①

$U = 100 \underline{|0°}$ (AC source)

1, R, 2, R, 5 (left branch)

X_C 1, R_L (middle branch, green)
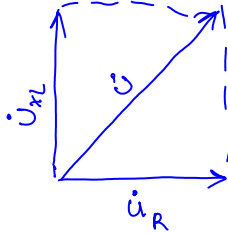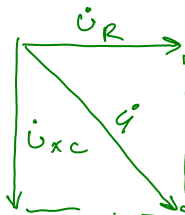
X_L 3, 4, R (right branch, blue)

$R = x_C = x_L$, $\psi = \sphericalangle(\dot{U}_{53}, \dot{U}_{23}) = ?$

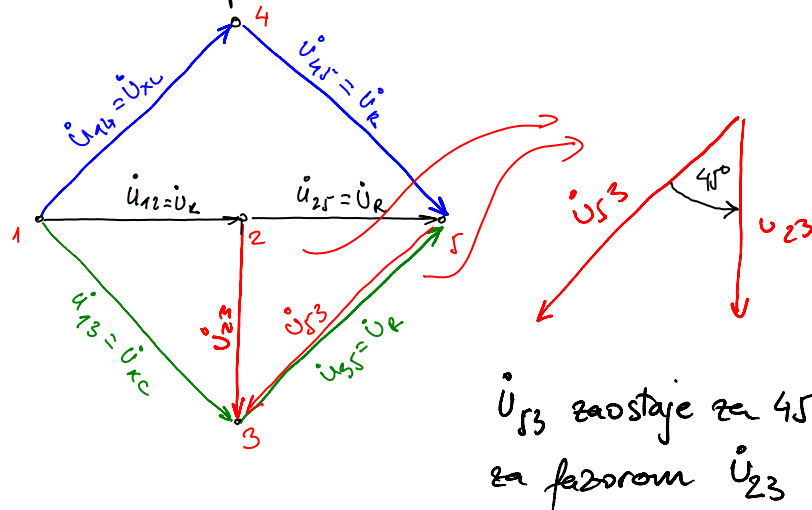- ako su otpori jednaki, onda su iznosi napona na pojedinim elementima u grani jednaki!

→ fazor napona na X_L rani za fazorom napona na R

→ fazor napona na X_C kasni za naponom na R

- odnos napona u grani sa X_L i R

$\dot{U}_{xL}$, $\dot{U}$, $\dot{U}_R$

- odnos napona u grani sa X_C i R

$\dot{U}_R$, $\dot{U}_{xc}$, $\dot{U}$

- rezultantni napon se mora poklapati (fazor $\dot{U}$)

4
$\dot{U}_{14} = \dot{U}_{xL}$ $\dot{U}_{45} = \dot{U}_R$
1   $\dot{U}_{12} = \dot{U}_R$ 2 $\dot{U}_{25} = \dot{U}_R$ 5
$\dot{U}_{13} = \dot{U}_{xc}$ $\dot{U}_{23}$ $\dot{U}_{53}$ $\dot{U}_{53} = \dot{U}_R$
3

$\dot{U}_{53}$ 45° U_{23}

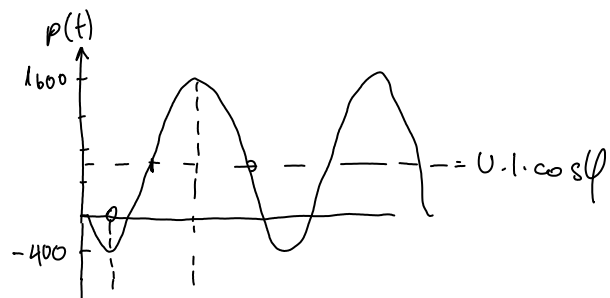$\dot{U}_{53}$ zaostaje za 45° za fazorom $\dot{U}_{23}$

