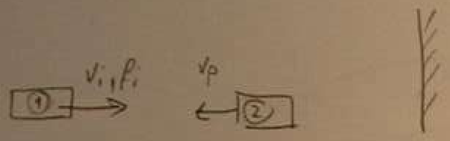
⑥ DOPPLER

2 MI 2010/2011 - 1. rok



$v_i = 60\,km/h$

$v_p = 120\,km/h$

$v_z = 1240\,km/h$

$f_i = 250\,Hz$

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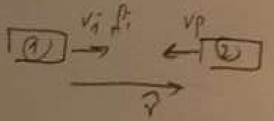
općenito $\dfrac{f_p}{f_i} = \dfrac{1 - \hat{n}\frac{v_p}{v_z}}{1 - \hat{n}\frac{v_i}{v_z}}$ $\hat{n}$ - jedinični vektor od izvora => prijemniku

a)

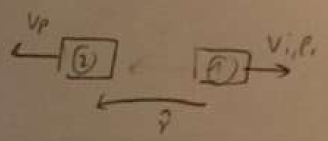


v_p i $\hat{n}$ u suprotnim smjerovima

$$\underline{f_p} = f_i \frac{1 - \frac{-v_p}{v_z}}{1 - \frac{+v_i}{v_z}} = f_i \frac{1 + \frac{v_p}{v_z}}{1 - \frac{v_i}{v_z}} = f_i \underline{\underline{\frac{v_z + v_p}{v_z - v_i}}}$$

v_i i $\hat{n}$ u istom smjeru

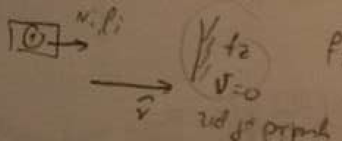
b)



$$\underline{f_p} = f_i \frac{1 - \frac{v_p}{v_z}}{1 - \frac{-v_i}{v_z}} = f_i \underline{\underline{\frac{v_z - v_p}{v_z + v_i}}}$$

c)

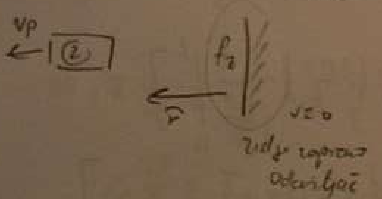
1) f_z - frekv. koju čuje zid



brzina zida = 0

$$f_z = f_i \frac{1 - \frac{v=0}{v_z}}{1 - \frac{v_i}{v_z}} = f_i \frac{v_z}{v_z - v_i}$$

2)



$$f_p = f_z \cdot \frac{1 - \frac{v_p}{v_z}}{1 - \frac{v=0}{v_z}} = f_z \frac{v_z - v_p}{v_z}$$

$$f_p = f_i \frac{v_z}{v_z - v_i} \cdot \frac{v_z - v_p}{v_z}$$

$$f_p = f_i \underline{\underline{\frac{v_z - v_p}{v_z - v_i}}}$$

⑥ VALOVI

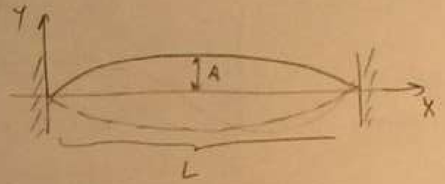
ZIR 2012/2013 3.rok

$d = 1\,mm$ $\quad f = 200\,Hz$

$L = 3\,m$ $\quad A = 2\,cm$

$\rho = 7500\,kg/m^3$

$E_{uk} = ?$



stojni val

$a(x) = A\sin\left(\frac{\pi x}{L}\right)$ maksimalna amplituda u ovisnosti o x

$y(x,t) = a(x)\cdot\sin(\omega t) = A\sin\left(\frac{\pi x}{L}\right)\sin(\omega t)$

$$v(x,t) = \frac{y(x,t)}{dt} = A\sin\left(\frac{\pi x}{L}\right)\cdot\omega\cos(\omega t)$$

$$v(x) = v(x,t)_{max} = A\omega\sin\left(\frac{\pi x}{L}\right)$$

općenito $E = \frac{mv^2}{2}$ $\quad dE = \frac{v^2}{2}dm$

$$\left.\begin{array}{l} m = \rho V = \rho\left(\frac{d}{2}\right)^2\pi\cdot L \qquad dm = \rho\left(\frac{d}{2}\right)^2\pi\,dx \end{array}\right\} dE = \rho\left(\frac{d}{2}\right)^2\pi\cdot\frac{v(x)^2}{2}dx$$

$$E = \int_0^L \rho\left(\frac{d}{2}\right)^2\pi\cdot\frac{1}{2}v(x)^2dx = \rho\left(\frac{d}{2}\right)^2\frac{\pi}{2}\cdot A^2\omega^2\int_0^L \sin^2\left(\frac{\pi x}{L}\right)dx =$$

