RLC krugovi

VII. tjedan predavanja

RLC krugovi

Fazni odnos između napona i struje na impedancijama

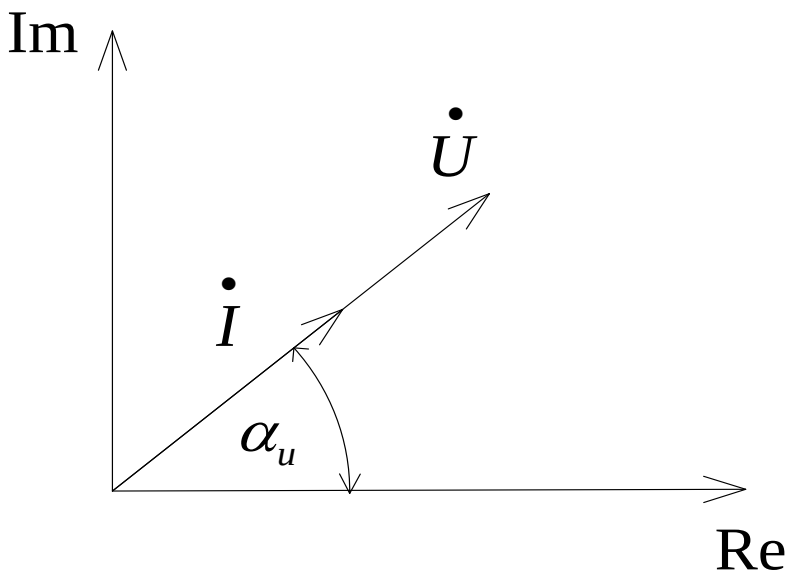
Zadani su $\dot{U}$ i $\underline{Z}$, a traži se $\dot{I}$

$$\left.\begin{aligned}\dot{U} &= U \cdot e^{j\alpha_u} \\ \underline{Z} &= Z \cdot e^{j\varphi}\end{aligned}\right\} \dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{Z \cdot e^{j\varphi}} = \frac{U}{Z} \cdot e^{j(\alpha_u - \varphi)}$$

$$\varphi = \alpha_u - \alpha_i$$

a) OMSKI OTPOR

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{\dot{U}}{R} = \frac{U \cdot e^{j\alpha_u}}{R \cdot e^{j0}} = \frac{U}{R} \cdot e^{j\alpha_u}$$



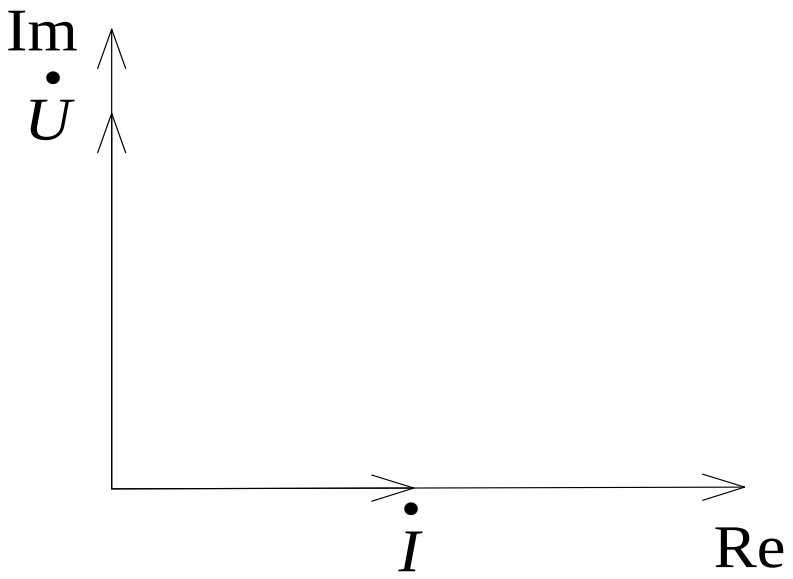
Napon i struja imaju <u>isti fazni kut</u> i kažemo da su na omskom otporu napon i struja u fazi.

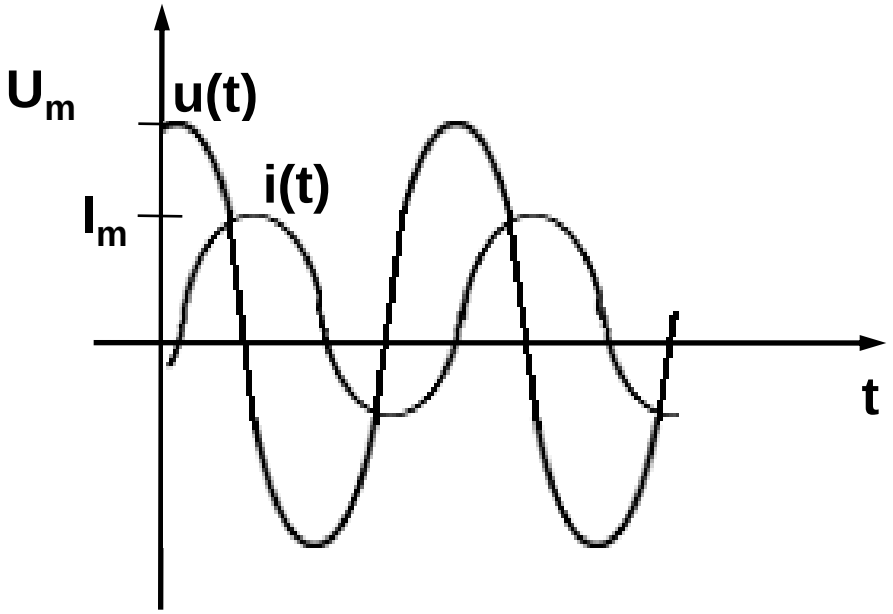
b) ZAVOJNICA

$$\underline{Z} = jX_L = j\omega L \qquad j = e^{j\frac{\pi}{2}}$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{jX_L} = \frac{U \cdot e^{j\alpha_u}}{X_L \cdot e^{j\frac{\pi}{2}}} = \frac{U}{X_L} \cdot e^{j\left(\alpha_u - \frac{\pi}{2}\right)}$$

Struja kasni za naponom za $\frac{\pi}{2}$.



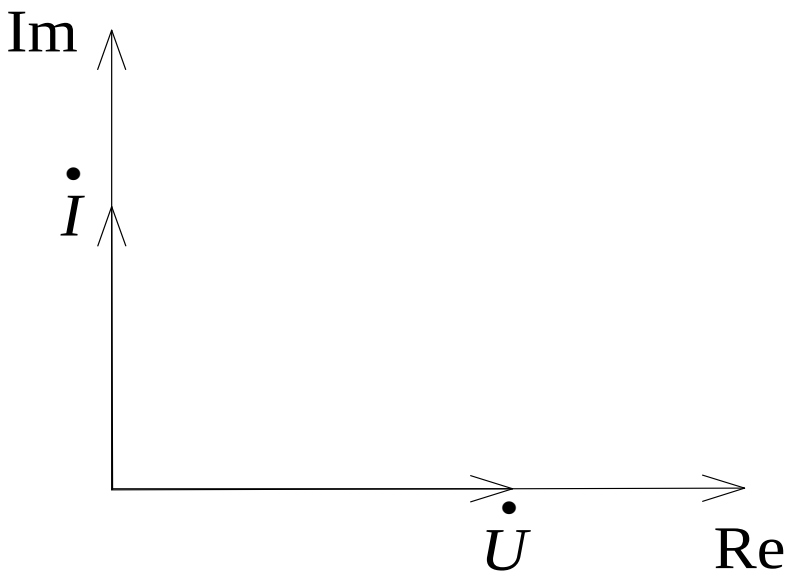


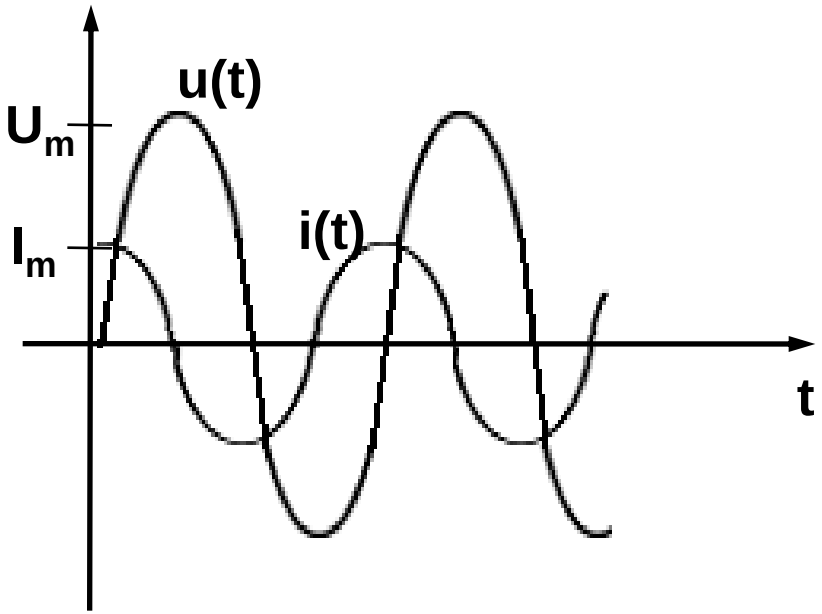
c) KAPACITET

$$\underline{Z} = -j \cdot \frac{1}{\omega C} = -jX_C$$

Struja prethodi naponu za $\frac{\pi}{2}$.

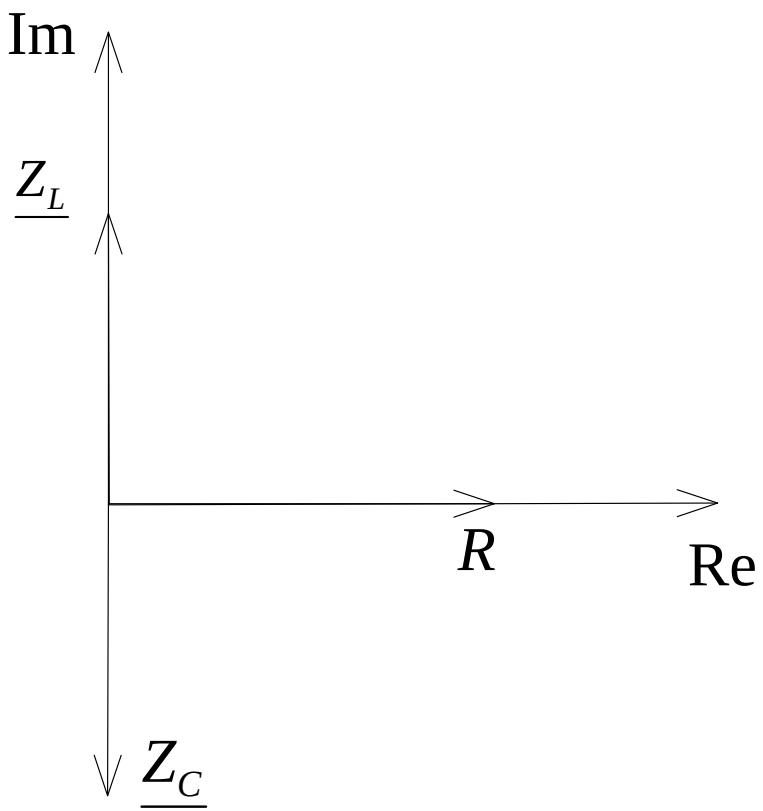
$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{-jX_C} = \frac{U \cdot e^{j\alpha_u}}{X_C \cdot e^{-j\frac{\pi}{2}}} = \frac{U}{X_C} \cdot e^{j\left(\alpha_u + \frac{\pi}{2}\right)}$$