$P_{MAX} = 1600 \text{ VA}$

$P_{min} = -400 \text{ VA}$

trenutna snaga trošila

$$p(t) = U \cdot I \cdot \cos\varphi - U \cdot I \cdot \cos(2\omega t - \varphi)$$
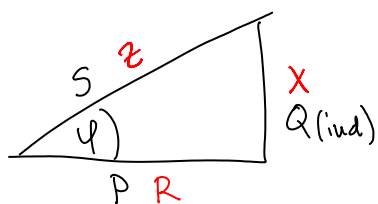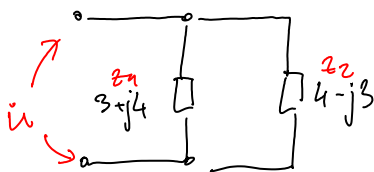
konstantan dio   vremenski ovisan dio

$P_{sr} = U \cdot I \cdot \cos\varphi = \dfrac{P_{MAX} + P_{MIN}}{2}$

$= \dfrac{1600 - 400}{2} = 600$

za $\varphi = 0$ $P \geq S = UI = P_{sr} = 1000 \text{ VA} \longrightarrow Q = 0 \text{ VAr}$ (tako dobiješ S)

za $\varphi \neq 0$ $Q = \sqrt{S^2 - P^2} = 800 \text{ VAr}$

$$z_{uk} = z_1 \| z_2 = \frac{z_1 \cdot z_2}{z_1 + z_2} = \frac{(3+j4)(4-j3)}{3+j4+4-j3} =$$

$$= \frac{12+j16-j9+12}{7+j} =$$

$$= \frac{24+j7}{7+j} \cdot \frac{7-j}{7-j} =$$
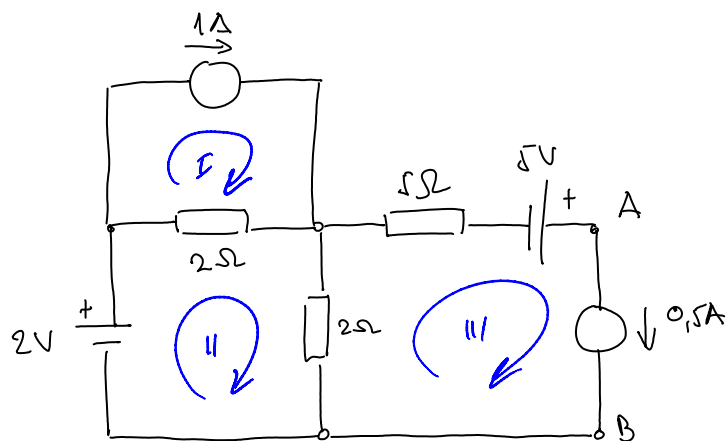
$$= \frac{168+j49-j24+7}{49+1} =$$

$$= \frac{175+j25}{50} \cdot \frac{\frac{1}{25}}{\frac{1}{25}} = \frac{7}{2} + j\frac{1}{2}$$

$$\Rightarrow tg\varphi = \frac{Im\{\dot{z}\}}{Re\{\dot{z}\}} = \frac{1}{7}$$

$$tg\varphi = \frac{Q}{P} = \frac{1}{7} \Rightarrow P = \frac{Q}{tg\varphi}$$

$$P = 700 \; W$$

4.

$U_{AB} = ?$

I: $\quad I_1 = 1\,A$

II: $\quad 2 = I_{II}(2+2) - I_{III} \cdot 2 - I_1 \cdot 2$

III: $\quad I_{III} = 0,5\,A$

$$2 = I_{II} \cdot 4 - 0,5 \cdot 2 - 1 \cdot 2$$

$$2 = 4I_{II} - 1 - 2$$

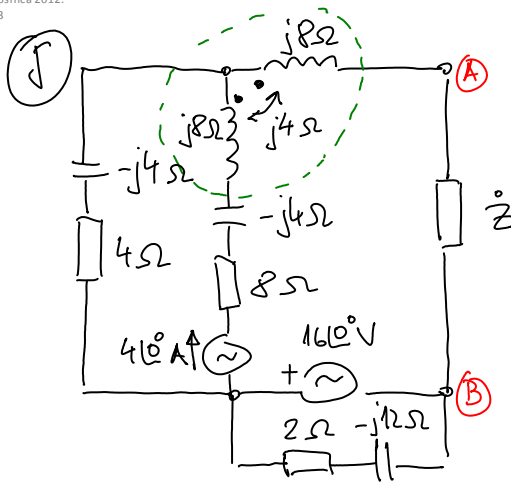$$4I_{II} = 5$$

$$I_{II} = \frac{5}{4} = 1,25\,A$$

kontura III:

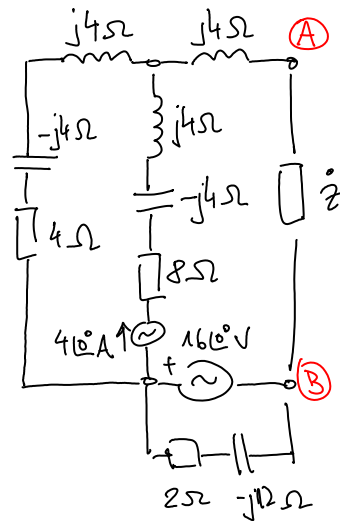$$5V = U_{AB} + I_{III} \cdot 5\Omega + (I_{III} - I_{II}) \cdot 2\Omega$$

$$5 = U_{AB} + 0,5 \cdot 5 + (0,5 - 1,25) \cdot 2$$
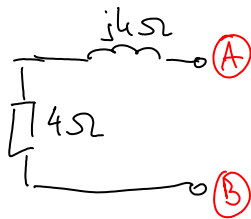
$$U_{AB} = 5 - 2,5 + 0,75 \cdot 2$$

$$U_{AB} = 4V$$

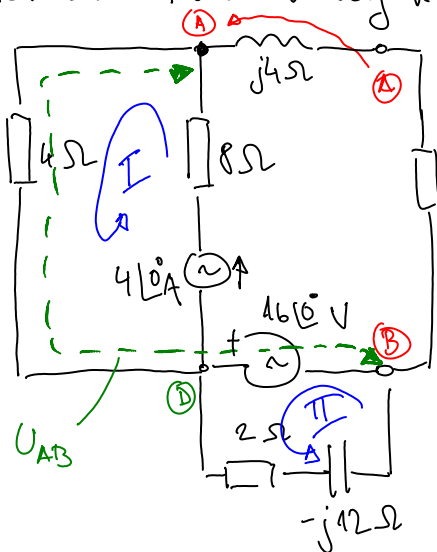$$\xrightarrow{\text{nadomjesna shema (zbog M)}}$$

- proračun Theveninove impedancije (strujni izvori se odspajaju, naponski izvori se kratko spajaju)

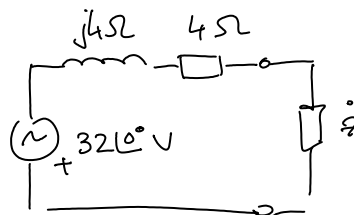$$\dot{Z}_{AB} = 4 + j4\,\Omega$$

- proračun Theveninovog napona ($j4\Omega$ je odspojen pa Ⓐ ide s njegove lijeve strane)

$$U_{AB} = +4\underline{/0°}\,A \cdot 4\,\Omega + 16\,V$$

$$= +32\,V = E_T$$

nadomjesna shema: $E_T, \dot{Z}_T$

- za P_{max}

$$\dot{Z} = \dot{Z}_i^* = 4 - j4$$

$$\operatorname{Re}\{\dot{Z}\} = R_{max} = 4\,\Omega$$

$$\dot{Z}_{uk} = \dot{Z}_i + \dot{Z} = 8\,\Omega$$

$$i = \frac{\dot{U}}{\dot{Z}_{uk}} = \frac{32\underline{/0°}}{8\underline{/0°}} = 4\underline{/0°}\,A = 4\,A$$

$$P_{max} = 4^2 \cdot R = 16 \cdot 4 = 64\,W$$
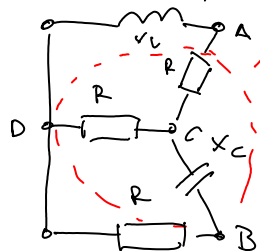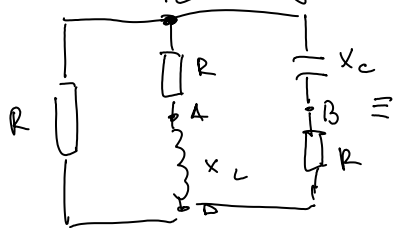
$$\boxed{P_{max} = 64\,W}$$

