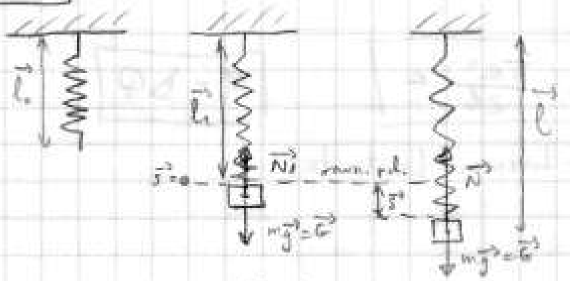
OPRUGA



$\vec{N_1} = -k(\vec{l_1} - \vec{l_0})$

$\vec{F} = \vec{G} + \vec{N_1} = 0$

$\vec{G} = -\vec{N_1}$

$\vec{F} = \vec{N} + \vec{G}$

$\vec{F} = -k(\vec{l} - \vec{l_1}) + k(\vec{l_1} - \vec{l_0})$

$\vec{F} = -k(\vec{l} - \vec{l_1})$

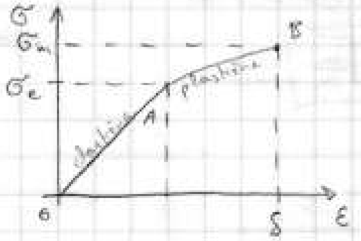
$\boxed{\vec{F} = -k\vec{s}}$

1 ELASTIČNOST MATERIJALA

naprezanje $\boxed{\sigma = \frac{F}{S}}$ $[N/m^2 = Pa]$ $\sigma = \frac{dF}{dS}$ = tenzor

- okomito $\sigma_n = \lim_{\Delta S \to 0} \frac{\Delta F \cos\varphi}{\Delta S}$ vlačno / tlačno
- tangencijalno $\tau = \lim_{\Delta S \to 0} \frac{\Delta F \sin\varphi}{\Delta S}$

relativna deformacija $\varepsilon = \lim_{x \to 0} \frac{\Delta x}{x}$ $\varepsilon = \frac{\Delta l}{l}$



Ⓐ – granica linearnosti

σ_e – granica elastičnosti [nakon njega ostaje rezidualna deformacija]

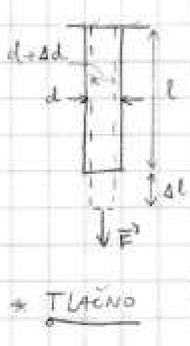
σ_m – granično naprezanje → kidanje

δ – prekidno rastezanje

1.1 HOOKEOV ZAKON
(linearna deformacija)
[do A]

$\frac{F}{S} = E\frac{\Delta l}{l}$ $\boxed{\sigma = E\varepsilon}$ * VLAČNO

└ Youngov modul elastičnosti



$\boxed{\frac{\Delta d}{d} = -\mu\frac{\sigma}{E}}$ $\mu = -\frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}$ $[0,2 - 0,5]$

└ Poissonov broj

* TLAČNO $\boxed{p = -\sigma = -B\frac{\Delta V}{V}}$ $\boxed{K = \frac{1}{B} = -\frac{1}{V}\frac{dV}{dp}}$

└ volumni modul elastičnosti └ stlačivost

$\Rightarrow \boxed{F = \frac{ES}{l}\Delta l = k \cdot \Delta l}$

$\vec{F_{el}} = -\vec{F} = -k \cdot \Delta\vec{l}$ $\boxed{\vec{F_{el}} = -k\Delta\vec{l}}$ elast. / harm. sila / opruge

1.2 SMICANJE I TORZIJA – F ∥ S

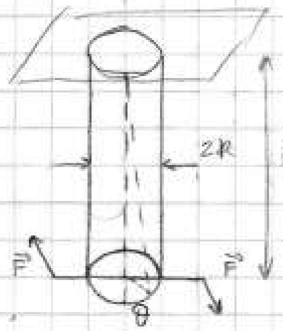
$\boxed{\tau = \frac{F}{S}}$ naprezanje smicanja $G = [N/m^2]$

$\boxed{\varepsilon = tg\,\beta}$ kutna deformacija $\beta \approx \boxed{\varepsilon = \frac{1}{G}\tau}$ $\boxed{\tau = G\varepsilon}$

(mali kutevi) └ modul smicanja └ Coulombov modul

$G = \frac{E}{2(1+\mu)}$

TORZIJA ŠTAPA



- par sila

$$\theta = \frac{1}{G}\frac{2l}{\pi R^4} M_p \qquad \boxed{D = \frac{\pi R^4}{2l} G} \qquad \boxed{M_p = D\theta}$$

torziona konstanta

2 JEDNOSTAVNO HARMONIČKO TITRANJE

2.1 HARM. OSCILATOR

- opruga – nepriguženo za mali $\vec{s}$, mT

$\vec{F} = -k\vec{s}$ $\qquad F = ma \qquad m\frac{d^2s}{dt^2} = -ks$

(1) $\boxed{\frac{d^2s}{dt^2} + \frac{k}{m}s = 0}$ (LDJ 2. r.)