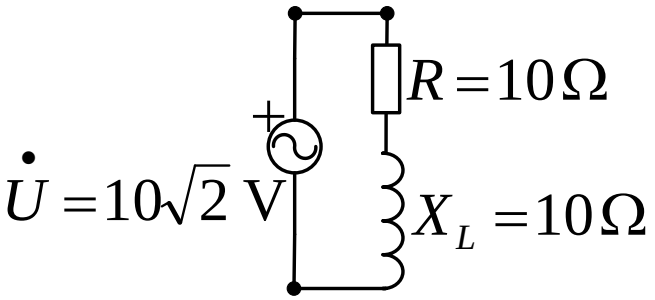
OPĆI SLUČAJ

$$\underline{Z} = Z\angle\varphi\,[\Omega]$$



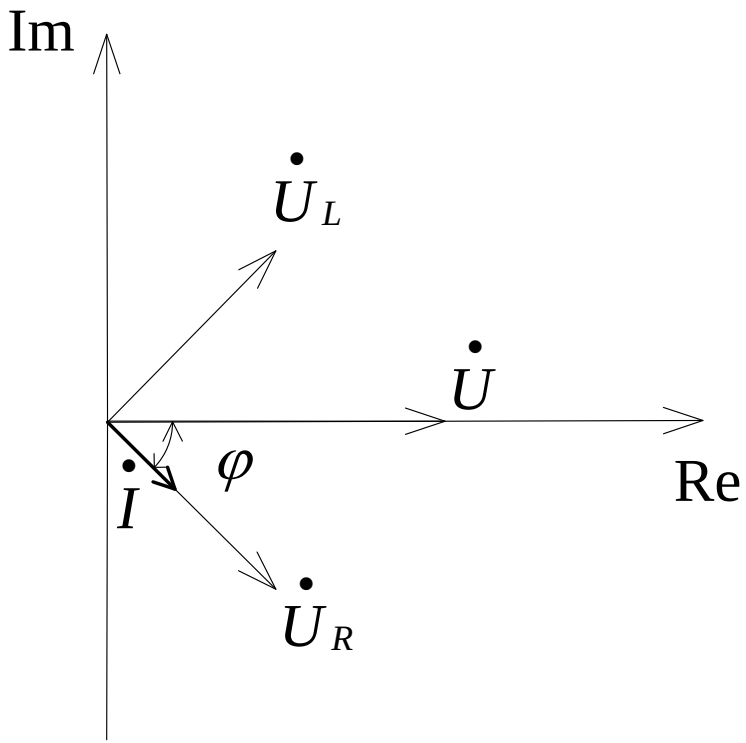
Vektor impedancije može se pojaviti u I. ili IV. kvadrantu.

RLC krugovi
RLC krug-Primjer 1.



$$\underline{Z}_{RL} = R + jX_L = 10 + j10 = 10\sqrt{2}\angle 45^\circ\,\Omega$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}_{RL}} = \frac{10\sqrt{2}}{10\sqrt{2}\angle 45^\circ} = 1\angle -45^\circ\ \mathrm{A}$$

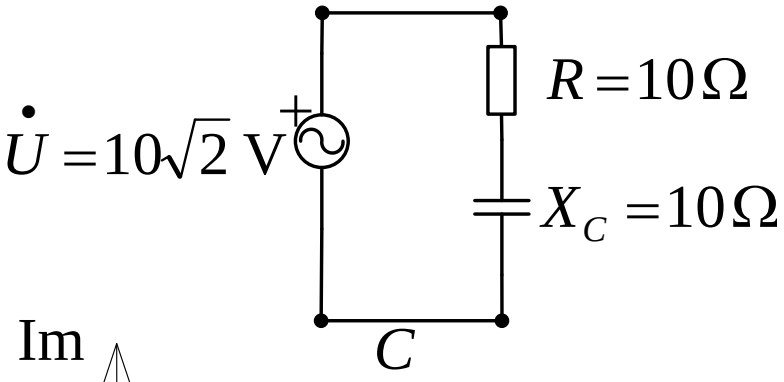


$$\dot{U}_R = \dot{I} \cdot R = 10\angle -45^\circ\ \mathrm{V}$$

$$\dot{U}_L = \dot{I} \cdot \underline{X}_L = 10\angle 45^\circ\ \mathrm{V}$$

Induktivno ponašanje kruga

RLC krugovi
RLC krug-Primjer 2.

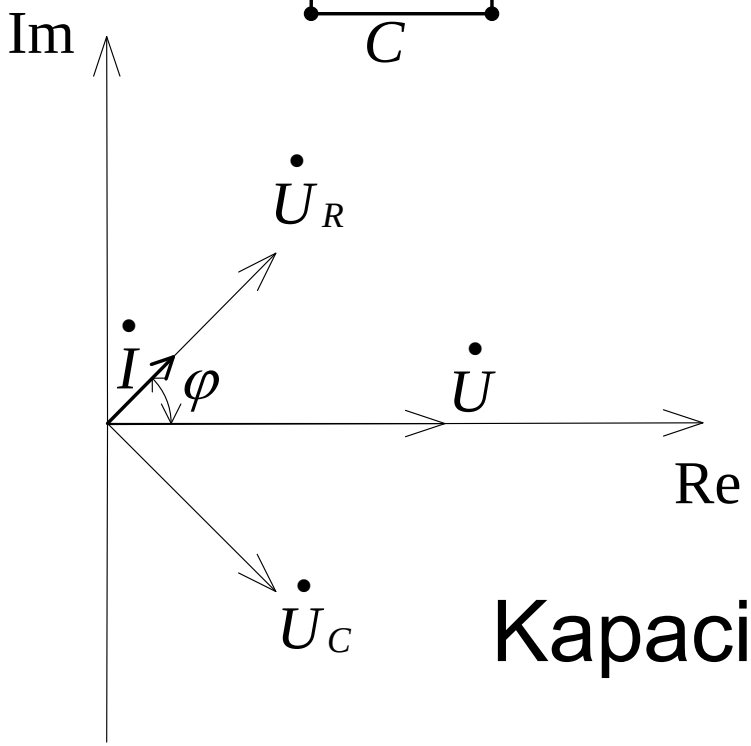


$$\underline{Z}_{RL} = R - jX_C = 10 - j10 = 10\sqrt{2}\angle -45^\circ\,\Omega$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}_{RC}} = \frac{10\sqrt{2}}{10\sqrt{2}\angle -45^\circ} = 1\angle 45^\circ\ \mathrm{A}$$

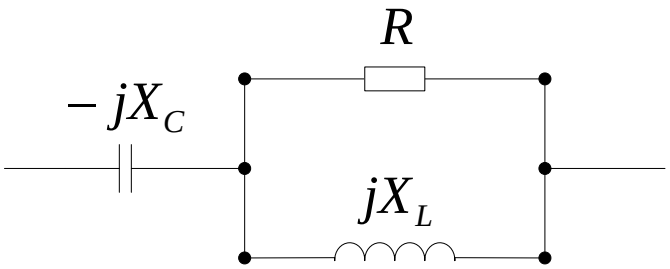
$$\dot{U}_R = \dot{I} \cdot R = 10\angle 45^\circ\ \mathrm{V}$$

$$\dot{U}_C = \dot{I} \cdot \underline{X}_C = 10\angle -45^\circ\ \mathrm{V}$$



Kapacitivno ponašanje kruga

RLC krugovi
Primjer 3.

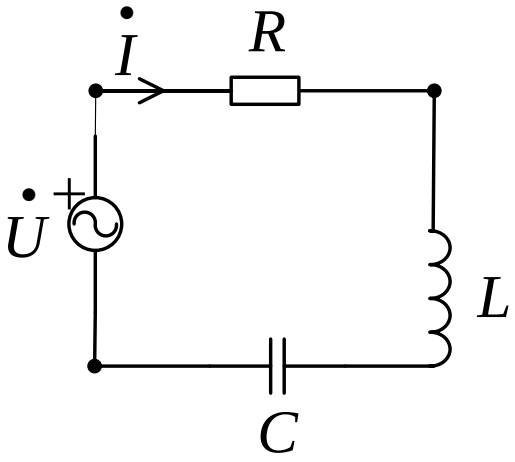


$$\underline{Z}_{RL} = \frac{R \cdot jX_L}{R + jX_L} \cdot \frac{R - jX_L}{R - jX_L} = \frac{jR^2 \cdot X_L + X_L^2 \cdot R}{R^2 + X_L^2} =$$

$$\frac{R \cdot X_L^2}{R^2 + X_L^2} + j\frac{R^2 \cdot X_L}{R^2 + X_L^2}$$

$$\underline{Z}_{uk} = \underline{Z}_C + \underline{Z}_{RL} = \frac{R \cdot X_L^2}{R^2 + X_L^2} + j\left(\frac{R^2 \cdot X_L}{R^2 + X_L^2} - X_C\right)$$

RLC krugovi
Serijski RLC krug

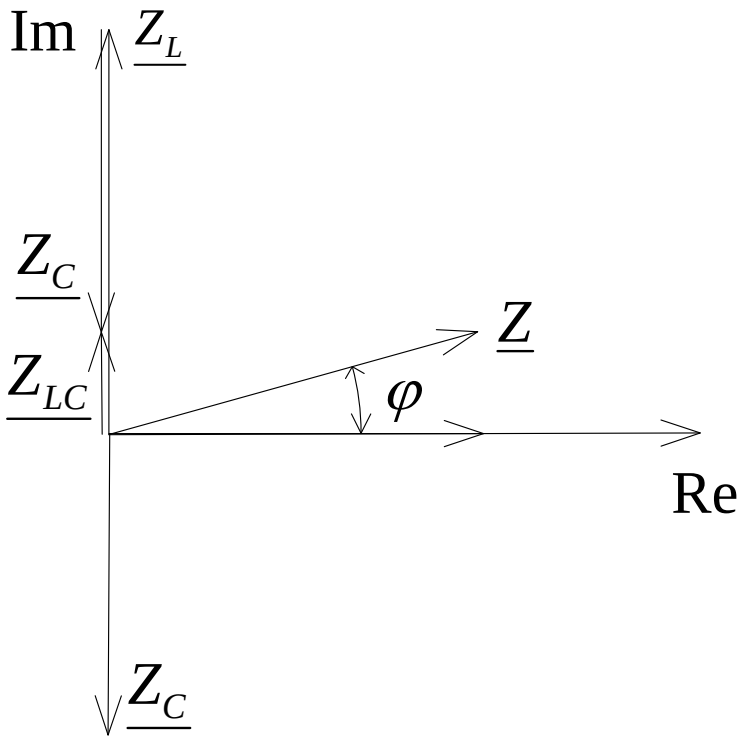


$$\underline{Z} = R + j\omega L - j \cdot \frac{1}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\varphi = arctg \frac{\omega L - \frac{1}{\omega C}}{R}$$

