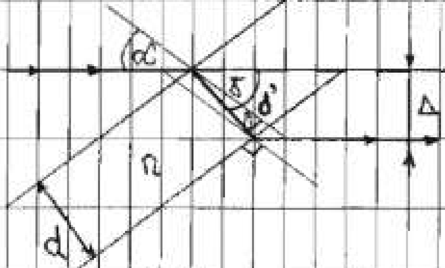
3. ciklus

4) Planparalelna ploča

$$n = \frac{\sin\alpha}{\sin\beta}, \quad \delta = (\alpha - \beta)$$



$$d = d' \cos\beta$$

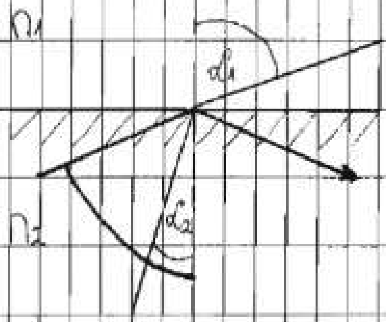
$$\Delta = d' \sin\delta$$

$$\Delta = \frac{d}{\cos\beta} \cdot \sin(\alpha-\beta) = d\,\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\beta} = d\sin\alpha\left[1 - \frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}\right] = d\sin\alpha\left[1 - \frac{\cos\alpha}{\sin\alpha}\cdot\frac{\frac{\sin\alpha}{n}}{\sqrt{1-\sin^2\beta}}\right]$$

$$= d\sin\alpha\left[1 - \frac{\cos\alpha}{\sqrt{n^2-(n\sin\beta)^2}}\right] = \boxed{d\sin\alpha\left[\frac{1-\cos\alpha}{\sqrt{n^2-\sin^2\alpha}}\right]}$$

5) Totalna refleksija

$$n_2 > n_1 \rightarrow \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{n_2}{n_1}$$

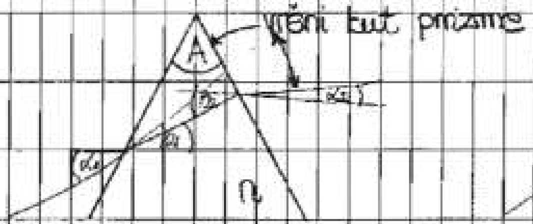


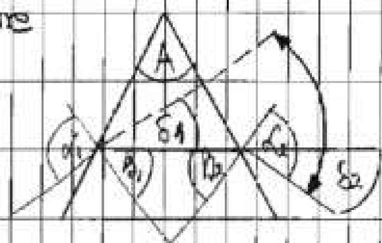
pretpostavka: $\alpha_1 \rightarrow \pi/2$

$$\sin\alpha_1 = n_1/n_2$$

granični $\alpha = \arcsin\frac{n_1}{n_2}$

6) Optička prizma





α - upadni kut

δ - kut devijacije

kut devijacije: $\delta = \delta_1 + \delta_2$

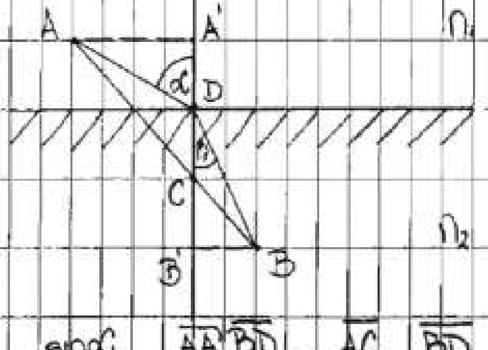
$\delta_1 = \alpha_1 - \beta_1$, $\sin\beta_1 = \frac{1}{n}\sin\alpha_1 \rightarrow \beta_1 = \arcsin\frac{\sin\alpha_1}{n}$

$\delta_2 = \alpha_2 - \beta_2$, $\sin\beta_2 = \frac{1}{n}\sin\alpha_2 \rightarrow \alpha_2 = \arcsin$ …

$$\pi = A + \left(\frac{\pi}{2} - \beta_1\right) + \left(\frac{\pi}{2} - \beta_2\right); \quad \beta_2 = A - \beta_1$$

7) Lom svjetlosti na sfernoj granici

Zakon loma u Möbiusovom obliku

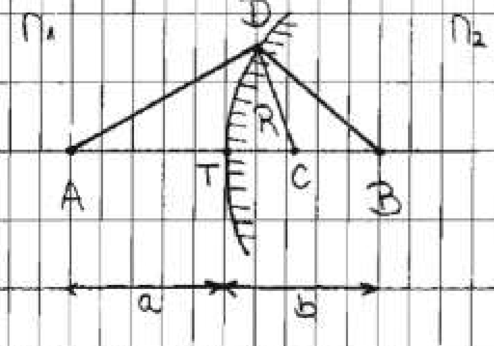


$\Delta AA'C \cong \Delta BB'C$

$\sin\alpha = \dfrac{\overline{AA'}}{\overline{AD}}$ $\quad\sin\beta = \dfrac{\overline{BB'}}{\overline{BD}}$

$$\frac{n_2}{n_1} = \frac{\sin\alpha}{\sin\beta} = \frac{\overline{AA'}}{\overline{BB'}}\cdot\frac{\overline{BD}}{\overline{AD}} = \frac{\overline{AC}}{\overline{BC}}\cdot\frac{\overline{BD}}{\overline{AD}} \Rightarrow n_1\frac{\overline{AC}}{\overline{AD}} = n_2\frac{\overline{BC}}{\overline{BD}}$$

Sferna granica



$$\frac{\overline{AC}}{\overline{AD}}\, n_1 = \frac{\overline{BC}}{\overline{BD}}\, n_2$$

$\overline{AD} \approx \overline{AT} = a$

$\overline{BD} \approx \overline{BT} = b$

Gaussove aproksimacije

$\overline{BC} = b - R$

$\overline{AC} = a + R$

Sferni dioptar

$$n_1\frac{a+R}{a} = n_2\frac{b-R}{b}$$

$$\left(1+\frac{R}{a}\right)n_1 = \left(1-\frac{R}{b}\right)n_2$$

$\boxed{\dfrac{n_1}{a} + \dfrac{n_2}{b} = \dfrac{n_2 - n_1}{R}}$ jednadžba sfernog dioptra

- slikovno žarište: $a \to \infty$

$$b \to \frac{n_2 R}{n_2 - n_1} \equiv f_b$$

- predmetno žarište: $b \to \infty$

$$a \to \frac{n_1 R}{n_2 - n_1} \equiv f_a$$

$$\frac{f_a}{a} + \frac{f_b}{b} = 1$$

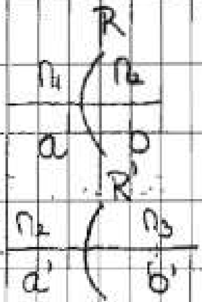
- povećanje: $m = \frac{Y'}{Y} = -\frac{\overline{BC}}{\overline{AC}} = -\frac{b-R}{a+R} = -\frac{b}{a}\frac{n_1}{n_2}$

⑧ Leće (tanke)

↳ = dvije sferne granice

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R} \quad \text{(prva granica)}$$

$$\frac{n_2}{a'} + \frac{n_3}{b'} = \frac{n_3 - n_2}{R'} \quad \text{(druga granica)}$$

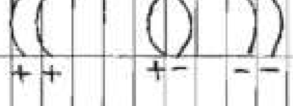


$$(1) \to \left[\frac{n_2}{b} = \frac{n_2 - n_1}{R} - \frac{n_1}{a} = -\frac{n_2}{a'}\right] \to (2)$$

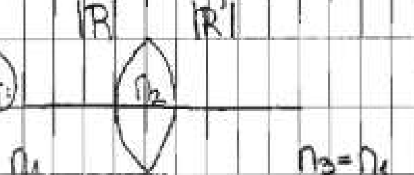
$$\frac{n_3}{b'} = \frac{n_3 - n_2}{R'} - \frac{n_2}{a'} = \frac{n_3 - n_2}{R'} + \frac{n_2}{b} = \frac{n_3 - n_2}{R'} + \frac{n_2 - n_1}{R} - \frac{n_1}{a}$$

$$\to \boxed{\frac{n_1}{a} + \frac{n_3}{b'} = \frac{n_2 - n_1}{R} + \frac{n_3 - n_2}{R'}}$$

Dogovor o predznacima:



Primjer:



$$\frac{n_1}{a} + \frac{n_1}{b} = \frac{n_2 - n_1}{+|R|} + \frac{n_1 - n_2}{-|R'|}$$

$$(n_2 - n_1)\left[\frac{1}{|R|} + \frac{1}{|R'|}\right]$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{|R|} + \frac{1}{|R'|}\right]$$

FIZIKALNA OPTIKA

1. Interferencija svjetlosti

- koherentni izvori – izvori elektromagn. titraja jednake frekvencije i stalnog faznog pomaka među njima

- superpozicija: $E_{1,2}(t,P) = E_0\cos\left[\omega\left(t - \frac{nr_{1,2}}{c}\right)\right]$ $\qquad \delta = nr$ ... duljina optičkog puta

$$\frac{\omega}{c} n r_{1,2} = 2\pi \frac{n r_{1,2}}{\lambda}$$

$E_1 + E_2 = \dots$ (korištenjem identiteta $\cos(a+b) + \cos(a-b) = 2\cos a \cos b$)

$$= \underbrace{2E_0 \underbrace{\cos\left[\omega \frac{n}{c}\frac{r_2-r_1}{2}\right]}_{\text{ne titra u vremenu}}}_{\text{Amplituda}} \underbrace{\cos\left[\omega\left(t - \frac{1}{c}\frac{n(r_1+r_2)}{2}\right)\right]}_{\text{oscilirajući faktor}}$$

Amplituda u P $= 2E_0\cos\left(\frac{\Delta\phi}{2}\right)$

$$\Delta\phi = \frac{\omega}{c} n(r_2-r_1) = 2\pi\frac{n(r_2-r_1)}{\lambda} = 2\pi \frac{\Delta\delta}{\lambda}$$

razlika u duljini optičkih putova

→ razlika u fazi vala uslijed prvog i drugog izvora

$$\cos\frac{\Delta\phi}{2} = \begin{cases} \pm 1, \text{ max. (konstruktivna interferencija)} \\ 0, \text{ min. (destruktivna interferencija)} \end{cases}$$

