

Zadatak 1

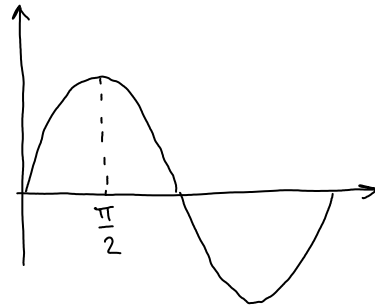
5. prosinca 2012.
16:03

$$(1) \quad U_{ef} = 200 \text{ V}, f = 10 \text{ Hz}$$

$$t_1 = \frac{1}{300} \text{ s}$$

$$u(t) = U_m \cdot \sin(\omega t)$$

$$U_{ef} = U_m \cdot \sqrt{2} = 282 \text{ V}$$



za maximum vrijedi:

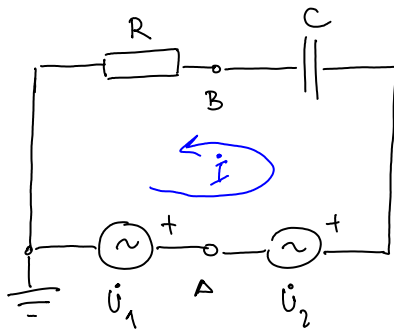
$$\sin(\omega t) = 1 \Rightarrow \omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega} = \frac{\pi}{2 \cdot 2\pi f} = \frac{1}{200} \text{ s} = 5 \text{ ms} = t_{\max}$$

$$\text{za } t = t_{\max} + t_1 = \frac{1}{200} + \frac{1}{300} = \frac{3+2}{600} = \frac{5}{600} \text{ s}$$

$$\begin{aligned} u(t_{\max} + t_1) &= U_m \cdot \sin(\omega(t_{\max} + t_1)) = 282 \cdot \sin\left(2\pi \cdot f \cdot \frac{5}{600}\right) = \\ &= 282 \cdot \sin\left(\frac{5}{6}\pi\right) = 282 \cdot 0,5 = 141 \end{aligned}$$

$$u(t_{\max} + t_1) = 141 \text{ V}$$

(2.)



$$\angle(\dot{U}_1, \dot{U}_{AB}) = 60^\circ$$

$$\omega = 1000 \text{ rad/s}$$

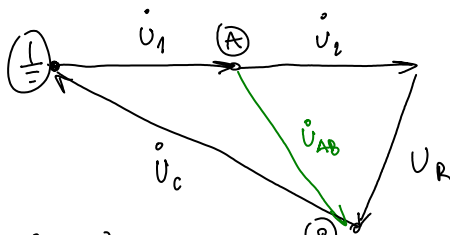
$$\dot{U}_1 = \dot{U}_2 = U \angle 0^\circ$$

$$R = 200\sqrt{3}$$

- skica fazorskog dijagrama (\dot{U}_C lomi na $\frac{\pi}{2}$ za \dot{U}_R)
(\rightarrow zbroj napona = 0, zatvaraju u istoj točki) da bi kut $\angle(\dot{U}_1, \dot{U}_{AB})$

bio 60° , mora i kut

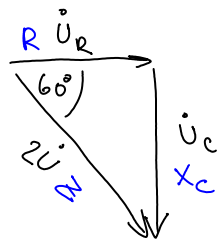
$$\varphi = \angle(\dot{U}_2, \dot{U}_{AB}) \text{ biti } 60^\circ$$



$$\begin{aligned} 1^\circ \quad U_C^2 &= U_2^2 + U_{AB}^2 - 2U_2 U_{AB} \cos 120^\circ = U^2 + U_{AB}^2 + U \cdot U_{AB} \\ 2^\circ \quad U_R^2 &= U_1^2 + U_{AB}^2 - 2U_1 U_{AB} \cos 60^\circ = U^2 + U_{AB}^2 - U \cdot U_{AB} \end{aligned} \quad \Bigg| +$$