RLC krugovi
Serijski RLC krug



$$\underline{Z} = Z\angle\varphi\,[\Omega]$$

$$\dot{U} = \dot{I}\cdot\underline{Z}$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U\cdot e^{j\alpha_u}}{Z\cdot e^{j\varphi}} = \frac{U}{Z}\cdot e^{j(\alpha_u-\varphi)}$$

φ>0 impedancija ima induktivni karakter (struja zaostaje za naponom)

φ<0 impedancija ima kapacitivni karakter (struja prethodi naponu)

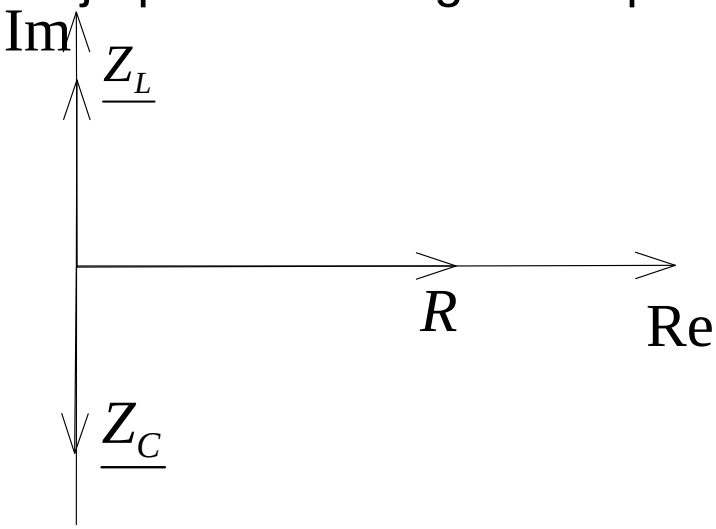
Uvjet rezonancije

$$\mathrm{Im}\{\underline{Z}\} = 0$$

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

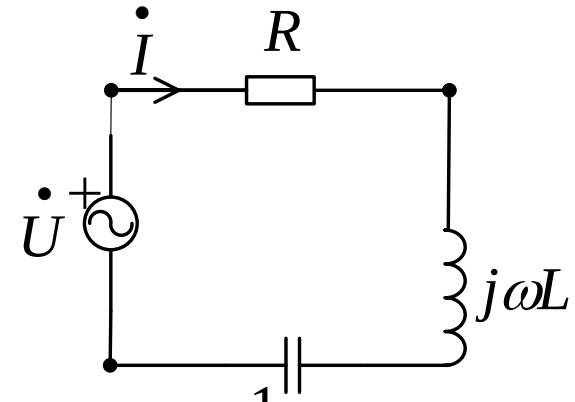
Napon i struja su u fazi, krug se ponaša radno, struja ima maksimalni iznos te je predana snaga na otporu maksimalna.



$\omega > \omega_0$, krug se ponaša induktivno

$\omega < \omega_0$, krug se ponaša kapacitivno

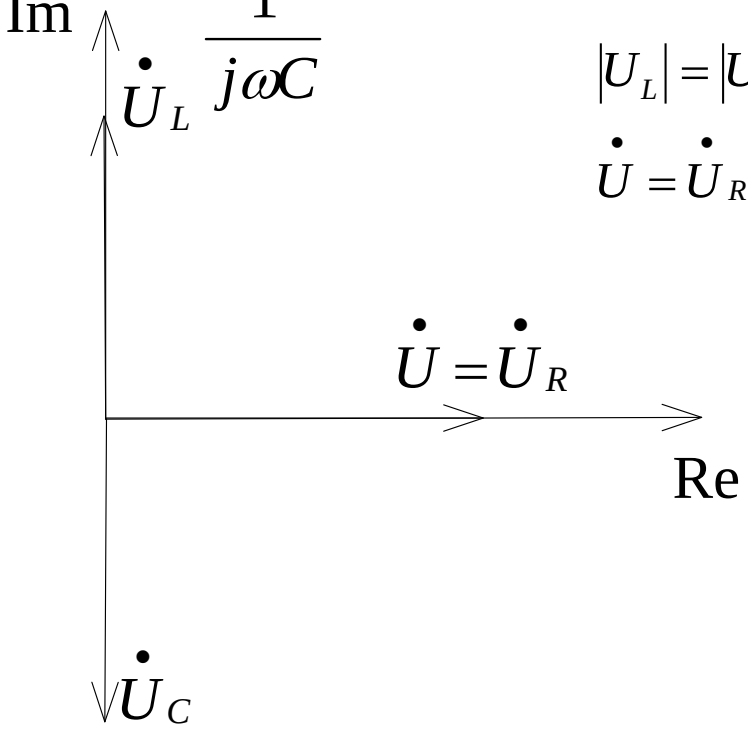
RLC krugovi
Serijski RLC krug



$$\dot{U}_R = \dot{I} \cdot R$$

$$\dot{U}_L = \dot{I} \cdot j\omega L$$

$$\dot{U}_C = \dot{I} \cdot \frac{1}{j\omega C}$$



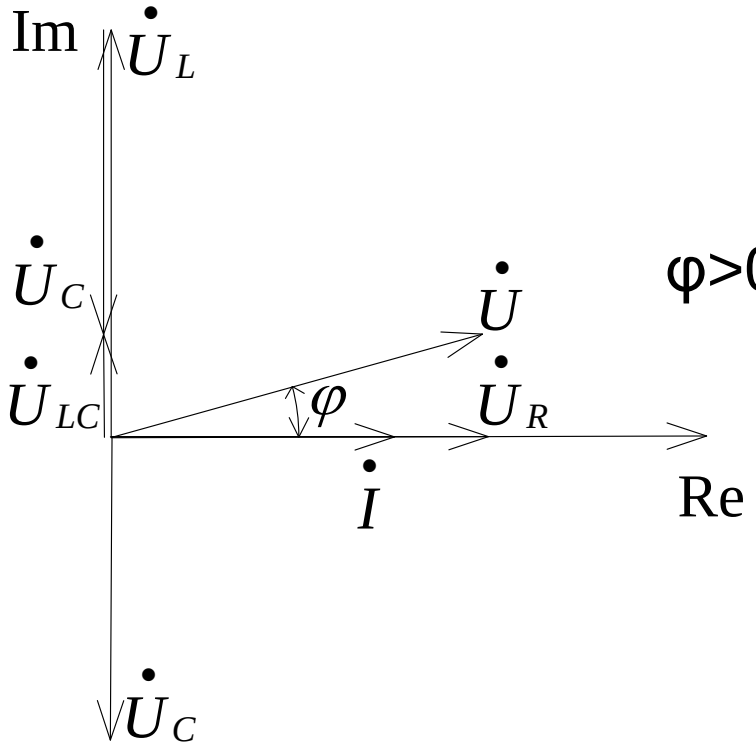
$$|U_L| = |U_C|$$

$$\dot{U} = \dot{U}_R$$

U postupku analize kruga prvo odredimo impedanciju, pa na temelju nje struju te napone na pojedinim elementima.

U rezonanciji napon na zavojnici jednak je naponu na kondenzatoru pa serijsku rezonanciju nazivamo i naponska rezonancija.

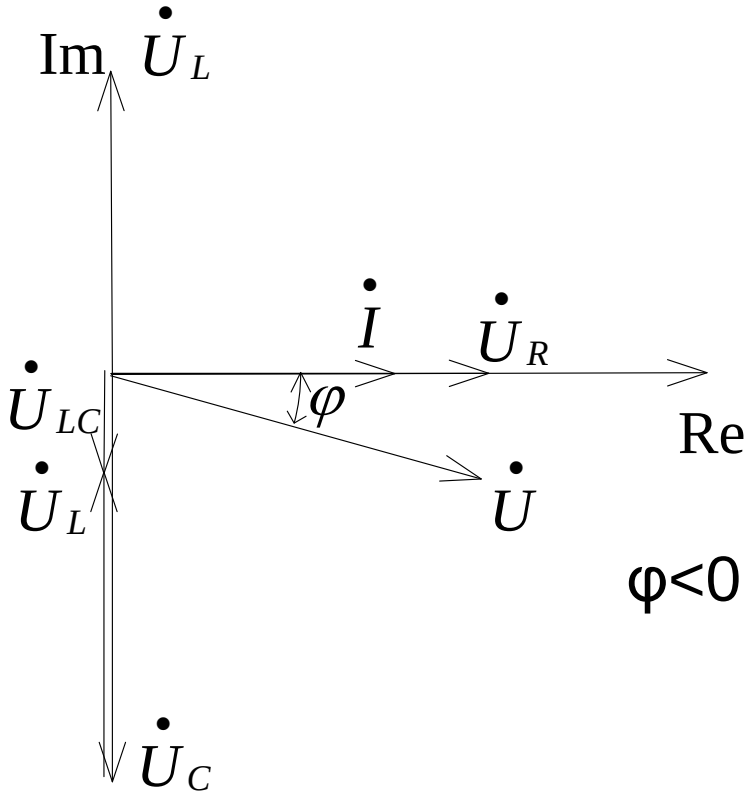
RLC krugovi
Serijski RLC krug



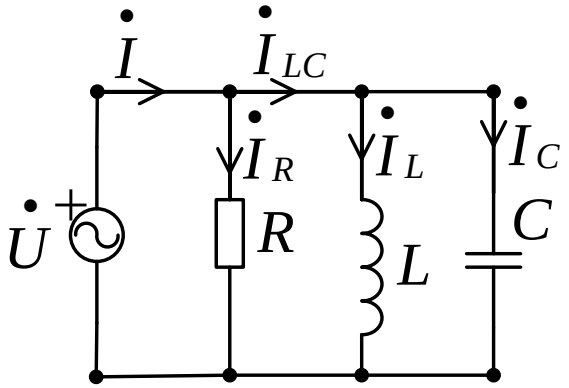
φ>0 induktivni karakter kruga

Kako je struja $\dot{I}$ zajednička veličina za ovaj krug, obično je postavimo po realnoj osi i od nje počnemo crtati vektorski dijagram.

