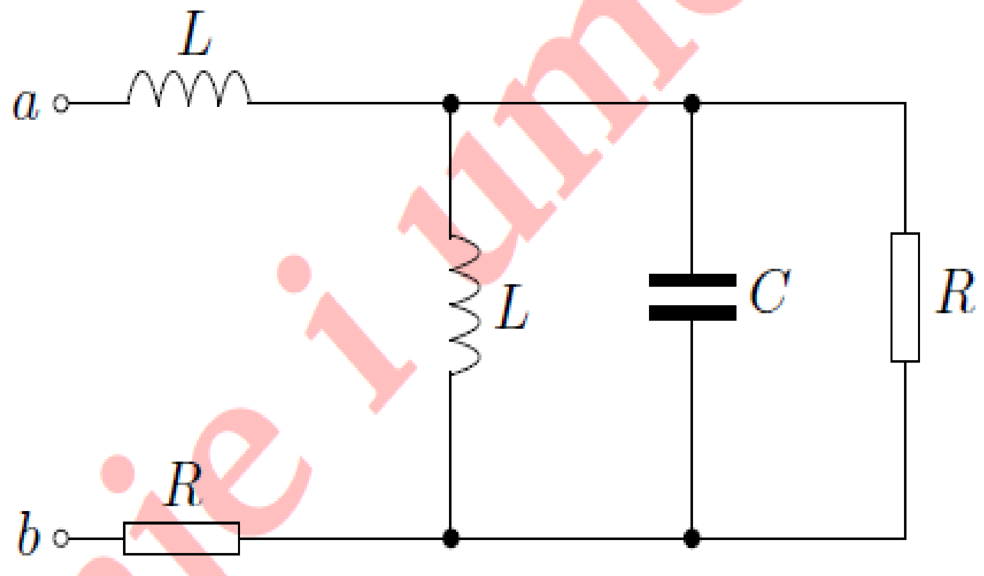
OE - konzultacije

8. svibnja 2020.

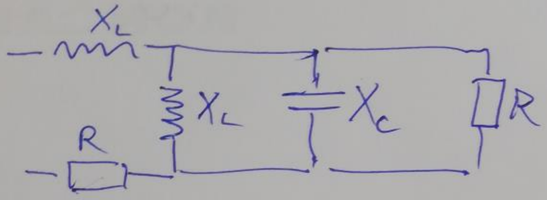
Zad.1.26. Odredite iznos i kut nadomjesne admitancije $\underline{Y}_{ab}$ gledano s točaka **a** i **b**. Zadano je: $R = X_L = X_C = 50\,\Omega$.

Rješenje: $\underline{Y}_{ab} = 8{,}94\angle{-26,565^\circ}$ mS



ZAD_1.26_4. KNJIŽ

$X_L = X_C = R = 50$



$$Z_{paralela} = jX_L \,||\, (-jX_C) \,||\, R$$

$$= \frac{jX_L\cdot(-jX_C)}{jX_L - jX_C} \,||\, R$$

$$= \frac{X_L X_C}{j(X_L - X_C)} \,||\, R$$

$$= \frac{\frac{X_L X_C}{j(X_L - X_C)}\cdot R}{\frac{X_L X_C}{j(X_L - X_C)} + R}$$

$$= \frac{X_L X_C \cdot R}{X_L X_C + R(jX_L - jX_C)}$$

$(\ldots\ X_L = X_C = R = 50\,)$

$$\boxed{Z_{paralela} = R}$$

ili $Z_{paralela} = \frac{1}{Y_{paralela}}$

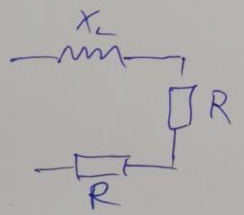
$$Y_{paralela} = \frac{1}{jX_L} + \frac{1}{-jX_C} + \frac{1}{R}$$

$(X_L = X_C = R)$

$$Y_{paralela} = \frac{1}{R}$$

$$\boxed{Z_{paralela} = R}$$

Z :



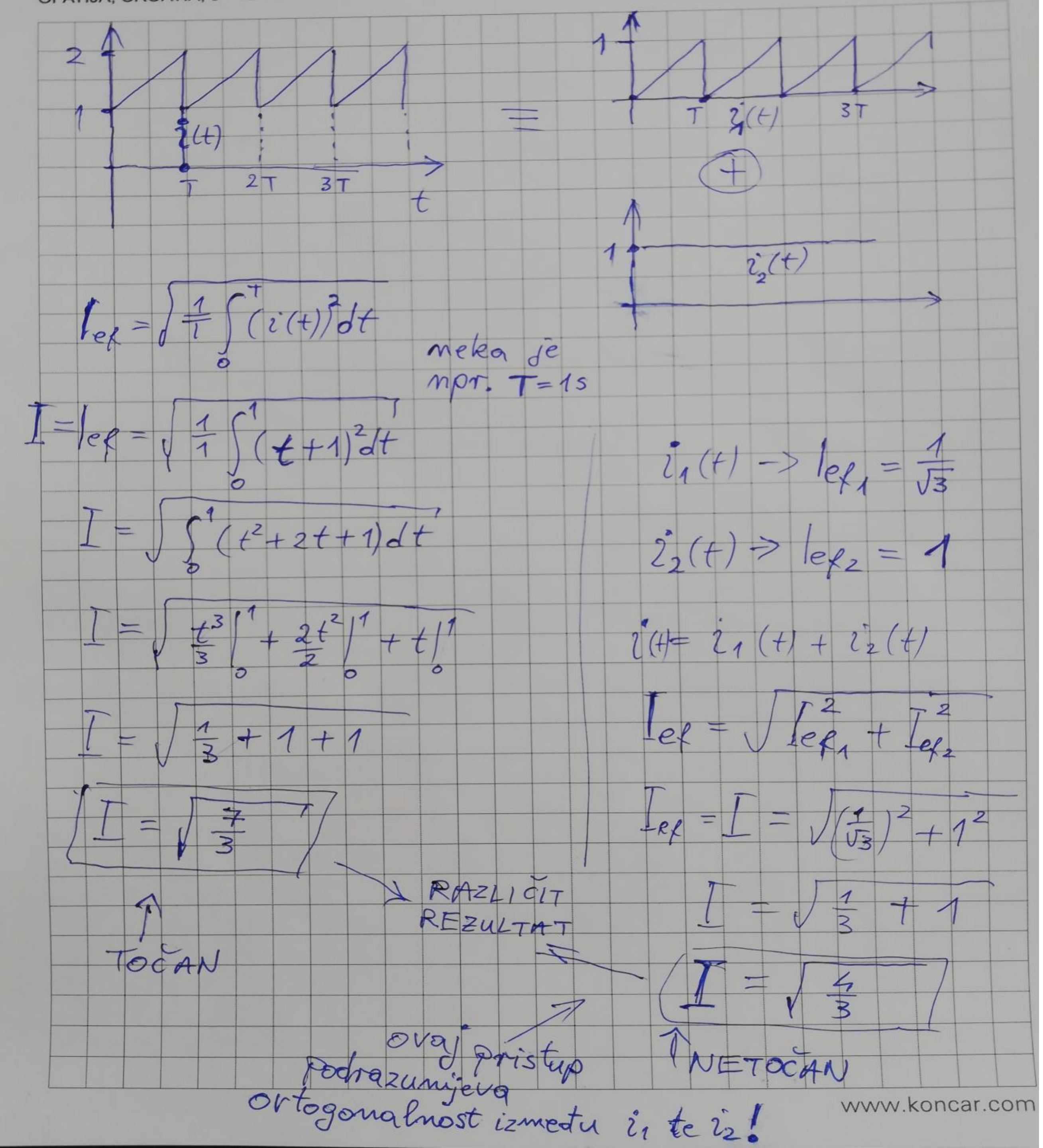
$$Z = jX_L + R + R$$

$$Z = Rj + 2R$$

$$Z = 50j + 100$$

$$Z = 50\sqrt{5}\,\angle 26{,}6^\circ\ \Omega$$

$$\boxed{Y = \frac{1}{Z} = 8{,}944\,\angle{-26{,}56^\circ}\ mS}$$



Na slici su prezentirana dva načina izračuna efektivne vrijednosti struje prikazane na lijevom grafu.