⑥
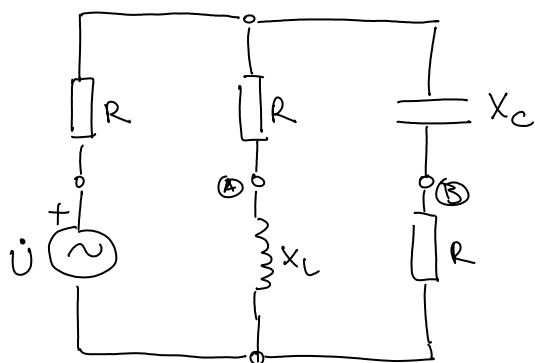
$R = K_L = X_C = 10 \ \Omega$

$\dot{I}_N = ?$

$\dot{Z}_N = ?$

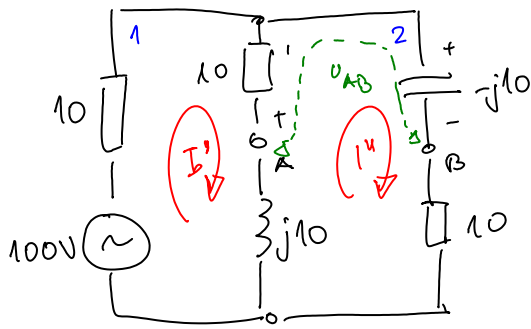1° impedancija $\dot{Z}_N$ (isto kao i Z_T)

pretvorba $\lambda-\Delta$
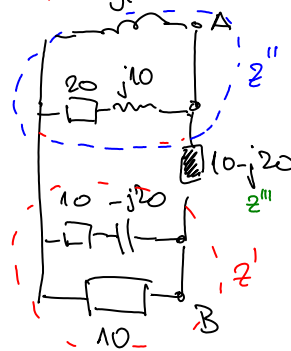
$Z''= j10 \parallel (20+j10) = \dfrac{j10(20+j10)}{20+j20} = \dfrac{10(-10+j20)}{10(2+j2):2} =$

$= \dfrac{-5+j10}{1+j} \cdot \dfrac{1-j}{1-j} = \dfrac{10-5+j5+j10}{2} = 2{,}5 + j7{,}5$

$Z' = (10-j20)\parallel 10 = \dfrac{10(10-j20):20}{20-j20 \quad :20} = \dfrac{5-j10}{1-j} \cdot \dfrac{1+j}{1+j} =$

$= \dfrac{5-j10+j5+10}{2} = 7{,}5 - j2{,}5$

$Z''' = 10 - j20$

$Z_{AB} = (Z'+Z'')\parallel Z''' = (2{,}5+j7{,}5+7{,}5-j2{,}5)\parallel Z''' = (10+j5)\parallel(10-j20)$

$= \dfrac{50(2+j)\cdot(1-j2):5}{20-j15 \quad :5} = \dfrac{10(4-j3)}{4-j3} = 10\ \Omega = Z_T = Z_N$