RLC krugovi
Serijski RLC krug



φ<0 kapacitivni karakter kruga



Zajednička veličina na svim elementima je napon.

$$\dot{U} = \dot{U}_R = \dot{U}_L = \dot{U}_C$$

$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C$$

$$\dot{I}_R = \frac{\dot{U}}{R} = \dot{U} \cdot G$$

$$\dot{I}_L = \frac{\dot{U}}{jX_L} = \dot{U} \cdot \left(-jB_L\right)$$

$$\dot{I}_C = \frac{\dot{U}}{-jX_C} = \dot{U} \cdot \left(jB_C\right)$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \dot{U} \cdot \underline{Y}$$

B_L – induktivna vodljivost

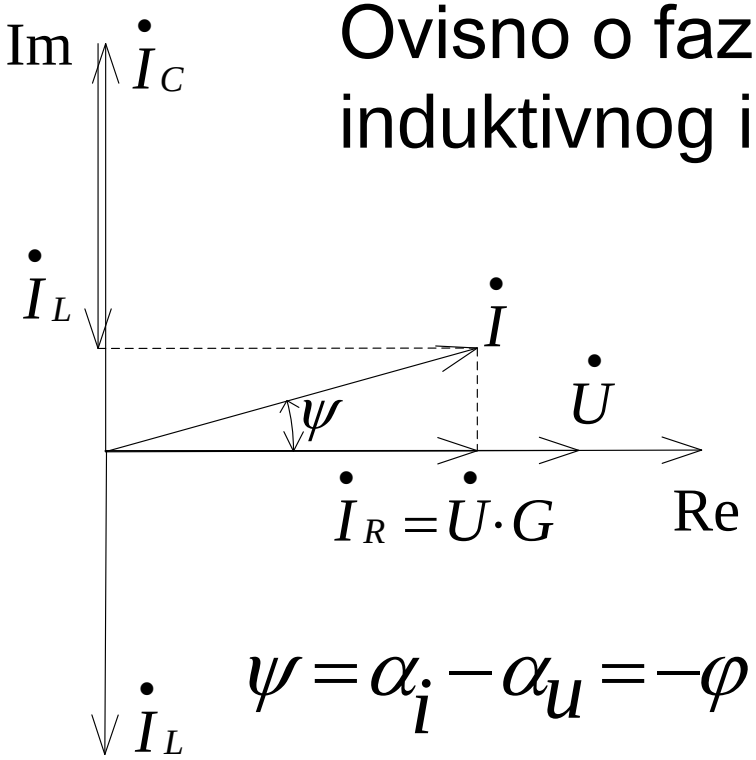
B_C – kapacitivna vodljivost

RLC krugovi
Paralelni RLC krug

$$\dot{U}\cdot\underline{Y}=\dot{U}\cdot G+\dot{U}\cdot\left(-jB_L\right)+\dot{U}\cdot\left(jB_C\right)$$

$$\underline{Y}=G+j\left(B_C-B_L\right)$$ Ukupna admitancija

Imaginarni dio (B_C-B_L) nazivamo susceptancija.



Ovisno o faznom kutu, krug može biti induktivnog ili kapacitivnog karaktera.

$$\dot{I}_C=\dot{U}\cdot jB_C$$

$$\dot{I}_R=\dot{U}\cdot G$$

$$\dot{I}_L=\dot{U}\cdot\left(-jB_L\right)$$

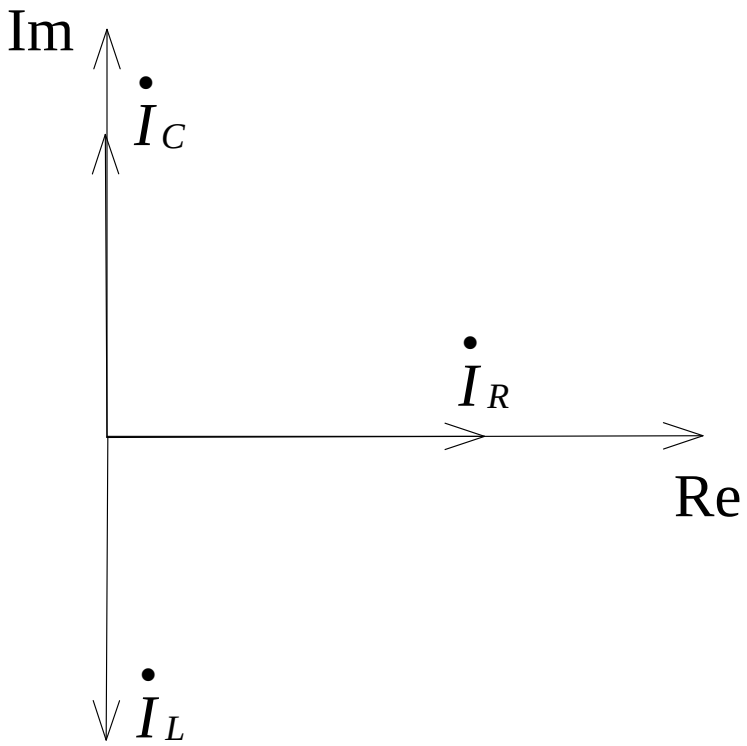
$$\dot{I}=\dot{I}_R+\dot{I}_L+\dot{I}_C$$

$$\psi=\alpha_i-\alpha_u=-\varphi$$

RLC krugovi
Paralelni RLC krug

$B_L = B_C$ specijalni slučaj

$\mathrm{Im}\{\underline{Y}\} = 0$ REZONANCIJA



$$|I_L| = |I_C|$$

$$\dot{I} = \dot{I}_R$$ (REZONANCIJA)!

U rezonanciji paralelnog RLC kruga struja je minimalna.

RLC krugovi
Paralelni RLC krug

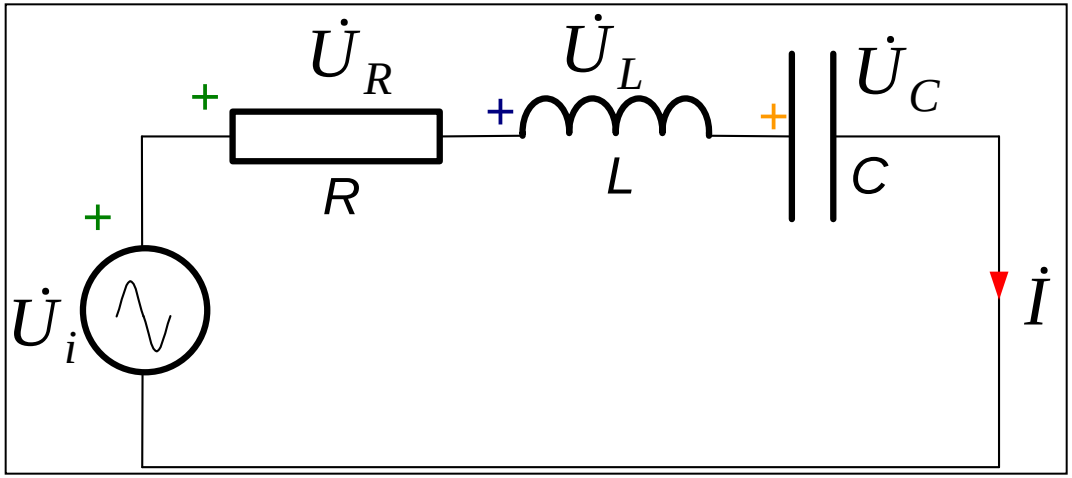
- Paralelnu rezonanciju nazivamo i strujna rezonancija
- Sa strane izvora kao da je LC dio odspojen. Struje I_L i I_C teku, ali su suprotnog smjera i istog iznosa. Dobijemo titrajni krug gdje energija neprestano prelazi s L na C i obratno (akumulirana elektrostatska energija na kondenzatoru, pretvara se u magnetsku energiju i obratno).
- Uvjet rezonancije:

$$B_C = B_L$$

$$\omega C = \frac{1}{\omega L} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega < \omega_0$ krug se ponaša induktivno

$\omega > \omega_0$ Krug se ponaša kapacitivno



- Analiziraju se veličine u krugu pri promjeni frekvencije od 0 do ∞

- Impedancija kruga je:

$$\underline{Z} = R + jX = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

- Pri promjeni frekvencije:

$$R = \text{konst.} \qquad X(\omega) = \omega L - \frac{1}{\omega C}$$

$$\underline{Z}(\omega) = |\underline{Z}(\omega)| e^{j\varphi}$$

$$|\underline{Z}(\omega)| = \sqrt{R^2 + X^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\dot{I} = \frac{\dot{U}_i}{\underline{Z}}$$

$$\varphi(\omega) = \text{arctg}\left(\frac{X}{R}\right) = \text{arctg}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

- Pri frekvenciji ω_0 imaginarni dio impedancije jednak je nuli. Frekvencija ω_0 naziva se **rezonantna frekvencija** i krug je **u rezonanciji**:

- U rezonanciji je:

$$X(\omega_0) = \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\underline{Z}(\omega_0) = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$X_{L0} = \omega_0 L = \frac{1}{\sqrt{LC}} L = \sqrt{\frac{L}{C}} = \rho$$

$$X_{C0} = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} = \rho$$

- ρ - **valni otpor;** γ - **valna vodljivost:**

$$\rho = \frac{1}{\gamma} = \sqrt{\frac{L}{C}} \quad (\Omega)$$

U rezonanciji je:

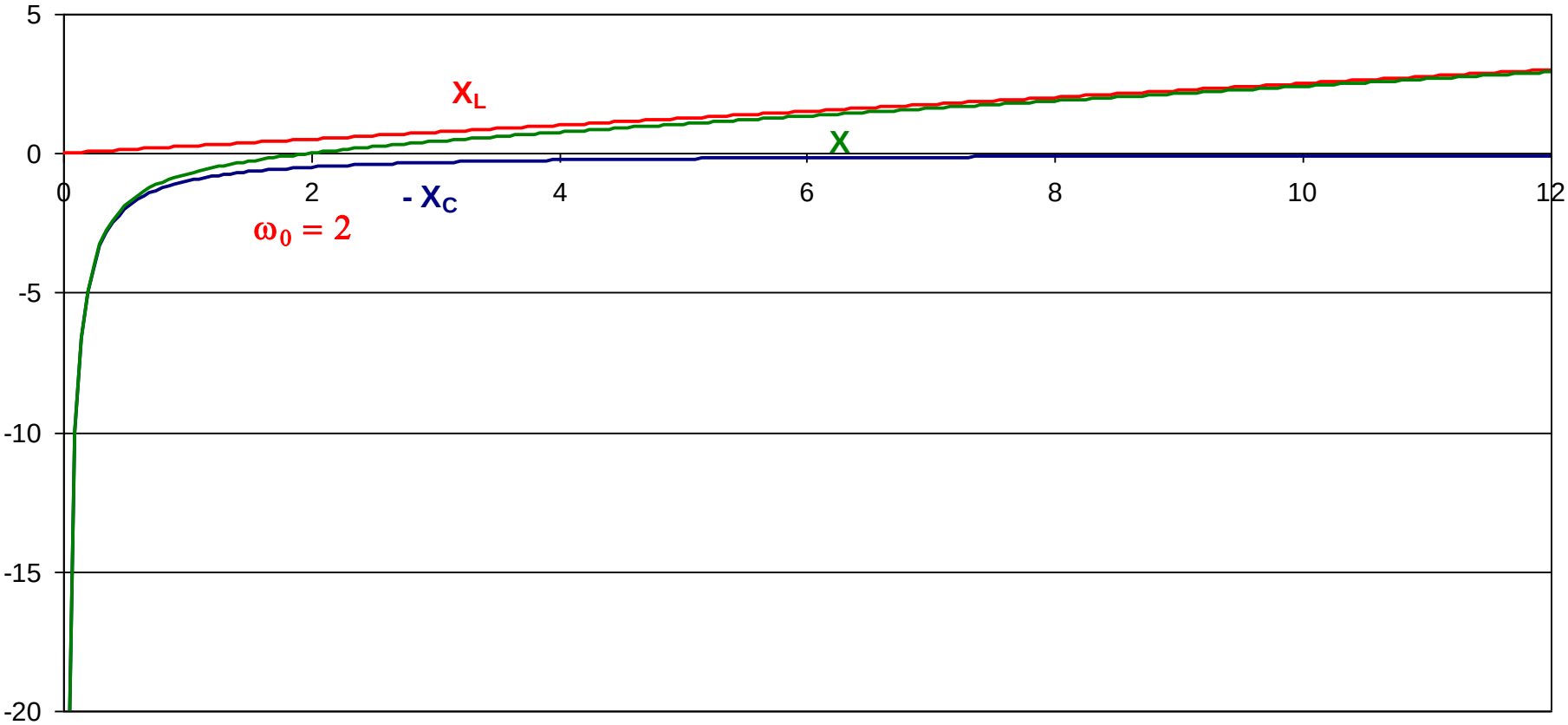
$$I_0 = \frac{U_i}{R}$$

$$U_{R0} = I_0 \cdot R = U_i$$

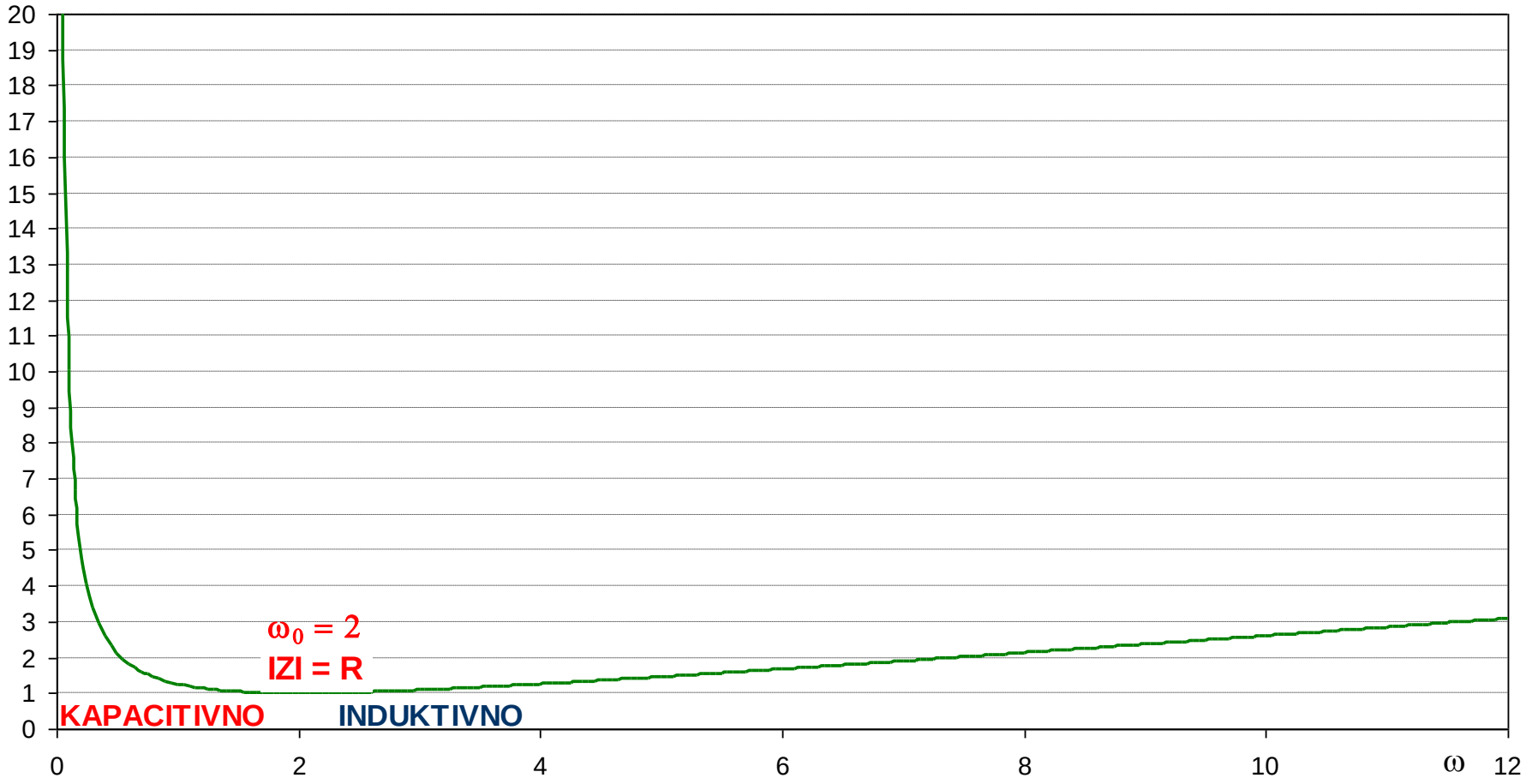
$$U_{L0} = I_0 X_{L0} = U_i \frac{\rho}{R}$$

