Unit 3

motion in one and two dimensions

Introduction to Acceleration

Acceleration is a key concept in physics that describes how the velocity of an object changes over time. Whether it's a car speeding up, a bike slowing down, or an object changing direction, acceleration is involved. Understanding this concept is crucial in analyzing motion in both everyday life and scientific contexts.

Brainstorming: Understanding Acceleration

1. **What is Acceleration?**
   - Acceleration occurs when there is a change in the velocity of an object. This change can be in:
     - The magnitude (speed) of the velocity.
     - The direction of the velocity.
     - Both the magnitude and direction.
2. **Why is Acceleration a Vector?**
   - Since velocity is a vector (having both magnitude and direction), acceleration is also a vector. This means it not only has a size (how much the velocity changes) but also a direction (in which the change occurs).
3. **Units of Acceleration**
   - The standard unit of acceleration is meters per second squared (**m/s^2**). This unit tells us how much the velocity changes per second.

Acceleration Explained

- **Positive vs. Negative Acceleration**:
  - When the velocity and acceleration have the same sign (both positive or both negative), the object speeds up.
  - When the velocity and acceleration have opposite signs, the object slows down. This is sometimes called deceleration, but not all negative accelerations are decelerations—it depends on the direction.

Average Acceleration

- **Definition**: The average acceleration is the total change in velocity divided by the total time taken.
- **Formula**: $a_{av}=\frac{vf-vi}{\Delta t}$ where:
  - v_f is the final velocity.
  - v_i is the initial velocity.
  - Δt is the time interval.

Instantaneous Acceleration

- **Definition**: Instantaneous acceleration is the acceleration of an object at a specific moment in time.
- **Calculation**: It's calculated by taking the limit of the average acceleration as the time interval approaches zero: $a_{ins}=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}$

Example Problem

Problem: A particle moves along a straight line with a velocity given by $v(t)=t^2-2t$. Find the acceleration at $t=2$ seconds.

Solution:

1. Differentiate the velocity equation $v(t)=t^2-2t$ with respect to time to find the acceleration.
2. Acceleration a(t) is given by: $a(t)=\frac{d}{dt}$ (t2−2t)=2t−2a(t) = \frac{d}{dt}(t^2 - 2t) = 2t - 2a(t)=dtd (t2−2t)=2t−2
3. At t=2t = 2t=2 seconds: $a(2)=2(2)-2=4-2=2$ m/s^2 So, the acceleration at t=2 seconds is 2 m/s^2.

Motion with Constant Acceleration

When an object has constant acceleration:

- The velocity changes by the same amount in each time interval.
- This is known as uniformly accelerated motion.
- **Equations of Motion**:
  1. **Velocity-Time Relation**: $v_f=v_i+at$
  2. **Displacement-Time Relation**: $s=v_it+\frac{1}{2}at^2$
  3. **Velocity-Displacement Relation**: $v_f^2=v_i^2+2as$

These equations apply only when the acceleration is constant.

Practical Example

Example: A car accelerates uniformly from rest to 20 m/s in 8 seconds.

- **Find the acceleration**: $a=\frac{vf-vi}{t}=\frac{20\,m/s-0}{8\,s}=2.5\ m/s^2$
- **Find the distance traveled**: $s=v_i t+\frac{1}{2}at^2=0+\frac{1}{2}(2.5\ m/s2)(8\ s)^2=80\ m$

Misconception Alert!

Not all negative accelerations are decelerations. Negative acceleration can also indicate a change in direction depending on the reference point. Always consider the context when analyzing motion.

Key Takeaways

- Acceleration is a measure of how quickly velocity changes.
- It can be positive, negative, or zero.
- The equations of motion for constant acceleration are crucial for solving problems related to motion.

Graphical Representation of Uniformly Accelerated Motion in 1D

Understanding motion in one dimension can be greatly enhanced by using graphs. These graphs help visualize how different quantities like displacement, velocity, and acceleration change over time. Let's break down each type of graph:

1. Displacement-Time Graphs

Displacement-Time Graph Overview:

- A displacement-time graph shows how an object's position changes with time.
- **Displacement** (how far an object has moved) is plotted on the Y-axis (vertical axis).
- **Time** is plotted on the X-axis (horizontal axis).

Slope of the Displacement-Time Graph:

- The slope of this graph tells you the **velocity** of the object.
- **Slope = rise/run = change in displacement/change in time = velocity (v).**

Types of Displacement-Time Graphs:

- **Uniform Motion (Constant Velocity):**
  - The graph is a straight line with a constant slope, indicating the object moves at a constant speed.
- **Uniformly Accelerated Motion:**
  - The graph is a curved line (parabola), indicating the object's speed is increasing over time.
- **Uniformly Decelerated Motion:**
  - The graph is a curved line where the slope decreases over time, showing that the object is slowing down.

