

Rješenja s postupcima
završnog ispita iz OE
12/13

Made by: giggs

1. Na sinusoidni izvor frekvencije 50 Hz paralelno su spojene dvije impedancije $Z_1 = 2 + j4 \Omega$ i $Z_2 = 4 - j2 \Omega$. Snaga na otporniku od 2Ω je 18 W. Odrediti potreban kapacitet kondenzatora koji treba spojiti paralelno impedancijama da bi ukupni faktor snage spoja bio $\cos \varphi = 1$.

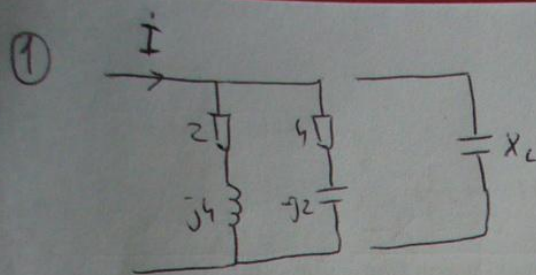
A) 141,6 μF

B) 198,2 μF

C) 232,4 μF

D) 280,8 μF

E) 318,3 μF



$$Z_1 = 2 + j4$$

$$Z_2 = 4 - j2$$

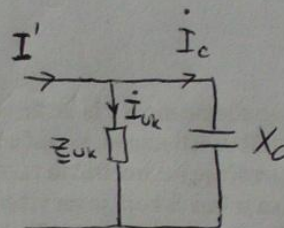
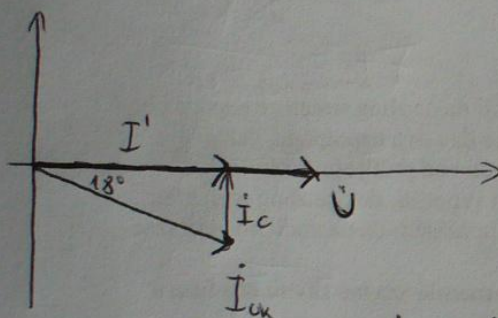
$$P_1 = 18 \text{ W}$$

$$f = 50 \text{ Hz}$$

$$P_1 = I_1^2 R_1 \Rightarrow I_1 = \sqrt{\frac{P_1}{R_1}} = 3 \text{ A}$$

$$U = I_1 Z_1 = 6\sqrt{5} \text{ V} \Rightarrow I_{\text{uk}} = \frac{U}{Z_{\text{uk}}} = 3\sqrt{2}$$

$$Z_{\text{uk}} = \frac{Z_1 Z_2}{Z_1 + Z_2} = 3 + j = \sqrt{10} \angle 18,44^\circ \Omega$$



$$I_c = I_{\text{uk}} \sin(18,44^\circ) = 1,342 \text{ A}$$

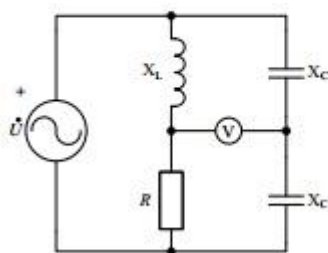
$$X_c = \frac{U}{I_c} = \frac{6\sqrt{5}}{1,342} = 9,99732$$

$$\omega = 2\pi f$$

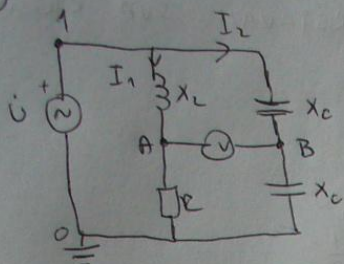
$$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_c} = \underline{\underline{3,184 \cdot 10^{-4} \text{ F}}}$$

2. U mreži prema slici zadano je $X_L = X_C = R = 10 \Omega$. Ukoliko voltmetar pokazuje $U_V = 10 \text{ V}$, odredite 2 boda napon izvora U .

