Složeni krugovi izmjenične struje

(uredio prof.dr.sc. Armin Pavić)

Sadržaj:

Mosni spoj

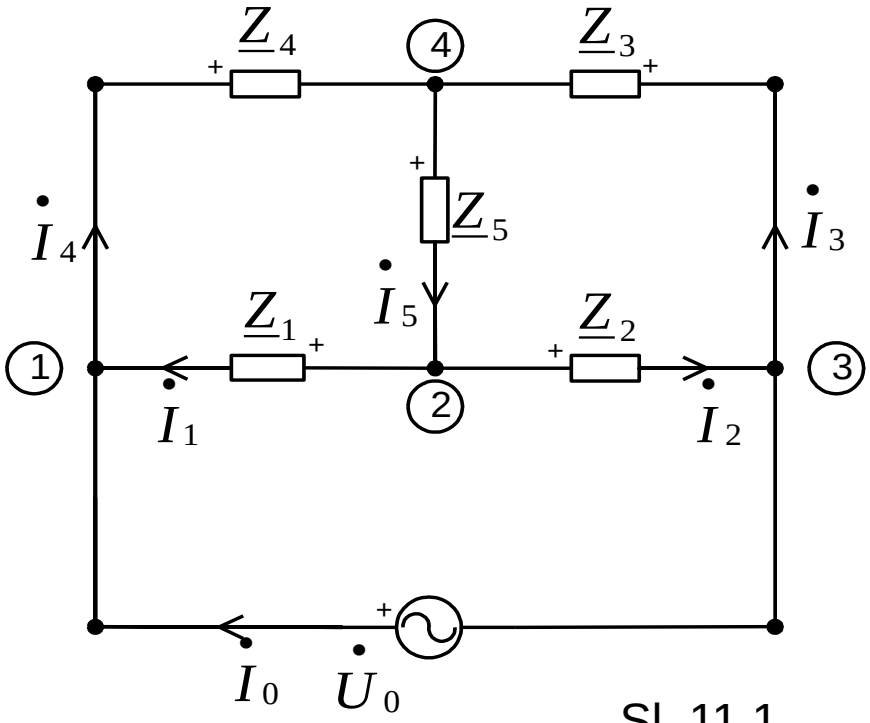
Pretvorbe spojeva trokuta i zvijezde

Rješavanje mreže jednadžbama KZ

Pretvorba međuinduktivne veze

Metoda superpozicije

Mosni spoj

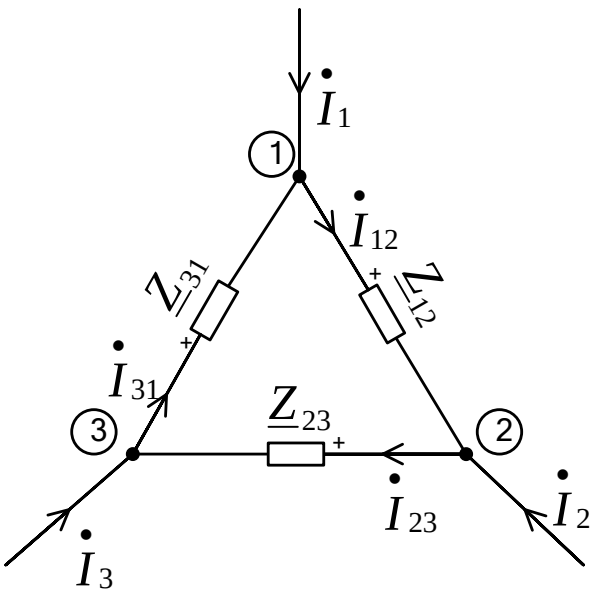


Sl. 11.1

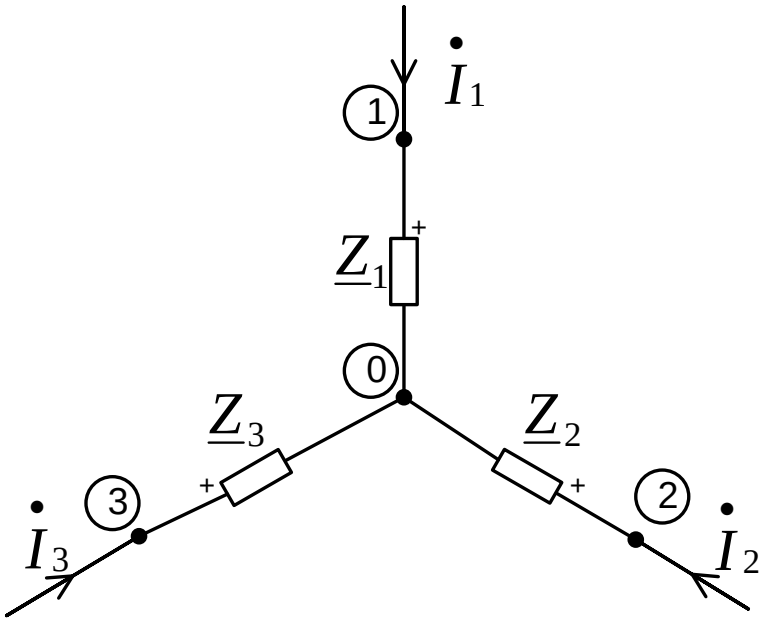
Uvjet ravnoteže mosta:

$$\underline{Z}_1 \cdot \underline{Z}_3 - \underline{Z}_2 \cdot \underline{Z}_4 = 0; \quad \frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_4}{\underline{Z}_3} \qquad (11.1)$$