$$= \rho\left(\frac{d}{2}\right)^2\frac{\pi}{2}A^2\omega^2\int_0^L\left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi x}{L}\right)\right]dx =$$

$$= \rho\left(\frac{d}{2}\right)^2\frac{\pi}{2}A^2\omega^2\left[\frac{1}{2}L - \frac{1}{2}\sin\left(\frac{2\pi x}{L}\right)\cdot\frac{1}{\frac{2\pi}{L}}\Bigg|_0^L\right] =$$

$$= \rho\left(\frac{d}{2}\right)^2\frac{\pi}{2}A^2\omega^2\left[\frac{1}{2}L - \frac{L}{4\pi}\left(\underbrace{\sin\left(2\pi\frac{L}{L}\right) - \sin(0)}_{0}\right)\right]$$

$$= \underbrace{\rho\left(\frac{d}{2}\right)^2\cdot L\cdot\pi}_{m}\cdot\frac{1}{4}A^2\omega^2 = \underline{\underline{\frac{1}{4}A^2\omega^2 m}} = \ldots$$

⑥ ZI 2010/11 2 zad

cijev $L_c = 1m$ zatvorena na 1 kraju

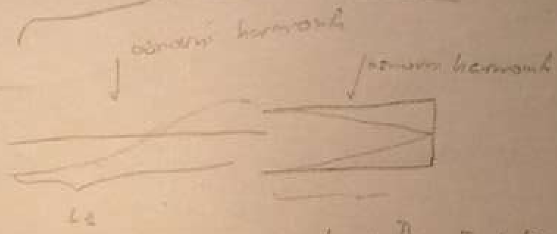
$L_ž = 0.3m$

$m_ž = 0.01 kg$

$v_z = 340 m/s$

$F = ?$ napetost žice

bitno dobro nacrtati !



$L_ž = \frac{\lambda_ž}{2}$

$L_c = \frac{\lambda}{4}$ $\lambda_c = 4 L_c$

$v = \sqrt{\frac{F}{\mu}}$ $\mu = \frac{m_ž}{L_ž}$ ⇒ treba nam v

$v = \frac{\lambda}{T} = \lambda \cdot f$ $v = \lambda_ž \cdot f = 2L_ž f$

isto vrijedi i za cijev samo što je $v = v_z$ $v_z = \lambda_c \cdot f \Rightarrow f = \frac{v_z}{4L_c}$

$v = 2L_ž \cdot \frac{v_z}{4L_c} = \frac{1}{2}\frac{L_ž}{L_c} \cdot v_z$

$F = v^2 \cdot \mu = \frac{m_ž}{L_ž} \cdot \left(\frac{L_ž}{2L_c} v_z\right)^2 = \frac{m_ž \cdot L_ž \cdot v_z^2}{2L_c^2} = \ldots$

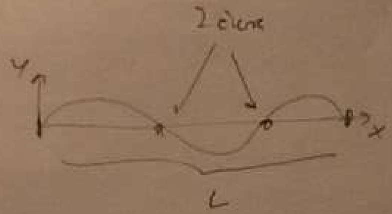
⑥ bolesnički 1 MI 2010/2011

$L = 3 m$ stojni val sa 2 čvora

$A = 1 cm$

$v = 100 m/s$

jednadžbe 2 vala koji tvore stojni val



$\lambda = \frac{2}{3}L = 2m$

$v = \lambda \cdot f \Rightarrow f = \frac{v}{\lambda} = \frac{v}{\frac{2}{3}L} = \frac{3}{2}\frac{v}{L} = \frac{3 \cdot 100}{2 \cdot 3} = 50 Hz$ $\omega = 2\pi f = 100\pi$

$k = \frac{2\pi}{\lambda} = \pi$

$y(x,t) = \underbrace{A \cdot \sin(kx)}_{\text{amplituda u ovisnosti o } x} \cdot \sin(\omega t)$ ili $\cos(\omega t)$ svejedno

$y(x,t) = y_1(x,t) + y_2(x,t) = \underbrace{B \sin(\omega t + kx)}_{\text{polazni}} + \underbrace{B \sin(\omega t - kx + \pi)}_{\text{reflektirani}}$ (+π: obrtanje)

$= B\left(\sin(\omega t + kx) - \sin(\omega t - kx)\right) = B \cdot 2 \cdot \sin\left(\frac{2kx}{2}\right)\cos\left(\frac{2\omega t}{2}\right)$

