Unit 2

Vectors

Introduction to Vectors

In the physical world, many quantities have both a magnitude (size) and a direction. For example, when describing the motion of an airplane, it's essential to specify not only how fast it's going (speed) but also the direction it's moving. Similarly, to describe a force, we need to know both how strong the force is and the direction in which it is applied. This unit will focus on how to describe these two-dimensional (2-D) vector quantities and perform mathematical operations with them.

Scalar and Vector Quantities

Some physical quantities, such as mass, speed, and distance, can be described completely by their magnitude and units. These are called scalar quantities. However, other quantities like force, velocity, acceleration, and displacement require both magnitude and direction to be fully described. These are known as vector quantities.

Quantity	Type	Quantity	Type
Area	Scalar	Density	Scalar
Distance	Scalar	Weight	Vector
Pressure	Scalar	Velocity	Vector

Representation of Vectors

Vectors are often represented graphically by arrows. The length of the arrow, drawn to scale, represents the magnitude of the vector, while the tip of the arrow indicates the direction. For example, a displacement of 10 km might be represented by a vector 1 cm long on a diagram, while a displacement of 20 km would be shown by a vector 2 cm long.

Types of Vectors

- **Parallel Vectors**: Vectors in the same direction.
- **Antiparallel Vectors**: Vectors in opposite directions.
- **Equal Vectors**: Vectors with the same magnitude and direction.
- **Collinear Vectors**: Vectors lying along the same line or parallel lines.

- **Coplanar Vectors**: Vectors lying in the same plane.
- **Zero Vector**: A vector with zero magnitude.
- **Orthogonal Vectors**: Vectors that are perpendicular to each other.
- **Unit Vector**: A vector with a magnitude of one unit.

Operations with Vectors

- **Negative of a Vector**: The negative of a vector has the same magnitude but the opposite direction. For example, if a displacement vector is 60 km toward the North, its negative would be 60 km toward the South.

Methods of Vector Addition

1. **Graphical Method**:
   - **Triangle Law**: When two vectors are added, they can be arranged head-to-tail. The resultant vector is drawn from the tail of the first vector to the head of the second vector.
   - **Parallelogram Law**: Two vectors can be added by constructing a parallelogram where the vectors form adjacent sides. The diagonal of the parallelogram represents the resultant vector.
   - **Polygon Law**: For more than two vectors, place each vector head-to-tail. The resultant is drawn from the tail of the first vector to the head of the last vector.
2. **Analytical Method**: Involves breaking down vectors into their components and adding the respective components mathematically.

Subtraction of Vectors

Subtracting a vector is equivalent to adding its negative. If vector A is subtracted from vector BBB, the result is the vector sum of A and −B.

Practical Applications of Vectors

Vectors are used in various real-life scenarios, such as:

- **Engineering**: Analyzing forces on structures like dams.
- **Boating**: Understanding the forces acting on a boat.
- **Sports**: Calculating the resultant force in a tug-of-war.

Algebraic Method of Addition of Vectors in Two Dimensions (2-D)

1. Adding Two Collinear Vectors

Vectors, unlike scalars, require consideration of both magnitude and direction when performing addition. Let's start by reviewing how to add vectors in one dimension, which will help us understand the process in two dimensions.

- **Vectors in the Same Direction:** When two vectors are in the same direction, the resultant vector $\vec{R}$ is found by simply adding their magnitudes. The direction of $\vec{R}$ is the same as that of the original vectors.

  For example, if two force vectors $\vec{F1}$ and $\vec{F2}$ act along the same line, the resultant force is:

  $\vec{R}=\vec{F1}+\vec{F2}$

  The magnitude of $\vec{R}$ is the sum of the magnitudes of $\vec{F1}$ and $\vec{F2}$, and it points in the same direction as $\vec{F1}$ and $\vec{F2}$.

- **Vectors in Opposite Directions:** If two vectors act in opposite directions, the resultant vector $\vec{R}$ has a magnitude equal to the absolute difference between the magnitudes of the vectors. The direction of $\vec{R}$ is along the direction of the larger vector.

  For example, if $\vec{F1}=200\ \text{N}$ and $\vec{F2}=100\ \text{N}$ act in opposite directions:

  $\vec{R}=200\ \text{N}-100\ \text{N}=100\ \text{N}$

  The resultant vector $\vec{R}$ points in the direction of $\vec{F1}$.