Max. $\frac{\Delta\phi}{2} = m\pi$

$\Delta\delta_{max} = n(r_2 - r_1) = m\lambda$

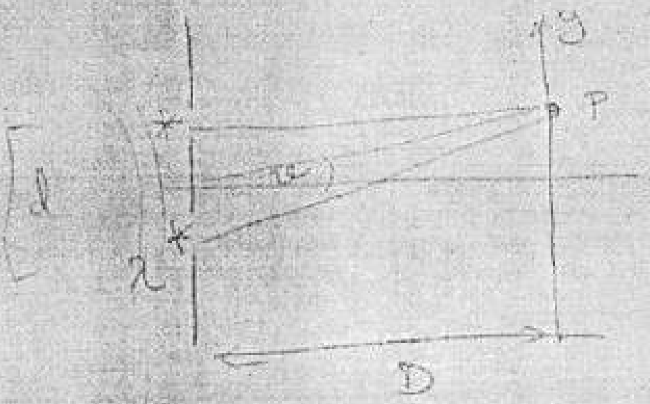
Min $\frac{\Delta\phi}{2} = (2m+1)\frac{\pi}{2}$

$\delta_{min} = \left(m + \frac{1}{2}\right)\lambda$

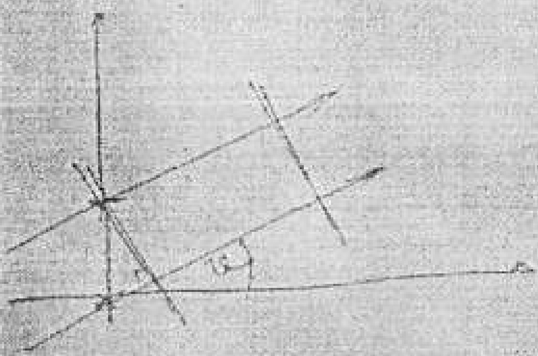
Dio koji fali u FIZIKALNOJ OPTICI

(u bilježnici je samo točka (1), ovo je ostatak)

(2.) Youngov pokus na 2 pukotine



$D >> d$



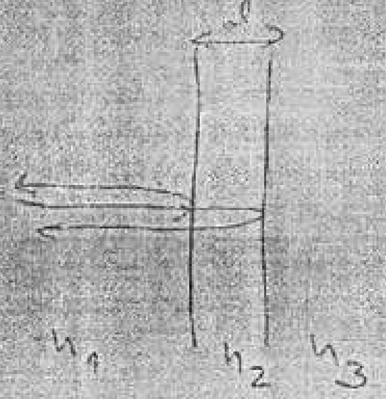
$$\left.\begin{array}{l}\delta = d \sin\vartheta \\ y = D \,\mathrm{tg}\,\vartheta\end{array}\right\} \quad \frac{\delta}{d} = \frac{y}{D}$$

Maksimumi

$\delta = m\lambda$

$$y_{max} = \frac{D}{d}\,\delta_{max} = \frac{D}{d}\, m\lambda \qquad m = 0, \pm 1, \ldots$$

③ TANKI LISTIĆI



$$\delta = 2dn_2 + \frac{\lambda}{2} + \frac{\lambda}{2}$$

Max

$$\delta_{MAX} = m\lambda$$

$$\forall\ n_1 < n_2 < n_3$$

$$2dn_2 + \lambda = m\lambda$$

$$d = \frac{\lambda}{2n}$$

Min

$$\delta_{MIN} = \left(m + \frac{1}{2}\right)\lambda$$

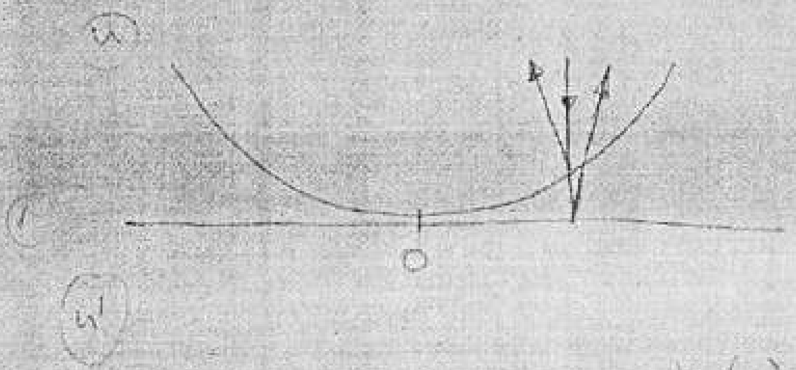
$$\forall\ n_1 < n_2 < n_3$$

$$2dn_2 + \lambda = \left(m + \frac{1}{2}\right)\lambda$$

najtanji listić

$$r = \frac{\lambda}{4n_2}$$

4. Newtonovi kolobari



$$u(r) = R - \sqrt{R^2 - r^2} \approx \frac{r^2}{2R}$$

REFLEKSIJA

$$r \ll R$$

$$\delta = 2u(r) + \frac{\lambda}{2} = \frac{r^2}{R} + \frac{\lambda}{2}$$

Max

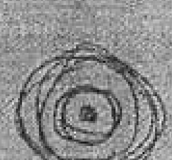
$$\delta = m\lambda$$

$$r_{max} = \sqrt{R\left(m - \frac{1}{2}\right)\lambda} \qquad m = 1, 2, \dots$$

Min

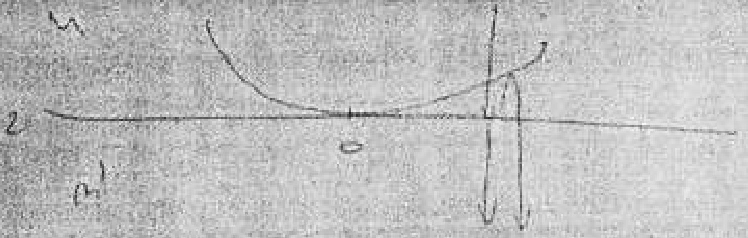
$$\delta = \left(m + \frac{1}{2}\right)\lambda$$

$$r_{min} = \sqrt{Rm\lambda} \qquad m = 0, 1, 2 \dots$$

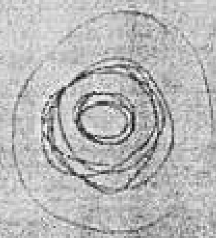


Tamna središnja broji se od 0 tamne

TRANSMISIJA



$$\delta = 2h(r) + 2\frac{\lambda}{2}$$



$$r_{max} = \sqrt{Rm\lambda} \qquad m = 0, 1, \ldots$$

$$r_{min} = \sqrt{R\left(m - \frac{1}{2}\right)\lambda} \qquad m = 1, 2, \ldots$$

5. OPTIČKA REŠETKA

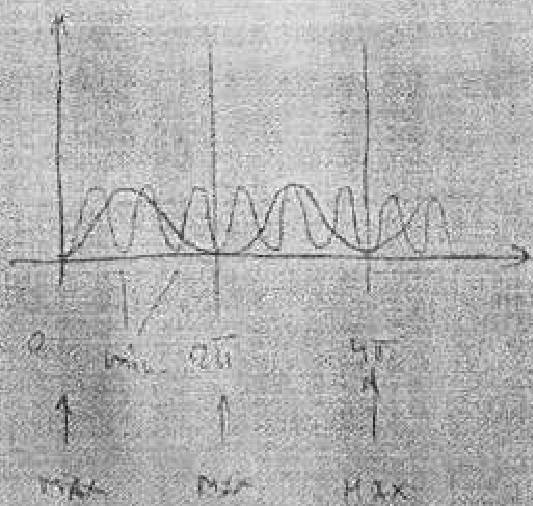
N koherentnih izvora

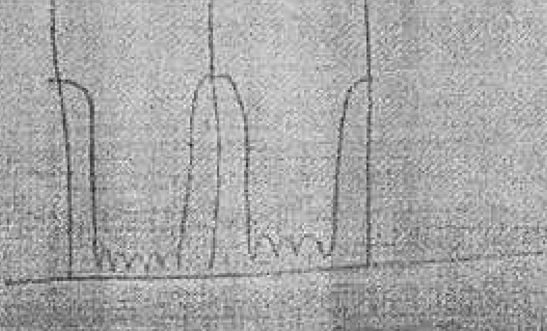
Rezultantno polje

$$E(\alpha) = E_0 \cdot \frac{\sin\left(N \frac{\Delta\phi}{2}\right)}{\sin \frac{\Delta\phi}{2}}$$

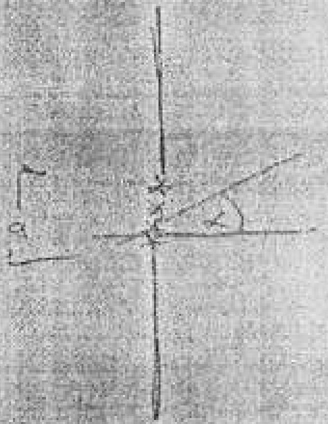
$$\Delta\phi = 2\pi \frac{d \sin\alpha}{\lambda}$$

$$I = I_0 \frac{\sin^2\left(N \frac{\Delta\phi}{2}\right)}{\sin^2 \frac{\Delta\phi}{2}}$$





6 Difrakcija na jednoj pukotini



Model: N toč. izvora na razmaku

$$d = \frac{a}{N}$$

1. Formula za rešetku

$$E(\alpha) = E_0 \frac{\sin\left(N \frac{\Delta\phi}{2}\right)}{\sin\frac{\Delta\phi}{2}}$$

$$\Delta\phi = 2\pi \frac{\delta}{\lambda} = 2\pi \frac{d \sin\alpha}{\lambda} = 2\pi \frac{a \sin\alpha}{N\lambda}$$

$$E(\alpha) = E_0 \frac{\sin\left(\pi \frac{a \sin\alpha}{\lambda}\right)}{\sin\left(\pi \frac{a \sin\alpha}{N\lambda}\right)}$$

2. Gledamo limes $N \to \infty$

$$z = \frac{\pi a \sin\alpha}{\lambda}$$

$$= \underbrace{N E_0}_{E_0'} \frac{\sin z}{z}$$

$$I(\alpha) = I_0 \cdot \frac{\sin^2 z}{z^2} \to \text{OGIB}$$

ACO

7. SPEKTROGRAF S REŠETKOM

N – broj pukotina

d – konst. rešetke

a – širina pukotine

$$I(\alpha) = I_0 \cdot \frac{\sin^2\left(\pi \frac{a \sin\alpha}{\lambda}\right)}{\left(\pi \frac{a \sin\alpha}{\lambda}\right)^2} \cdot \frac{\sin^2\left(N\pi \frac{d \sin\alpha}{\lambda}\right)}{\sin^2\left(\pi \frac{d \sin\alpha}{\lambda}\right)}$$

Maksimumi

$$d \sin\alpha = m\lambda$$

$$d \sin\alpha = \frac{m}{N}\lambda$$

Minimum mora biti pri nekom max.

