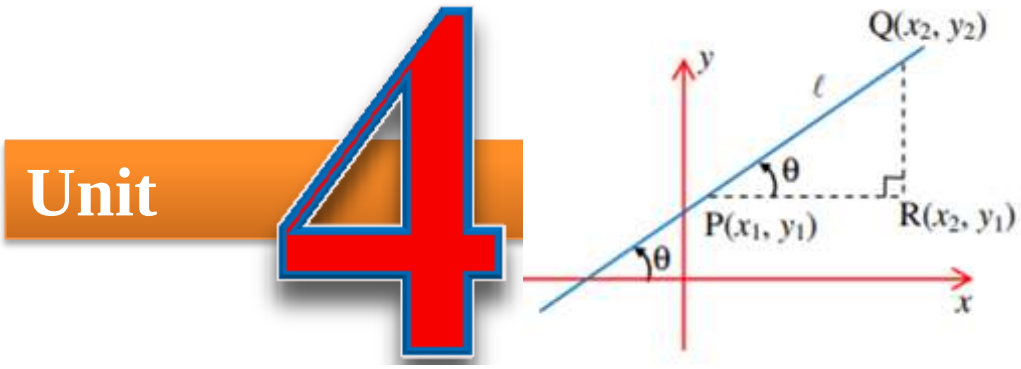


COORDINATE GEOMETRY

Unit Outcomes:

After completing this unit, you should be able to:

- *apply the distance formula to find the distance between any two given points in the coordinate plane.*
- *formulate and apply the section formula to find a point that divides a given line segment in a given ratio.*
- *write different forms of equations of a line and understand related terms.*
- *describe parallel or perpendicular lines in terms of their slopes.*

Main Contents: (Practice on Questions at the end)

4.1. Division of a line segment

4.1. DIVISION OF A LINE SEGMENT

The point $R(x_0, y_0)$ dividing the line segment PQ internally in the ratio $m:n$ is given by
$R(x_0, y_0) = \left(\frac{nx_1+mx_2}{n+m}, \frac{ny_1+my_2}{n+m}\right)$.
This is called the **section formula**.

Example 1: Find the coordinate of a point R that divides the line segments with end-points $A(-3,3)\ and\ B(12,-7)$ in the ratio$2:3$.

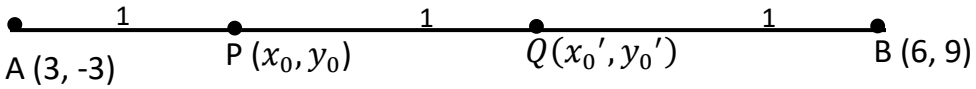
Solution: Put$(x_1, y_1) = (-3,3), (x_2, y_2) = (12,-7), m = 2\ and\ n = 3$, using the section formula ,you have:

$R(x_0, y_0) = \left(\frac{nx_1+mx_2}{n+m}, \frac{ny_1+my_2}{n+m}\right) = \left(\frac{3\times-3+2\times12}{3+2}, \frac{3\times3+2\times-7}{3+2}\right) = \left(\frac{-9+24}{5}, \frac{9-14}{5}\right) = (3,-1)$

Therefore,$R\ is\ (3,-1)$

Example 2: A line segment has end- points $(3,-3)$ and $(6,9)$ and it is divided into three equal parts. Find the coordinate of the points that trisect the segment.

Solution: Let $P(x_0, y_o)\ and\ Q(x_0', y_0')$ be points which trisect the line segment joining the points $(3,-3)\ and\ (6,9)$



The first point $P(x_0, y_0)$ divides the line segment in the ratio $1:2$ and hence

$(x_1, y_1) = (3,-3), (x_2, y_2) = (6,9), m = 1\ and\ n = 2$

$P(x_0, y_0) = \left(\frac{nx_1+mx_2}{n+m}, \frac{ny_1+my_2}{n+m}\right) = \left(\frac{2\times3+1\times6}{2+1}, \frac{2\times-3+1\times9}{2+1}\right) = \left(\frac{12}{3}, \frac{3}{3}\right) = (4,1)$

Therefore the first point $P(x_0, y_0) = (4,1)$

The second point $Q(x_0', y_0')$ divides the line segment in the ratio$2:1$.

Thus

$(x_1, y_1) = (3,-3), (x_2, y_2) = (6,9), m = 2\ and\ n = 1$

$Q(x_0', y_0') = \left(\frac{nx_1+mx_2}{n+m}, \frac{ny_1+my_2}{n+m}\right) = \left(\frac{1\times3+2\times6}{2+1}, \frac{1\times-3+2\times9}{2+1}\right) = \left(\frac{15}{3}, \frac{15}{3}\right) = (5,5)$

Therefore the second point $Q(x_0', y_0') = (5,5)$

PRACTICE QUESTIONS ON UNIT 4

CHOOSE THE BEST ANSWER FROM THE GIVEN ALTERNATIVES

1. If line l_1 passes through the points $(5,x)$ and $(-1,3)$ and line l_2 contains the points $(x,6)$ and $(2,0)$, then the value of x for which the two lines are perpendicular is:

   A. $\frac{2}{5}$ B. $\frac{5}{2}$ C. 5 D. $\frac{1}{2}$

2. If a line passes through $(2,8)$ and $(-5,15)$, then what is the degree measure of the angle of inclination that this line makes with positive $x-$axis?

