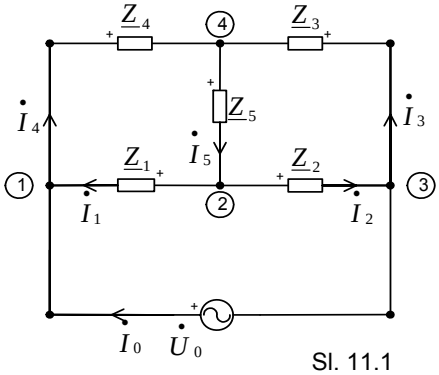
11. predavanje iz OE

Složeni krugovi izmjenične struje

(uredio prof.dr.sc. Armin Pavić)

Mosni spoj

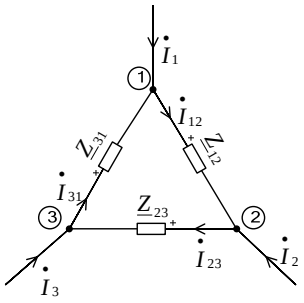


Sl. 11.1

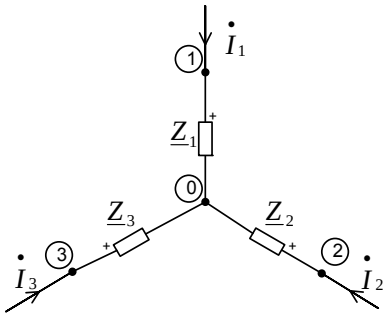
Uvjet ravnoteže mosta:

$$\underline{Z}_1 \cdot \underline{Z}_3 - \underline{Z}_2 \cdot \underline{Z}_4 = 0; \quad \frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_4}{\underline{Z}_3} \qquad (11.1)$$

Pretvorbe trokut-zvijezda



Sl. 11.2a: Trokut



Sl. 11.2b: Zvijezda

Pretvorba trokuta u zvijezdu

$$\underline{Z}_1 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{31}}{\underline{Z}_\Delta}; \quad \underline{Z}_2 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{23}}{\underline{Z}_\Delta}; \quad \underline{Z}_3 = \frac{\underline{Z}_{23} \cdot \underline{Z}_{31}}{\underline{Z}_\Delta} \qquad (11.2)$$

Gdje je:

$$\underline{Z}_\Delta = \underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31} \qquad (11.2a)$$

Pretvorba zvijezde u trokut

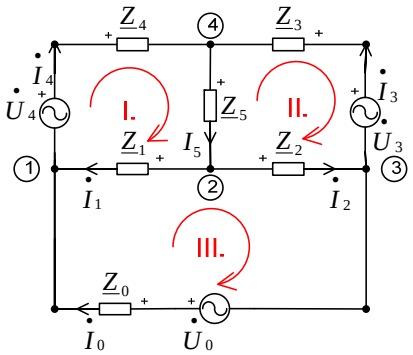
$$\underline{Z}_{12} = \underline{Z}_1 + \underline{Z}_2 + \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_3} \qquad (11.3a)$$

$$\underline{Z}_{23} = \underline{Z}_2 + \underline{Z}_3 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_1} \qquad (11.3b)$$

$$\underline{Z}_{31} = \underline{Z}_3 + \underline{Z}_1 + \frac{\underline{Z}_3 \cdot \underline{Z}_1}{\underline{Z}_2} \qquad (11.3c)$$

Mreže izmjenične struje

Izravna primjena jednadžbi Kirchhoffovih zakona:



Sl. 11.3

Izravna primjena jednadžbi Kirchhoffovih zakona

Jednadžbe Kirchhoffovih zakona:

$$\left.\begin{aligned} +\dot{I}_0 \quad +\dot{I}_1 \qquad\qquad -\dot{I}_4 \qquad &= 0 \\ -\dot{I}_1 \quad -\dot{I}_2 \qquad\qquad +\dot{I}_5 \qquad &= 0 \\ +\dot{I}_3 \quad +\dot{I}_4 \quad -\dot{I}_5 \qquad &= 0 \end{aligned}\right\} \textit{strujne}$$

$$\left.\begin{aligned} +\underline{Z}_1\cdot\dot{I}_1 \qquad\qquad +\underline{Z}_4\cdot\dot{I}_4+\underline{Z}_5\cdot\dot{I}_5 &= \dot{U}_4 \\ -\underline{Z}_2\cdot\dot{I}_2-\underline{Z}_3\cdot\dot{I}_3 \qquad -\underline{Z}_5\cdot\dot{I}_5 &= -\dot{U}_3 \\ +\underline{Z}_0\cdot\dot{I}_0-\underline{Z}_1\cdot\dot{I}_1+\underline{Z}_2\cdot\dot{I}_2 \qquad &= \dot{U}_0 \end{aligned}\right\} \textit{naponske}$$

Jednadžbe Kirchhoffovih zakona u matričnom obliku

One se u matričnom obliku pišu:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & \underline{Z}_1 & 0 & 0 & \underline{Z}_4 & \underline{Z}_5 \\ 0 & 0 & -\underline{Z}_2 & -\underline{Z}_3 & 0 & -\underline{Z}_5 \\ \underline{Z}_0 & -\underline{Z}_1 & \underline{Z}_2 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_0 \\ \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \\ \dot{I}_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{U}_4 \\ -\dot{U}_3 \\ \dot{U}_0 \end{bmatrix} \qquad (11.4a)$$

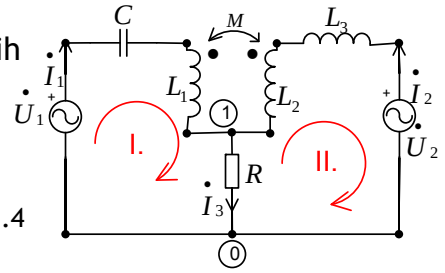
ili $\underline{\underline{Z}}\cdot\underline{\dot{I}}=\underline{\dot{U}}$ (11.4b)

Rješenje sustava jednadžbi je:

$$\underline{\dot{I}}=\underline{\underline{Z}}^{-1}\cdot\underline{\dot{U}} \qquad (11.5)$$

Mreža s međuinduktivnom vezom – jednadžbe KZ

Napisati jednadžbe Kirchhoffovih zakona za mrežu na slici desno.



Sl. 11.4

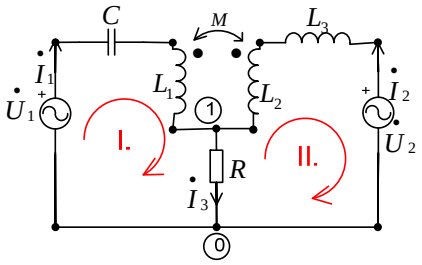
KZS (čvor 1):

KZN (petlja I):

KZN (petlja II):

Mreža s međuinduktivnom vezom – jednadžbe KZ (2)

Napisati jednadžbe Kirchhoffovih zakona za mrežu na slici desno.



Sl. 11.4

KZS (čvor 1): $$\dot{I}_1+\dot{I}_2-\dot{I}_3=0 \qquad (11.6)$$

KZN (petlja I): $$\dot{I}_1\cdot\frac{1}{j\omega C}+\dot{I}_1\cdot j\omega L_1+\dot{I}_2\, j\omega M+\dot{I}_3 R=\dot{U}_1 \qquad (11.7)$$

KZN (petlja II): $$-\dot{I}_1\cdot j\omega M-\dot{I}_2\, j\omega(L_2+L_3)-\dot{I}_3 R=-\dot{U}_2 \qquad (11.8)$$

Transformacija međuinduktivne veze

$$\dot{U}_{13} = \dot{I}_1 \cdot j\omega L_1 + \dot{I}_2\, j\omega M \qquad \dot{I}_2 = \dot{I}_3 - \dot{I}_1$$