Pretvorbe trokut-zvijezda



Sl. 11.2a: Trokut



Sl. 11.2b: Zvijezda

Pretvorba trokuta u zvijezdu

$$\underline{Z}_1 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{31}}{\underline{Z}_\Delta}; \quad \underline{Z}_2 = \frac{\underline{Z}_{12} \cdot \underline{Z}_{23}}{\underline{Z}_\Delta}; \quad \underline{Z}_3 = \frac{\underline{Z}_{23} \cdot \underline{Z}_{31}}{\underline{Z}_\Delta} \qquad (11.2)$$

Gdje je:

$$\underline{Z}_\Delta = \underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31} \qquad (11.2a)$$

Pretvorba zvijezde u trokut

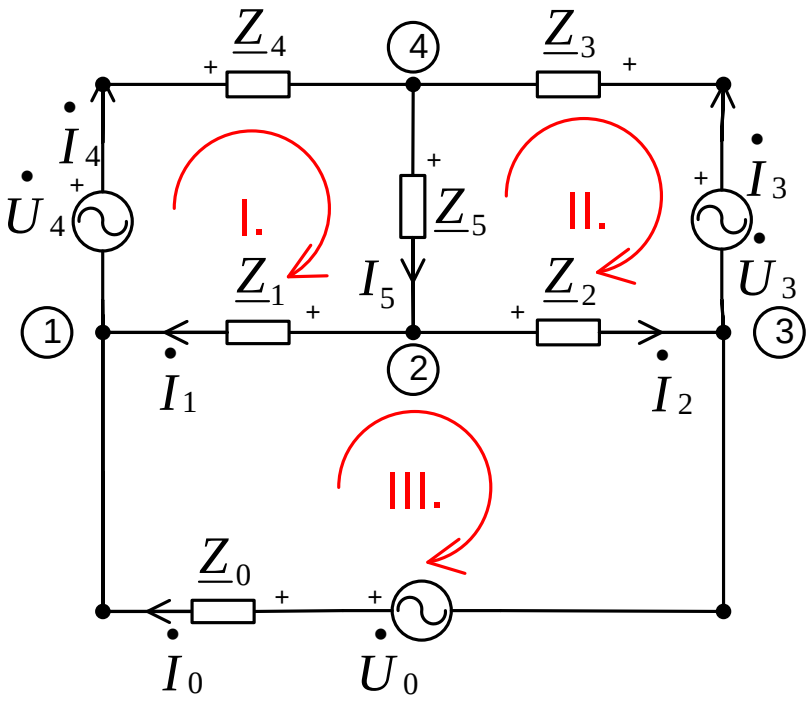
$$\underline{Z}_{12} = \underline{Z}_1 + \underline{Z}_2 + \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_3} \qquad (11.3a)$$

$$\underline{Z}_{23} = \underline{Z}_2 + \underline{Z}_3 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_1} \qquad (11.3b)$$

$$\underline{Z}_{31} = \underline{Z}_3 + \underline{Z}_1 + \frac{\underline{Z}_3 \cdot \underline{Z}_1}{\underline{Z}_2} \qquad (11.3c)$$

Mreže izmjenične struje

Izravna primjena jednadžbi Kirchhoffovih zakona:



Sl. 11.3

Izravna primjena jednadžbi Kirchhoffovih zakona

Jednadžbe Kirchhoffovih zakona:

$$\left.\begin{aligned} +\dot{I}_0 \quad +\dot{I}_1 \quad -\dot{I}_4 &= 0 \\ -\dot{I}_1 \quad -\dot{I}_2 \quad +\dot{I}_5 &= 0 \\ +\dot{I}_3 \quad +\dot{I}_4 \quad -\dot{I}_5 &= 0 \end{aligned}\right\} \textit{strujne}$$

$$\left.\begin{aligned} +\underline{Z}_1\cdot\dot{I}_1 \quad +\underline{Z}_4\cdot\dot{I}_4+\underline{Z}_5\cdot\dot{I}_5 &= \dot{U}_4 \\ -\underline{Z}_2\cdot\dot{I}_2-\underline{Z}_3\cdot\dot{I}_3 \quad -\underline{Z}_5\cdot\dot{I}_5 &= -\dot{U}_3 \\ +\underline{Z}_0\cdot\dot{I}_0-\underline{Z}_1\cdot\dot{I}_1+\underline{Z}_2\cdot\dot{I}_2 &= \dot{U}_0 \end{aligned}\right\} \textit{naponske}$$

