Unit 2

Two-dimensional motion

Introduction to Kinematics

Kinematics is the branch of physics that studies the motion of objects without considering the forces that cause the motion. For example, when analyzing the motion of a football, we focus only on its trajectory and speed, not on the forces that caused it to move. In Grade 11, you studied one-dimensional kinematics, which dealt with motion along a straight line. Now, in two-dimensional kinematics, we extend these concepts to include motion along curved paths, such as a ball kicked by a football player or the orbit of planets.

Projectile Motion

Projectile motion refers to the motion of an object that is thrown, fired, or otherwise launched into the air and moves only under the influence of gravity. Examples of projectile motion include:

- A ball kicked by a football player.
- A cannonball fired from a cannon.
- A bullet fired from a gun.
- A jet of water from a hose.

When analyzing projectile motion, we make the following assumptions:

1. **Constant Acceleration**: The acceleration due to gravity, g=9.8 m/s^2, is constant and always directed downward.
2. **Negligible Air Resistance**: We ignore the effects of air resistance.
3. **Independent Components**: The horizontal and vertical motions are independent of each other.

Under these assumptions, the path of a projectile, called its **trajectory**, is a parabola.

Horizontal Projection

In horizontal projection, the object is launched horizontally from a certain height. Initially, it has a horizontal velocity but no vertical velocity. As time progresses, gravity causes the object to gain vertical velocity, while its horizontal velocity remains constant.

Horizontal Component of Motion:

- The projectile has no acceleration in the horizontal direction, so its horizontal velocity v0remains constant:
- $vx=v_0x$ (constant)
- The horizontal distance traveled by the projectile is:
- $\Delta x=v_0x\cdot t$

Vertical Component of Motion:

- The vertical motion is a constant accelerated motion due to gravity. The vertical velocity at any time t is:
- $vy=v_0y+g\cdot t$
- Since the initial vertical velocity $v_0y=0$, this simplifies to:
- $vy=g\cdot t$
- The vertical displacement is:

- $\Delta y=1/2gt^2$

Time of Flight and Range:

- **Time of Flight**: The time taken for the projectile to hit the ground is calculated using the vertical motion equation:
- $t=\sqrt{(2\Delta y/g)}$
- **Range**: The maximum horizontal distance traveled by the projectile is:
- $R=v0x\cdot t$

Example Problems

Example 1: A rifle is aimed horizontally at a target 30 m away. The bullet hits the target 2 cm below the aiming point. Find:

(a) The bullet's time of flight.

(b) The initial velocity of the bullet.

Solution: Given: $\Delta x=30$ m, $\Delta y=-0.02$ m, $g=10$ m/s^2 g = 10.

(a) Vertical displacement equation:

$\Delta y=1/2gt^2$

$-0.02=1/2(-10)t^2$

$t=0.06$ s

(b) Horizontal motion equation: $\Delta x=v_0x\cdot t$

$v_0x=\Delta x/t=30$ m$/0.06$ s$=500$ m/s

Inclined Projectile Motion

In inclined projectile motion, the object is launched at an angle θ with an initial velocity v_0. The initial velocity can be resolved into horizontal and vertical components:

- Horizontal: $v_0x=v_0\cos\theta$
- Vertical: $v_0y=v_0\sin\theta$

The motion can be analyzed using similar principles as horizontal projection, with the horizontal velocity remaining constant and the vertical velocity affected by gravity.

Horizontal Range and Maximum Height of a Projectile

In projectile motion, the **horizontal range** (R) and the **maximum height** (H) of the projectile are two key aspects that describe the motion. Let's analyze these terms by considering a projectile launched from the origin and landing back at the same horizontal level.

Range (R)

The range of a projectile is the maximum horizontal distance it covers. There is no horizontal acceleration since gravity acts only vertically. The formula for the horizontal displacement Δx is:

$\Delta x=v_0\cos\theta\times t$

Where:

- v_0 is the initial velocity,
- θ is the angle of projection,
- t is the time of flight.

To find the range, we substitute the time of flight, t_total, into the equation:

$t\_total = 2v_0 \sin\theta / g$

Thus, the range R is given by:

$R = v_0 2 \sin 2\theta / g$

This equation shows that the range is directly proportional to the square of the initial speed v_0 and $\sin 2\theta$. The range is maximized when the angle of projection θ is 45°.

