

# VIII TITIKAR PREDAVANJA

## VIII.1 - RLC KROGOVI

①  $R = 7.5 \Omega$

$U = 25 V$

$f = 50 Hz$

$L = ?$

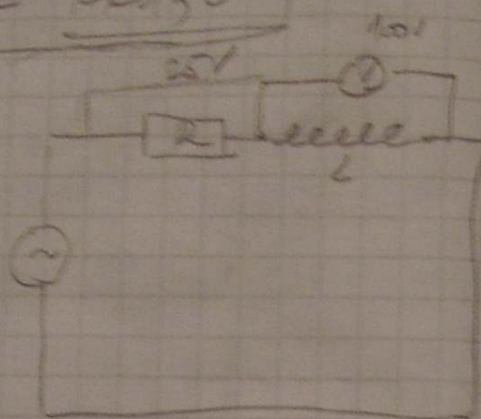
$I = ?$

$$I = \frac{U}{R} = \frac{25 V}{7.5 \Omega} = 3.33 A$$

$$X_L = \frac{U}{I} = \frac{100 V}{10 A} = 10 \Omega$$

$\omega L = 10 \Omega$

$$L = \frac{10 \Omega}{2\pi \cdot 50} = 0.032 \Omega$$



$U_{ef} = 25 V$

INDUKTIVNO  
PONAŠANJE KROGA

②  $R = 100 \Omega$

$C = 43 \mu F$

$U = 220 V$

$f = 50 Hz$

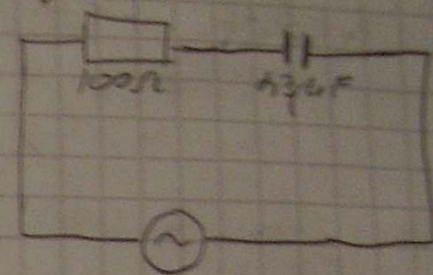
$Z = ?$

$U_C = ?$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{10000 + 5479.72}$$

$= 124.41 \Omega$

KAPACITIVNO  
PONAŠANJE KROGA

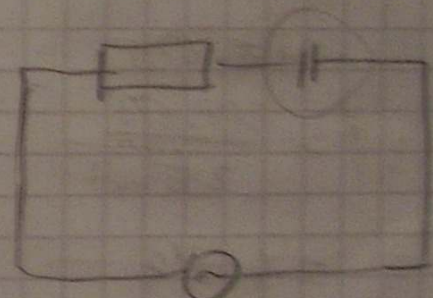


③ ZARADJE 160 V / 40 W

220 V / 50 Hz

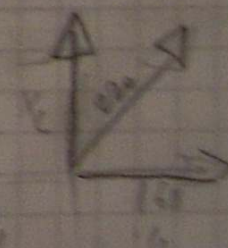
$P = U \cdot I \Rightarrow I = \frac{P}{U}$

