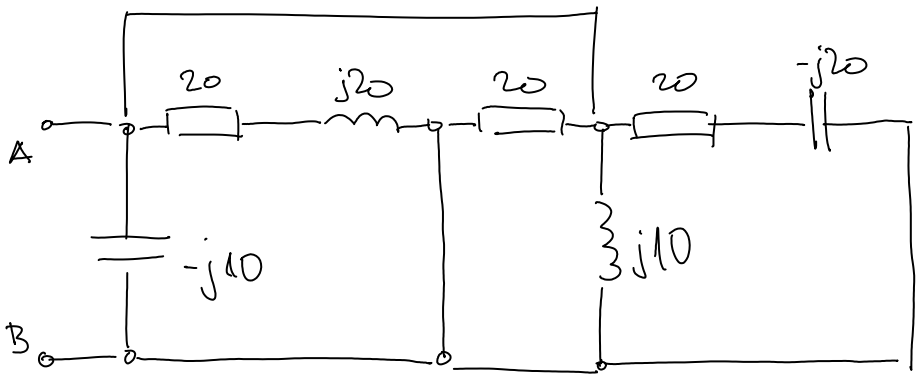
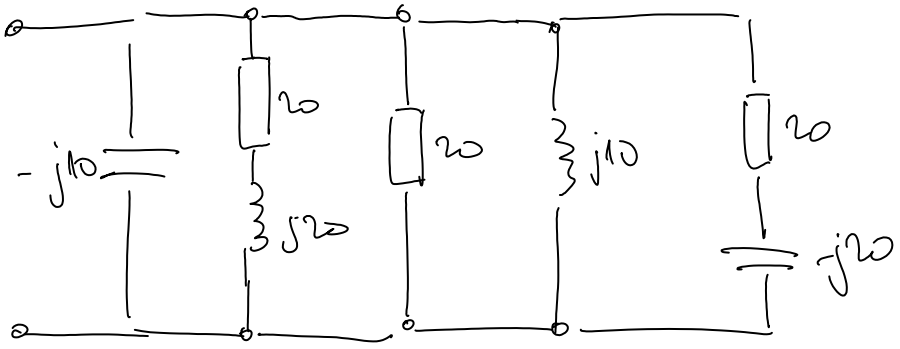
Zadatak 1

31. siječnja 2013. 13:14

1.



$Z_{AB} = ?$



$$Z_{AB} = -j10 \,\|\, (20+j20) \,\|\, 20 \,\|\, j10 \,\|\, 20-j20$$

$$\frac{1}{Z_{AB}} = \frac{1}{-j10} + \frac{1}{20+j20} + \frac{1}{20} + \frac{1}{j10} + \frac{1}{20-j20}$$

$$= \frac{1}{20}\left(\frac{1}{1+j} + 1 + \frac{1}{1-j}\right) =$$

$$= \frac{1}{20} \cdot \frac{1-j+2+1+j}{2} = \frac{1}{20} \cdot \frac{4}{2} = \frac{1}{10}$$

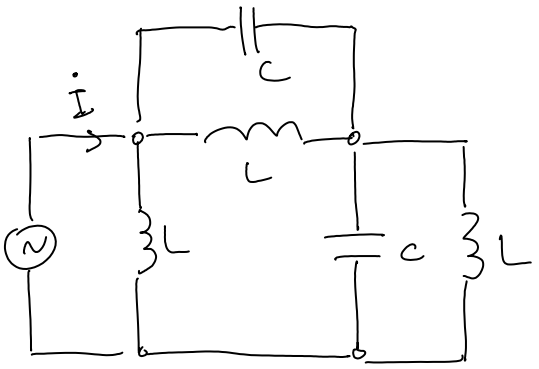
$$\Downarrow$$

$$\dot{Z}_{AB} = 10\,\Omega$$

Zadatak 2

31. siječnja 2013. 13:27

2.