Maks. $d \sin\alpha = m\lambda \quad (\lambda+\Delta\lambda)$

min: $d \sin\alpha = \frac{m(N+1)}{N}\lambda$

$$m(\lambda+\Delta\lambda) = \frac{mN+1}{N}\lambda$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{mN}$$

Moć razlučivanja rešetke

$$R = \frac{\lambda}{\Delta\lambda} = m \cdot N$$

8 POLARIZACIJA SVJETLOSTI

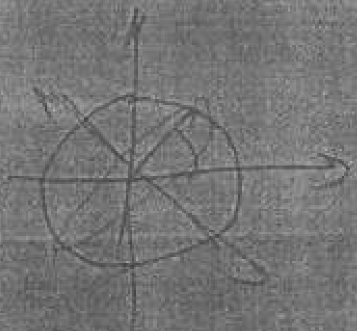
- linearno polarizirani val

$$\vec{E}(t,\vec{r}) = \vec{j}\,\underbrace{E_0}_{\text{Amp}} \cos(\omega t - \underbrace{\vec{k}}_{\text{val. vektor}}\vec{r})$$

vrijeme, položaj vektor

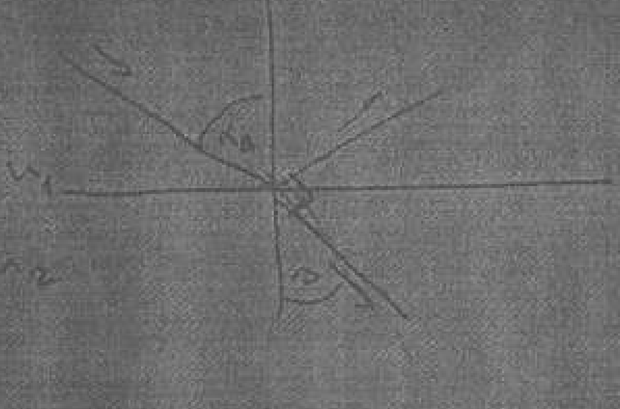
- kružno polarizirani val

$$\vec{E}(t,\vec{r}) = \vec{j}\,E_0 \cos(\omega t - \vec{k}\vec{r}) + \vec{i}\,E_0 \cos\left(\omega t - \vec{k}\vec{r} - \frac{\pi}{2}\right)$$



Brewsterov kut

Ako je kut između lomljene i ~~polarizirane~~ reflektirane zrake $= \frac{\pi}{2}$, zraka je polarizirana

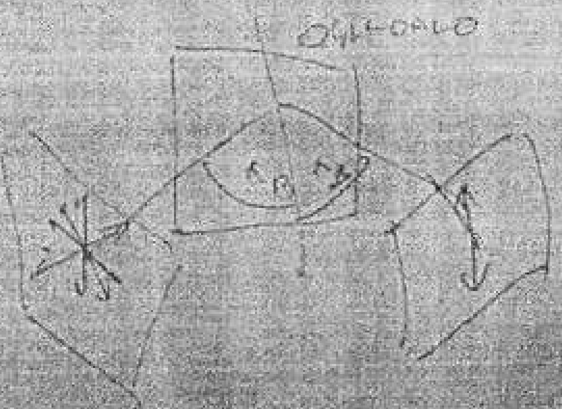


Snelov zakon

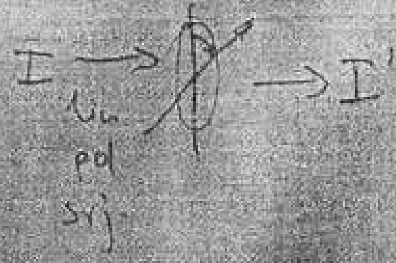
$$\frac{\sin \alpha_B}{\sin \beta} = \frac{n_2}{n_1}$$

$$\frac{\sin \beta}{\sin\left(\frac{\pi}{2} - \alpha_B\right)} = tg\, \alpha_B \qquad \alpha_B = ctg \frac{n_2}{n_1}$$

Malusov zakon



$$I' = I \cos^2 \theta$$



7.01.2009.

"MODERNA" FIZIKA

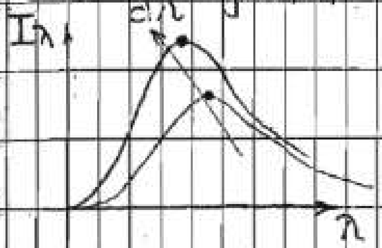
1) Zračenje crnog tijela

Štefan-Boltzmannov zakon (experimentalno)

Tijelo ugrijano na T zrači: $I = \sigma T^4$, $\sigma = 5.67 \cdot 10^{-8}\ W/m^2K^4$

Raspodjela intenziteta po valnim duljinama

$I_\lambda = \frac{dI}{d\lambda} = f(\lambda, T)$



- maximumi se pomiču sve manjim valnim duljinama

Planckov zakon

$$I_\lambda = f(\lambda, T) = \underbrace{\left(\frac{c}{4}\frac{8\pi}{\lambda^4}kT\right)}\underbrace{\left(\frac{hc}{\lambda kT}\right)\frac{1}{e^{\frac{hc}{\lambda kT}}-1}}$$

$$I_\nu = \underbrace{\frac{c}{4}\cdot\overbrace{\frac{8\pi\nu^2 kT}{c^3}}^{R-J}}_{U_\nu}\cdot\overbrace{\frac{h\nu/kT}{e^{\frac{h\nu}{kT}}-1}}^{Planck}$$

U_ν - spektralna gustoća energije

→ Rayleigh-Jeansova formula, KLASIČNA FIZIKA

"Ultraljubičasta katastrofa"

Planck $\frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$

k → Boltzmannova konst. $k = 1.381 \cdot 10^{-23}\ J/K$

h → Planckova konst. $h = 6.626 \cdot 10^{-34}\ Js$

$$I = \int dI = \int_0^\infty I_\lambda d\lambda = \int_0^\infty f(\lambda, T)\, d\lambda = \frac{2\pi^5 k^4}{15 c^2 h^3} T^4$$

Pokazujemo da je R-J zakon limes Planckova zakona pri $\lambda \to \infty$

Planckov faktor $\frac{hc}{\lambda kT}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} = \left\{\frac{hc}{\lambda kT} = x\right\} = \frac{x}{e^x - 1}$

lim $\lambda \to \infty$ → odg limes $\lambda \to 0$

$$\lim_{x\to 0}\frac{x}{e^x-1} = \lim_{x\to 0}\frac{x}{(1+x)-1} = \lim\frac{x}{x} = 1$$

Oprez $\lambda = \frac{c}{\nu}$

$$dI = f(\lambda,T)\,d\lambda$$

$$= \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{k\lambda T}}-1} = \frac{2\pi hc^2}{\left(\frac{c}{\nu}\right)^5}\cdot\frac{1}{e^{\frac{hc}{k\lambda T}}-1} = \frac{2\pi h\nu^3}{c^2}\cdot\frac{1}{e^{\frac{hc}{k\lambda T}}-1}$$

$$\boxed{I_\nu = \frac{2\pi h\nu^3}{c^2}\cdot\frac{1}{e^{\frac{hc}{k\lambda T}}-1}}$$

Wienov zakon

$\lambda_{max}\cdot T = k_W \rightarrow$ Wienova konst. 0,0029 mK

$I_\lambda = f(\lambda,T) \propto \frac{x^5}{e^x-1}$ $\qquad x = \frac{hc}{k\lambda T}$

$\frac{dI_\lambda}{dx} = \frac{df}{dx} = 0$ $\qquad$ $-f$ nikad nije 0

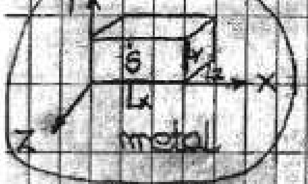
$\underbrace{\left(\frac{5}{x} - \frac{x}{e^x-1}\right)}_{=0} f$ $\qquad x = 4.965$

$$x = \frac{hc}{\lambda_{max}kT}$$

10.01.2009

Izvod zakona zračenja crnog tijela

I. Spektralna gustoća elektromagn. modova



EM zračenje zadovoljava rubne uvjete: $\vec{E} = 0$

$\Rightarrow \vec{E}(\vec{r},t) = \vec{E}_0 e^{i\omega t}\left(e^{i\vec{k}\vec{r}} + e^{-i\vec{k}\vec{r}}\right) = \vec{E}_0 e^{i\omega t} 2\cos(\vec{k}\vec{r})$ gdje je $\vec{k}$ valni vektor

$\vec{k} = k_x\vec{i} + k_y\vec{j} + k_z\vec{k}$

Zbog rubnih uvjeta: $k_x = \frac{2\pi}{\lambda_x}$, $L_x = n_x\frac{\lambda_x}{2} \Rightarrow k_x = \frac{n_x}{L_x}\pi$, $n_x = 1,2,3,\ldots$

analogno k_y, k_z

Razmatramo el. val u n-prostoru $dn_x\,dn_y\,dn_z = d^3n$

k-prostoru $dk_x\,dk_y\,dk_z = d^3k$

$$\left.\begin{aligned} dk_x &= \frac{\pi}{L_x} dn_x \\ dk_y &= \frac{\pi}{L_y} dn_y \\ dk_z &= \frac{\pi}{L_z} dn_z \end{aligned}\right\} * \rightarrow dk_x dk_y dk_z = \frac{\pi^3}{L_x L_y L_z} dn_x dn_y dn_z$$

$$\boxed{d^3k = \left(\frac{\pi^3}{V}\right) d^3n} \rightarrow \text{gustoća dopuštenih valnih vektora u k-prost}$$

Broj elektromagn. modova u V takvih da $|\vec{k}| < k$

$$N(k) = \underbrace{2}_{\text{2 moguća smjera polarizacije}} \cdot \underbrace{\frac{1}{8}}_{\text{oktant}} \cdot \underbrace{\frac{4}{3} k^3 \pi}_{\text{kugla polumjera } k} \cdot \underbrace{\frac{V}{\pi^3}}_{\text{gustoća}} = \frac{1}{3} k^3 \frac{V}{\pi^2}$$