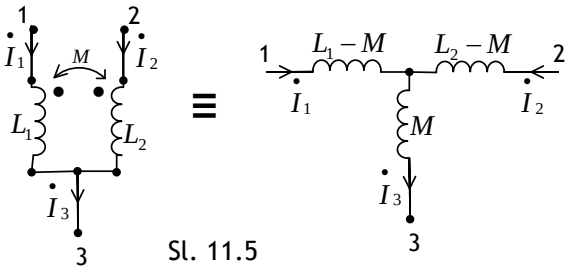
$$\dot{U}_{23} = \dot{I}_2\, j\omega L_2 + \dot{I}_1\, j\omega M \qquad \dot{I}_1 = \dot{I}_3 - \dot{I}_2 \qquad (\dot{I}_1 + \dot{I}_2 = \dot{I}_3)$$

$$\dot{U}_{13} = \dot{I}_1\, j\omega L_1 + (\dot{I}_3 - \dot{I}_1) j\omega M = \dot{I}_1\, j\omega (L_1 - M) + \dot{I}_3\, j\omega M$$

$$\dot{U}_{23} = \dot{I}_2\, j\omega L_2 + (\dot{I}_3 - \dot{I}_2) j\omega M = \dot{I}_2\, j\omega (L_2 - M) + \dot{I}_3\, j\omega M$$

Transformacija međuinduktiviteta

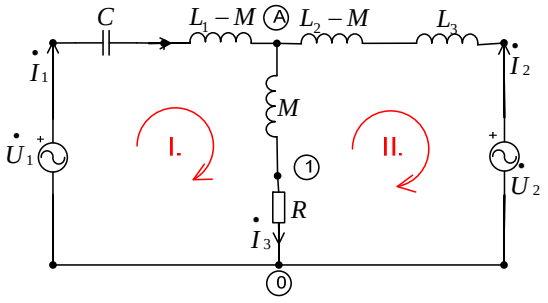
(spoj u 3 točke)



Sl. 11.5

Mreža s transformiranom međuinduktivnom vezom

Napisati jednadžbe KZN za petlje I. i II. te jednadžbu KZS za čvor A, u transformiranoj mreži (iz primjera 1) na slici.



Sl. 14.6

❖ Vrijede li ovdje jednadžbe napisane za izvornu mrežu?

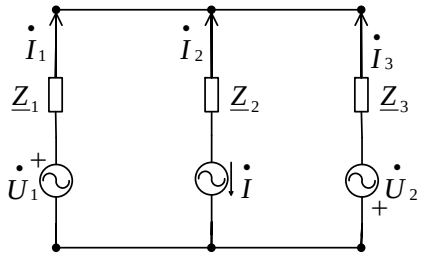
Metoda superpozicije

Isto kao i u mrežama istosmjerne struje

- samo se računa s kompleksnim izrazima!

Superpozicija: primjer

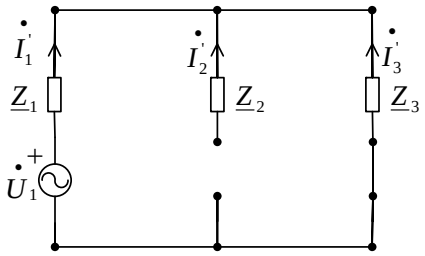
Odredi struje $\dot{I}_1, \dot{I}_2$, i $\dot{I}_3$ u mreži prema slici.



Sl. 11.7

Superpozicija: primjer (2)

a) Prvi korak



Sl. 11.7a

$$\dot{I}_1' = \frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_3}$$

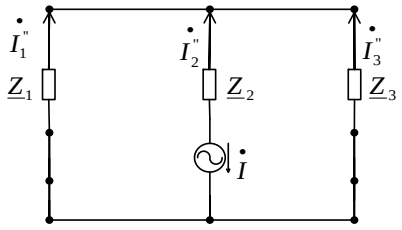
$$\dot{I}_2' = 0$$

$$\dot{I}_3' = -\frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_3}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1+\underline{Z}_3=0$?

