

Složeni krugovi izmjenične struje

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Sadržaj:

Mosni spoj

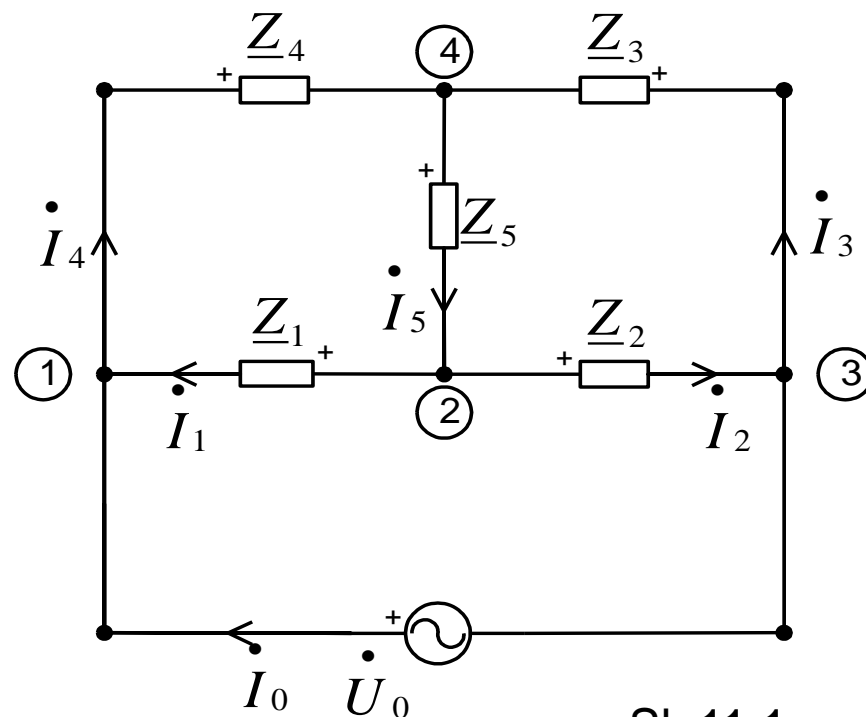
Pretvorbe spojeva trokuta i zvijezde

Rješavanje mreže jednažbama KZ

Pretvorba međuinduktivne veze

Metoda superpozicije

Mosni spoj

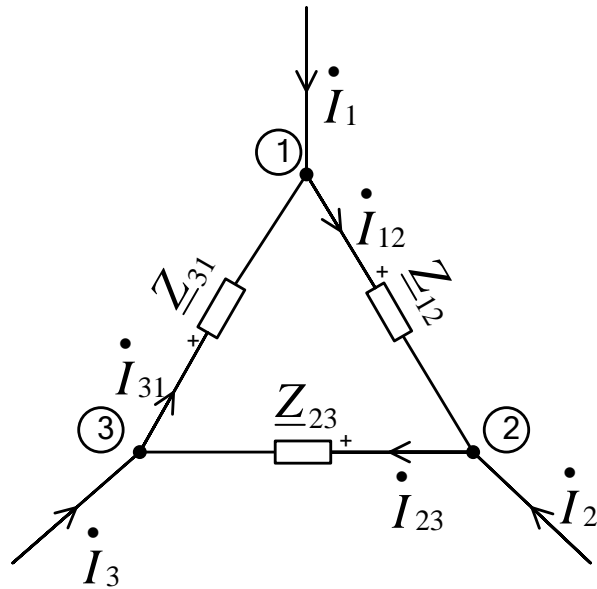


Sl. 11.1

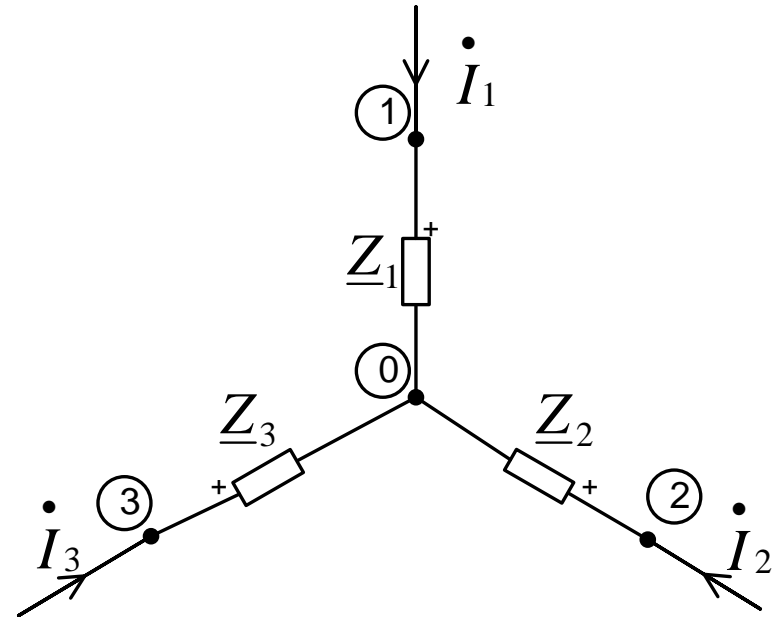
Uvjet ravnoteže mosta:

$$\underline{Z}_1 \cdot \underline{Z}_3 - \underline{Z}_2 \cdot \underline{Z}_4 = 0; \quad \frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_4}{\underline{Z}_3} \quad (11.1)$$