$= \frac{40 W}{220 V} = 0.18 A$



$U^2 - U_R^2 = U_C^2$

$U_C = 151 V$



$X_C = \frac{U}{I}$

$C = \frac{1}{2\pi f X_C} = 5.27 \mu F$



④  $R=5\Omega$   $L=0.03H$

$U=U_{max}(\sin \omega t)$

$I=I_{max}(\sin \omega t - \frac{\pi}{3})$

$Z = \sqrt{R^2 + \omega^2 L^2}$

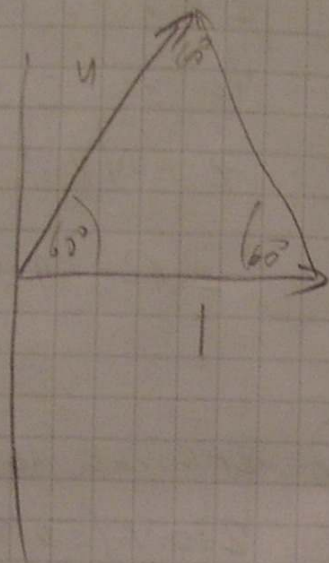
$3R^2 = \omega^2 L^2$

$\frac{75 \Omega^2}{9 \cdot 10^{-4} H^2} = \omega^2$

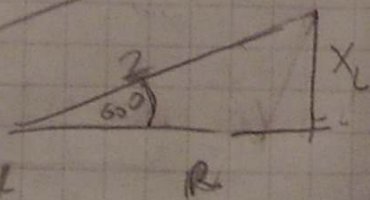
$\omega = 288.675$

$f = \frac{288.675}{2\pi}$

$= 46 Hz$



$Z = \frac{U}{I} =$



$\cos 60^\circ = \frac{R}{Z}$

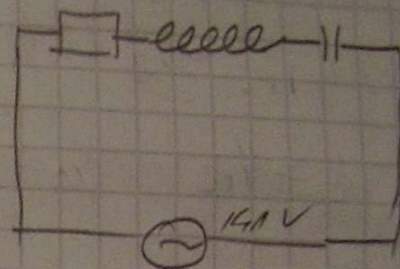
$L = \frac{1}{\omega} Z$

$Z = 2R$

$Z^2 = R^2 + X_L^2$

⑤  $R=4\Omega$   $X_L=3\Omega$   $X_C=6\Omega$

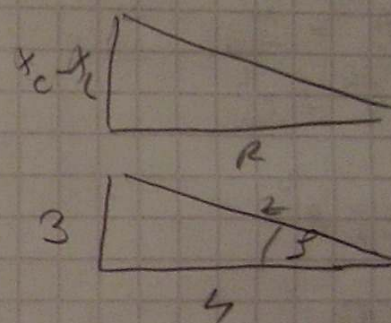
$U_{eff} = 100 V$



$Z^2 = 3^2 + 4^2$

$Z^2 = 25$

$Z = 5\Omega$



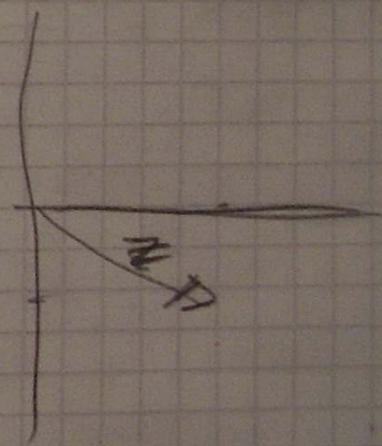
$\cos \varphi = \frac{R}{Z}$

$\varphi = 36.9^\circ$

$Z = 5 \angle -36.9^\circ$

$I = \frac{U}{Z} = \frac{100}{5} = 20$

$I = 20 \angle 36.9^\circ$



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⑥  $R_1 = 1\Omega$   $R_2 = 2\Omega$   $X_L = 1\Omega$   $X_C = 0.5\Omega$   $X_C = 1\Omega$

$$I_1 = \frac{U}{R} = \frac{2V}{1\Omega} = 2A \quad I_{R_2} = 1A \quad I_{X_L} = 2A$$

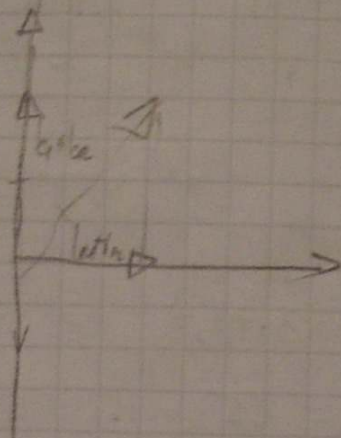
$$I_{X_C} = 4A \quad X_C = 2A$$

$$I^2 = (I_1 + I_{X_C} - I_L)^2 + (I_{R_2} + I_{X_C})^2$$

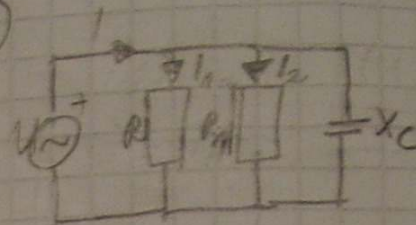
$$I^2 = 4^2 + 3^2$$

$$I^2 = 25A^2$$

$$I = 5A$$



⑦



$$I = 3 \quad I_1 = 2A \quad I_2 = 2A \quad R_1 = 4\Omega$$

$$P_R = ?$$

$$I_2^2 = I_1^2 + I_{X_C}^2$$

$$I^2 = (I_1 + I_{X_C})^2 + I_{X_C}^2$$

$$G = \frac{1}{I_1^2} + 2 \frac{1}{I_1 R_1} + \frac{1}{R_1^2} + \frac{1}{X_C^2}$$