Superpozicija: primjer (3)

b) Drugi korak



Sl. 11.7b

$$\dot{I}_1'' = \dot{I} \cdot \frac{\underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3}$$

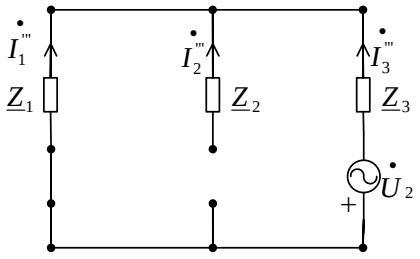
$$\dot{I}_2'' = -\dot{I}$$

$$\dot{I}_3'' = \dot{I} \cdot \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_3}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1 = -\underline{Z}_3$?

Superpozicija: primjer (4)

c) Treći korak



$$\dot{I}_1''' = \frac{\dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

$$\dot{I}_2''' = 0$$

$$\dot{I}_3''' = -\frac{\dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

Sl. 11.7c

❖ Pitanje: Što se događa ako je $\underline{Z}_1+\underline{Z}_3=0$?

Superpozicija: primjer (5)

♦ Završetak superpozicije

$$\dot{I}_1 = \dot{I}_1' + \dot{I}_1'' + \dot{I}_1''' = \frac{\dot{U}_1 + \dot{U}_2 + \dot{I}\cdot\underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3}$$

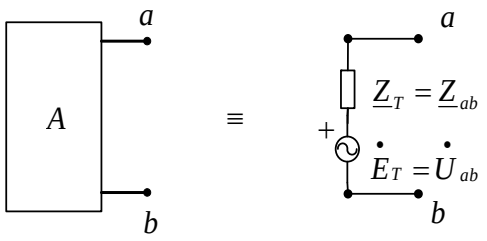
$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' + \dot{I}_2''' = -\dot{I}$$

$$\dot{I}_3 = \dot{I}_3' + \dot{I}_3'' + \dot{I}_3''' = \frac{\dot{I}\cdot\underline{Z}_1 - \dot{U}_1 - \dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

Napomena uz rješenje:

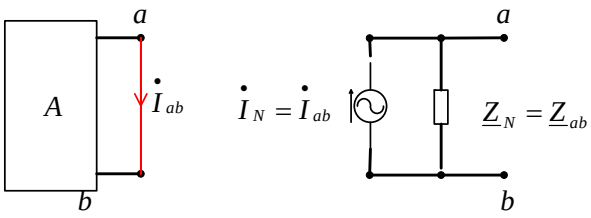
Strujni krug nije definiran za slučaj kada je $\underline{Z}_1+\underline{Z}_3=0$!

Theveninov teorem



Sl. 11.8

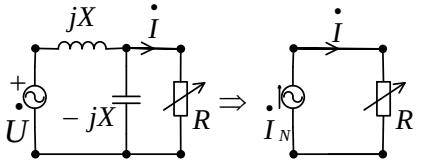
Nortonov teorem



Sl. 11.9

Opravdanje potrebe Nortonovog teorema

- Odredite kako se struja $\dot{I}$ mijenja s porastom R!



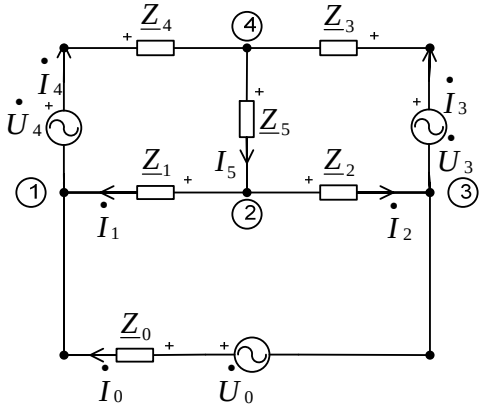
Sl. 11.10

Ovdje je jedino moguće nadomještanje po Nortonu, gdje je nadomjestak aktivne mreže idealni strujni izvor (Z_N=∞).

$$\dot{I}_N = \frac{\dot{U}}{jX} \quad .$$

Metoda potencijala čvorova

Isto kao i u mrežama istosmjerne struje!