- A) $U = 20 \text{ V}$
 B) $U = 15 \text{ V}$
 C) $U = 30 \text{ V}$
 D) $U = 10 \text{ V}$
 E) $U = 25 \text{ V}$



②



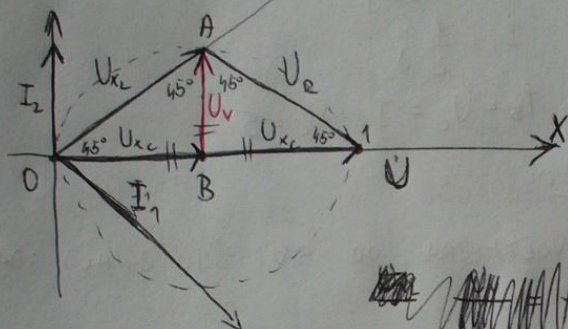
$$R = X_L = X_C = 10 \Omega$$

$$U_V = 10 \text{ V}$$

$$U = ?$$

$$Z = ?$$

Crteno topografski diagram



Postavimo \dot{U} na x-os. U prvoj grani je $Z_1 = 10 + 10j = 10\sqrt{2} \angle 45^\circ$ što znači da I_1 zaostaje za 45° za \dot{U} . Znamo da na induktivitetu napon prethodi struji za 90° . Znamo i da je napon na otporu u fazi sa strujom. To sve nacrtamo. Druga grana je čisti kapacitet što znači da I_2 prethodi \dot{U} za 90° . Pošto imamo dva ista kapaciteta vidljivo je da će oni dijeliti napon \dot{U} na pola. Sada vrtamo \dot{U}_V .

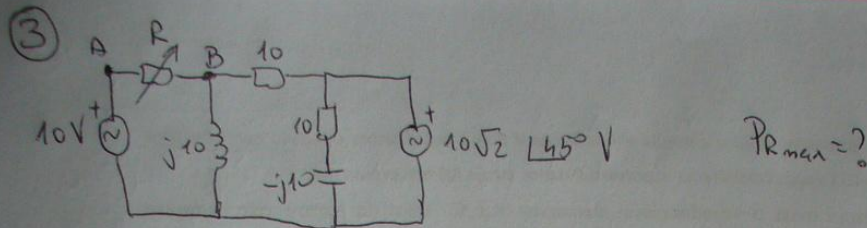
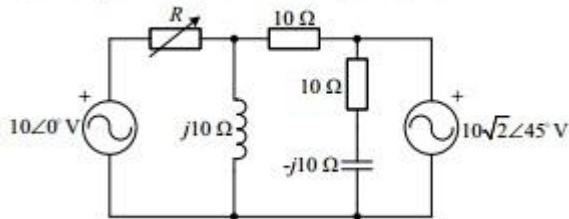
Pošto su svi kutovi 45° jasno je da su to dva jednakokrakna trokuta sa krakovima 10 V . (označeni na slici sa //)

I dobije se da je $\dot{U} = 20 \text{ V}$.

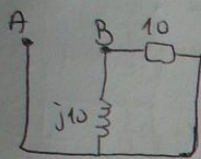
NAPOМЕНА: Mogu se uvrštavati rješenja i vidjeti kaj se poklapa... ako ovo neznate

3. Odredite maksimalnu snagu koja se može razvijati na otporu R u mreži prema slici.
3 boda

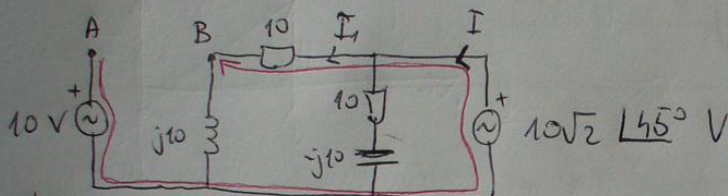
- A) $P = 4,6 \text{ W}$
B) $P = 6,2 \text{ W}$
C) $P = 8,3 \text{ W}$
D) $P = 9,7 \text{ W}$
E) $P = 11,4 \text{ W}$