U lijevom stupcu se efektivna vrijednost računa po definiciji.

U desnom stupcu je pokazan izračun efektivne vrijednosti korištenjem izraza:

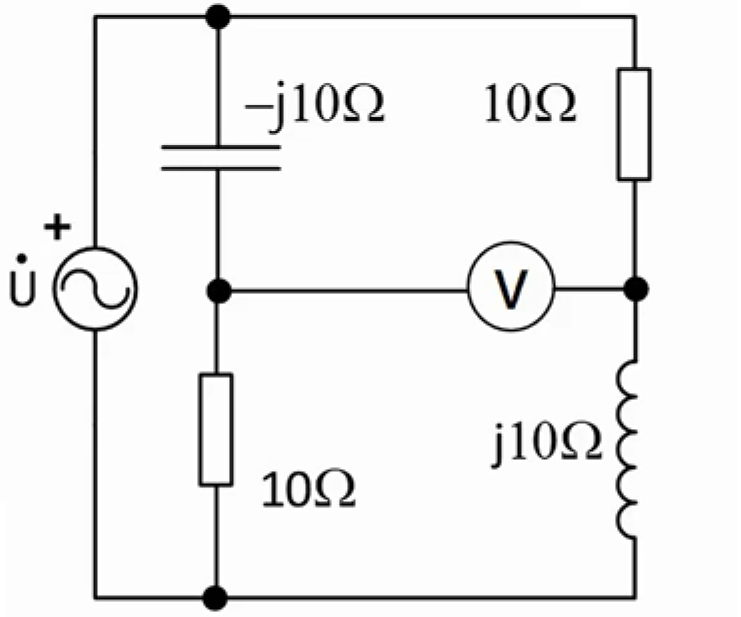
$$I_{ef} = \sqrt{I_{ef1}^2 + I_{ef2}^2 + \ldots + I_{efn}^2}$$

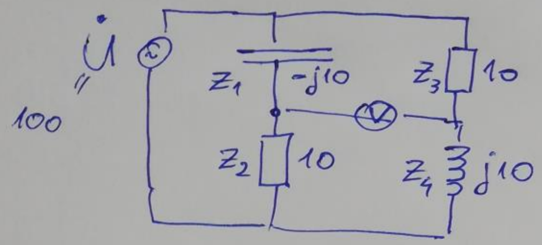
Vidimo da moramo paziti pri korištenju te formule, jer ona podrazumijeva da su valni oblici međusobno ortogonalni. Da bismo na ispravan način koristili ovo formulu u ovom primjeru, bilo bi potrebno struju *i*(*t*) prikazati kao:

$$i(t) = i_1(t) + i_2(t) = 1{,}5 + 0{,}5\,p(t)$$

gdje je *p*(*t*) čisto izmjenična pilasta funkcija.

- Odredite napon kojeg mjeri voltmetar. Provjerite da li je ispunjen uvjet za most u ravnoteži. Zadano: $\dot{U} = 100$ V .





PROVJERA UVJETA MOSTA

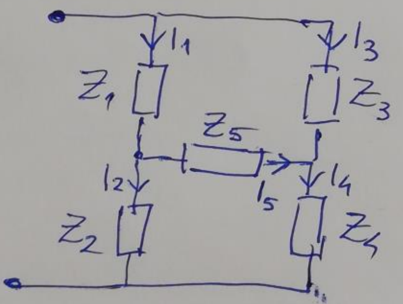
$\boxed{\dot{Z}_1 \cdot \dot{Z}_4 = \dot{Z}_2 \cdot \dot{Z}_3}$

$-j10 \cdot j10 = 10 \cdot 10$

$\boxed{100 = 100}$ ✓

$\Downarrow$ $\boxed{U_V = 0}$

OPĆENITO VRIJEDI:



Ako je $I_5 = 0$ tada možemo izbaciti Z_5!

KADA je $\dot{I}_5 = 0$?

↳ $\dot{I}_5 = 0$ ako je $\varphi_2 = \varphi_4$, tj. nema razlike potencijala koja bi potjerala struju, tj. $\dot{I}_5 = 0$

- $\dot{\varphi}_2 = \dot{\varphi}_4 \Rightarrow \boxed{\dot{U}_{Z2} = \dot{U}_{Z4}}$
- $(\dot{U}_{Z2} = \dot{U}_{Z4}) \Rightarrow \dot{U} - \dot{U}_{Z2} = \dot{U} - \dot{U}_{Z4}$
  $\boxed{\dot{U}_{Z1} = \dot{U}_{Z3}}$
- Ako $I_5 = 0 \Rightarrow \boxed{I_1 = I_2}$ i $\boxed{I_3 = I_4}$

$U_{Z2} = U_{Z4}$

$Z_2 \cdot I_2 = Z_4 \cdot I_4$

$Z_2 \cdot I_1 = Z_4 \cdot I_3$

$Z_2 \cdot \frac{U_{Z1}}{Z_1} = Z_4 \cdot \frac{U_{Z3}}{Z_3}$

$\frac{Z_2}{Z_1} \cdot \cancel{U_{Z1}} = \frac{Z_4}{Z_3} \cdot \cancel{U_{Z3}}$

$\boxed{Z_2 Z_3 = Z_1 Z_4}$ ← UVJET

- U mreži prema slici odredite primjenom metode konturnih struja efektivnu vrijednost napona na otporniku R_2 ako je zadano:

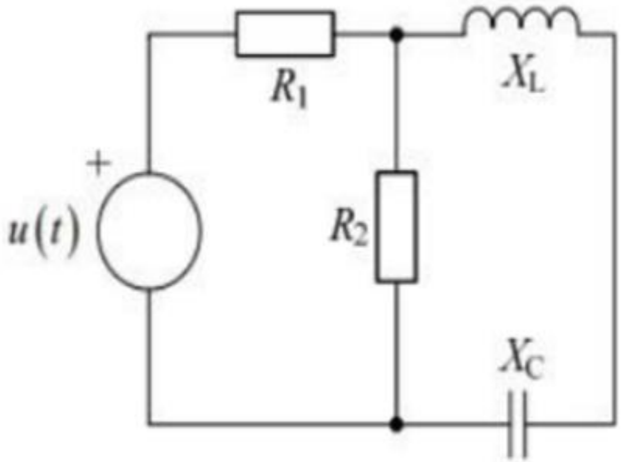
$u(t) = 10\sqrt{2}\sin(\omega t)\mathrm{V}$,

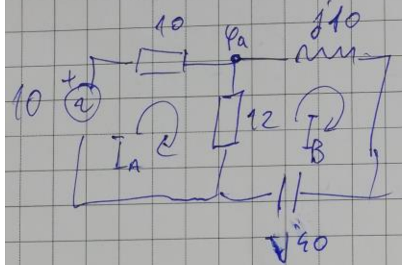
$R_1 = 10\Omega$,

$R_2 = 12\Omega$,

$X_L = 10\Omega$,

$X_C = 40\Omega$





PRIMJER 4
(predavanje)

METODA KONTURNIH STRUJA:

$I_A(10+12) - I_B\cdot 12 = 10$

$I_B(12-30j) - I_A\cdot 12 = 0$

$I_A\cdot 22 - 12 I_B = 10$

$I_A = \frac{10+12 I_B}{22}$

$I_B(12-30j) - \frac{12}{22}\cdot(10+12 I_B) = 0 \quad /\cdot 22$

$22 I_B(12-30j) - 12(10+12 I_B) = 0$

$264 I_B - 660j I_B - 120 - 144 I_B = 0$

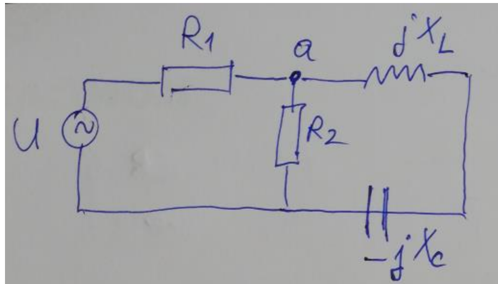
$120 I_B - 660j I_B = 120$

$I_B = \frac{120}{120-660j} = \frac{1}{1-5{,}5j} = \frac{2\sqrt{5}}{25}\angle 79{,}695^\circ$

$I_A = \frac{\sqrt{145}}{25}\angle 11{,}4965^\circ$

$I_3 = 0{,}4472$

$\boxed{\varphi_a = 5{,}366\angle -10{,}3}$



$R_1 = 10\Omega$
$R_2 = 12\Omega$
$X_L = 10\Omega$
$X_c = 40\Omega$

METODA NAPONA ČVOROVA:

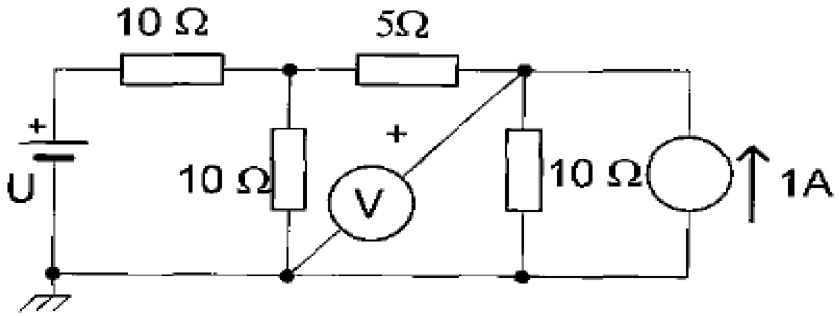
$\varphi_a\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{jX_L - jX_c}\right) = \frac{U}{R_1}$

$\varphi_a\left(\frac{1}{10}+\frac{1}{12}+\frac{1}{j\cdot 10-30j}\right) = \frac{10}{10} = 1$

$\boxed{\varphi_a = 5{,}366\angle -10{,}3^\circ}$

Na slici iznad, zadatak je radi usporedbe riješen i na alternativan način, korištenjem metode napona čvorova

IV.-8. Koliki je napon izvora (slika lijevo) ako je U_V=15 V označenog polariteta? Koliko će pokazivati voltmeter ako napon izvora smanjimo na polovinu prvotnog iznosa?



a-dio zadatka

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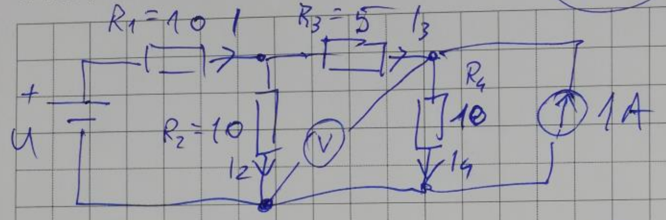
"TRANSFORMER RESEARCH AND ASSET MANAGEMENT"
OPATIJA, CROATIA, 9 - 12 OCTOBER 2019