Interpreting the Graphs:

- **Flat Line:** The object is at rest (zero velocity).
- **Upward Straight Line:** The object is moving forward at a constant speed.
- **Upward Curved Line:** The object is accelerating.
- **Downward Curved Line:** The object is decelerating.

2. Velocity-Time Graphs

Velocity-Time Graph :

- A velocity-time graph shows how an object's velocity changes with time.
- **Velocity** is plotted on the Y-axis.
- **Time** is plotted on the X-axis.

Slope of the Velocity-Time Graph:

- The slope of this graph tells you the **acceleration** of the object.
- **Slope = change in velocity/change in time = acceleration (a).**

Area Under the Curve:

- The area between the velocity-time graph and the time axis represents the **displacement** of the object.

Types of Velocity-Time Graphs:

- **Uniform Motion:**
  - The graph is a straight horizontal line, indicating constant velocity (zero acceleration).
- **Uniformly Accelerated Motion:**

    - The graph is a straight line with a positive slope, indicating constant acceleration.
- **Uniformly Decelerated Motion:**
    - The graph is a straight line with a negative slope, indicating constant deceleration.

3. Acceleration-Time Graphs

Acceleration-Time Graph Overview:

- An acceleration-time graph shows how an object's acceleration changes with time.
- **Acceleration** is plotted on the Y-axis.
- **Time** is plotted on the X-axis.

Interpreting the Graph:

- For uniformly accelerated or decelerated motion, the acceleration-time graph is a horizontal line parallel to the time axis, indicating constant acceleration or deceleration.

Example Problem: Motion of a Car

Let's consider a car that undergoes different phases of motion: it accelerates, moves at a constant speed, decelerates, and finally moves in the opposite direction. We can represent each phase using displacement-time, velocity-time, and acceleration-time graphs.

Motion Phases:

1. **Acceleration:** The car accelerates from rest at 2.0 m/s22.0 \, \text{m/s}^22.0m/s2 for 5 seconds.
2. **Constant Speed:** The car maintains a constant speed for 4 seconds.
3. **Deceleration:** The car decelerates to a stop within 2 seconds.
4. **Opposite Direction:** The car moves back to the starting point at a constant speed of 15 m/s.

Graph Interpretation:

- **Displacement-Time Graph:** Shows the changing position of the car during each phase.
- **Velocity-Time Graph:** Illustrates the car's increasing velocity during acceleration, constant velocity during uniform motion, and decreasing velocity during deceleration.
- **Acceleration-Time Graph:** Highlights constant acceleration, zero acceleration during constant speed, and negative acceleration during deceleration.

By studying these graphs, we can understand the car's motion, calculate its displacement, and even verify that it returns to its starting point.

3.4 Vertical Motion

Freely Falling Bodies

Free Fall:
Free fall occurs when an object moves vertically under the influence of gravity alone. This means the object is accelerating downward at a constant rate, known as acceleration due to gravity (g). Near Earth's surface, g is approximately 9.8 m/s^2.

Equations of Motion for Free Fall:

1. **Velocity-Time Relationship:**

   $$v=u+gt$$

   - v is the final velocity.
   - u is the initial velocity.
   - t is the time.
   - g is the acceleration due to gravity (negative when moving upwards).
2. **Displacement-Time Relationship:**

   $$y=ut+\frac{1}{2}gt^2$$

   - y is the displacement.
   - u is the initial velocity.
   - t is the time.
   - g is the acceleration due to gravity.
3. **Velocity-Displacement Relationship:**

   $$v^2=u^2+2gy$$

   - v is the final velocity.
   - u is the initial velocity.
   - y is the displacement.
   - g is the acceleration due to gravity.

Note: These equations are vector equations, so direction matters. Positive direction is typically upward, and gravity acts downward, thus ggg is negative when an object is thrown upward.

Free Fall Examples:

- **Object Thrown Upward:**
  - Initial velocity (u) is positive.
  - Gravity (g) is negative.

  $v=u-gt$

  $y=ut-\frac{1}{2}gt^2$

  $v^2=u^2-2gy$

- **Object Thrown Downward:**
  - Initial velocity (u) is positive.
  - Gravity (g) is positive.

  v $v=u+gt$

  $y=ut+\frac{1}{2}gt^2$

  $v^2=u^2+2gy$

Multiflash Photography:
When an object falls freely, its displacement increases uniformly over time, indicating constant acceleration. This is shown in a multiflash photo where displacement covered in equal time intervals grows.