Circuit diagram labels: 1, 2, 10, 10, U_{AB}, $-j10$, I', I'', $j10$, 10, $100V$, A, B

kontura 1: $100 = I'(20+j10) - I''(10+j10) \quad /:10$

kontura 2: $0 = I'' \cdot 20 - I'(10+j10) \quad /:10$

$$10 = (2+j)\cdot I' - (1+j)I''$$

$$I'' = \frac{1+j}{2}\cdot I'$$

$$10 = (2+j)I' - \frac{1+j}{2}\cdot(1+j)I'$$

$$10 = (2+j)I' - jI' = I'(2+j-j)$$

$$2I' = 10 \Rightarrow I' = 5A$$

$$I'' = 2,5 + j2,5 A$$

$$\dot{U}_{AB} = 10(I''-I') - j10\cdot I''$$

$$\dot{U}_{AB} = -25 + j25 - j25 + 25$$

$$\dot{U}_{AB} = 0 = E_T$$

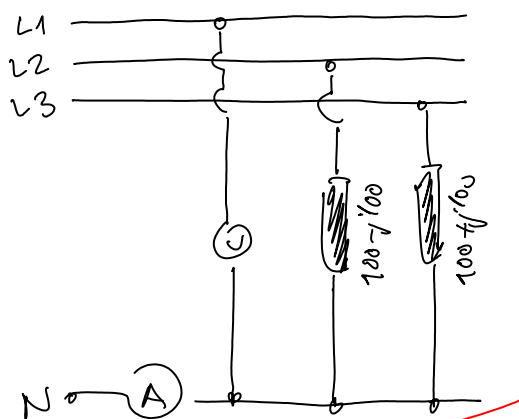$$I_N = \frac{E_T}{Z} = \frac{0}{10} = 0 \; A$$

$$\boxed{I_N = 0A, \; Z_N = 10\,\Omega}$$

$$\dot{z}_1 = \infty \qquad\qquad \dot{U}_1 = 220\,\underline{|0°}$$

$$\dot{z}_2 = 100\sqrt{2}\,\underline{|-45°} \qquad \dot{U}_2 = 220\,\underline{|-120°}$$

$$\dot{z}_3 = 100\sqrt{2}\,\underline{|45°} \qquad \dot{U}_3 = 220\,\underline{|120°}$$

$$\dot{i}_1 = \frac{\dot{U}_1}{\dot{z}_1} = 0 \quad A$$

$$\dot{i}_2 = \frac{\dot{U}_2}{\dot{z}_2} = 1{,}1\sqrt{2}\,\underline{|-75°} = 0{,}4 - j1{,}5$$

$$\dot{i}_3 = \frac{\dot{U}_3}{\dot{z}_3} = 1{,}1\sqrt{2}\,\underline{|75°} = 0{,}4 + j1{,}5$$

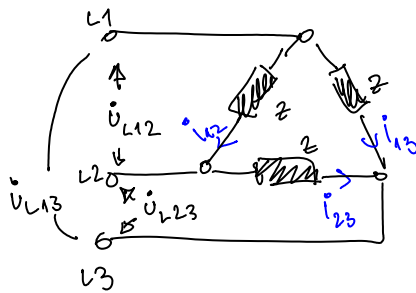$$\dot{i}_N = \dot{i}_1 + \dot{i}_2 + \dot{i}_3 =$$

$$= 0{,}4 - j1{,}5 + 0{,}4 + j1{,}5 =$$

$$= 0{,}8 + j0 = 0{,}8\,\underline{|0°}\ A \quad \longrightarrow \quad I_N = 0{,}8A$$

⑧.

→ fazorski dijagram izvora:
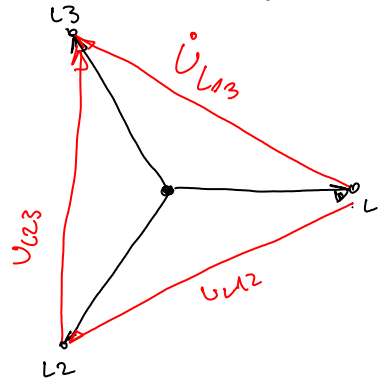
$U_{L12} = 380 \lfloor 210°$

$U_{L13} = 380 \lfloor 150°$

$U_{L23} = 380 \lfloor 90°$

$Z = 80 - j60 = 100 \lfloor -36,87$

$I_{12} = 3,8 \lfloor 246,87 \longrightarrow S_{12} = \dot{U}_{12} \cdot \dot{I}_{12}^* = 1444 \lfloor -36,87° = 1155,2 - j866,4$