IV.-8



$U_V = 15V$

$U_V = R_4 \cdot I_4 \to I_4 = \frac{15}{10} = 1{,}5A$

$I_3 + 1A = I_4 \to I_3 = 1{,}5 - 1 = 0{,}5A$

$U_3 = 5 \cdot I_3 = 5 \cdot 0{,}5 = 2{,}5V$

$U_2 = 2{,}5 + 15 = 17{,}5V$

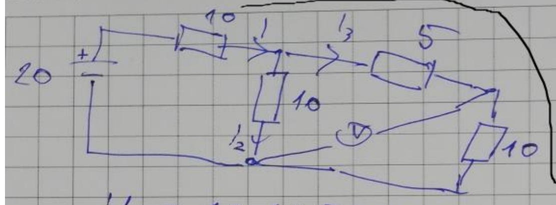
$I_2 = \frac{17{,}5}{10} = 1{,}75A$

$I_1 = I_2 + I_3 = 1{,}75 + 0{,}5 = 2{,}25A$

$U_1 = 22{,}5V \to U = 17{,}5 + 22{,}5$

$\boxed{U = 40V}$

b-dio zadatka



$Z_p = \frac{10 \cdot 15}{10+15} = \frac{150}{25} = 6$

$I = \frac{20}{10+6} = \frac{5}{4} = 1{,}25A$

$U_1 = 10 \cdot 1{,}25 = 12{,}5V$

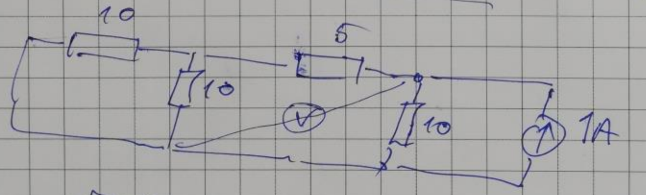
$U_2 = 20 - 12{,}5V = 7{,}5V$

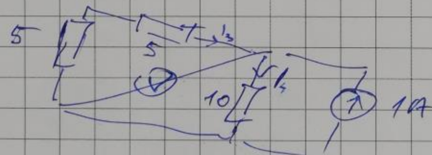
$I_2 = \frac{7{,}5}{10} = 0{,}75A$

$I_3 = I_1 - I_2 = 1{,}25 - 0{,}75 = 0{,}5A$

$U_3 = I_3 \cdot 5 = 0{,}5 \cdot 5 = \boxed{2{,}5V}$

$U_V' = 7{,}5V - 2{,}5V = \boxed{5V}$





$\Rightarrow I_4 = 0{,}5A$

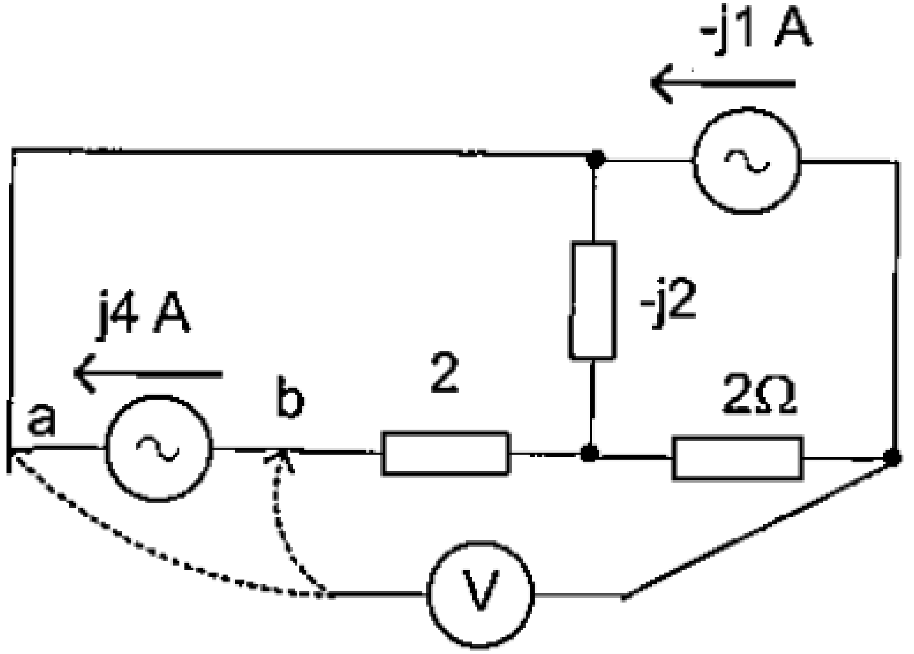
$U_4 = U_V'' = I_4 \cdot 10\Omega$

$\boxed{U_V'' = 5V}$

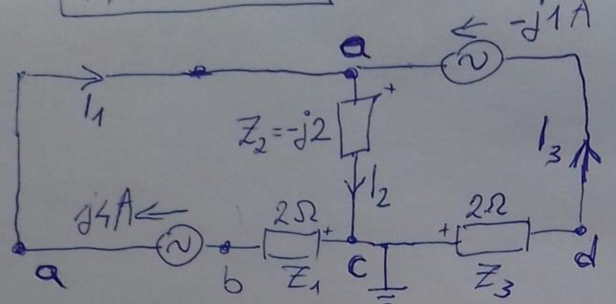
$\boxed{U_V = 10V}$

IX.2-5. Koliki napon mjeri voltmetar (efektivna vrijednost) u spoju prema slici desno ako je spojen na točku a odnosno na točku b?

Rezultat: $2\sqrt{10}$; 10 V



$\boxed{IX-2-5}$



čvor e:

$I_2 = I_1 + I_3$

$I_2 = 4j - 1j$

$\boxed{I_2 = 3j}$

Sad imamo struje u svim granama

$\varphi_c = 0$ (uzemljenje)

$\varphi_a - \varphi_c = U_{Z_2} = I_2 \cdot Z_2 = 3j \cdot (-2j) = 6\ V$

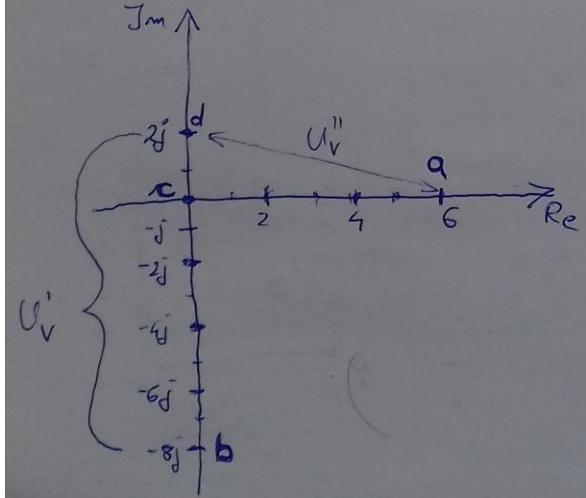
$\boxed{\varphi_a} = 6 + \varphi_c = \boxed{6\angle 0^\circ\ V}$

$\varphi_c - \varphi_d = U_{Z3} = I_3 \cdot 2 = (-j) \cdot 2 = -2j\ V$

$\boxed{\varphi_d} = 2j + \varphi_c = \boxed{2j\ V}$

$\varphi_c - \varphi_b = U_{Z1} = I_1 \cdot Z_1 = 4j \cdot 2 = 8j$

$\varphi_b = \varphi_c - 8j = \boxed{-8j\ V}$



$\varphi_d - \varphi_b = \dot{U}'_V = j\,10\ V$

$U'_V = 10\ V$

$\varphi_a - \varphi_b = 6 - 2j = \dot{U}''_V$

$U''_V = \sqrt{6^2 + 2^2} = \sqrt{40}$

$U''_V = 2\sqrt{10}$

XIII-5. Analizom valnih oblika struje i napona na priključnicama jednog dvopola dobiveno je da se napon i struja s dovoljnom točnošću mogu prikazati u obliku:

$$u(t)=50+50\sin(5\cdot 10^3 t)+30\sin(10^4 t)+20\sin(2\cdot 10^4 t)\ \mathrm{V},\ \text{te}$$

$$i(t)=11{,}2\sin(5\cdot 10^3 t+63{,}4^0)+10{,}6\sin(10^4 t+45^0)+8{,}97\sin(2\cdot 10^4 t+26{,}6^0)\ \mathrm{A}.$$

Kolika je srednja snaga tog dvopola ?

$$u(t)=50+50\sin(5000t)+30\sin(10000t)+20(\sin(20000t))$$

$$i(t)=0+11{,}2\sin(5000t+63{,}4^\circ)+10{,}6\sin(10000t+45^\circ)+8{,}97\sin(20000t+26{,}6^\circ)$$

$$P=\int_0^T p(t)\,dt=\int_0^T u(t)\cdot i(t)\,dt$$

ZA sinusne funkcije vrijedi (ORTOGONALNOST):

$$\int_0^T \sin(n\cdot\omega t)\cdot\sin(m\cdot\omega t)\,dt=0 \quad \ldots \text{ako je } n\neq m$$

$$n,m\in\mathbb{N} \ \text{i}\ T=\frac{2\pi}{\omega}$$

... pa slijedi

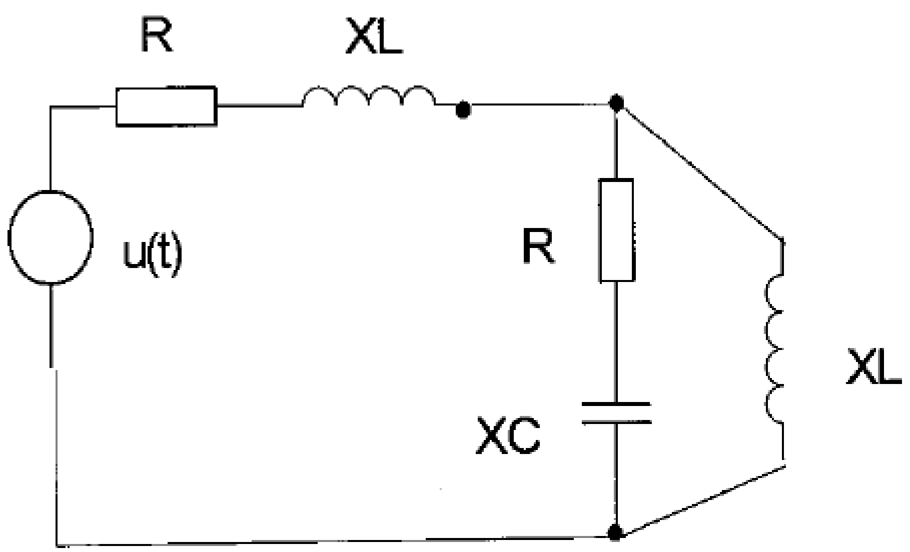
$$P=U_0\cdot I_0+\frac{U_1\cdot I_1}{2}\cos\varphi_1+\frac{U_2 I_2}{2}\cos\varphi_2+\frac{U_3 I_3}{2}\cdot\cos\varphi_3$$

$$P=U_0 I_0+U_1 I_1\cdot\cos\varphi_1+U_2 I_2\cos\varphi_2+U_3 I_3\cdot\cos\varphi_3$$

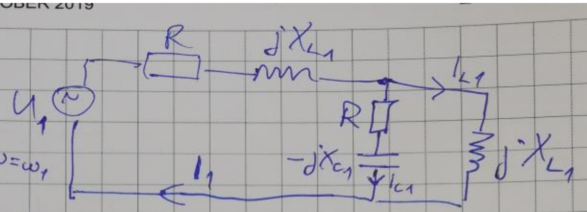
$$P=50\cdot 0+\frac{50}{\sqrt2}\cdot\frac{11}{\sqrt2}\cdot\cos 63{,}4^\circ+\frac{30}{\sqrt2}\cdot\frac{10{,}6}{\sqrt2}\cdot\cos 45^\circ+\frac{20}{\sqrt2}\cdot\frac{8{,}97}{\sqrt2}\cdot\cos 26{,}6^\circ$$

$$\boxed{P=318\ \mathrm{W}}$$

XIII-7. Napon izvora u krugu na slici ima prvi i peti harmonik. Efektivna vrijednost struje prvog (osnovnog) harmonika u lijevoj grani iznosi I_{C1}=2 A, a petog harmonika u desnoj grani I_{L5}=0,1 A. Ako su vrijednosti otpora za osnovni harmonik $R=X_L=X_C=5\ \Omega$, odredite efektivnu vrijednost priključenog napona.



