1) $m = 60\,g$

$\theta(t) = \theta_0 \cos(\omega t + \varphi)$

$\theta_0 = 0{,}08$

$\omega = 4{,}43\ s^{-1}$

$E_{k_{max}} = ?$

$\Omega(t) = \dot{\theta}(t) = -\omega\theta_0 \sin(\omega t + \varphi)$

$\omega^2 = \frac{g}{l} \qquad l = \frac{g}{\omega^2}$

$v = \Omega l = \Omega \frac{g}{\omega^2} = \cancel{\omega}\theta_0 \frac{g}{\omega^{\cancel{2}}}$

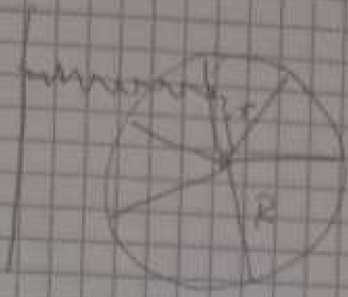
$E_{k_{max}} = \frac{m v^2}{2} = \frac{m}{2}\left(\frac{\theta_0 g}{\omega}\right)^2$ ✓

2) $M = 2{,}6\ kg$

R

$m = 0{,}1\ kg$

$r = \frac{3}{4} R$



$I \ddot{\vartheta} = - F r$

$I \ddot{\vartheta} = - k \Delta x\, r$

$\Delta x = r \vartheta$

$I \ddot{\vartheta} = - k r^2 \vartheta$

$I = M R^2 + 6 \cdot \frac{m R^2}{3}$

$I = R^2 (M + 2m)$

$R^2 (M + 2m) \ddot{\vartheta} + k r^2 \vartheta = 0$

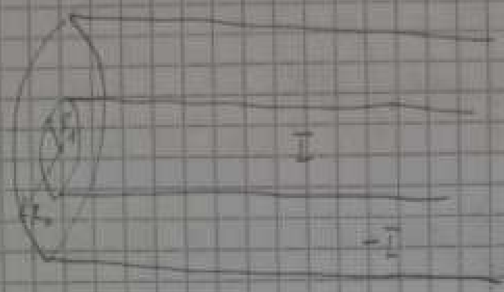
$$\ddot{\vartheta} + \frac{k r^2}{R^2 (M + 2m)} \vartheta = 0$$

$$\omega^2 = \frac{k\, 3 \cancel{R^2}}{4 \cancel{R^2} (M + 2m)} = \frac{3}{4} \frac{k}{M + 2m}$$

$$\omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{4}{3} \frac{M + 2m}{k}}$$

1.

3)



a) $r > R_2$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

$$r B(r) \int_0^{2\pi} d\varphi = \mu_0 (I - I) = 0$$

$$B(r) = 0 \qquad r > R_2$$

b) $R_1 < r < R_2$

$$\oint \vec{B}\, d\vec{s} = \mu_0 I$$

$$2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad R_1 < r < R_2$$

c) $r < R_1$

$$j = \frac{dI}{dA}$$

$$\oint \vec{B}\, d\vec{s} = \mu_0 I$$

$$2\pi r B = \mu_0 \iint \vec{j}\, d\vec{A}$$

$$2\pi r B = \mu_0 j \int_0^r r\,dr \int_0^{2\pi} d\varphi$$

$$2\pi \cancel{r} B = \mu_0 j \frac{r^{\cancel{2}}}{2} 2\pi \qquad j = \frac{I}{R_1^2 \pi}$$

$$2\cancel{\pi} B = \mu_0 \frac{I}{R_1^2 \pi} r \cancel{\pi}$$

$$B = \frac{\mu_0 I r}{2\pi R_1^2} \qquad r < R_1$$

7.1) $\vec{v} = v_0 \hat{x}$

$v_0 = 7{,}5 \cdot 10^6 \; m/s$

$\vec{B} = (B_x, B_y, B_z) = (14{,}5,\ 12{,}7,\ 5{,}5) \cdot 10^{-4}\, T$

$$\vec{F} = -e\, \vec{v} \times \vec{B} =$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_0 & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix} =$$

