# Unit 4
## Dynamics

## Introduction to Dynamics and Newton's Laws of Motion

### 1. Introduction to Dynamics

In Physics, motion is a fundamental concept. Whether it's the running of athletes, the flight of birds, or the trajectory of a soccer ball, motion surrounds us. Dynamics is the branch of Physics that studies motion and its causes—forces. This unit will explore how forces affect the motion of objects and how they can alter the speed, direction, or shape of objects.

### 2. The Concept of Force

- **Definition:** Force is a push or pull that can change an object's motion or shape. It is a vector quantity, meaning it has both magnitude and direction. The SI unit of force is the Newton (N), where 1 N = 1 kg·m/s².

  **Examples:**

  - Pushing a table or kicking a ball involves applying force.
  - Deforming a tennis ball by hitting it shows the effect of force on shape.
- **Types of Forces:**
  - **Contact Forces:** Occur when objects touch each other (e.g., pulling a box, kicking a ball).
  - **Field Forces:** Act over a distance without physical contact (e.g., gravitational force, magnetic force).

  The four fundamental forces in nature are:

  3. **Strong Nuclear Force:** Holds the nucleus of an atom together.
  4. **Electromagnetic Force:** Acts between charged particles.
  5. **Weak Nuclear Force:** Responsible for radioactive decay.
  6. **Gravitational Force:** Attracts objects towards each other.

### 3. Newton's Laws of Motion

#### Newton's First Law of Motion (Law of Inertia)

- **Statement:** An object will remain at rest or continue to move in a straight line at constant velocity unless acted upon by a net external force. This is known as inertia.

  **Examples:**

  - A book on a desk stays at rest until pushed.
  - A moving puck slows down due to friction, but in a frictionless environment, it would keep moving indefinitely.

**Applications:**

- **Seat Belts:** According to Newton's First Law, if a car suddenly stops, passengers continue moving at the car's speed. Seat belts keep passengers in place, preventing injuries.

**Newton's Second Law of Motion**

- **Statement:** The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. Mathematically, F=ma, where F is the net force, m is mass, and aaa is acceleration.

  **Observations:**

  - Increasing the force on an object increases its acceleration.
  - Increasing the mass of an object decreases its acceleration for the same force.

**Examples:**

- Pushing a car with greater force makes it accelerate more.
- A heavier object accelerates less compared to a lighter object when the same force is applied.

**4. Frame of Reference**

- **Inertial Frame of Reference:** A frame that is either at rest or moves with constant velocity. Newton's laws are applicable here (e.g., a car traveling at a constant speed).
- **Non-inertial Frame of Reference:** A frame that is accelerating. Newton's laws do not apply directly here (e.g., a car accelerating or decelerating).

**5. Summary**

In summary, dynamics is the study of how forces cause motion. Newton's laws describe the relationship between forces and motion:

- The First Law explains the concept of inertia.
- The Second Law quantifies the effect of forces on motion.

Understanding these principles helps explain and predict how objects move under the influence of forces, both in everyday life and in complex systems.

## Newton's Laws of Motion and Friction:

***Newton's Third Law of Motion***

**Concept:** Newton's Third Law states that for every action, there is an equal and opposite reaction. This means if one object exerts a force on a second object, the second object exerts a force of equal magnitude but in the opposite direction on the first object.

**Examples:**

- **Hammer and Nail:** When you hit a nail with a hammer, the nail pushes back on the hammer with the same force.
- **Rocket Propulsion:** A rocket moves upward by expelling gases downward.

***Weight and Gravitational Force***

**Weight Definition:** Weight is the force due to gravity acting on an object. It is given by: Weight=mass×acceleration due to gravity
$W=mg$ On Earth, $g \approx 9.8\ m/s^2$.

**Example:**

- A block with a mass of 6 kg has a weight of: $W=6\times9.8=58.8\ N$

**Free Fall:** An object in free fall accelerates downward due to gravity alone.

***Normal Force***

**Definition:** The normal force is the force exerted by a surface to support the weight of an object resting on it. It acts perpendicular to the surface.

**On a Flat Surface:** The normal force equals the weight of the object.

**On an Inclined Plane:** The normal force is reduced by the cosine of the incline angle: $F_N=W\cos\theta$ where θ is the angle of the incline.

**Example:**

- For a 6 kg block on a 37° incline, the normal force is:

  $F_N=58.8\cos37^\circ\approx47$ N

***Friction***

**Definition:** Friction opposes the motion between two surfaces in contact.

**Types:**

- **Static Friction:** Prevents motion between surfaces at rest relative to each other.
- **Kinetic Friction:** Occurs when surfaces are sliding past each other.

**Coefficients of Friction:**

- **Static Friction Coefficient ($\mu_s$):** Maximum value when an object is about to move.
- **Kinetic Friction Coefficient ($\mu_k$):** Value when an object is moving.