Jednadžbe Kirchhoffovih zakona u matričnom obliku

One se u matričnom obliku pišu:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & \underline{Z}_1 & 0 & 0 & \underline{Z}_4 & \underline{Z}_5 \\ 0 & 0 & -\underline{Z}_2 & -\underline{Z}_3 & 0 & -\underline{Z}_5 \\ \underline{Z}_0 & -\underline{Z}_1 & \underline{Z}_2 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_0 \\ \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \\ \dot{I}_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{U}_4 \\ -\dot{U}_3 \\ \dot{U}_0 \end{bmatrix} \qquad (11.4a)$$

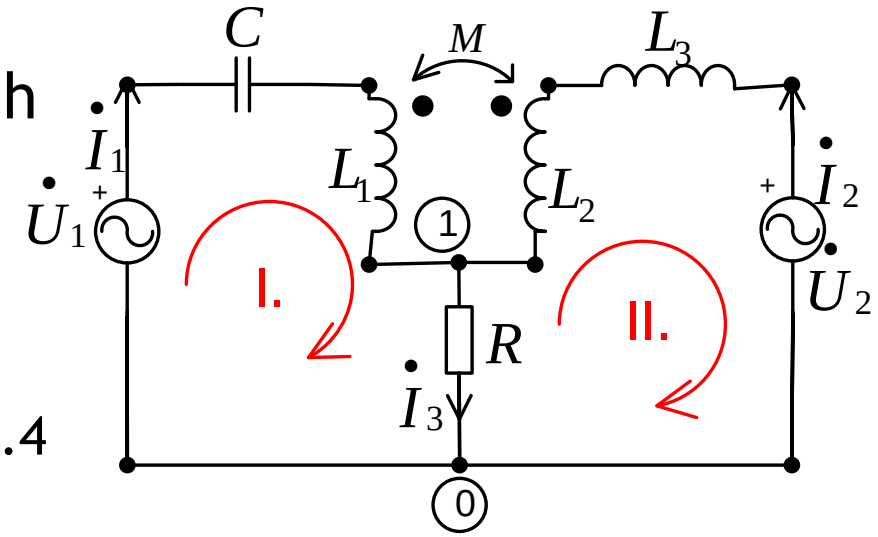
ili $$\underline{\underline{Z}} \cdot \underline{\dot{I}} = \underline{\dot{U}} \qquad (11.4b)$$

Rješenje sustava jednadžbi je:

$$\underline{\dot{I}} = \underline{\underline{Z}}^{-1} \cdot \underline{\dot{U}} \qquad (11.5)$$

Mreža s međuinduktivnom vezom – jednadžbe KZ

Napisati jednadžbe Kirchhoffovih zakona za mrežu na slici desno.



Sl. 11.4

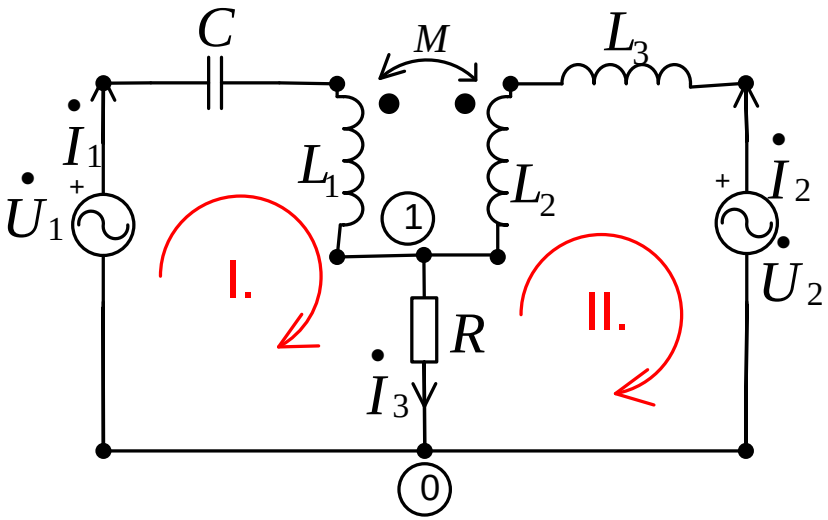
KZS (čvor 1):

KZN (petlja I):

KZN (petlja II):

Mreža s međuinduktivnom vezom – jednadžbe KZ (2)

Napisati jednadžbe Kirchhoffovih zakona za mrežu na slici desno.