Pretvorbe trokut-zvijezda



Sl. 11.2a: Trokut



Sl. 11.2b: Zvijezda

Pretvorba trokuta u zvijezdu

$$\underline{Z}_1 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{31}}{\underline{Z}_{\Delta}}; \quad \underline{Z}_2 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{23}}{\underline{Z}_{\Delta}}; \quad \underline{Z}_3 = \frac{\underline{Z}_{23} \cdot \underline{Z}_{31}}{\underline{Z}_{\Delta}} \quad (11.2)$$

Gdje je:

$$\underline{Z}_{\Delta} = \underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31} \quad (11.2a)$$

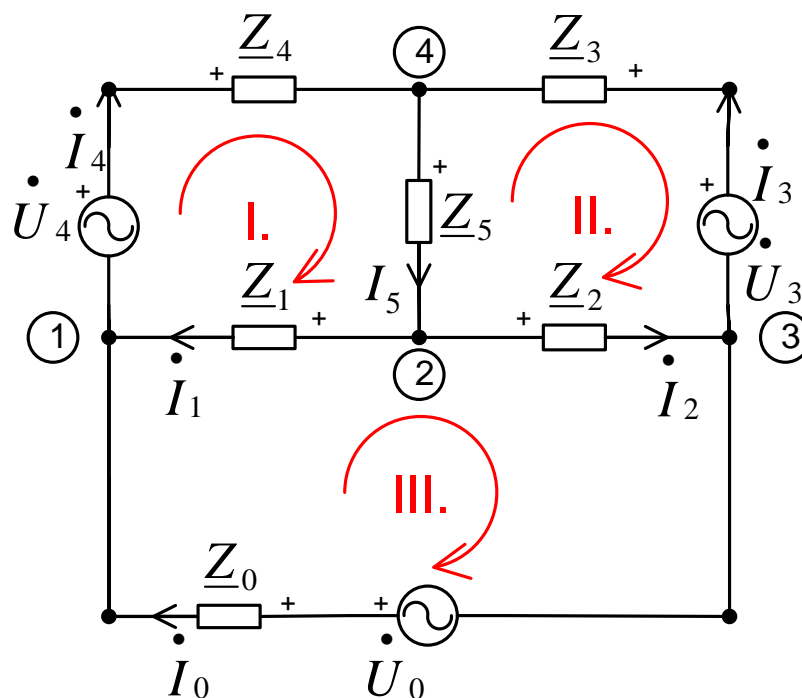
Pretvorba zvijezde u trokut

$$\underline{Z}_{12} = \underline{Z}_1 + \underline{Z}_2 + \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_3} \quad (11.3a)$$

$$\underline{Z}_{23} = \underline{Z}_2 + \underline{Z}_3 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_1} \quad (11.3b)$$

$$\underline{Z}_{31} = \underline{Z}_3 + \underline{Z}_1 + \frac{\underline{Z}_3 \cdot \underline{Z}_1}{\underline{Z}_2} \quad (11.3c)$$

Izravna primjena jednačbi Kirchhoffovih zakona:



Sl. 11.3

Jednažbe Kirchhoffovih zakona:

$$\left. \begin{array}{rclcl} +\dot{I}_0 & +\dot{I}_1 & & -\dot{I}_4 & = 0 \\ & -\dot{I}_1 & -\dot{I}_2 & & +\dot{I}_5 = 0 \\ & & +\dot{I}_3 & +\dot{I}_4 & -\dot{I}_5 = 0 \end{array} \right\} \text{strujne}$$

$$\left. \begin{array}{rcl} +\underline{Z}_1 \cdot \dot{I}_1 & & +\underline{Z}_4 \cdot \dot{I}_4 + \underline{Z}_5 \cdot \dot{I}_5 = \dot{U}_4 \\ & -\underline{Z}_2 \cdot \dot{I}_2 - \underline{Z}_3 \cdot \dot{I}_3 & -\underline{Z}_5 \cdot \dot{I}_5 = -\dot{U}_3 \\ +\underline{Z}_0 \cdot \dot{I}_0 - \underline{Z}_1 \cdot \dot{I}_1 + \underline{Z}_2 \cdot \dot{I}_2 & & = \dot{U}_0 \end{array} \right\} \text{naponske}$$

One se u matričnom obliku pišu:

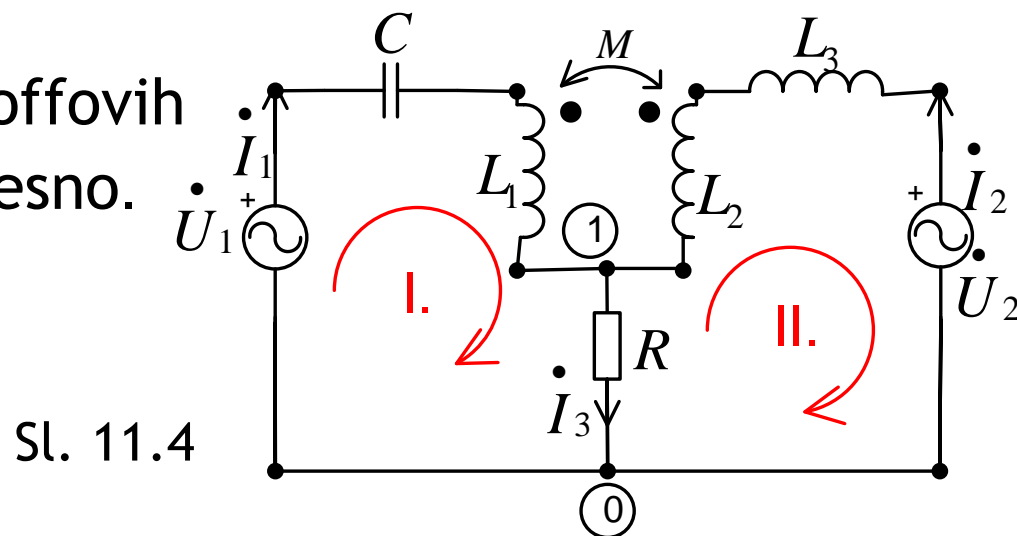
$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & \underline{Z}_1 & 0 & 0 & \underline{Z}_4 & \underline{Z}_5 \\ 0 & 0 & -\underline{Z}_2 & -\underline{Z}_3 & 0 & -\underline{Z}_5 \\ \underline{Z}_0 & -\underline{Z}_1 & \underline{Z}_2 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_0 \\ \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \\ \dot{I}_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{U}_4 \\ -\dot{U}_3 \\ \dot{U}_0 \end{bmatrix} \quad (11.4a)$$

$$\text{ili} \quad \underline{\underline{Z}} \cdot \dot{\underline{I}} = \dot{\underline{U}} \quad (11.4b)$$

Rješenje sustava jednažbi je:

$$\dot{\underline{I}} = \underline{\underline{Z}}^{-1} \cdot \dot{\underline{U}} \quad (11.5)$$

Napisati jednađžbe Kirchhoffovih zakona za mrežu na slici desno.



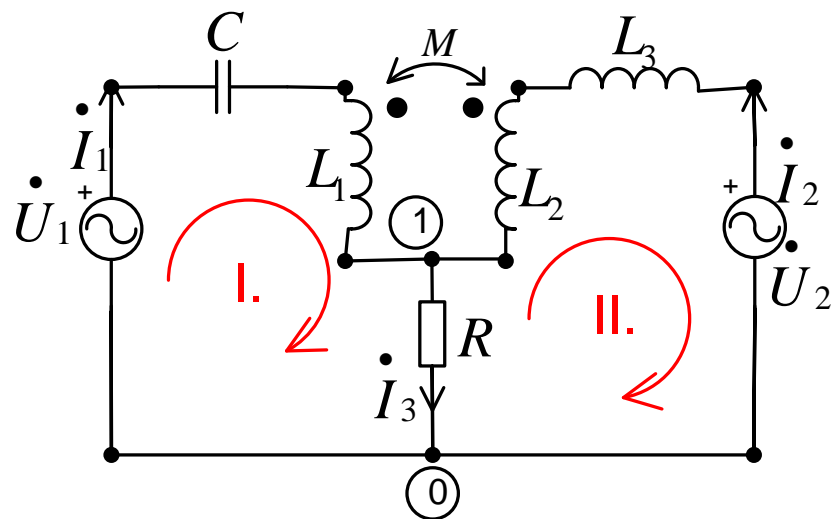
KZS (čvor 1):

KZN (petlja I):

KZN (petlja II):

Napisati jednađžbe Kirchhoffovih zakona za mrežu na slici desno.