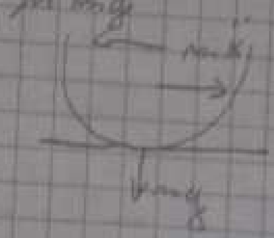
$$= \hat{x}(0) - \hat{y}(v_0 B_z - 0) + \hat{z}(v_0 B_y - 0)$$

$$\vec{F} = -e\, v_0 (B_y \hat{z} - B_z \hat{y})$$

$$|F| = \sqrt{\vec{F} \cdot \vec{F}} = \sqrt{e^2 v_0^2 (B_y^2 + B_z^2)} = e\, v_0 \sqrt{B_y^2 + B_z^2}$$

5) $A = 0{,}5\ m$

$\mu = 0{,}42$



$m\ddot{x} \leq \mu m g$

$x(t) = A \sin(\omega t + \varphi)$

$\dot{x}(t) = A\omega \cos(\omega t + \varphi)$

$\ddot{x}(t) = -A\omega^2 \sin(\omega t + \varphi)$

$a_{MAX} = A\omega^2$

$m\, a_{MAX} = \mu m g$

$\cancel{m} A\omega^2 = \mu \cancel{m} g$

$\omega^2 = \frac{\mu g}{A}$

$\nu = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$

6)

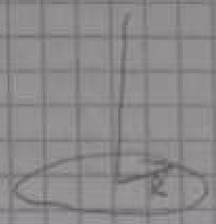
$m = 200\,g$

$R = 20\,cm$

$T_D = 2{,}2\,s$

$T_S = 3{,}8\,s$

$I = 0$



$$T_D = 2\pi\sqrt{\frac{I_D}{D}}$$

$$\left(\frac{T_D}{2\pi}\right)^2 = \frac{I_D}{D} \Rightarrow D = I_D \left(\frac{2\pi}{T_D}\right)^2$$

$$T_S = 2\pi\sqrt{\frac{I_D + I}{D}}$$

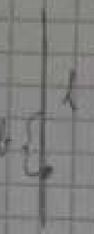
$$\left(\frac{T_S}{2\pi}\right)^2 = \frac{I_D + I}{D}$$

$$I_D + I = \left(\frac{T_S}{2\pi}\right)^2 \cdot I_D \left(\frac{2\pi}{T_D}\right)^2$$

$$I = I_D \left(\frac{T_S}{T_D}\right)^2 - I_D \qquad I_D = \frac{m}{2} R^2$$

$$I = \frac{m}{2} R^2 \left(\left(\frac{T_S}{T_D}\right)^2 - 1\right)$$

7.)



$T_{min} = ?$

$b = ?$

$$T = 2\pi \sqrt{\frac{I}{mgb}} \qquad I_{Steiner} = \frac{m\,l^2}{12} + m\,b^2$$

$$\frac{T}{2\pi} = \sqrt{\frac{\frac{\cancel{m} l^2}{12} + \cancel{m} b^2}{\cancel{m} g b}}$$

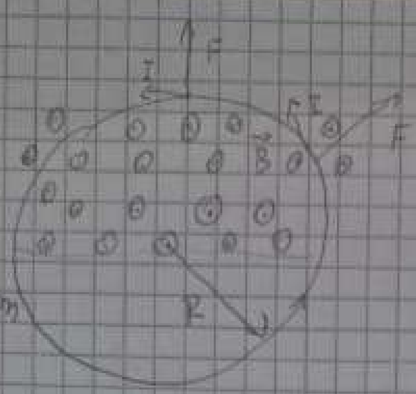
$$\frac{T}{2\pi} = \frac{1}{\sqrt{g}} \sqrt{\frac{l^2}{12 b} + b}$$

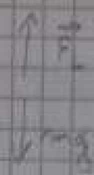
$$\left(\frac{T}{2\pi} g\right)^2 = \frac{l^2}{12 b} + b$$

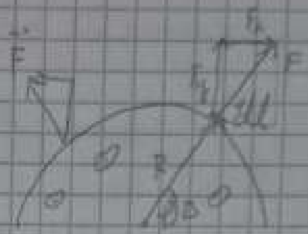
$$\frac{d\left(\frac{Tg}{2\pi}\right)^2}{db} = 0 = -\frac{l^2}{12 b^2} + 1$$

$$b = \frac{l}{\sqrt{12}}$$

8)