Sl. 11.4

KZS (čvor 1): $$\dot{I}_1 + \dot{I}_2 - \dot{I}_3 = 0 \qquad (11.6)$$

KZN (petlja I): $$\dot{I}_1 \cdot \frac{1}{j\omega C} + \dot{I}_1 \cdot j\omega L_1 + \dot{I}_2\, j\omega M + \dot{I}_3 R = \dot{U}_1 \qquad (11.7)$$

KZN (petlja II): $$-\dot{I}_1 \cdot j\omega M - \dot{I}_2\, j\omega (L_2 + L_3) - \dot{I}_3 R = -\dot{U}_2 \qquad (11.8)$$

Transformacija međuinduktivne veze

$$\dot{U}_{13} = \dot{I}_1 \cdot j\omega L_1 + \dot{I}_2\, j\omega M$$

$$\dot{U}_{23} = \dot{I}_2\, j\omega L_2 + \dot{I}_1\, j\omega M$$

$$\dot{I}_2 = \dot{I}_3 - \dot{I}_1$$

$$\dot{I}_1 = \dot{I}_3 - \dot{I}_2$$

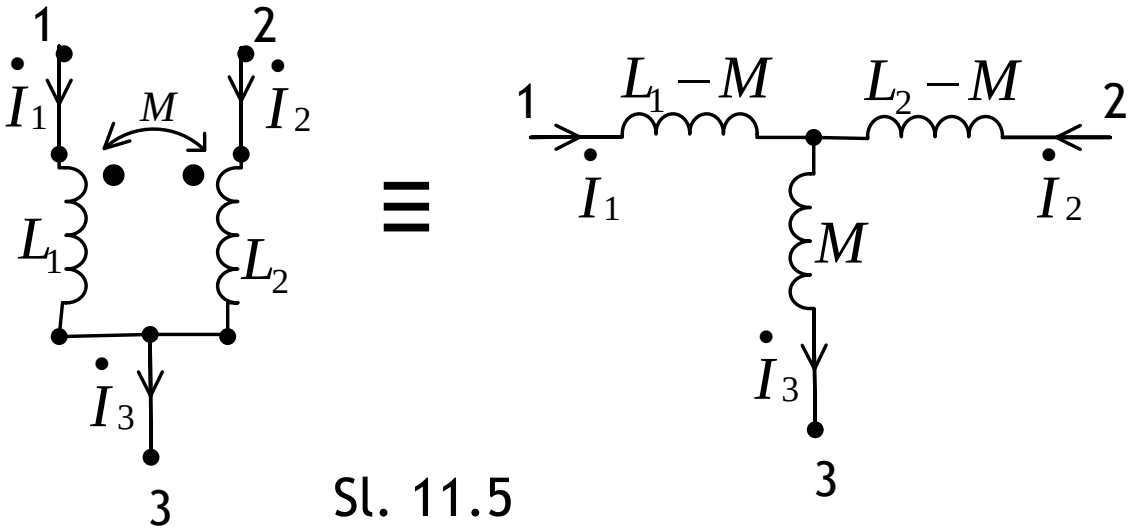
$$(\dot{I}_1 + \dot{I}_2 = \dot{I}_3)$$

$$\dot{U}_{13} = \dot{I}_1\, j\omega L_1 + (\dot{I}_3 - \dot{I}_1)\, j\omega M = \dot{I}_1\, j\omega (L_1 - M) + \dot{I}_3\, j\omega M$$

$$\dot{U}_{23} = \dot{I}_2\, j\omega L_2 + (\dot{I}_3 - \dot{I}_2)\, j\omega M = \dot{I}_2\, j\omega (L_2 - M) + \dot{I}_3\, j\omega M$$

Transformacija međuinduktiviteta

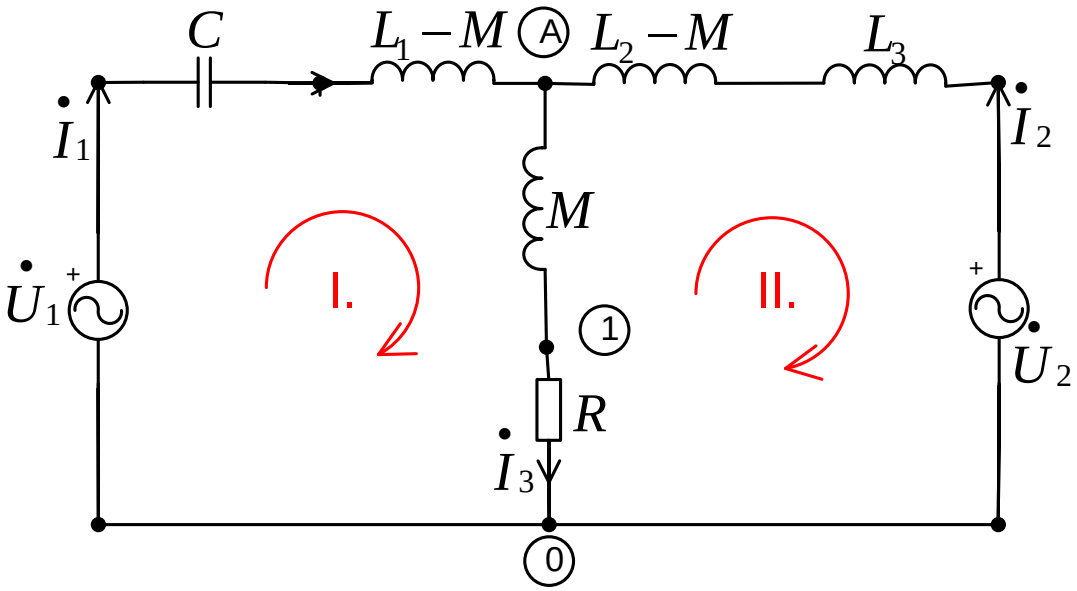
(spoj u 3 točke)