Postavljanje jednadžbi potencijala čvorova

- Za mrežu s prethodnog *slidea*, uz izbor četvrtoga čvora kao referentnog ($\varphi_4 = 0$), jednadžbe potencijala čvorova glase:

$$+\varphi_1\cdot(\underline{Y}_0+\underline{Y}_1+\underline{Y}_4) - \varphi_2\cdot\underline{Y}_1 - \varphi_3\cdot\underline{Y}_0 = +U_0\cdot\underline{Y}_0 - U_4\cdot\underline{Y}_4 \quad \text{(čvor 1)}$$

$$-\varphi_1\cdot\underline{Y}_1 + \varphi_2\cdot(\underline{Y}_1+\underline{Y}_2+\underline{Y}_5) - \varphi_3\cdot\underline{Y}_2 = 0 \quad \text{(čvor 2)}$$

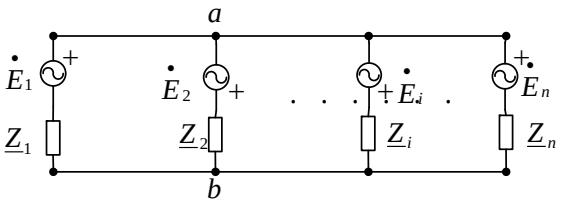
$$-\varphi_1\cdot\underline{Y}_0 - \varphi_2\cdot\underline{Y}_2 + \varphi_3\cdot(\underline{Y}_0+\underline{Y}_2+\underline{Y}_3) = - U_0\cdot\underline{Y}_0 - U_3\cdot\underline{Y}_3 \quad \text{(čvor 3)}$$

Gdje je:

$$\underline{Y}_i = 1/\underline{Z}_i,\ i = 0,1,\ldots,5$$

admitancija odgovarajuće grane.

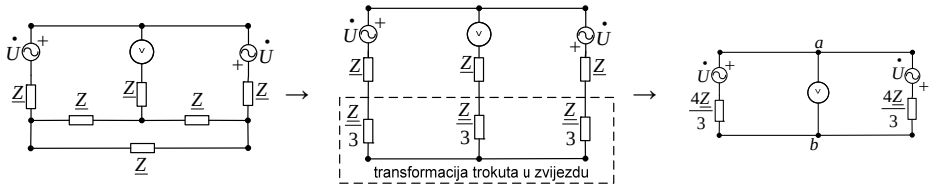
Millmanov teorem



Sl. 11.11

$$\dot{U}_{ab} = \frac{\operatorname{alg}\sum_{i=1}^{n}\frac{\dot{E}_i}{\underline{Z}_i}}{\sum_{i=1}^{n}\frac{1}{\underline{Z}_i}}$$

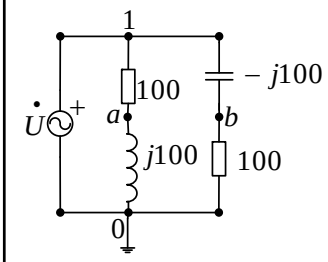
Primjer 1 - Odredite napon voltmetra!



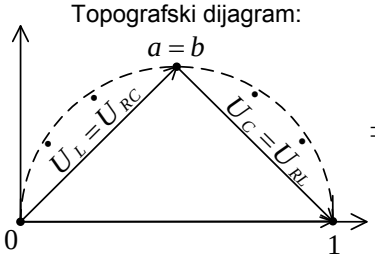
Primjenom Millmanova teorema

$$\dot{U}_{ab} = \frac{\frac{3\dot{U}}{4\underline{Z}} - \frac{3\dot{U}}{4\underline{Z}}}{\frac{3}{4\underline{Z}} + \frac{3}{4\underline{Z}}} = 0 \qquad U_V = \left|\dot{U}_{ab}\right| = 0$$