$$U_{C0} = I_0 X_{C0} = U_i \frac{\rho}{R}$$

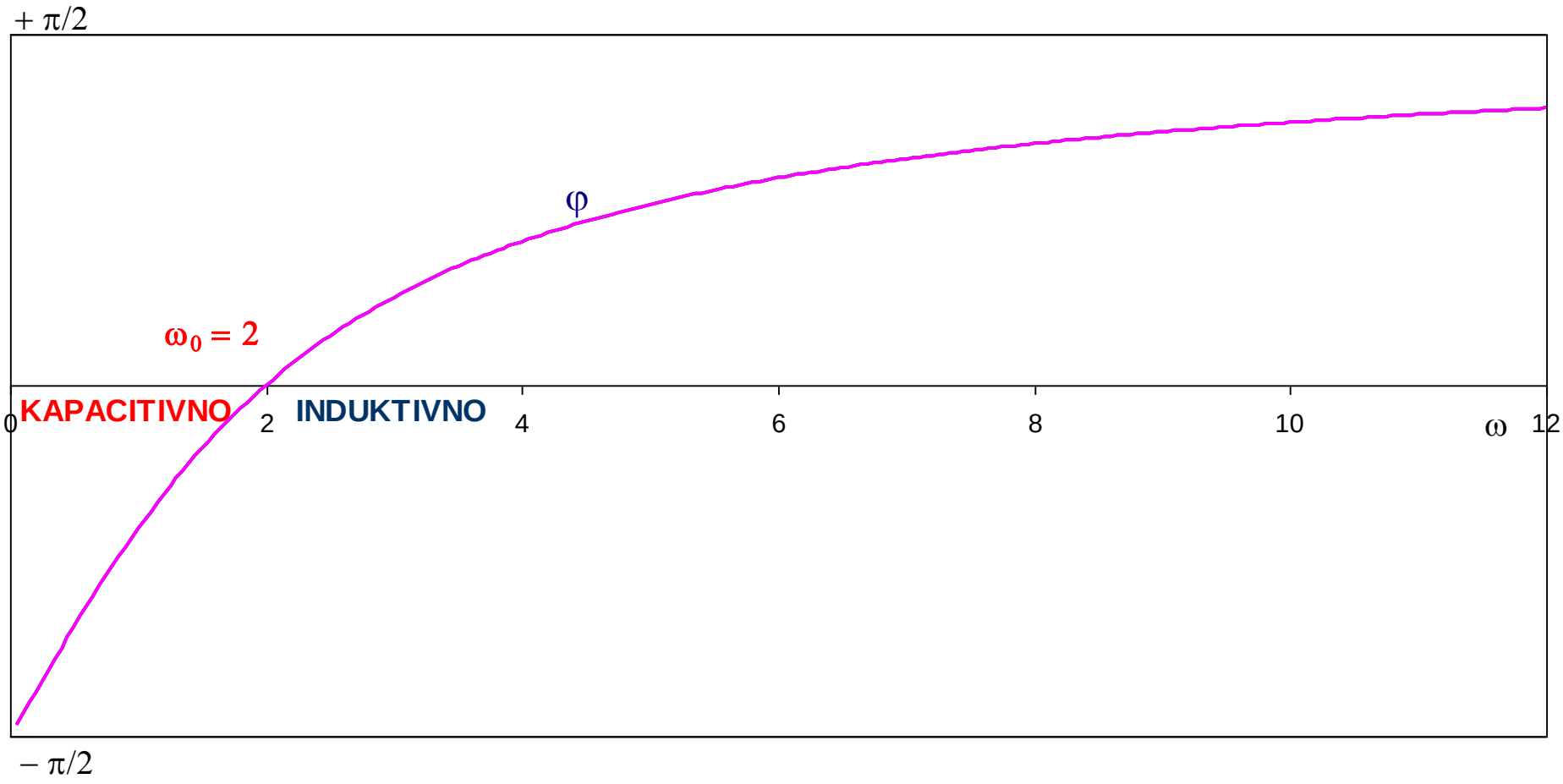
Reaktancije

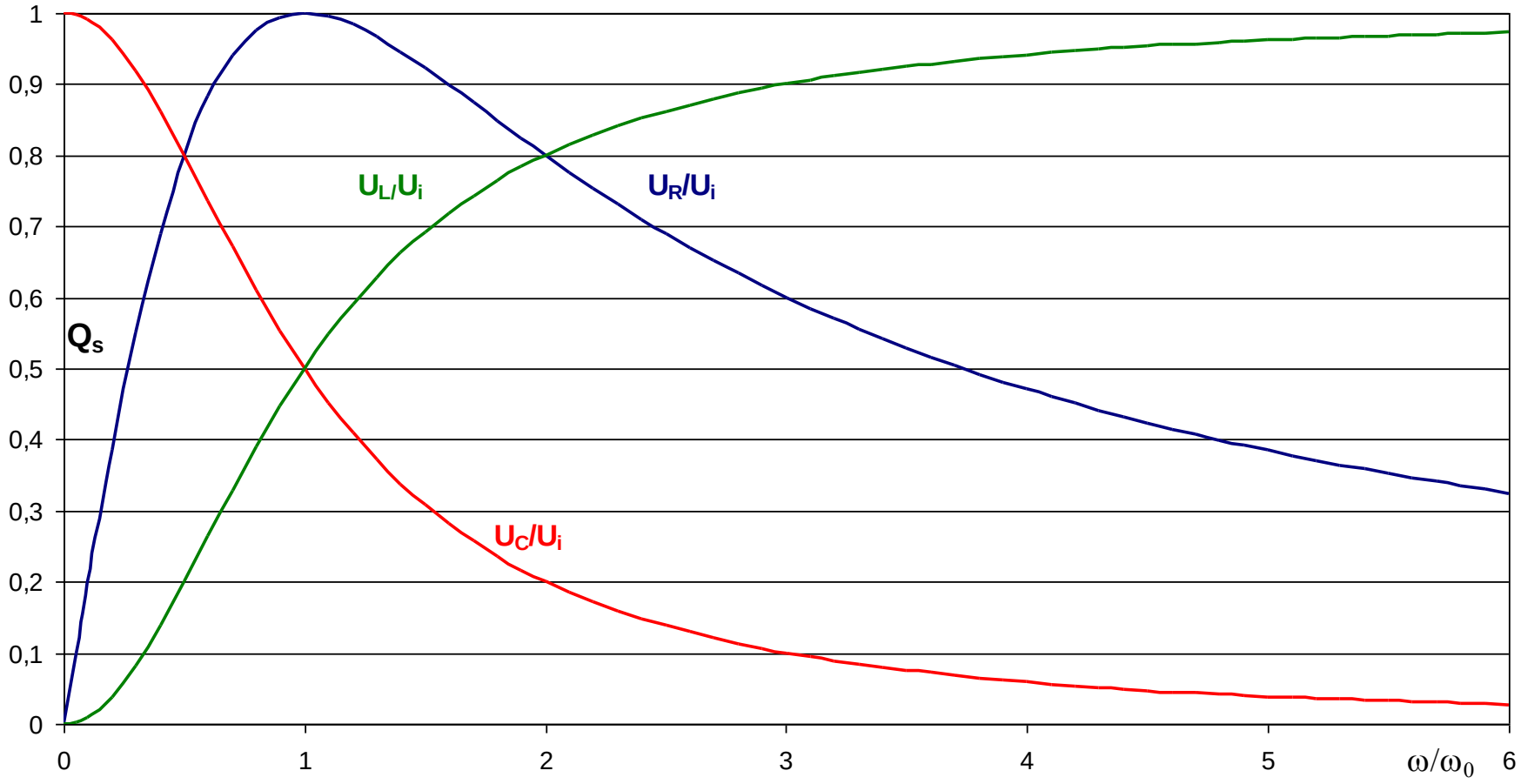


Modul impedancije

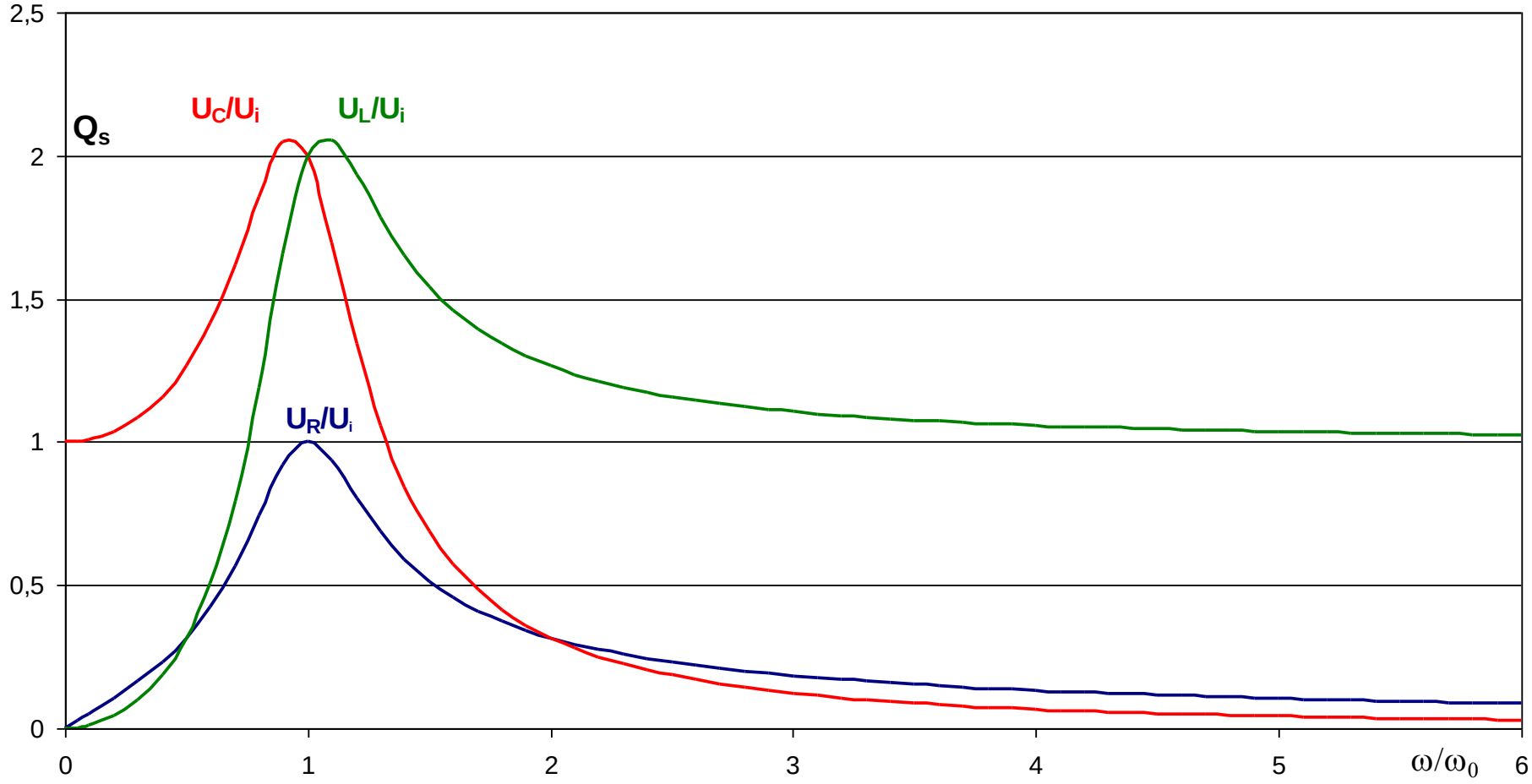


Kut impedancije



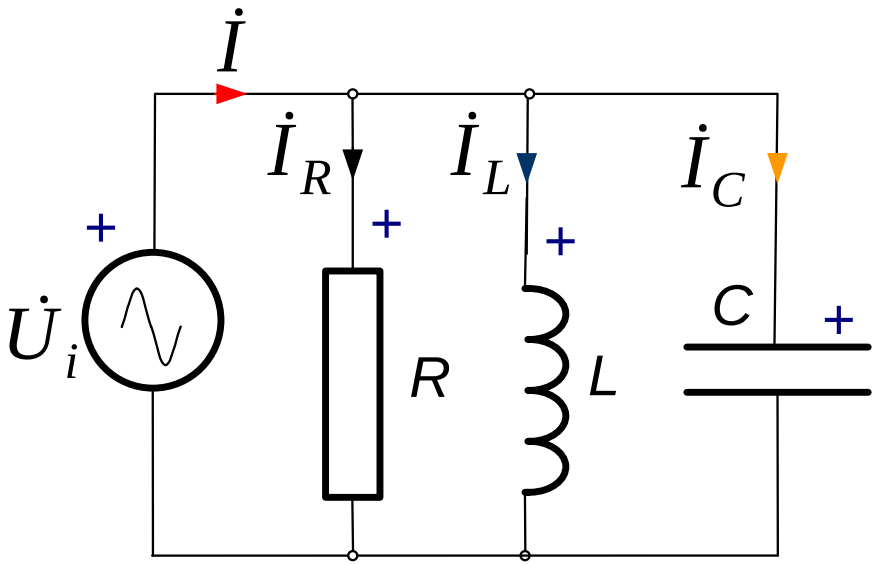


Primjer 1: napona na elementima



Primjer 2: naponi na elementima

Frekvencijske karakteristike paralelnog R,L,C kruga priključenog na naponski izvor



- Analiziraju se veličine u krugu pri promjeni frekvencije od 0 do ∞

- Admitancija kruga je:

$$\underline{Y} = G + jB = G + j(B_C - B_L) = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

- Pri promjeni frekvencije:

$$G = \text{konst.} \qquad B(\omega) = \omega C - \frac{1}{\omega L}$$

$$\underline{Y}(\omega) = |\underline{Y}(\omega)| e^{j\psi}$$

$$|\underline{Y}(\omega)| = \sqrt{G^2 + B^2} = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\psi(\omega) = \text{arctg}\left(\frac{B}{G}\right) = \text{arctg}\left(\frac{\omega C - \frac{1}{\omega L}}{G}\right)$$

- Pri frekvenciji ω_0 imaginarni dio admitancije jednak je nuli. Frekvencija ω_0 naziva se **rezonantna frekvencija** i krug je **u rezonanciji**:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

U rezonanciji je: $B(\omega_0) = \omega_0 C - \frac{1}{\omega_0 L} = 0$ $\quad \underline{Y}(\omega_0) = G = \frac{1}{R}$

$$B_{C0} = \omega_0 C = \frac{1}{\sqrt{LC}} C = \sqrt{\frac{C}{L}} = \gamma \qquad B_{L0} = \frac{1}{\omega_0 L} = \frac{\sqrt{LC}}{L} = \sqrt{\frac{C}{L}} = \gamma$$

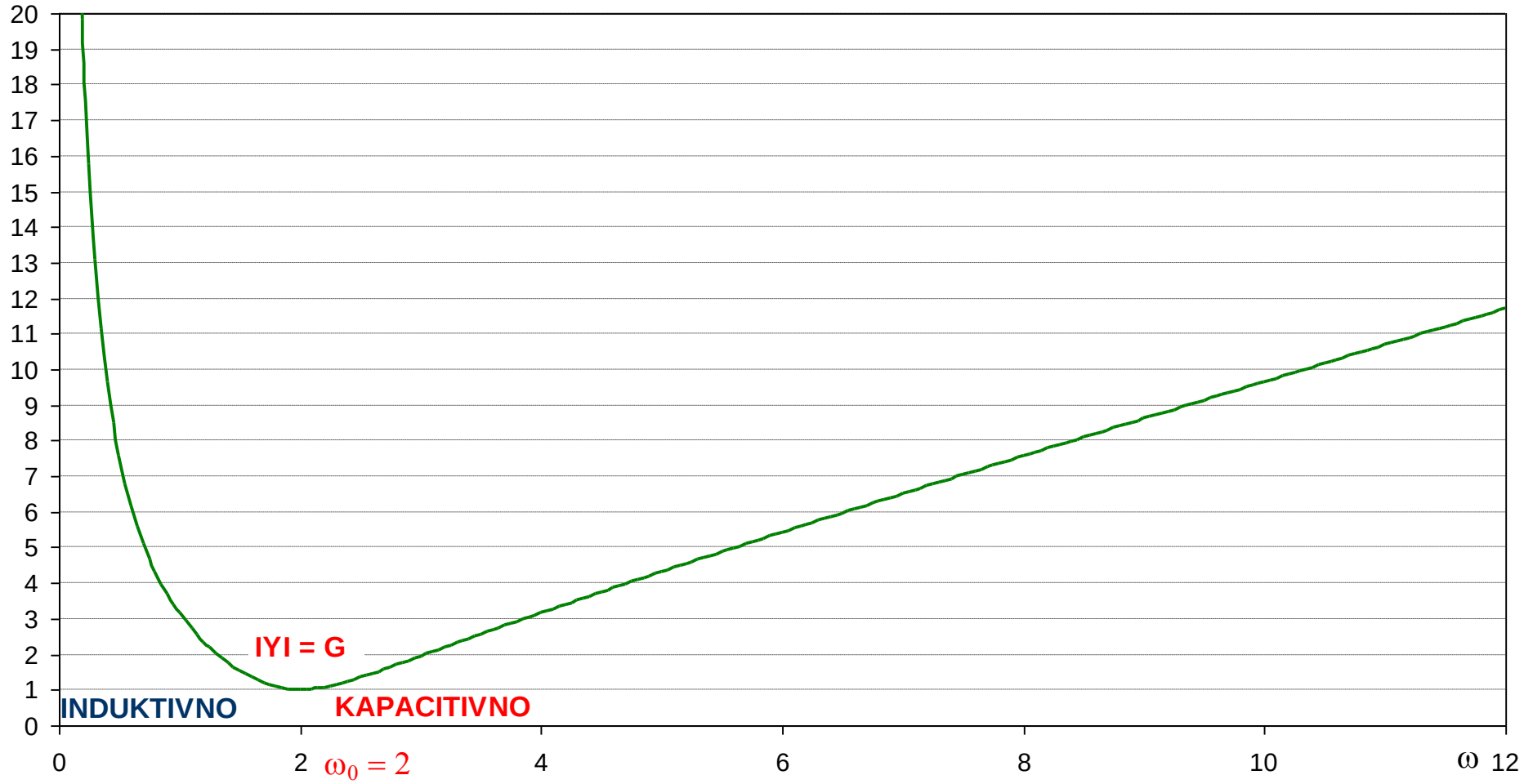
- γ - **valna vodljivost**; ρ - **valni otpor:**