$\Rightarrow s(t) = a\sin\omega t + b\cos\omega t \qquad a = A\cos\varphi_0, \quad b = A\sin\varphi_0$

$s(t) = A\cos\varphi_0 \sin\omega t + A\sin\varphi_0\cos\omega t$

(2) $\boxed{s(t) = A\sin(\omega t + \varphi_0)} \rightarrow (1) \Rightarrow -A\omega^2\cancel{\sin(\omega t+\varphi_0)} + \frac{k}{m}A\cancel{\sin(\omega t + \varphi_0)} = 0$

$\boxed{\omega = \sqrt{\frac{k}{m}}}$

$\boxed{s(t) = A\sin\left(\sqrt{\frac{k}{m}}t + \varphi_0\right)}$

$\sqrt{\frac{k}{m}}(t+T) = \sqrt{\frac{k}{m}}t + 2\pi$

$T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{T} \qquad \boxed{\omega = \frac{2\pi}{T} = 2\pi f}$ kružna frekvencija $[s^{-1}]$

$v(t) = \frac{ds}{dt} = A\omega\cos(\omega t + \varphi_0)$ (3)

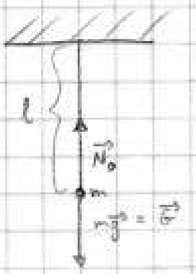
$a(t) = \frac{d^2s}{dt^2} = -A\omega^2\sin(\omega t + \varphi_0) = \underline{-\omega^2 s(t)}$ (4)

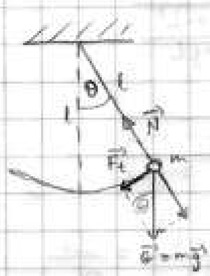
- $t = 0$, $s(0) = A\sin\varphi_0$, $v(0) = A\omega\cos\varphi_0$ $\quad |^2 \; |+$

$\boxed{A^2 = s^2(0) + \frac{v^2(0)}{\omega^2}} \qquad \boxed{\mathrm{tg}\,\varphi_0 = \frac{\omega s(0)}{v(0)}}$

* serija opruga ($\sim C$) $\qquad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$

(2.2) MATEMATIČKO NJIHALO





$N = mg\cos\theta$

$F_t = -mg\sin\theta$ — smjer suprotan povećanju kuta

$\theta \ll \;\rightarrow\; \sin\theta \approx \theta \;\Rightarrow\; \boxed{F_t = -mg\theta}$

harmonička sila

$m a_t = F_t = -mg\sin\theta \qquad a_t = l\alpha = l\frac{d^2\theta}{dt^2}$

$m l \frac{d^2\theta}{dt^2} = -mg\theta$

$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0} \quad (1) \quad \Rightarrow \quad \boxed{\theta = \theta_0 \sin(\omega t + \varphi_0)} \quad (2)$

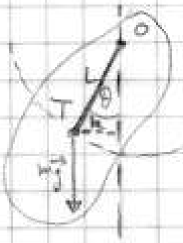
θ_0 – amplitude

$\boxed{\omega = \sqrt{\frac{g}{l}}}$

$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

(2.3) FIZIČKO NJIHALO

– kruto tijelo koje se zbog utjecaja sile teže njiše oko hor. osi koja ne prolazi težištem



$\boxed{M = -mgL\sin\theta} \qquad [\,M = -hF,\; h = L\sin\theta\,]$

$\sin\theta \approx \theta \qquad \boxed{M = -mgL\theta}$

$M = I\alpha = -mgL\theta$

(I) – moment tromosti s obzirom na O

$I\frac{d^2\theta}{dt^2} + mgL\theta = 0$

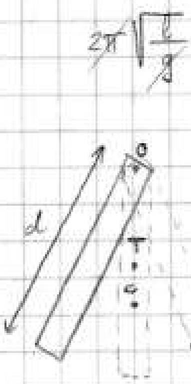
$\boxed{\frac{d^2\theta}{dt^2} + \frac{mgL}{I}\theta = 0} \quad (1)$