$= 2B \sin(kx)\cos(\omega t)$

$2B \sin(kx)\cos(\omega t) = A \sin(kx)\cos(\omega t)$ $B = \frac{A}{2}$

$y_1 = 0.5 cm \sin(100\pi t + \pi x)$

$y_2 = 0.5 cm \sin(100\pi t - \pi x + \pi)$

⑥ 1 MI 2010/2011 zad

$d = 1\ mm$

$L = 3m$

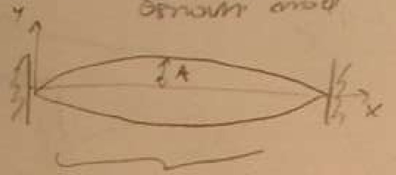
$F = 2200\ N$

$A = 2cm$

$\rho = 7800\ kg/m^3$

$v_{max} = ?$

osnovni mod



$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$

Bio u ZIR 2012/2013

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$

$$y(x,t) = A\sin(kx)\sin(\omega t) = A\sin\left(\frac{\pi}{L}x\right)\sin(\omega t)$$

$$\frac{dy}{dt} = v(x,t) = A\sin\left(\frac{\pi x}{L}\right)\cdot\omega\cos(\omega t)$$

$$v_{max} = A\cdot\omega$$

$\omega = ?$ $\quad \omega = 2\pi f = 2\pi\frac{v}{\lambda}$

$v = \sqrt{\frac{F}{\mu}}$

$\mu = \frac{m}{L} = \frac{\rho\left(\frac{d}{2}\right)^2\pi\cdot L}{L} = \rho\left(\frac{d}{2}\right)^2\pi$

$$\omega = \frac{2\pi}{2L}\cdot\sqrt{\frac{F}{\rho\left(\frac{d}{2}\right)^2\pi}}$$

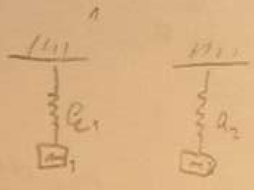
$$\omega = \frac{1}{L}\sqrt{\frac{F\pi}{\rho\left(\frac{d}{2}\right)^2}}$$

$$v_{max} = A\cdot\omega = \underline{\underline{\frac{2A}{Ld}\sqrt{\frac{F\pi}{\rho}}}} = 12.35\ m/s$$

OSCILACIJE

JIR 2013/2014 1 rok

$m_1 = 1\,kg$ $m_2 = 1.1\,kg$ $T_1 : T_2 = 1 : 1.1$



$\Delta m = ?$ da $T_1 = T_2$

$T = 2\pi\sqrt{\frac{m}{k}}$ $\frac{T_1}{T_2} = \frac{\sqrt{\frac{m_1}{k_1}}}{\sqrt{\frac{m_2}{k_2}}} = \sqrt{\frac{m_1}{m_2}\frac{k_2}{k_1}}$

$m_2 = 1.1\,m_1$ $\frac{1}{1.1} = \frac{T_1}{T_2} = \sqrt{\frac{1}{1.1}\frac{k_2}{k_1}}$ $/^2$

$\left(\frac{1}{1.1}\right)^2 = \frac{1}{1.1}\frac{k_2}{k_1} \Rightarrow \frac{k_2}{k_1} = \frac{1}{1.1}$ $k_1 = 1.1\,k_2$

želimo

$T_1 = T_2$ $\sqrt{\frac{m_1'}{k_1}} = \sqrt{\frac{m_2'}{k_2}} \Rightarrow \sqrt{\frac{m_1 + \Delta m}{1.1\,\cancel{k_2}}} = \sqrt{\frac{m_2 - \Delta m}{\cancel{k_2}}}$

$m_1 + \Delta m = 1.1\,(m_2 - \Delta m)$

$\Delta m \cdot 2.1 = 1.1\,m_2 - m_1$

$\underline{\Delta m = 0.1\,kg}$

ZIR 2011/2012

$v_{1max} = v_{2max}$ $f_1 = 200\,Hz$ $f_2 = 300\,Hz$ $F_1 = F_2 = F$

$\omega_r = ?$

općenito $x(t) = A\sin(\omega t)$ $v(t) = \frac{dx(t)}{dt} = A\omega\cos(\omega t)$ $v_{max} = A\cdot\omega$

$A = \frac{F/m}{\sqrt{(\omega_0^2-\omega^2)^2+\delta^2\omega^2}}$ prigušeno titranje

$v_{1max} = v_{2max}$

$A_1\cdot\omega_1 = A_2\omega_2$

$\frac{\omega_1\cdot\frac{F}{m}}{\sqrt{(\omega_0^2-\omega_1^2)^2+\delta^2\omega_1^2}} = \frac{\omega_2\frac{F}{m}}{\sqrt{(\omega_0^2-\omega_2^2)^2+\delta^2\omega_2^2}}$