**Formulas:**

- **Static Friction:** $F_s\leq\mu_s F_N$
- **Kinetic Friction:** $F_k=\mu_k F_N$

**Example Problem:**

- A 6 kg block on a 30° incline remains at rest. To find:
  - **Static Friction ($F_s$):** Use $F_s=mg\sin\theta$ where $\theta=30^\circ$.
  - **Coefficient of Static Friction ($\mu_s$):** Use $\mu_s=\frac{mg\sin\theta}{mg\cos\theta}$, which simplifies to $\mu_s=\tan\theta$.

**Solution:** For a block with mass 6 kg on a 30° incline:

- Weight: $W=6\times9.8=58.8$ N
- Normal Force: $F_N=58.8\cos30^\circ=58.8\times0.866=50.9$N
- Static Friction: $F_s=mg\sin30^\circ=58.8\times0.5=29.4$ N
- Coefficient of Static Friction: $\mu_s=\frac{29.4}{50.9}\approx0.58$

*Free-Body Diagrams*

**Purpose:** Free-body diagrams represent all forces acting on an object. They help in applying Newton's laws to solve problems by visualizing the forces and their components.

**Steps to Draw:**

1. Draw the object.
2. Indicate all forces acting on it.
3. Resolve forces into x and y components if necessary.

**Example:**

- For a block on an inclined plane, show forces like weight, normal force, and friction. Resolve weight into components parallel and perpendicular to the incline.

***Example Problems:***

1. **System of Masses:**
   - For blocks with masses $m_1$=4 kg and $m_2$=8 kg, and force F=36 N:
   - Acceleration aaa is calculated using: $a=\frac{F}{m_1+m_2}=\frac{36}{4+8}=3\ m/s^2$
   - Tension $T=m_2\times a=8\times 3=24$ N
2. **With Kinetic Friction:**
   - If $\mu_k$=0.25 for the same blocks:

   Calculate frictional forces and use them to find acceleration: $a=\frac{F-(F_{f1}+F_{f2})}{m_1+m_2}$

3. **Atwood's Machine:**
   - For masses $m_1$=4 kg and $m_2$=6 kg:
   - Acceleration a: $a=\frac{g(m_2-m_1)}{m_1+m_2}=1.96 m/s^2$
   - Tension: $T=m_1(g+a)=4(9.8+1.96)=47$ N.

**Work Done by the Gravitational Force**

## Introduction

The work done by a force is a measure of the energy transferred when the force acts on an object. When dealing with gravitational forces, it's important to understand how this force affects the energy of an object as it moves.

## Work Done by Gravitational Force

**1. Work Done During an Object's Ascent:**

When an object of mass mmm is thrown upward with an initial speed v0v_0v0 , it moves against the gravitational force FgF_gFg . The gravitational force FgF_gFg is given by mgmgmg, where ggg is the acceleration due to gravity. As the object rises, the gravitational force does negative work on it because it opposes the object's motion.

- **Formula:**

$W=-mgd$, where d is the vertical distance.

**2. Work Done During an Object's Descent:**

When the object falls back down, the gravitational force does positive work as it aids the object's motion.

- **Formula:** $W_g=mgd$.

## Example

**Example 4.12:**

A 5 kg bag is raised to a height of 2.5 m above the ground.

- **Work Done by the Applied Force:**

  $W_f=mgh=(5\ \text{kg})\times(9.8\ \text{m/s}^2)\times(2.5\ \text{m})=122.5\ \text{J}$

- **Work Done by Gravitational Force:**

  $W_g=-mgh=-122.5\ \text{J}$

If the bag is lowered from the same height, the work done by the gravitational force becomes positive.

## Work Done by a Variable Force

When a force varies with displacement, you can't use the simple formula for work. Instead, you calculate the infinitesimal work done and integrate over the displacement.

- **Graphical Method:** The work done by a variable force can be found by calculating the area under the force vs. displacement graph.

## Kinetic Energy

Kinetic energy (KE) is the energy an object possesses due to its motion.

- **Formula:** $KE=\frac{1}{2}mv^2$

## Work-Energy Theorem

The work-energy theorem states that the total work done on an object is equal to the change in its kinetic energy.

- **Formula:** $W=\Delta KE=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2$.

## Potential Energy

- **Gravitational Potential Energy (PE):**

  $PE=mgh$

- **Elastic Potential Energy (for springs):**

  $PE=\frac{1}{2}kx^2$

## Conservative and Non-Conservative Forces

- **Conservative Forces:** The work done is path-independent and zero for a closed path (e.g., gravitational force).
- **Non-Conservative Forces:** The work done depends on the path and causes energy dissipation (e.g., friction).

## Power

Power measures how quickly work is done.

- **Average Power:**

  $P_{avg}=\frac{W}{t}$.

- **Instantaneous Power:**

$P = F \cdot v$

Where F is the force and v is the instantaneous velocity.