$\boxed{\theta = \theta_0 \sin(\omega t + \varphi_0)} \quad (2) \qquad \boxed{\omega = \sqrt{\frac{mgL}{I}}} \qquad \boxed{T = 2\pi\sqrt{\frac{I}{mgL}}}$

* (mat. ~ fizičko)

$T_m = T_f$

$2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{I}{mgL}} \qquad \boxed{l_r = \frac{I}{mL}}$ reducirana duljina fiz. njihala



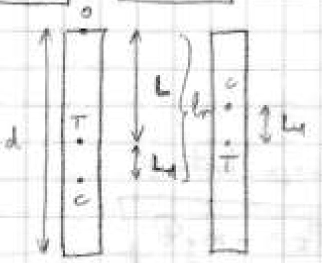
$I_T = \frac{md^2}{12}$

$\boxed{I_O = \frac{md^2}{12} + m\left(\frac{d}{2}\right)^2 = \frac{md^2}{12} + \frac{md^2}{4} = \frac{md^2}{3}}$

$\boxed{l_r = \frac{\frac{\cancel{m}d^{\cancel{2}}}{3}}{\cancel{m}\frac{\cancel{d}}{2}} = \frac{2}{3}d}$

(C) – središte titranja

dokaz | $\boxed{O - C}$



$\boxed{O}$ $T_o = 2\pi\sqrt{\dfrac{I_o}{mgL}} = 2\pi\sqrt{\dfrac{I_T + mL^2}{mgL}}$

$\boxed{C}$ $T_c = 2\pi\sqrt{\dfrac{I_c}{mgL_1}} = 2\pi\sqrt{\dfrac{I_T + mL_1^2}{mgL_1}}$

$$L_1 = l_r - L = \frac{I_o}{mL} - L = \frac{I_o - mL^2}{mL} = \frac{I_T}{mL}$$

$$T_c = 2\pi\sqrt{\dfrac{I_T + \dfrac{mI_T^2}{m^2L^2}}{mg\dfrac{I_T}{mL}}} = 2\pi\sqrt{\dfrac{\dfrac{\cancel{I_T}\, m^2L^2 + \cancel{m}I_T^2}{m^2L^2}}{\dfrac{\cancel{mg}\,\cancel{I_T}}{\cancel{mL}}}} = 2\pi\sqrt{\frac{I_T + mL^2}{mgL}}$$

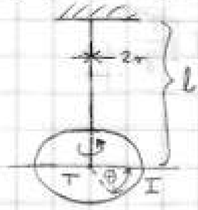
$\Rightarrow$ REVERZIONO NJIHALO

- udarac u C – rotacija
- udarac u T – translacija
- udarac u $\setminus\{T, C\}$ – trans + rot

$$\boxed{T_c = T_o}$$

2.4 TORZIONO TITRANJE

– momentom vanjskog para sila zakrenemo tijelo za kut Θ iz položaja ravnoteže, pa naglo ukloniмо M_p



$M_p = -M_{\check{z}}$

$$\boxed{M_{\check{z}} = -D\Theta}$$

$$M = I\alpha = -D\Theta$$

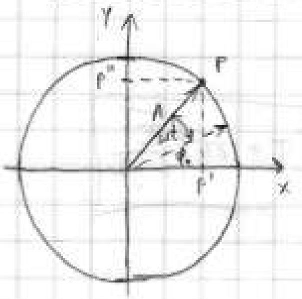
$$I\frac{d^2\Theta}{dt^2} + D\Theta = 0$$

$$\boxed{\frac{d^2\Theta}{dt^2} + \frac{D}{I}\Theta = 0} \quad (1)$$

$$\boxed{\Theta = \Theta_o \sin(\omega t + \varphi_o)} \quad (2)$$

$$\boxed{\omega = \sqrt{\frac{I}{D}}} \quad \boxed{T = 2\pi\sqrt{\frac{D}{I}}}$$

2.5 HARM. TIT. POMOĆU ROTIRAJUĆEG VEKTORA



$\varphi = \varphi_o + \omega t$ (ω = konst kutna brzina)

P': $x = A\cos(\omega t + \varphi_o)$

P'': $y = A\sin(\omega t + \varphi_o)$

$\vec{OP}$ – rotvektor, fazor

2.6 ENERGIJA TITRANJA

$E_k \leftrightarrow E_p$

$$s = A\sin(\omega t + \varphi_o)$$

$$v = \frac{ds}{dt} = A\omega\cos(\omega t + \varphi_o)$$

- $E_k = \dfrac{mv^2}{2} = \dfrac{1}{2}\underline{m}A^2\underline{\omega^2}\cos^2(\omega t + \varphi_o) = \boxed{\dfrac{1}{2}kA^2\cos^2(\omega t + \varphi_o)}$

$$\boxed{E_k = \frac{1}{2}k(A^2 - s^2)}$$

- $F = -ks$

$$\boxed{E_p = -W = -\int_0^s(-ks)\,ds = \frac{1}{2}ks^2} \qquad \boxed{E_p = \frac{1}{2}kA^2\sin^2(\omega t + \varphi_o)}$$