$\omega_1^2\cdot\left((\omega_0^2-\omega_2^2)^2+\delta^2\omega_2^2\right) = \omega_2^2\left((\omega_0^2-\omega_1^2)^2+\delta^2\omega_1^2\right)$

$\omega_1^2(\omega_0^2-\omega_2^2)^2 = \omega_2^2(\omega_0^2-\omega_1^2)^2$ $/\sqrt{\ }$

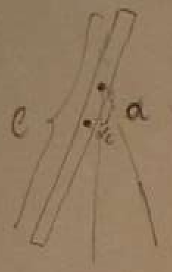
$\omega_1(\omega_0^2-\omega_2^2) = \pm\,\omega_2(\omega_0^2-\omega_1^2)$

$\frac{\omega_1}{\omega_0^2-\omega_1^2} = \pm\,\frac{\omega_2}{\omega_0^2-\omega_2^2}$

$\omega_0^2(\omega_1+\omega_2) = \omega_1^2\cdot\omega_2 + \omega_1\omega_2^2$

$\omega_0 = \sqrt{\omega_1\cdot\omega_2}$

⑥ 1 MI 2009/2010 1. zad



$$M = \frac{1}{12} m l^2 + m a^2$$

$$T = 2\pi\sqrt{\frac{M}{mga}} = 2\pi\sqrt{\frac{\frac{l^2}{12}+a^2}{g\cdot a}}$$

$$0 = \frac{dT}{da} = 2\pi \left(\frac{\frac{l^2}{12}+a^2}{ga}\right)\cdot\left(-\frac12\right)\cdot\left[-\frac{l^2}{12g}\cdot\frac{1}{a^2}+\frac1g\right]$$

$$\frac{\frac{l^2}{12}+a^2}{ga}\cdot \left(\underbrace{\frac1g - \frac{l^2}{12g}\cdot\frac{1}{a^2}}_{=0}\right) = 0$$

$$\frac{l^2}{12g}\cdot\frac{1}{a^2}=\frac1g$$

$$a^2 = \frac{l^2}{12} \qquad a=\frac{l}{\sqrt{12}}$$

⑥ POW Z1 2008/2009 1. zad

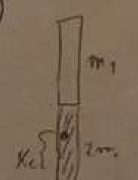
$\frac{m_1}{m_2}=\frac12$ $\quad m_2=2m_1$

$l_1 = l_2$

$$y_c = \frac{\frac{l}{2}\cdot m_2 + \frac{3l}{2}m_1}{m_1+m_2} = \frac{\frac{l}{2}\cdot 2m_1 + \frac{3l}{2}m_1}{m_1+2m_1} = \frac{l+\frac{3l}{2}}{3} = \frac{(2+3)l}{6} = \frac56 l$$

$$T = 2\pi\sqrt{\frac{I_{uk}}{mgl_{cm}}}$$

1)



$$I_{uk_1} = \frac{1}{12}m_1l^2 + m_1\left(\frac l2\right)^2 + \frac1{12}2m_1l^2 + 2m_1\left(\frac{3l}{2}\right)^2 = m_1l^2\left(\frac1{12}+\frac14+\frac2{12}+\frac{18}{4}\right) = 5m_1l^2$$

$$(mgl_{cm})_1 = 3m_1 g\cdot(2l-y_c) = 3m_1g\left(2l-\frac56 l\right) = \frac{3\cdot7}{6}m_1gl = \frac72 m_1gl$$

$$T_1 = 2\pi\sqrt{\frac{5m_1l^2}{\frac72 m_1 gl}} = 2\pi\sqrt{\frac{10}{7}\frac lg}$$

2)

$$I_{uk_2} = \frac1{12}2m_1l^2 + 2m_1\left(\frac l2\right)^2 + \frac1{12}m_1l^2 + m_1\left(\frac{3l}{2}\right)^2 = m_1l^2\left(\frac2{12}+\frac24+\frac1{12}+\frac94\right) = 3m_1l^2$$

$$(mgl_{cm})_2 = 3m_1g\cdot\frac56 l$$

$$T_2 = 2\pi\sqrt{\frac{3m_1l^2}{3m_1g\frac56 l}} = 2\pi\sqrt{\frac65\frac lg}$$

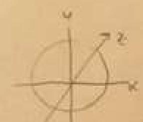
$$\frac{T_1}{T_2} = \frac{\sqrt{\frac{10}{7}}}{\sqrt{\frac65}} = \sqrt{\frac{10}{7}\cdot\frac56} = \sqrt{\frac{50}{42}}$$

⑥ PON 1.11 2008/2009 1.20 d

a)  $R = 35\,cm$ $T = 2\pi\sqrt{\frac{I_{CM}}{mRg}} = 2\pi\sqrt{\frac{\frac{3}{2}mR^2}{mRg}} = 2\pi\sqrt{\frac{3R}{2g}}$