Sl. 11.5

Mreža s transformiranom međuinduktivnom vezom

Napisati jednadžbe KZN za petlje I. i II. te jednadžbu KZS za čvor A, u transformiranoj mreži (iz primjera 1) na slici.



Sl. 11.6

❖ Vrijede li ovdje jednadžbe napisane za izvornu mrežu?

Metoda superpozicije

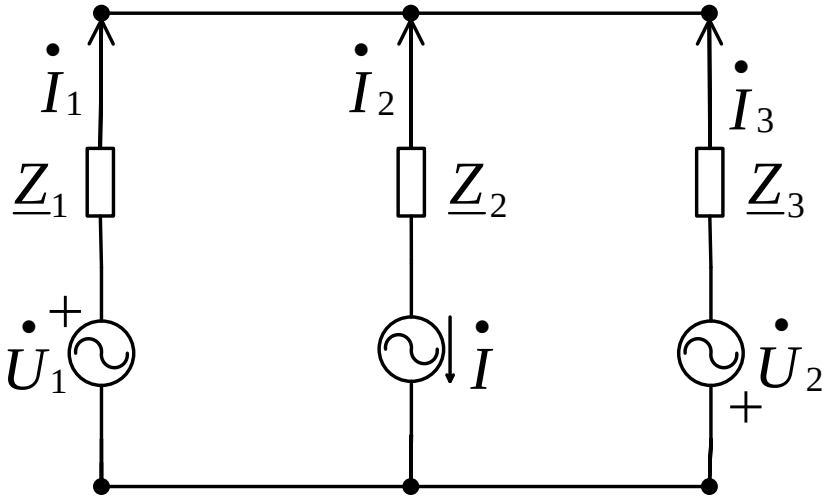
Kao i u mrežama istosmjerne struje,

uz slijedeće RAZLIKE:

- ✓ UMJESTO OTPORA, RABE SE IMPEDANCIJE
- ✓ UMJESTO IZNOSA NAPONA I STRUJA, RABE SE NJIHOVI KOMPLEKSNI IZRAZI (FAZORI)

Superpozicija: Primjer

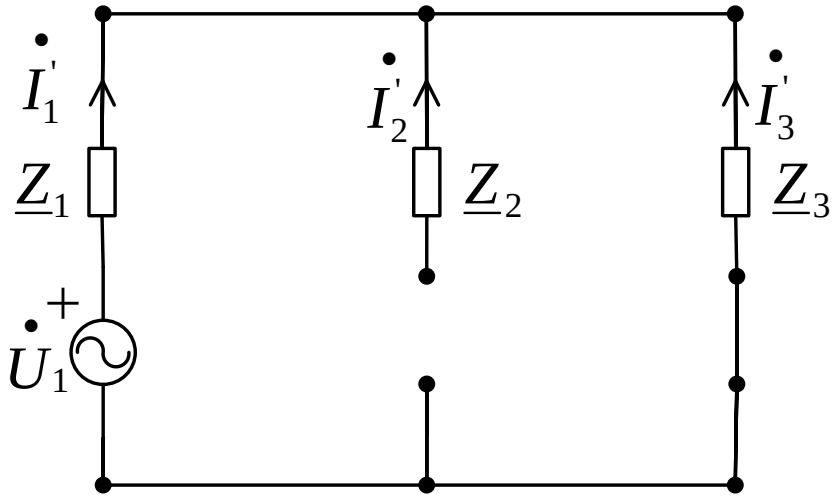
Odredi struje $\dot{I}_1, \dot{I}_2$, i $\dot{I}_3$ u mreži prema slici.



Sl. 11.7

Superpozicija: Primjer (a)