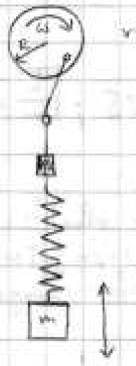
- $E = E_k + E_p = \dfrac{1}{2}kA^2\left[\cos^2(\omega t + \varphi_o) + \sin^2(\omega t + \varphi_o)\right] = \boxed{\dfrac{1}{2}kA^2}$

$$\boxed{E = E_k + E_p = \text{konst.}}$$

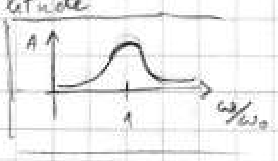
④ $\boxed{\text{PRISILNO TITRANJE, REZONANCIJA}}$

- vanjska F djeluje na titrajni sistem te se tako nadoknađuje E izgubljena zbog trenja

$$\boxed{F_v = F_0 \sin \omega t}$$



- $\omega < \omega_0$ – male amplitude
- $\boxed{\omega = \omega_0}$ – rezonancija, max amplitude
- $\omega > \omega_0$ – smanjenje amplituda



$\boxed{\text{3 sile}}$ harmonička $F = -ks$

sila trenja $F_{tr} = -b \frac{ds}{dt}$

vanjska periodična $F_v = F_0 \sin \omega t$

$$m \frac{d^2 s}{dt^2} = -ks - b\frac{ds}{dt} + F_0 \sin \omega t$$

$$\boxed{\frac{d^2 s}{dt^2} + 2\delta \frac{ds}{dt} + \omega_0^2 s = A_0 \sin \omega t} \quad (1) \qquad \left(A_0 = \frac{F_0}{m}\right) \left(\delta = \frac{b}{2m}\right) \left(\omega_0^2 = \frac{k}{m}\right)$$

$$\boxed{s(t) = A(\omega) \sin(\omega t - \varphi)} \quad (2)$$

$A(\omega)$ – modulirana amplituda

φ – kašnjenje u fazi iza vanjskog osc.

$$\frac{ds}{dt} = A(\omega)\, \omega \cos(\omega t - \varphi)$$

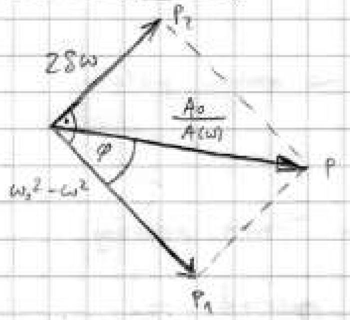
$$\frac{d^2 s}{dt^2} = -A(\omega)\, \omega^2 \sin(\omega t - \varphi) = -\omega^2 s(t)$$

$$\Rightarrow \quad -A(\omega)\, \omega^2 \sin(\omega t - \varphi) + 2\delta A(\omega)\, \omega \cos(\omega t - \varphi) + A(\omega)\, \omega_0^2 \sin(\omega t - \varphi) = A_0 \sin \omega t$$

$$(\omega_0^2 - \omega^2) \sin(\omega t - \varphi) + 2\delta \omega \underbrace{\cos(\omega t - \varphi)}_{\sin(\omega t - \varphi + \frac{\pi}{2})} = \frac{A_0}{A(\omega)} \sin \omega t$$

- 2 međusobno okomita titranja amplituda $(\omega_0^2 - \omega^2)$ i $2\delta\omega$

$\Rightarrow$ fazorski prikaz



$$\left[\frac{A_0}{A(\omega)}\right]^2 = 4\delta^2\omega^2 + (\omega_0^2 - \omega^2)^2$$

$$\left[A(\omega) = \frac{A_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}} = \frac{A_0}{\omega_0^2 \sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\frac{\delta^2 \omega^2}{\omega_0^2}}} \right]$$

$$\left[\operatorname{tg} \varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2} = \frac{2\delta\omega / \omega_0^2}{1 - \omega^2/\omega_0^2} \right]$$