XIII-7



$R = X_{L1} = X_{C1} = 5\,\Omega$ za $\omega = \omega_1$

$I_{C1} = 2A$ za $\omega = \omega_1$

$U_{CR1} = 2(\underset{R}{5} - \underset{X_C}{5}j) = 10 - 10j = U_{L1} \Rightarrow I_{L1} = \frac{U_{L1}}{jX_{L1}}$

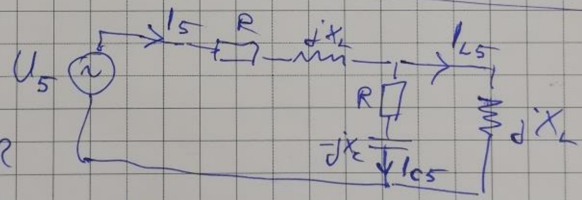
$I_{L1} = \frac{10-10j}{5j} = \boxed{-2-2j}$

$I_1 = I_{C1} + I_{L1} = 2 - 2 - 2j = \boxed{-2j}$

$U_1 = I_1(R + jX_{L1}) + U_{CR1} = -2j\cdot(5+5j) + (10 - 10j)$

$U_1 = (-10j + 10) + (10 - 10j) = 20 - 20j = 20\sqrt{2}\angle -$

$\boxed{I_{L5} = 0{,}1}$



$\omega = 5\omega_1$

$X_{L5} = 5\cdot 5 = 25\,\Omega$

$X_{C5} = \frac{X_{C1}}{5} = 1\,\Omega$

$R = 5\,\Omega$

$U_{L5} = I_{L5}\cdot jX_{L5} = 0{,}1\angle 0^\circ \cdot 25\angle 90^\circ$

$U_{L5} = 2{,}5j = U_{CR5}$

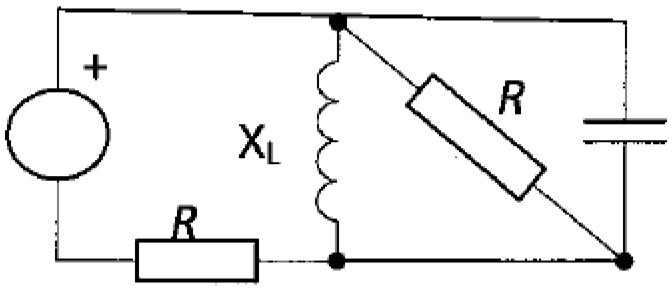
$I_{C5} = \frac{U_{CR5}}{R - jX_{C5}} = \frac{2.5j}{5-1j} = -\frac{5}{52} + j\frac{25}{52}$

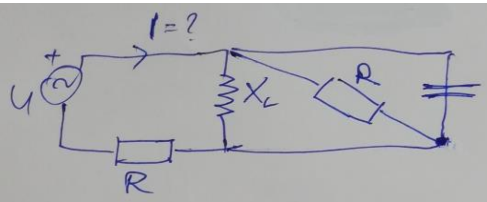
$I_5 = I_{L5} + I_{C5} = 0{,}1 - \left(-\frac{5}{52} + \frac{25}{52}j\right) = \frac{1}{260} + \frac{25}{52}j$

$U_5 = I_5(5 + 25j) + 2{,}5j = -12 + 5j = 13\angle 157{,}3^\circ$

$U_{ef} = \sqrt{13^2 + (20\sqrt{2})^2} = \boxed{31{,}1}$

XIII-12 Spoj prema slici priključen je na napon $u(t)=40\sin\omega t+16\sin 2\omega t$. Zadan je R=1,33 Ω i reaktivni otpori za kružnu frekvenciju frekvenciju ω $X_L=1\ \Omega$ $X_C=4\ \Omega$ .Izračunajte efektivnu vrijednost struje izvora.





$u(t) = 40\sin\omega_0 t + 16\sin 2\omega_0 t$

$R = 1{,}33\ \Omega$

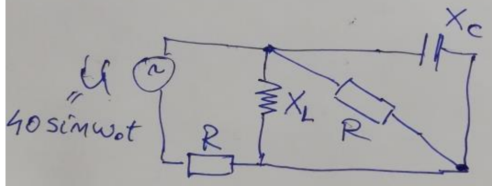
$X_L = 1\ \Omega$ za $\omega=\omega_0$

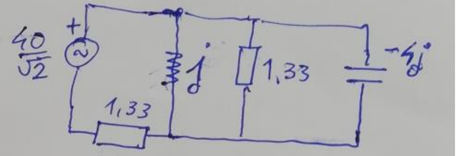
$X_C = 4\ \Omega$ za $\omega=\omega_0$

$X_L = L\cdot\omega \rightarrow X_L = 2\ \Omega$ za $\omega = 2\omega_0$

$X_C = \frac{1}{C\omega} \rightarrow X_C = 2\ \Omega$ za $\omega = 2\omega_0$

za $\omega=\omega_0$:





$Y_p = \frac{1}{j} + \frac{1}{1{,}33} + \frac{1}{-4j}$

$Y_p = \frac{100}{133} - \frac{3}{4}j$

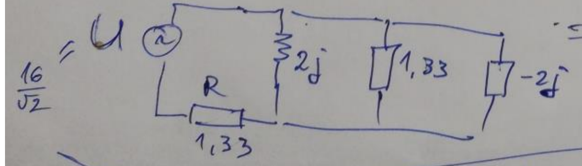
$Z_p = \frac{1}{Y_p} = 0{,}667+0{,}665j$

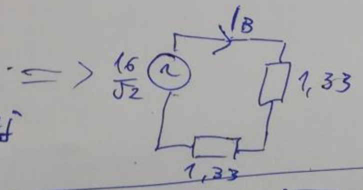
$Z = 1{,}33 + Z_p$

$Z = 2 + 0{,}665j$

$I_A = \frac{\frac{40}{\sqrt{2}}}{2+0{,}665j} = 13{,}4197\angle -18{,}39$

Za $\omega = 2\omega_0$:





$I_B = 4{,}253$

$I_{ef} = \sqrt{I_A^2+I_B^2} = \boxed{14{,}07\ A}$

XIII-13. Otpornik R=5 Ω spojen je u seriju sa paralelnim spojem L i C. Na frekvenciji ω=500 rad/s reaktancije su X_L=2 Ω, X_C=8 Ω. Odredite efektivnu vrijednost ukupne struje ako je spoj priključen na napon: u(t)=50+66,57sin(500t)+50,5 sin(1000t) V.