a) Prvi korak



Sl. 11.7a

$$\dot{I}_1' = \frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_3}$$

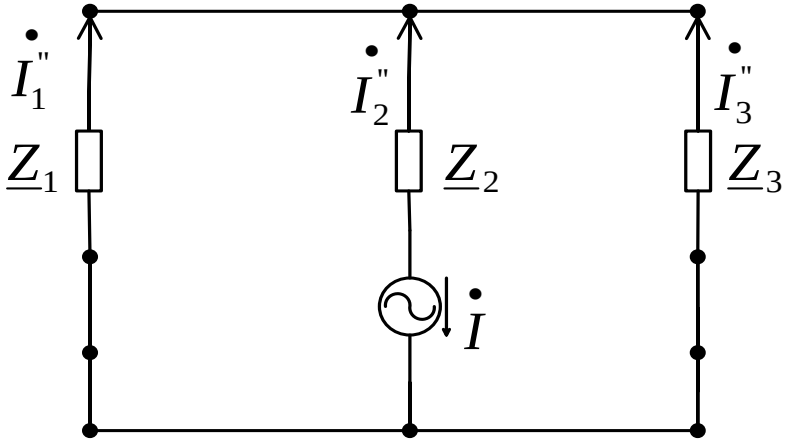
$$\dot{I}_2' = 0$$

$$\dot{I}_3' = -\frac{\dot{U}_1}{\underline{Z}_1 + \underline{Z}_3}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1+\underline{Z}_3=0$ ($Z_1= -Z_3$) ?

Superpozicija: Primjer (b)

b) Drugi korak



Sl. 11.7b

$$\dot{I}_1'' = \dot{I} \cdot \frac{\underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3}$$

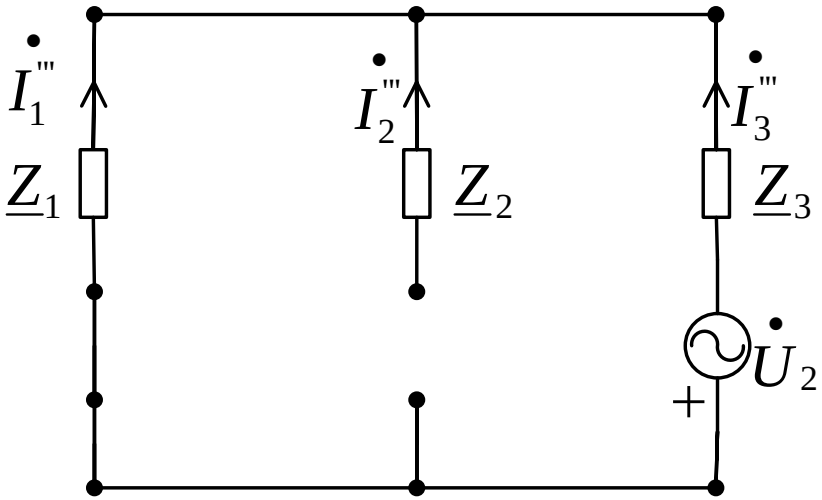
$$\dot{I}_2'' = -\dot{I}$$

$$\dot{I}_3'' = \dot{I} \cdot \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_3}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1 = -\underline{Z}_3$?

Superpozicija: Primjer (c)

c) Treći korak



Sl. 11.7c

$$\dot{I}_1''' = \frac{\dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

$$\dot{I}_2''' = 0$$

$$\dot{I}_3''' = -\frac{\dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

❖ Pitanje: Što se događa ako je $\underline{Z}_1+\underline{Z}_3=0$?

Superpozicija: Primjer (d)

- Završetak superpozicije

$$\dot{I}_1 = \dot{I}_1' + \dot{I}_1'' + \dot{I}_1''' = \frac{\dot{U}_1 + \dot{U}_2 + \dot{I}\cdot \underline{Z}_3}{\underline{Z}_1 + \underline{Z}_3}$$

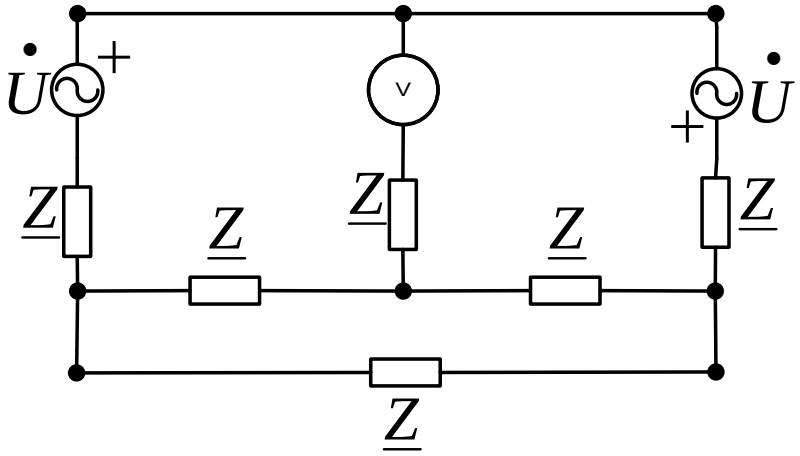
$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' + \dot{I}_2''' = -\dot{I}$$

$$\dot{I}_3 = \dot{I}_3' + \dot{I}_3'' + \dot{I}_3''' = \frac{\dot{I}\cdot \underline{Z}_1 - \dot{U}_1 - \dot{U}_2}{\underline{Z}_1 + \underline{Z}_3}$$

Napomena uz rješenje:

Metoda ovdje nije primjenjiva u slučaju kada je $\underline{Z}_1+\underline{Z}_3=0$!