$A(\omega)$ – $\boxed{\max}$ pri rezonantnoj frekvenciji ω_r $\quad \left(\frac{dA(\omega)}{d\omega} = 0\right)$

$$\frac{d}{d\omega}\left[(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2\right]^{-\frac{1}{2}} = -\frac{1}{2}\left[(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2\right] \cdot \underbrace{2(\omega_0^2 - \omega^2)(-2\omega) + 4\delta^2 \cdot 2\omega}_{=0} = 0$$

$$-\cancel{4\omega}(\omega_0^2 - \omega^2) + \overset{2}{\cancel{8}}\delta^2 \cancel{\omega} = 0$$

$$\omega^2 = \omega_0^2 - 2\delta^2 \qquad \boxed{\omega_r = \sqrt{\omega_0^2 - 2\delta^2}} \text{ rezonantna frekv. – max } A(\omega)$$

$[\text{bez } F_{tr} \rightarrow \omega_r = \omega_0]$

6

③ PRIGUŠENO TITRANJE

$$\frac{dE}{dt} < 0$$

$$\boxed{\vec{F}_{tr} = -b\vec{v} = -b\frac{d\vec{s}}{dt}}$$ (b) – konstanta trenja (>0)

$$m\vec{a} = \vec{F}_{opr} + \vec{F}_{tr}$$

$$m\frac{d^2s}{dt^2} = -ks - b\frac{ds}{dt}$$

$$\boxed{\frac{d^2s}{dt^2} + \frac{b}{m}\frac{ds}{dt} + \frac{k}{m}s = 0}$$

$$\frac{b}{m} = 2\delta \; ; \; \sqrt{\frac{k}{m}} = \omega_0$$

$$\boxed{\frac{d^2s}{dt^2} + 2\delta\frac{ds}{dt} + \omega_0^2 s = 0} \quad (1)$$

(ω_0) – vlastita frekv. nepriguśenog tit.

(δ) – faktor prigušenja

$$\boxed{s(t) = a(t)\sin(\omega t + \varphi_0) = Ae^{-\delta t}\sin(\omega t + \varphi_0)} \quad (2)$$

$$\frac{ds}{dt} = -A\delta e^{-\delta t}\sin(\omega t + \varphi_0) + Ae^{-\delta t}\omega\cos(\omega t + \varphi_0)$$

$$\frac{d^2s}{dt^2} = A\delta^2 e^{-\delta t}\sin(\omega t + \varphi_0) - A\delta e^{-\delta t}\omega\cos(\omega t + \varphi_0) - A\delta e^{-\delta t}\omega\cos(\omega t + \varphi_0) - Ae^{-\delta t}\omega^2\sin(\omega t + \varphi_0)$$

$$A\delta^2 e^{-\delta t}\sin(\omega t + \varphi_0) - \cancel{A\omega\delta e^{-\delta t}\cos(\omega t + \varphi_0)} - \cancel{A\omega\delta e^{-\delta t}\cos(\omega t + \varphi_0)} - A\omega^2 e^{-\delta t}\sin(\omega t + \varphi_0) -$$
$$- 2A\delta^2 e^{-\delta t}\sin(\omega t + \varphi_0) + \cancel{2A\omega\delta e^{-\delta t}\cos(\omega t + \varphi_0)} + \omega_0^2 Ae^{-\delta t}\sin(\omega t + \varphi_0) = 0$$

$$\underbrace{(A\delta^2 - A\omega^2 - 2A\delta^2 + A\omega_0^2)}_{=0} e^{-\delta t}\sin(\omega t + \varphi_0) = 0$$

$$\omega^2 = \omega_0^2 - \delta^2 \qquad \boxed{\omega = \sqrt{\omega_0^2 - \delta^2}}$$ pulzacija prigušenih titranja

– amplituda $\boxed{a(t) = Ae^{-\delta t}}$

- $\boxed{\text{Q – faktor}}$ (faktor dobrote) – omjer dviju susjednih amplituda koje se razlikuju za period T je konstantan i iznosi:

$$\frac{a(t)}{a(t+T)} = \frac{A\cancel{e^{-\delta t}}}{A\cancel{e^{-\delta t}}e^{-\delta T}} = \boxed{e^{\delta T}}$$

- logaritamski dekrement titranja $\quad \lambda = \ln\left[\frac{a(t)}{a(t+T)}\right] = \delta T$

$$[b = 2m\delta]$$

$$\boxed{Q = \frac{\pi}{\lambda} = \frac{\pi}{\delta T} = \frac{\omega}{2\delta}}$$ faktor dobrote

– neprigušeni oscilator $\quad (\delta = 0) \quad Q \to \infty$

$\boxed{\text{APERIODIČNO TITRANJE}}$ – trenje preveliko, umjesto titranja → ravn. položaj

$$\delta^2 > \omega_0^2 \quad \to \quad \omega \text{ – imaginarna}$$

– što je $\frac{\delta}{\omega_0}$ veće, to se tijelo sporije vraća u ravn. položaj

– $\boxed{\delta = \omega_0}$ kritično gušenje (granica prigušenog i aperiodičnog)
 – najbrži povratak u R.P.