Broj elektrom. modova tako da $|\vec{k}| \in (k, k+dk)$

$$dN = \frac{dN(k)}{dk} dk = k^2 \frac{V}{\pi^2} dk$$

Broj elektromagn. modova tako da $\nu \in (\nu, \nu + d\nu)$: $\left(k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu\right)$

$$dN = \left(\frac{2\pi}{c}\nu\right)^2 \frac{V}{\pi^2} \frac{2\pi}{c} d\nu = g(\nu)\, d\nu$$

$$\boxed{g(\nu) = \frac{8\pi^3}{c^3}\nu^2} \text{ spektralna gustoća elektromagn. modova}$$

(I) Rayleigh-Jeans

Pretpostavke: - termodinamička ravnoteža

- Boltzmannova raspodjela

$$f(\epsilon) = Ce^{-\epsilon/kT} = \frac{1}{kT} e^{-\epsilon/kT}$$

Normalizacija vjerojatnosti: $\int_0^\infty f(\epsilon)\, d\epsilon = 1 \Rightarrow C = \frac{1}{kT}$

Srednja energija: $\langle\epsilon\rangle = \int_0^\infty \epsilon f(\epsilon)\, d\epsilon = \int_0^\infty \epsilon \frac{1}{kT} e^{-\epsilon/kT} d\epsilon = kT$

Spektralna gustoća energije: $u(\nu) = \langle\epsilon\rangle \cdot g(\nu) = kT \frac{8\pi^3}{c^3}\nu^2$

(II) Planck

Pretpostavke: - termodinamička ravnoteža

- Boltzmannova raspodjela

- Hipoteza o kvantizaciji

$\langle \epsilon \rangle = \sum_n \epsilon_n f_n \qquad f_n = C e^{-\frac{\epsilon_n}{kT}}$

$\epsilon_n = nh\nu$

Normalizacija vjerojatnosti: $\sum_n f_n = 1 = C\sum_n e^{-nx} \qquad x = \frac{h\nu}{kT}$

$= C\left[e^{-x} + (e^{-x})^2 + \dots\right]$

$= C\,\frac{1}{1-e^{-x}}$

$\Rightarrow C = 1 - e^{-x} = 1 - e^{-h\nu/kT}$ ← uvrstimo

$\langle \epsilon \rangle = \sum_n nh\nu\,(1-e^{-h\nu/kT})\, e^{-\frac{nh\nu}{kT}}$

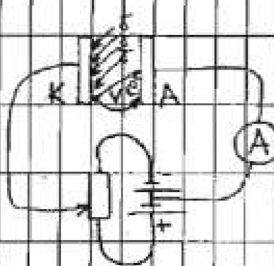
$= x(1-e^{-x})kT\sum_n n e^{-nx} = \dots$

$= x\,\frac{kT}{e^{x}-1} = \frac{h\nu}{\cancel{k}T}\,\frac{\cancel{k}T}{e^{\frac{h\nu}{kT}}-1} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

Spektralna gustoća energije:

$u(\nu) = \langle \epsilon(\nu) \rangle\, g(\nu) = \frac{8\pi h\nu^3}{c^3}\,\frac{1}{e^{\frac{h\nu}{kT}}-1}$

2) Fotoelektrični efekt



i) Struja razmjerna intenzitetu zračenja. [OK]

ii) Ako ν ide ispod neke ν_c struja nestaje [PROBLEM]

iii) Max kin. energija electrona ne ovisi o intenzitetu [PROBLEM]

$E_{k,max} = \frac{mv^2_{max}}{2} = eV_z$ zaustavni napon

19.01.2009.

Objašnjenje: Albert Einstein (1905)

$E_\gamma = \hbar\omega = h\nu = h\frac{c}{\lambda}$

$\hbar$: $h/2\pi$ — Planckova konst.

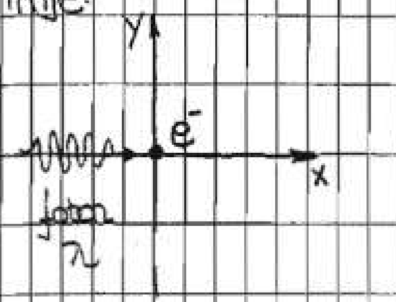
Energija $E_\gamma \geq W + E_{kin}$

W → Izlazni rad

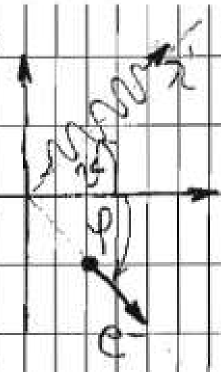
3) "Comptonovo" raspršenje

- fotoni nalijeću na česticu koja miruje, "raspršenje fotona na e^-"

Prije:



Poslije:



$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\vartheta)$$

$$\frac{1}{h\nu'} - \frac{1}{h\nu} = \frac{1}{m_0c^2}(1-\cos\vartheta)$$

Energija $\left[E = \sqrt{(m_0c^2)^2 + (pc)^2} = \frac{m_0c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right]$

$$E = \underbrace{h\nu + m_0c^2}_{\text{prije}} = \underbrace{h\nu' + \sqrt{(m_0c^2)^2 + \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}c\right)^2}}_{\text{poslije}} \qquad (1)$$

Impuls $\left[\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \text{ za } m>0 \,;\; p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \text{ za } m=0\right]$

$$p_x = \underbrace{\frac{h\nu}{c}}_{\text{prije}} = \underbrace{\frac{h\nu'}{c}\cos\vartheta + \frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\cos\varphi}_{\text{poslije}} \qquad (2)$$

$$p_y = 0 = \frac{h\nu'}{c}\sin\vartheta - \frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\sin\varphi \qquad (3)$$

$^2, +$

Iz (2) i (3) eliminiramo φ:

$$\left(\frac{h\nu}{c}\right)^2 - 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\vartheta + \left(\frac{h\nu'}{c}\right)^2 = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2 \qquad (4)$$

Prepisujemo (1):

$$\left(\frac{h\nu}{c} - \frac{h\nu'}{c} + m_0c\right)^2 - (m_0c)^2 = \left(\frac{m_0 v}{\sqrt{\;}}\right)^2$$

$$\left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + 2\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right)m_0c - 2\frac{h\nu}{c}\frac{h\nu'}{c} = \left(\frac{m_0 v}{\sqrt{\;}}\right)^2 \qquad (5)$$

(5) − (4)

$$2\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right)m_0c - 2\frac{h\nu}{c}\frac{h\nu'}{c} + 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\vartheta = 0$$

$$\left(\frac{1}{h\nu'/c} - \frac{1}{h\nu/c}\right) m_0 c = 1-\cos\vartheta$$

uz $\lambda\nu = c$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\vartheta)$$

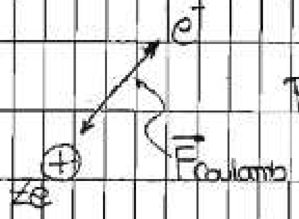
4. Bohrov model atoma (1913)

Prethodnik: * Thomson

* Rutherford
* Balmerova formula $\lambda = (364.56\ \mathrm{nm})\frac{n^2}{n^2-4}$ $n=3,4,5,\ldots$

ili $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n=3,4,5,\ldots$ $R_H = 1.097373\times10^{7}\ \mathrm{m^{-1}}$

Rydbergova konstanta



$Z = 1, 2, 3, 4, \ldots$

$$P = \frac{e^2 a^2}{6\pi\varepsilon_0 c^3} = \frac{dE}{dt}$$

Prvi Bohrov postulat: Postoje dozvoljene staze na kojima e^- ne zrači.

P2: Kutna kol. gibanja je klasična veličina kod Bohra. $L = n\hbar = n\frac{h}{2\pi} = m v_n r_n$, $n = 1,2,3,\ldots$

P3: Elektron može iz jednog dozvoljenog stanja prijeći u drugo.

Pri prelasku: $n_i \to n_f$, zrači se foton, $E_{fot} = E_i - E_f = h\nu$

Jednakost sila: $F_{cp} = \frac{m v_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{Ze\cdot e}{r_n^2}$

iz P2: $L_n = n\hbar = m v_n r_n \rightarrow v_n = \frac{n\hbar}{m r_n}$

$$\Rightarrow \frac{m}{r_n}\left(\frac{n\hbar}{m r_n}\right)^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r_n^2}$$

$$\boxed{r_n = \frac{4\pi\varepsilon_0\hbar^2}{Ze^2 m}n^2} \qquad \boxed{v_n = \frac{Ze^2}{4\pi\varepsilon_0\hbar}\cdot\frac{1}{n}}$$

Ukupna energija e^-

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} + \frac{m}{2}v_n^2 = \ldots = -\frac{mZ^2e^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{n^2}$$

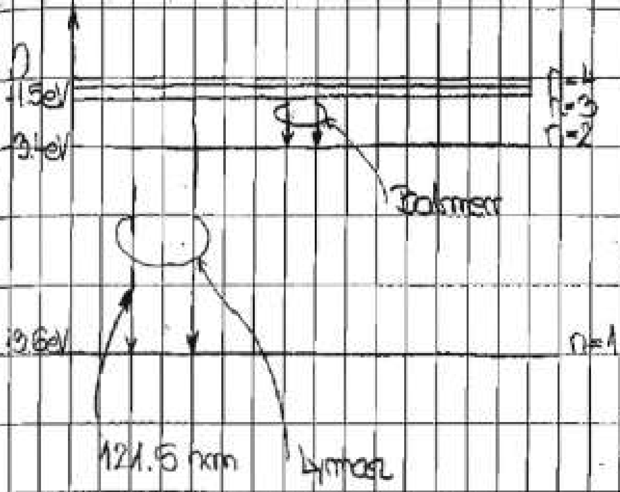
$E_1 = -13.58\ \mathrm{eV}$

$E_n = E_1\cdot\frac{1}{n^2}$

→ energija ionizacije osnovnog stanja

Iz (P3) $h\nu = E_i - E_f = E_1\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Balmer $n_f = 2$