$I_{cm} = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$

b)  $I_z = I_x + I_y$



teorem o okomitim osima

$I_x = I_y$

$I_z = I_x + I_y = 2I_x$

$I_x = \frac{I_z}{2} = \frac{1}{4}mR^2$ $T = 2\pi\sqrt{\frac{\frac{5}{4}mR^2}{mRg}} = 2\pi\sqrt{\frac{5}{4}\frac{R}{g}}$

$I_{cl} = I_x + mR^2 = \frac{1}{4}mR^2 + mR^2 = \frac{5}{4}mR^2$

c)  $I_{cm}' = \frac{1}{4}mR^2 + ma^2$

$T = 2\pi\sqrt{\frac{\frac{1}{4}mR^2 + ma^2}{mag}} = 2\pi\sqrt{\frac{\frac{1}{4}R^2 + a^2}{ag}}$

$\sqrt{\frac{\frac{1}{4}R^2 + a^2}{ag}} = \sqrt{\frac{3R}{2g}}$ $\left(\frac{1}{4}R^2 + a^2\right)\cdot 2 = 3Ra$

$\frac{1}{2}R^2 + 2a^2 - 3Ra = 0$

$2a^2 - 3Ra + \frac{1}{2}R^2 = 0$

$a_{1,2} = \frac{3R \pm \sqrt{9R^2 - 4R^2}}{4} = \frac{3R \pm \sqrt{5}R}{4}$

$= \frac{+3-\sqrt{5}}{4}R \approx \underline{0.2R}$

ZI 2008/2009 1.20 d

1) $R = 10\,cm$ kugla 2) druga kugla

$a = 5\,cm$ $b = ?$ tako da $\frac{T_1}{T_2} = 0.5$

 $I_{cm} = \frac{2}{5}mR^2$

$T_1 = 2\pi\sqrt{\frac{\frac{2}{5}mR^2 + ma^2}{mag}}$ $T_2 = 2\pi\sqrt{\frac{\frac{2}{5}mR^2 + mb^2}{mbg}}$

$a = \frac{R}{2}$

$= 2\pi\sqrt{\frac{\frac{2}{5}mR^2 + m\frac{R^2}{4}}{m\frac{R}{2}g}} = \sqrt{\frac{13R}{10g}}$

1) $\frac{T_1}{T_2} = \frac{1}{2}$ $\frac{1}{2} = \sqrt{\frac{13R}{10g}}\cdot\sqrt{\frac{bg}{\frac{2}{5}R^2 + b^2}}$ $\frac{1}{4} = \frac{13Rb}{10(\frac{2}{5}R^2 + b^2)}$ $10\left(\frac{2}{5}R^2 + b^2\right) = 52Rb$

$b^2 - \frac{26}{5}Rb + \frac{2}{5}R^2 = 0$

2) $\frac{T_2}{T_1} = \frac{1}{2}$ $4 = \frac{13Rb}{10(\frac{2}{5}R^2 + b^2)}$ $\Rightarrow$ $40\left(\frac{2}{5}R^2 + b^2\right) = 13Rb$

$b^2 - \frac{13}{40}Rb + \frac{2}{5}R^2 = 0$

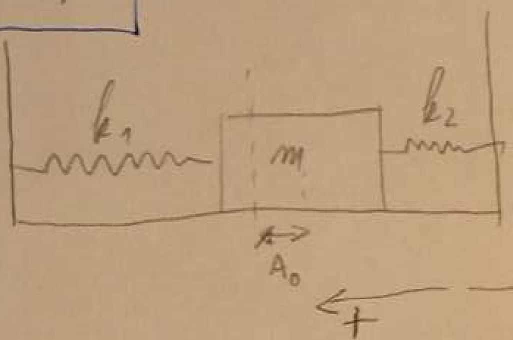
nema nj

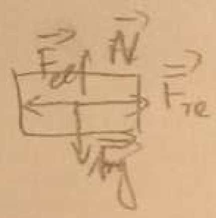
$b = 0.078\cdot R = \underline{0.78\,cm}$