BRZINA PRISILNOG OSCILATORA

$$v(t) = \frac{d}{dt}\left[A(\omega)\sin(\omega t - \varphi)\right]$$

kašnjenje u fazi

$$\boxed{\operatorname{tg}\varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2}}$$

- $\omega \ll \omega_0 \rightarrow \operatorname{tg}\varphi = 0 \rightarrow \boxed{\varphi = 0}$ titranje u fazi
- $\omega = \omega_0 \rightarrow \operatorname{tg}\varphi = \infty \rightarrow \boxed{\varphi = \frac{\pi}{2}}$ sustav kasni za vanjskim osc. za $\frac{T}{4}$
- $\omega \gg \omega_0 \rightarrow \boxed{\varphi = \pi}$

JEDN. GIBANJA PRISILNOG OSC

$$\ddot{s} + 2\delta\dot{s} + \omega_0^2 s = A_0 \sin\omega t \qquad (\text{nehomogena})$$

$\rightarrow$ opće (hom) $\quad s_1 = s(t) = A e^{-\delta t}\sin(\omega_p t + \varphi_0) \quad // \quad \omega_p = \sqrt{\omega_0^2 - \delta^2}$ (~ prigušeno)

$\rightarrow$ posebno (nehom) $\quad s_2 = s(t) = A(\omega)\sin(\omega t - \varphi)$

$$s(t) = s_1(t) + s_2(t)$$

$$\boxed{s(t) = A_1 e^{-\delta t}\sin(\omega_p t + \varphi_0) + A(\omega)\sin(\omega t - \varphi)}$$

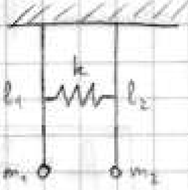
- sistem počne titrati vlastitom frekv. ($\omega_p = \omega_0$) te nastoji slijediti titranje vanjskog osc. $\rightarrow$ superpozicija 2 titranja
- vlastito titranje zbog prigušenja gotovo trne $\rightarrow$ sistem titra frekv. ω (vanjski osc)

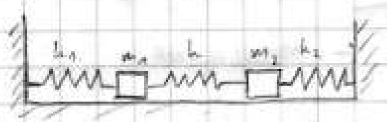
$\Rightarrow$ STACIONARNO RJEŠENJE J.P.O. $\quad s_2(t) = A(\omega)\sin(\omega t - \varphi)$

5 VEZANI OSCILATORI

- Oberbeckovo njihalo

$(m_1 = m_2,\ l_1 = l_2)$





$0 < t < T/2$ — 1. polutitraj drugog, 1. strane
$T/2 < t < T$ — 2. polutitraj prvog, 2. strane

SVAKI OSCILATOR

$$m_1 \frac{ds_1^2}{dt^2} = -k_1 s_1 + k(s_2 - s_1) \qquad (1.1)$$

$$m_2 \frac{ds_2^2}{dt^2} = -k_2 s_2 - k(s_2 - s_1) \qquad (1.2)$$

- male amplitude $\rightarrow$ vrijedi i za O. Nj.

$$\boxed{k_1 = \frac{m_1 g}{l_1}} \quad \boxed{k_2 = \frac{m_2 g}{l_2}}$$

$$[m_1 = m_2,\ l_1 = l_2 \Rightarrow k_1 = k_2]$$

rješenja: $\quad s_1(t) = A\sin(\omega_1 t + \varphi_{01})$

$$s_2(t) = B\sin(\omega_2 t + \varphi_{02}),$$

$$\frac{ds_1}{dt} = A\omega_1\cos(\omega_1 t + \varphi_{01}) \qquad \frac{ds_2}{dt} = B\omega_2\cos(\omega t + \varphi_{02})$$

$$\frac{d^2 s_1}{dt^2} = -A\omega_1^2\sin(\omega_1 t + \varphi_{01})$$

$$\frac{d^2 s_2}{dt^2} = -B\omega_2^2\sin(\omega_2 t + \varphi_{02})$$

$$-m_1 A \omega_1^2 \sin(\omega_1 t + \varphi_{01}) = -k_1 A \sin(\omega_1 t + \varphi_{01}) + k\left[B \sin(\omega_2 t + \varphi_{02}) - A \sin(\omega_1 t + \varphi_{01})\right]$$

$$-m_2 B \omega_2^2 \sin(\omega_2 t + \varphi_{02}) = -k_2 B \sin(\omega_2 t + \varphi_{02}) + k\left[B \sin(\omega_2 t + \varphi_{02}) - A \sin(\omega_1 t + \varphi_{02})\right]$$

odnos s_1 i s_2 – faza ili protufaza [fazori s_1, $\frac{d s_1^2}{dt}$ leže na istom pravcu → f ili protuf]