$$U_C^2 + U_R^2 = 2U^2 + 2U_{AB}^2$$

$$(2U)^2 = 2U^2 + 2U_{AB}^2 \rightarrow U = U_{AB} \rightarrow U_C = \sqrt{3}U, U_R = U$$



odnosi napona i impedancija
su jednaki jer se radi o
serijskom spoju, gdje snaga
kroz svaki element jednaka

$$\frac{U_C}{U_R} = \tan \varphi = \frac{X_C}{R} \rightarrow X_C = \frac{U_C}{U_R} \cdot R \rightarrow X_C = \frac{\sqrt{3}U}{U} \cdot 200\sqrt{3} = 600$$

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{1000 \cdot 600} = 1,67 \mu\text{F}$$

$$\boxed{C = 1,67 \mu\text{F}}$$

3. serija RLC

$$R = 50$$

$$L = 0,05 \text{ H}$$

$$C = 20 \mu\text{F}$$

$$I' = \frac{1}{\sqrt{2}}$$

1. rezonantna frekvencija:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \mu \cdot 0,05}} = \frac{1}{10^{-5}} = 10^5 \text{ rad/s}$$

$$I = \frac{U}{R} = I = \frac{U}{50}$$

$$\frac{1}{I'} = \sqrt{2} \rightarrow \frac{\frac{U}{R}}{\frac{U}{|Z|}} = \sqrt{2} \Rightarrow \frac{|Z|}{R} = \sqrt{2} \Rightarrow |Z| = \sqrt{2} R$$

$$\Rightarrow R^2 + (X_L - X_C)^2 = 2R^2$$

$$(X_L - X_C)^2 = R^2 \quad | \sqrt{}$$

$$\pm (X_L - X_C) = R \quad \rightarrow \text{kap. karakter} \Rightarrow X_C > X_L$$

odabiremo predznak -

$$X_C - X_L = R$$

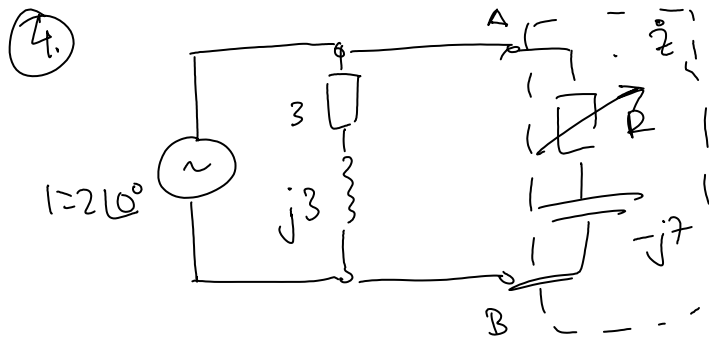
$$\frac{1}{\omega C} - \omega L = R \quad | \cdot \omega C$$

$$1 - \omega^2 LC = \omega RC \rightarrow \omega^2 LC + \omega RC - 1 = 0$$

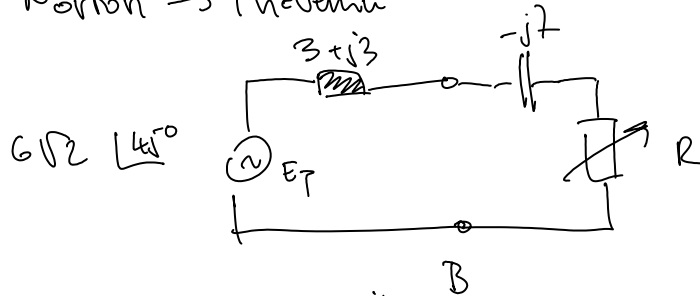
$$\omega_{1,2} = \frac{-RC \pm \sqrt{(RC)^2 + 4LC}}{2LC} = \frac{-0,001 \pm 0,00224}{0,00002} \cdot 10^6$$