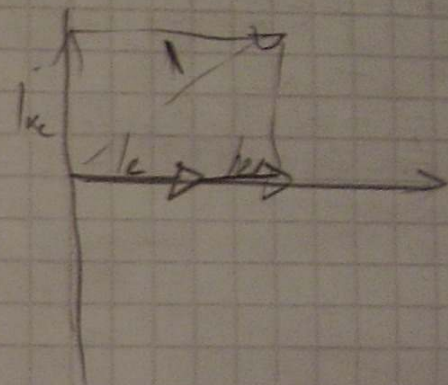
$$G = 4 + 4 + 2 \frac{1}{1 \cdot 4}$$

$$\frac{1}{2} = \frac{1}{1} \cdot \frac{1}{R_1}$$

$$\boxed{\frac{1}{4} = \frac{1}{R_1}}$$

$$U = I \cdot R = 4 \cdot \frac{1}{4} = 1V$$

$$R = \frac{U}{I} = \frac{1V}{2A} = \frac{1}{2}\Omega$$



$$P = I^2 \cdot R = 4 \cdot \frac{1}{2} = 2W$$

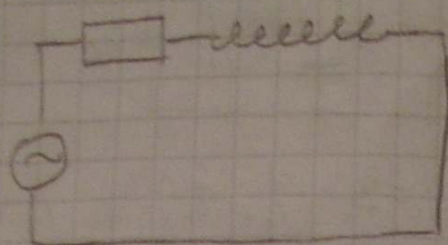


⑧  $\underline{U} = 50 \angle 45^\circ$

$\underline{I} = 2.5 \angle -15^\circ$

$R = 5 \Omega$

$\underline{X}_L = j8 \Omega$



$\underline{Z} = \frac{\underline{U}}{\underline{I}} = 20 \angle 60^\circ = 10 + j17.32$

$\underline{Z}_1 = 10 + j17.32 - 5 - j8 \Omega = 5 + j9.32$

$30^\circ$

$115^\circ$

⑨

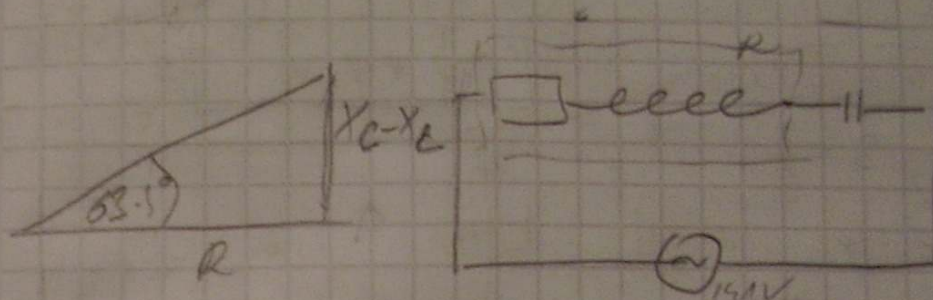
$L = 25 \text{ mH}$

$C = 50 \mu\text{F}$

$U = 120 \text{ V}$

$\varphi = 63.4^\circ$

$\omega = 400 \frac{\text{rad}}{\text{s}}$



$X_L = \omega \cdot L = 400 \cdot 25 \cdot 10^{-3} = 10 \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{400 \cdot 50 \cdot 10^{-6}} = 50 \Omega$

$\tan 63.4^\circ = \frac{X_C - X_L}{R}$

$R = \frac{40 \Omega}{\tan 63.4^\circ} = 20 \Omega$

$\underline{Z} = 20 \Omega + j10 \Omega - j50 \Omega = 20 \Omega - j40 \Omega$

$\underline{Z} = 44.72 \angle -63.43^\circ$

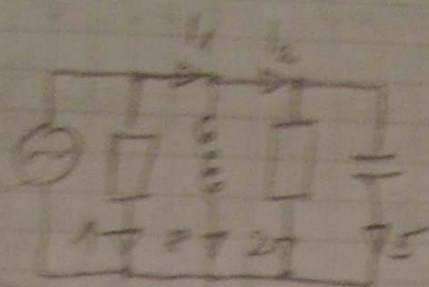
$I = \frac{U}{Z} = 2.6833 \angle 63.43^\circ$