Sl. 11.4



KZS (čvor 1):

$$\dot{I}_1 + \dot{I}_2 - \dot{I}_3 = 0 \quad (11.6)$$

KZN (petlja I): $\dot{I}_1 \cdot \frac{1}{j\omega C} + \dot{I}_1 \cdot j\omega L_1 + \dot{I}_2 j\omega M + \dot{I}_3 R = \dot{U}_1 \quad (11.7)$

KZN (petlja II): $-\dot{I}_1 \cdot j\omega M - \dot{I}_2 j\omega(L_2 + L_3) - \dot{I}_3 R = -\dot{U}_2 \quad (11.8)$

Transformacija međuinduktivne veze

$$\dot{U}_{13} = \dot{I}_1 \cdot j\omega L_1 + \dot{I}_2 j\omega M$$

$$\dot{I}_2 = \dot{I}_3 - \dot{I}_1$$

$$\dot{U}_{23} = \dot{I}_2 j\omega L_2 + \dot{I}_1 j\omega M$$

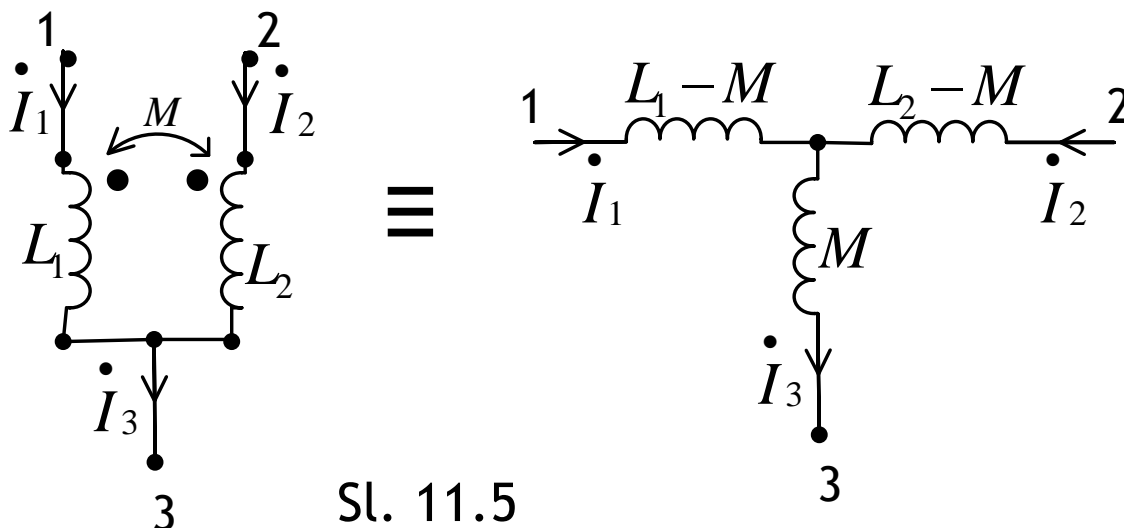
$$\dot{I}_1 = \dot{I}_3 - \dot{I}_2$$

$$(\dot{I}_1 + \dot{I}_2 = \dot{I}_3)$$

$$\dot{U}_{13} = \dot{I}_1 j\omega L_1 + (\dot{I}_3 - \dot{I}_1) j\omega M = \dot{I}_1 j\omega(L_1 - M) + \dot{I}_3 j\omega M$$

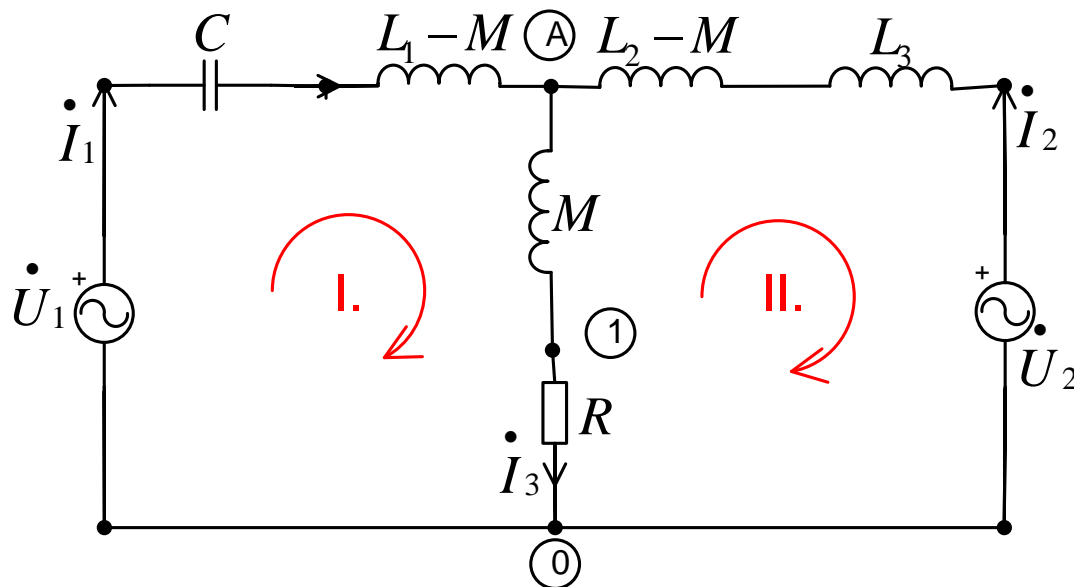
$$\dot{U}_{23} = \dot{I}_2 j\omega L_2 + (\dot{I}_3 - \dot{I}_2) j\omega M = \dot{I}_2 j\omega(L_2 - M) + \dot{I}_3 j\omega M$$

Transformacija
međuinduktiviteta
(spoj u 3 točke)



Sl. 11.5

Napisati jednadžbe KZN za petlje I. i II. te jednadžbu KZS za čvor A, u transformiranoj mreži (iz primjera 1) na slici.



