENJA

$$I = \int_0^\infty f(\lambda; T)\, d\lambda = \sigma T^4 \qquad E_0 = h \cdot \nu \qquad h = 6.626 \cdot 10^{-34}\ \mathrm{Js}$$

$$\text{R-J FORM} \Rightarrow f(\lambda; T) = \frac{2\pi c}{\lambda^4} \bar{E} = \frac{2\pi c}{\lambda^4} kT \qquad E = n h \nu$$

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-\frac{E_n}{kT}}}{\sum_{n=0}^{\infty} e^{-\frac{E_n}{kT}}} = \frac{h\nu \sum_{n=0}^{\infty} n e^{-\frac{nh\nu}{kT}}}{\sum e^{-\frac{nh\nu}{kT}}} \qquad e^{-\frac{nh\nu}{kT}} \to x$$

$$= \frac{h\nu \sum n x^n}{\sum x^n} =$$

$$= h\nu \frac{x + 2x^2 + \dots n x^n}{1 + x + x^2 \dots x^n} = h\nu x \frac{1 + 2x + 3x^{n-1}}{1 + x + x^2 \dots x^n} = \qquad = \frac{1}{(1-x)^2} \qquad = \frac{1}{1-x}$$

$$= \frac{h\nu x}{1-x} = h\nu \frac{e^{-\frac{h\nu}{kT}}}{1 - e^{-\frac{h\nu}{kT}}} : e^{-\frac{h\nu}{kT}} = h\nu \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

//

Z. MILIKAN ~

$\uparrow F_{OT}$

$\uparrow \vec{U} = \rho_0 g V$

$\downarrow \vec{G} = m\vec{g}$

$$\vec{U} + \vec{F}_{OT} + G = 0$$

$$\rho_0 g V + 6\pi\eta r v - mg = 0$$

$$6\pi\eta r v_1 = mg - \rho_0 g V = \frac{4}{3} r^3 \pi g (\rho - \rho_0)$$

$\uparrow F_e = q \cdot \vec{E}$

$$\underbrace{\vec{G} + \vec{U}}_{6\pi\eta r v_1} + \underbrace{\vec{F}_{OT}}_{6\pi\eta r v_2} + \underbrace{\vec{F}_e}_{qE} = 0$$

$$q = \frac{1}{E} \left[6\pi\eta r (v_1 - v_2) \right]$$

COMPTONOVO RASPRŠENJE

ν, λ

$E = h\nu$

$p = \frac{h\nu}{c}$

$E_0 = m c_0^2$

$v_0 = 0$

$\vec{p}_e = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

$E = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}$

$E = E_0 + E_k$

1. KOLIČINA GIBANJA

$\vec{p} = \vec{p}\,' + \vec{p}_e$

$\Delta\nu = \nu - \nu'$

$\Delta\lambda = \lambda - \lambda'$

$\vec{p}_e = \vec{p} \quad \vec{p}\,' \, /^2$

$\vec{p}_e = p^2 + p'^2 - 2pp'\cos\vartheta$

$$\frac{m^2v^2}{1-\frac{v^2}{c^2}} = \frac{h^2\nu^2}{c^2} + \frac{h^2\nu'^2}{c^2} - 2\,\frac{h^2\nu\cdot\nu'}{c^2}\cos\vartheta \qquad \Delta\nu = \nu-\nu' \Rightarrow \nu' = \nu - \Delta\nu$$

$$\frac{m^2v^2}{1-\frac{v^2}{c^2}} = \text{——//——} + \frac{h^2}{c^2}(\nu-\Delta\nu) - \frac{2h^2}{c^2}\nu(\nu-\Delta\nu) \quad /: m^2c^2$$

$$\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = 2\frac{h^2\nu^2}{m^2c^4}\left(1-\frac{\Delta\nu}{\nu}\right)(1-\cos\vartheta) + \frac{h^2(\Delta\nu)^2}{m^2c^4} \quad (I)$$

2\. BNG. $h\nu + E_0 = h\nu' + E_e$ $\qquad E_e = E_k + E_0$

$$h\Delta\nu = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \Big/ : mc^2$$

$$\frac{h\Delta\nu}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \quad \Big/^2$$

$$\frac{1}{1-\frac{v^2}{c^2}} = 1 + \frac{h^2(\Delta\nu)^2}{m^2c^4} + \frac{2h\Delta\nu}{mc^2}$$

$$\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} = \text{---} \;/\!\!\leftarrow \qquad (2)$$

$II = I$

$$\Delta\nu = \frac{h\nu\nu'}{mc^2}(1-\cos\vartheta) \qquad \Delta\lambda = \lambda' - \lambda = \frac{c}{\nu'} - \frac{c}{\nu} = \frac{c}{\nu - \Delta\nu} - \frac{c}{\nu} =$$

$\lambda_C = 2,426 \cdot 10^{-12}$ m COMPTONOVA VALNA DULJINA

$$\Delta\lambda = \frac{h}{mc}(1-\cos\vartheta)$$

• ZAKON RADIOAKTIVNOG RASPADA

aktivnost $= -\frac{dN}{dt}$ [Bq]

$$-\frac{dN}{dt} = \lambda N$$

$$\frac{dN}{N} = -\lambda dt / \int$$

$$\int_{N_0}^{N(t)} \frac{dN}{N} = \int_0^t -\lambda dt$$

$$\ln \frac{N(t)}{N_0} = -\lambda t \quad / e$$

$$N(t) = N_0 e^{-\lambda t}$$

$$A(t) = -\frac{dN}{dt} = \lambda N(t)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$