Thevenin



$$\underline{Z}_T = 5 + 5j = 5\sqrt{2} \angle 45^\circ$$



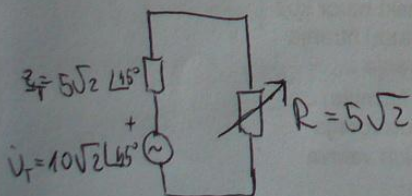
gledamo samo desni dio $\rightarrow \underline{Z} = \frac{(10 + j10)(10 - j10)}{20} = 10 \Rightarrow$ ovo nam zapravo ni ne treba

$$I_1 = \frac{10\sqrt{2} \angle 45^\circ}{10 + j10} = 1 \text{ A}$$

CRVENO JE OZNAČEN PUT PO KOJEMU ĆU RAČUNATI \dot{U}_T

$$\dot{U}_T = -10 \text{ V} + 10\sqrt{2} \angle 45^\circ + \underbrace{1 \cdot 10}_{I_1 \cdot R} = 10\sqrt{2} \angle 45^\circ \text{ V}$$

NOVA SKICA



$P_{Rmax} = ?$

$$\underline{Z} = 13,07 \angle 22,5^\circ$$

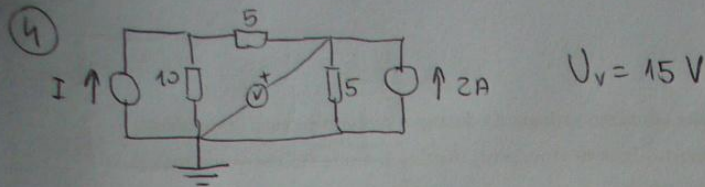
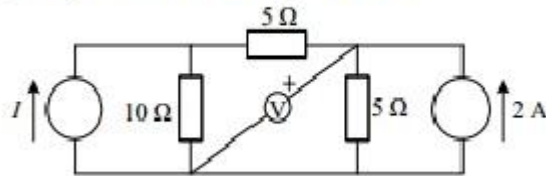
$$I = 1,08 \angle 22,5^\circ$$

$$P_{max} = I^2 \cdot R = 1,08^2 \cdot 5\sqrt{2} = 8,3 \text{ W}$$

4. Kolika je struja izvora I ako voltmetar pokazuje $U_v = 15 \text{ V}$ označenog polariteta?

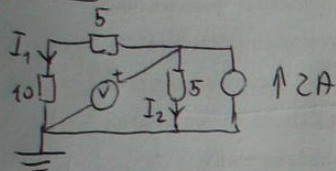
3 boda

- A) $I = 1,0 \text{ A}$
- B) $I = 2,5 \text{ A}$
- C) $I = 3,0 \text{ A}$**
- D) $I = 4,5 \text{ A}$
- E) $I = 5,7 \text{ A}$



Primjetimo da ako donju točku uzemljimo voltmetar mjeri samo pad napona na desnom otporu. To se lako izračuna superpozicijom.