2. Adding Two Perpendicular Vectors

When adding two vectors that are perpendicular to each other, the process involves more than simple arithmetic. Here's how you do it:

- **Using the Pythagorean Theorem:** The magnitude of the resultant vector $\vec{R}$ can be found using the Pythagorean theorem:

  $R=\sqrt{A^2+B^2}$

  where A and B are the magnitudes of the two perpendicular vectors.

- **Finding the Direction:** To find the direction of the resultant vector, use trigonometry. The angle θ that the resultant makes with one of the vectors can be calculated using:

  $\tan\theta = \frac{\textit{opposite side}}{\text{adjacent side}} = BA$

3. Components of a Vector

When dealing with vectors in two dimensions, it's often useful to break them down into components along the x-axis and y-axis. This method is known as the **Component Method**.

- **Vector Components:** A vector $\vec{A}$ can be resolved into two perpendicular components: the x-component A_x and the y-component A_y . If the vector makes an angle θ with the x-axis, the components are:

  $A_x = A\cos\theta$ and $A_y = A\sin\theta$

- **Resultant Vector by Component Method:** To find the resultant vector when adding two or more vectors using the component method:
  1. Resolve each vector into its x and y components.
  2. Sum all the x-components to get the resultant x-component Rx.
  3. Sum all the y-components to get the resultant y-component Ry .
  4. The magnitude of the resultant vector is given by: $R=\sqrt{\mathrm{Rx}^2 + \mathrm{Ry}^2}$
  5. The direction is given by: $\theta=\tan^{-1}(\frac{Ry}{\mathrm{Rx}})$

4. Unit Vectors

A **unit vector** is a vector that has a magnitude of one and is used to indicate direction. In a 2D coordinate system:

- $\hat{i}$ is the unit vector along the x-axis.
- $\hat{j}$ is the unit vector along the y-axis.

Any vector $\vec{A}$ can be expressed in terms of its components using unit vectors:

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

By understanding and applying these methods, you can efficiently solve problems involving the addition of vectors in two dimensions.

2.4 Product of Vectors

In this section, we'll explore the concept of vector multiplication. Unlike regular numbers, vectors require special rules for multiplication. There are two primary ways to multiply vectors: **Scalar (Dot) Product** and **Vector (Cross) Product**.

1. **Scalar (Dot) Product**: This product results in a scalar quantity (a number without direction).
2. **Vector (Cross) Product**: This product results in another vector.

Multiplying or Dividing a Vector by a Scalar

When you multiply a vector by a scalar (a regular number), the result is another vector. The new vector has a magnitude equal to the absolute value of the scalar multiplied by the magnitude of the original vector. The direction of the new vector depends on the sign of the scalar:

- If the scalar is positive, the new vector points in the same direction as the original vector.
- If the scalar is negative, the new vector points in the opposite direction.

Example:

- If vector **A** is multiplied by 2, the result is a vector twice as long as **A** in the same direction.
- If **A** is multiplied by -2, the result is a vector twice as long as **A** but in the opposite direction.

Dot Product

The dot product of two vectors is a scalar quantity. It is denoted by **A ⋅B** and calculated using the formula: $A \cdot B = AB\cos\theta$ where:

- **A** and **B** are the magnitudes of vectors **A** and **B**.
- **θ** is the angle between the vectors.

This product is a measure of how much one vector extends in the direction of another. Depending on the angle between the vectors, the dot product can be positive, negative, or zero.

Alternative Calculation: The dot product can also be found by multiplying corresponding components of the vectors and adding them up:

$A \cdot B = A_x B_x + A_y B_y$ where:

- A_x and B_x are the x-components of **A** and **B**.
- A_y and B_y are the y-components.

Example: Given vectors A=i+j and B=−2i+3j, the dot product is calculated as: $A \cdot B=(1)(-2)+(1)(3)=-2+3=1$

The angle between the vectors can be found using:

$\cos\theta=\frac{A \cdot B}{\mathrm{AB}}$ where AB is the product of the magnitudes of A and B.

This gives the angle θ as: $\theta=\cos^{-1}(0.196)=78.7^{\circ}$

Summary

- **Scalar Product**: A number (scalar) obtained from the dot product of two vectors.
- **Multiplying a Vector by a Scalar**: Produces a vector with a magnitude scaled by the scalar.
- **Dot Product Formula**: $A \cdot B=AB\cos\theta$ or $A \cdot B=A_xB_x+A_yB_y$
- **Angle Calculation**: $\theta=\cos^{-1}(\frac{A \cdot B}{\mathrm{AB}})$