$I = 0\ A$

$U = 10\ V$

$L = 3\ mH$

$C = 10\ \mu F$

$$Z_{uk} = X_L \,||\, \left[2 \cdot (X_L \,||\, X_C)\right] = X_L \,||\, \left(2 \cdot \frac{X_L X_C}{X_L + X_C}\right) =$$

$$= \frac{\frac{2X_L^2 X_C}{X_L + X_C}}{X_L + \frac{2X_L X_C}{X_L + X_C}} = \frac{\frac{2X_L^2 X_C}{X_L + X_C}}{\frac{X_L^2 + X_L X_C + 2X_L X_C}{X_L + X_C}} =$$

$$= \frac{2X_L^2 X_C \,:X_L}{X_L^2 + 3X_L X_C \,:X_L} = \frac{2X_L X_C}{X_L + 3X_C}$$

- za $I = 0$ vrijedi $Z_{uk} = \infty$, dakle nazivnik od Z_{uk} mora biti $= \phi$

$$X_L + 3X_C = 0$$

$$\omega L - 3 \cdot \frac{1}{\omega C} = 0 \quad / \cdot \omega C$$

$$\omega^2 LC - 3 = 0$$

$$\omega = \sqrt{\frac{3}{LC}} = \sqrt{\frac{3}{3 \cdot 10^{-3} \cdot 10 \cdot 10^{-6}}} = \sqrt{\frac{1}{10^{-10}}} = 10^5$$

$$\omega = 10\,000\ rad/s$$

Zadatak 3

31. siječnja 2013. 14:19

(3.) $z_S = z_P = 8 + j6 = R_P \| jX_P$

$$8 + j6 = \frac{jR_PX_P}{R_P + jX_P} \cdot \frac{R_P - jX_P}{R_P - jX_P} =$$

$$8 + j6 = \frac{jR_P^2X_P + R_PX_P^2}{R_P^2 + X_P^2} = \frac{R_PX_P^2}{R_P^2 + X_P^2} + j\frac{R_P^2X_P}{R_P^2 + X_P^2} \quad (*)$$

- izjednačimo imaginarne dijelove jednadžbe (*):

$$\frac{R_P^2X_P}{R_P^2 + X_P^2} = 6$$

$$R_P^2X_P - 6R_P^2 - 6X_P^2 = 0 \rightarrow R_P = \frac{X_P\sqrt{6}}{\sqrt{X_P - 6}}$$

- izjednačimo realne dijelove jednadžbe (*):

$$\frac{R_PX_P^2}{R_P^2 + X_P^2} = 8$$

$$R_PX_P^2 - 8R_P^2 - 8X_P^2 = 0$$

$$\frac{\sqrt{6} \cdot X_P \cdot X_P^2}{\sqrt{X_P - 6}} - \frac{48X_P^2}{X_P - 6} - 8X_P^2 = 0 \quad / : X_P^2$$

$$\frac{\sqrt{6X_P - 36} \cdot X_P - 48 - 8X_P + 48}{X_P - 6} = 0 \quad (\text{brojnik} = 0)$$

$$\sqrt{6X_P - 36} \cdot X_P - 8X_P = 0 \quad / : X_P \quad / ^2$$

$$6X_P - 36 - 64 = 0$$

$$X_P = \frac{100}{6}$$

$$R_P = \frac{X_P\sqrt{6}}{\sqrt{X_P - 6}} = \frac{\frac{100}{6} \cdot \sqrt{6}}{\sqrt{\frac{100}{6} - 6}} =$$

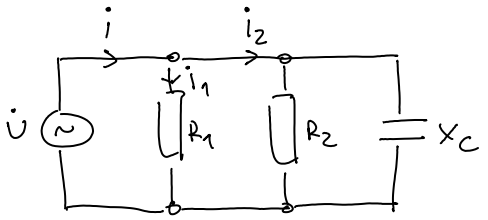
$$= \frac{\frac{100}{\sqrt{6}}}{\sqrt{\frac{64}{6}}} = \frac{\frac{100}{\sqrt{6}}}{\frac{8}{\sqrt{6}}} = 12{,}5\,\Omega$$

$$R_p = 12{,}5\,\Omega$$

Zadatak 4

31. siječnja 2013. 15:32

4.