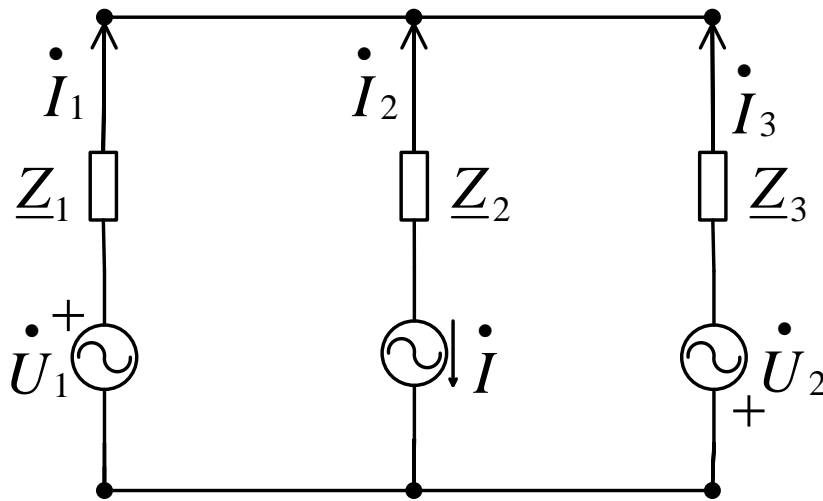
Sl. 11.6

❖ Vrijede li ovdje jednadžbe napisane za izvornu mrežu?

Kao i u mrežama istosmjerne struje,
uz slijedeće RAZLIKE:

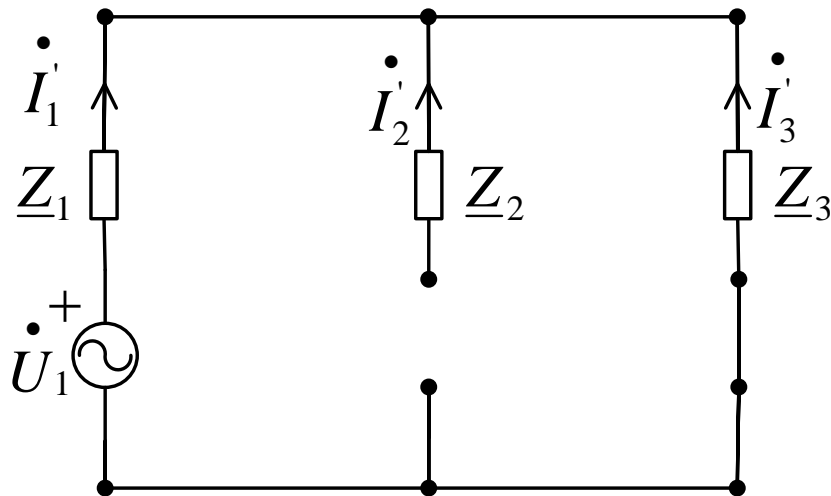
- ✓ UMJESTO OTPORA, RABE SE IMPEDANCIJE
- ✓ UMJESTO IZNOSA NAPONA I STRUJA, RABE SE NJIHOVI KOMPLEKSNI IZRAZI (FAZORI)

Odredi struje \dot{I}_1 , \dot{I}_2 , i \dot{I}_3 u mreži prema slici.



Sl. 11.7

a) Prvi korak

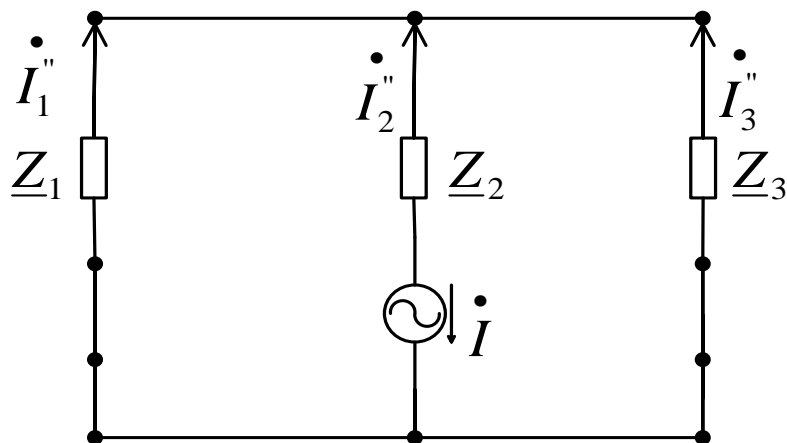


$$\begin{aligned}\dot{I}_1' &= \frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_3} \\ \dot{I}_2' &= 0 \\ \dot{I}_3' &= -\frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_3}\end{aligned}$$