$I_{13} = 3,8 \lfloor 186,87 \longrightarrow S_{13} = \dot{U}_{L13} \cdot \dot{I}_{13}^* = 1444 \lfloor -36,87 = 1155,2 - j866,4$

$I_{23} = 3,8 \lfloor 126,87 \longrightarrow S_{23} = \dot{U}_{L23} \cdot \dot{I}_{23}^* = 1444 \lfloor -36,87 = 1155,2 - j866,4$

ΣP   ΣQ

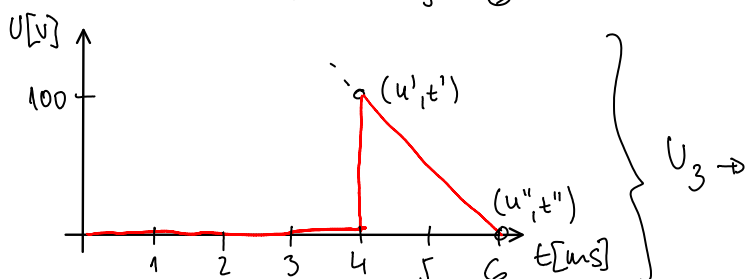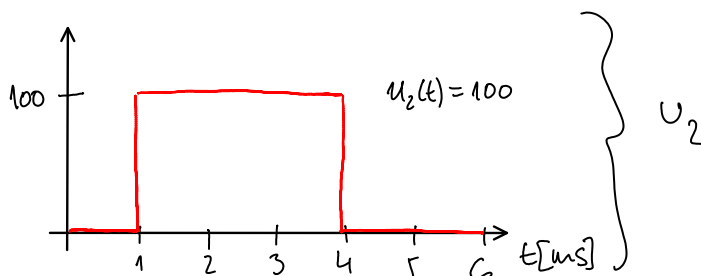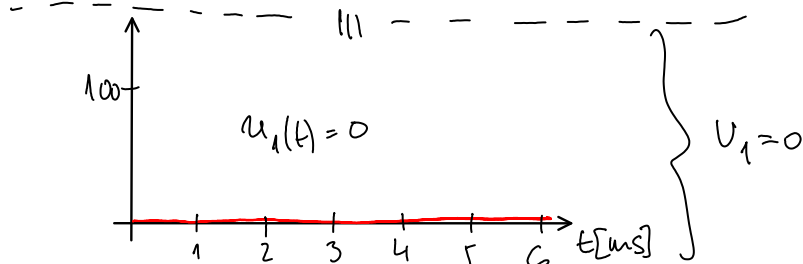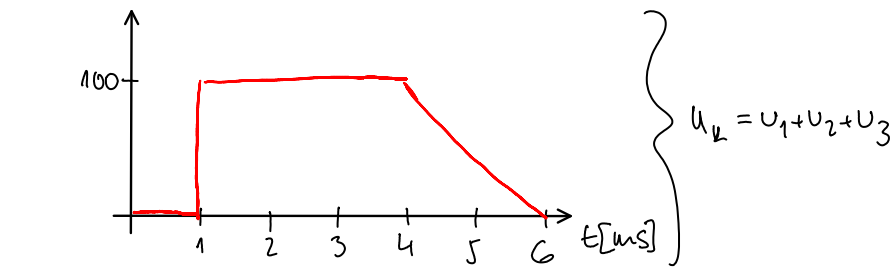$S_{uk} = \Sigma P + \Sigma Q = 3465,6 - j2599,2 \ [VA]$

↓

$\boxed{P_{uk} = 3465,6 \ W}$

(lakši način: budući da je TROŠILO SIMETRIČNO izračunaš S
u samo jednoj grani trokuta, nađeš Re{S} koji
je jednak djelatnoj snazi P i pomnožiš ga
sa 3. Tako dobiješ ukupnu djelatnu snagu)

⑨.