Maximum Height (H)

The maximum height is the highest point the projectile reaches during its flight. At this point, the vertical component of the velocity (vy) becomes zero. The vertical displacement Δy at maximum height is:

$\Delta y = v_0 \sin\theta \times t - (1/2) g t^2$

The time to reach the maximum height t_max is:

$t\_max = v_0 \sin\theta / g$

Substituting t_max into the equation for vertical displacement gives the maximum height H:

$H = V_0 2 \sin^2\theta / 2g$

Relation Between Range and Maximum Height

The relationship between the range RRR and the maximum height HHH can be derived by dividing the equation for HHH by the equation for RRR:

$H/R = \sin\theta / 4 \cos\theta = R \tan\theta / 4$

This shows that for a given angle of projection, the maximum height is directly related to the range.

Examples

Example 1:

A football player kicks a ball at a 37° angle with an initial velocity of 40 m/s.

- **a)** Find the maximum height.
- **b)** Find the horizontal range.

Solution: Given $v_0=40$ m/s, $\theta=37^\circ$, and $g=10$ m/s^2.

- **a)** Maximum height:

$$H=\frac{(40\ \text{m/s})^2\times\sin237^\circ}{2\times10\ \text{m/s}^2}=28.8\ \text{m}$$

- **b)** Horizontal range:

$$R=\frac{(40\ \text{m/s})^2\times\sin74^\circ}{10\ \text{m/s}^2}=153.8\ \text{m}$$

Rotational Motion

Rotational motion occurs when an object moves in a circular path around a fixed axis. Examples include the Earth's rotation and the spinning of a ceiling fan.

Key concepts include **angular displacement**, **angular velocity**, and **angular acceleration**:

- **Angular displacement** is the angle through which an object moves on a circular path.
- **Angular velocity** (ω) is the rate of change of angular displacement, measured in radians per second (rad/s).
- **Angular acceleration** (α) is the rate of change of angular velocity, measured in radians per second squared (rad/s^2).

Direction of Angular Velocity and Angular Acceleration

Angular velocity (ω) and angular acceleration (α) are vector quantities, which means they have both magnitude and direction. For objects rotating around a fixed axis, the direction of these vectors is along the axis of rotation.

Right-Hand Rule

To determine the direction of the angular velocity vector (ω)(\omega)(ω), we use the **right-hand rule**:

- Imagine wrapping the fingers of your right hand around the direction of rotation (the way the object is rotating).
- Your thumb, when extended, points in the direction of the angular velocity vector (ω).

For angular acceleration (a), its direction depends on how the angular speed is changing:

- If the angular speed is **increasing**, a points in the same direction as ω.
- If the angular speed is **decreasing**, a points in the opposite direction (antiparallel) to ω.

Equations of Motion for Constant Angular Acceleration

When an object rotates with constant angular acceleration, its motion can be described by a set of kinematic equations that are analogous to those used for linear motion. These equations help us calculate various aspects of rotational motion, such as angular velocity, angular displacement, and angular acceleration.

1. **Angular Velocity**

   $\omega f=\omega_0+a\Delta t$

   Where:

   - ωf is the final angular velocity.
   - ω_0 is the initial angular velocity.
   - A is the angular acceleration.
   - Δt is the time interval.
2. **Angular Displacement**

   $\Delta\theta=\omega_0\Delta t+(1/2)a\Delta t^2$

   Where:

   - $\Delta\theta$ is the angular displacement.

- ω_0 is the initial angular velocity.
- a is the angular acceleration.
- Δt is the time interval.

3. **Relationship Between Angular Velocity and Displacement**

   $\Omega f^2=\omega_0^2+2a\Delta\theta$

   This equation relates the final angular velocity to the initial angular velocity, angular acceleration, and angular displacement.

Example Problems

Example 1: Calculate the average angular velocity of a rotating wheel if its angular speed changes from 30 rad/s to 50 rad/s in 2 seconds.

Solution:

Given: $\omega i=30$ rad/s, $\omega f=50$ rad/s, $\Delta t=2$ s
$a=(\omega f-\omega i)/\Delta t$
$=(50 \text{ rad/s}-30 \text{ rad/s})/2 \text{ s}$
$=10 \text{ rad/s}^2$

Relationship Between Angular and Linear Motion

Angular motion has a direct relationship with linear motion:

- **Tangential Speed** (v) at a point on the rotating object is given by:
- $v=\omega r$
- Where r is the radius of the circular path.
- **Tangential Acceleration** (a) is given by:
- $a=ar$

These relationships allow us to connect the rotational motion of an object to the linear motion of points on that object, providing a complete picture of how objects move when rotating.