Sl. 11.7a

❖ Pitanje: Što se događa ako je $\underline{Z}_1 + \underline{Z}_3 = 0$ ($\underline{Z}_1 = -\underline{Z}_3$) ?

b) Drugi korak

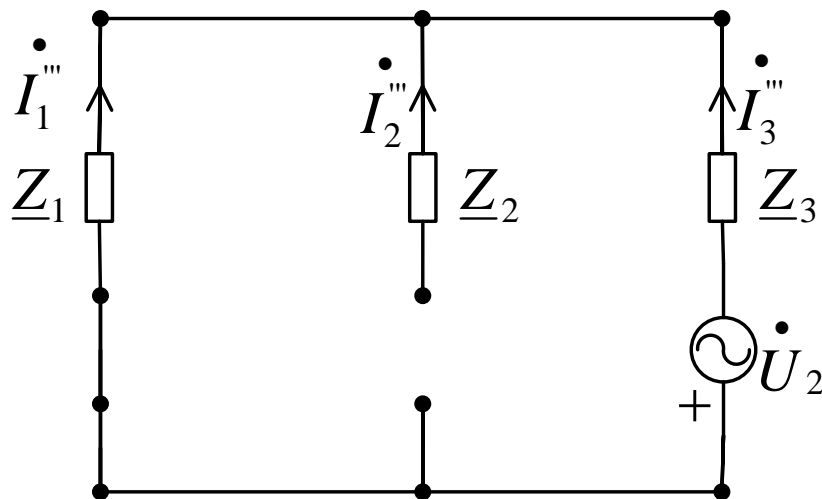


Sl. 11.7b

$$\begin{aligned} \dot{I}_1'' &= \dot{I} \cdot \frac{\underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3} \\ \dot{I}_2'' &= -\dot{I} \\ \dot{I}_3'' &= \dot{I} \cdot \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_3} \end{aligned}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1 = -\underline{Z}_3$?

c) Treći korak



Sl. 11.7c

$$\begin{aligned} \dot{I}_1''' &= \frac{\dot{U}_2}{\underline{Z}_1 + \underline{Z}_3} \\ \dot{I}_2''' &= 0 \\ \dot{I}_3''' &= -\frac{\dot{U}_2}{\underline{Z}_1 + \underline{Z}_3} \end{aligned}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1 + \underline{Z}_3 = 0$?

- ◆ Završetak superpozicije

$$\dot{I}_1 = \dot{I}_1' + \dot{I}_1'' + \dot{I}_1''' = \frac{\dot{U}_1 + \dot{U}_2 + \dot{I} \cdot \underline{Z}_3}{\dot{\underline{Z}}_1 + \underline{Z}_3}$$

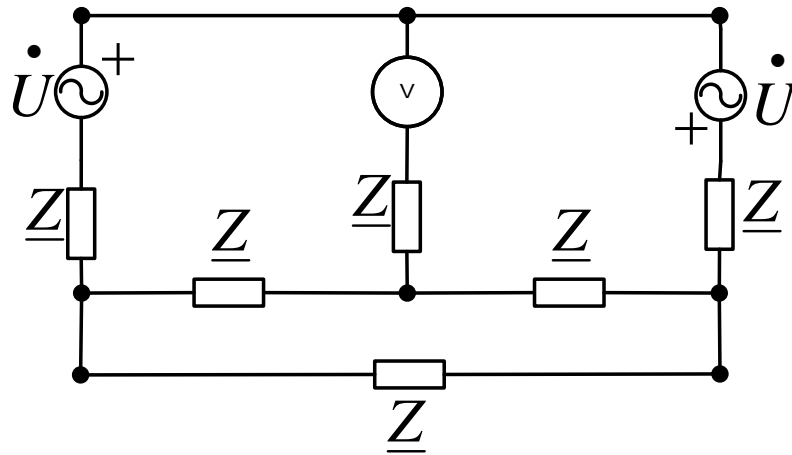
$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' + \dot{I}_2''' = -\dot{I}$$

$$\dot{I}_3 = \dot{I}_3' + \dot{I}_3'' + \dot{I}_3''' = \frac{\dot{I} \cdot \underline{Z}_1 - \dot{U}_1 - \dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

Napomena uz rješenje:

Metoda ovdje nije primjenjiva u slučaju kada je $\underline{Z}_1 + \underline{Z}_3 = 0$!