1. slučaj

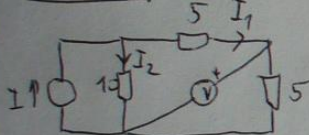


$$I_1 = I \cdot \frac{5}{20} = 0,5 \text{ A}$$

$$I_2 = 1,5 \text{ A}$$

$$U_v = I_2 \cdot 5 = 7,5 \text{ V}$$

2. slučaj



$$I_1 = I_2 = \frac{I}{2}$$

$$U_v = \frac{I}{2} \cdot 5$$

sada zbrojimo ta dva slučaja i izjednačimo sa zadanim

$$7,5 + \frac{I}{2} \cdot 5 = 15$$

$$\frac{I}{2} \cdot 5 = 15 - 7,5 = 7,5$$

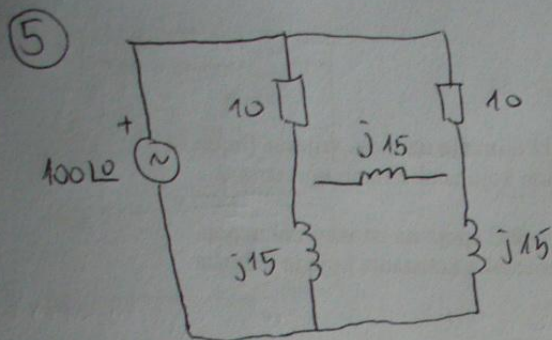
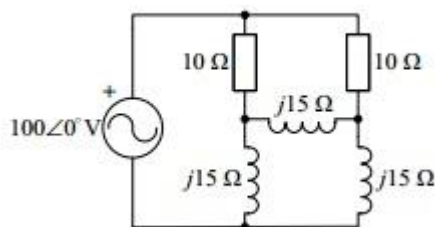
$$5I = 15 \cdot 2$$

$$I = 3 \text{ A}$$

5. Odrediti struju izvora u mreži prema slici.

3 boda

- A) $I = 12,5 \text{ A}$
- B) $I = 11,1 \text{ A}$**
- C) $I = 9,7 \text{ A}$
- D) $I = 7,4 \text{ A}$
- E) $I = 6,1 \text{ A}$



$I = ?$

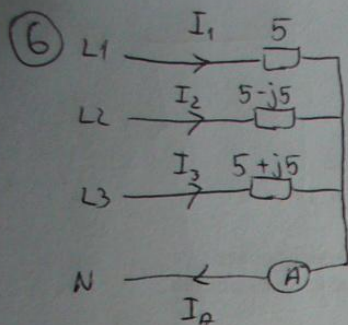
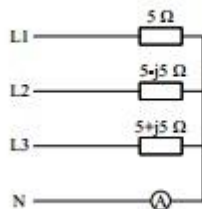
Most je u ravnoteži tako da se srednja grana može odspojiti

$$Z_{uk} = \frac{(10 + j15)^2}{20 + j30} = 5 + \frac{15}{2}j$$

$$I_{uk} = \frac{\dot{U}}{Z} = \frac{100 \angle 0^\circ}{5 + \frac{15}{2}j} = \underline{\underline{11,09 \angle -56^\circ \text{ A}}}$$

6. Trošilo prema slici priključeno je na trofazni simetrični izvor linijskog napona $U_L = 380 \text{ V}$. Odredite 3 boda pokazivanje ampermetra.

- A) $I = 40 \text{ A}$
- B) $I = 50 \text{ A}$
- C) $I = 60 \text{ A}$**
- D) $I = 70 \text{ A}$
- E) $I = 80 \text{ A}$



$$U_L = 380 \text{ V}$$

$$I_A = ?$$

$$U_\phi = 220 \text{ V}$$

$$I_A = I_1 + I_2 + I_3$$

$$I_1 = \frac{U_\phi}{5} = 44$$

$$I_2 = \frac{220 \angle -120^\circ}{5 - j5} = 22\sqrt{2} \angle -75^\circ$$

$$I_3 = \frac{220 \angle -240^\circ}{5 + j5} = 22\sqrt{2} \angle 75^\circ$$

PAZI!!!