   A. 30^0 B. 45^0 C. 135^0 D. 225^0

3. If the line passing through points $(2,8)$ and $(-7,t+4)$ is parallel to the line passing through points $(1,t)$ and $(4,-2)$, then what is the value of t?

   A. $-\frac{1}{2}$ B. $\frac{5}{2}$ C. -5 D. 1

4. Which one of the following is the equation of a line that is perpendicular to the line with equation $2x+3y+4=0$?

   A. $3y-2x+4=0$ C. $3x-2y+4=0$

   B. $-3x-2y-4=0$ D. $2x+3y-4=0$

5. What are the co-ordinates of appoint that divides the line segment joining points A (2, 3) and B (5, -7) in the ratio 3:4?

   A. $\left(\frac{23}{7},\frac{9}{7}\right)$ B. $\left(\frac{2}{7},-\frac{9}{7}\right)$ C. $\left(-\frac{23}{7},-\frac{2}{7}\right)$ D. $\left(\frac{23}{7},-\frac{9}{7}\right)$

6. If a line with x-intercept 4 and y-intercept -6 is given, then its slope is equal to_____.

   A. $-\frac{2}{3}$ B. $-\frac{3}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

7. The distance between P (2,3) and Q (1, -1) is:

   A. 17 units B. 16 units C. $\sqrt{17}$ units D. 9 units

8. Which one of the following pairs of equations represents perpendicular lines?

   A. $x+y=0$ and $-x+y=1$ C. $x+y=1$ and $y-2x=2$

   B. $2x+y=1$ and $-2x-y=1$ D. $3x-2y=0$ and $3x+2y=2$

9. Which one of the following lines is parallel to the line $5x-2y=0$?

   A. $y=-\frac{5}{2}$ C. $-5x-2y=1$

   B. $2x+5y=-4$ D. $-5x+2y=6$

10. Which one of the following is **true** about a second quadrant angle θ in standard position whose terminal side lies on the line $2x+y=0$?

    A. $\sin\theta=\frac{1}{\sqrt{5}}$ B. $\cos\theta=\frac{2}{\sqrt{5}}$ C. $\sin\theta=\frac{2}{\sqrt{5}}$ D. $\cos\theta=\frac{1}{\sqrt{5}}$

11. The equation of the line that passes through the point (2, -1) and perpendicular to the line $2x+4y+3=0$ is given by:
    A. $x+2y=0$ C. $x-2y+2=0$
    B. $-2x+y-4=0$ D. $-2x+y+5=0$
12. Which one of the following is the equation of a line whose x-intercept and y-intercept are 1 and -2 respectively?
    A. $-2x+y-1=0$ B. $x-y+1=0$ C. $2x-y-2=0$ D. $x-2y-1=0$
13. What is the equation of the line passing through mid-point of the line segment with end points (-1, 3) and (3, 1), and perpendicular to the line whose angle of inclination is double the angle of inclination of the line $2x+y=7$?
    A. $3y+4x=10$ C. $2y-x=3$
    B. $4y-5x=3$ D. $4y+3x=11$
14. What is the coordinate of the point R on the line segment with end points P(-3, 0) and Q(0, -3) such that $\frac{PR}{RQ}=\frac{2}{3}$?
    A. $(-1,-1)$ B. $\left(-1,\frac{7}{5}\right)$ C. $\left(-\frac{9}{5},-\frac{6}{5}\right)$ D. $\left(-\frac{7}{5},0\right)$
15. Let (a,b) and (b,a) be points such that $a \neq b$. Which of the following is the equation of the line through (a,b) and perpendicular to the line containing the given points?
    A. $y=x+b-a$ C. $y=-x+b+a$
    B. $y=x-a-b$ D. $y=-x-a+b$
16. The equation straight line passing through P (-2, 1) and perpendicular to the line with equation $6x+5y=10$ is:
    A. $6y+5x+16=0$ C. $y=5x-10$
    B. $6y-5x+16=0$ D. $6y-5x-16=0$
17. If P (2, -1) and Q (-3, 5) are points on the coordinates plane, so that, P is the midpoint of $\overline{QR}$, then what are the coordinates of R?
    A. (7, 7) B. (7, -7) C. (6, 7) D. (10, 11)
18. Line l passes through $(0, 5)$ and $(-5, 0)$. What is the angle between the $y-axis$ and l in radian measure?
    A. $\frac{\pi}{4}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{3}{4}\pi$
19. Suppose l_1 and l_2 are perpendicular lines intersecting at $(2, 1)$. If the angle of inclination of l_2 is 45^0, what is the equation of l_1?
    A. $y=-x+2$ B. $y=-x+3$ C. $y=x-1$ D. $y=-2x+5$
20. If a line with angle of inclination of $\frac{3}{4}\pi$ passes through $(0, 1)$, which one of the following is the equation of the line?
    A. $y=-x+1$ B. $y=x+1$ C. $y=-x-1$ D. $y=x-1$