$U_R = 53.66 \text{ V} \quad U_L = 26.833 \text{ V}$

$U_C = 134.165 \text{ V}$



⑥

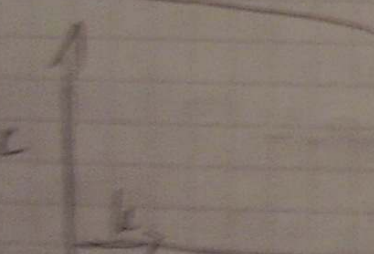
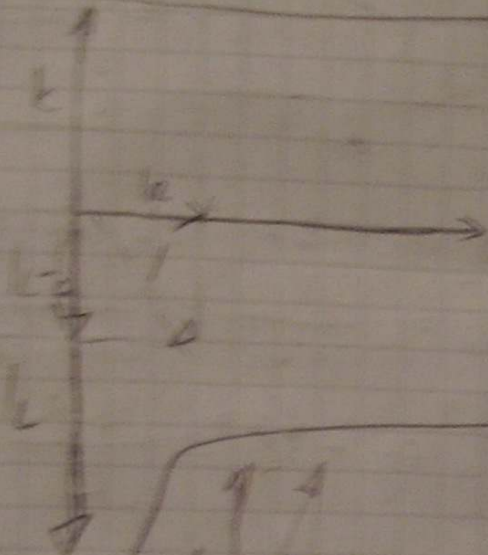


$$I^2 = 3^2 + 3^2$$

$$I^2 = 18$$

$$I = 4.2426$$

$$I =$$



$$I^2 = 2^2 + 2^2$$

$$I^2 = 8$$

$$I = 2.83A$$

$$I^2 = 5^2 + 2^2$$

$$I^2 = 29$$

$$I = 5.39A$$

⑦

$$I = 100V, R_1 = 20\Omega, R_2 = 20\Omega, R_3 = 5\Omega$$

$$I_1 = 5A, I_2 = 5A$$

$$I_3 = 20A$$

$$I^2 = 20^2 - 15^2$$

$$I^2 = 500$$

$$I = 22.36A$$

$$I = \frac{100}{25} = 4A$$

$$U_1 = 80V$$

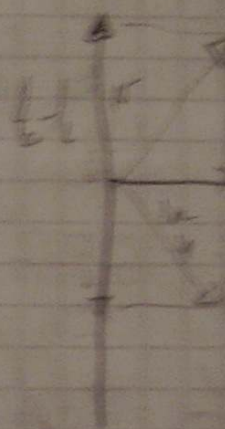
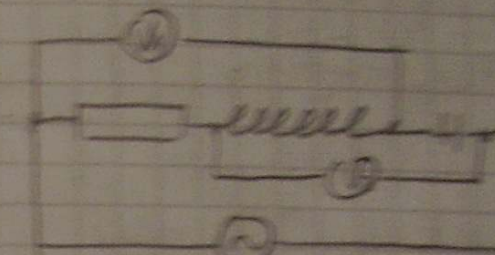
$$U_2 = 80V - U_1 = 10V$$

$$U_3 = U_1 - U_2 = 60V$$

$$U_1^2 = U_2^2 + U_3^2$$

$$U_1^2 = 12800V^2$$

$$U_1 = 113V$$





(12)  $U = 220V / 50Hz$

$I = 11A$

$U = U_L = 660V$

$U_C = U_C = 500V$

$U^2 = U_R^2 + (U_L - U_C)^2$   
 $220^2 = U_R^2 + 160^2$

$U_R = 151V$

$R = \frac{U_R}{I} = \frac{151V}{11A} = 13.7\Omega$

$X_L = \frac{U_L}{I} = \frac{660V}{11A} = 60\Omega$

$60\Omega = \omega L \Rightarrow L = \frac{60}{2\pi \cdot 50} = 0.191H$

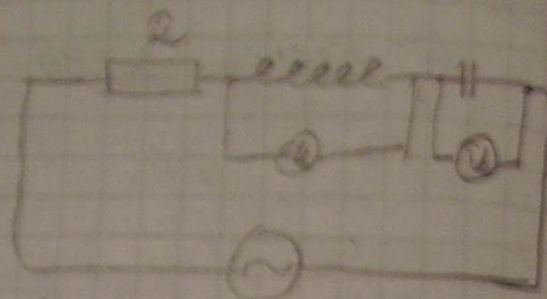
$X_C = \frac{U_C}{I} = \frac{500V}{11A} = 45.45\Omega$

$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = 70\mu F$

$\cos \phi = \frac{U_L - U_C}{U_R} = \frac{14.54V}{13.7V} = 1.0613$

$\phi = 46.7^\circ$

(inductive  
power)



