Optimum Loft Angle for Greatest Carry Distance

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1 Abstract

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2 Response

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3 Theory and model

3.1 Assumptions

The following assumptions are made throughout the report and model:

· golf course is level and has no effect on trajectory;

· height of the tee is negligible;

· gravitational field strength is constant (9.81 $N \cdot kg^{-1}$) and does not flucuate with height;

· driver is roughly a flat plate and strikes the ball precisely at the center, with no draw or fade.

3.2 Impact

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3.3 Flight

3.3.1 Simple golf ball

A simple golf ball experiencing only weight may be modelled by the following system of differential equations;

$$a_x = \frac{\partial v_x}{\partial t} = 0 \tag{1}$$

$$a_y = \frac{\partial v_y}{\partial t} = -g \tag{2}$$

where g is the gravitional field strength.

Assuming the initial velocity is v_0 and the launch angle is θ, the above equations can be solved to give:

$$v_x = v_0 \cos\theta \tag{3}$$

$$v_y = v_0 \sin\theta - gt \tag{4}$$

Integrating the above equations w.r.t. time gives the displacement as a function of time:

$$x = v_0 t \cos\theta \tag{5}$$

$$y = v_0 t \sin\theta - \frac{1}{2} g t^2 \tag{6}$$

Figure 1: Golf ball trajectory under no air resistance or lift

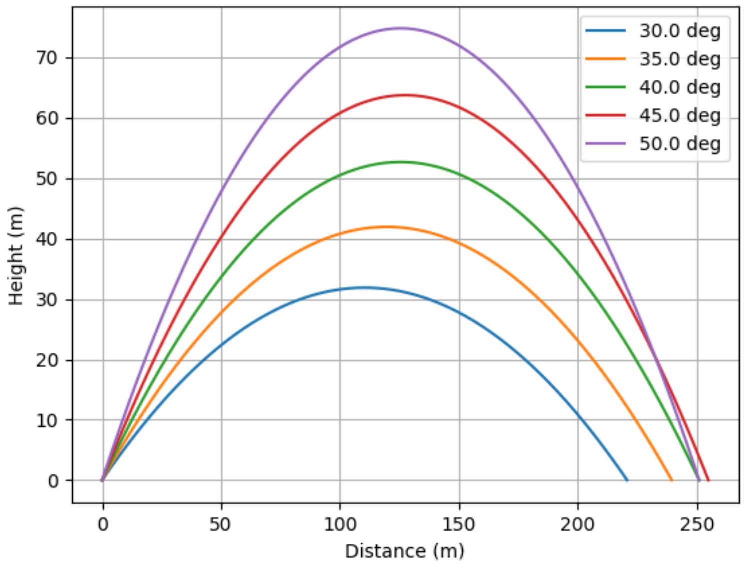
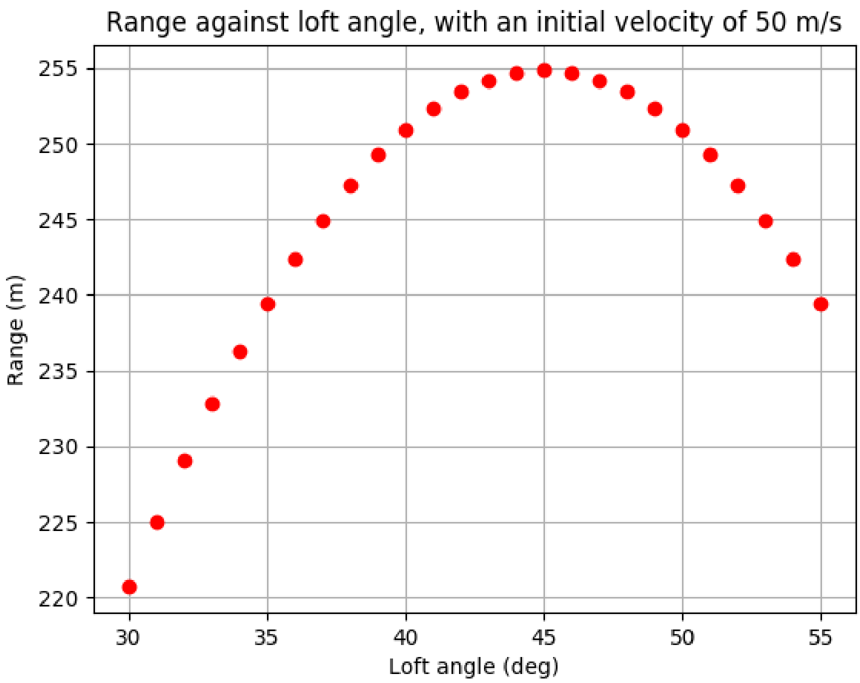


Figure 2: Range as a function of loft angle, no drag or lift



Figures (1) and (2) show the maximum range is when the loft angle at 45°, as predicted by the equations of projectile motion.

3.3.2 Smooth golf ball experiencing drag

The drag equation is:

$$\vec{F_d} = \frac{1}{2} A C_d \rho_{air} |\vec{v}| \vec{v} \tag{7}$$

where ρ_{air} is the density of air; A is the reference area, which in the case of a smooth sphere of radius r, is the cross-sectional area πr^2; C_d is the coefficient of drag, which is dependent on the Reynolds number; and $\vec{v}$ is the flow velocity relative to the golf ball.

Applying equation (7) to equations (1)-(2), we get

$$\frac{\partial v_x}{\partial t} = -k|v_x|v_x \tag{8}$$

$$\frac{\partial v_y}{\partial t} = -g - k|v_y|v_y \tag{9}$$

where $k = \frac{1}{2} A C_d \rho$.