$$= \frac{-1000 \pm 2240}{2} \Rightarrow \left\{ \omega \text{ je uvijek } > 0 \right\} \Rightarrow \omega = \frac{1240}{2} = 620 \text{ s}^{-1}$$

$$\boxed{\omega = 619 \text{ rad/s}}$$

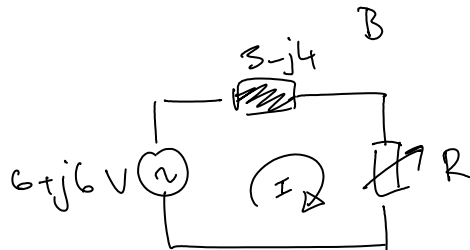


- Norton \rightarrow Thevenin



$$(1+j)^2 = 1^2 + 2j - 1$$

$$2j$$



$$P = I^2 \cdot R = \frac{U^2}{|Z|^2} \cdot R = \left(\frac{6\sqrt{2}}{\sqrt{(3+R)^2 + 16}} \right)^2 \cdot R = \frac{72R}{(3+R)^2 + 16}$$

$$\frac{\partial P}{\partial R} = 0 = \frac{72 \cdot [(3+R)^2 + 16] - 2(3+R) \cdot 72R}{((3+R)^2 + 16)^2} \Rightarrow \text{brzina} = 0$$

$$72(9 + 6R + R^2 + 16) - (432R + 144R^2) = 0$$

$$1800 + 432R + 72R^2 - 432R - 144R^2 = 0 \quad | :72$$

$$25 - R^2 \Rightarrow R = 5 \Omega$$

$$P = \frac{72 \cdot 5}{(3+5)^2 + 16} \Rightarrow$$

$$P = 4,5 \text{ W}$$

$$\textcircled{J.} \quad \left. \begin{aligned} \dot{z} &= 20 \angle 30^\circ \\ u &= 120 \angle 0^\circ \end{aligned} \right\} \quad \dot{Y} = \frac{1}{20} \angle -30^\circ$$

(HINT: zadatak je puno
lakše riješiti preko
admitancija)

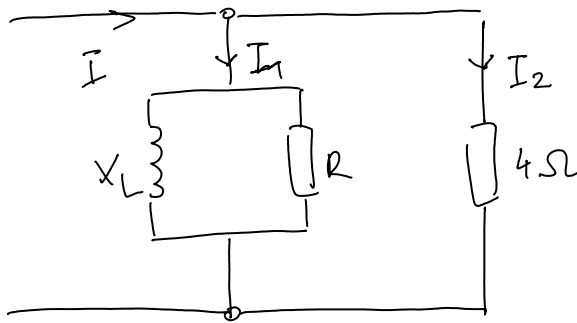
$$\dot{Y} = \frac{1}{20} \angle -18^\circ = \frac{1}{20} \angle -30^\circ + jB_c = \frac{\sqrt{3}}{40} - j\frac{1}{40} + jB_c$$

$$\tan(-18^\circ) = \frac{-\frac{1}{40} + B_c}{\frac{\sqrt{3}}{40}} = \frac{-1 + 40B_c}{\sqrt{3}} \Rightarrow B_c = \frac{\sqrt{3}\tan(-18^\circ) + 1}{40} = 0,011$$

$$B_c = \omega C \Rightarrow C = \frac{0,011}{314} = 3,48 \cdot 10^{-5} \text{ F}$$

$$C = 34,8 \mu\text{F}$$

6.