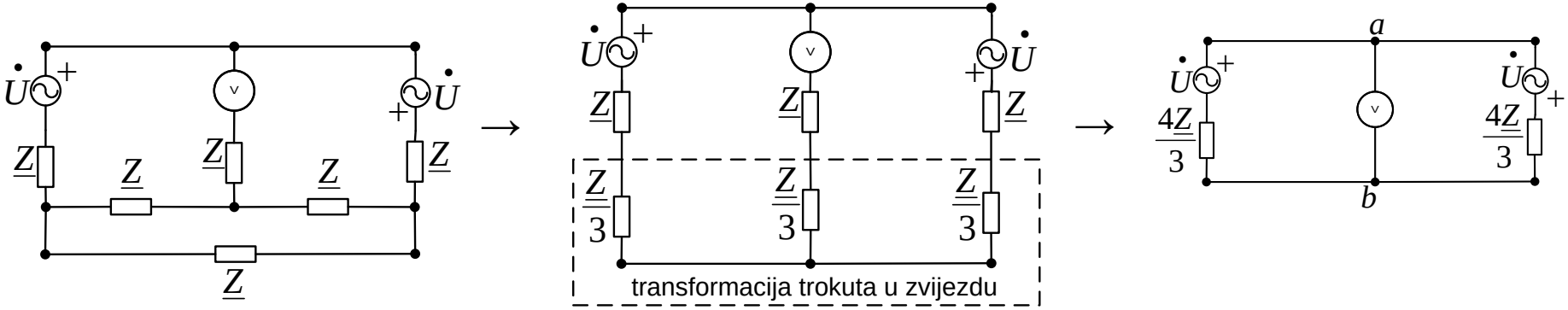
Primjer 1 – Odredite napon voltmetra u spoju na slici



Sl. 11.8

Naputak: Koristite pretvorbu trokut-zvijezda

Primjer 1 - Rješenje

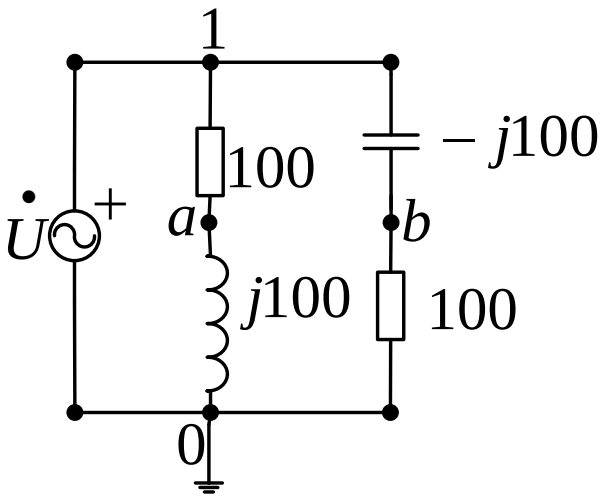


Sl. 11.8a Sl. 11.8b Sl. 11.8c

Kroz impedanciju spojenu serijski s voltmetrom ne teče struja, pa se dobiva serijski krug na slici 11.8c. Ovdje se ukupni napon $2\mathring{U}$ dijeli na dvije jednake impedancije (od $4/3Z$) tako da je na svakoj od njih 1/2 ukupnog napona od $2\mathring{U}$ (tj. napon $\mathring{U}$) pa je

$$\dot{U}_{ab} = \dot{U} - \dot{U} = 0 \qquad U_V = \left|\dot{U}_{ab}\right| = 0$$

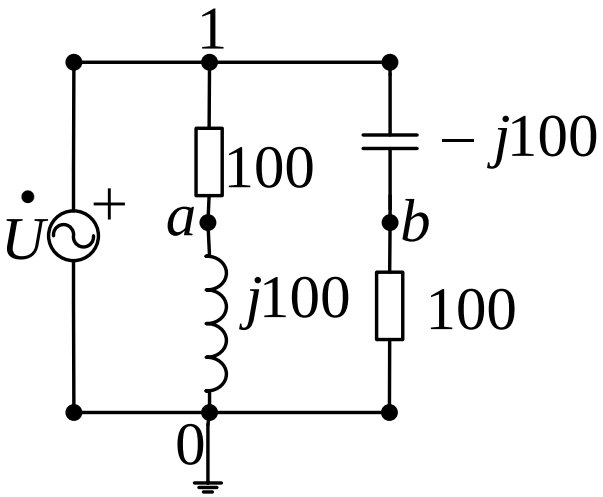
Primjer 2 – Odredite napon U_{ab} u spoju na slici