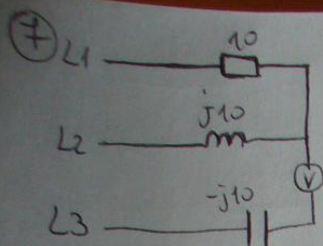
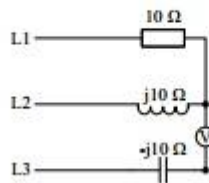
Kojevi trofaznog sustava

~~$$I_A = 44 + 22\sqrt{2} \angle -75^\circ + 22\sqrt{2} \angle 75^\circ$$~~

$$I_A = 44 + 22\sqrt{2} \angle -75^\circ + 22\sqrt{2} \angle 75^\circ = \underline{\underline{60,105 \text{ A}}}$$

7. Na simetrični trofazni izvor linijskog napona $U_L = 400\text{ V}$ priključen je spoj prema slici. Odredite 3 boda pokazivanje voltmetra.

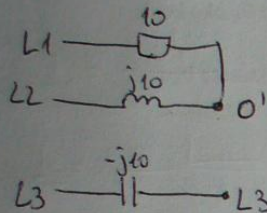
- A) $U = 102,8\text{ V}$
- B) $U = 115,3\text{ V}$
- C) $U = 127,1\text{ V}$
- D) $U = 146,4\text{ V}$**
- E) $U = 191,5\text{ V}$



$$U_L = 400\text{ V}$$

$$U_f = 230,94\text{ V} = 231\text{ V}$$

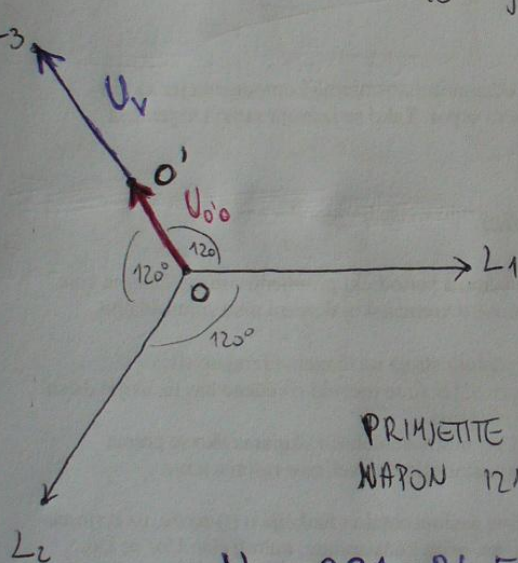
VOLTMETAR IMA BESKONAČNI OTPOR PA JE DOLJNJA GRANA ODSPOJENA.



MORAMO NAĆI POLOŽAJ O' PO MILIMETRU

$$U_{O'O} = \frac{\frac{231}{10} + \frac{231 \angle -120^\circ}{j10}}{\frac{1}{10} + \frac{1}{j10}} = 84,55 \angle 120^\circ$$

$$L_3 = 231 \angle 120^\circ$$



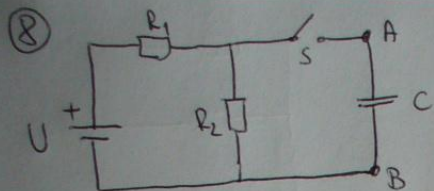
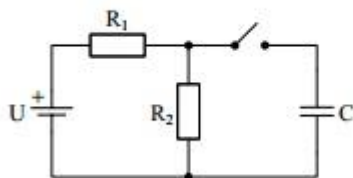
PRIJETITE DA VOLTMETAR ZAPRAVO MJERI NAPON IZMEĐU TOČKA O' I L_3 .

$$U_v = 231 - 84,55 = \underline{\underline{146,45}}$$

VOLTMETAR NE POKAZUJE KUTOVE

8. Kondenzator je prazan. U trenutku $t_0=0$ zatvara se sklopka. Odredite napon na kondenzatoru u trenutku $t=50\mu s$. Zadano je $R_1=2\text{ k}\Omega$, $R_2=2\text{ k}\Omega$, $C=20\text{ nF}$, $U=100\text{ V}$.