$$\rho = \frac{1}{\gamma} = \sqrt{\frac{L}{C}} \quad (\Omega)$$

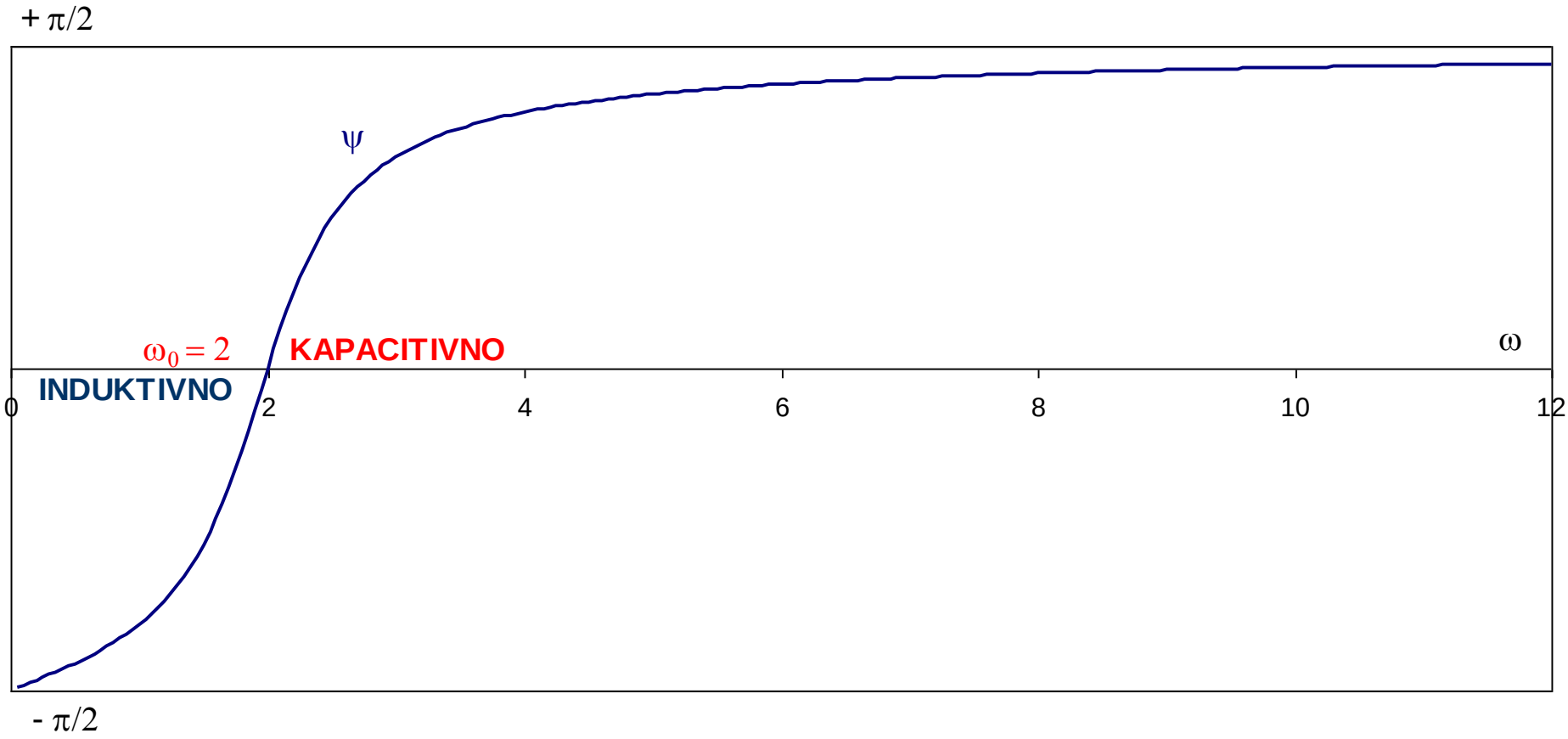
- Struje pri rezonanciji su:

$$I_{R0} = I_0 = U_i G \qquad I_{L0} = U_i B_{L0} = U_i \gamma \qquad I_{C0} = U_i B_{C0} = U_i \gamma$$

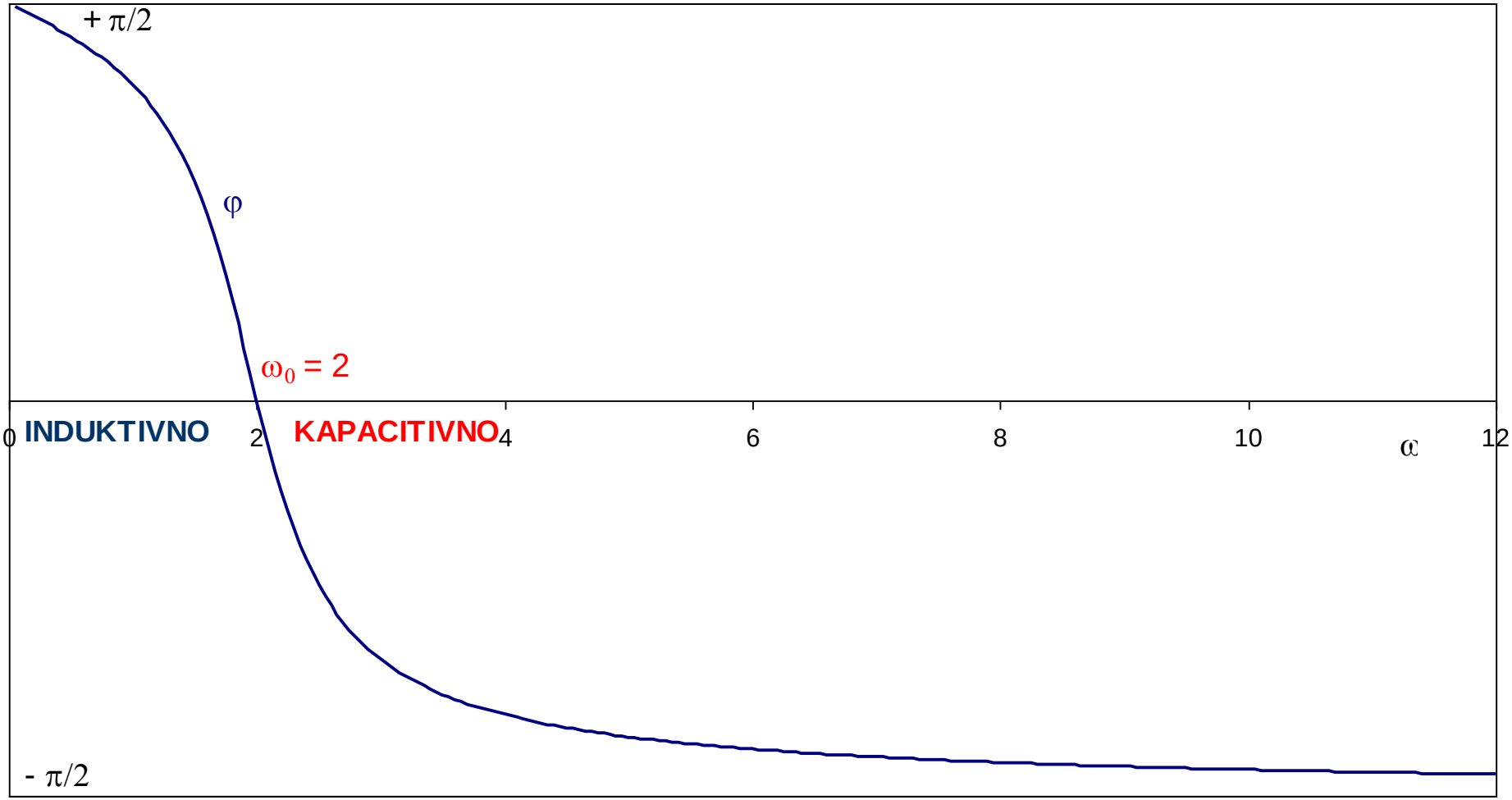
Modul admitancije



Kut admitancije



Kut impedancije



- Impedancija je:

$$\underline{Z}(\omega)=\frac{1}{\underline{Y}(\omega)}=\frac{G}{G^2+B^2(\omega)}-j\frac{B(\omega)}{G^2+B^2(\omega)}=|\underline{Z}|e^{j\varphi}$$