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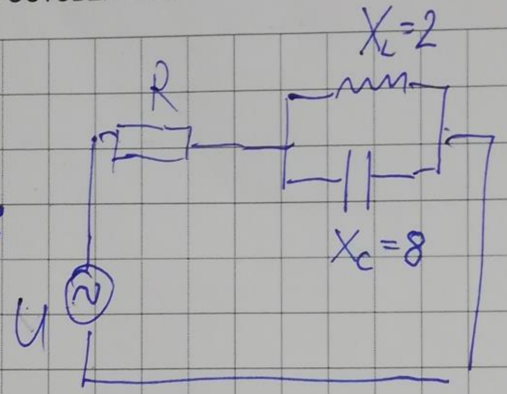
$R=5$

Za $\omega=500$:

$X_L=2$

$X_C=8$



$u(t)=u_A+u_B+u_C$

$u_A(t)=50$

$u_B(t)=66{,}57\sin(500t)$

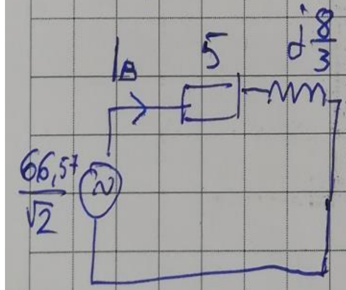
$u_C(t)=50{,}5\sin(1000t)$

$$Z_p=\frac{j2\cdot(-j8)}{2j-8j}$$

$$Z_p=\frac{16}{-6j}=\boxed{j\frac{8}{3}}$$

$$Z=R+Z_p=5+j\frac{8}{3}\quad(\omega=500)$$

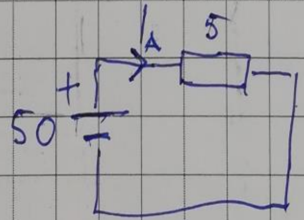
$\omega=500$:



$$I_B=\frac{\frac{66{,}57}{\sqrt{2}}}{5+j\frac{8}{3}}$$

$$\boxed{I_B=8{,}3\angle{-28^\circ}}$$

$\omega=0$:



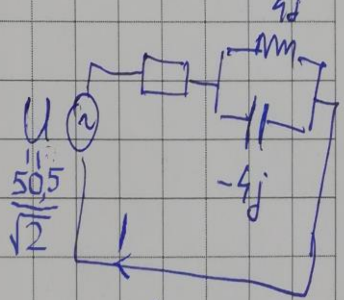
$$I_A=\frac{50}{5}$$

$$\boxed{I_A=10A}$$

$\omega=1000$:

$X_L=4$

$X_C=4$



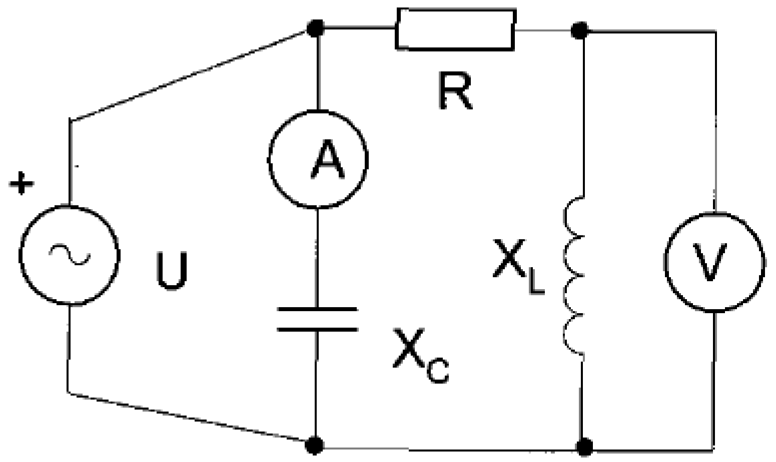
$$Y_p=\frac{1}{4j}-\frac{1}{4j}=0$$

$$\boxed{I=0}$$

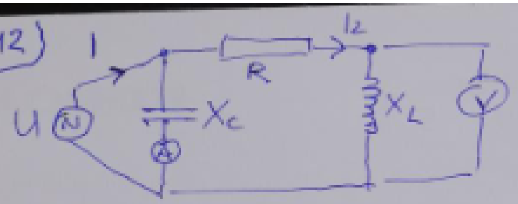
$$\boxed{I_{uk}=\sqrt{8{,}3^2+10^2}=13A}$$

VII.2-12. U spoju na slici voltmetar mjeri napon U_V, a ampermetar struju I_A. Ako su struja i napon izvora u fazi, odredite induktivni otpor X_L.

ZADANO: U=10 V; U_V=6V; I_A=1 A;



VII-2-12



$I_A = 1A$
$U_V = 6V$
$U = 10V$

$\boxed{U \text{ i } I \text{ su u fazi}}$ ⇔ REZONANCIJA

$$X_c = \frac{U}{I_A} = \frac{10}{1} = \boxed{10\Omega}$$

$$Z = \frac{-jX_c \cdot (R + jX_L)}{-jX_c + R + jX_L} = \frac{X_c X_L - j \cdot R X_c}{R + j(X_L - X_c)} = \frac{(X_c X_L - jRX_c)(R - j(X_L - X_c))}{R^2 + (X_L - X_c)^2}$$

$$= \frac{RX_L X_c - RX_c(X_L - X_c)}{R^2 + (X_L - X_c)^2} - j \cdot \underbrace{\frac{X_c X_L (X_L - X_c) + R^2 X_c}{R^2 + (X_L - X_c)^2}}_{=0}$$

U i I u fazi (rezonancija) ⇒

$$X_c X_L^2 - X_c^2 X_L + R^2 X_c = 0$$

$$10 X_L^2 - 100 X_L + R^2 \cdot 10 = 0$$

$$10 X_L^2 - 100 X_L + \frac{160}{9} X_L^2 = 0$$

$$\frac{250}{9} X_L^2 - 100 X_L = 0$$

$$\frac{250}{9} X_L = 100$$

$$\boxed{X_L = 3{,}6\,\Omega}$$

$$\dot{U} = \dot{I}_2 \cdot R + jX_L \cdot \dot{I}_2$$

$$U = \sqrt{\left(\frac{U_V}{X_L} \cdot R\right)^2 + \left(X_L \cdot \frac{U_V}{X_L}\right)^2}$$

$$10 = \sqrt{\left(\frac{6 \cdot R}{X_L}\right)^2 + 6^2}$$

$$\left(\frac{6 \cdot R}{X_L}\right)^2 = 100 - 36 = 64$$

$$\frac{6R}{X_L} = 8$$

$$\boxed{R = \frac{4}{3} X_L}$$

DRUGI (BRŽI) NAČIN:

U i I su u fazi ⇔ REZONANCIJA ⇒ $Q_{IZV} = Q_{TROŠ} = 0$

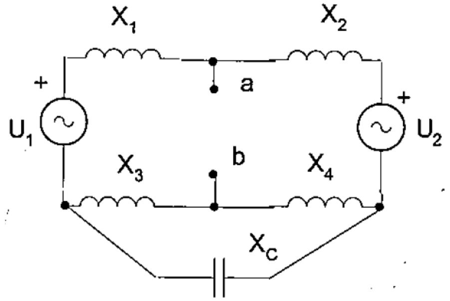
$$jQ_L - jQ_C = 0 \Rightarrow \boxed{Q_L = Q_C}$$

$$Q_C = U \cdot I_A = 10$$

$$Q_C = Q_L = 10 \rightarrow Q_L = 10 = \frac{U_V^2}{X_L} \Rightarrow X_L = \frac{U_V^2}{Q_L} = \frac{6^2}{10} \Rightarrow \boxed{X_L = 3{,}6\,\Omega}$$