(13)  $U = 25V \quad \omega = 100 \frac{rad}{s} \quad R = 12.5\Omega$

$C = 200\mu F \quad L = 100mH$

$I = ?$

$X_C = 50\Omega$

$X_L = 10\Omega$

$X_C > X_L$  inductive  
power

$I_C = \frac{U}{X_C} = \frac{25}{50} = 0.5A$

$I_L = \frac{U}{X_L} = \frac{25}{10} = 2.5A$

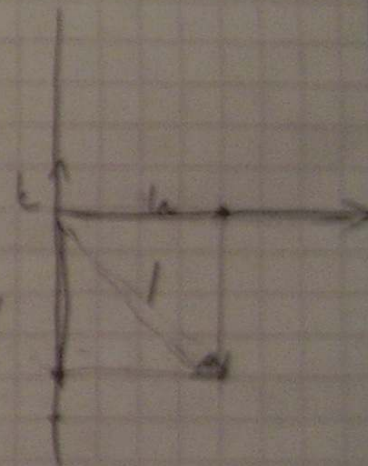
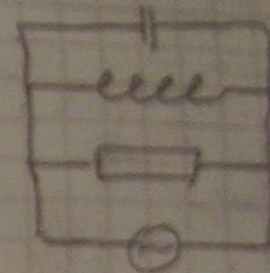
$I_R = \frac{U}{R} = \frac{25}{12.5} = 2A$

$I = 2 - j2A$

$I^2 = 2^2 + 2^2$

$I^2 = P$

$I = 2.82A$





16)  $U = 110V$   $f = 50Hz$   $R = 10\Omega$   $X = 60\Omega$

$$Z = 10 + j60$$

$$Y = \frac{1}{Z} = \frac{1}{10 + j60} \cdot \frac{10 - j60}{10 - j60} =$$

$$= \frac{10 - j60}{100 + 3600} = \frac{10 - j60}{3700}$$

$$= 0.0027 - j0.016$$

$$G = 0.0027S \quad B = -0.016S$$

$$I = Z \cdot U = 0.01644 \cdot 100 = 1.644A$$

$$R = \frac{1}{G} = \frac{1}{0.0027S} = 370.37\Omega$$

$$X_L = \frac{1}{0.016} = -62.5$$

$$L = \frac{62.5}{2\pi \cdot 50} = 0.197H$$

17)  $R = 10\Omega$   $C = 50\mu F$

$$\tan 30^\circ = \frac{X_C}{R}$$

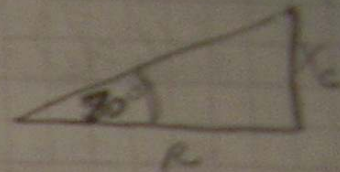
$$R \tan 30^\circ = X_C$$

$$27.4757 = \frac{1}{2\pi f C}$$

$$f = \frac{1}{2\pi C \cdot R \cdot \tan 30^\circ}$$

$$f_1 \cdot \tan 30^\circ = f_2 \cdot \tan 70^\circ$$

$$f_2 = 115Hz$$





(18)  $L = 0.015 \text{ H}$   $R = 620 \Omega$   $U = 10 \text{ V}$   
 $f(I_R = I_L) = ?$   $I_R = I_L = ?$   $I_L = ?$

$$I_R = \frac{U}{R} = \frac{10}{620} = 0.01613 \text{ A}$$

$$= 10 \text{ mA}$$

$$I_L = 2(16 \cdot 10^{-3})^2$$

$$I_L = 22.56 \text{ mA}$$

$$U = \frac{I}{R}$$

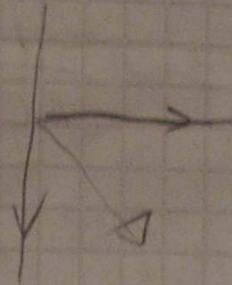
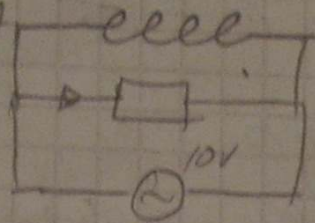
$$620 = 2\pi f \cdot L$$

$$f = \frac{620}{2\pi \cdot 0.015} = 6578 \text{ Hz}$$

$$\tan \varphi = \frac{610 \Omega}{620 \Omega} = 45^\circ$$

$$\varphi = 45^\circ$$

Struga razlike za ugraditi  
 $45^\circ$



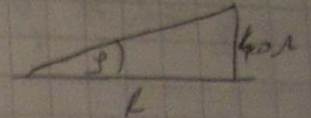
(19)  $U = 12 \text{ V}$   $\omega = 400 \frac{\text{rad}}{\text{s}}$   $L = 25 \text{ mH}$   
 $C = 50 \mu\text{F}$   $\varphi = 63.5^\circ$