Figure 3: Golf ball trajectory when experiencing drag but no lift, $C_d = 0.5$

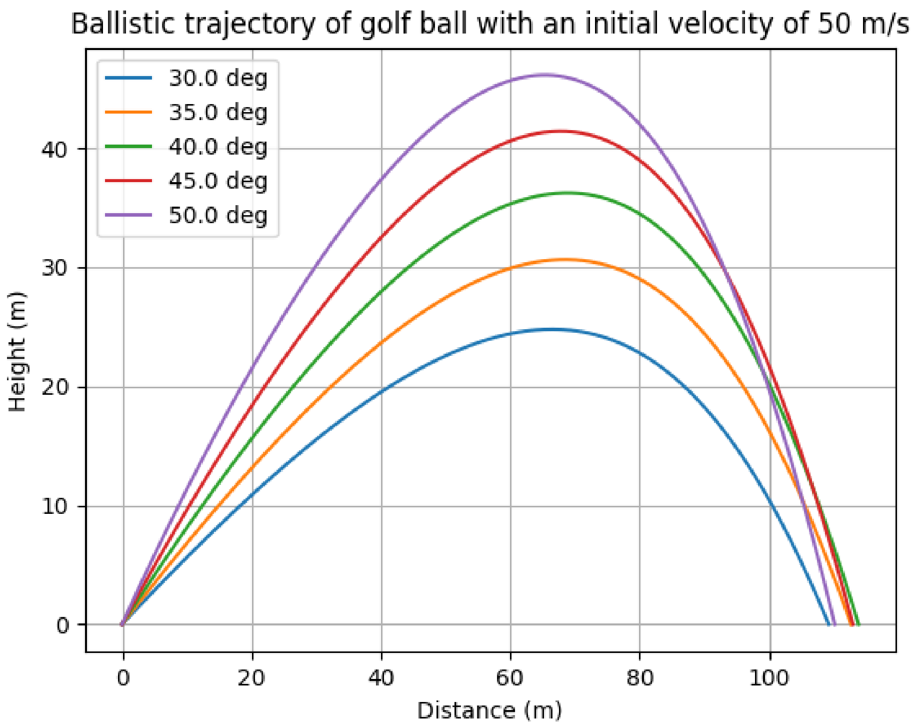


Figure 4: Range as a function of loft angle, $C_d = 0.5$

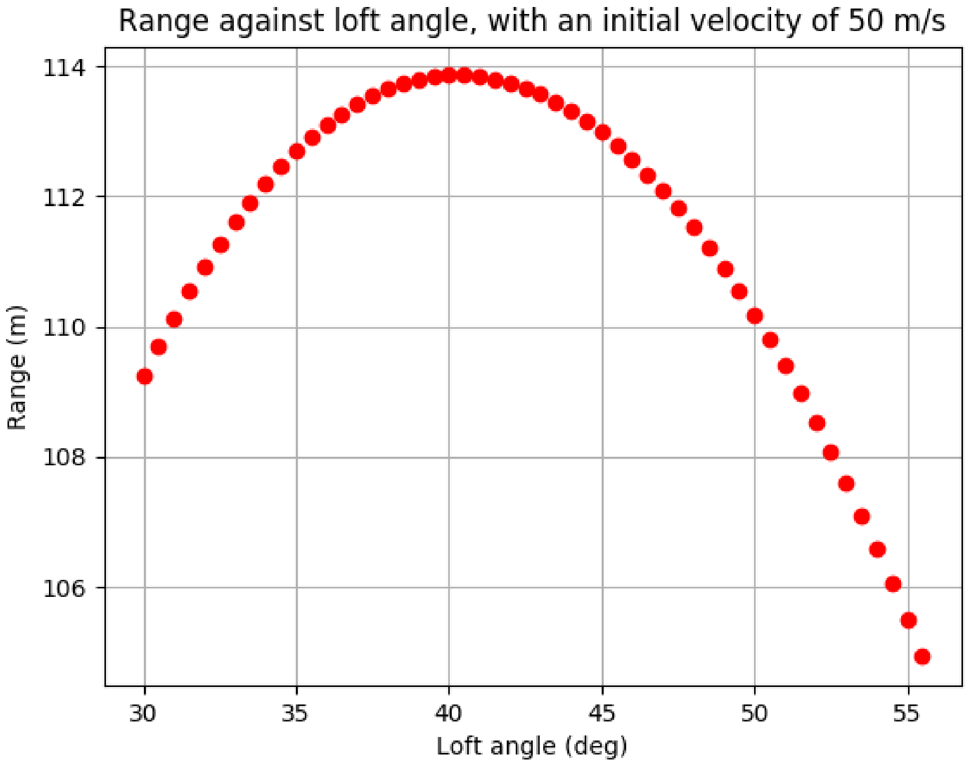


Figure (4) shows that the maximum range is achieved at 40.2°, for an initial velocity of 50 $m \cdot s^{-1}$. Additionally, the range is significantly decreased when drag was added to the model. The greatest range with drag is about 114 m versus the previous range of 255 m at 45.0°. However, C_d is not constant and depends on the Reynolds number, which is proportional to the velocity of the golf ball. The Reynolds number is given by:

$$Re = \frac{2vr}{\nu} \tag{10}$$

where ν is the kinematic viscosity of air.

Using data from [whereever we found the Reynolds number data], we used Python to find a fourth-order polynomial approximation as shown in Figure (5).

Figure 5: Coefficient of drag as a function of Reynolds number with curve of best fit