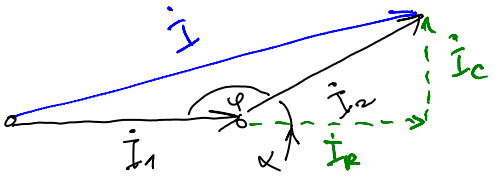
$I = 3\,A$

$I_1 = 2\,A$

$I_2 = 2\,A$

$R_1 = 4\,A$

- skica fazorskog dijagrama struja:



$$I^2 = I_1^2 + I_2^2 - 2 I_1 I_2 \cos\varphi$$

$$\cos\varphi = -\frac{I^2 - I_1^2 - I_2^2}{2 I_1 I_2} = -\frac{1}{8} \Rightarrow \varphi = 97{,}18^\circ$$

$$\alpha = 180^\circ - 97{,}18^\circ = 82{,}82^\circ$$

$$I_R = I_2 \cos\alpha = 0{,}25\,A$$

$$U = I_1 \cdot 4 = 8\,V$$

$$P = U \cdot I_R + U \cdot I_1 = U(I_R + I_1) = 8 \cdot (2 + 0{,}25) = 18\,W$$

$$\boxed{P = 18\,W}$$

Zadatak 5

31. siječnja 2013. 15:43

5. $i(t) = I_m \cdot \sin(\omega t + \alpha_i^\circ)$, $t_1 = 15\,ms$

$\dot{I} = 5\angle 45^\circ = I_{ef}\angle \varphi_i^\circ$

$$i(t_1) = 5\sqrt{2} \cdot \sin\left(1000\pi \cdot 0{,}015 + \frac{\pi}{4}\right) = 5\sqrt{2} \cdot \sin\left(\frac{3\pi}{2} + \frac{\pi}{4}\right)$$

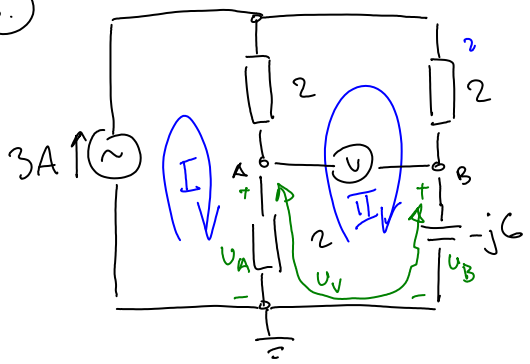
$$= 5\sqrt{2} \sin\left(\frac{7\pi}{4}\right) = 5\sqrt{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) = -5A$$

$$\boxed{i(t_1) = -5A}$$

Zadatak 6

31. siječnja 2013. 15:46

6.



I: $I = 3A$

II: $0 = (2+2)(I_{II} - I) + (2-j6)I_{II}$

$0 = 4I_{II} - 4 \cdot 3 + 2I_{II} - j6I_{II}$

$12 = (6-j6)I_{II} \quad / : 6 \quad / : (1-j)$

$\frac{2}{1-j} = I_{II} = \frac{2(1+j)}{2} = 1+j$

$\dot{U}_V = (\dot{I} - \dot{I}_{II}) \cdot 2 - \dot{I}_{II} \cdot (-j6)$

$\dot{U}_V = (3 - 1 - j) \cdot 2 - (1+j)(-j6)$

$\dot{U}_V = 4 - 2j + j6 - 6 = -2 + j4$

$|U_V| = \sqrt{(-2)^2 + 4^2} = \sqrt{20} = 4{,}47$

$U_V \approx 4{,}5\ V$

Zadatak 7

31. siječnja 2013. 17:24

7. serija RLC: $R^2 = L/C$, $P = 100\,W$ pri ω_{rez}

za ω_{rez}: $X_L = \omega \cdot L = \sqrt{\frac{L^2}{LC}} = \sqrt{\frac{L}{C}} = R$ $\quad P = U \cdot I$ - snaga izvora