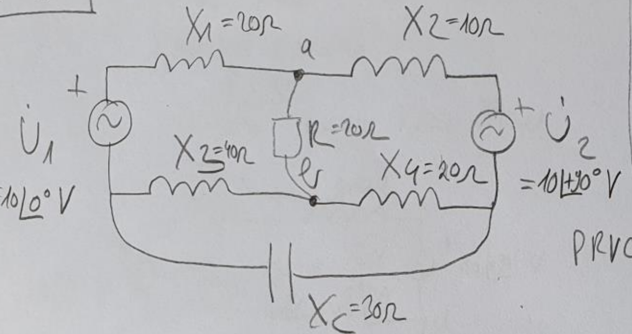
Pavić-Felja 2. dio: X-18

X-18. Mrežu prema slici nadomjestite prema Nortonovom teoremu s točaka a i b. Kolika struja bi tekla kroz otpornik R=20 Ω koji priključimo između tih točaka?
Zadano: $X_1=X_4=20$, $X_3=40$, $X_2=10$, $X_C=30$ Ω, $U_1=10$ V (početni fazni kut 0) $U_2=10$ V (početni fazni kut $+90^0$).



Rezultat : $\underline{Z}_N$=j20 Ω $\underline{I}_N=0{,}37\angle -26{,}5^0$; 0,26 A

X-18

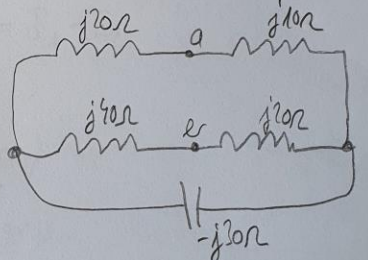


NORTONOV TEOREM

TRAŽI SE IZNOS $|\dot{I}_R|$

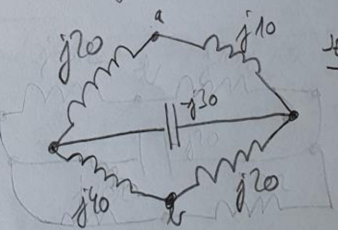
TRAŽI SE $\dot{I}_N$ i $\underline{Z}_N$ IZMEĐU TOČAKA a i b

PRVO $\underline{Z}_N$: NAPONSKE IZVORE GASIMO I NADOMJESTIMO S KRATKIM SPOJEM



← PRECRTAMO DOLJE DA UOČIMO MOST

$\boxed{\underline{Z}_N = j20\Omega}$



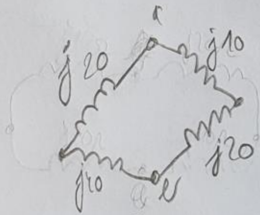
PROVJERIMO JE LI U RAVNOTEŽI

$j10 \cdot j40 = j20 \cdot j20$

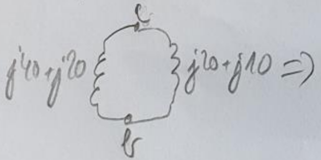
$-400 = -400$

JE

-j30 MOŽEMO ODSPOJITI ILI KRATKO SPOJITI



ODSPOJIMO -j30Ω

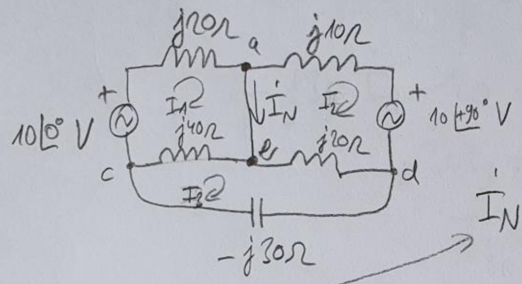


$\Rightarrow \underline{Z}_N = \underline{Z}_{ab} = j30 \| j60 = \frac{j30 \cdot j60}{j30+j60} = \frac{-1800}{j90} = -\frac{20}{j} = j20\Omega$

TRAŽIMO $\dot{I}_N = \dot{I}_{ab}$ NEKA JE REF. SMJER a→b

KRATKO SPOJIMO a i b

I TRAŽIMO STRUJU KROZ ŽICU



$\dot{I}_N = \dot{I}_1 - \dot{I}_2$

KONTURNE STRUJE:

$\dot{I}_1(j20+j40) - \dot{I}_2 \cdot 0 - \dot{I}_3 j40 = 10\angle 0° V$

$-\dot{I}_1 \cdot 0 + \dot{I}_2(j20+j10) - \dot{I}_3 j20 = -10\angle 90° V$

$-\dot{I}_1 j40 - \dot{I}_2 j20 + \dot{I}_3(j40+j20-j30) = 0$

$j60\dot{I}_1 - j40\dot{I}_3 = 10 \Rightarrow j60\dot{I}_1 = 10 + j40\dot{I}_3 \Rightarrow \dot{I}_1 = \frac{10}{j60} + \frac{j40}{j60}\dot{I}_3 = -j\frac{1}{6} + \frac{2}{3}\dot{I}_3$

$j30\dot{I}_2 - j20\dot{I}_3 = -j10 \Rightarrow j30\dot{I}_2 = -j10 + j20\dot{I}_3 \Rightarrow \dot{I}_2 = \frac{-j10}{j30} + \frac{j20}{j30}\dot{I}_3 = -\frac{1}{3} + \frac{2}{3}\dot{I}_3$

$-j40\dot{I}_1 - j20\dot{I}_2 + j30\dot{I}_3 = 0$

$-j40\left(-j\frac{1}{6} + \frac{2}{3}\dot{I}_3\right) - j20\left(-\frac{1}{3} + \frac{2}{3}\dot{I}_3\right) + j30\dot{I}_3 = 0 \Rightarrow$

Pavić-Felja 2. dio: X-18

$$-\frac{20}{3} - j\frac{80}{3}\dot{I}_3 + j\frac{20}{3} - j\frac{40}{3}\dot{I}_3 + j30\dot{I}_3 = 0 \quad /\cdot 3$$

$$-20 - j80\dot{I}_3 + j20 - j40\dot{I}_3 + j90\dot{I}_3 = 0$$

$$-j30\dot{I}_3 - 20 + j20 = 0$$

$$j30\dot{I}_3 = -20 + j20$$

$$\dot{I}_3 = -\frac{20}{j30} + \frac{j20}{j30}$$

$$\dot{I}_3 = +j\frac{2}{3} + \frac{2}{3} = \frac{2}{3} + j\frac{2}{3}\ \text{A}$$

ONDA $\dot{I}_1 = -j\frac{1}{6} + \frac{2}{3}\cdot\dot{I}_3 = -j\frac{1}{6} + \frac{4}{9} + j\frac{4}{9} = +\frac{4}{9} + j\frac{-3+8}{18} =$

$$= +\frac{4}{9} + j\frac{5}{18}\ \text{A}$$

ONDA $\dot{I}_2 = -\frac{1}{3} + \frac{2}{3}\dot{I}_3 = -\frac{1}{3} + \frac{4}{9} + j\frac{4}{9} = \frac{-3+4}{9} + j\frac{4}{9} = \frac{1}{9} + j\frac{4}{9}\ \text{A}$

ONDA: $\dot{I}_N = \dot{I}_1 - \dot{I}_2 = +\frac{4}{9} + j\frac{5}{18} - \frac{1}{9} - j\frac{4}{9} = \frac{1}{3} - j\frac{1}{6} = 0.37268\angle{-26.565^\circ}\ \text{A}$

DAKLE:

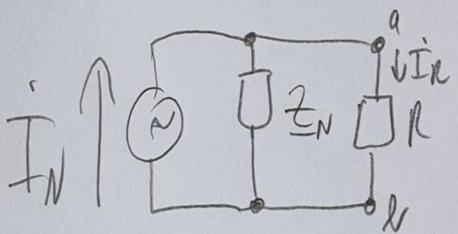
$$\boxed{\underline{Z}_N = j20\,\Omega}$$

$$\dot{I}_N = 0.37268\angle{-26.565^\circ}\ \text{A}$$
ili
$$\dot{I}_N = \frac{1}{3} - j\frac{1}{6}\ \text{A}$$

TRAŽI SE $|\dot{I}_R|$ KROZ

$R = 20\,\Omega$ SPOJEN IZMEĐU a i b!