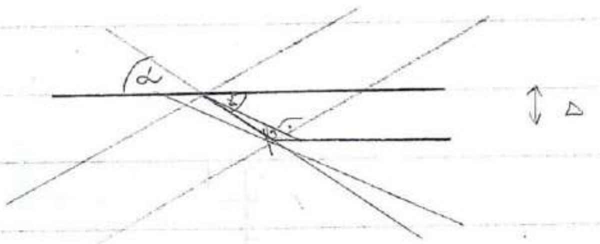
3. CIKLUS

4. PLANPARALELNA PLOČA

sloj optički gušćeg sredstva omeđenog 2. materijalima

d - debljina ploče

n - indeks loma



$$n = \frac{\sin\alpha}{\sin\beta}$$

$$d = d'\cos\beta$$

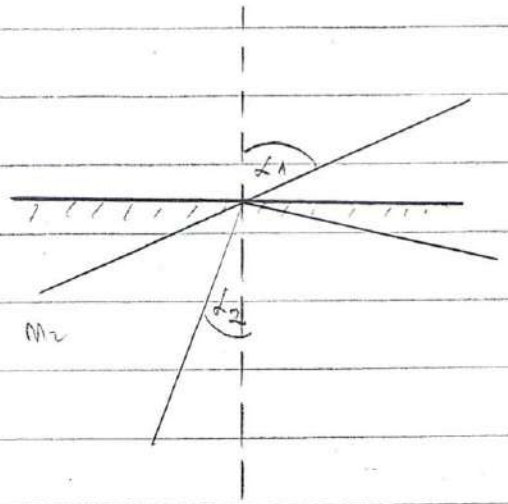
$$\Delta = d'\sin(\alpha - \beta)$$

$$\Delta = \frac{d}{\cos\beta} \cdot \sin(\alpha-\beta) = d\,\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\beta} =$$

$$= d\sin\alpha\left[1 - \frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}\right] = d\sin\alpha\left[1 - \cos\alpha\,\frac{\frac{\sin\alpha}{n}}{\sin\alpha\sqrt{1-\sin^2\beta}}\right] =$$

$$= d\sin\alpha\left[1 - \frac{\cos\alpha}{\sqrt{n^2-(n\sin\beta)^2}}\right] = d\sin\alpha\left[\frac{1-\cos\alpha}{\sqrt{n^2-\sin^2\alpha}}\right]$$

5. TOTALNA REFLEKSIJA



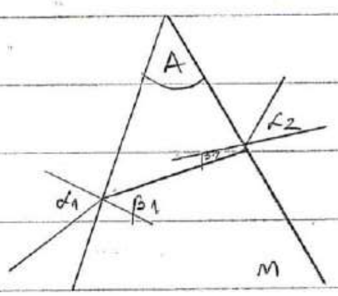
$$n_2 > n_1 \to \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{n_2}{n_1}$$

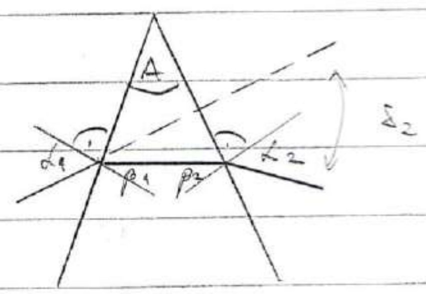
pretpostavka: $\alpha_1 \to \frac{\pi}{2}$

$$\sin\alpha_1 = \frac{n_1}{n_2}$$

granični $\alpha_2 = \arcsin\frac{n_1}{n_2}$

6. OPTIČKA PRIZMA





α - upadni kut

δ - kut devijacije

kut devijacije $\delta = \delta_1 + \delta_2$

$$\pi = A + \left(\frac{\pi}{2} - \beta_1\right) + \left(\frac{\pi}{2} - \beta_2\right)$$

$$\beta_2 = A - \beta_1$$

$\delta_1 = \alpha_1 - \beta_1$, $\sin\beta_1 = \frac{1}{n}\sin\alpha_1$

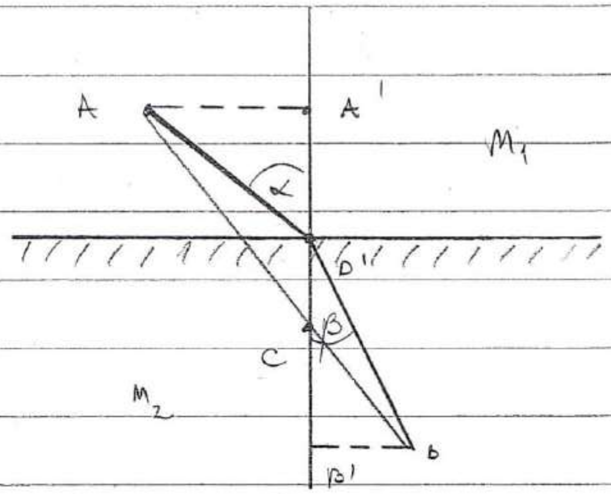
$$\beta_1 = \arcsin\frac{\sin\alpha_1}{n}$$

$\delta_2 = \alpha_2 - \beta_2$, $\sin\beta_2 = \frac{1}{n}\sin\alpha_2$

$$\alpha_2 = \arcsin(n\sin\beta_2)$$

7. LOM SVJETLOSTI NA SFERNOJ GRANICI

Zakon loma u Möbiusovom obliku



$$\Delta(AA'C) \cong \Delta(BB'C)$$

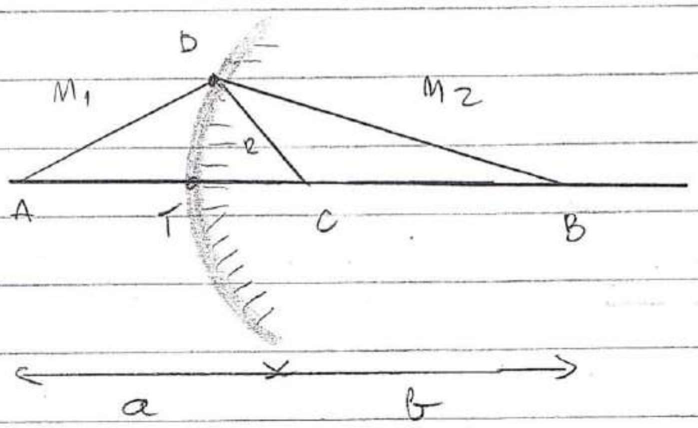
$$\sin\alpha = \frac{\overline{AA'}}{\overline{AD}}$$

$$\sin\beta = \frac{\overline{BB'}}{\overline{BD}}$$

$$\frac{M_2}{M_1} = \frac{\sin\alpha}{\sin\beta} = \frac{\frac{\overline{AA'}}{\overline{AD}}}{\frac{\overline{BB'}}{\overline{BD}}} = \frac{\overline{AA'}}{\overline{BB'}} \cdot \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{AC}}{\overline{BC}} \cdot \frac{\overline{BD}}{\overline{AD}}.$$

$$\boxed{M_1 \frac{\overline{AC}}{\overline{AD}} = \frac{\overline{BC}}{\overline{BD}} M_2}$$ $\Rightarrow$ Zakon loma prema Möbiusu

Sferna granica



$$\frac{\overline{AC}}{\overline{AD}} \cdot n_1 = \frac{\overline{BC}}{\overline{BD}} n_2$$

$\overline{AD} \simeq \overline{AT} = a$
$\overline{BD} \simeq \overline{BT} = b$
Gaussove aproksimacije

$\overline{BC} = b - R$

$\overline{AC} = a + R$

Jednadžba sfernog dioptra

$$n_1 \frac{a+R}{a} = n_2 \frac{b-R}{b}$$

$$\left(1 + \frac{R}{a}\right) n_1 = \left(1 - \frac{R}{b}\right) n_2$$

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$ jednadžba sfernog dioptra

slikovno žarište: $a \to \infty$

$$b \to \frac{n_2 R}{n_2 - n_1} \equiv f_b$$

predmetno žarište: $b \to \infty$

$$a \to \frac{n_1 R}{n_2 - n_1} \equiv f_a$$

$$\frac{f_a}{a} + \frac{f_b}{b} = 1$$

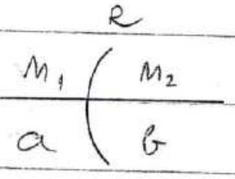
povećanje:

$$m = \frac{y'}{y} = \frac{-\overline{BC}}{\overline{AC}} = -\frac{b-R}{a+b} = -\frac{b}{a}\frac{n_1}{n_2}$$

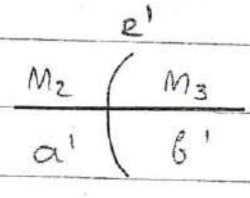
8. LEĆE

=> sloj omeđen dvjema sfernim granicama

$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R} \quad \text{(prva granica)}$$



$$\frac{n_2}{a'} + \frac{n_3}{b'} = \frac{n_3 - n_2}{R'} \quad \text{(druga granica)}$$



$$(1) \Rightarrow \frac{n_2}{b} = \frac{n_2 - n_1}{R} - \frac{n_1}{a} = \frac{-n_2}{a'} \Rightarrow (2)$$

$$\frac{n_3}{b'} = \frac{n_3 - n_2}{R'} - \frac{n_2}{a'} = \frac{n_3 - n_2}{R'} + \frac{n_2}{b} = \frac{n_3 - n_2}{R'} + \frac{n_2 - n_1}{R} - \frac{n_1}{a}$$

$$\frac{n_1}{a'} + \frac{n_3}{b'} = \frac{n_2 - n_1}{R} + \frac{n_3 - n_2}{R'}$$

dogovor o predznacima:

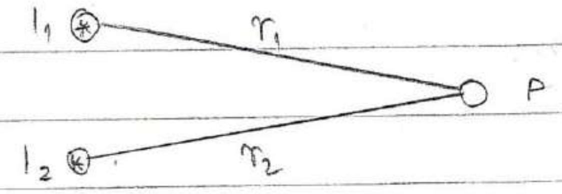
((()   ))

++   +-   --

FIZIKALNA OPTIKA

1. INTERFERENCIJA SVJETLOSTI

<u>koherentni izvori</u> - izvori elektromagnetskog titraja jednake frekvencije i stalnog faznog pomaka među njima



- superpozicija:

$$E_{1,2}(t,p) = E_0 \cos\left[\omega\left(t + \frac{n r_{1,2}}{c}\right)\right]$$

jednaka frekv.