$X_c = \frac{1}{\omega C} = \frac{1}{\sqrt{\frac{C^2}{LC}}} = \sqrt{\frac{L}{C}} = R$ $\quad I = \frac{U}{R}$ $\quad U = I \cdot R$

za $2\omega_{rez}$: $X_L' = 2X_L = 2R$

$X_c' = \frac{1}{2} X_c = 0{,}5R$

$\left.\begin{array}{l} \end{array}\right\} X' = X_L' - X_c' = 1{,}5R \Rightarrow Z = R + j1{,}5R$

$I' = \frac{U}{Z} = \frac{U}{R(1+j1{,}5)} = I \cdot \frac{1}{1+j1{,}5}$

→ prividna snaga izvora za $2\omega_{rez}$:

$S = U \cdot I' = \underbrace{U \cdot I}_{P} \frac{1}{1+j1{,}5} = 100 \cdot \frac{1}{1+j1{,}5} \cdot \frac{1-j1{,}5}{1-j1{,}5}$

$= \frac{100}{3{,}25} - j\frac{150}{3{,}25}$

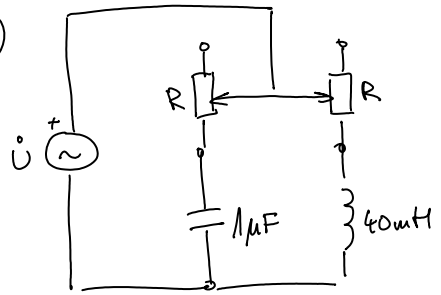
$P = Re\{S\} = \frac{100}{3{,}25} = 30{,}77\,W$

$P(2\omega_{rez}) \approx 30\,W$

Zadatak 8

31. siječnja 2013. 17:52

8.



$R = 0 \div 1000\,\Omega$

za rezonanciju vrijedi $\operatorname{Im}\{\underline{Z}\} = 0$

$$\underline{Z}_{uk} = (R - jX_C) \,||\, (R + jX_L) =$$

$$= \frac{R^2 + X_L X_C + j(X_L - X_C)R}{2R + j(X_L - X_C)} \cdot \frac{2R - j(X_L - X_C)}{2R - j(X_L - X_C)}$$

$$\underline{Z}_{uk} = \frac{[R^2 + X_L X_C + jR(X_L - X_C)][2R - j(X_L - X_C)]}{4R^2 + (X_L - X_C)^2} =$$

$$\operatorname{Im}\{\underline{Z}_{uk}\} = \frac{2R^2(X_L - X_C) - R^2(X_L - X_C) - X_L X_C (X_L - X_C)}{4R^2 + (X_L - X_C)^2} = 0$$

$\Rightarrow$ {brojnik mora biti $= \phi$} $\Downarrow$

$$2R^2(X_L - X_C) - R^2(X_L - X_C) - X_L X_C (X_L - X_C) = 0 \quad / : (X_L - X_C)$$

$$2R^2 - R^2 - X_L X_C = 0$$

$$R^2 = X_L X_C \quad / \sqrt{}$$

$$R = \sqrt{X_L X_C} = \sqrt{\omega L \cdot \frac{1}{\omega C}} = \sqrt{\frac{L}{C}}$$

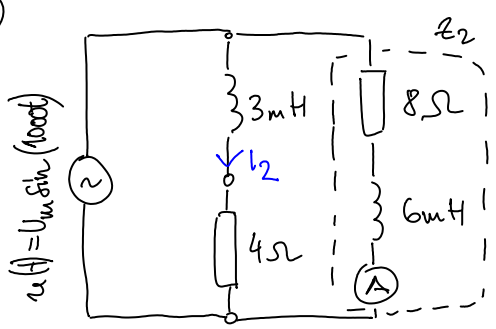
$$= \sqrt{\frac{40 \cdot 10^{-3}}{10^{-6}}} = \sqrt{4 \cdot 10^4} = 2 \cdot 10^2\,\Omega$$

$$\boxed{R = 200\,\Omega}$$

Zadatak9

31. siječnja 2013. 18:14

9.