- A) $U = 23,3\text{ V}$
- B) $U = 38,5\text{ V}$
- C) $U = 60,2\text{ V}$
- D) $U = 12,7\text{ V}$
- E) $U = 45,9\text{ V}$



$$t_0 = 0$$

$$t = 50\mu s$$

$$R_1 = 2\text{ k}\Omega$$

$$R_2 = 2\text{ k}\Omega$$

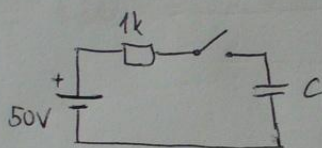
$$C = 20\text{ nF}$$

$$U = 100\text{ V}$$

PRVO ćemo ostatak mreže zamijeniti Theveninovim nadomjestnim izvorom

$$R_{th} = R_T = 1\text{ k}\Omega$$

$$U_{th} = U_T = 50\text{ V}$$



I SADA PO FORMULI $U_c(t) = U \left(1 - e^{-\frac{t}{\tau}} \right)$

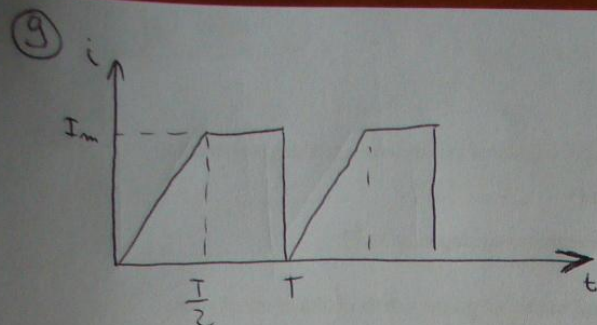
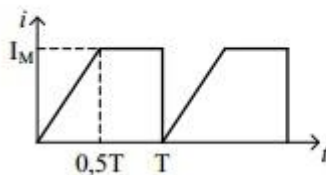
$$\tau = RC = 2 \cdot 10^{-5}\text{ s}$$

$$\frac{t}{\tau} = \frac{5}{2}$$

$$U_c = 50 \left(1 - e^{-\frac{5}{2}} \right) = \underline{\underline{45,896\text{ V}}}$$

9. Ako je srednja vrijednost struje valnog oblika prema slici $I_{sr}=2$ A, odredite njezinu efektivnu vrijednost.

- A) $I_{ef} = 1,55$ A
- B) $I_{ef} = 1,76$ A
- C) $I_{ef} = 1,92$ A
- D) $I_{ef} = 2,18$ A**
- E) $I_{ef} = 2,35$ A



$$I_{sr} = 2 \text{ A}$$

$$I_{ef} = ?$$

PRVO MORAMO IZRAČUNATI I_m PREKO I_{se}

$$\text{od } 0 \text{ do } \frac{T}{2}$$

$$I_{se1} = \frac{1}{T} \int_0^{\frac{T}{2}} \frac{2I_m}{T} t dt = \frac{1}{T} \cdot \frac{I_m}{T} t^2 \Big|_0^{\frac{T}{2}} = \frac{I_m}{T^2} \frac{T^2}{4} = \frac{I_m}{4}$$

$$\text{od } \frac{T}{2} \text{ do } T$$

$$I_{se2} = \frac{I_m}{2}$$

SLIJEDI:

$$I_{se} = I_{se1} + I_{se2}$$

$$2 = \frac{3}{4} I_m \Rightarrow I_m = \frac{8}{3}$$

SADA TRAŽIMO EFEKTIVNE VRIJEDNOSTI

$$I_{ef1} = \frac{I_m}{\sqrt{3}} \sqrt{\frac{T_i}{T}} \Rightarrow \text{ZA PILASTI OBLIK UVIJEK IDE } \frac{I_m}{\sqrt{3}}$$

