

RLC krugovi

VII. tjedan predavanja

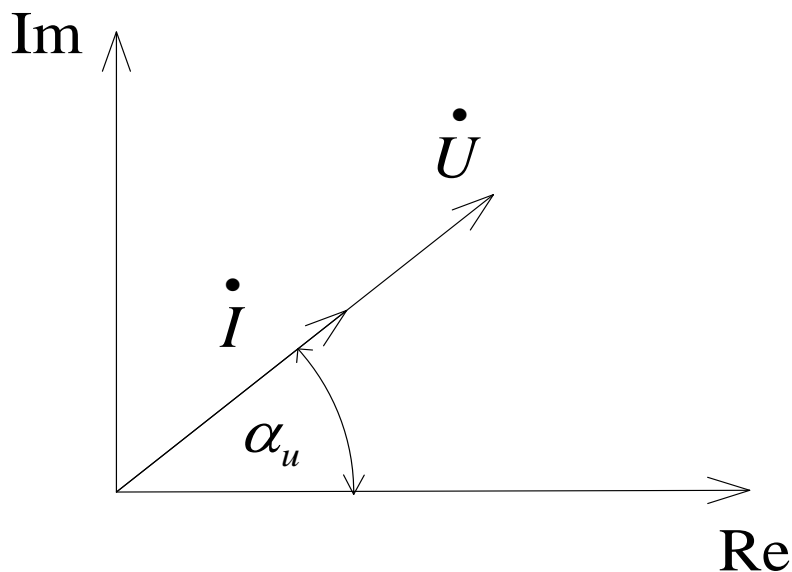
Zadani su \dot{U} i \underline{Z} , a traži se \dot{I}

$$\left. \begin{array}{l} \dot{U} = U \cdot e^{j\alpha_u} \\ \underline{Z} = Z \cdot e^{j\varphi} \end{array} \right\} \dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{Z \cdot e^{j\varphi}} = \frac{U}{Z} \cdot e^{j(\alpha_u - \varphi)}$$

$$\varphi = \alpha_u - \alpha_i$$

a) OMSKI OTPOR

$$\underline{\dot{I}} = \frac{\underline{\dot{U}}}{\underline{Z}} = \frac{\underline{\dot{U}}}{R} = \frac{U \cdot e^{j\alpha_u}}{R \cdot e^{j0}} = \frac{U}{R} \cdot e^{j\alpha_u}$$

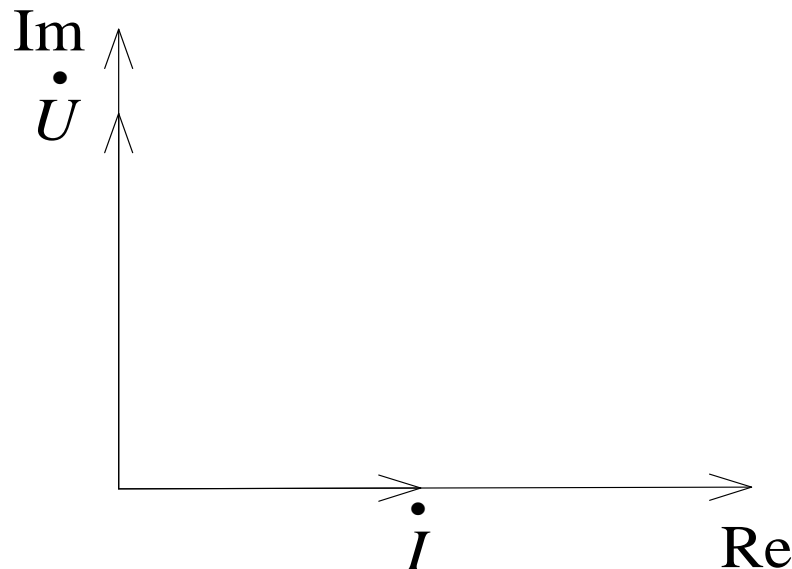


Napon i struja imaju isti fazni kut i kažemo da su na omskom otporu napon i struja u fazi.

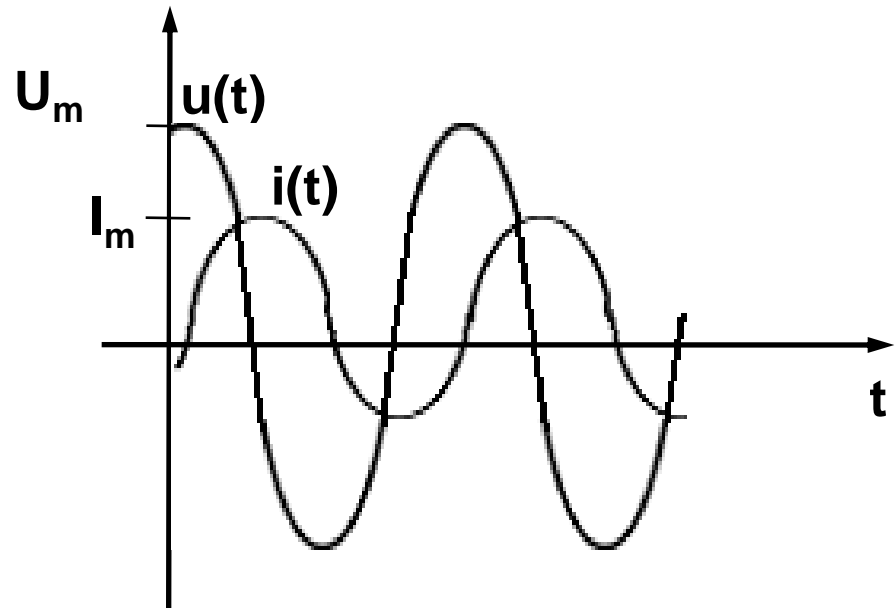
b) ZAVOJNICA

$$\underline{Z} = jX_L = j\omega L \quad j = e^{j\frac{\pi}{2}}$$

$$\dot{\underline{I}} = \frac{\dot{\underline{U}}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{jX_L} = \frac{U \cdot e^{j\alpha_u}}{X_L \cdot e^{j\frac{\pi}{2}}} = \frac{U}{X_L} \cdot e^{j\left(\alpha_u - \frac{\pi}{2}\right)}$$



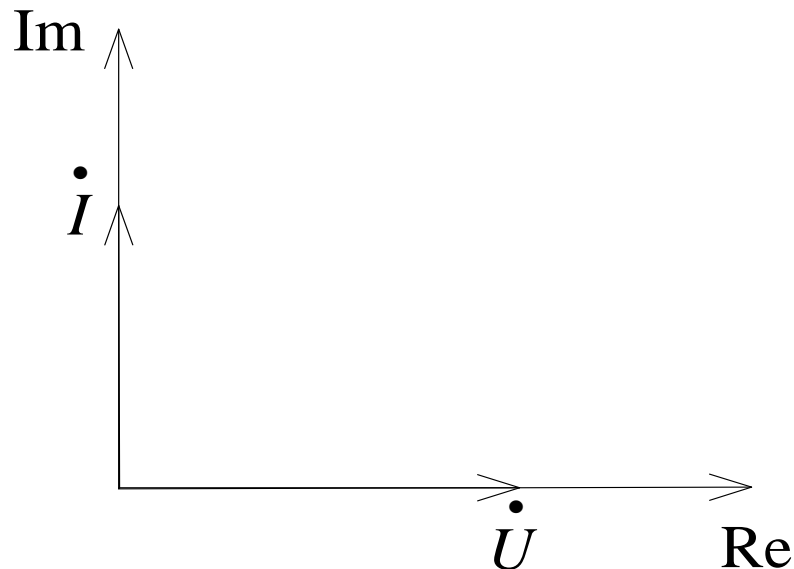
Struja kasni za naponom za $\frac{\pi}{2}$.



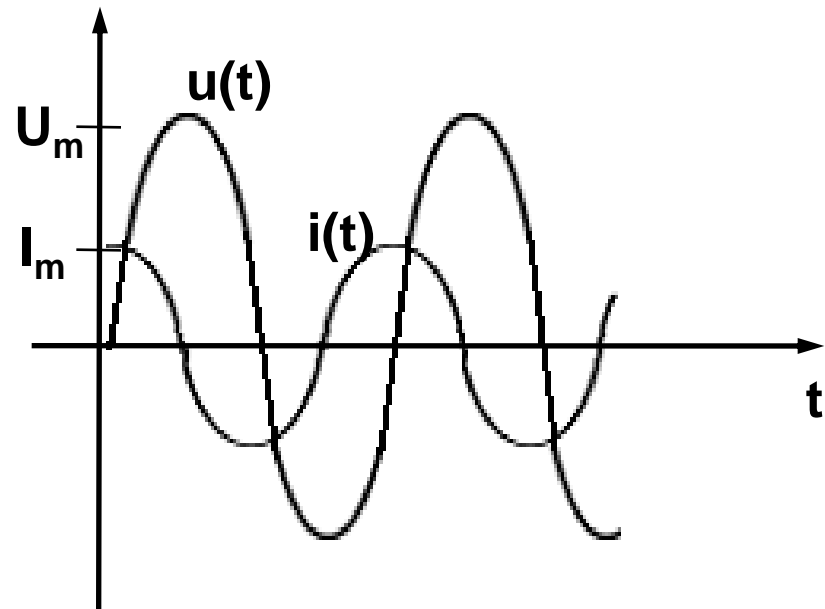
c) KAPACITET

$$\underline{Z} = -j \cdot \frac{1}{\omega C} = -jX_C$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{-jX_C} = \frac{U \cdot e^{j\alpha_u}}{X_C \cdot e^{-j\frac{\pi}{2}}} = \frac{U}{X_C} \cdot e^{j\left(\alpha_u + \frac{\pi}{2}\right)}$$

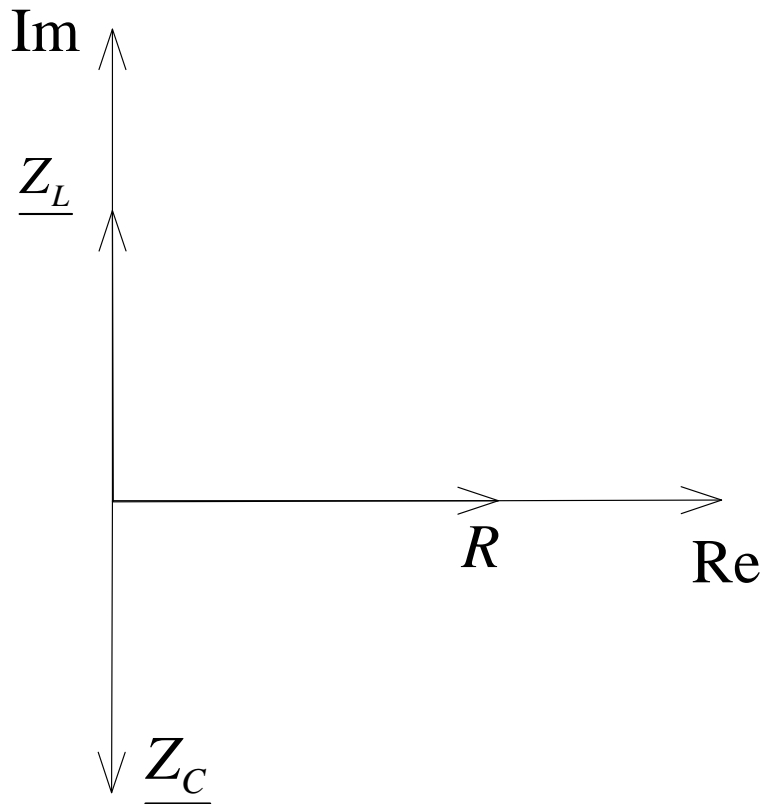


**Struja prethodi
naponu za $\frac{\pi}{2}$.**



OPĆI SLUČAJ

$$\underline{Z} = Z \angle \varphi [\Omega]$$

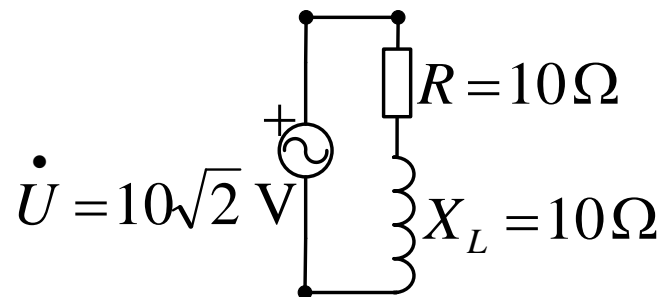


**Vektor impedancije
može se pojaviti u I. ili
IV. kvadrantu.**

RLC krugovi

RLC krug-Primjer 1.