~~$b_2 = 5.1\cdot R$~~

GUŠENE OSCILACIJE

$\boxed{\text{1 MI 2009/10}}$





$F_g = N$

$F_{TR} = \mu \cdot N = mg\mu$

$\vec{F}_{el} = -k\vec{x}$

PREKO ENERGIJE

$$E_0 = \frac{1}{2} k A_0^2$$

$$E_1 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 + W \longrightarrow W = F_{TR} \cdot s$$

$$E_k = \frac{1}{2} k A_k^2 + F_{TR}(A_k + A_0)$$

OČ. EN

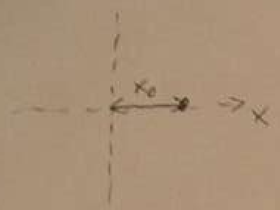
$$E_0 = E_k$$

$$\frac{1}{2} k A_0^2 = \frac{1}{2} k A_k^2 + F_{TR}(A_k + A_0)$$

$$0 = \frac{1}{2} k A_k^2 + mg\mu A_k + mg\mu A_0 - \frac{1}{2} k A_0^2$$

$$A_{k_{1,2}} = \frac{-mg\mu \pm \sqrt{(mg\mu)^2 - 4 \cdot \frac{1}{2} k \left(mg\mu A_0 - \frac{1}{2} k A_0^2\right)}}{2 \cdot \frac{1}{2} k} = 0{,}08 \text{ m}$$

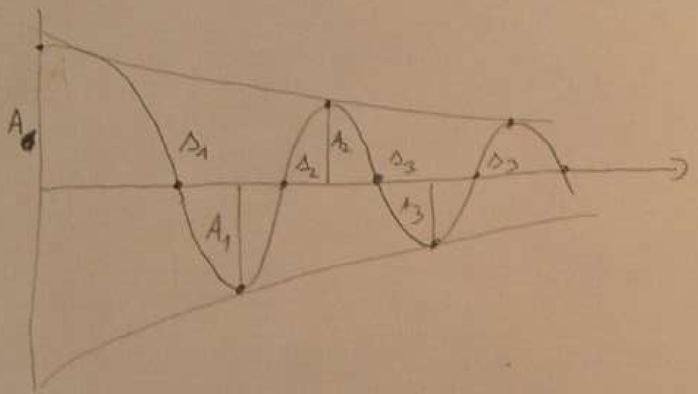
PON 1 MI 2008/09



$$\Lambda = \ln \frac{A_n}{A_{n+1}}$$

$$A_n = A_{n+1} e^{\Lambda}$$

$$A_{n+1} = A_n e^{-\Lambda}$$



$$\Delta_1 = A_0$$

$$\Delta_2 = A_0 + 2 \cdot A_1 = A_0 + 2 A_0 e^{-\Lambda} = A_0 \left(1 + 2 e^{-\Lambda}\right)$$

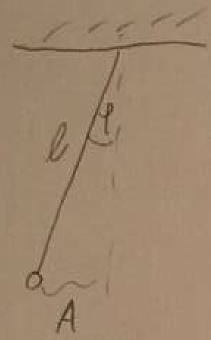
$$\Delta_3 = A_0 + 2A_1 + 2A_2 = A_0 + 2A_1 + 2A_1 e^{-\Lambda} = A_0 + 2A_0 e^{-\Lambda} + 2A_0 e^{-2\Lambda} = A_0\left(1 + 2e^{-\Lambda} + 2e^{-2\Lambda}\right)$$

$$\vdots$$

$$\Delta_n = A_0 \left(1 + 2e^{-\Lambda} + 2e^{-2\Lambda} + \dots 2e^{-(n-1)\Lambda}\right)$$

$$\Delta_U = 2A_0 \sum_{n=0}^{\infty} \left(e^{-n\Lambda}\right) - A_0 = 2A_0 \frac{1}{1 - e^{-\Lambda}} - A_0 = 10 \ m$$

2. BOL 1MI 2010/11



$$m\ddot{x} = -6\pi\eta r\dot{x} - mg\frac{x}{l}$$

$$\ddot{x} + \frac{6\pi\eta r}{\rho\frac{4}{3}r^3\pi}\dot{x} + \frac{g}{l}x = 0$$

$$\ddot{x} + \underbrace{\frac{9\eta}{\rho 2 r^2}}_{2\delta}\dot{x} + \underbrace{\frac{g}{l}}_{\omega_0^2}x = 0$$

$$A_{Al}(t) = A_{0_{Al}}e^{-\delta t}$$

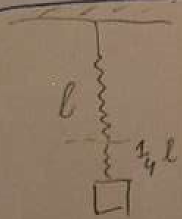
$$\frac{1}{t}\ln 2 = \frac{9\eta}{2\rho_{Al}r^2} \Rightarrow \eta = \frac{2\rho_{Al}r^2}{9t}\ln 2$$

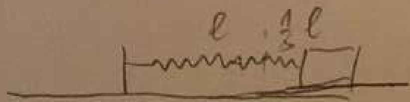
$$A_B(t) = A_{0_B}e^{-\frac{9\frac{2\rho_{Al}r^2}{9t}\ln 2}{2\rho_B r^2}t} = A_{0_B}e^{-\frac{\rho_{Al}}{\rho_B}\ln 2} = A_{0_B}\,0{,}81036$$