$$X_C = 50 \Omega$$

$$X_L = 10 \Omega$$

$$\tan \varphi = \frac{X_C}{R}$$

$$R = \frac{X_C}{\tan 63.5^\circ} = 20 \Omega$$



$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 44.72 \Omega$$

$$I = \frac{U}{Z} = \frac{12}{44.72} = 0.2683 \text{ A}$$

$$U_R = I \cdot R = 5.4 \text{ V}$$

$$U_L = I \cdot X_L = 2.7 \text{ V}$$

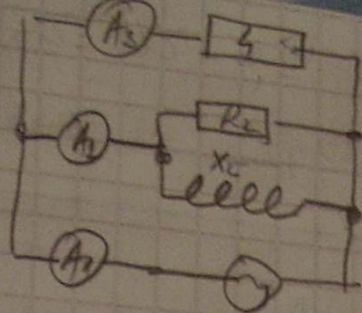
$$U_C = I \cdot X_C = 13.4 \text{ V}$$



$$I_1 = 30 \text{ A}$$

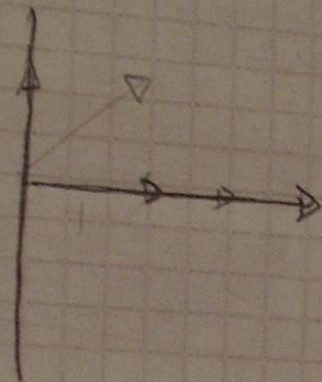
$$I_2 = 18 \text{ A}$$

$$I_3 = 1 \text{ A}$$



$$U = I_3 \cdot R = 60 \text{ V}$$

$$Z = \frac{U}{I} = \frac{60 \text{ V}}{18 \text{ A}} = 3.33 \Omega$$



$$(I_R + I_R)^2 + I_{X_L}^2 = I^2$$

$$(I_R + 15)^2 + I_{X_L}^2 = 30^2$$

$$18^2 + 30 I_R + 225 = 900$$

$$I_R = 11.7$$

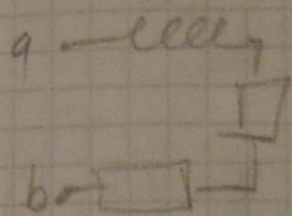
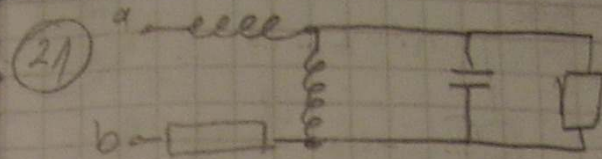
$$R = \frac{60 \text{ V}}{11.7 \text{ A}} = 5.13 \Omega$$

$$I_{X_L}^2 = 18^2 - I_R^2$$

$$I_{X_L} = 18.11$$

$$I_{X_L} = 13.67$$

$$X_L = \frac{U}{I} = \frac{60 \text{ V}}{13.6788 \text{ A}} = 4.3863 \Omega$$



$$Z = 100 + 50j$$

$$\frac{1}{Y} = \frac{1}{100 + 50j} \cdot \frac{100 - 50j}{100 - 50j} =$$

$$= \frac{100 - 50j}{12500} = \frac{1}{125} - \frac{j}{250}$$

$$\angle \phi = \frac{-\frac{1}{250}}{\frac{1}{125}} = -\frac{1}{2} \quad \phi = -26.5^\circ$$

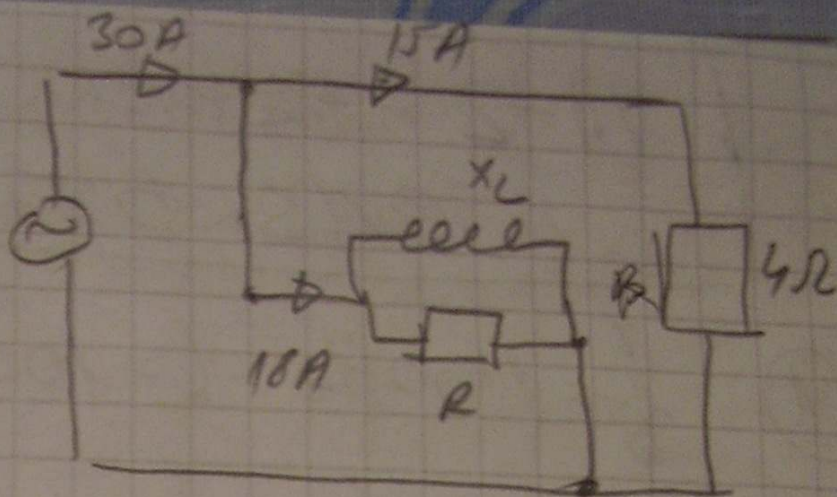
$$Y^2 = \left(\frac{1}{125}\right)^2 + \left(-\frac{1}{250}\right)^2 = 9$$