OSNOVE ELEKTROTEHNIKE

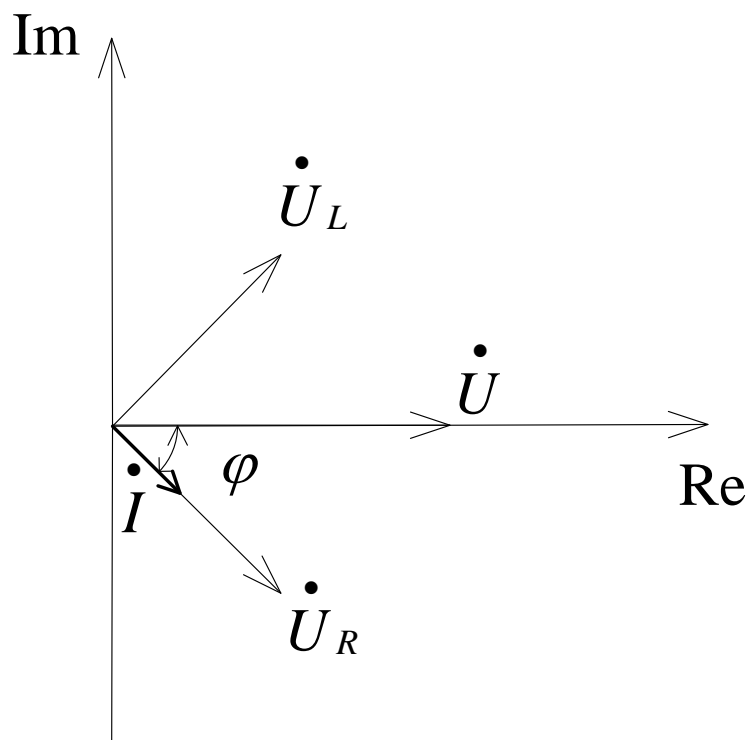


$$\underline{Z}_{RL} = R + jX_L = 10 + j10 = 10\sqrt{2} \angle 45^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}_{RL}} = \frac{10\sqrt{2}}{10\sqrt{2} \angle 45^\circ} = 1 \angle -45^\circ \text{ A}$$

$$\dot{U}_R = \dot{I} \cdot R = 10 \angle -45^\circ \text{ V}$$

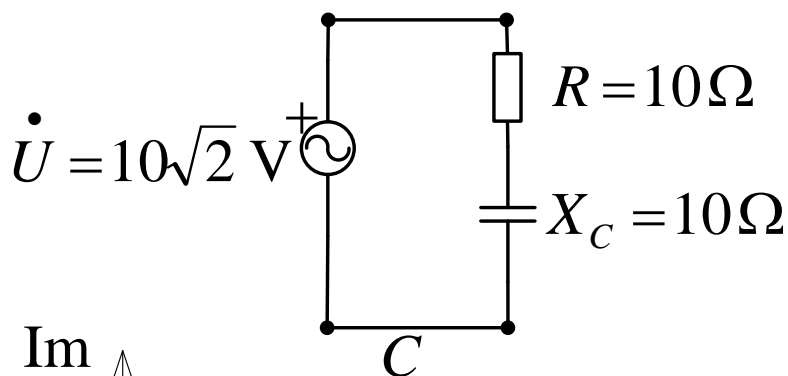
$$\dot{U}_L = \dot{I} \cdot \underline{X}_L = 10 \angle 45^\circ \text{ V}$$



Induktivno ponašanje kruga

RLC krugovi

RLC krug-Primjer 2.

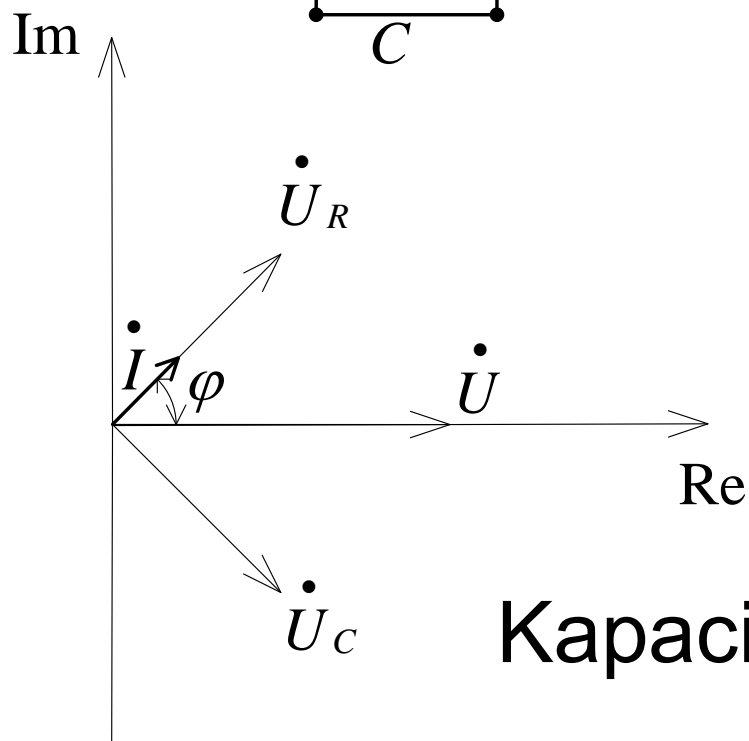


$$\underline{Z}_{RL} = R - jX_C = 10 - j10 = 10\sqrt{2} \angle -45^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}_{RC}} = \frac{10\sqrt{2}}{10\sqrt{2} \angle -45^\circ} = 1 \angle 45^\circ \text{ A}$$

$$\dot{U}_R = \dot{I} \cdot R = 10 \angle 45^\circ \text{ V}$$

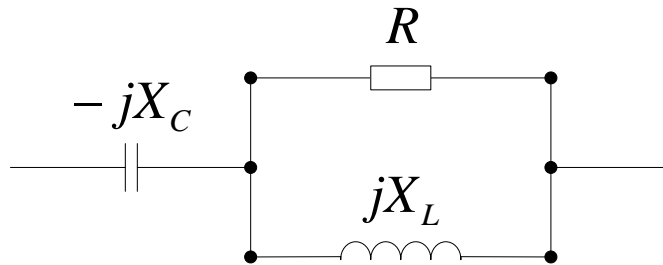
$$\dot{U}_C = \dot{I} \cdot \underline{X}_C = 10 \angle -45^\circ \text{ V}$$



Kapacitivno ponašanje kruga

RLC krugovi

Primjer 3.



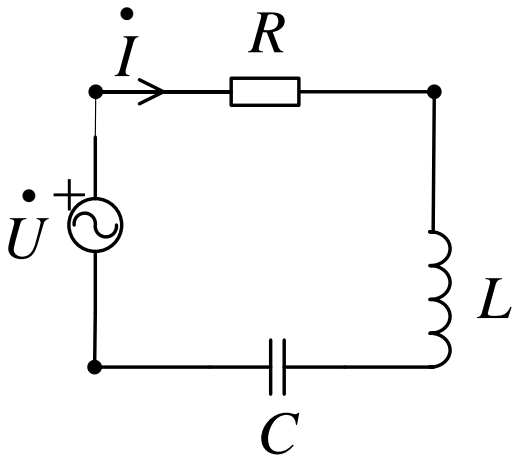
$$\underline{Z}_{RL} = \frac{R \cdot jX_L}{R + jX_L} \cdot \frac{R - jX_L}{R - jX_L} = \frac{jR^2 \cdot X_L + X_L^2 \cdot R}{R^2 + X_L^2} =$$

$$\frac{R \cdot X_L^2}{R^2 + X_L^2} + j \frac{R^2 \cdot X_L}{R^2 + X_L^2}$$

$$\underline{Z}_{uk} = \underline{Z}_C + \underline{Z}_{RL} = \frac{R \cdot X_L^2}{R^2 + X_L^2} + j \left(\frac{R^2 \cdot X_L}{R^2 + X_L^2} - X_C \right)$$

RLC krugovi

Serijski RLC krug



$$\underline{Z} = R + j\omega L - j \cdot \frac{1}{\omega C} = R + j \left(\omega L - \frac{1}{\omega C} \right)$$

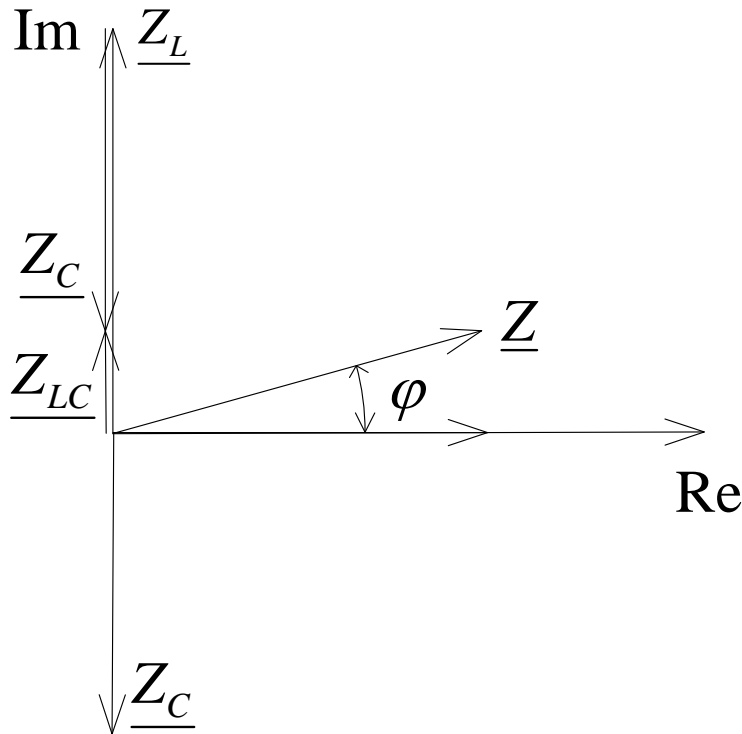
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$\varphi = \arctg \frac{\omega L - \frac{1}{\omega C}}{R}$$