period $T = 6$ ms

$R = 10\,\Omega$

$u_k = U_1 + U_2 + U_3$

$u_1(t) = 0$ — $U_1 = 0$

$u_2(t) = 100$ — U_2

$U[V]$, (u', t'), (u'', t'') — $U_3 \to$

$U_{ef} = \sqrt{U_{1ef}^2 + U_{2ef}^2 + U_{3ef}^2}$

$U_{1ef} = 0$

$u_{2ef} = \sqrt{\dfrac{1}{6} \int_1^{4} u_2(t)^2 \, dt}$

$u_{3ef} = \sqrt{\dfrac{1}{6} \int_4^{6} u_3(t) \, dt}$

$u_3(t)$ — jednadžba pravca kroz dvije točke

$u_3(t) - u' = \dfrac{u'' - u'}{t'' - t'}\left(t - t'\right)$

$u_3(t) - 100 = \dfrac{0 - 100}{6 - 4}\left(t - 4\right)$

⑦ $u_3(t) = -50t + 300$

$u_3(t)^2 = 2500t^2 - 30000t + 90000$

$u_{2ef}(t) = \sqrt{\dfrac{1}{6} \cdot \int_1^{4} 100^2 \, dt} = \sqrt{\dfrac{1}{6} \cdot 100^2 (4-1)} = 100\dfrac{\sqrt{2}}{2} = 50\sqrt{2} \ V$

$u_{3ef}(t) = \sqrt{\dfrac{1}{6} \int_4^{6} \left[2500t^2 - 30000t + 90000\right] dt} =$

$= \sqrt{\dfrac{1}{6}\left[2500\int_4^{6} t^2 \, dt - 30000 \int_4^{6} t \, dt + 90000 \int_4^{6} dt\right]} =$

$= \sqrt{\dfrac{1}{6}\left[2500\left(\dfrac{6^3}{3} - \dfrac{4^3}{3}\right) - 30000\left(\dfrac{6^2}{2} - \dfrac{4^2}{2}\right) + 90000(6-4)\right]}$

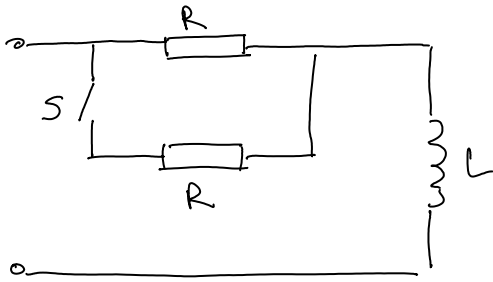$= \sqrt{\dfrac{1}{6}\left[180000 - 53\,333,\dot{3} - 540000 + 240000 + 180000\right]}$

$= \sqrt{\dfrac{1}{6} \cdot 6\,666,\dot{6}7} = 33,\dot{3}3$

$$u_{ef} = \sqrt{u_{1ef}^2 + u_{2ef}^2 + u_{3ef}^2} = \sqrt{0^2 + (50\sqrt{2})^2 + 33{,}33^2} = 78{,}1736 \ V$$
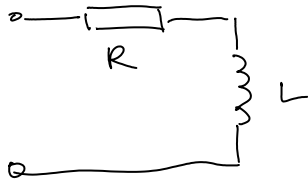
→ za ekvivalentni sinus vrijedi:

$$U_m = u_{ef} \cdot \sqrt{2} = 110{,}55 \ V$$
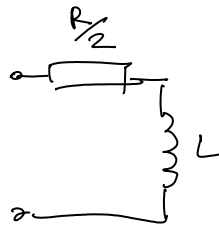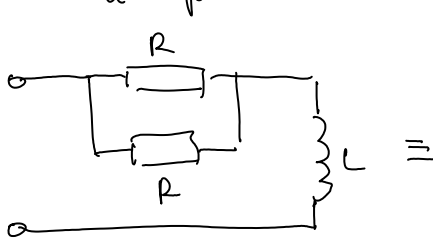
10.

1° prije uklopa S

$$\tau_1 = \frac{L}{R}$$

2° nakon uklopa S

$$\tau_2 = \frac{L}{\frac{R}{2}} = \frac{2L}{R} = 2 \cdot \tau_1$$

– vremenska konstanta će se **povećati 2 puta**