

# Optimum Loft Angle for Greatest Carry Distance

## Group D

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# **1 Abstract**

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# **2 Response**

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# **3 Theory and model**

## **3.1 Assumptions**

The following assumptions are made throughout the report and model:

- golf course is level and has no effect on trajectory;
- height of the tee is negligible;
- gravitational field strength is constant ( $9.81 \text{ N} \cdot \text{kg}^{-1}$ ) and does not fluctuate with height;
- driver is roughly a flat plate and strikes the ball precisely at the center, with no draw or fade;
- the ball used is a Titleist Pro V1, with a mass of 45.93 g, diameter of 42.67 mm.

## **3.2 Impact**

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### 3.3 Flight

#### 3.3.1 Simple golf ball

A simple golf ball experiencing only weight may be modelled by the following system of differential equations;

$$a_x = \frac{\partial v_x}{\partial t} = 0 \quad (1)$$

$$a_y = \frac{\partial v_y}{\partial t} = -g \quad (2)$$

where  $g$  is the gravitational field strength.

Assuming the initial velocity is  $v_0$  and the launch angle is  $\theta$ , equations (1-2) can be solved to give equations (3-4):

$$v_x = v_0 \cos \theta \quad (3)$$

$$v_y = v_0 \sin \theta - gt \quad (4)$$

Integrating equations (3-4) with respect to time yields the displacement as a function of time:

$$x = v_0 t \cos \theta \quad (5)$$

$$y = v_0 t \sin \theta - \frac{1}{2}gt^2 \quad (6)$$

Figure 1: Golf ball trajectory, no drag or lift

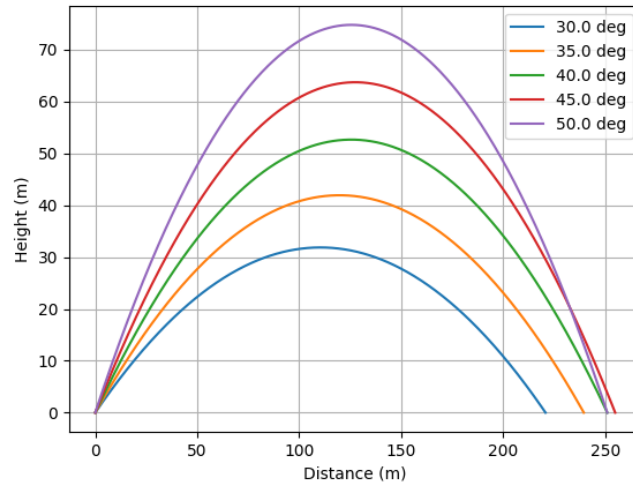
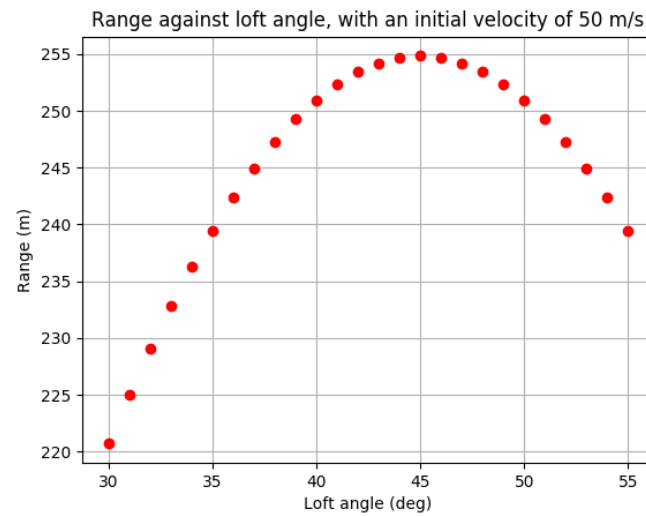


Figure 2: Range as a function of loft angle, no drag or lift



Figures (1-2) show the maximum range is when the loft angle at  $45^\circ$ , as predicted by the equations of projectile motion.