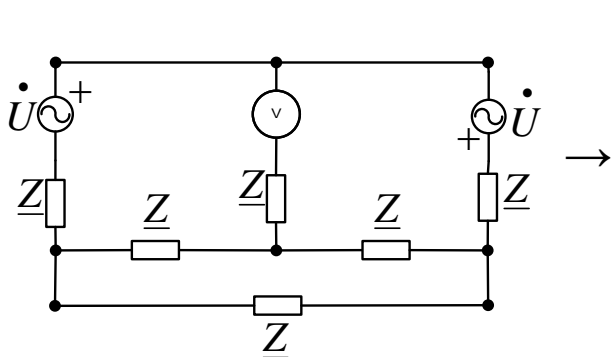
Primjer 1 - Odredite napon voltmetra u spoju na slici



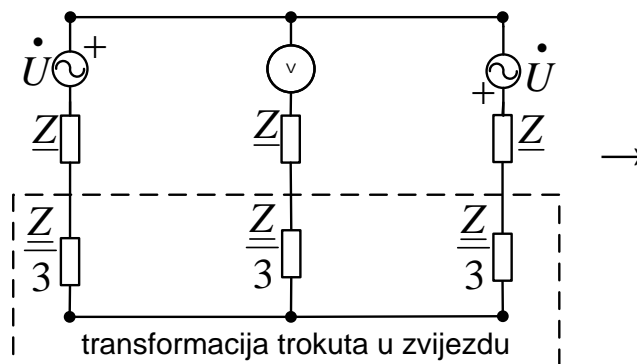
Sl. 11.8

Naputak: Koristite pretvorbu trokut-zvijezda

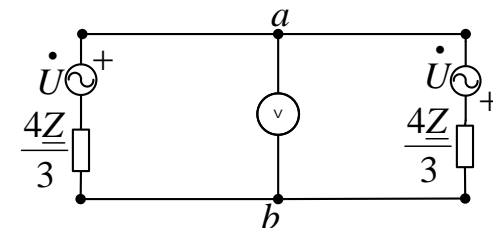
Primjer 1 - Rješenje



Sl. 11.8a



Sl. 11.8b

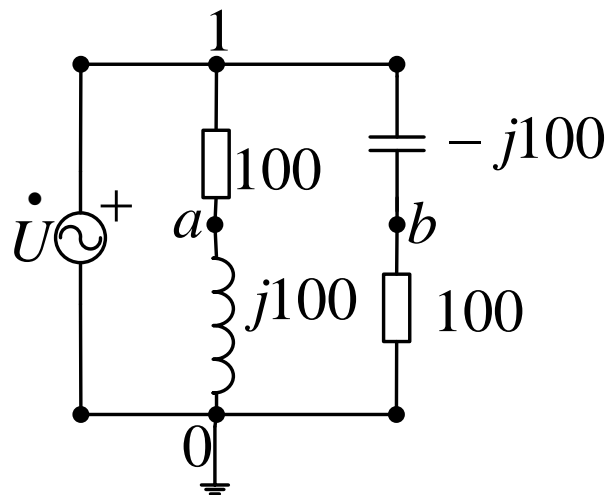


Sl. 11.8c

Kroz impedanciju spojenu serijski s voltmetrom ne teče struja, pa se dobiva serijski krug na slici 11.8c. Ovdje se ukupni napon $2\dot{U}$ dijeli na dvije jednake impedancije (od $4/3Z$) tako da je na svakoj od njih $1/2$ ukupnog napona od $2\dot{U}$ (tj. napon \dot{U}) pa je

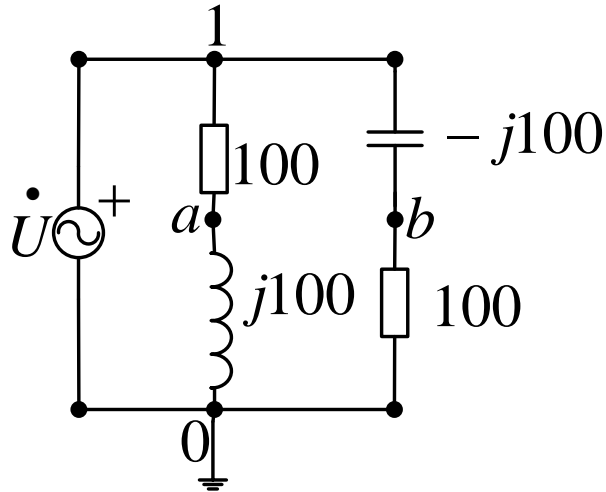
$$\dot{U}_{ab} = \dot{U} - \dot{U} = 0 \qquad U_V = \left| \dot{U}_{ab} \right| = 0$$