$u(t) = U_m \cdot \sin(1000t) \Rightarrow$ nema faznog pomaka napona

$\dot{U} = \frac{U_m}{\sqrt{2}} \angle 0°$

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impedancija 1. grane je dvostruko manja od druge → struja je onda dvostruko veća → $I_2 = 10$ A

$|U| = |I|_A \cdot |z| = 5A \cdot \sqrt{8^2 + (1000 \cdot 0{,}006)^2}$

$= 5A \cdot \sqrt{8^2+6^2} = 50$ V

$\hookrightarrow \dot{U} = 50 \angle 0°$, $\dot{z}_2 = 10 \angle 36{,}87°$

$\dot{I}_A = \frac{\dot{U}}{\dot{z}_2} = \frac{50 \angle 0°}{10 \angle 36{,}87°} = 5 \angle -36{,}87° \Rightarrow i_A(t) = 5\sqrt{2} \cdot \sin(1000t - 36{,}87°)$

$\downarrow$

$i_2(t) = 10\sqrt{2} \sin(1000t - 36{,}87°)$ } +

$\downarrow$

$i_{uk}(t) = 15\sqrt{2} \sin(1000t - 36{,}87°)$

$p(t) = |U| \cdot |I| \cdot (\cos\varphi - \cos(2\omega t - \varphi))$ $\quad \varphi = \alpha_u - \alpha_i = 36{,}87°$

p_{max} vrijedi za trenutak kada je $\cos(2\omega t - \varphi) = -1$

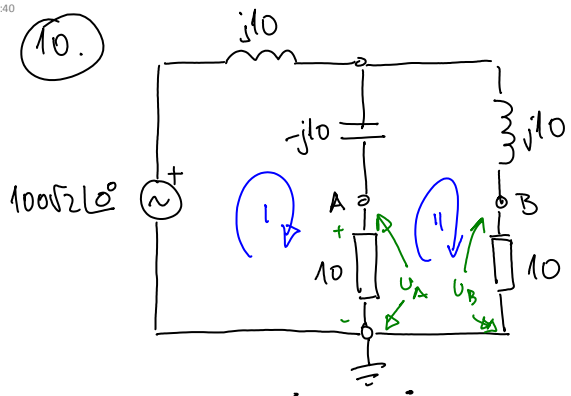
$p_{max} = 50 \cdot 15 \cdot (\cos 36{,}87° + 1) = 1350$ VA

$p_{max} = 1350$ VA

Zadatak 10

31. siječnja 2013. 18:40

10.



$\dot{U} = 100\sqrt{2}\angle 0^\circ = 100\sqrt{2} + j0$

kontura I: $100\sqrt{2} = \dot{I}_1(j10 - j10 + 10) - \dot{I}_{II}(-j10 + 10)$

kontura II: $0 = \dot{I}_{II}(10 + j10 + 10 - j10) - \dot{I}_1(-j10 + 10)$

$100\sqrt{2} = 10\dot{I}_1 + 10\dot{I}_{II}(j-1)$

$0 = 20\dot{I}_{II} + 10\dot{I}_1(j-1) \Rightarrow I_{II} = 0{,}5\dot{I}_1(1-j)$

$100\sqrt{2} = 10\dot{I}_1 - 5\dot{I}_1(1-j)^2$

$100\sqrt{2} = 10\dot{I}_1 - 5\dot{I}_1(1 - 2j - 1)$

$100\sqrt{2} = 10\dot{I}_1(1+j) \Rightarrow \dot{I}_1 = \frac{100\sqrt{2}}{10(1+j)} \cdot \frac{1-j}{1-j} = \frac{10\sqrt{2}(1-j)}{2} =$

$= 5\sqrt{2}(1-j)$

$I_{II} = \frac{1}{2} \cdot 5\sqrt{2}(1-j)\cdot(1-j) = 5\frac{\sqrt{2}}{2}\cdot(-2j) = -j5\sqrt{2}$

$U_A = (I_1 - I_{II})\cdot 10 = (5\sqrt{2} - 5\sqrt{2}j + j5\sqrt{2})\cdot 10 = 50\sqrt{2}$

$U_B = I_{II}\cdot 10 = -j50\sqrt{2}$

$U_{AB} = U_A - U_B = 50\sqrt{2}(1+j) = 100\angle 45^\circ\ V$

$U_{AB} = 100\angle 45^\circ\ V$