$$I_1 = 18 \text{ A}$$

$$I_2 = 15 \text{ A}$$

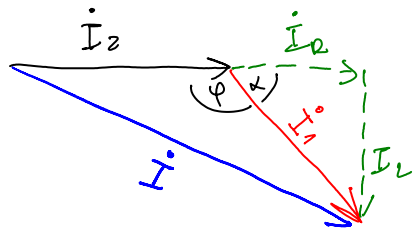
$$I = 30 \text{ A}$$

$$P = ? = I_1^2 \cdot R + I_2^2 \cdot (4 \Omega)$$

$$= U \cdot I_2 + U \cdot I_R$$

$$= U(I_2 + I_R)$$

- skica fazorskog dijagrama struja



$$I^2 = I_1^2 + I_2^2 - 2I_1I_2 \cos \phi$$

$$\cos \phi = \frac{I^2 - I_1^2 - I_2^2}{2I_1I_2} = -0,65$$

$$\phi = 130,54^\circ$$

$$\alpha = 180^\circ - \phi = 49,46^\circ$$

$$I_R = I_1 \cdot \cos \alpha = 11,7$$

$$U = I_2 \cdot R = 15 \cdot 4 = 60 \text{ V}$$

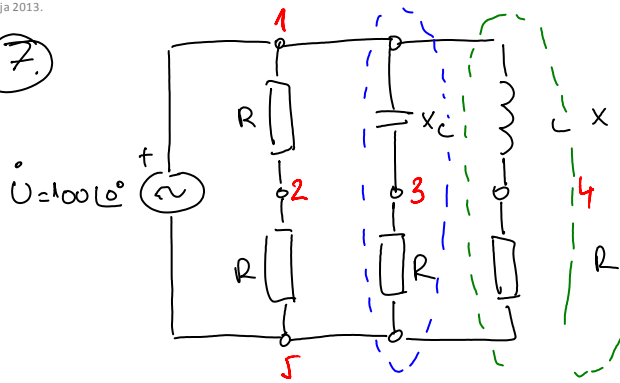
$$P = U \cdot I_R + U \cdot I_2 = U(I_2 + I_R) = 60 \cdot (15 + 11,7) = 1601,98 \text{ W}$$

$$P = 1602 \text{ W}$$

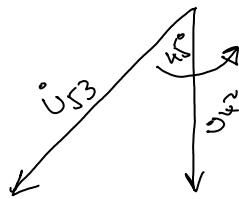
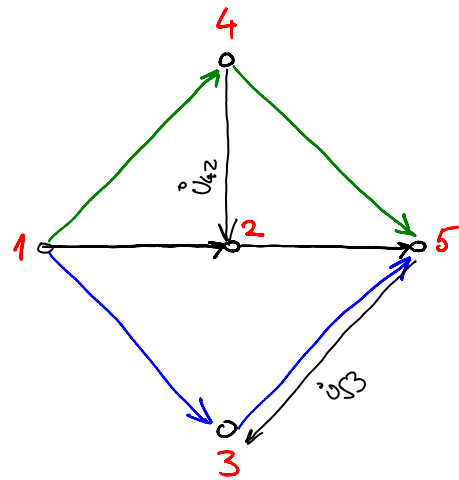
Zadatak 7

30. siječnja 2013.
22:41

7.



$$R = X_L = X_C$$



$\rightarrow \dot{u}_{42}$ prethodi naponu \dot{u}_{53} za 45°

8.

$$\dot{Y}_S = \dot{Y}_P$$

$$\dot{Z}_S = 10 + j10 \, \Omega = 10\sqrt{2} \angle 45^\circ$$

↓

$$Y_S = \frac{1}{10\sqrt{2}} \angle -45^\circ = \frac{1}{20} - j\frac{1}{20} \Rightarrow ?$$

$$Z_P = R \parallel jX = \frac{RjX}{R+jX} \Rightarrow \frac{1}{Z_P} = \frac{R+jX}{jRX} = Y_S = \frac{1-j}{20} \quad | \cdot$$

$$\frac{j \cdot \frac{R+jX}{jRX}}{j \cdot \frac{R+jX}{jRX}} = \frac{1-j}{20}$$

$$\frac{jR - X}{-RX} = \frac{1-j}{20} \Rightarrow \frac{X-jR}{RX} = \frac{1-j}{20} \quad | \cdot 20$$

$$\Rightarrow \left(\frac{20}{R} \right) \angle \left(j \frac{20}{X} \right) = 1-j$$

$$\rightarrow \text{realni: } \frac{20}{R} = 1 \Rightarrow R = 20$$

$$\rightarrow \text{imaginarni: } -\frac{20}{X} = -1 \Rightarrow X = 20$$

$$R = 20 \, \Omega, X_L = 20 \, \Omega$$

(9.)