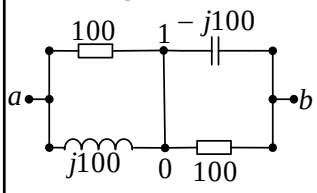
Primjer 2 - Odredite $\dot{E}_T = \dot{U}_{ab}$ i $\underline{Z}_T = \underline{Z}_{ab}$



Topografski dijagram:



$$\Rightarrow \dot{U}_{ab} = \dot{E}_T = 0$$

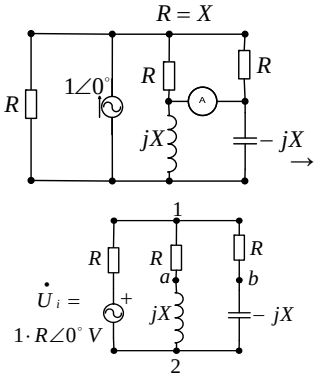


$$\underline{Z}_{ab} = \underline{Z}_T = (100 \,||\, j100) + [100 \,||\, (-j100)] = 100 + j \cdot 0$$

$$\underline{Z}_T = 100\angle 0^\circ$$

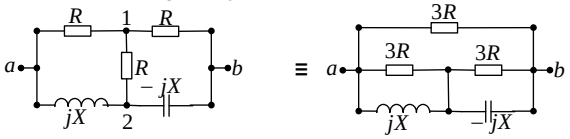
❖ Što se dogodi ako u jednoj grani omski i reaktivni element zamijene mjesta?

Primjer 3 – Odredite struju ampermetra!



Određivanjem struje ampermetra svodi se na traženje iznosa Nortonove struje. Primijenimo Theveninov teorem i $\dot{I}_N = \frac{\dot{E}_T}{\underline{Z}_T}$

Iz prethodnog primjera znamo postupak za $\underline{Z}_{ab}$



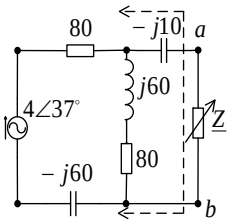
$$\underline{Z}_{ab} = 3R \,\|\, [3R \,\|\, jX + 3R \,\|\, (-jX)] = \frac{R}{2} = \underline{Z}_T$$

Kako je $\underline{Z}_{12} = (R + jX) \,\|\, (R - jX) = R \Rightarrow \dot{U}_{12} = \frac{1}{2}\dot{U}_i = \frac{R}{2}\angle 0^\circ \text{ V}$

Iz prethodnog primjera znamo da je $\dot{U}_{ab} = \dot{U}_{12}\angle 90^\circ = \frac{R}{2}\angle 90^\circ = \dot{E}_T$

$$\dot{I}_N = \frac{\dot{E}_T}{\underline{Z}_T} = 1\angle 90^\circ \text{ A} \Rightarrow I_A = 1 \text{ A}$$

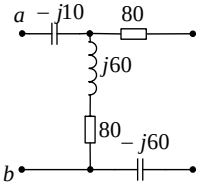
Primjer 4 – Odredite najveću moguću snagu na $\underline{Z}$



Theveninov teorem i teorem maksimalne snage na promjenjivoj impedanciji.

$$\dot{E}_T = \dot{U}_{ab} = 4\angle 37^\circ \cdot (80 + j60) = 400\angle 74^\circ \text{ V}$$

$\underline{Z}_T = \underline{Z}_{ab}$



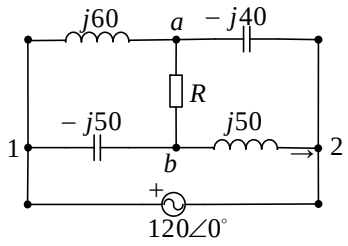
$$\underline{Z}_T = 80 + j50$$

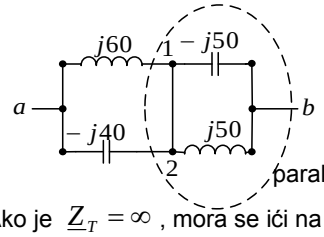
$$\underline{Z} = \underline{Z}_T^* = 80 - j50$$

(na njoj se razvija najveća snaga)

$$P_{\max} = \left(\frac{\dot{E}_T}{2\,\text{Re}\{\underline{Z}_T\}} \right)^2 \cdot \text{Re}\{\underline{Z}\} = 500 \text{ W}$$

Primjer 5 – Odredite snagu na otporu R=10 Ω!