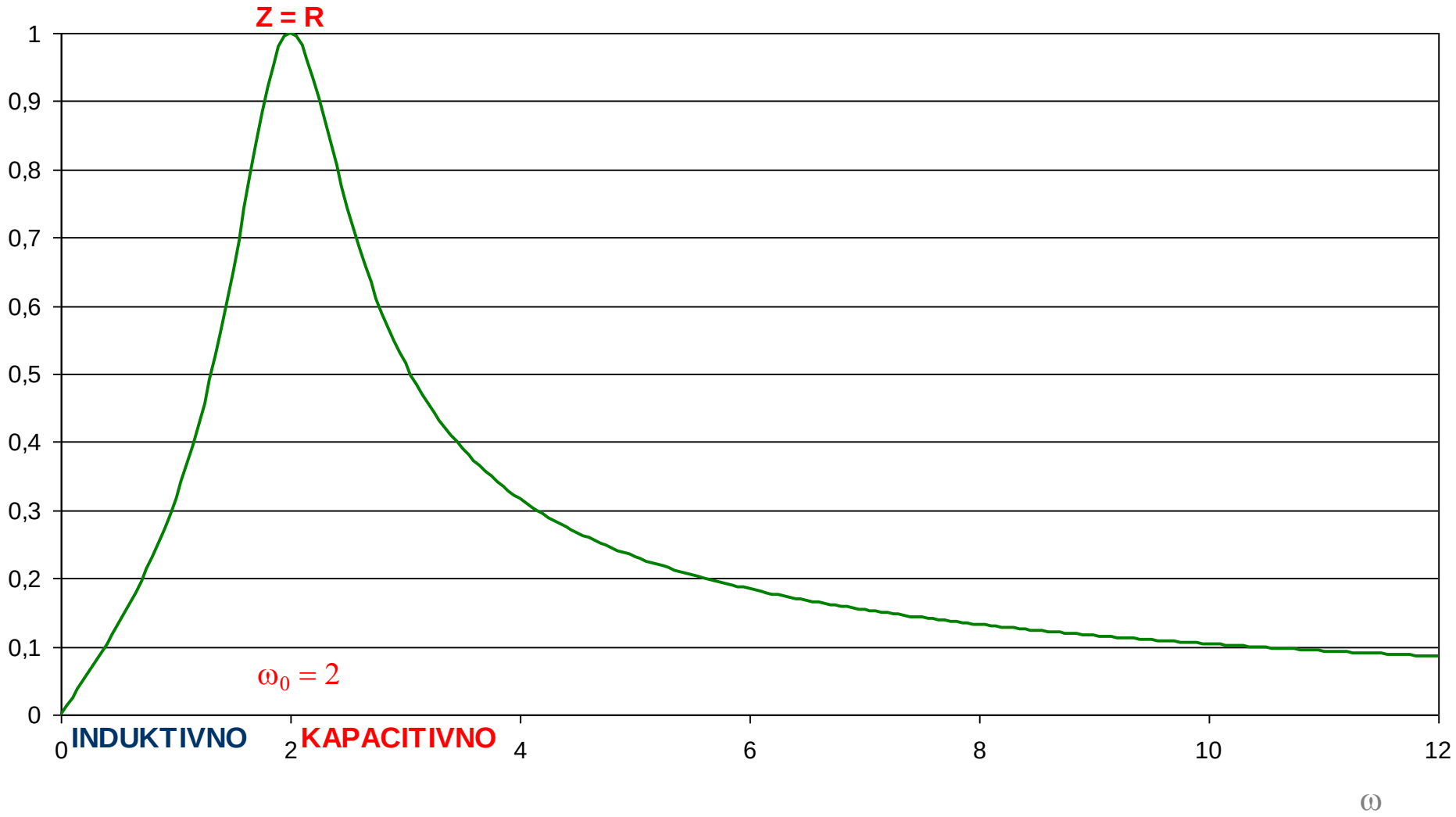
$$|\underline{Z}(\omega)|=\frac{1}{|\underline{Y}(\omega)|}=\frac{1}{\sqrt{G^2+\left(\omega C-\frac{1}{\omega L}\right)^2}}$$

$$\varphi=\operatorname{arctg}\left(-\frac{B}{G}\right)=-\psi$$

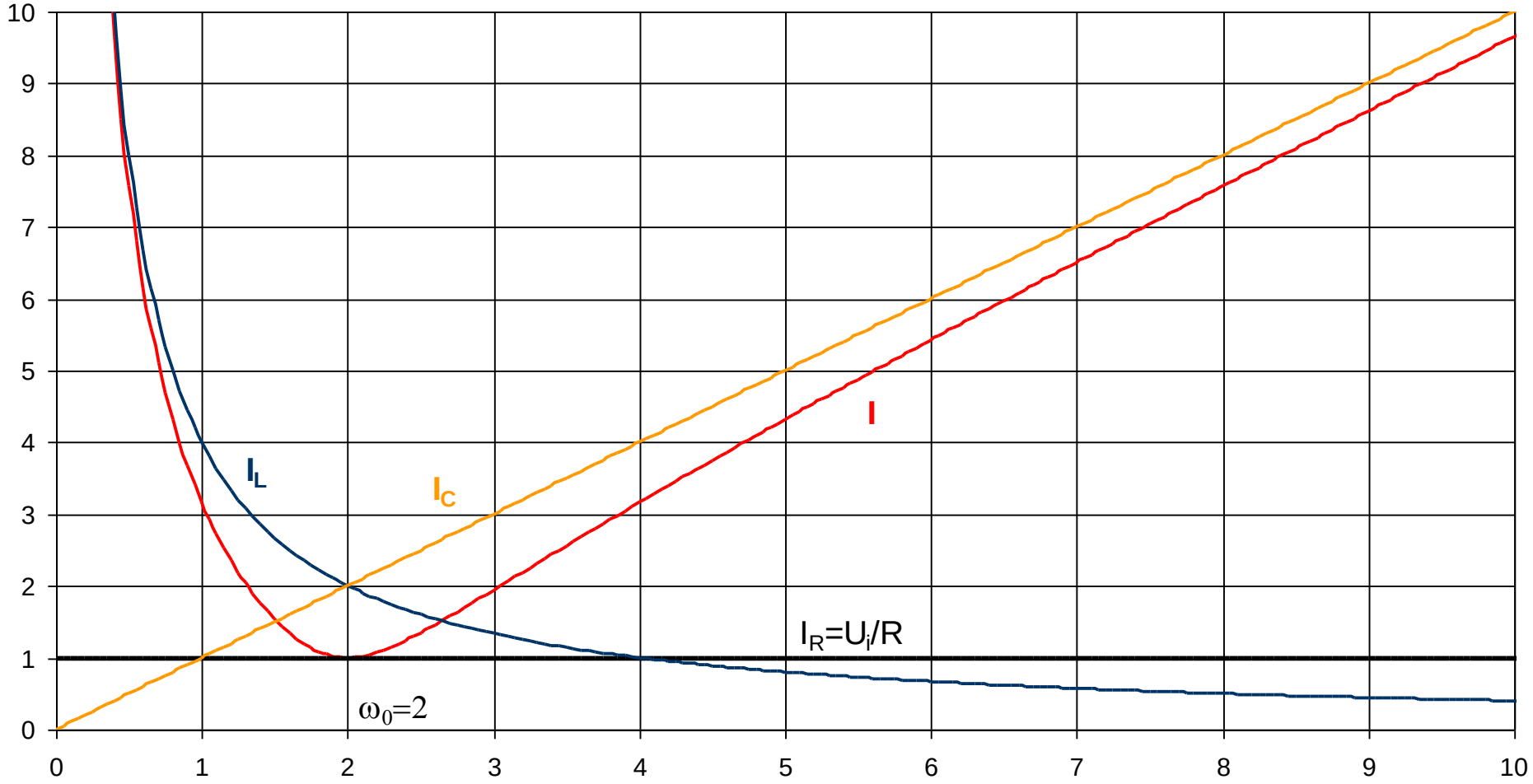
- Struja izvora je: $\dot{I}=\dot{U}_i\cdot\underline{Y}\ ;\ \frac{\dot{I}}{\dot{I}_0}=\frac{\dot{U}_i\cdot\underline{Y}}{\dot{U}_i\cdot G}=\frac{\underline{Y}}{G}$

  - u rezonanciji je minimalna $I_0=U_iG$ pa se ova rezonancija naziva i **antirezonancija**

Modul impedancije



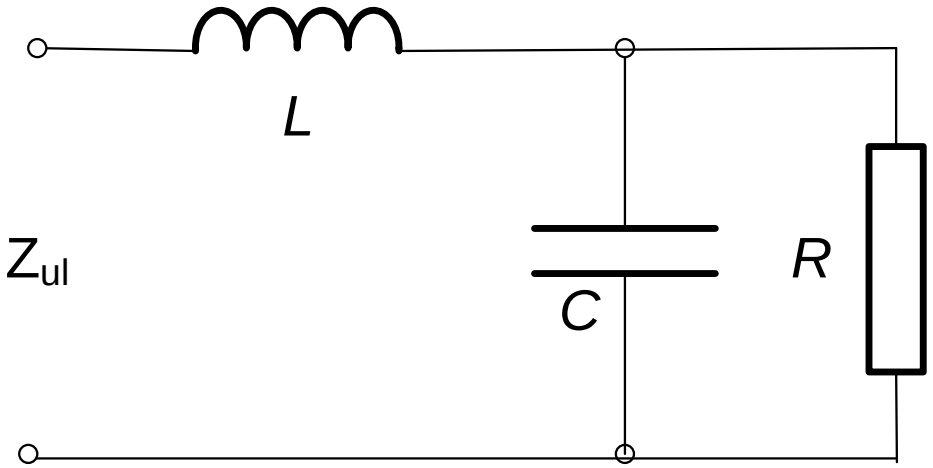
Struje u krugu



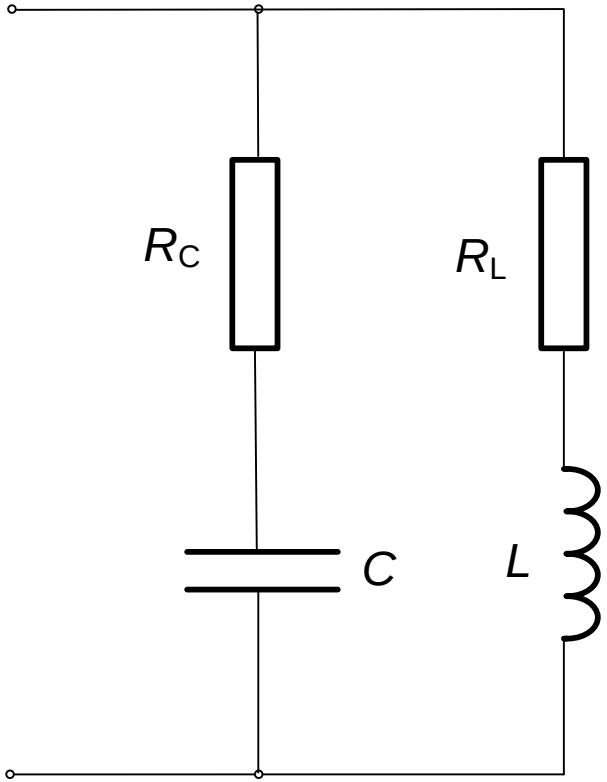
Primjer 4.

Za spoj prema slici ulazna impedancija kod frekvencije ω=0 iznosi 5 Ω, a kod rezonantne frekvencije je 2,5 Ω. Koliki je X_C?

(Rješenje: Xc=5 Ω)



5. Odrediti rezonantnu frekvenciju za krug prema slici.



$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - j\dfrac{1}{\omega C}}$$

$$\underline{Y} = \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right) +$$

$$+ j\left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right)$$

Rezonancija:

$$\mathrm{Im}\{\underline{Y}\} = 0 \Rightarrow \frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\frac{\frac{1}{\omega_0 C}}{R_C^2+\frac{1}{(\omega_0 C)^2}}=\frac{\omega_0 L}{R_L^2+(\omega_0 L)^2}\Rightarrow\omega_0=\frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2-\rho^2}{R_C^2-\rho^2}}\quad;\quad\rho^2=\frac{L}{C}$$

- Da bi se ispunio uvjet za rezonanciju mora biti:

$$R_L^2>\frac{L}{C}\ \text{ i }\ R_C^2>\frac{L}{C}\quad\text{ili}\quad R_L^2<\frac{L}{C}\ \text{ i }\ R_C^2<\frac{L}{C}$$

- Specijalni slučaj:

$$R_L^2=R_C^2=\frac{L}{C}\Rightarrow\omega_0=\frac{0}{0}$$ krug rezonira na svim frekvencijama