ELASTIČNOST

1. BOL 1 MI 2009/10

$v = 1{,}8\ m/s$





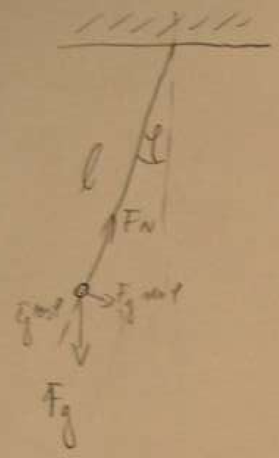
$$mg = k\frac{l}{4}$$

$$\frac{mv^2}{\frac{6}{5}l} = k\frac{1}{5}l$$

$$mv^2 = \frac{6}{25}kl^2$$

$$\frac{mv^2}{mg} = \frac{\frac{6}{25}kl^2}{k\frac{l}{4}} \Rightarrow l = \frac{v^2}{g}\frac{1}{4}\frac{25}{6} = \frac{25}{24}\frac{v^2}{g} = 0{,}344\ m$$

1.



$$I\ddot{\varphi} = -b\dot{\varphi}\cdot l - F_g \sin\varphi \cdot l \qquad I = m l^2$$

$$\ddot{\varphi} + \frac{bl}{I}\dot{\varphi} + \frac{mgl}{I}\varphi = 0$$

$$\ddot{\varphi} + \frac{b}{ml}\dot{\varphi} + \frac{g}{l}\varphi = 0$$

$$\varphi(t) = \varphi_0 e^{-\delta t}\cos(\omega t)$$

$$\frac{\varphi(6\,\text{min})}{\varphi_0} = e^{-\delta\cdot 6\,\text{min}} \Rightarrow \delta = \frac{1}{6\,\text{min}}\ln\frac{5{,}5}{4{,}4} = 6{,}2\cdot 10^{-4}\,s^{-1}$$

$$\omega = \sqrt{\omega_0^2 - \delta^2} \Rightarrow \frac{2\pi}{T} = \sqrt{\omega_0^2 - \delta^2} \Rightarrow T = 2\pi\frac{1}{\sqrt{\omega_0^2 - \delta^2}} = 2\pi\frac{1}{\sqrt{\left(\frac{g}{l}\right) - \delta^2}} = 2{,}837$$

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2. MI 2012/13

$\omega_0 = 30\,s^{-1}$

$\delta = 0{,}057\,s^{-1}$

za $\omega = 27\,s^{-1}$ $A = 1\,\text{mm}$

$$A = \frac{F_0}{m\sqrt{(\omega_0^2-\omega^2)^2+4\delta^2\omega^2}} \qquad \frac{A_{max}}{A} = \frac{\sqrt{(\omega_0^2-\omega^2)^2+4\delta^2\omega^2}}{\sqrt{(\omega_0^2-\omega_r^2)^2+4\delta^2\omega_r^2}} = 50$$

$$A_{max} = \frac{F_0}{m\sqrt{(\omega_0^2-\omega_r^2)^2+4\delta^2\omega_r^2}} \qquad A_{max} = 50\,\text{mm}$$

$$\omega_r = \sqrt{\omega_0^2 - 2\delta^2} = 29{,}9998$$

$\boxed{\text{1. LJIR 2011/12}}$

$\Delta x_1 = 9,9 \text{ cm}$

$A(t) = A_0 e^{-\delta t}$

$\Delta x_1 = \frac{mg}{k}$

$\Lambda = \ln \frac{A_0}{A_1} = \ln\left(e^{+\delta T}\right) = \delta T = \delta \frac{2\pi}{\omega} \Rightarrow \omega = \frac{\delta 2\pi}{\Lambda}$

$\Lambda = 3,4$

$T = \frac{2\pi}{\omega}$

$\omega^2 = \omega_0^2 - \delta^2$

$\delta^2\left(\left(\frac{2\pi}{\Lambda}\right)^2 + 1\right) = \sqrt{\frac{k}{m}}^2 = \sqrt{\frac{g}{\Delta x_1}}^2$

$\delta = \sqrt{\frac{\frac{g}{\Delta x_1}}{\left(\frac{2\pi}{\Lambda}\right)^2 + 1}} = 4,785 \quad \rightarrow \quad T = \frac{\Lambda}{\delta} = 0,71 \text{ s}$

$\boxed{\text{1. ZI 2010/11.}}$

$E \sim A^2$

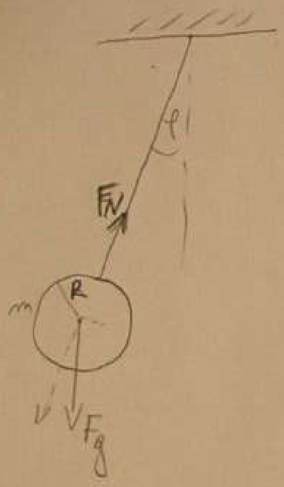
$\frac{E_0}{E_n} = e \Rightarrow \frac{A_0}{A_n} = \sqrt{e}$