RLC krugovi

Serijski RLC krug

OSNOVE ELEKTROTEHNIKE



$$\underline{Z} = Z \angle \varphi [\Omega]$$

$$\dot{U} = \dot{I} \cdot \underline{Z}$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \frac{U \cdot e^{j\alpha_u}}{Z \cdot e^{j\varphi}} = \frac{U}{Z} \cdot e^{j(\alpha_u - \varphi)}$$

**$\varphi > 0$ impedancija ima induktivni karakter
(struja zaostaje za naponom)**

**$\varphi < 0$ impedancija ima kapacitivni karakter
(struja prethodi naponu)**

RLC krugovi

Serijski RLC krug

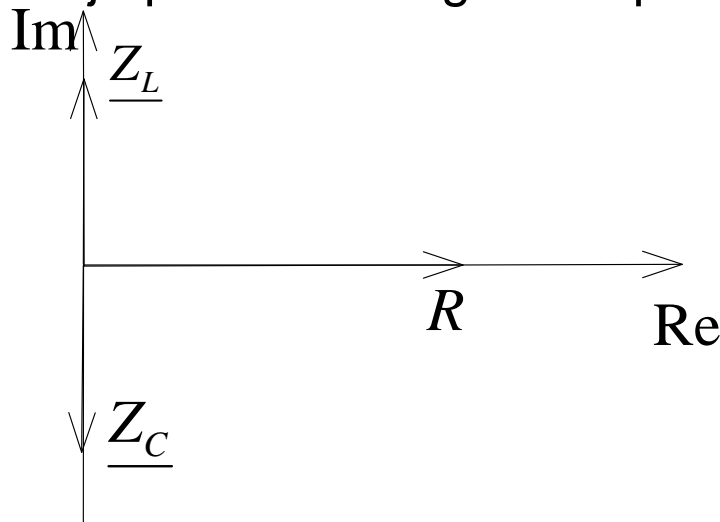
Uvjet rezonancije

$$\text{Im}\{\underline{Z}\} = 0$$

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Napon i struja su u fazi, krug se ponaša radno, struja ima maksimalni iznos te je predana snaga na otporu maksimalna.

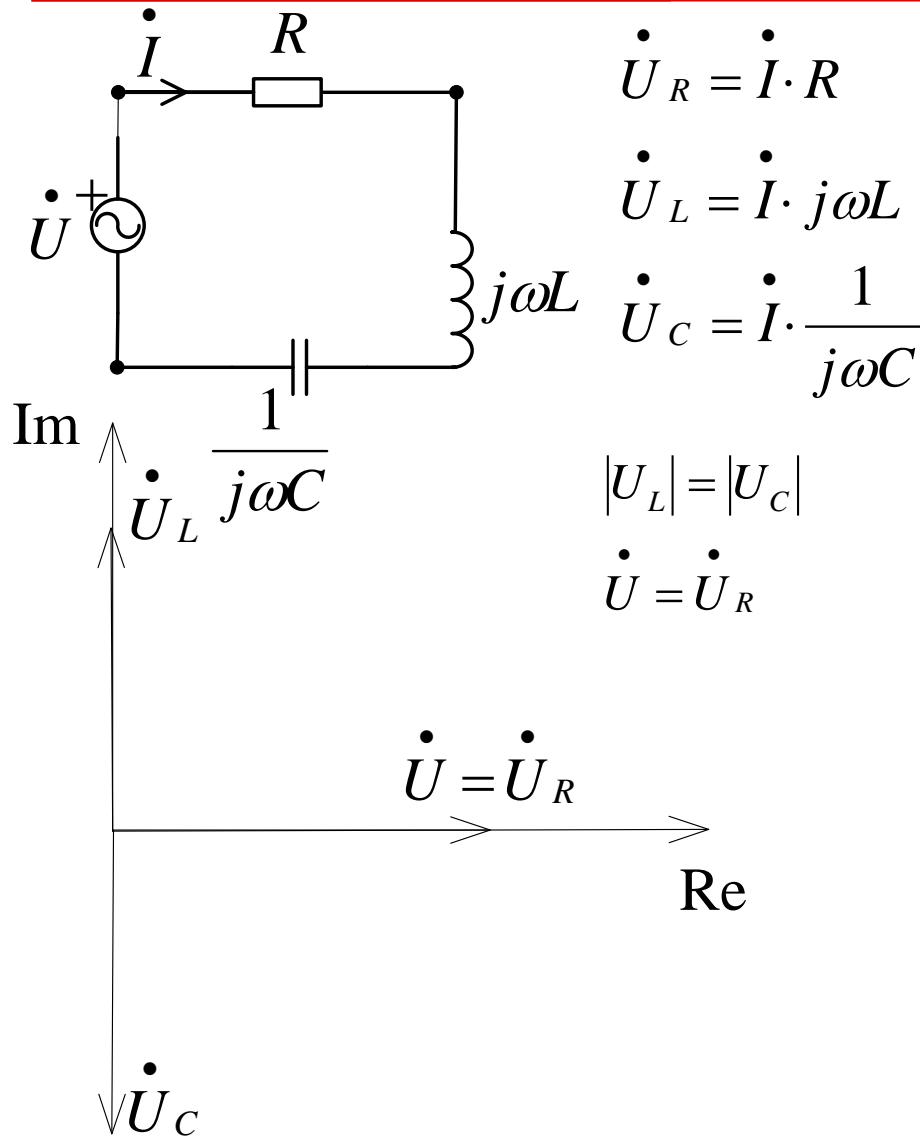


$\omega > \omega_0$, krug se ponaša induktivno

$\omega < \omega_0$, krug se ponaša kapacitivno

RLC krugovi

Serijski RLC krug



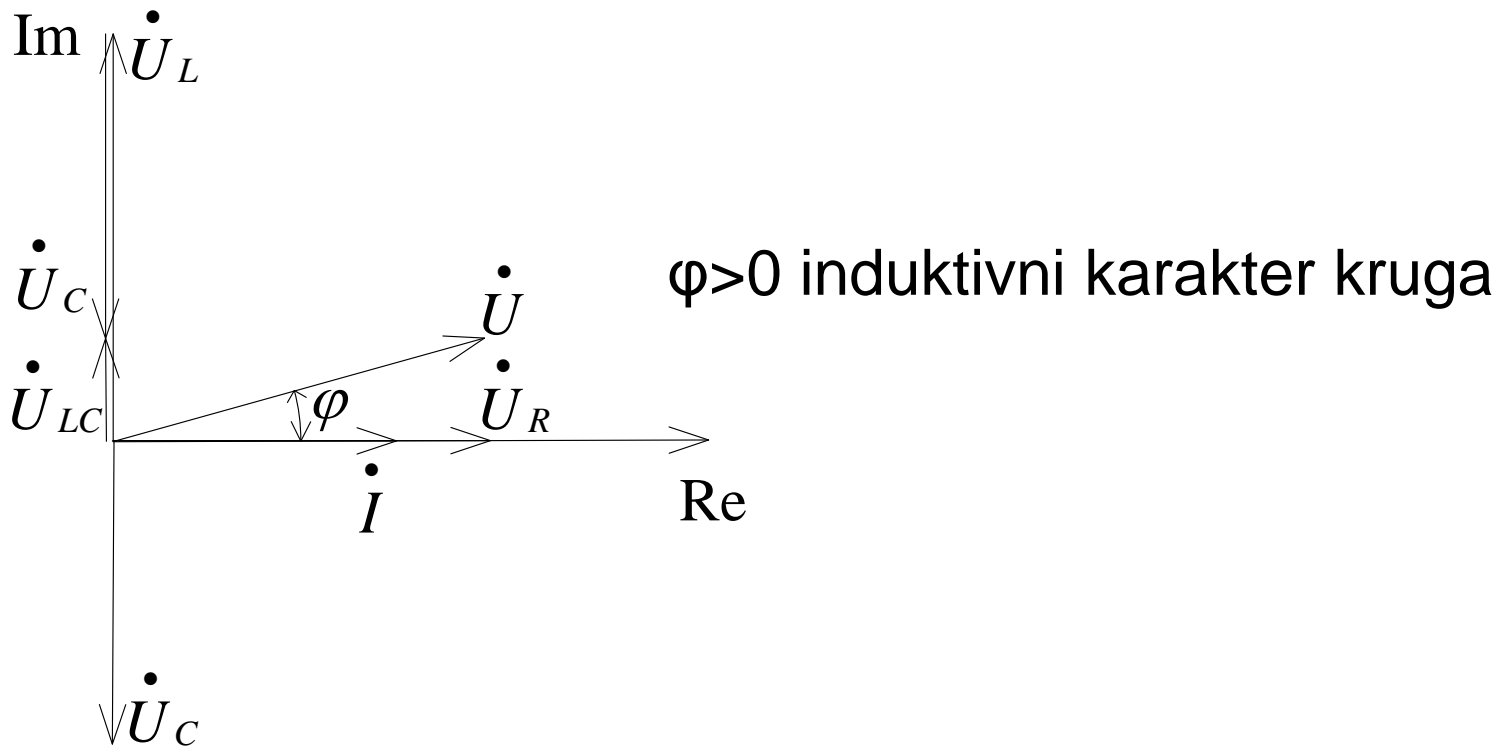
U postupku analize kruga prvo odredimo impedanciju, pa na temelju nje struju te napone na pojedinim elementima.

U rezonanciji napon na zavojnici jednak je naponu na kondenzatoru pa serijsku rezonanciju nazivamo i naponska rezonancija.

RLC krugovi

Serijski RLC krug

OSNOVE ELEKTROTEHNIKE

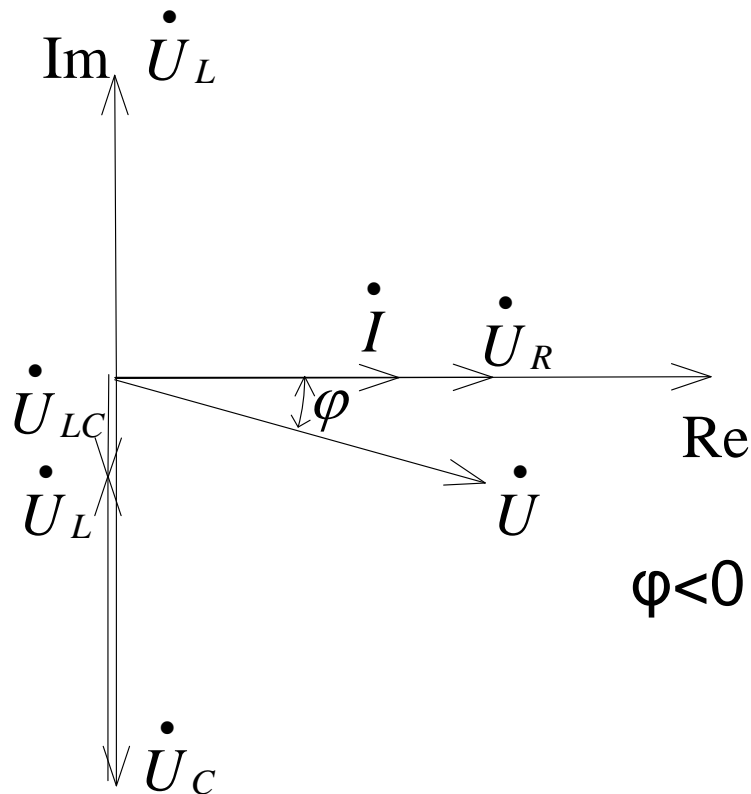


Kako je struja \dot{I} zajednička veličina za ovaj krug, obično je postavimo po realnoj osi i od nje počnemo crtati vektorski dijagram.