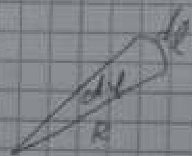
$\vec{F} = I\vec{L}\times\vec{B}$

$F_x = 0$

$dF = I\,dl\,B$

$F_y = F\sin\varphi$

$dF_y = I\,dl\,B\sin\varphi$



$F_y = IB\int dl\sin\varphi$ $\qquad dl = R\,d\varphi$

$$F_y = IBR\int_0^\pi \sin\varphi\,d\varphi$$

$$F_y = IBR(-\cos\varphi)\Big|_0^\pi = 2IBR$$

$mg = 2IBR$

$$I = \frac{mg}{2BR}$$

9.) $A_1 = 44\,mm$

φ $\qquad \omega_1 = \omega_2 = \omega$

$A_2 = 36\,mm$

$$s_1 = A_1 \sin(\omega t - kx)$$

$$s_2 = A_1 \sin(\omega t - kx + \varphi)$$

$$s_1 + s_2 = A_1 \left(\sin(\omega t - kx) + \sin(\omega t - kx + \varphi)\right)$$

$$s_1 + s_2 = A_1 \left(2 \sin\left(\omega t - kx + \frac{\varphi}{2}\right) \cos \frac{\varphi}{2}\right)$$

$$s_1 + s_2 = \underbrace{2A_1 \cos \frac{\varphi}{2}}_{A_2} \sin\left(\omega t - kx + \frac{\varphi}{2}\right)$$

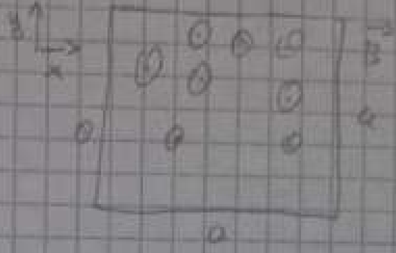
$$2A_1 \cos \frac{\varphi}{2} = A_2$$

$$\cos \frac{\varphi}{2} = \frac{A_2}{2A_1}$$

10.)

$B = A\,y\,t^2$

$a = 1{,}20\ \mathrm{m}$



$$\mathcal{E} = -\frac{d}{dt}\iint \vec{B}\,d\vec{A} = -\frac{d}{dt}\int_0^a A\,y\,t^2\,dy\int_0^a dx$$

$$\mathcal{E} = -\frac{d}{dt}\left(\frac{a^2}{2}A\,t^2\cdot a\right)$$

$$\mathcal{E} = -A\,a^3\,t$$

11.) $y(x,t) = A\,e^{(ax-bt)}$

$A = 0{,}5\ cm$

$a = 3\ cm^{-1} = 300\ m^{-1}$

$b = 4\ s^{-1}$

$y(x,t) = A\,e^{a(x-\frac{b}{a}t)}$

$v = \frac{b}{a} = 0{,}013\dot{3}\ m/s$

12)

$d = 1\,mm$

$L = 2\,m$

$\nu = 200\,Hz$

$A = 2\,cm$

$\rho = 7800\,kg/m^3$

$$y(x,t) = A \sin \omega t \sin \frac{\pi x}{L}$$

$$\dot{y}(x,t) = A\omega \cos \omega t \sin \frac{\pi x}{L}$$

$$\dot{y}_M(x,t) = A\omega \sin \frac{\pi x}{L}$$

$$dm = \lambda\, dx$$

$$dE_k = \frac{dm\, v^2}{2} = \frac{\lambda}{2} A^2\omega^2 \sin^2 \frac{\pi x}{L}\, dx$$

$$E_k = \frac{\lambda}{2} A^2\omega^2 \int_0^L \sin^2 \frac{\pi x}{L}\, dx = \frac{\lambda}{2} A^2\omega^2 \frac{1}{2} L$$

$$\int_0^m dm = \int_0^L dx\, \lambda \qquad m = L\lambda \qquad \lambda = \frac{m}{L}$$

$$E_k = \frac{m}{\cancel{L}} \frac{A^2\omega^2}{4} \cancel{L} \qquad m = \rho V$$

$$m = \rho \left(\frac{d}{2}\right)^2 \pi L$$

$$E_k = \rho \pi L \left(\frac{A\omega d}{4}\right)^2$$

13.)

$R = 3\,cm$

$j = a\,r$

$r < R$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{ob}$$

$dA = r\,dr\,d\varphi$

$ds = r\,d\varphi$

$$B\,2\pi r = \mu_0 \iint \vec{j} \cdot d\vec{A} = \mu_0 \int_0^r a\,r^2\,dr \int_0^{2\pi} d\varphi$$

$$B\,\cancel{2\pi}\,\cancel{r} = \mu_0\,a\,\frac{r^{\cancel{3}\,2}}{3} \cdot \cancel{2\pi}$$

$$B = \mu_0\,a\,\frac{r^2}{3}$$