This summary provides a foundational understanding of angular velocity and acceleration, along with their equations of motion and their connection to linear motion.

Rotational Dynamics

Rotational dynamics deals with the forces and torques that cause objects to rotate. It is the rotational equivalent of linear dynamics, where torque plays a role similar to force in linear motion.

Torque

Torque (τ\tauτ) is the rotational equivalent of force. It is what causes an object to rotate. The torque is calculated using the formula:

$\tau = rF\sin\theta$

Where:

- r is the distance from the axis of rotation to the point of force application.
- F is the force applied.
- θ the angle between the force vector and the position vector.

Torque is a vector quantity, meaning it has both magnitude and direction. The direction of the torque is determined by the right-hand rule: if you curl the fingers of your right hand in the direction of rotation, your thumb points in the direction of the torque. The SI unit of torque is Newton-meter (Nm).

Example: Consider three forces acting on an object at different distances from a pivot point, with different angles. The net torque can be found by calculating the individual torques and then summing them up.

Moment of Inertia (I)

The moment of inertia (III) is the rotational equivalent of mass in linear motion. It represents an object's resistance to changes in its rotational motion. The moment of inertia depends on the mass distribution relative to the axis of rotation. For a point mass, it is given by:

$I = mr^2$

Where:

- m is the mass of the object.
- r is the distance from the axis of rotation.

For multiple particles, the total moment of inertia is the sum of the individual moments of inertia.

Example: If three particles are connected by rods along the y-axis and rotate about the x-axis, the moment of inertia about the x-axis can be calculated by summing the contributions from each particle.

Torque and Angular Acceleration

The net torque acting on an object is related to its angular acceleration (α\alphaα) through the equation:

$τ_{net}=Iα$

This is analogous to Newton's second law of motion (F=ma) in linear dynamics, where mass (m) is replaced by the moment of inertia (I) and linear acceleration (aaa) is replaced by angular acceleration (α).

Example: If a wheel experiences a torque of 36 Nm and has an angular acceleration of 24 rad/s^2, its moment of inertia can be calculated using the equation above.

Planetary Motion and Kepler's Laws

Kepler's laws describe the motion of planets around the Sun:

1. **First Law (Law of Ellipses):** Planets orbit the Sun in elliptical paths, with the Sun at one focus of the ellipse.
2. **Second Law (Law of Equal Areas):** A line drawn from the Sun to a planet sweeps out equal areas in equal time intervals. This implies that a planet moves faster when it is closer to the Sun (at perihelion) and slower when it is farther from the Sun (at aphelion).
3. **Third Law (Law of Harmonies):** The square of the orbital period (T) of a planet is proportional to the cube of its average distance (R) from the Sun:

$T^2/R^3=K$

Where K is a constant for all planets.

Example: Using Kepler's third law, if you know the orbital period and distance of one planet from the Sun, you can calculate the orbital period of another planet at a different distance.

Newton's Law of Universal Gravitation

Newton’s law of universal gravitation states that every two masses in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

$F=Gm1m2/r^2$

Where:

- F is the gravitational force.
- G is the gravitational constant (6.67×10^{-11} Nm^2/kg^2).
- m1and m2 are the masses of the two objects.
- r is the distance between the centers of the two masses.

Example: If you have two masses, say 10 kg and 100 kg, separated by a distance of 1 meter, you can calculate the gravitational force between them using the formula above.

Centripetal Force and Kepler’s Third Law

Centripetal force is the force that keeps an object moving in a circular path, directed towards the center of the circle. For planets orbiting the Sun, the gravitational force provides this centripetal force:

$Fc=(mv^2)/r$

Where:

- m is the mass of the planet.
- v is the orbital speed.
- r is the radius of the orbit.

Using Newton's law of gravitation and centripetal force, you can derive Kepler’s third law. The relationship between the orbital period and distance from the Sun can be expressed, leading to the conclusion that the square of the period is proportional to the cube of the distance.

Example: If you know the distance of a planet from the Sun, you can calculate its orbital period using Kepler’s third law.