RLC krugovi

Serijski RLC krug

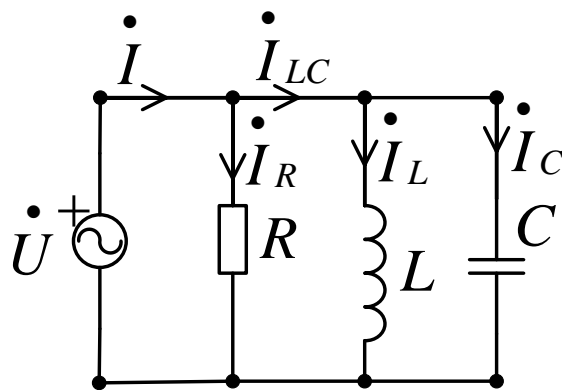
OSNOVE ELEKTROTEHNIKE



$\varphi < 0$ kapacitivni karakter kruga

RLC krugovi

Paralelni RLC krug



Zajednička veličina na svim elementima je napon.

$$\dot{U} = \dot{U}_R = \dot{U}_L = \dot{U}_C$$

$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C$$

B_L – induktivna vodljivost

B_C – kapacitivna vodljivost

$$\dot{I}_R = \frac{\dot{U}}{R} = \dot{U} \cdot G$$

$$\dot{I}_L = \frac{\dot{U}}{jX_L} = \dot{U} \cdot (-jB_L)$$

$$\dot{I}_C = \frac{\dot{U}}{-jX_C} = \dot{U} \cdot (jB_C)$$

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} = \dot{U} \cdot \underline{Y}$$

RLC krugovi

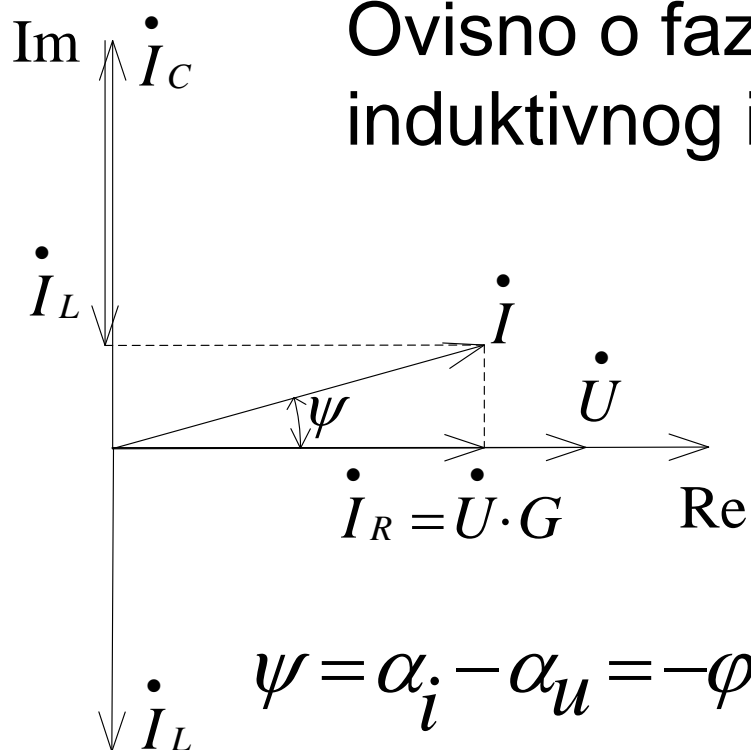
Paralelni RLC krug

$$\dot{U} \cdot \underline{Y} = \dot{U} \cdot G + \dot{U} \cdot (-jB_L) + \dot{U} \cdot (jB_C)$$

$$\underline{Y} = G + j(B_C - B_L) \quad \text{Ukupna admitancija}$$

Imaginarni dio ($B_C - B_L$) nazivamo susceptancija.

Ovisno o faznom kutu, krug može biti induktivnog ili kapacitivnog karaktera.



$$\dot{I}_C = \dot{U} \cdot jB_C$$

$$\dot{I}_R = \dot{U} \cdot G$$

$$\dot{I}_L = \dot{U} \cdot (-jB_L)$$

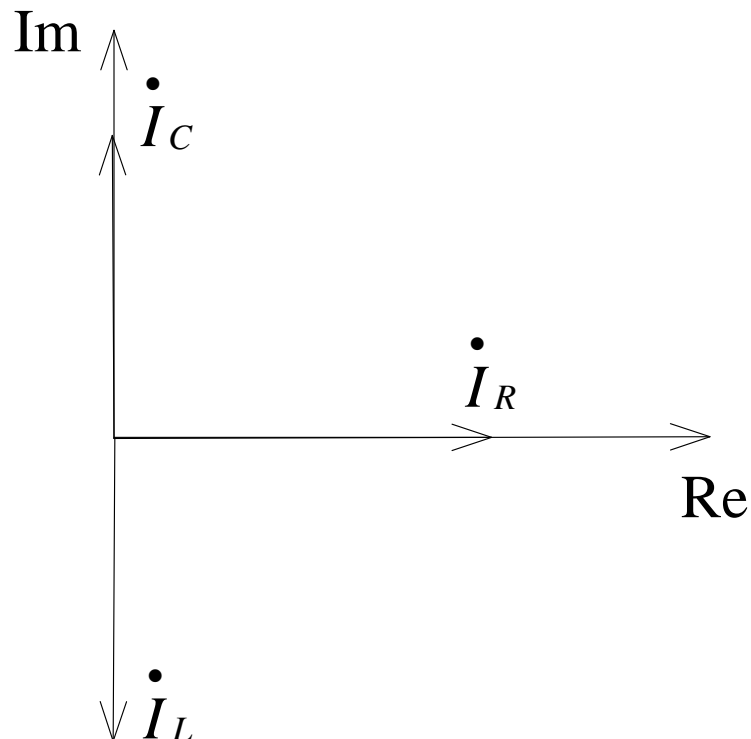
$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C$$

RLC krugovi

Paralelni RLC krug

$B_L = B_C$ specijalni slučaj

$\text{Im}\{\underline{Y}\} = 0$ REZONANCIJA



$$|I_L| = |I_C|$$

$$\dot{I} = \dot{I}_R \quad (\text{REZONANCIJA})!$$

U rezonanciji paralelnog RLC kruga struja je minimalna.

RLC krugovi

Paralelni RLC krug

- ◆ Paralelnu rezonanciju nazivamo i strujna rezonancija
- ◆ Sa strane izvora kao da je LC dio odspojen. Struje I_L i I_C teku, ali su suprotnog smjera i istog iznosa. Dobijemo titrajni krug gdje energija neprestano prelazi s L na C i obratno (akumulirana elektrostatska energija na kondenzatoru, pretvara se u magnetsku energiju i obratno).
- ◆ Uvjet rezonancije:

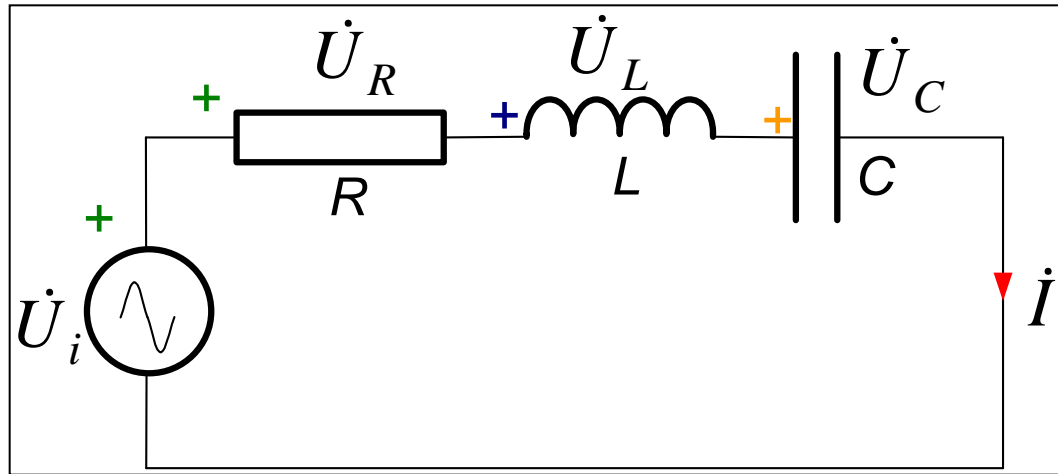
$$B_C = B_L$$

$$\omega C = \frac{1}{\omega L} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega < \omega_0$ krug se ponaša induktivno

$\omega > \omega_0$ Krug se ponaša kapacitivno

Frekvencijske karakteristike u serijskom R,L,C krugu priključenom na naponski izvor



- ♦ Analiziraju se veličine u krugu pri promjeni frekvencije od 0 do ∞

- Impedancija kruga je:

$$\underline{Z} = R + jX = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

- ◆ Pri promjeni frekvencije:

$$R = \text{konst.} \quad X(\omega) = \omega L - \frac{1}{\omega C}$$

$$\underline{Z}(\omega) = |\underline{Z}(\omega)| e^{j\varphi}$$

$$|\underline{Z}(\omega)| = \sqrt{R^2 + X^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\underline{I} = \frac{\underline{U}_i}{\underline{Z}}$$

$$\varphi(\omega) = \text{arctg}\left(\frac{X}{R}\right) = \text{arctg}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

- ◆ Pri frekvenciji ω_0 imaginarni dio impedancije jednak je nuli. Frekvencija ω_0 naziva se rezonantna frekvencija i krug je u rezonanciji:

- ♦ U rezonanciji je:

$$X(\omega_0) = \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\underline{Z}(\omega_0) = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$X_{L0} = \omega_0 L = \frac{1}{\sqrt{LC}} L = \sqrt{\frac{L}{C}} = \rho$$

$$X_{C0} = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} = \rho$$

- ♦ ρ - valni otpor; γ - valna vodljivost:

$$\rho = \frac{1}{\gamma} = \sqrt{\frac{L}{C}} \quad (\Omega)$$

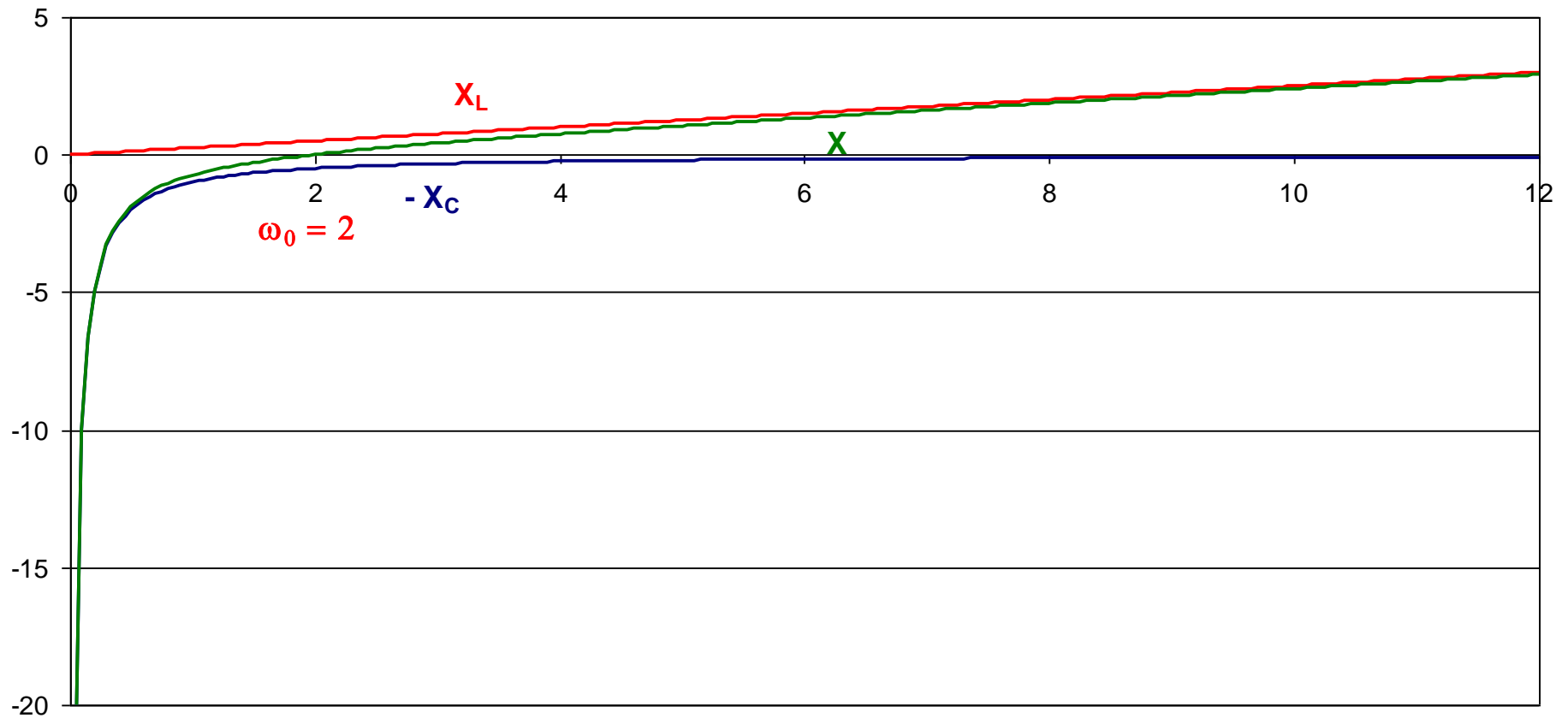
U rezonanciji je:

$$I_0 = \frac{U_i}{R}$$

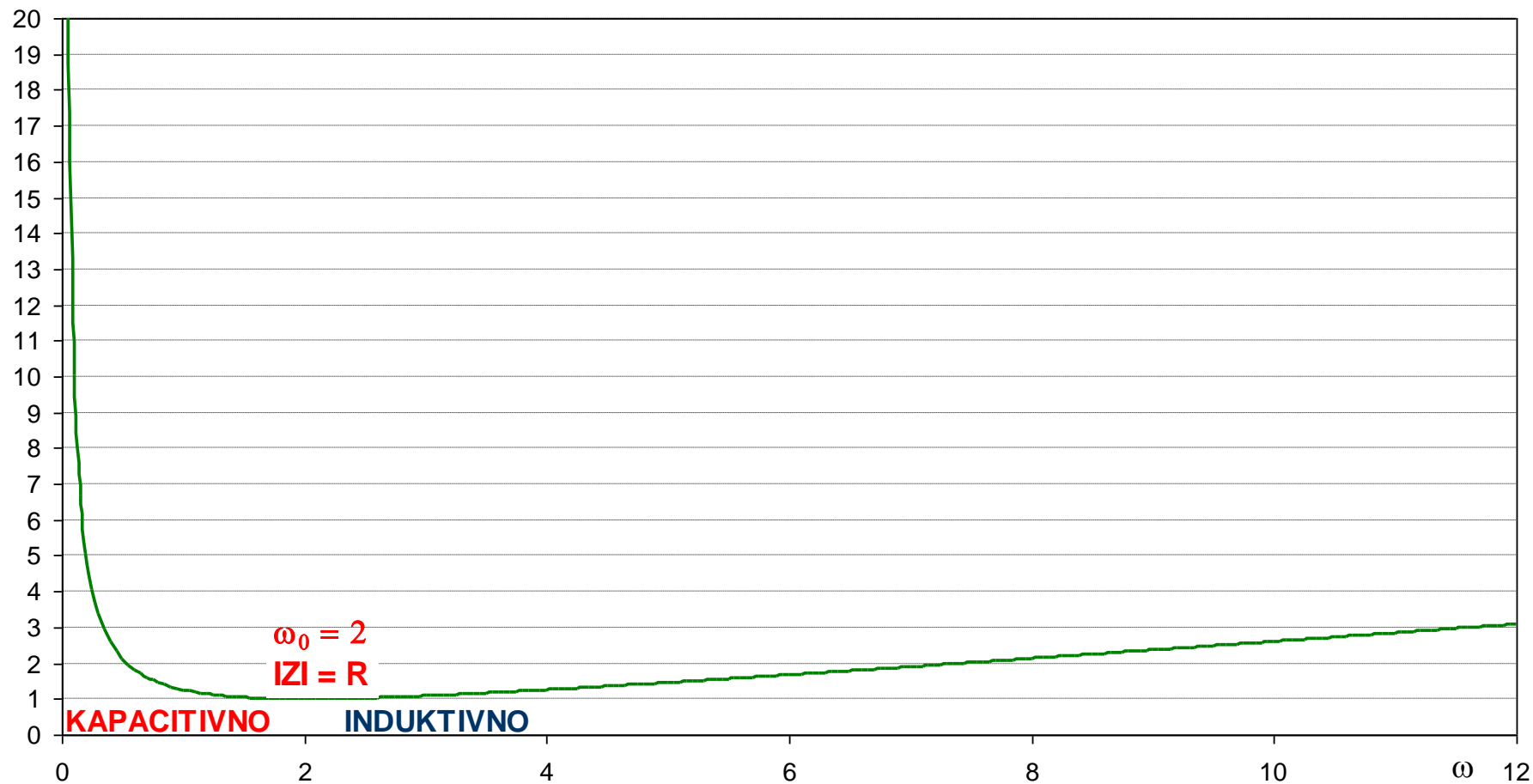
$$U_{R0} = I_0 \cdot R = U_i$$

$$U_{L0} = I_0 X_{L0} = U_i \frac{\rho}{R}$$

$$U_{C0} = I_0 X_{C0} = U_i \frac{\rho}{R}$$

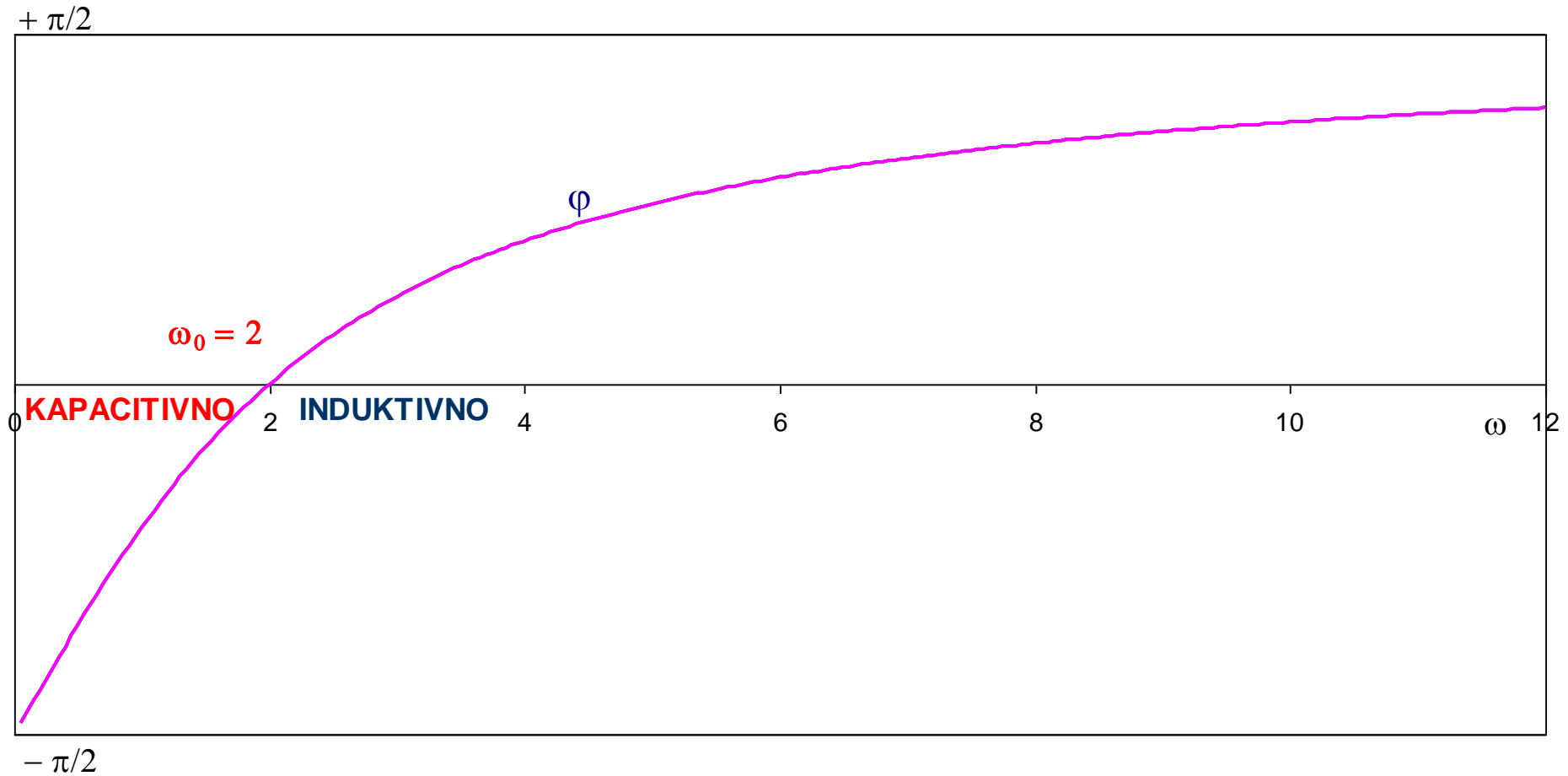


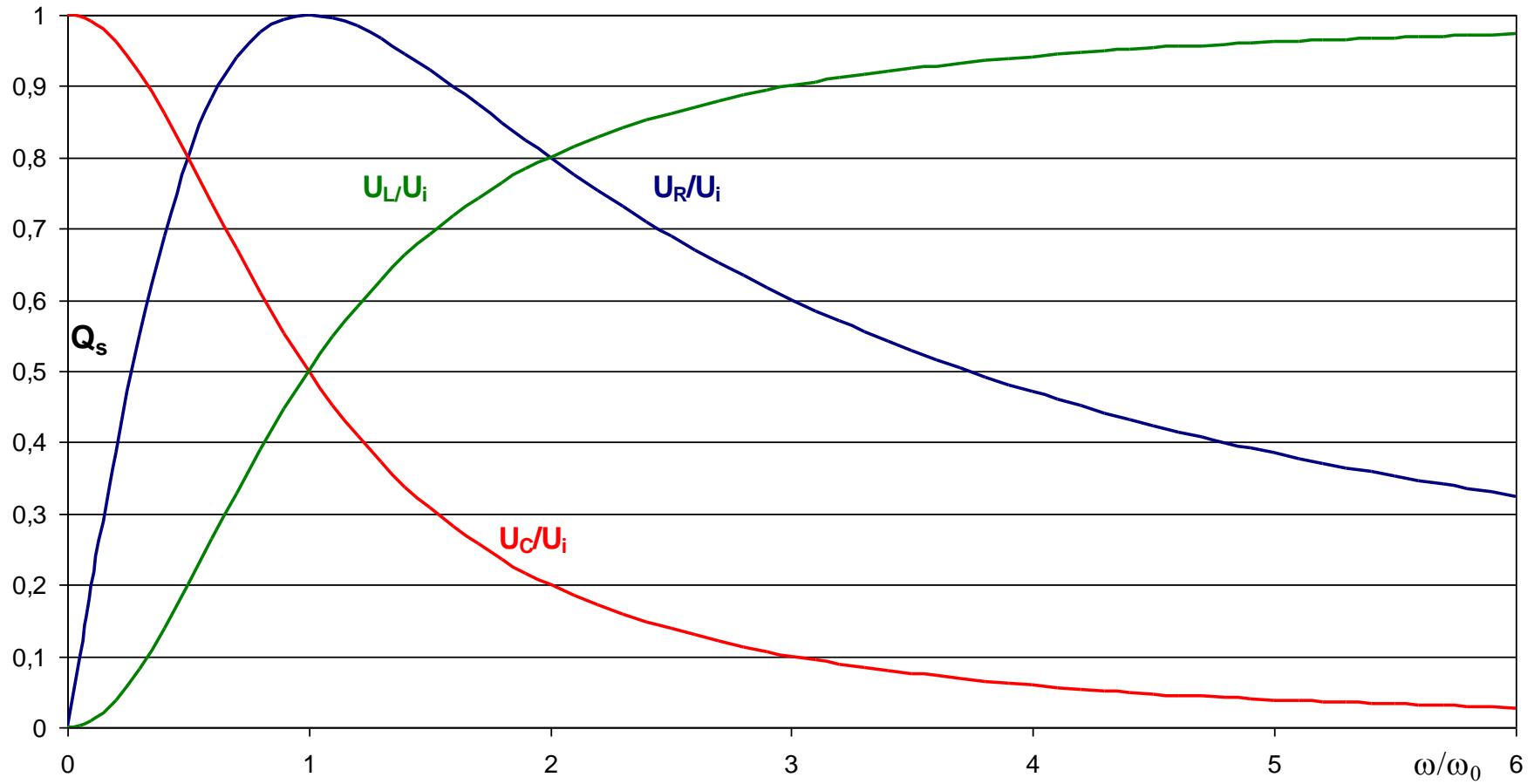
Modul impedancije



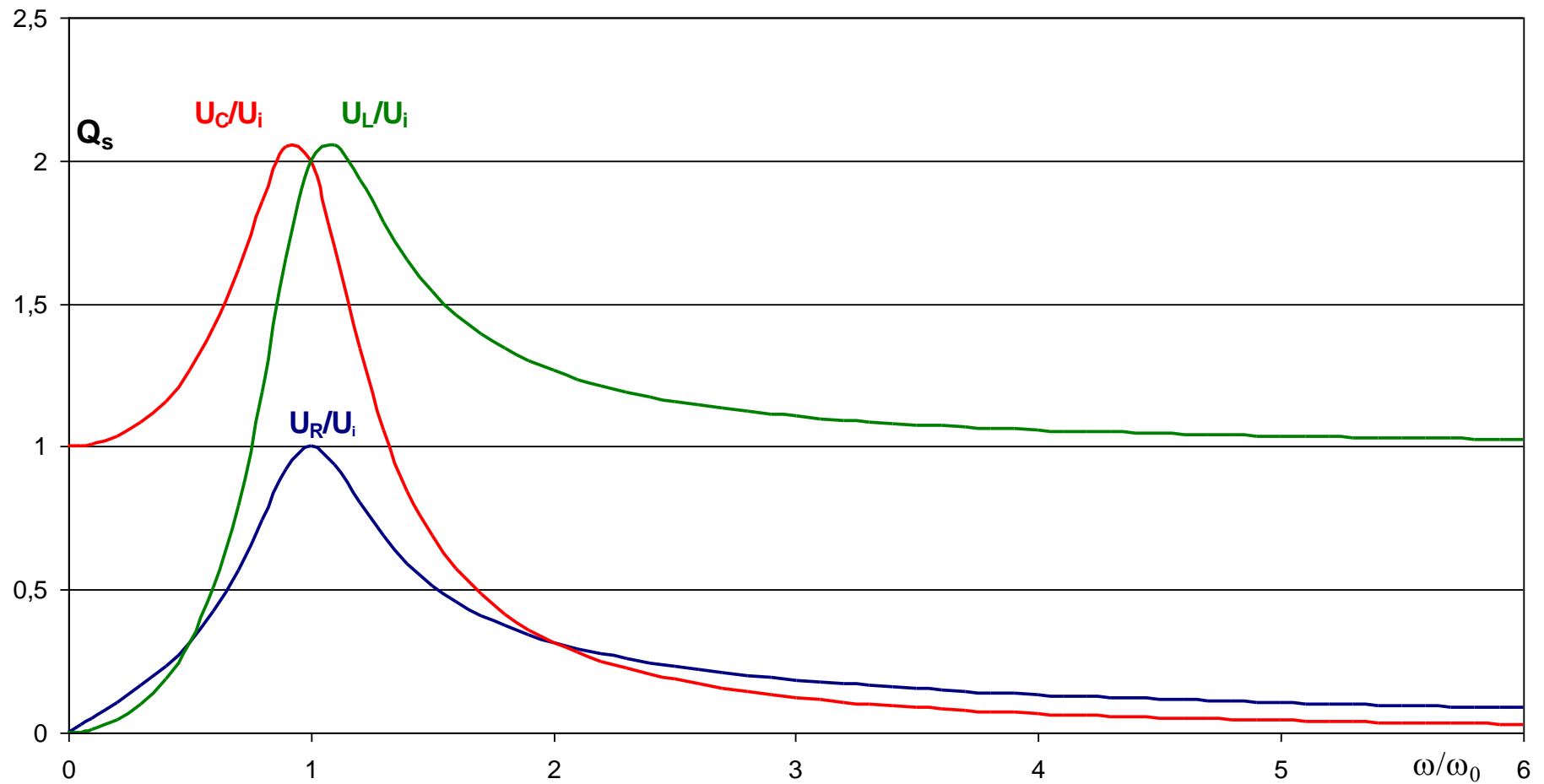
Kut impedancije

OSNOVE ELEKTROTEHNIKE



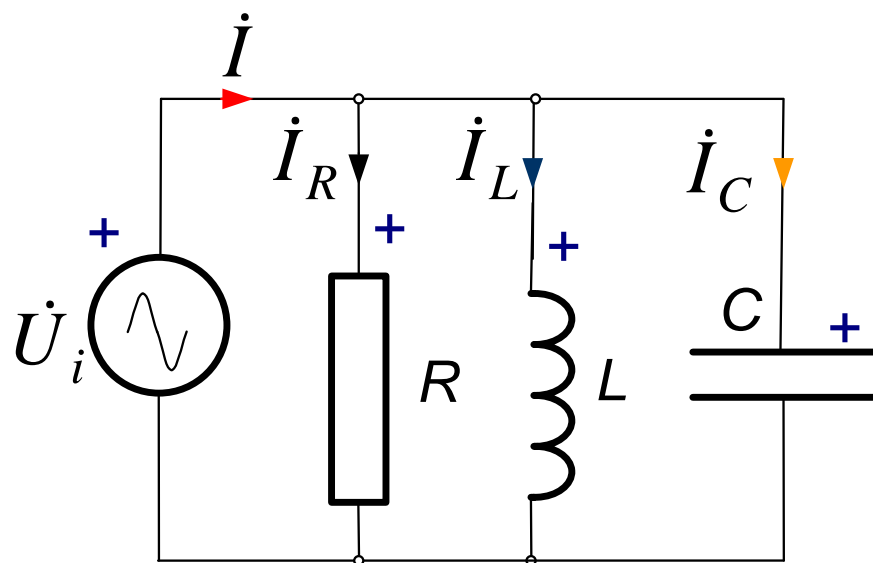


Primjer 1: napona na elementima



Primjer 2: naponi na elementima

Frekvencijske karakteristike paralelnog R,L,C kruga priključenog na naponski izvor



- ♦ Analiziraju se veličine u krugu pri promjeni frekvencije od 0 do ∞

- Admitancija kruga je:

$$\underline{Y} = G + jB = G + j(B_C - B_L) = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

- ◆ Pri promjeni frekvencije:

$$G = \text{konst.} \quad B(\omega) = \omega C - \frac{1}{\omega L}$$

$$\underline{Y}(\omega) = |\underline{Y}(\omega)|e^{j\psi}$$

$$|\underline{Y}(\omega)| = \sqrt{G^2 + B^2} = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\psi(\omega) = \arctg\left(\frac{B}{G}\right) = \arctg\left(\frac{\omega C - \frac{1}{\omega L}}{G}\right)$$

- ◆ Pri frekvenciji ω_0 imaginarni dio admitancije jednak je nuli. Frekvencija ω_0 naziva se rezonantna frekvencija i krug je u rezonanciji:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

U rezonanciji je: $B(\omega_0) = \omega_0 C - \frac{1}{\omega_0 L} = 0$ $Y(\omega_0) = G = \frac{1}{R}$

$$B_{C0} = \omega_0 C = \frac{1}{\sqrt{LC}} C = \sqrt{\frac{C}{L}} = \gamma$$

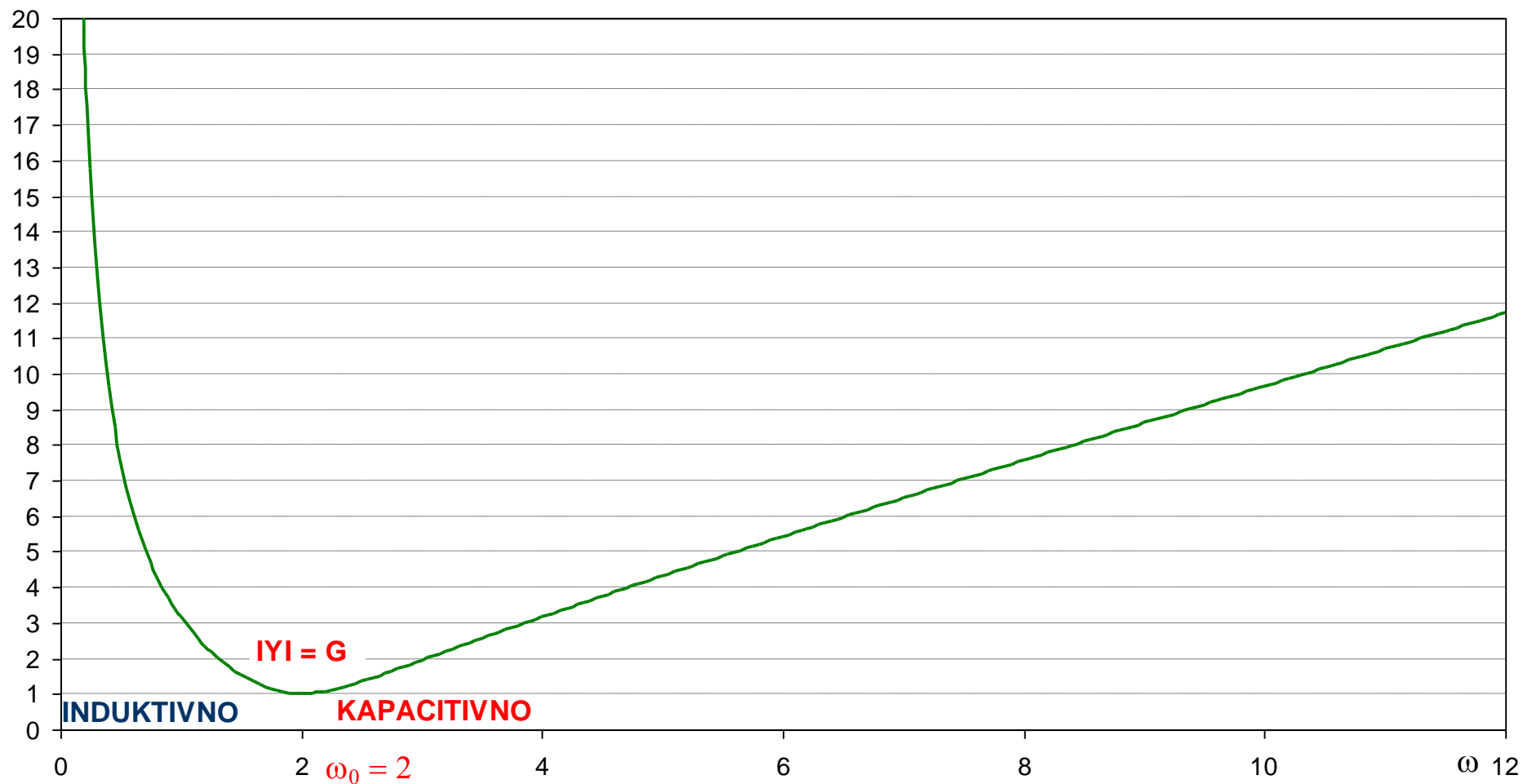
$$B_{L0} = \frac{1}{\omega_0 L} = \frac{\sqrt{LC}}{L} = \sqrt{\frac{C}{L}} = \gamma$$