$T_i \rightarrow$ KOLIKO DUGO JE PILASTI OBLIK

$T \rightarrow$ PERIOD

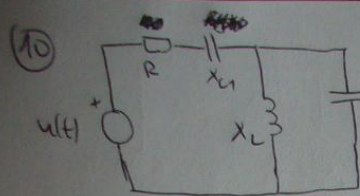
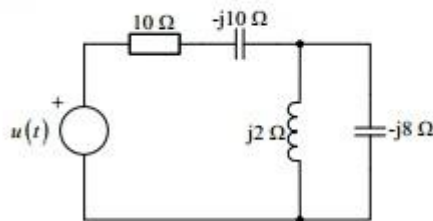
$$I_{ef1} = \frac{\frac{8}{3}}{\sqrt{3}} \sqrt{\frac{\frac{T}{2}}{T}} = \frac{8}{3\sqrt{6}}$$

$$I_{ef2} = I_m \sqrt{\frac{T_i}{T}} = \frac{8}{3\sqrt{2}} \Rightarrow \text{ZA PRAVOKUTNI OBLIK}$$

$$I_{ef} = \sqrt{I_{ef1}^2 + I_{ef2}^2} = 2,177 \text{ A}$$

10. U spoju prema slici napon izvora je složenog valnog oblika danog izrazom
2 boda $u(t) = 100 + \sqrt{2} \sin(\omega t) + 2\sqrt{2} \sin(2\omega t)$ V. Odredite efektivnu vrijednost struje kroz otpornik. Sve reaktancije su dane za kružnu frekvenciju ω .

- A) $I_{ef} = 38,1$ mA
B) $I_{ef} = 59,2$ mA
C) $I_{ef} = 80,6$ mA
D) $I_{ef} = 97,0$ mA
E) $I_{ef} = 105,5$ mA



$$u(t) = 100 + \sqrt{2} \sin(\omega t) + 2\sqrt{2} \sin(2\omega t)$$

$$I_{ef} = ?$$

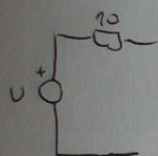
$$X_{C1} = \frac{1}{\omega C_1} = 10 \, \Omega \quad R = 10 \, \Omega$$

$$X_{C2} = \frac{1}{\omega C_2} = 8 \, \Omega$$

$$\omega L = 2 \, \Omega = X_L$$

i) harmonik

$$U = 100$$



$$I_{ef1} = 0$$

NAPOМЕНА: kod istosmjerne struje kapaciteti se brišu inuktivitati se kratko spajaju

ii) harmonik

$$U = \sqrt{2} \sin(\omega t)$$

$$U_{ef} = 1$$

$$Z = 10 - \frac{22}{3} j$$

$$I_{ef2} = \frac{U_{ef}}{Z} = 0,08064 \, A$$

iii) harmonik

$$U = 2\sqrt{2} \sin(2\omega t)$$

PAZI !!!

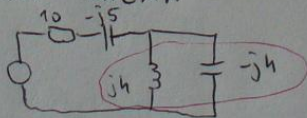
TREBA IZRAČUNATI REAKTANCIJE

$$X'_{C1} = \frac{1}{2\omega C_1} \Rightarrow \frac{1}{2} \frac{1}{\omega C_1} = X'_{C1} \Rightarrow X'_{C1} = \frac{1}{2} X_{C1} = 5 \, \Omega$$

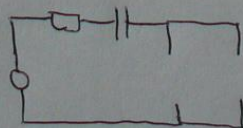
$$X'_{C2} = \frac{1}{2} \frac{1}{\omega C_2} = \frac{1}{2} X_{C2} = 4 \, \Omega$$

$$X'_L = 2\omega L = 2 X_L = 4 \, \Omega$$

NOVA SHEMA



PARALELNA REZONANCIJA



NEMA ZATVORENOG STRUJNOG KRUGA

$$I_{ef3} = 0$$

$$I_{efn} = I_{ef1} + I_{ef2} + I_{ef3}$$

$$I_{ef} = 0 + 0,08064 + 0$$

$$I_{ef} = 0,08064 \, A$$