$$u(t) = 100 \sin(\omega t + \pi) \Rightarrow \dot{U} = 10\sqrt{2} \angle 180^\circ$$

$$i(t) = 1 \cdot \sin\left(\omega t + \frac{2\pi}{3}\right) \Rightarrow \dot{I} = \frac{\sqrt{2}}{2} \angle 120^\circ$$

$$\dot{z} = \frac{\dot{U}}{\dot{I}} = \frac{10\sqrt{2} \angle 180^\circ}{\frac{\sqrt{2}}{2} \angle 120^\circ} = 100 \angle 60^\circ = 50 + j50\sqrt{3}$$

$$\dot{z}' = 50 + j50\frac{\sqrt{3}}{3} \rightarrow |z| = \sqrt{50^2 + 50^2 \cdot \frac{1}{3}} = 50\sqrt{\frac{3+1}{3}} = \frac{100}{\sqrt{3}}$$

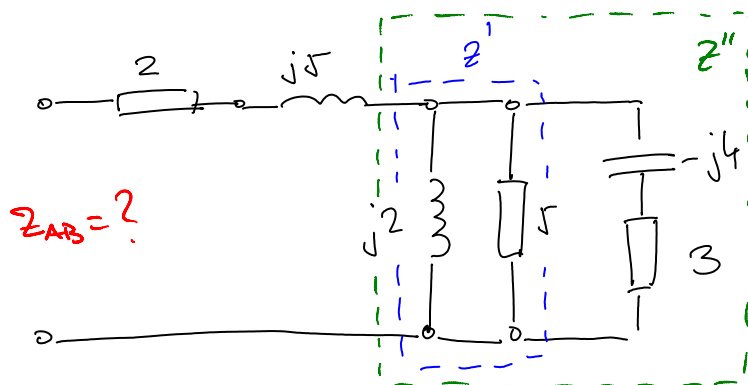
$$\rightarrow \tan \varphi = \frac{50\frac{\sqrt{3}}{3}}{50} = \frac{\sqrt{3}}{3} \Rightarrow \varphi = 30^\circ$$

$$\dot{z}' = \frac{100}{\sqrt{3}} \angle 30^\circ$$

$$\dot{I} = \frac{\dot{U}}{\dot{z}} = \frac{10\sqrt{2} \angle 180^\circ}{\frac{100}{\sqrt{3}} \angle 30^\circ} = \frac{\sqrt{6}}{2} \angle 150^\circ$$

$$\dot{I} = \frac{\sqrt{6}}{2} \angle 150^\circ$$

(10.)

 $Z_{AB} = ?$

$$Z' = j2 \parallel 5 = \frac{j10}{5+j2} \cdot \frac{5-j2}{5-j2} = \frac{20+j50}{29} = 0,7 + j1,72$$

$$Z'' = Z' \parallel (3-j4) = \frac{(0,7+j1,72)(3-j4)}{3,7-j2,28} = \frac{8,98+j2,36}{3,7-j2,28} \cdot \frac{3,7+j2,28}{3,7+j2,28} =$$

$$= \frac{27,78 + j29,2064}{18,89} = 1,47 + j1,55$$

$$Z = 2 + j5 + Z'' = 3,47 + j6,55 \quad \left\{ \begin{array}{l} |Z| = \sqrt{3,47^2 + 6,55^2} = 7,41 \\ \varphi = \arctan\left(\frac{6,55}{3,47}\right) = 62,1^\circ \end{array} \right.$$

$$Z_{AB} = 7,42 \angle 62,1^\circ$$