♦ γ - valna vodljivost; ρ - valni otpor:

$$\rho = \frac{1}{\gamma} = \sqrt{\frac{L}{C}} \quad (\Omega)$$

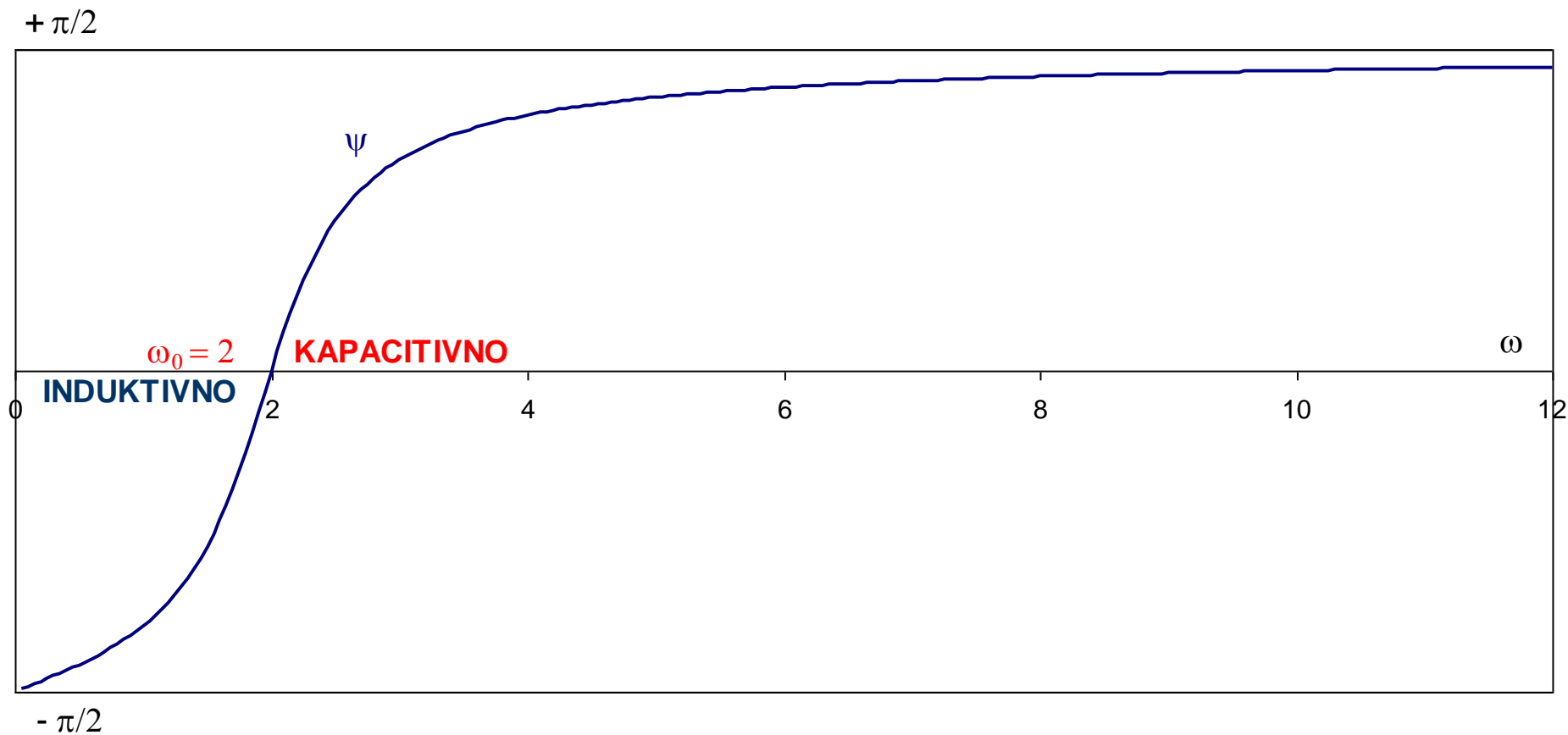
♦ Struje pri rezonanciji su:

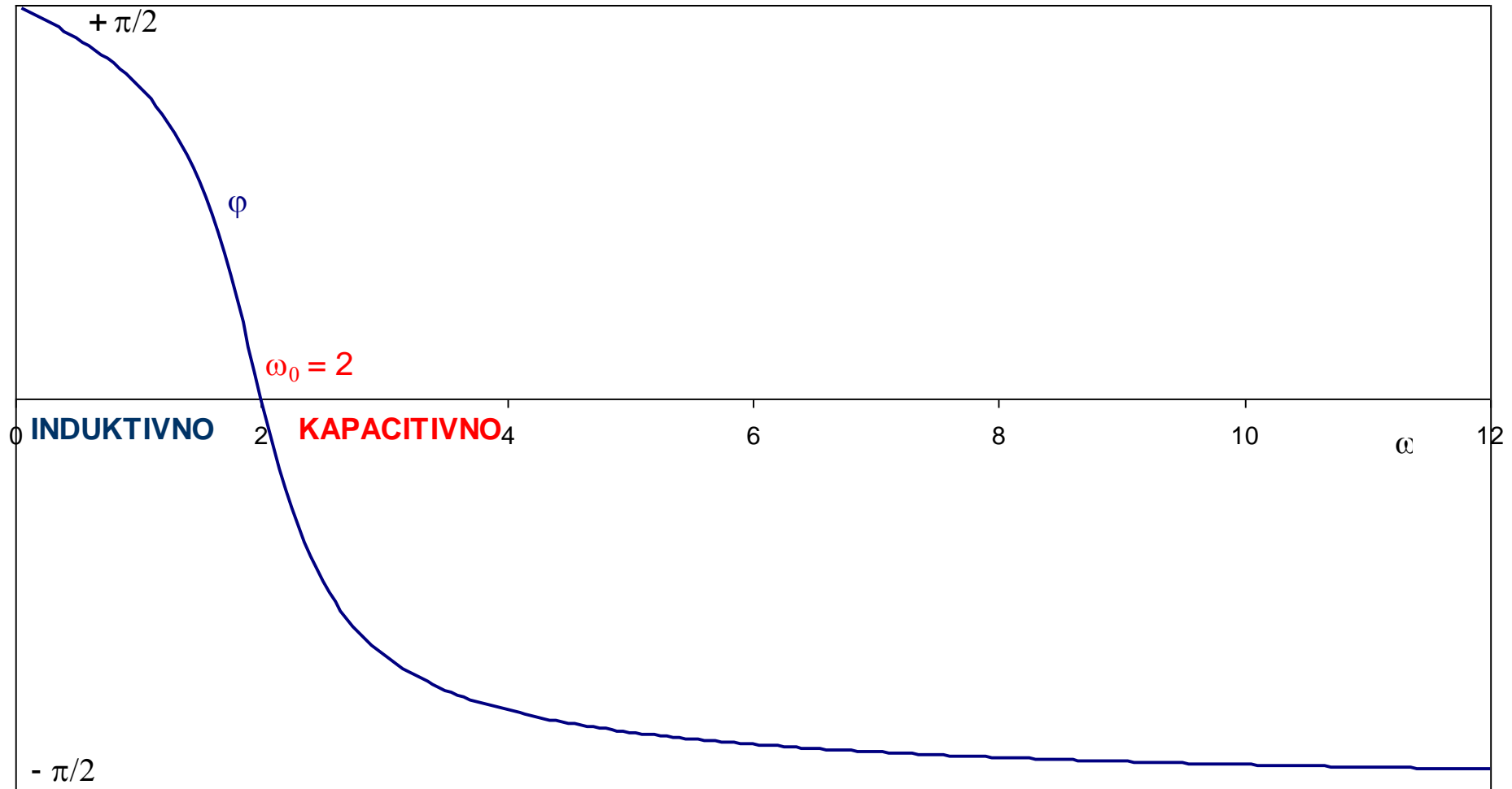
$$I_{R0} = I_0 = U_i G \quad I_{L0} = U_i B_{L0} = U_i \gamma \quad I_{C0} = U_i B_{C0} = U_i \gamma$$



Kut admitancije

OSNOVE ELEKTROTEHNIKE





♦ Impedancija je:

$$\underline{Z}(\omega) = \frac{1}{\underline{Y}(\omega)} = \frac{G}{G^2 + B^2(\omega)} - j \frac{B(\omega)}{G^2 + B^2(\omega)} = |\underline{Z}| e^{j\varphi}$$