NORTONOV NADOMJESNI SPOJ:



STRUJNO DIJELILO:

$$\dot{I}_R = \dot{I}_N \frac{\underline{Z}_N}{\underline{Z}_N + R}$$

$$|\dot{I}_R| = \left|0.37268\angle{-26.565^\circ}\ \frac{j20}{j20+20}\right|$$

$$|\dot{I}_R| = 0.37268\cdot\frac{20}{\sqrt{400+400}} = 0.37268\ \frac{20}{20\sqrt{2}} \Rightarrow$$

$$\Rightarrow |\dot{I}_R| = \frac{0.37268}{\sqrt{2}} = 0.2635\ \text{A}$$

Pavić-Felja 2. dio: X-18 (drugi pristup – primjena Theveninovog teorema pa onda određivanje parametara Nortonovog izvora)

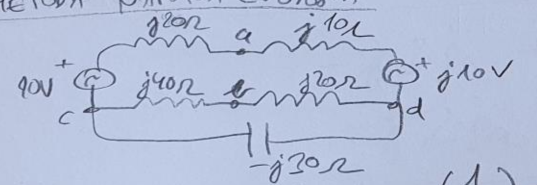
DRUGI NAČIN ZA NORTONOVU STRUJU:

NAĆI $\dot{U}_T$ PA ONDA $\dot{I}_N = \frac{\dot{U}_T}{\underline{Z}_T}$

ZNAMO DA

$\underline{Z}_T = \underline{Z}_N$ ZNAČI $\underline{Z}_T = j20\Omega$ ISTO!

METODA NAPONA ČVOROVA



$\dot{U}_T = \dot{U}_{ae}$

NEKA JE d REF. TOČKA

$$\dot{\varphi}_a \cdot \left(\frac{1}{j20} + \frac{1}{j10}\right) - \dot{\varphi}_e\left(\frac{1}{\infty}\right) - \dot{\varphi}_c\left(\frac{1}{j20}\right) = \frac{10}{j20} + \frac{j10}{j10}$$

$$-\dot{\varphi}_a \cdot \left(\frac{1}{\infty}\right) + \dot{\varphi}_e\left(\frac{1}{j40} + \frac{1}{j20}\right) - \dot{\varphi}_c\left(\frac{1}{j40}\right) = 0$$

$$-\dot{\varphi}_a\left(\frac{1}{j20}\right) - \dot{\varphi}_e\left(\frac{1}{j40}\right) + \dot{\varphi}_c\left(\frac{1}{j20} + \frac{1}{j40} + \frac{1}{-j30}\right) = -\frac{10}{j20}$$

$$-j\frac{3}{20}\dot{\varphi}_a + j\frac{1}{20}\dot{\varphi}_c = 1 - j\frac{1}{2}$$

$$-j\frac{3}{40}\dot{\varphi}_e + j\frac{1}{40}\dot{\varphi}_c = 0 \Rightarrow j\frac{1}{40}\dot{\varphi}_c = j\frac{3}{40}\dot{\varphi}_e \Rightarrow \dot{\varphi}_c = 3\dot{\varphi}_e$$

$$j\frac{1}{20}\dot{\varphi}_a + j\frac{1}{40}\dot{\varphi}_e - j\frac{1}{24}\dot{\varphi}_c = j\frac{1}{2}$$

$$-j\frac{3}{20}\dot{\varphi}_a + j\frac{3}{20}\dot{\varphi}_e = 1 - j\frac{1}{2} \quad /\cdot 20$$

$$+j\frac{1}{20}\dot{\varphi}_a + j\frac{1}{40}\dot{\varphi}_e - j\frac{1}{8}\dot{\varphi}_e = j\frac{1}{2} \quad /\cdot 40$$

$$-j3\dot{\varphi}_a + j3\dot{\varphi}_e = 20 - j10 \quad /\cdot 2$$

$$+j2\dot{\varphi}_a + j\dot{\varphi}_e - j5\dot{\varphi}_e = j20 \quad /\cdot 3 \quad /+$$

$$j6\dot{\varphi}_e - j12\dot{\varphi}_e = 40 - j20 + j60$$

$$-j6\dot{\varphi}_e = 40 + j40$$

$$\dot{\varphi}_e = \frac{40}{-j6} + \frac{j40}{-j6}$$

$$\dot{\varphi}_e = j\frac{20}{3} - \frac{20}{3} = -\frac{20}{3} - j\frac{20}{3}\,V$$

$$-j\frac{3}{20}\dot{\varphi}_a + \cancel{1} - j = \cancel{1} - j\frac{1}{2}$$

$$-j\frac{3}{20}\dot{\varphi}_a = j\frac{1}{2}$$

$$\dot{\varphi}_a = \frac{j\frac{1}{2}}{-j\frac{3}{20}} = -\frac{10}{3}\,V$$

$$\dot{U}_T = \dot{\varphi}_a - \dot{\varphi}_e = \frac{10}{3} + j\frac{20}{3}\,V = \frac{10}{3}(1+j2) = \frac{10}{3}\sqrt{5}\,\angle 63.435^\circ\,V$$

Theveninov izvor = realni naponski izvor
Nortonov izvor = realni strujni izvor

Pretvorba realni struni ↔ realni naponski:

$$\dot{I}_N = \frac{\dot{U}_{Th}}{\underline{Z}_{Th}} \qquad \dot{U}_{Th} = \dot{I}_N \underline{Z}_N \qquad \underline{Z}_{Th} = \underline{Z}_N$$

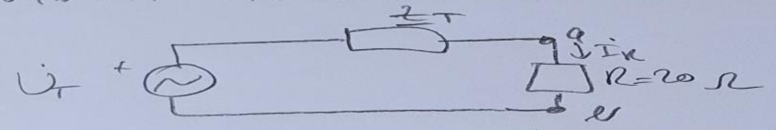
Pazite na rubne slučajeve! Ako je:
$\underline{Z}_{Th} = 0$ onda koristite samo Thevenina
$\underline{Z}_{Th} = \infty$ onda koristite samo Nortona

ONDA: $\dot{I}_N = \frac{\dot{U}_T}{\underline{Z}_T} = \frac{\frac{10}{3}\sqrt{5}\,\angle 63.435^\circ}{20\,\angle 90^\circ} = \frac{\sqrt{5}}{6}\,\angle -26.565^\circ\,A$

$\boxed{\dot{U}_T = \frac{10\sqrt{5}}{3}\,\angle 63.435^\circ\,V}$

$\dot{I}_N = 0.3728\,\angle -26.565^\circ\,A$

NADOMJESNI THEVENINOV IZVOR:



$$\dot{I}_R = \frac{\dot{U}_T}{\underline{Z}_T + R} \Rightarrow |\dot{I}_R| = \frac{|\dot{U}_T|}{\sqrt{(R)^2 + (j20)^2}} = \frac{\frac{10\sqrt{5}}{3}}{20\sqrt{2}} = \frac{\sqrt{5}}{6\sqrt{2}}$$

$$|\dot{I}_R| \approx 0.2635\,A$$