$$Y = 9 \text{ mS} \angle -26.5^\circ$$



23

$$U = 15A \cdot 4\Omega = 60V$$



$$\left( \frac{U}{R} + I_{R_1} \right)^2 + I_{x_L}^2 = I^2$$

$$\frac{U^2}{R^2} + 2 \frac{U}{R} I_{R_1} + I_{R_1}^2 + I_{x_L}^2 = I^2$$

$$225 + 324 + 20 I_R = 900$$

$$I_R = 11.7$$

$$I_{x_L}^2 = 324 - I^2$$

$$R = \frac{U}{I_R} = \frac{60V}{11.7A} = 5.13\Omega$$

$$I_{x_L} = 13.68$$

$$X_L = \frac{U}{I_L} = \frac{60V}{13.68A} = 4.39\Omega$$



①  $f = 1100 \text{ Hz}$   $L = 100 \text{ mH}$

$$R = 2\pi fL = 2\pi \cdot 1100 \text{ Hz} \cdot 100 \cdot 10^{-3}$$

$$= 690 \Omega$$

②  $R = 2000 \Omega$   $C = 22 \text{ nF}$   $U = 10 \text{ V}$

$$U_C = U_R$$

$$I \cdot R = I \cdot X_C$$

$$R = \frac{1}{\omega C}$$

$$f = \frac{1}{2\pi CR} = 3818 \text{ Hz}$$

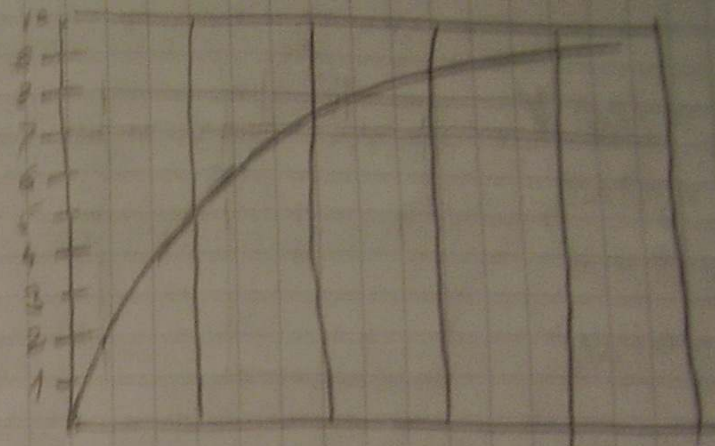
$$U_C^2 + U_R^2 = U^2$$

$$2U_C^2 = U^2$$

$$U_C \sqrt{2} = 10 \text{ V}$$

$$U_C = U_R = 7.07 \text{ V}$$

③



$$U_R = \frac{1}{\omega C}$$

$$f = \frac{1}{2\pi C U_R}$$

④  $U = 3.5 \text{ V}$

$$\omega = \frac{1}{\sqrt{2LC}}$$



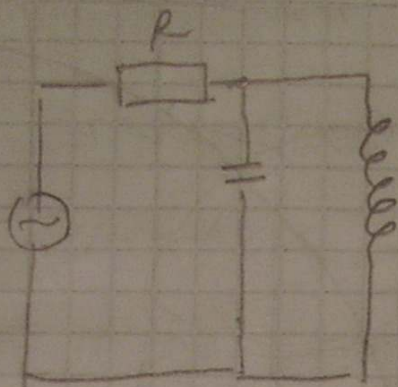
5

$$U = 120 \text{ V}$$

$$R = 100 \Omega$$

$$L = 0.1 \text{ H}$$

$$C = 40 \mu\text{F}$$



a)  $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \text{ H} \cdot 40 \mu\text{F}}} = 80 \text{ Hz}$

b) Kako je kraj u rezonanciji onda je stav  
napon na paralelnom spoju LC.  
 $U_{\text{pr}} = 120 \text{ V}$