Primjer 2 - Odredite napon U_{ab} u spoju na slici

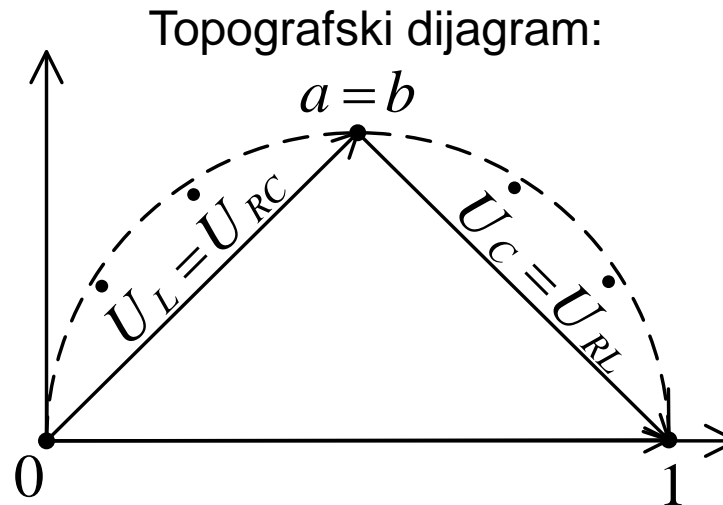


Sl. 11.9

Naputak: Koristite topografski dijagram



Sl. 11.9

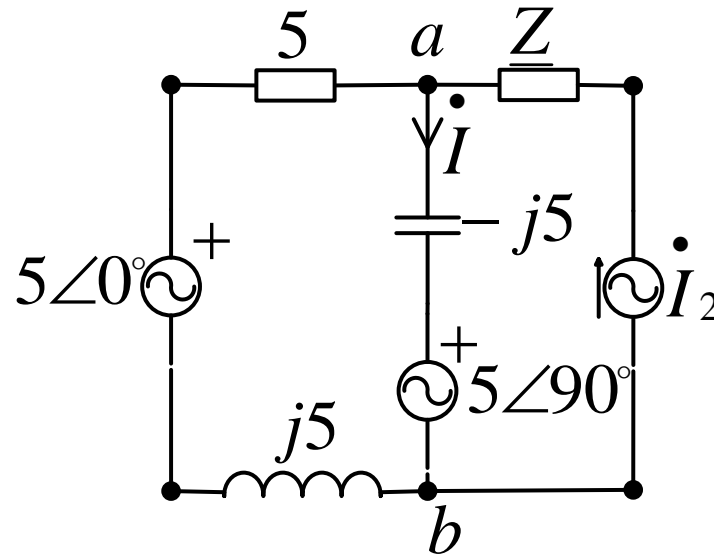


Sl. 11.9a

$$\Rightarrow \dot{U}_{ab} = 0$$

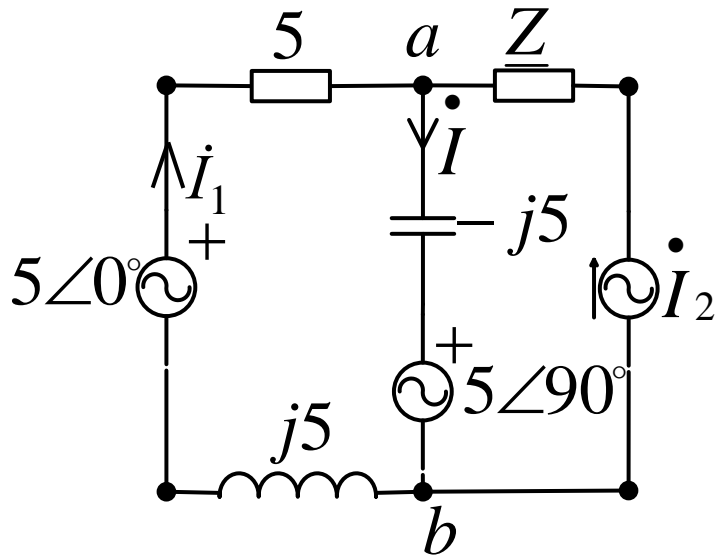
❖ Što se dogodi ako u jednoj grani omski i reaktivni element zamijene mjesta?

Primjer 3 - Ako je u spoju na slici $\dot{I} = 2\angle 0^\circ$ A, kolika je \dot{I}_2



Sl. 11.10

Naputak: Koristite KZS za čvor a



Sl. 11.11

Pomoću zadane struje $\dot{I} = 2\angle 0^\circ$ A određuje se napon U_{ab} kako slijedi:

$$\dot{U}_{ab} = 5\angle 90^\circ + \dot{I} \cdot (-j5) = -j5 \text{ V}$$

iz kojega se može odrediti struja I_1

$$\dot{I}_1 = \frac{5\angle 0^\circ \text{ V} - \dot{U}_{ab}}{5\Omega - j5\Omega} = \frac{5 + j5}{5 + j5} \text{ A} = 1\angle 0^\circ \text{ A}$$

Iz KZS za čvor a dobiva se konačno: $\dot{I}_2 = \dot{I} - \dot{I}_1 = 1\angle 0^\circ \text{ A}$

❖ Utječe li Z na rješenje? Zašto?