Z rj: a) $A = B = A_1$

$$\omega_1 = \omega_0 = \sqrt{\frac{k_1}{m_1}}$$

$$s_1(t) = A_1 \sin(\omega_1 t + \varphi_{01})$$
$$s_2(t) = A_1 \sin(\omega_1 t + \varphi_{01})$$

$\Rightarrow$ u fazi (kao nepovezani)

b) $A = -B = A_2$

$$\omega_2 = \sqrt{\frac{k_1}{m_1} + \frac{2k}{m_1}} = \sqrt{\omega_0^2 + \frac{2k}{m_1}}$$

$$s_1(t) = A_2 \sin(\omega_2 t + \varphi_{02})$$
$$s_2(t) = -A_2 \sin(\omega_2 t + \varphi_{02})$$

$\Rightarrow$ u protufazi, ω malo veća nego kad su nepovezani

OPĆENITO RJ

$$s_1(t) = A_1 \sin(\omega_1 t + \varphi_{01}) + A_2 \sin(\omega_2 t + \varphi_{02})$$
$$s_2(t) = A_1 \sin(\omega_1 t + \varphi_{01}) - A_2 \sin(\omega_2 t + \varphi_{02})$$

⊛ jednake amplitude $\Rightarrow$ $(A_1 = A_2)$

$$s_1(t) = 2A \cos\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right) \sin\left(\frac{\omega_1 + \omega_2}{2} t + \frac{\varphi_{01} + \varphi_{02}}{2}\right)$$
$$s_2(t) = 2A \sin\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right) \cos\left(\frac{\omega_1 + \omega_2}{2} t + \frac{\varphi_{01} + \varphi_{02}}{2}\right)$$

→ frekvencija titranja $\frac{f_1 + f_2}{2}$

→ amplituda → $[0, 2A]$ varira u vremenu frekvencijom $f_A = \frac{f_1 - f_2}{2}$

– MODULIRANA

• nije harmoničko titranje – UDARI $\boxed{f_u = \frac{\omega_1 - \omega_2}{2\pi} = f_1 - f_2}$ $(f_u = 2 f_A)$

1. osc. $2A \cos\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right)$

2. osc. $2A \sin\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2}\right) = 2A \cos\left(\frac{\omega_1 - \omega_2}{2} t + \frac{\varphi_{01} - \varphi_{02}}{2} - \frac{\pi}{2}\right)$

$\boxed{\Delta\varphi = \frac{\pi}{2}}$ $\left(\frac{T_A}{4}\right)$ – energija se prenese s 1 osc na drugi

– amp. prolazi kroz 0, cos mijenja predznak, a njihalo fazu (za π)

– njihalo koje prima energiju kasni za $\frac{\pi}{2}$ iza njihala koje daje E

$\left[t = 0;\ s_1(0) = A,\ s_2(0) = 0 \rightarrow \varphi_{01} = \varphi_{02} = \frac{\pi}{2} \Rightarrow \begin{matrix} s_1 = A(\cos\omega_1 t + \cos\omega_2 t) \\ s_2 = A(\cos\omega_1 t - \cos\omega_2 t) \end{matrix} \right]$

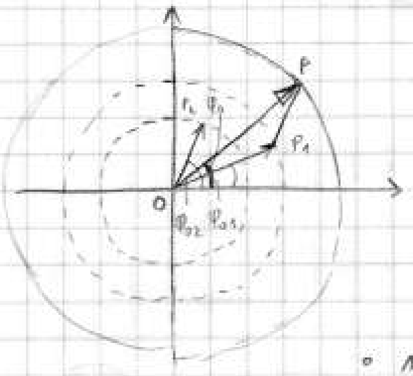
6 ZBRAJANJE HARM. TITRAJA

6.1 NA ISTOM PRAVCU – superpozicija / interferencija

a) JEDNAKE FREKVENCIJE

$$\left.\begin{aligned} s_1 &= A_1 \sin(\omega t + \varphi_{01}) \\ s_2 &= A_2 \sin(\omega t + \varphi_{02}) \end{aligned}\right\} \quad \text{konst. } \Delta\varphi \text{ – koherentna titranja}$$

$$s = s_1 + s_2 = A_1 \sin(\omega t + \varphi_{01}) + A_2 \sin(\omega t + \varphi_{02}) \qquad [\text{metodom rot-vektora}]$$



$$\vec{OP} = \vec{OP_1} + \vec{OP_2} \qquad \boxed{s = A \sin(\omega t + \varphi_0)}$$