### 3.3.2 Smooth golf ball experiencing drag

The drag equation is given by:

$$\vec{F}_d = \frac{1}{2}AC_d\rho_{air}|\vec{v}|\vec{v} \quad (7)$$

where  $\rho_{air}$  is the density of air;  $A$  is the reference area, which in the case of a smooth sphere of radius  $r$ , is the cross-sectional area  $\pi r^2$ ;  $C_d$  is the coefficient of drag, which is dependent on the Reynolds number; and  $\vec{v}$  is the flow velocity relative to the golf ball. In this case, we assume the air is stationary and the golf ball is moving through the air with velocity  $\vec{v}$ .

Applying equation (7) to equations (1-2), we get

$$\frac{\partial v_x}{\partial t} = -k|v_x|v_x \quad (8)$$

$$\frac{\partial v_y}{\partial t} = -g - k|v_y|v_y \quad (9)$$

where  $k = \frac{1}{2}AC_d\rho_{air}$ .

Figure 3: Golf ball trajectory when experiencing drag but no lift,  $C_d = 0.5$

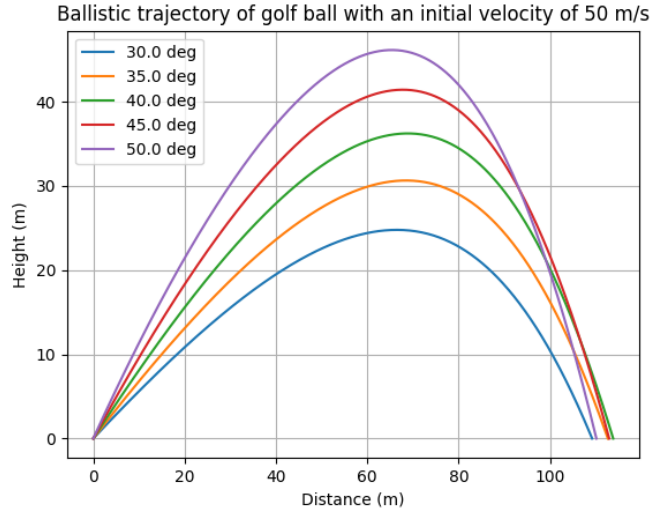


Figure 4: Range as a function of loft angle,  $C_d = 0.5$

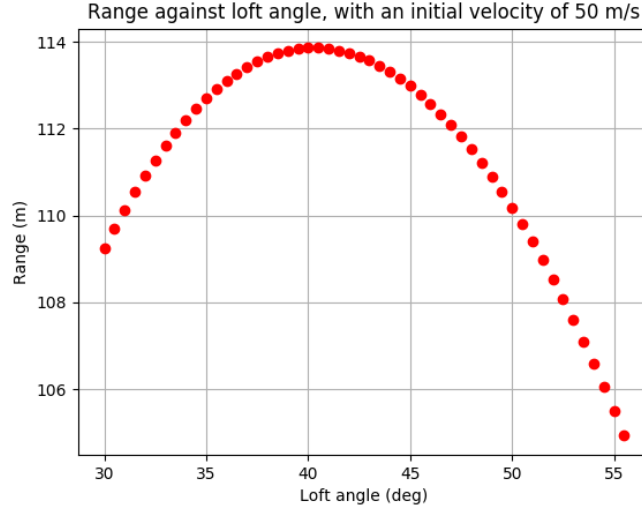


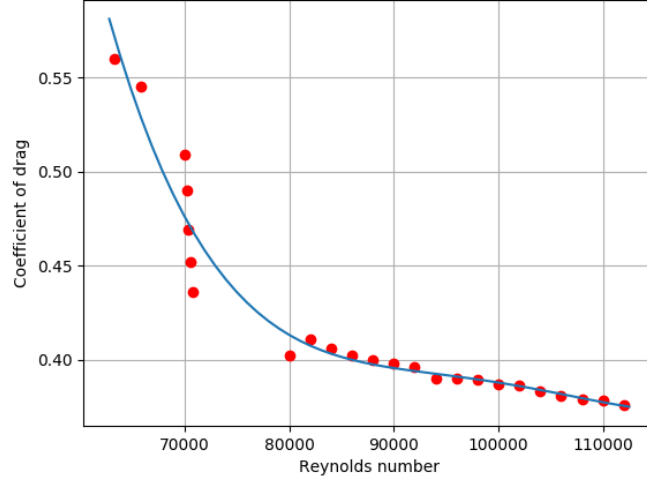
Figure (4) shows that the maximum range is achieved at  $40.2^\circ$ , for an initial velocity of  $50 \text{ m} \cdot \text{s}^{-1}$ . Additionally, the range is significantly decreased when drag was added to the model. The greatest range with drag is about 114 m versus the previous range of 255 m at  $45.0^\circ$ . However,  $C_d$  is not constant and depends on the Reynolds number, which is proportional to the velocity of the golf ball. The Reynolds number is given by:

$$R = \frac{2vr}{\nu} \quad (10)$$

where  $\nu$  is the kinematic viscosity of air.

Using data from [whereever we found the Reynolds number data], we used Python to find a fourth-order polynomial approximation as shown in Figure (5).

Figure 5: Coefficient of drag as a function of Reynolds number with curve of best fit



$$C_D = 9.46410458 \times 10^{-20} R^4 - 3.80736984 \times 10^{-14} R^3 + 5.72048806 \times 10^{-9} R - 3.81337408 \times 10^{-4} R + 9.92620 \quad (11)$$

Applying the approximation for the coefficient of drag, the optimum loft angle decreases down to about  $34.5^\circ$ , shown in figures (6-7).

Figure 6: Trajectory of golf ball with drag considering Reynolds number,  $v_i = 50m \cdot s^{-1}$

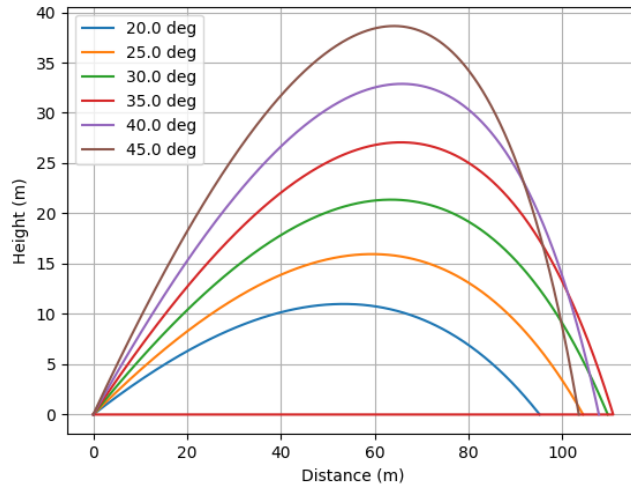
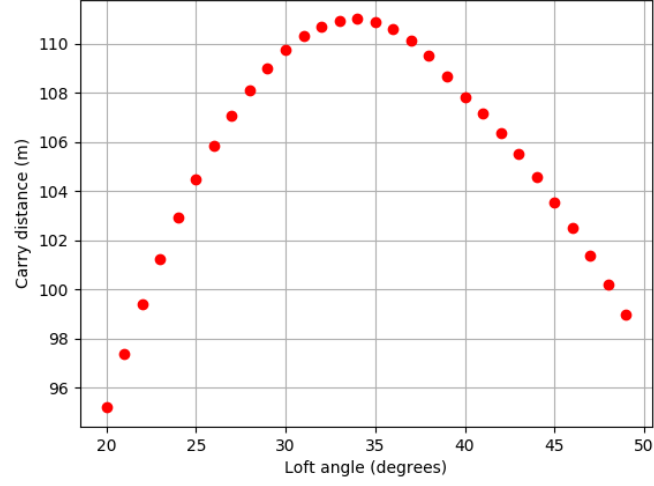


Figure 7: Range against loft angle for golf ball with drag considering Reynolds number,  $v_i = 50m \cdot s^{-1}$



### 3.3.3 Smooth golf ball experiencing lift

Lift on a golf ball is caused by the Magnus effect, which is dependent on the backspin of the ball. The equation used is:

$$F_l = \frac{1}{2} C_l \rho_{air} |v|^2 (\hat{\omega} \times \hat{v}) \quad (12)$$

where  $C_L$  is dependent on the spin parameter of the ball:

$$C_L = -3.25S^2 + 1.99S \quad (13)$$

The spin parameter is given by

$$S = \frac{r|\omega|}{|v|} \quad (14)$$