Example Problem: For an object thrown from a height:

- Calculate the time to return to the starting height using y=0.
- Use the kinematic equations to solve for time at different levels.

Terminal Velocity:

Concept:
When an object falls through a fluid (like air), it experiences two forces: gravitational force (downward) and drag force (upward). As velocity increases,

drag force increases until it balances the gravitational force. At this point, the object falls at a constant speed, called terminal velocity.

Factors Affecting Terminal Velocity:

- Weight of the object.
- Shape and surface area.
- Nature of the fluid.

Galileo's Contribution: Galileo Galilei demonstrated that, in a vacuum, all objects fall at the same rate regardless of mass. His work laid the foundation for understanding uniform acceleration and the effects of gravity.

Reaction Time Calculation:

1. Measure the distance between where your fingers were initially and where you catch the falling object.
2. Calculate reaction time using: Reaction Time=$\frac{Distance}{\text{Speed}}$

Summary

- **Free Fall:** Motion under gravity alone with acceleration g.
- **Equations:** Used to calculate velocity, displacement, and time.
- **Terminal Velocity:** Constant speed when drag force equals gravitational force.
- **Galileo's Discovery:** All objects fall at the same rate in a vacuum, regardless of mass.

Centripetal Acceleration: Acceleration in Uniform Circular Motion

Introduction: When an object moves in a circle at constant speed, it experiences centripetal acceleration. This acceleration is always directed towards the center of the circular path. Even though the object's speed remains constant, its direction changes continuously, which means it is accelerating.

Understanding Centripetal Acceleration:

- **Definition:** Centripetal acceleration (a_c) is the acceleration of an object moving in uniform circular motion, directed towards the center of the circle.
- **Direction:** The direction of centripetal acceleration is always radial, meaning it points directly towards the center of the circular path.
- **Magnitude:** The formula for centripetal acceleration is: $a_c=\frac{v^2}{\text{r}}$ where v is the tangential speed and r is the radius of the circle.

Deriving Centripetal Acceleration:

1. **Average Acceleration:** For uniform circular motion, the average acceleration can be calculated using:

   $a_{av}=\frac{\Delta v}{\Delta t}$

   where Δv is the change in velocity and Δt is the time interval.

2. **Velocity Vectors:** At two points, P and Q, on the circular path, the velocities are v_i and v_f respectively. Since these velocities are equal in magnitude but differ in direction, their change (Δv) is directed towards the center of the circle.
3. **Instantaneous Acceleration:** As Δt approaches zero, the average acceleration becomes the instantaneous centripetal acceleration:

   $a_c=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{v^2}{r}$

   where v is the constant speed of the object.

Angular Velocity Relation:

- **Tangential Velocity:** $v=\omega r$ is the angular velocity.
- **Centripetal Acceleration in Terms of Angular Velocity:** $a_c=\omega^2 r$

Example Problem: Given: An automobile moves at 50.4 km/h around a circular track with a diameter of 40 m.

1. **Convert Speed:**

   $v=50.4\ \text{km/h}=14\ \text{m/s}$

2. **Calculate Radius:**

   $r=\frac{40\ m}{2}=20\ \text{m}$

3. **Find Angular Speed (ω):**

   $\omega=\frac{v}{r}=\frac{14\ m/s}{20\ \text{m}}=0.7\ \text{rad/s}$

4. **Calculate Period (T):**

$T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 20\, m}{14\, m/s} \approx 8.97$ s

Centripetal Force:

- **Definition:** The force causing centripetal acceleration is called centripetal force (F_C). It acts towards the center of the circular path and is given by: $F_C = ma_C = \frac{mv^2}{r}$ where m is the mass of the object.

Example Problem: Given: A 100 g ball is whirled in a circle of radius 40 cm, with a maximum rod strength of 50 N.

1. **Convert Mass:**

   $m = 100\, g = 0.1$ kg

2. **Calculate Maximum Speed:**

   $F_C = \frac{mv^2}{r} \Longrightarrow v = \sqrt{\frac{Fc \times r}{m}} = \sqrt{\frac{50\, N \times 0.4\, m}{0.1\, kg}} = 14.14\, m/s$

Applications:

- **Centrifuge:** Uses high-speed rotation to separate particles based on density.
- **Merry-Go-Round:** Shows high radial acceleration with smaller radii and faster speeds.

Key Points to Remember:

- Centripetal acceleration is always directed towards the center of the circular path.
- It depends on the speed of the object and the radius of the circle.
- The centripetal force is responsible for maintaining circular motion and is perpendicular to the object's velocity.