$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{02} - \varphi_{01})}}$$

$$\boxed{\operatorname{tg}\varphi_0 = \frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}}}$$

- razlika u fazi $\Delta\varphi = \varphi_{02} - \varphi_{01}$

$= 0, 2\pi, 4\pi, \ldots \Rightarrow A = A_1 + A_2$ – max KONSTRUKTIVNA

$= \pi, 3\pi, 4\pi, \ldots \Rightarrow A = A_1 - A_2$ – min DESTRUKTIVNA

b) RAZLIČITE FREKVENCIJE $(A_1 = A_2)$

$$\left.\begin{aligned} s_1 &= A \sin(\omega_1 t + \varphi_{01}) \\ s_2 &= A \sin(\omega_2 t + \varphi_{02}) \end{aligned}\right\} + \quad s = A\left[\sin(\omega_1 t + \varphi_{01}) + \sin(\omega_2 t + \varphi_{02})\right]$$

$$s = \underbrace{2A \cos\left(\frac{\omega_1 - \omega_2}{2}t + \frac{\varphi_{01} - \varphi_{02}}{2}\right)}_{\substack{\text{modulirana amp.} \\ [0,\,2A]}} \sin\left(\frac{\omega_1 + \omega_2}{2}t + \frac{\varphi_{01} + \varphi_{02}}{2}\right) \qquad \omega = \frac{\omega_1 + \omega_2}{2}$$

– udari $(f_u = f_1 - f_2)$.

6.2 OKOMITI. LISSAJOUSOVE KRIVULJE

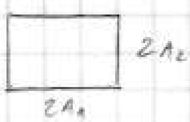
$x = A_1 \sin \omega_1 t$

$y = A_2 \sin(\omega_2 t + \Delta\varphi)$

$\Delta\varphi$ – fazna razlika titranja u x i y smjeru

→ putanja = 2D krivulja zadana param. jednadžbama

– upisana u pravokutnik



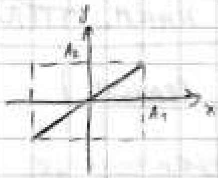
– ovisi o $\frac{\omega_1}{\omega_2}$, $\Delta\varphi$

a) $\boxed{\omega_1 = \omega_2}$

① $\Delta\varphi = 0$:

$$x = A_1 \sin \omega t$$
$$y = A_2 \sin \omega t$$

$$y = \frac{A_2}{A_1} x$$



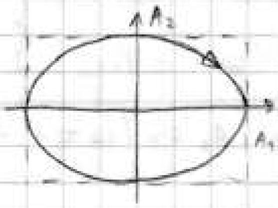
② $\Delta\varphi = \pi$:

$$y = -\frac{A_2}{A_1} x$$

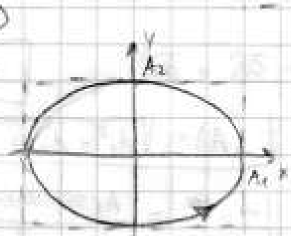
③ $\Delta\varphi = \frac{\pi}{2}$:

$$x = A_1 \sin \omega t$$
$$y = A_2 \cos \omega t$$

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$$



④ $\Delta\varphi = \frac{3\pi}{2}$:



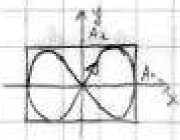
b) $\boxed{\omega_1 \neq \omega_2}$

* $\frac{\omega_1}{\omega_2} = \frac{1}{2}$

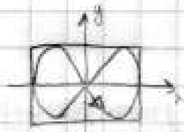
$$x = A \sin \omega_1 t$$
$$y = A \sin (2\omega_1 t + \Delta\varphi)$$

– oblik krivulje ovisi o $\Delta\varphi$

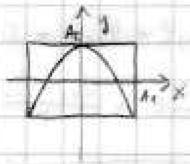
① $\Delta\varphi = 0$



② $\Delta\varphi = \pi$



③ $\Delta\varphi = \frac{\pi}{2}$



④ $\Delta\varphi = \frac{3\pi}{2}$