$$\frac{\omega}{c}\, n\, r_{1,2} = 2\pi \frac{n\, r_{1,2}}{\lambda}$$

$\delta = n \cdot r \Rightarrow$ "duljina" optičkog puta

$E_1 + E_2 = \dots$ korištenjem identiteta $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$

$$= \underbrace{\underbrace{2E_0 \cos\left(\omega \frac{n}{c} \cdot \frac{r_2 - r_1}{2}\right)}_{\text{ne titra u vremenu}}}_{\text{amplituda}} \cdot \underbrace{\cos\left[\omega\left(t - \frac{1}{c}\frac{n(r_1+r_2)}{2}\right)\right]}_{\text{oscilirajući faktor}}$$

$$\cos x + \cos y = \cos\left[\underbrace{\frac{x+y}{2}}_{\alpha} - \underbrace{\frac{x-y}{2}}_{\beta}\right] + \cos\left[\underbrace{\frac{y+x}{2}}_{\alpha} + \underbrace{\frac{y-x}{2}}_{\beta}\right]$$

$$= \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$\Rightarrow \quad 2\cos\alpha\cos\beta$$

$$\alpha = \omega\left(t - \frac{n(r_1+r_2)}{2c}\right) \qquad \beta = -\frac{n(r_1-r_2)}{2c}\omega$$

Amplituda u P

$$P = 2E_0 \cos\left(\frac{\Delta\Phi}{2}\right)$$

$$\Delta\Phi = \frac{\omega}{c} n(r_2-r_1) = 2\pi \frac{n(r_2-r_1)}{\lambda} = \frac{2\pi\Delta\delta}{\lambda}$$ ⇒ razlika u duljini optičkih puteva

↓

razlika u fazi ~~između izvora~~

$$\cos\frac{\Delta\Phi}{2} = \begin{cases} \pm 1, & \text{max (konstruktivna interferencija)} \\ 0, & \text{min (destruktivna interferencija)} \end{cases}$$

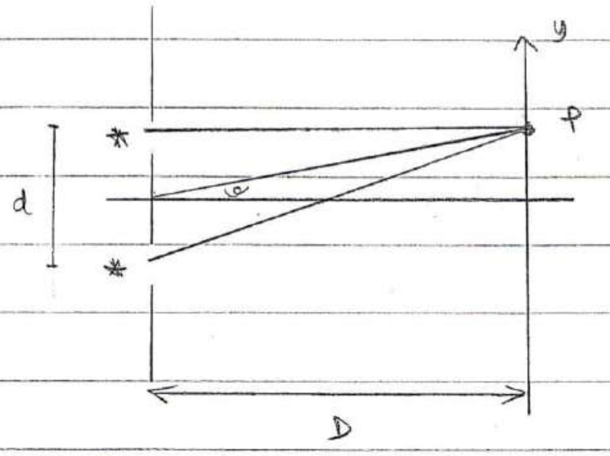
MAX: $\frac{\Delta\Phi}{2} = m\pi, \quad m = 0, \pm 1, \pm 2 \ldots$

$$\Delta\delta_{MAX} = (r_2-r_1)n = m\lambda$$

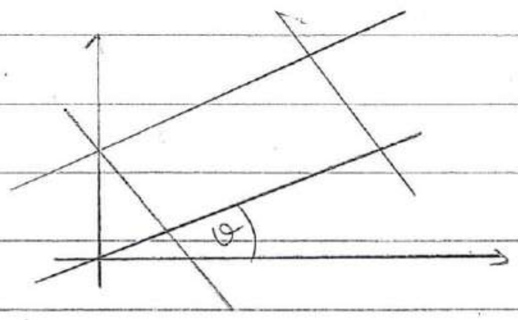
MIN: $\frac{\Delta\Phi}{2} = (2m+1)\frac{\pi}{2}, \quad m = 0, \pm 1, \pm 2 \ldots$

$$\delta_{MIN} = \left(m+\frac{1}{2}\right)\lambda$$

2. YOUNGOV POKUS NA DVIJE PUKOTINE



$D >> d$



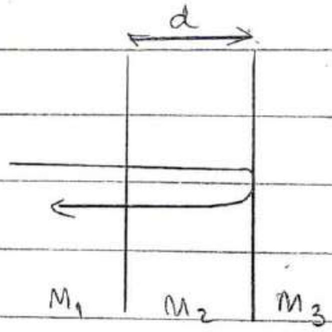
$$\left.\begin{array}{l} \delta = d \sin \vartheta \\ y = d \,\mathrm{tg}\, \vartheta \end{array}\right\} \quad \frac{\delta}{d} = \frac{y}{D}$$

Maximum

$$\delta = m \lambda$$

$$y_{MAX} = \frac{D}{d} \delta_{MAX} = \frac{D}{d} m \lambda \qquad m = 0, \pm 1, \pm 2 \ldots$$

3. TANKI LISTIĆI



$$\delta = 2dn_2 + \frac{\lambda}{2} + \frac{\lambda}{2}$$

Max: $\delta_{MAX} = m\lambda$

$$\forall\ n_1 < n_2 < n_3$$

$$2dn_2 + \lambda = m\lambda$$

$$d = \frac{\lambda}{2n}$$

Min:

$$\delta_{MIN} = \left(m + \frac{1}{2}\right)\lambda$$

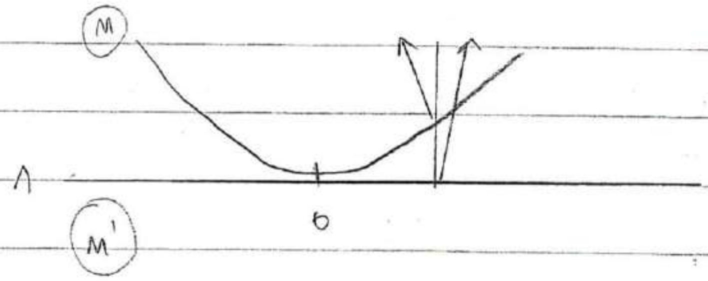
$$\forall\ n_1 < n_2 < n_3$$

$$2dn_2 + \lambda = \left(m + \frac{1}{2}\right)\lambda$$

najtanji listić

$$r = \frac{\lambda}{4n_2}$$

4. NEWTONOVI KOLOBARI



Refleksija

$$m(r) = R - \sqrt{R^2 - r^2} \simeq \frac{r^2}{2R}$$

$$r << R$$

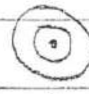
$$\delta = 2m(r) + \frac{\lambda}{2} = \frac{r^2}{R} + \frac{\lambda}{2}$$

Max: $\delta = m\lambda$.

$r_{MAX} = \sqrt{R(m-\frac{1}{2})\lambda}$ $m = 1, 2 \ldots$

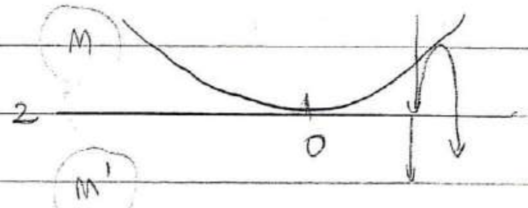
Min: $\delta = (m + \frac{1}{2})\lambda$

$r_{MIN} = \sqrt{Rm\lambda}$ $m = 0, 1, 2 \ldots$



tamna sredina,
broji se od 0

Transmisija



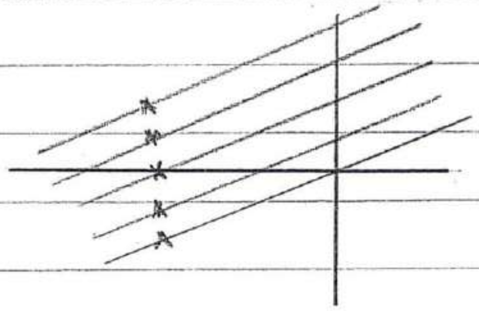
$$\delta = 2m(r) + 2\frac{\lambda}{2}$$

$r_{MAX} = \sqrt{Rm\lambda}$ $m = 0, 1, 2 \ldots$

$r_{MIN} = \sqrt{R(m-\frac{1}{2})\lambda}$ $m = 1, 2, \ldots$

5. OPTIČKA REŠETKA

N - koherentnih izvora

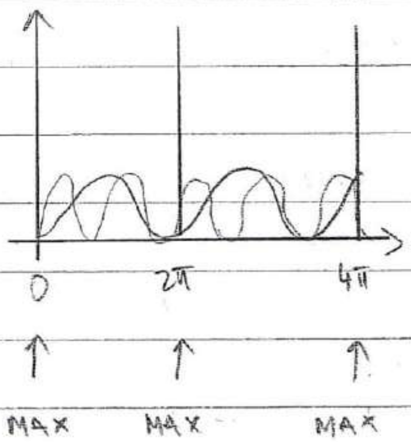


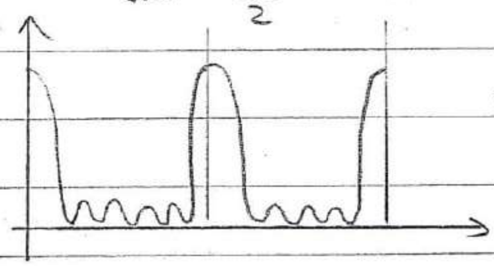
Rezultantno polje

$$E(\alpha) = E_0 \cdot \frac{\sin\left(N \frac{\Delta\phi}{2}\right)}{\sin \frac{\Delta\phi}{2}}$$

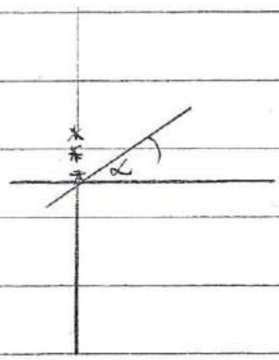
$$\Delta\phi = 2\pi \frac{d \sin\alpha}{\lambda}$$

$$I = I_0 \frac{\sin^2\left(N \frac{\Delta\phi}{2}\right)}{\sin^2 \frac{\Delta\phi}{2}}$$





6. DIFRAKCIJA NA JEDNOJ PUKOTINI



Model: N koherentnih izvora na razmak

$$d = \frac{a}{N}$$

Formula za rešetku

$$E(\alpha) = E_0 \frac{\sin\left(N \frac{\Delta\phi}{2}\right)}{\sin \frac{\Delta\phi}{2}}$$

$$\Delta\phi = 2\pi \frac{\delta}{\lambda} = 2\pi \frac{d \sin\alpha}{\lambda} = 2\pi \frac{a \sin\alpha}{N\lambda}$$

$$E(\alpha) = E_0 \frac{\sin\left(\pi \frac{a \sin\alpha}{\lambda}\right)}{\sin\left(\pi \frac{a \sin\alpha}{N\lambda}\right)}$$