6

$$R = 5 \Omega \quad L = 20 \text{ mH}$$

$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 \cdot 1000^2 \cdot 20 \cdot 10^{-3}} = 1.27 \mu\text{F}$$

7

$$L = 0.5 \text{ H}$$

$$u(t) = 70.7 \sin(500t + \pi/6)$$

$$i(t) = 2.83 \sin(500t + \frac{\pi}{6})$$

$$2\pi f = 500$$

$$f = 80 \text{ Hz} \approx 79.57747 \text{ Hz}$$

$$U = 70.7 \angle 60^\circ$$

$$i = 2.83 \angle 60^\circ$$

$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 \cdot 80^2 \cdot 0.5} = 8 \mu\text{F}$$

$$Z = \frac{U}{i} = \frac{70.7 \angle 60^\circ}{2.83 \angle 60^\circ} = 25 \angle 0^\circ$$

$$= 25 + j0$$

$$Z = 25 \Omega$$

8

$$L = 1 \text{ mH}$$

$$X_L = 7 \Omega$$

$$C = 20 \mu\text{F}$$

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{-jX_C} = \frac{R_L - jX_L}{R_L^2 + X_L^2} + j\frac{X_C}{X_C^2}$$

$$= 1/4 + j0 - j0.05 - 0.05^2$$



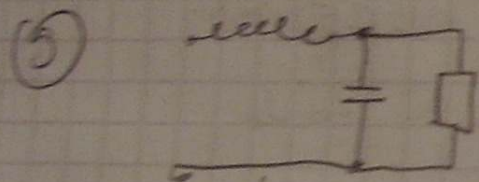
$$\frac{X_L}{X_L^2 - R^2} = \frac{1}{X_C}$$

$$\omega^2 L^2 = R^2 - \frac{L}{C}$$

$$\omega^2 = 10000000$$

$$\omega = 10000$$

$$f = 159 \text{ Hz}$$



$$Z = \frac{1}{R} - \frac{j}{X_C} + jX_L$$

$$R = \frac{1}{f \cdot C} = 0.2 \Omega$$

$$Z = \frac{1}{R} - \frac{j}{X_C} + jX_L$$

$$-\frac{j}{X_C} = X_L$$

(10) Napun rastu ( $f_1 < f_r$ ), pada kada je ( $f_2 > f_r$ )

(11) Sumo stigne pada kad je ( $f_1 < f_r$ ), a rastu kad je  $f_2 > f_r$ .

(12)  $R = X_L$   
 $R = 2\pi f L$

$$f = \frac{R}{2\pi L} = \frac{620}{2\pi \cdot 0.015} = 6578 \text{ Hz}$$

$$U_C = U_R$$

$$U_C^2 + U_C^2 = U^2$$

$$\sqrt{2} U_C = U$$

$$U_C = U_R = 7.07 \text{ V}$$

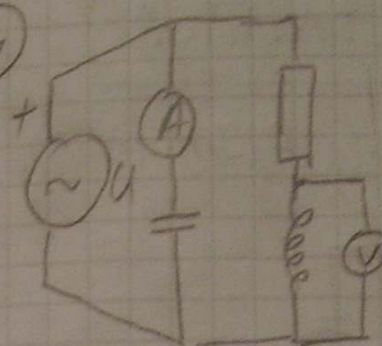
fazi kut je  $45^\circ$

$$I = \frac{7.07}{620} = 11.4 \text{ mA}$$



13

14



$$U = 10 \text{ V}$$

$$U_V = 6 \text{ V}$$

$$I_A = 1 \text{ A}$$

$$Y = \frac{1}{R + jX_L} + \frac{j}{X_C}$$

$$\dot{Y} = \frac{R - jX_L}{R^2 + X_L^2} - \frac{j}{X_C}$$

$$\frac{-X_L}{X_L^2 + R^2} = -\frac{1}{X_C}$$

$$X_L X_C = X_L^2 + R^2$$

$$\frac{U_L}{I_1} \cdot 10 \Omega = \frac{U_L^2}{I_1^2} + \frac{U_R^2}{I_1^2}$$

$$\frac{60}{1_1} = \frac{36}{1_1^2} + \frac{64}{1_1^2}$$

$$\frac{60}{1_1} = \frac{100}{1_1^2}$$

$$1_1 = \frac{5}{3} = 1.666$$

$$X_L = \frac{U}{I} = \frac{6 \text{ V}}{1.666} = 3.6 \Omega$$