**Units:**

- 1 watt (W) = 1 joule per second (J/s)

## Conservation of Mechanical Energy

**Concept:**

The law of conservation of mechanical energy states that in the absence of dissipative forces (like friction and air resistance), the total mechanical energy of a system remains constant. This means the sum of kinetic energy (KE) and potential energy (PE) does not change.

**Mathematical Formulation:**

$\Delta ME = 0$

$\Delta KE + \Delta PE = 0$

$KE_i + PE_i = KE_f + PE_f$

Where:

- $KE_i$ = Initial kinetic energy
- $PE_i$ = Initial potential energy
- $KE_f$ = Final kinetic energy
- $PE_f$ = Final potential energy

**Example 1: Block on an Inclined Plane**

A 10 kg block is released from rest at the top of a smooth inclined plane 10 m in length.

1. **Find the speed of the block at the bottom:**

   **Solution:**

   - Initial kinetic energy $KE_i = 0$ (since the block starts from rest)
   - Initial potential energy $PE_i = mgh$
   - Final kinetic energy $KE_f = \frac{1}{2}mv^2$

- Final potential energy $PE_f=0$ (at the bottom of the incline)

Applying conservation of energy:

$KE_i+PE_i=KE_f+PE_f$

$0+mgh=\frac{1}{2}mv^2+0$

$mgh=\frac{1}{2}mv^2$

Given:

- $m=10$ kg
- $g=9.8$ m/s$^2$
- $h=5$ m (since $h=l\sin\theta$ and $\sin30^\circ=0.5$)

Substitute these values:

$v=\sqrt{2gh}$

$v=\sqrt{2\times 9.8\ \mathrm{m/s^2}\times 5\ \mathrm{m}}\approx 9.9$ m/s

**Key Points:**

- **Mechanical Energy Conservation**: Total mechanical energy (KE + PE) remains constant without dissipative forces.
- **Friction and Other Forces**: These forces can convert mechanical energy into other forms (like heat), and must be accounted for in real scenarios.
- **Calculations**: Always check if forces like friction are present and adjust your calculations accordingly.

This note covers how to apply the conservation of mechanical energy and includes examples to illustrate how to solve related problems.

## Law of Conservation of Linear Momentum

**Definition:** The Law of Conservation of Linear Momentum states that if two bodies interact (e.g., collide or push each other apart) and no external forces are acting on them, the total momentum of the system remains constant. This means the total momentum before the interaction is equal to the total momentum after the interaction.

**Mathematical Expression:** $m_1v_{1i}+m_2v_{2i}=m1v_{1f}+m2v_{2f}$

where:

- $m_1$ and $m_2$ are the masses of the two bodies.
- $v_{1i}$ and $v_{2i}$ are the initial velocities of the two bodies.
- $v_{1f}$ and $v_{2f}$ are the final velocities of the two bodies.

**Impulse and Momentum Change:** Impulse is the change in momentum resulting from a force applied over a time interval Δt. The impulse experienced by each car during the collision is equal and opposite, which follows from Newton's third law. Thus: $\Delta p_1=F_{12}\Delta t$

where:

- $\Delta p_1$ and $\Delta p_2$ are the changes in momentum of Car 1 and Car 2, respectively.
- $F_{12}$ is the force exerted by Car 2 on Car 1 (and vice versa).

**Collisions:**

- **Elastic Collision:** Both momentum and kinetic energy are conserved. Examples include collisions between atomic particles or billiard balls.
- **Inelastic Collision:** Momentum is conserved, but kinetic energy is not. If the objects stick together after the collision, it's perfectly inelastic.

**Elastic Collision Example:** Two masses m1m_1m1 and m2m_2m2 collide elastically. The conservation equations are:

1. Momentum: $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$
2. Kinetic Energy: $\frac{1}{2}m_1v_{1i}+\frac{1}{2}m_2v_{2i}=\frac{1}{2}m_1v_{1f}+\frac{1}{2}m_2v_{2f}$

**Perfectly Inelastic Collision Example:** When two objects stick together after collision: $m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f$

**Center of Mass:** The center of mass of a system is the point where the total mass of the system can be considered to be concentrated. It moves as though all the system's mass were located there and all external forces were applied at that point.

**Center of Mass Equations:** For a system of particles:

- In the x-direction: $x_{cm}=\frac{m_1x_1+m_2x_2+\ldots+m_nx_n}{m_1+m_2+\ldots+m_n}$
- In the y-direction: $y_{cm}=\frac{m_1y_1+m_2y_2+\ldots+m_ny_n}{m_1+m_2+\ldots+m_n}$

**Rocket Propulsion:** In a rocket, the conservation of momentum explains why it moves. The rocket ejects gas backward, which propels it forward. For a rocket and gas system:

$m_{rocket} \cdot v_{rocket} = -m_{gas} \cdot v_{gas}$

By understanding these concepts, you can analyze and solve problems involving collisions, momentum conservation, and the motion of systems.