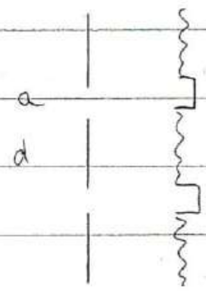
Gledamo limes $N \to \infty$

$$z = \frac{\pi a \sin\alpha}{\lambda} \qquad = N E_0 \frac{\sin z}{z}$$

$$E_0{}'$$

$$I(\alpha) = I_0 \frac{\sin^2 z}{z^2} \Rightarrow \text{ogib}$$

7. SPEKTOGRAF S REŠETKOM

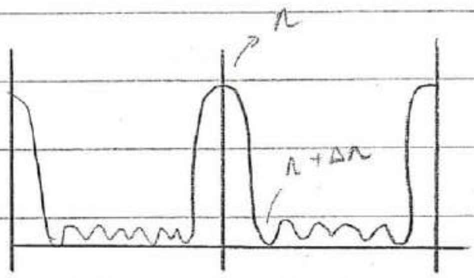


N = broj pukotina
d = konst. rešetke
a = širina rešetke

$$I(\alpha) = I_0 \cdot \frac{\sin^2\left(\pi \frac{a \sin\alpha}{\lambda}\right)}{\left(\pi \frac{a \sin\alpha}{\lambda}\right)^2} \cdot \frac{\sin^2\left(N\pi \frac{d \sin\alpha'}{\lambda}\right)}{\sin^2\left(\pi \frac{d \sin\alpha}{\lambda}\right)}$$

MAXIMUM:

$$d \sin\alpha = m\lambda$$
$$d \sin\alpha = \frac{m}{N}\lambda$$



minimum mora biti prvi nakon maximum.

MAX: $d \sin\alpha = m \cdot \lambda \;(\lambda + \Delta\lambda)$

MIN: $d \sin\alpha = \frac{m(N+1)}{N}\lambda$

moć razlučivanja rešetke

$$m(\lambda + \Delta\lambda) = \frac{mN+1}{N}\lambda$$

$$R = \frac{\lambda}{\Delta\lambda} = m \cdot N$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{mN}$$

8. POLARIZACIJA SVJETLOSTI

linearno polarizirani val

$$\underbrace{\vec{E}(t, \vec{r})}_{} = \underbrace{\vec{j}}_{\text{amp. vektor}} E_0 \cos(\omega t - \underbrace{\vec{k}}_{\text{val. vektor}}\vec{r})$$

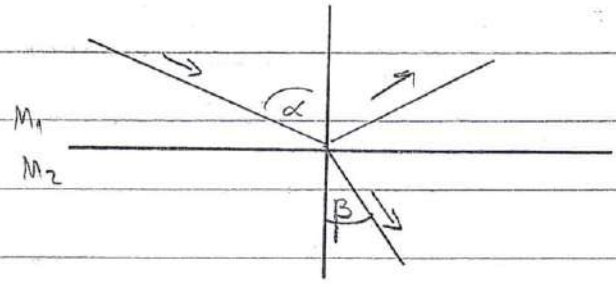
vrijeme (t), položaj ($\vec{r}$), amp. vektor, val. vektor

kružno polarizirani val

$$\vec{E}(t, \vec{r}) = \vec{j} E_0 \cos(\omega t - \vec{k}\vec{r}) + \vec{k} E_0 \cos\left(\omega t - \vec{k}\vec{r} - \frac{\pi}{2}\right)$$

Brewsterov kut

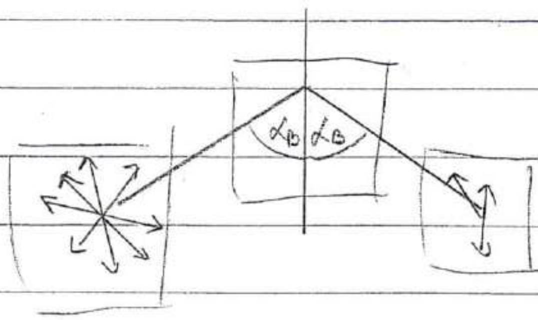
- ako je kut između lomljene i reflektirane zrake $= \frac{\pi}{2}$, zraka je polarizirana



Snellov zakon

$$\frac{\sin \alpha_B}{\sin \beta} = \frac{n_2}{n_1} \qquad \frac{\sin \beta}{\sin\left(\frac{\pi}{2} - \alpha_B\right)} = \operatorname{tg} \alpha_B \qquad \alpha_B = \operatorname{ctg} \frac{n_2}{n_1}$$

Malusov zakon



$$I' = I \cos^2 \Theta$$

$$I \rightarrow \oslash \rightarrow I'$$

lin. polar. svjetlost

MODERNA FIZIKA

1. ZRAČENJE CRNOG TIJELA

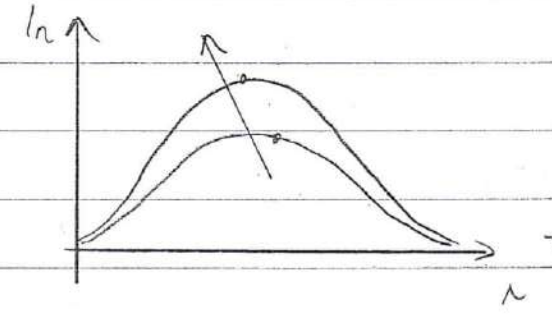
Štefan - Boltzmanov zakon (experimentalno)

- tijelo ugrijamo na T zraci : $I = \sigma T^4$

$\sigma = 5.67 \cdot 10^{-8}\ W/m^2K^4$

Raspodjela intenziteta po valnim duljinama

$$I_\lambda = \frac{dI}{d\lambda} = f(\lambda, T)$$



→ maximumi se pomiču sve manjim valnim duljinama

Planckov zakon

$$I_\lambda = f(\lambda, T) = \underbrace{\left(\frac{c}{4} \cdot \frac{8\pi}{\lambda^4} kT\right)}_{\text{Rayleigh - Jeansova formula}} \left(\frac{hc}{\lambda kT} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)$$

"ultraljubičasta katastrofa"

$$\text{planck} \rightarrow \frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

$k \rightarrow$ boltzmanova konst. $k = 1.381 \cdot 10^{-23}\ J/K$

$h \rightarrow$ Planckova konst. $h = 6.626 \cdot 10^{-34}\ Js$

$$I_\lambda = \int dI = \int_0^\infty I_\lambda \, d\lambda = \int_0^\infty f(\lambda,T)\, d\lambda = \frac{2\pi^5 k^4}{15 c^2 h^3} T^4$$

* pokazujemo da je R-J zakon, limes Planckova zakona pri $\lambda \to \infty$

Planckov faktor : $\frac{hc}{\lambda kT} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = \left\{ \frac{hc}{\lambda kT} = x \right\} = \frac{x}{e^x - 1}$

$\lim \lambda \to \infty \to$ odg. limes $\lambda \to 0$

$$\lim_{x \to 0} \frac{x}{e^x - 1} = \lim_{x \to 0} \frac{x}{(1+x)-1} = \lim \frac{x}{x} = 1$$

* oprez $\quad \lambda = \frac{c}{\nu}$

$$dI = f(\lambda, T) d\lambda$$

$$= \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} = \frac{2\pi h c^2}{\left(\frac{c}{\nu}\right)^5} \cdot \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}$$

$$\boxed{I_\nu = \frac{2\pi h \nu^3}{c^2} \cdot \frac{1}{e^{\frac{hc}{kT\lambda}} - 1}}$$

WIENOV ZAKON

$\lambda_{MAX} \cdot T = k_W \to$ Wienova konstanta 0.0029 mK

$I_\lambda = f(\lambda, T) \propto \frac{x^5}{e^x - 1} \qquad x = \frac{hc}{k\lambda T}$

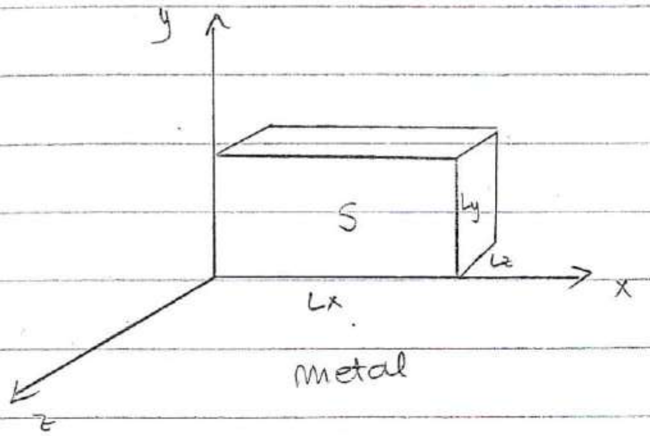
$\frac{dI_\lambda}{dx} = \frac{df}{dx} = 0 \qquad$ jer f sigurno nije 0

$\underbrace{\left(\frac{5}{x} - \frac{x}{e^x - 1} \right)}_{=0} f \qquad x = 4.965$

$$x = \frac{hc}{\lambda_{MAX} k T}$$

Izvod zakona zračenja crnog tijela

I. Spektralna gustoća elektrom. modova



EM zračenje zadovoljava rubne uvjete : $\vec{E} = 0$

$$\Rightarrow \vec{E}(\vec{r}, t) = \vec{E}_0 e^{i\omega t} \left(e^{i\vec{k}\vec{r}} + e^{-i\vec{k}\vec{r}} \right) = \vec{E}_0 e^{i\omega t} 2\cos(\vec{k}\vec{r})$$ gdje je