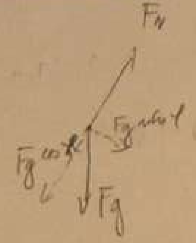
$\Lambda = \frac{1}{n} \ln \frac{A_0}{A_n} = \frac{1}{10} \ln \sqrt{e} = \frac{1}{20}$

$\uparrow$

$\ln\left(\frac{A_0}{A_1} \cdot \frac{A_1}{A_2} \cdots \frac{A_{n-1}}{A_n}\right) = \overbrace{\underset{\Lambda}{\ln\left(\frac{A_0}{A_1}\right)} + \underset{\Lambda}{\ln\left(\frac{A_1}{A_2}\right)} + \cdots \underset{\Lambda}{\ln\left(\frac{A_{n-1}}{A_n}\right)}}^{n} = n\Lambda$

2. BOL 1. MI 2009/10





$F_N = F_g \cos\varphi$

$\sin\varphi \approx \varphi$

$I = I_d + m(R+l)^2 =$

$= \frac{1}{2} m R^2 + m(R+l)^2$

$$I \ddot{\varphi} = -F_g \varphi \cdot (R+l) - b \overset{\dot{\varphi}}{\omega} (l+R)$$

$$\ddot{\varphi} + \frac{b(l+R)}{\frac{1}{2}mR^2 + m(R+l)^2} \dot{\varphi} + \underbrace{\frac{mg(R+l)}{\frac{1}{2}mR^2 + m(R+l)^2}}_{\omega_0^2} \varphi = 0$$

$$\varphi(t) = \varphi_0 e^{-\delta t} \cos(\omega t + \phi)$$

za $t=0$ $\quad \varphi(t) = \varphi_0 \Rightarrow \phi = 0$

nakon 2 titraja $\varphi(t) = \frac{\varphi_0}{3}$

$$\frac{\varphi(t)}{\varphi_0} = \frac{1}{3} = e^{-\delta t_2} \underbrace{\cos(\omega t_2)}_{1}$$

$\delta t_2 = \ln 3$

$\omega t_2 = \underset{\substack{\uparrow \\ 2 \text{ titraja}}}{2} \cdot 2\pi$

$$\Rightarrow \frac{\omega}{\delta} = \frac{4\pi}{\ln 3}$$

$$\omega = \sqrt{\omega_0^2 - \delta^2}$$

$$\frac{\sqrt{\omega_0^2 - \delta^2}}{\delta} = \frac{4\pi}{\ln 3} \Rightarrow \frac{\omega_0^2}{\delta^2} = \left(\frac{4\pi}{\ln 3}\right)^2 + 1$$

$$\delta^2 = \frac{\omega_0^2}{\left(\frac{4\pi}{\ln 3}\right)^2 + 1} = \frac{g(l+R)}{\left(\frac{1}{2}R^2 + (R+l)^2\right)\left[\left(\frac{4\pi}{\ln 3}\right)^2 + 1\right]}$$

$$\delta = \sqrt{\frac{g(l+R)}{\left(\frac{R^2}{2} + (R+l)^2\right)\left[\left(\frac{4\pi}{\ln 3}\right)^2 + 1\right]}} = 0{,}287$$

1. ZI 2006/07

$$s(t) = A e^{-\delta t} \sin(\omega t) \qquad \omega = 2\pi \ \text{rad/s}$$

$$v(t) = -A\delta e^{-\delta t} \sin(\omega t) + A e^{-\delta t} \cos(\omega t)\omega$$

$v(0{,}16) = 0$

$$\cancel{A}\delta e^{-\delta t} \sin(\omega t) \overset{t=0{,}160}{=} \cancel{A} e^{-\delta t} \cos(\omega t)\omega$$

$$\delta = \operatorname{ctg}(2\pi \cdot 0{,}16)\, 2\pi = 3{,}9899$$

$$\omega^2 = \omega_0^2 - \delta^2$$

$$\omega_0 = \sqrt{\omega^2 + \delta^2} = 7{,}44 \ \text{rad/s}$$

$\boxed{\text{1. ROK 2013/14}}$

$$x = A \sin \omega t$$

$$v = \omega A \cos \omega t$$

$$a = -\omega^2 A \sin \omega t$$

$$a_{max} = \omega^2 A = (2\pi f)^2 A$$

$$\cancel{m} g \mu = \cancel{m} a_{max}$$

$$\mu = \frac{(2\pi f)^2 A}{g} = 0{,}201$$