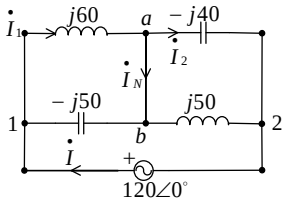




$\underline{Z}_{ab} = \underline{Z}_T = \infty$

paralelna rezonancija

Ako je $\underline{Z}_T = \infty$, mora se ići na Nortonov teorem!



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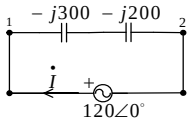
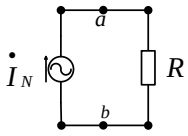


Kapacitivno naponsko djelilo!

$$\dot{I}_1 = \frac{3 \cdot 120}{5} \cdot \frac{1}{j60} = -j \cdot 1{,}2 \text{ A}$$

$$\dot{I}_2 = \frac{2 \cdot 120}{5} \cdot \frac{1}{-j40} = j \cdot 1{,}2 \text{ A}$$

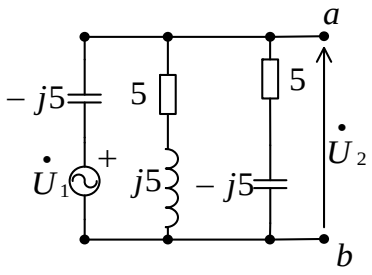
$$\dot{I}_N = \dot{I}_1 - \dot{I}_2 = -j \cdot 2{,}4 \text{ A}$$



Nadomjesna shema po Nortonu!

$$P = I_N^2 \cdot R = 57{,}6 \text{ W}$$

Primjer 6 – Odredite $\dot{U}_2/\dot{U}_1$!

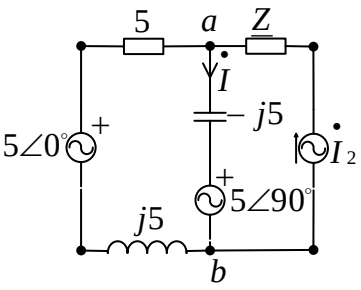


Millmanov teorem!

$$\dot{U}_{ab} = \dot{U}_2 = \frac{\dfrac{\dot{U}_1}{-j5}}{\dfrac{1}{-j5} + \dfrac{1}{5+j5} + \dfrac{1}{5-j5}} = \dot{U}_1 \frac{j}{1+j}$$

$$\frac{\dot{U}_2}{\dot{U}_1} = 0{,}5 + j0{,}5 = \frac{1}{\sqrt{2}} \angle 45°$$

Primjer 7 - Ako je je $\dot{I} = 2\angle 0^\circ$ A , kolika je $\dot{I}_2$



(1) $\dot{U}_{ab} = 5\angle 90^\circ + \dot{I}\cdot(-j5) = -j5$ V

Millmanov teorem (jednadžba po $\dot{I}_2$):

(2) $$\dot{U}_{ab} = -j5 = \frac{\frac{5\angle 0^\circ}{5+j5} + \frac{5\angle 90^\circ}{-j5} + \dot{I}_2}{\frac{1}{5+j5} + \frac{1}{-j5} + \left(+\frac{1}{\infty}\right)}$$

Rješavanjem jednadžbe (2) dobiva se: $\dot{I}_2 = 1\angle 0^\circ$ A

❖ Utječe li $\underline{Z}$ na rješenje? Zašto?