Sl. 11.9

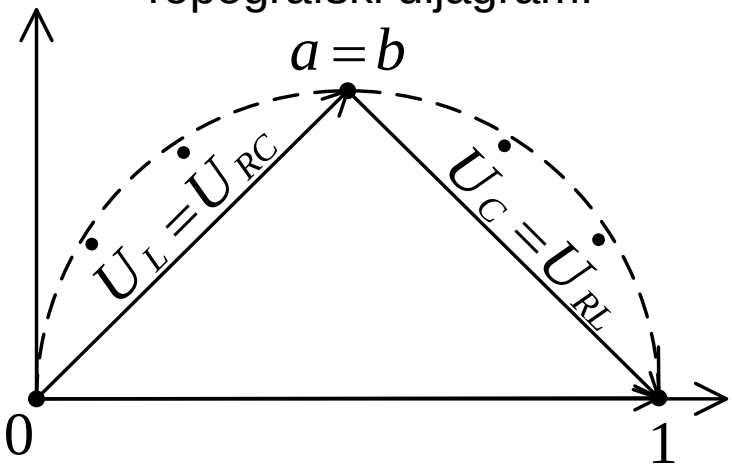
Naputak: Koristite topografski dijagram

Primjer 2 - Rješenje



Sl. 11.9

Topografski dijagram:

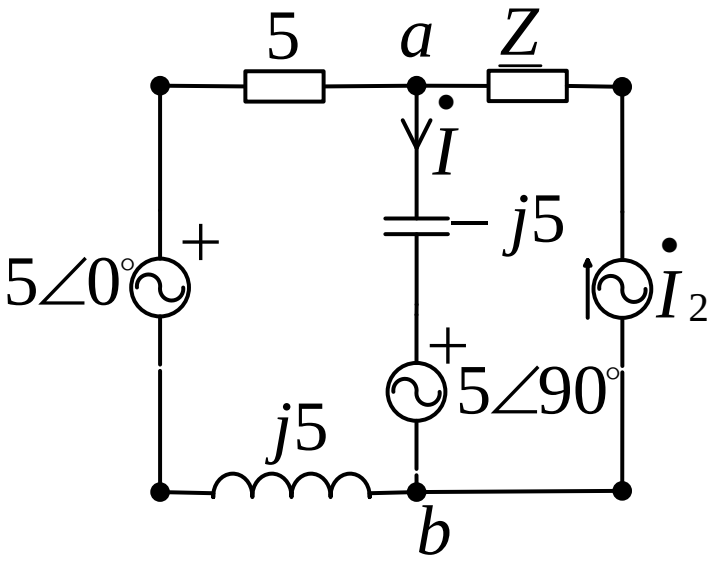


$$\Rightarrow \dot{U}_{ab} = 0$$

Sl. 11.9a

- Što se dogodi ako u jednoj grani omski i reaktivni element zamijene mjesta?

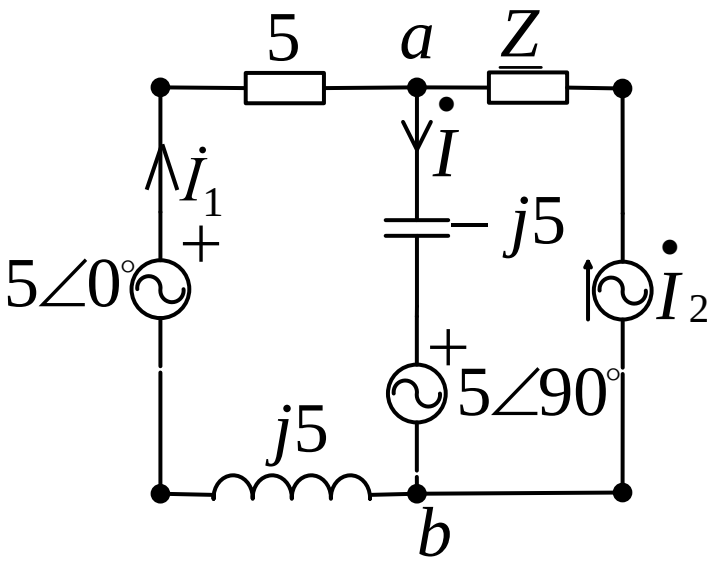
Primjer 3 - Ako je u spoju na slici $\dot{I} = 2\angle 0^\circ$ A, kolika je $\dot{I}_2$



Sl. 11.10

Naputak: Koristite KZS za čvor *a*

Primjer 3 – Rješenje



Sl. 11.11

Pomoću zadane struje $\dot{I} = 2\angle 0^\circ$ A određuje se napon U_{ab} kako slijedi:

$$\dot{U}_{ab} = 5\angle 90^\circ + \dot{I}\cdot(-j5) = -j5\ \text{V}$$

iz kojega se može odrediti struja I_1

$$\dot{I}_1 = \frac{5\angle 0°\,\text{V} - \dot{U}_{ab}}{5\Omega - j5\Omega} = \frac{5+j5}{5+j5}\,\text{A} = 1\angle 0°\,\text{A}$$

Iz KZS za čvor *a* dobiva se konačno: $\dot{I}_2 = \dot{I} - \dot{I}_1 = 1\angle 0^\circ$ A

❖ Utječe li $\underline{Z}$ na rješenje? Zašto?