$\vec{k}$ valni vektor

$$\vec{k} = k_x \vec{i} + k_y \vec{j} + k_z \vec{k}$$

Zbog rubnih uvjeta: $k_x = \frac{2\pi}{\lambda_x}$, $L_x = m_x \frac{\lambda_x}{2} \Rightarrow k_x = \frac{m_x}{L_x} \pi$, $m_x = 1, 2, 3.$

analogno k_y, k_z

Razmatramo EM. val u m-prostoru $dm_x \, dm_y \, dm_z \equiv d^3 m$

k- prostoru $dk_x \, dk_y \, dk_z = d^3 k$

$$\left. \begin{aligned} dk_x &= \frac{\pi}{L_x} \\ dk_y &= \frac{\pi}{L_y} \\ dk_z &= \frac{\pi}{L_z} \end{aligned} \right\} * \rightarrow dk_x \, dk_y \, dk_z = \frac{\pi^3}{L_x L_y L_z} dm_x \, dm_y \, dm_z$$

$$d^3 k = \frac{\pi^3}{V} \cdot d^3 m$$ → gustoća dopuštenih valnih vektora u k-prostoru

Broj elektromagnetskih modova u V takvih da $|\vec{k}| < k$

$$N(k) = 2 \cdot \frac{1}{8} \cdot \left(\frac{4}{3} k^3 \pi\right) \cdot \frac{V}{\pi^3} = \frac{1}{3} k^3 \frac{V}{\pi^2}$$

2 moguća smjera polarizacije; oktant; kugla polumjera k

Broj elektromagnetskih modova tako da $|\vec{k}| \in (k, k+dk)$

$$dN = \frac{dN(k)}{dk} \cdot dk = k^2 \frac{V}{\pi^2} dk$$

Broj elektromag. modova tako da $\nu \in (\nu, \nu + d\nu)$: $\left(k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu\right)$

$$dN = \left(\frac{2\pi}{c}\nu\right)^2 \frac{V}{\pi^2} \cdot \frac{2\pi}{c} d\nu = g(\nu) d\nu$$

$$g(\nu) = \frac{8\pi^3}{c^3} \nu^2$$ $\Rightarrow$ spektralna gustoća elektrom. modova

II. Rayleigh - Jeans

Pretpostavke: - termodinamička ravnoteža
- Boltzmanova raspodjela

$$f(\epsilon) = Ce^{\frac{-\epsilon}{kT}} = \frac{1}{kT} e^{\frac{-\epsilon}{kT}}$$

Normalizacija vrijednosti: $\int_0^\infty f(\epsilon) d\epsilon = 1 \Rightarrow C = \frac{1}{kT}$

Srednja energija: $\langle \epsilon \rangle = \int_0^\infty \epsilon f(\epsilon) d\epsilon = \int_0^\infty \epsilon \frac{1}{kT} e^{\frac{-\epsilon}{kT}} d\epsilon = kT$

Spektralna gustoća energije: $U(\nu) = \langle \epsilon \rangle g(\nu) = kT \cdot \frac{8\pi^3}{c^3} \nu^2$

III. Planck

Pretpostavke: - termodinamička ravnoteža

- Boltzmanova raspodjela

- hipoteza o kvantizaciji

$$\langle \epsilon \rangle = \sum_n E_n f_n \quad \rightarrow \quad f_n = C e^{-\frac{E_n}{kT}}$$

$$\downarrow$$

$$E_n = nh\nu$$

Normalizacija vjerojatnosti

$$\sum_n f_n = 1 = C \sum_n e^{-nx} \qquad x = \frac{h\nu}{kT}$$

$$= C\left[e^{-x} + (e^{-x})^2 + \dots \right]$$

$$= C \frac{1}{1-e^{-x}}$$

$$\Rightarrow \quad C = 1 - e^{-x} = 1 - e^{-\frac{h\nu}{kT}}$$

$$\langle \epsilon \rangle = \sum_n nh\nu \left(1 - e^{-\frac{h\nu}{kT}}\right) e^{-\frac{nh\nu}{kT}}$$

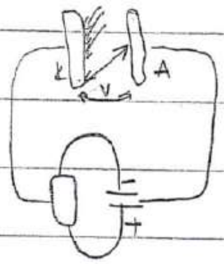
$$= x\left(1-e^{-x}\right) kT \sum_n n e^{-nx} = \dots$$

$$= x \frac{kT}{e^x - 1} = \frac{h\nu}{\cancel{kT}} \cdot \frac{\cancel{kT}}{e^{\frac{h\nu}{kT}}-1} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

Spektralna gustoća energije

$$u(\nu) = \langle \epsilon(\nu) \rangle g(\nu) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}}-1}$$

2. FOTOELEKTRIČNI EFEKT



a) struja razmjerna intenzitetu zračenja [OK]
b) ako ν ide ispod neke ν_c struja nestaje [PROBLEM]
c) max. kin. en. elektrona ne ovisi o intenzitetu [PROBLEM]

$$E_{k.MAX} = \frac{m v^2_{MAX}}{2} = e V_z \rightarrow \text{zaustavni napon}$$

objašnjenje: A. Einstein (1905)

$$E_\gamma = \hbar \omega = h \nu = h \cdot \frac{c}{\lambda}$$

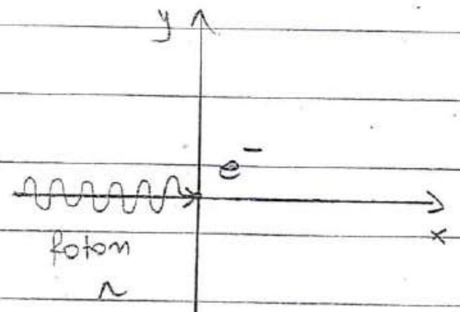
$\hbar = \frac{h}{2\pi}$; h → Planckova konst.

energija : $E_\gamma \geq W + E_{kin}$; W → izlazni rad

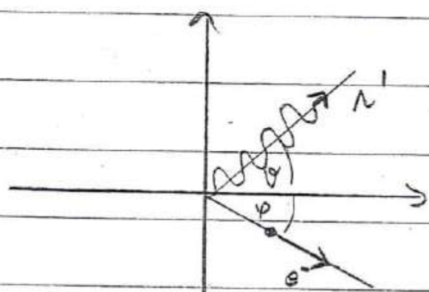
3. "COMPTONOVO" RASPRŠENJE

- foton nalijeće na česticu koja miruje, "raspršenje fotona na e^-"

prije :



poslije:



$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\vartheta)$$

$$\frac{1}{h\nu'} - \frac{1}{h\nu} = \frac{1}{m_0 c^2}(1 - \cos\vartheta)$$

Energija: $\left[E = \sqrt{(m_0c^2)^2 + (pc)^2} = \frac{m_0c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \right]$

$$E = h\nu + m_0c^2 = h\nu' + \sqrt{(m_0c^2)^2 + \left(\frac{mv \cdot c}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2} \qquad (1)$$

Impuls: $\left[\vec{p} = \frac{m \cdot \vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \quad \text{za } m>0, \quad p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \text{za } m=0 \right]$

$$p_x = \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\vartheta + \frac{m_0 \cdot v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\cos\varphi \qquad (2)$$

$$p_y = 0 = \frac{h\nu'}{c}\sin\vartheta - \frac{m_0 \cdot v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\sin\varphi \qquad (3)$$

$+$

Iz (2) i (3) eliminiramo φ:

$$\left(\frac{h\nu}{c}\right)^2 - 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c}\cos\vartheta + \left(\frac{h\nu'}{c}\right)^2 = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2 \qquad (4)$$

prepisujemo (1)

$$\left(\frac{h\nu}{c} - \frac{h\nu'}{c} + m_0c\right)^2 - (m_0c)^2 = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2$$

$$\left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu}{c}\right)^2 + 2\left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right)m_0c - 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c} = \left(\frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2 \quad (5)$$

(5) . (4)

$$2 \cdot \left(\frac{h\nu}{c} - \frac{h\nu'}{c}\right)m_0 \cdot c - 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c} + 2\frac{h\nu}{c} \cdot \frac{h\nu'}{c}\cos\vartheta = 0$$

$$\left(\frac{1}{\frac{h\nu'}{c}} - \frac{1}{\frac{h\nu}{c}}\right)m_0c = 1 - \cos\vartheta$$

uz $\lambda\nu = c$

$$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\vartheta)$$

4. BOHROV MODEL ATOMA (1913)

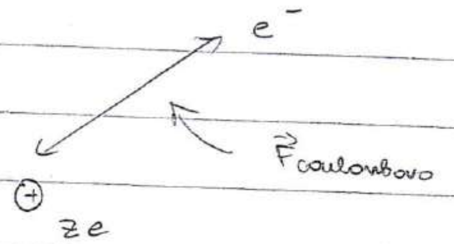
Prethodnik: * Thomson

* Rutherford

* Balmerova formula $\lambda = (364.55\,nm)\frac{n^2}{n^2-4}$ $n=3,4,5$

tj. $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n=3,4,5\ldots$ $R_H = 1.097373\cdot 10^7\, m^{-1}$

↓ Rydbergova konst.



$$P = \frac{e^2 a_{cp}}{6\pi\varepsilon_0 c^3} = \frac{dE}{dt}$$

$P_1 \Rightarrow$ I. B postulat : postoje dozvoljene staze na kojima e^- ne zrači

$P_2 \Rightarrow$ II. B postulat : kutna količina gibanja je klasična veličina kod Bohra. $L = n\hbar = n\frac{h}{2\pi} = m \cdot v_n \cdot r_n$; $n = 1,2,3\ldots$

$P_3 \Rightarrow$ III B. postulat: elektron može iz jednog dozvoljenog stanja prijeći u drugo. Pri prelasku $n_i \rightarrow n_f$; zrači se foton.; $E_{phot} = E_i - E_f = h\nu$

Jednakost sila: $F_{cp} = \frac{mv^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{ze\cdot e}{r_n^2}$

Iz P_2 $L_n = n\hbar = m v_n r_n \rightarrow v_n = \frac{n\hbar}{m r_n}$

$$\frac{m}{r_n}\left(\frac{n\hbar}{m r_n}\right)^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{ze^2}{r_n^2}$$

$$\boxed{r_n = \frac{4\pi\varepsilon_0 \hbar}{ze^2\cdot m}\cdot n^2}$$

$$v_n = \frac{ze^2}{4\pi\varepsilon_0\hbar}\cdot\frac{1}{n}$$

Ukupna energija

$$E_n = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r_n} + \frac{m}{2}v_n^2 = \ldots = \frac{-mZ^2e^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{n^2}$$

energija ionizacije osnovnog stanja

$$E_1 = -13.58\ eV$$

$$E_n = E_1\cdot\frac{1}{n_2}$$

Iz P3 $\quad h\nu = E_i - E_f = E_1\left(\frac{1}{n_f} - \frac{1}{n_i}\right)$

Balmer $n_f = 2$