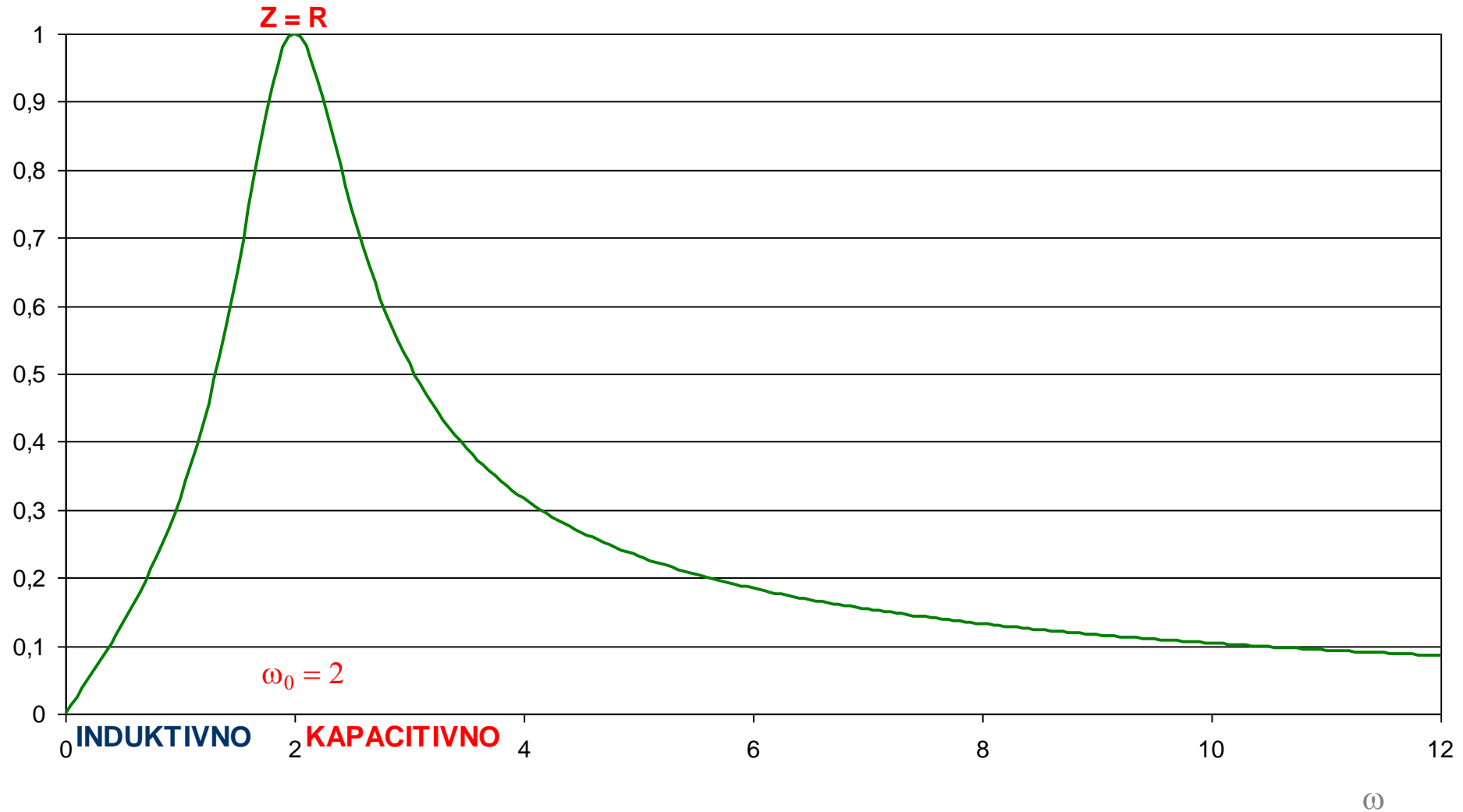
$$|\underline{Z}(\omega)| = \frac{1}{|\underline{Y}(\omega)|} = \frac{1}{\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

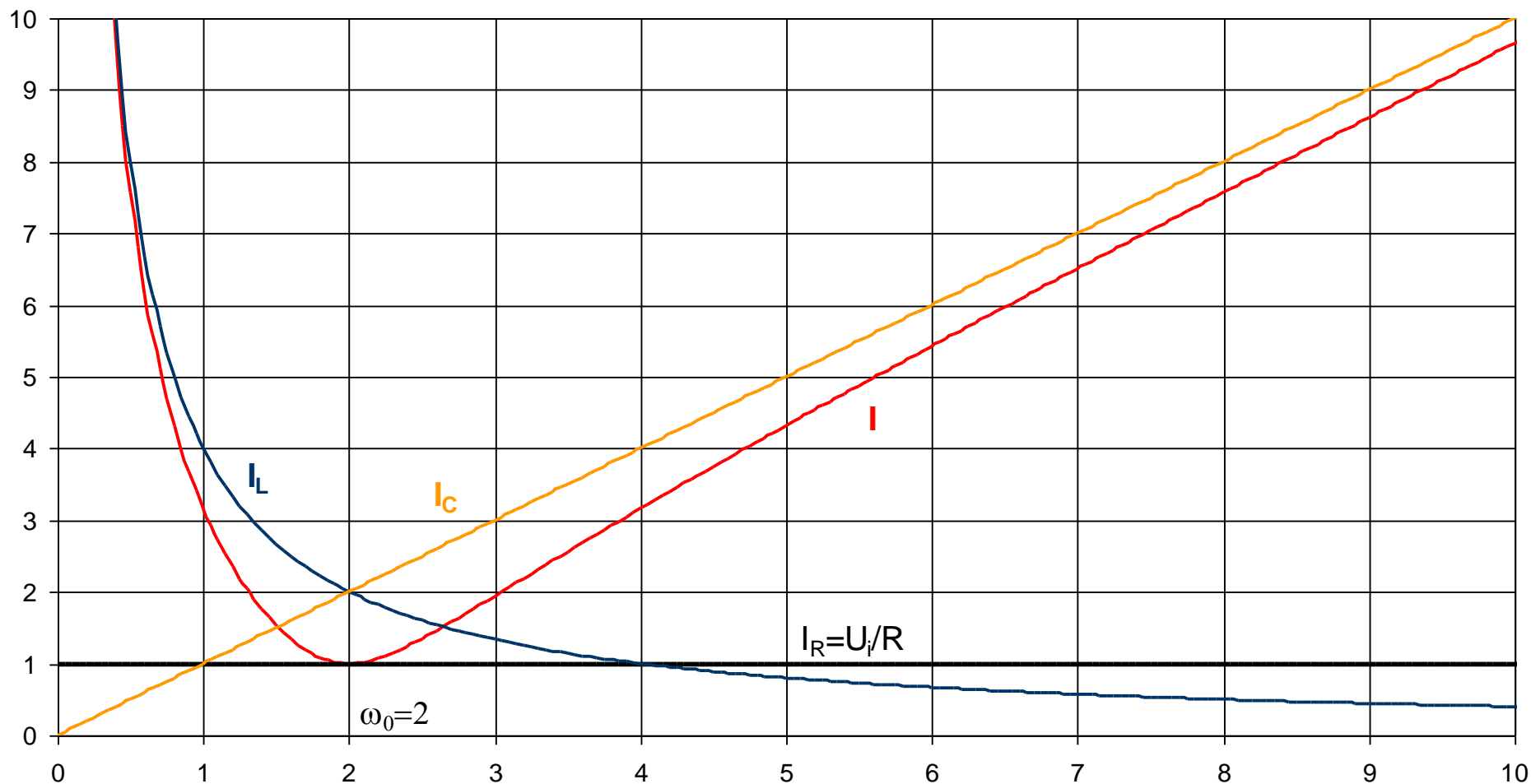
$$\varphi = \operatorname{arctg}\left(-\frac{B}{G}\right) = -\psi$$

♦ Struja izvora je: $\dot{I} = \dot{U}_i \cdot \underline{Y} \quad ; \quad \frac{\dot{I}}{\dot{I}_0} = \frac{\dot{U}_i \cdot \underline{Y}}{\dot{U}_i \cdot G} = \frac{\underline{Y}}{G}$

- u rezonanciji je minimalna $I_0 = U_i G$ pa se ova rezonancija naziva i **antirezonancija**

Modul impedancije

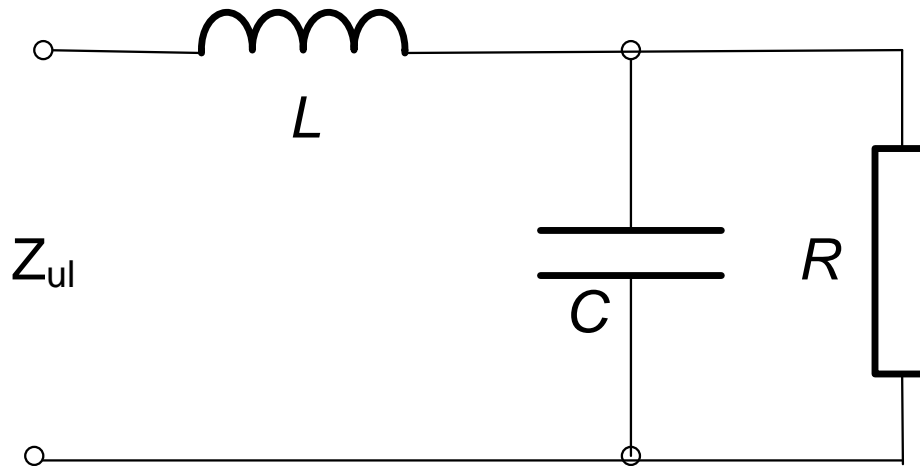




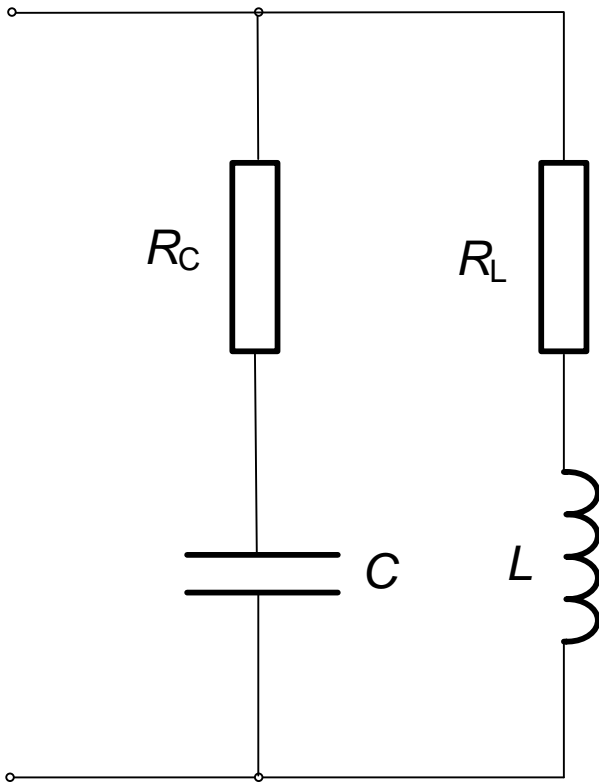
Primjer 4.

Za spoj prema slici ulazna impedancija kod frekvencije $\omega=0$ iznosi $5\ \Omega$, a kod rezonantne frekvencije je $2,5\ \Omega$. Koliki je X_C ?

(Rješenje: $X_C=5\ \Omega$)



5. Odrediti rezonantnu frekvenciju za krug prema slici.



$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - j\frac{1}{\omega C}}$$

$$\underline{Y} = \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} \right) + j \left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} \right)$$

Rezonancija:

$$\text{Im}\{\underline{Y}\} = 0 \Rightarrow \frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\frac{\frac{1}{\omega_0 C}}{R_C^2 + \frac{1}{(\omega_0 C)^2}} = \frac{\omega_0 L}{R_L^2 + (\omega_0 L)^2} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \rho^2}{R_C^2 - \rho^2}} \quad ; \quad \rho^2 = \frac{L}{C}$$

- Da bi se ispunio uvjet za rezonanciju mora biti:

$$R_L^2 > \frac{L}{C} \text{ i } R_C^2 > \frac{L}{C} \quad \text{ili} \quad R_L^2 < \frac{L}{C} \text{ i } R_C^2 < \frac{L}{C}$$

- Specijalni slučaj:

$$R_L^2 = R_C^2 = \frac{L}{C} \Rightarrow \omega_0 = \frac{0}{0} \quad \text{krug rezonira na svim frekvencijama}$$